Foundations and Applications of Granular Computing

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Outline

- Introductory Comments and Motivation
- Information granulation as a central pursuit of abstraction
- Defining Granular Computing
- Formal Models of Information Granules (sets, fuzzy sets, rough sets, shadowed sets)
- Communication Issues: Encoding and Decoding mechanisms
- Concluding note

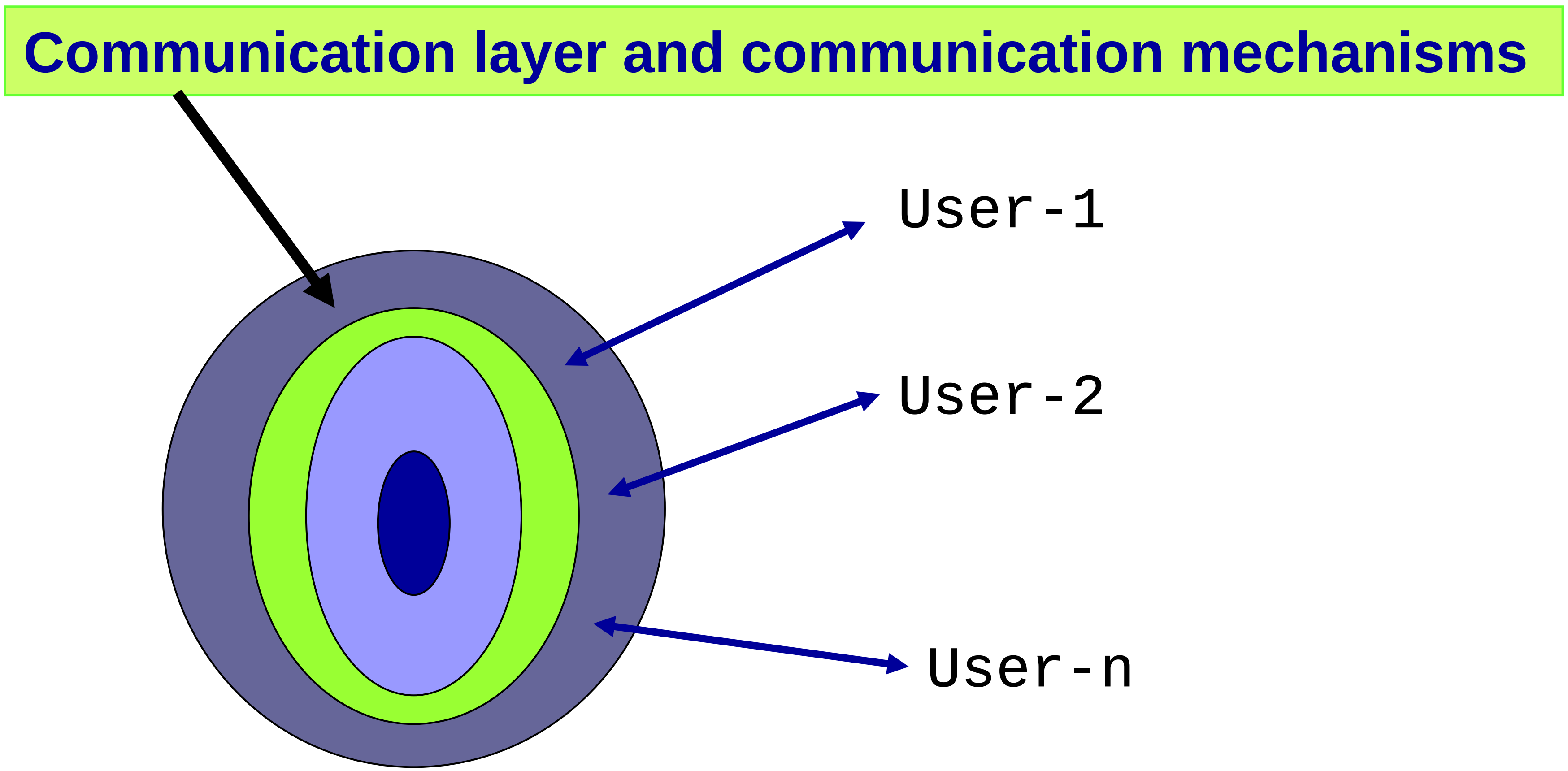
Granular Computing as a vehicle of human-centric pursuits

Human	**semantics**
	abstraction and levels of abstraction
	conflicting requirements
	decision making
	conflict resolution
	classification
	interpretation

Granular Computing as a vehicle of human-centric pursuits

Computing	**syntax**
	precision
	numeric processing
	hardware and software
	system of equations
	two-valued logic

Human-centric computing: communication framework



Granular Computing

Information granules: entities composed of elements drawn together on a basis of their
similarity,
functional closeness,
spatial neighborhood, etc.

Information granulation: processes that support the Development of information granules

Granular Computing: Motivation

Information granules as basic mechanisms of abstraction

Customized, user-centric and business-centric approach to problem description and problem solving

Processing at the level of information granules optimized with respect to the specificity of the problem

Granular Computing: diversity of formal environments

Set theory, interval analysis

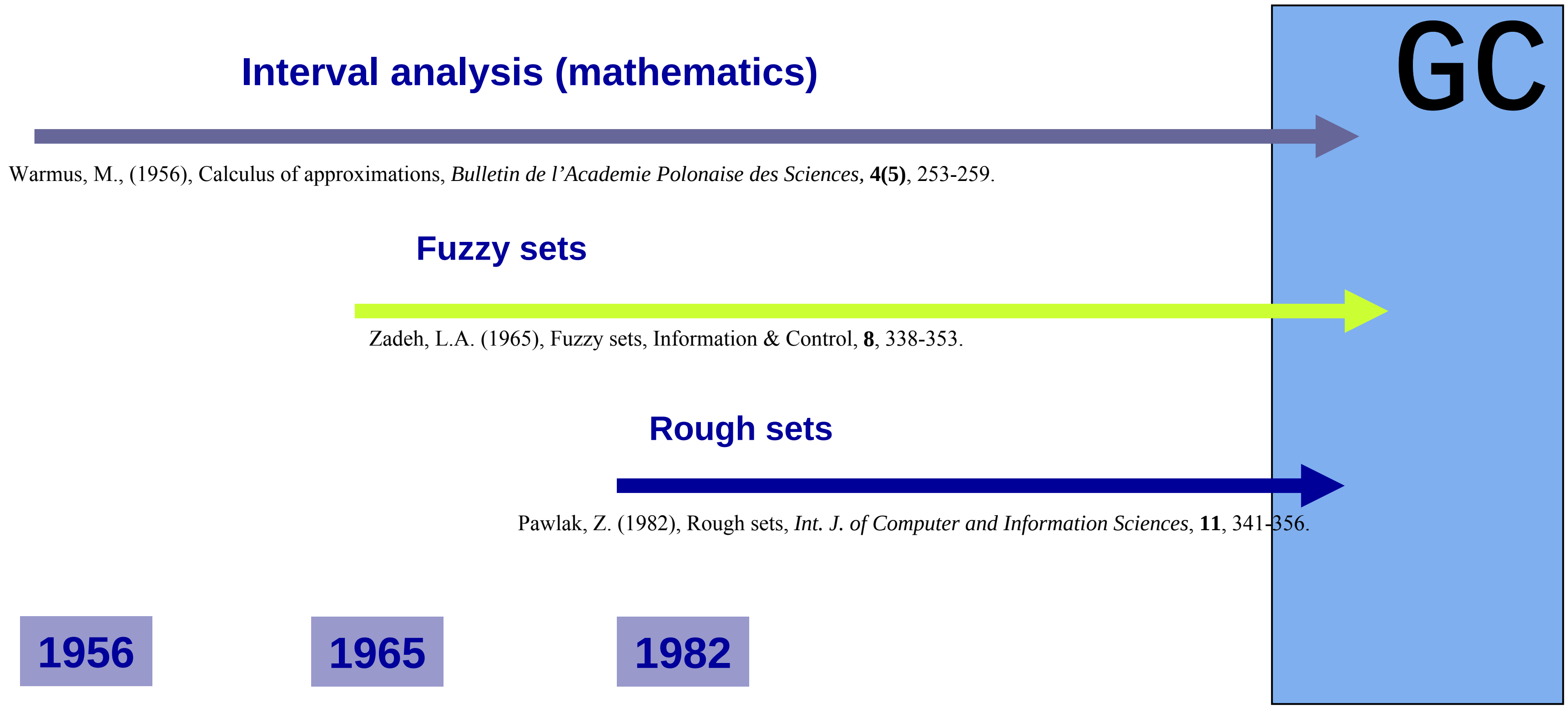
Probabilistic granules

Fuzzy sets

Rough sets

Shadowed sets

Granular Computing (GC)



Time and information granulation

Based on cultural, legal and business orientation of the users

Granularity:Years, months, days, …. Microseconds…

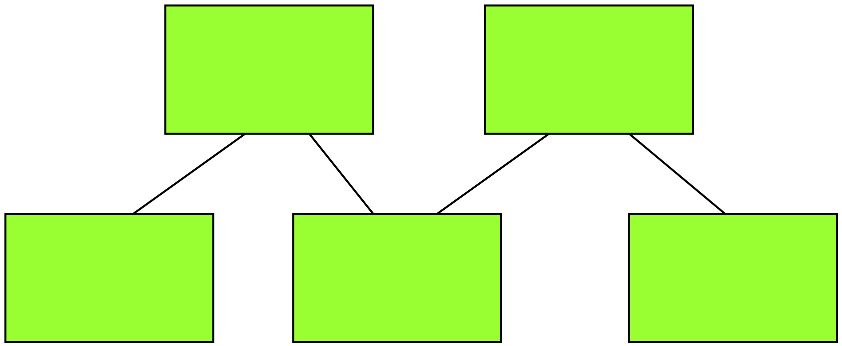
The granularity of information is user-oriented and problem-directed

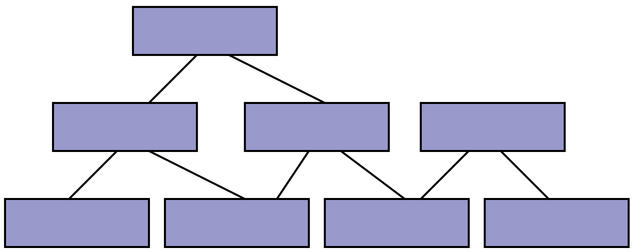
Information granularity

19th century: grains of silver emulsion in photography

20th century: grains (pixels) of digital images

Functional granulation





Modules as meaningful functional entities

Criteria of granulation (cohesion, coupling, comprehension, maintainability…)

Sets

- Notion of Membership

$\in$

belongs to

$\notin$

excluded from

Characteristic function

$$x \in A \iff A(x) = 1$$

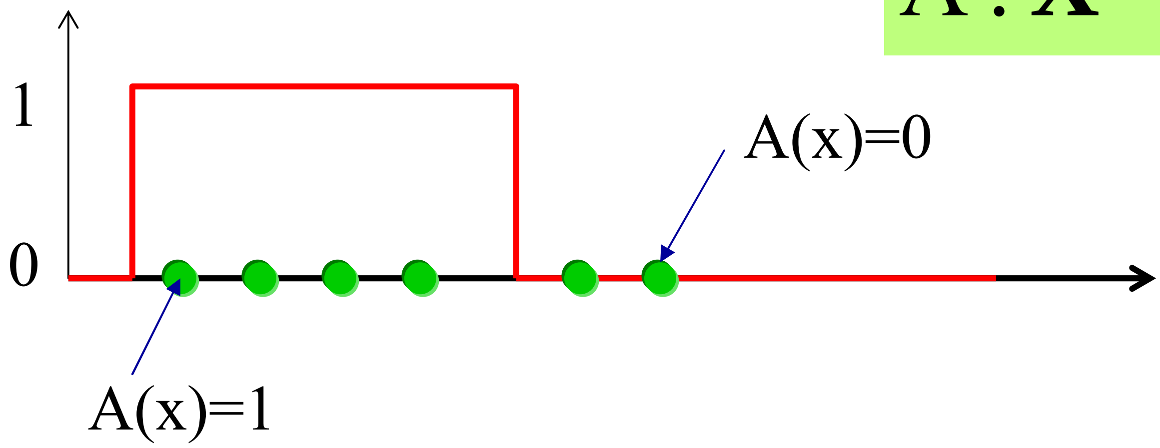
$$x \notin A \iff A(x) = 0$$

Concept of **dichotomy**

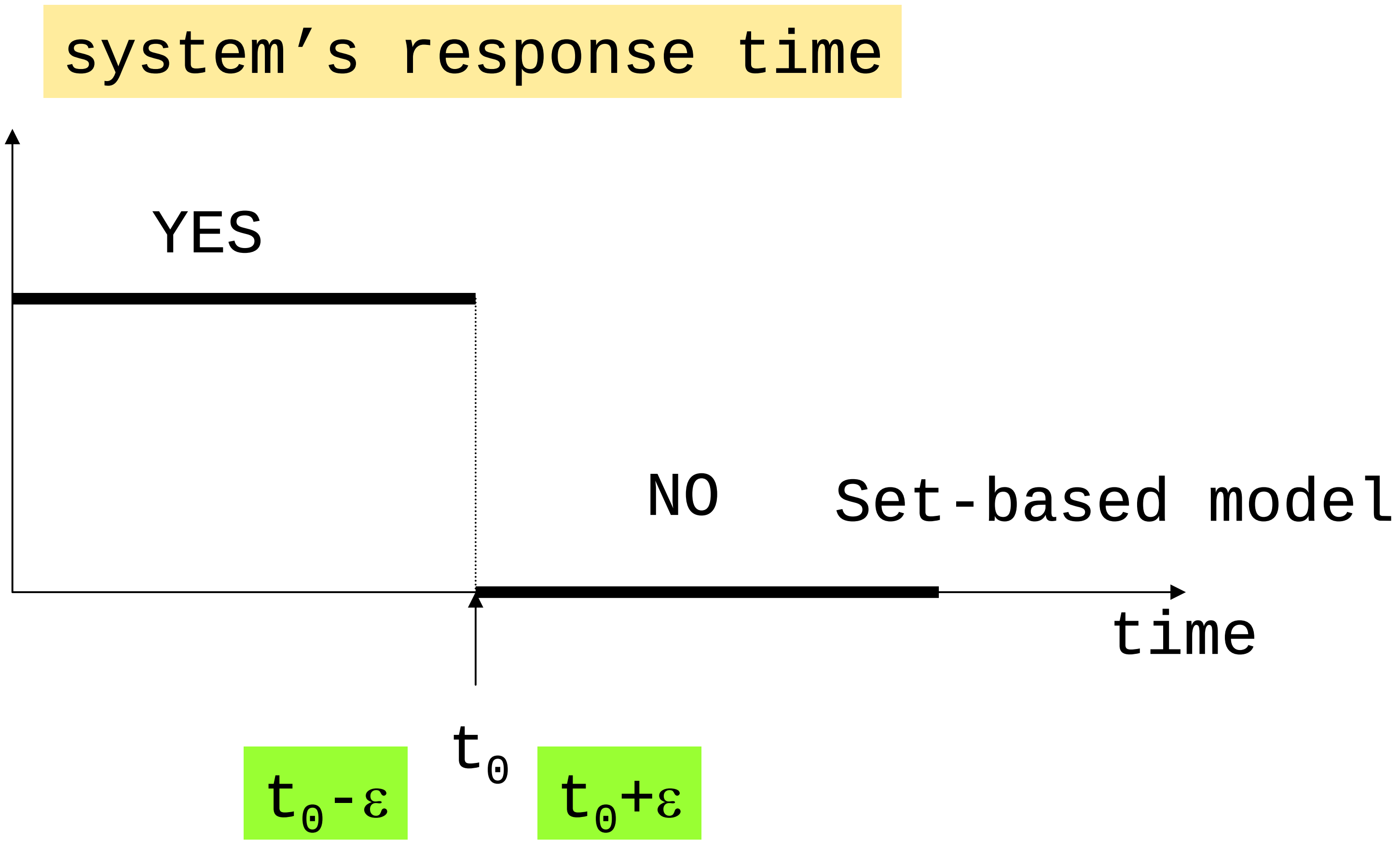
Description of a set

- Membership
  - *-enumerate elements belonging to the set*
- Characteristic function

$$A : \mathbf{X} \rightarrow \{0,1\}$$



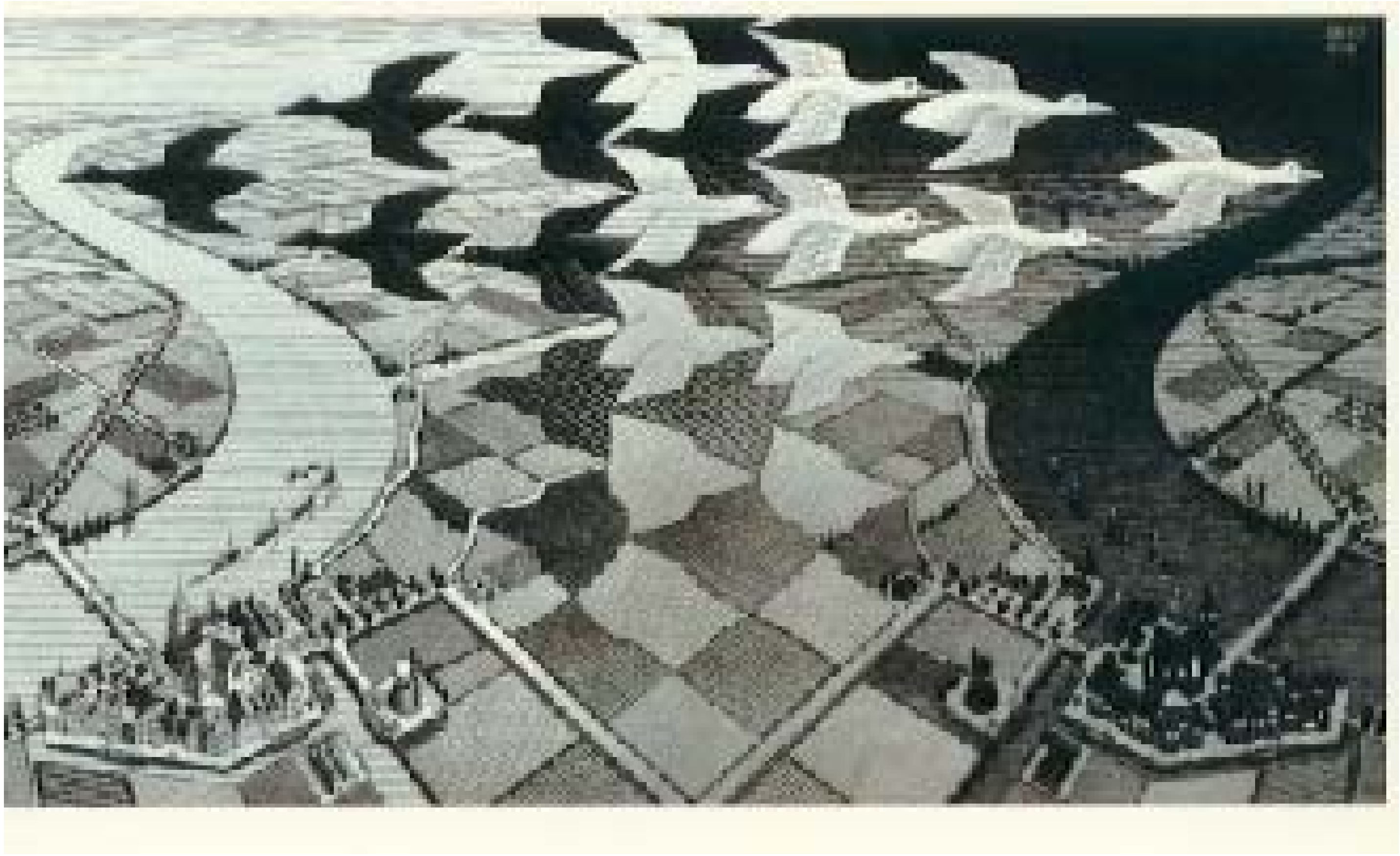
Expressing specifications (1)



Granular Computing:
Set Theory and Interval Analysis

- **Support basic processes of abstraction by employing an idea of dichotomization**
- **Two-valued logic as a formal means of computing**
- **Basic mechanism of abstraction**
- **Information hiding**
- **Level of specificity of information granules reflected (quantified) by set cardinality**

Sets – Fuzzy Sets



M.C.Escher

Challenge: three-valued logic

Lukasiewicz (~1920)

true (0)

false (1)

don’t know (1/2)

Three valued logic and databases

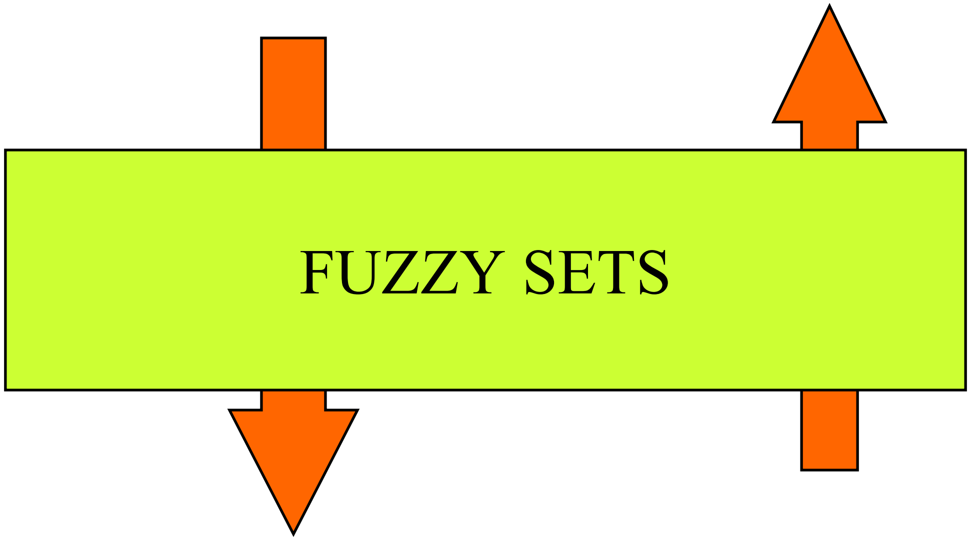
Granular Computing: Non-Aristotelian View

..in analyzing the Aristotelian codification, I had to deal with the two-valued, "either-or" type of orientation. In living, many issues are not so sharp, and therefore a system that posits the general sharpness of "either-or" and so objectifies "kind" , is unduly limited; it must be revised and more flexible in terms of "degree"…

A. Korzybski, 1933

“Impedence” Mismatch

Designer/User: linguistic terms, design objectives, conflicting requirements



Computer Systems: two-valued logic

Granular Computing: Fuzzy Sets

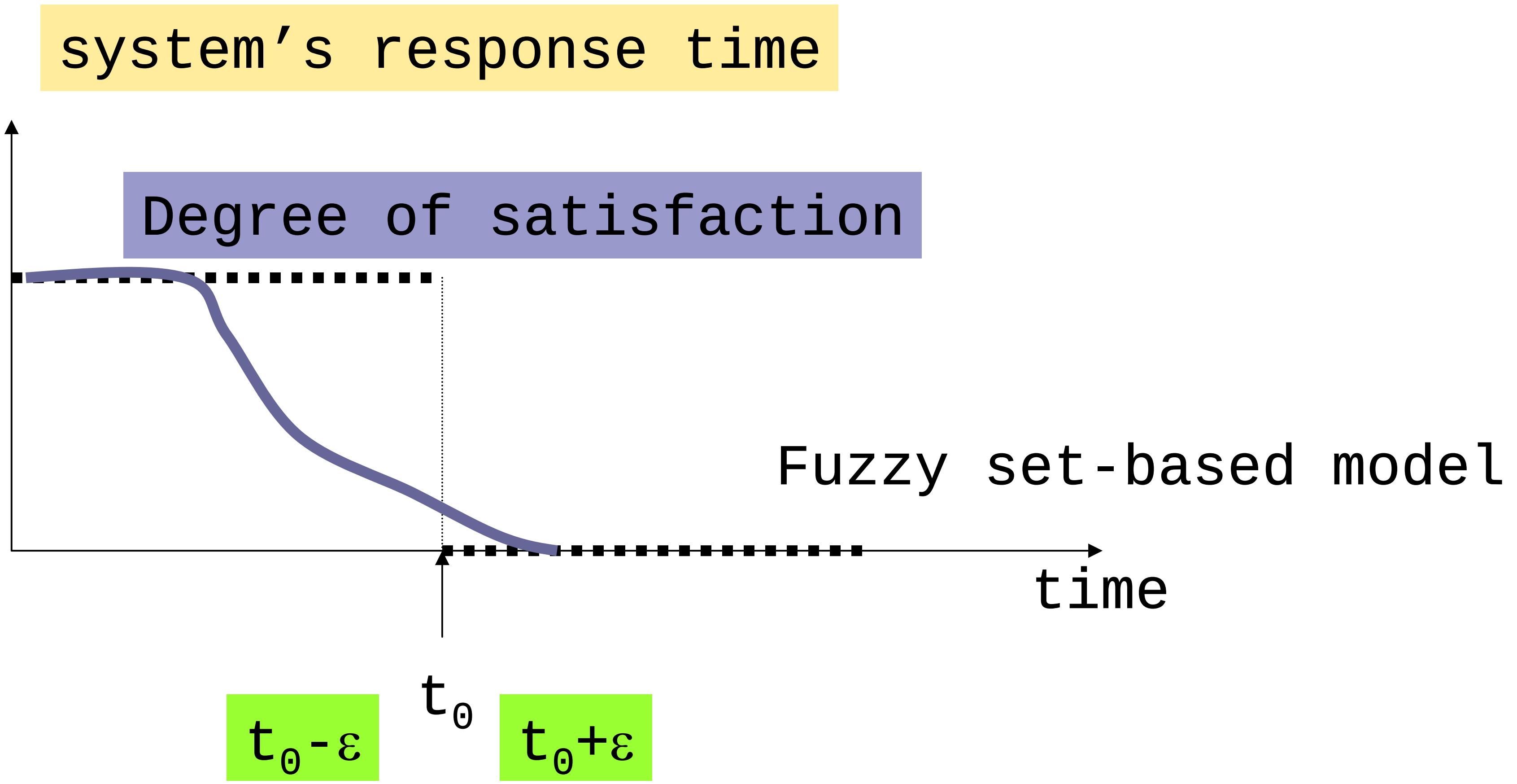
- departure from dichotomization (*yes-no*)
- refinement of concepts by accepting *continuous* membership grades
- based on ideas of multivalued (fuzzy) logic
- mechanism of abstraction capturing *qualitative* as well as *quantitative* facet of concepts

Fuzzy Sets: Membership functions

- Partial membership of element to the set – *membership degree* A(x)

- The higher the value of A(x), the more typical the element "x" (as a representative of A)

Expressing specifications (2)

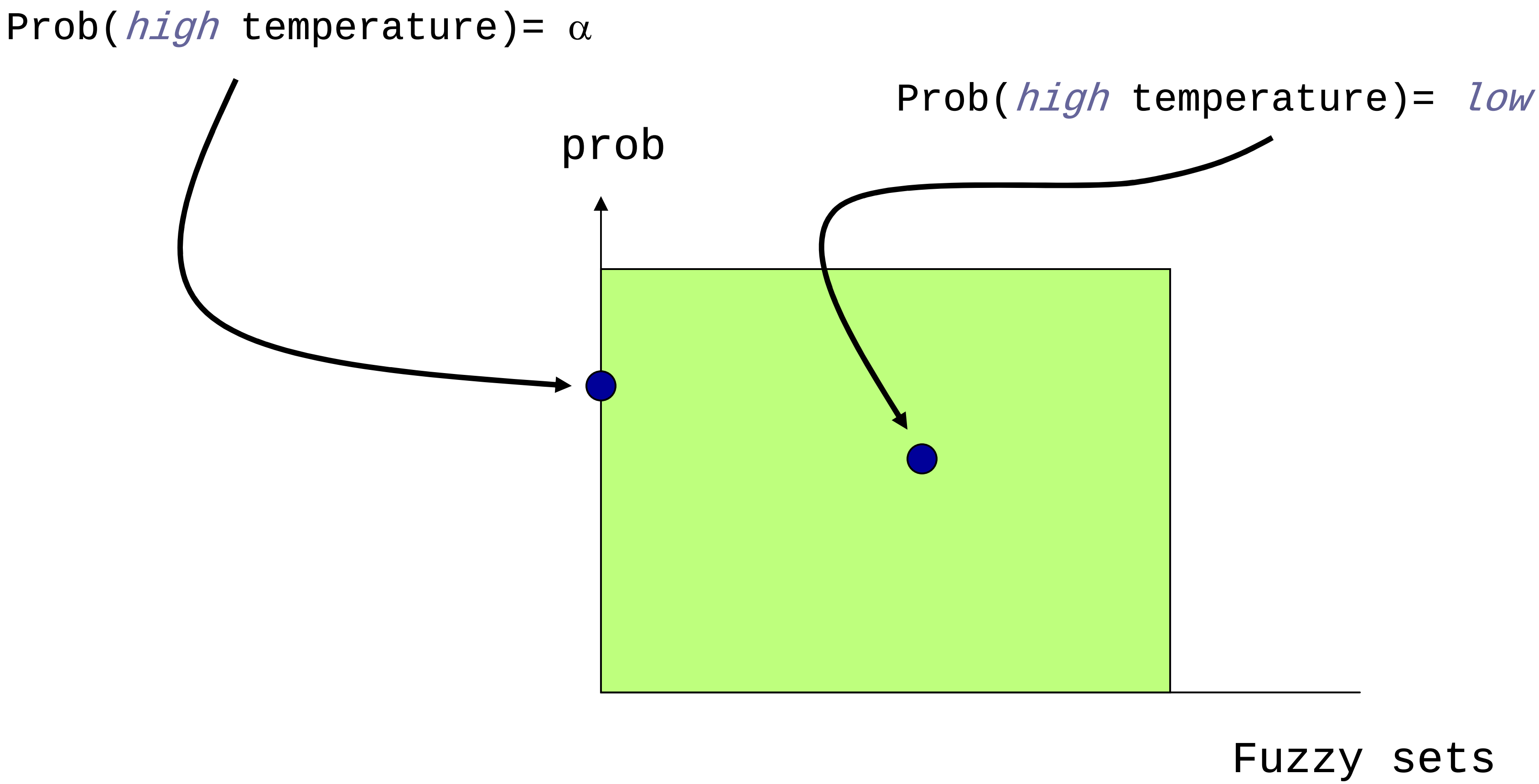


Probability and fuzzy sets

Prob(*high* temperature)= α

Prob(*high* temperature)= *low*

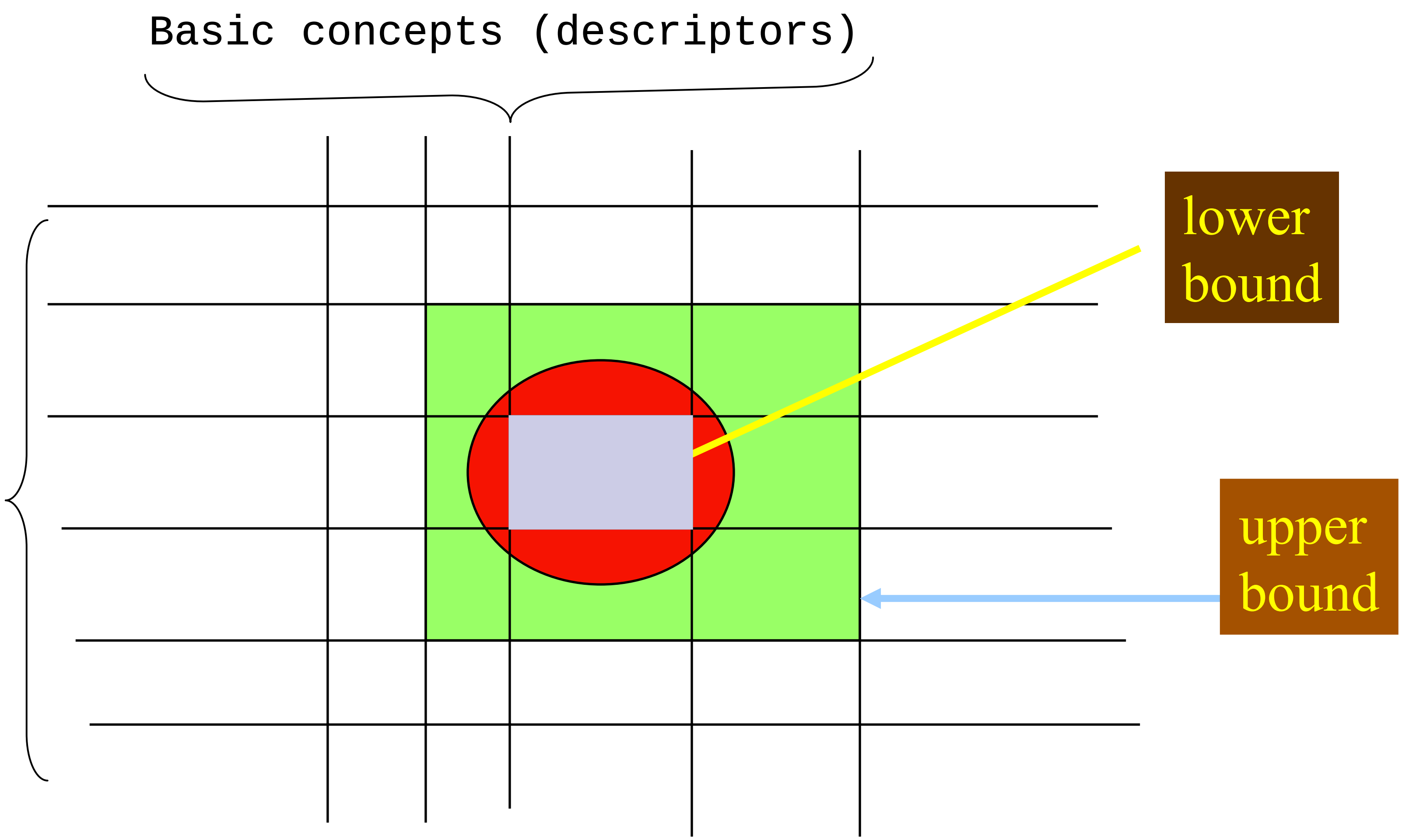
Probability and fuzzy sets



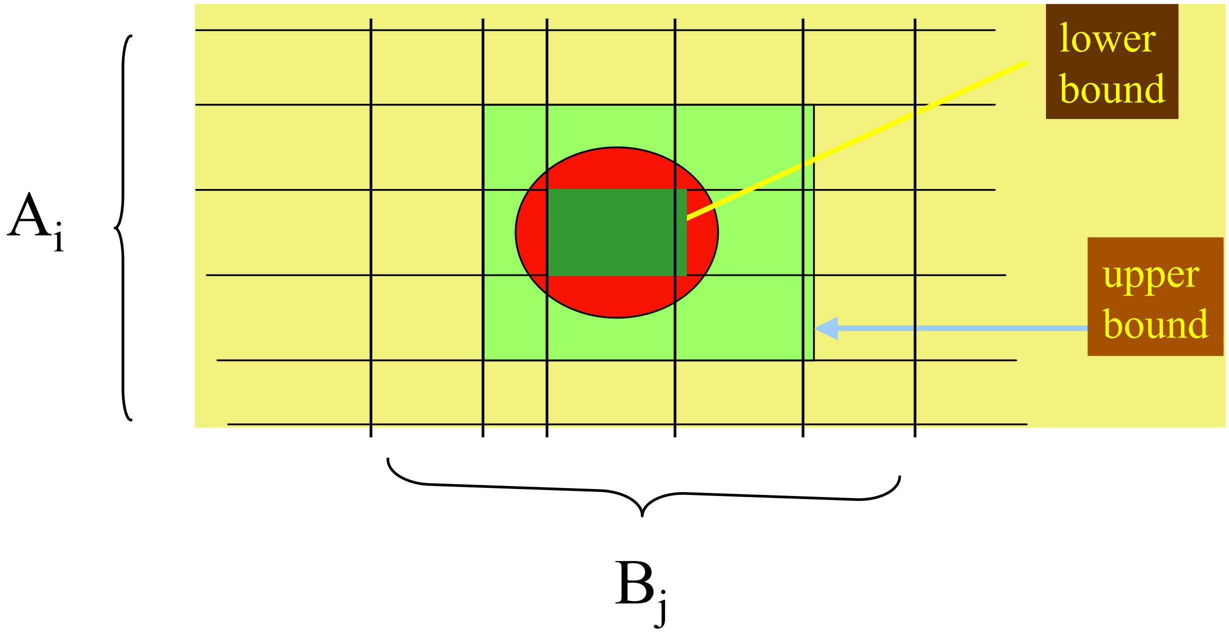
Granular Computing: Rough Sets

- **defining information granules through their lower and upper bounds**
- **identifying regions with a lack of knowledge about concept**
- **expressing aspects of uncertainty through “rough” boundaries**

Granular Computing: Rough Sets



Granular Computing: Rough Sets

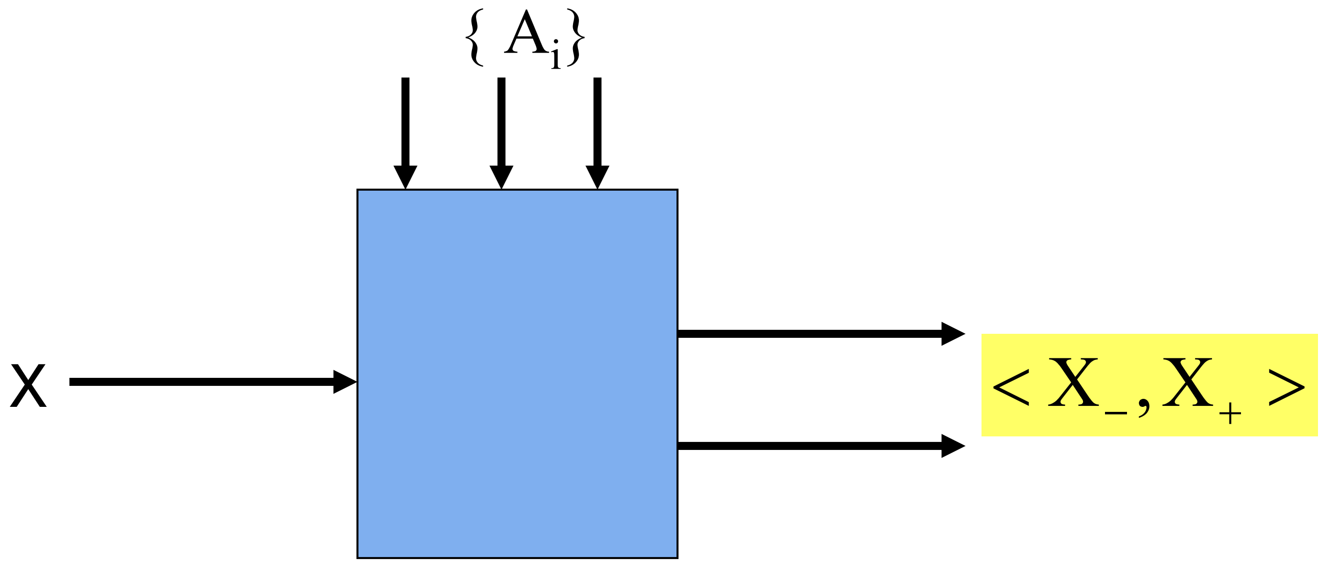


$$< X_-, X_+ >$$

$$\text{lower bound}: X_- = \{(A_i, B_j) \mid X \supseteq A_i \times B_j\}$$

$$\text{upper bound}: X_+ = \{(A_i, B_j) \mid X \cap (A_i \times B_j) \neq \emptyset\}$$

Communication mechanisms: Rough Sets



Description of X in the language of $\{A_i\}$

Shadowed sets and fuzzy set constructs

Interval-valued fuzzy sets

Type -2 fuzzy sets

Conceptual developments

Shadowed sets

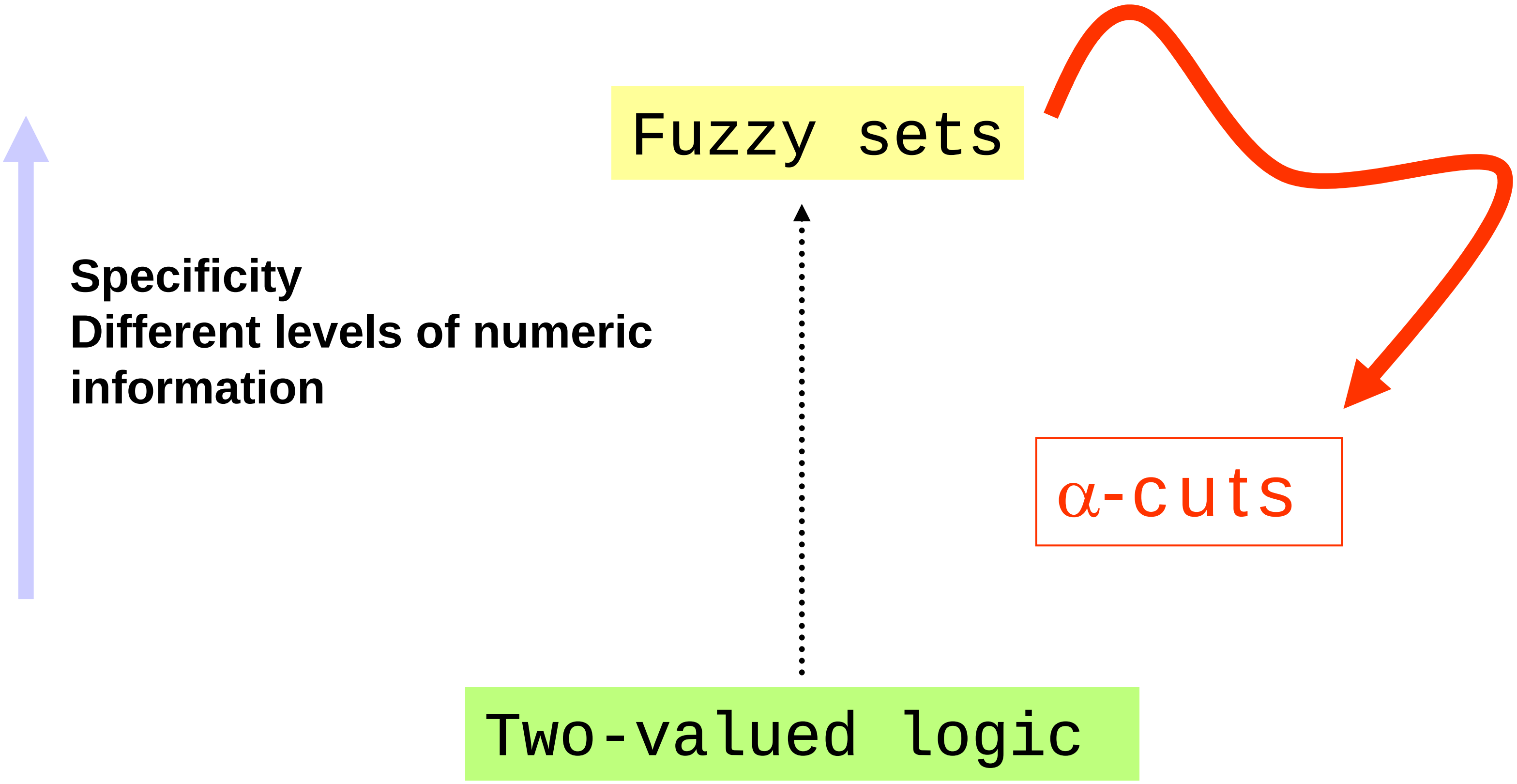
Induced by fuzzy sets,
Result of some design process

Fuzzy Sets: open questions (design, analysis, and interpretation)

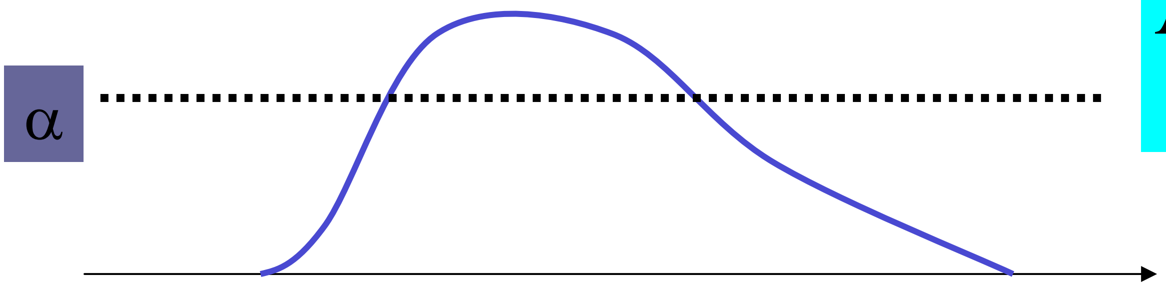
Fuzzy sets → processing → computing overhead

Fuzzy sets → interpretation (detailed numeric membership grades and their semantics)

Fuzzy Sets and some retrospective views



Fuzzy set and sets (α-cuts)



$$A = \bigcup_{\alpha \in (0,1]} \alpha A_{\alpha}$$

A(x) < α reduce to 0 otherwise return 1

* Choice of α

* no reflection of “quality” of conversion of membership grades to zero or one

Shadowed sets

$$A : \mathbf{X} \rightarrow \{0,\ 1,\ [0,1]\}$$

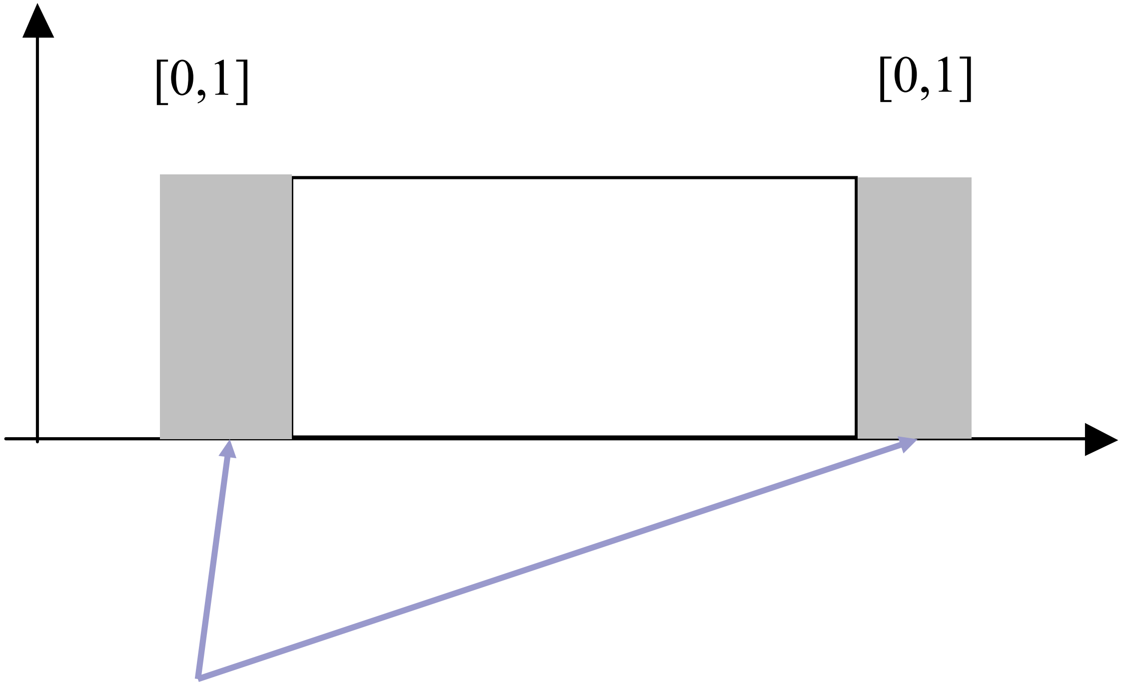
Exclusion

inclusion

No numeric commitment
(no single membership degree)

Shadowed sets

$$A : \mathbf{X} \rightarrow \{0,\ 1,\ \ [0,1]\}$$



Shadows- “concentration” of intermediate membership grades in some regions of X

Operations on shadowed sets (1)

union

	0	1	[0,1]
0	0	1	[0,1]
1	1	1	1
[0,1]	[0,1]	1	[0,1]

intersection

	0	1	[0,1]
0	0	0	0
1	0	1	[0,1]
[0,1]	0	[0,1]	[0,1]

Operations on shadowed sets (2)

complement

$$\begin{matrix} 0 \\ 1 \\ [0,1] \end{matrix} \begin{bmatrix} 1 \\ 0 \\ [0,1] \end{bmatrix}$$

Development of shadowed sets induced by fuzzy sets

Reallocation of membership degrees and maintaining their balance

REDUCTION OF MEMBERSHIP (to 0) +

+ELEVATION OF MEMBERSHIP (to 1) =

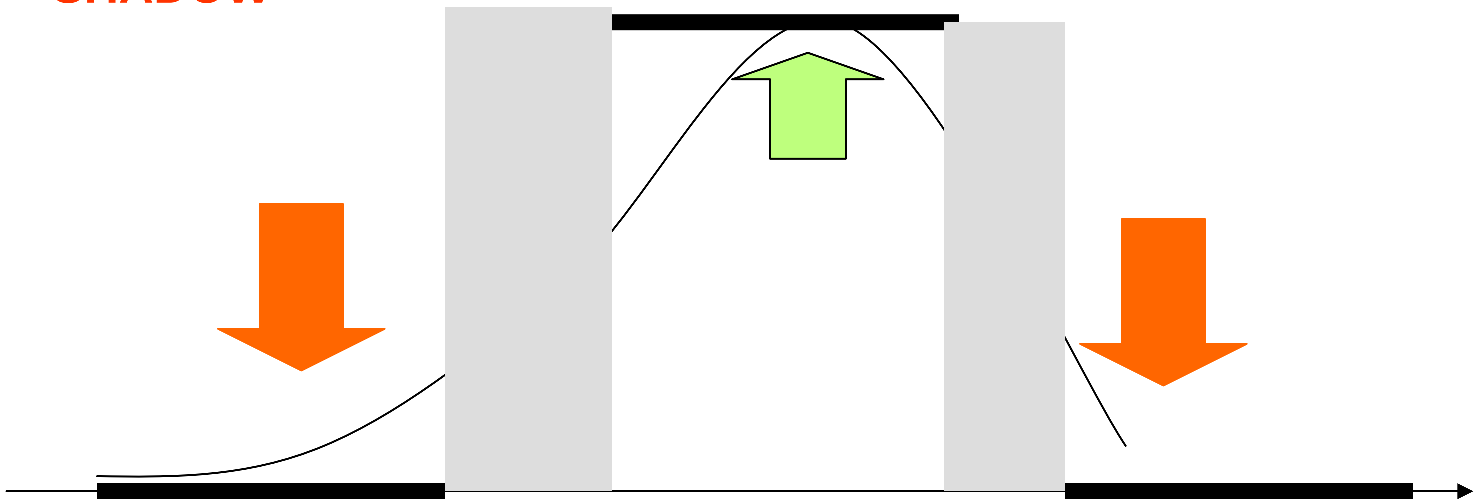
= SHADOW

Development of shadowed sets induced by fuzzy sets

REDUCTION OF MEMBERSHIP (to 0) +

+ELEVATION OF MEMBERSHIP (to 1) =

= SHADOW



Development of shadowed sets induced by fuzzy sets

Reduction of membership $\int_{x:A(x)\leq\beta} A(x)dx$

elevation of membership $\int_{x:A(x)\geq 1-\beta} (1-A(x))dx$

Shadow-localization of membership $\int_{x:\beta<A(x)<1-\beta} dx$

Development of shadowed sets as an optimization problem

REDUCTION OF MEMBERSHIP (to 0) + ELEVATION OF MEMBERSHIP (to 1) =

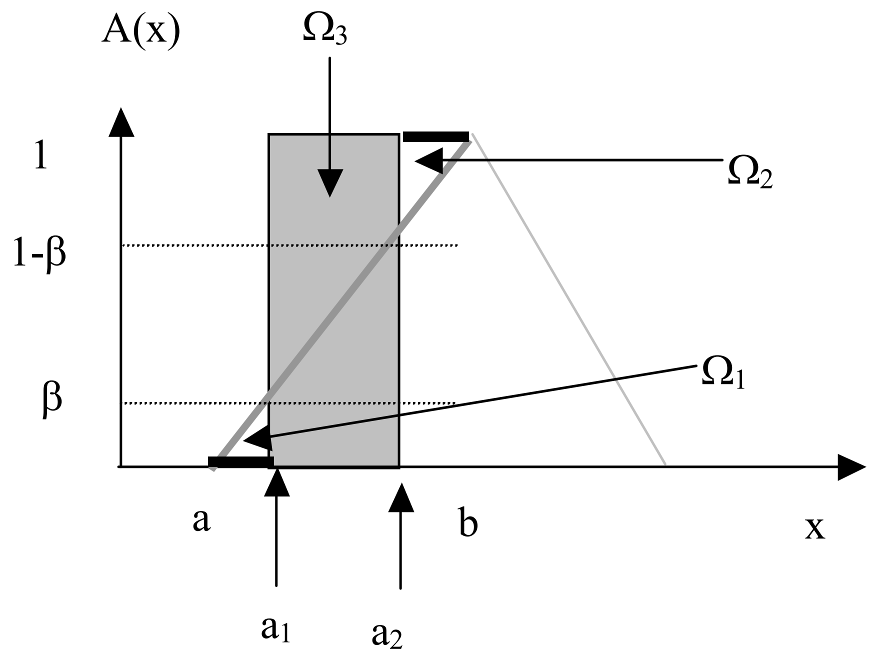
= SHADOW

$$V(\beta) = \left| \int_{x:A(x)\leq\beta} A(x)dx + \int_{x:A(x)\geq 1-\beta} (1 - A(x))dx - \int_{x:\beta<A(x)<1-\beta} dx \right|$$

Min $V(\beta)$ wrt. to β

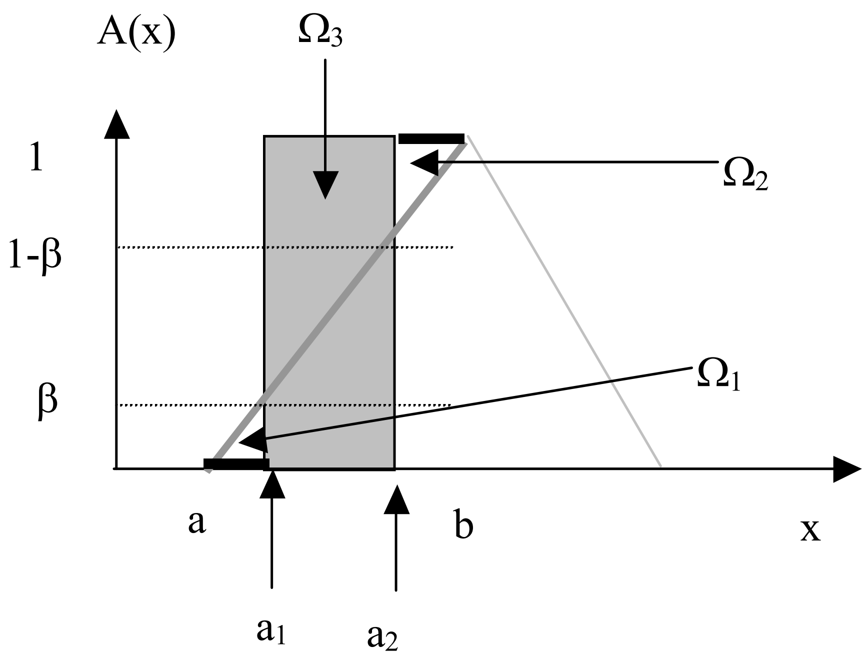
From fuzzy sets to shadowed sets

Design criterion:
reflect the amount of intermediate membership grades transformed into 0 or 1



$$\beta \in (0,1/2)$$

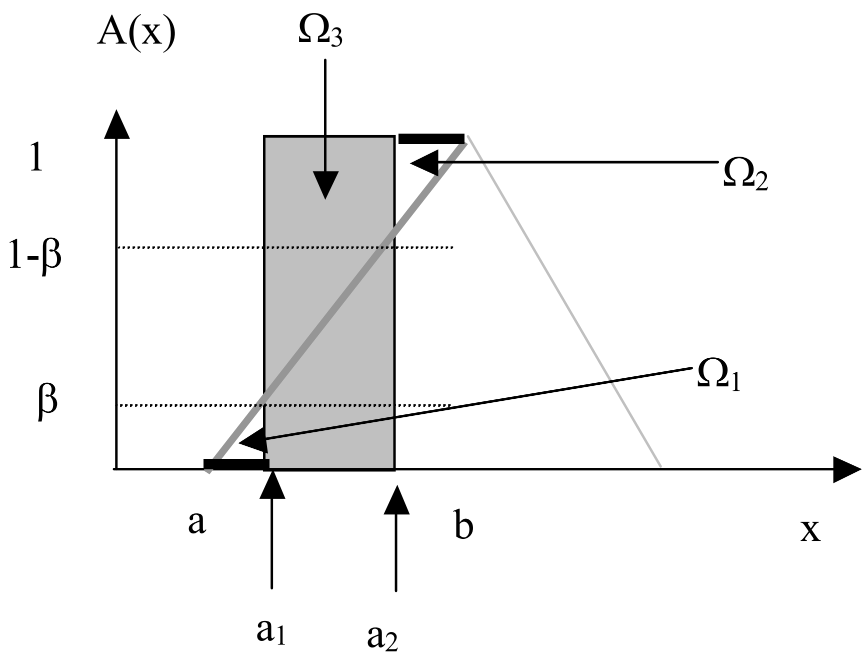
Development of shadowed sets



$$\Omega_1 + \Omega_2 = \Omega_3$$

$$\int_{a}^{a_1} A(x)dx + \int_{a_2}^{b} (1 - A(x))dx = \int_{a_1}^{a_2} dx$$

Triangular fuzzy sets



$$\beta = \frac{2^{3/2} - 2}{2} = 0.4142$$

Discrete shadowed sets

Fuzzy set with U_k , k=1, 2,…, N

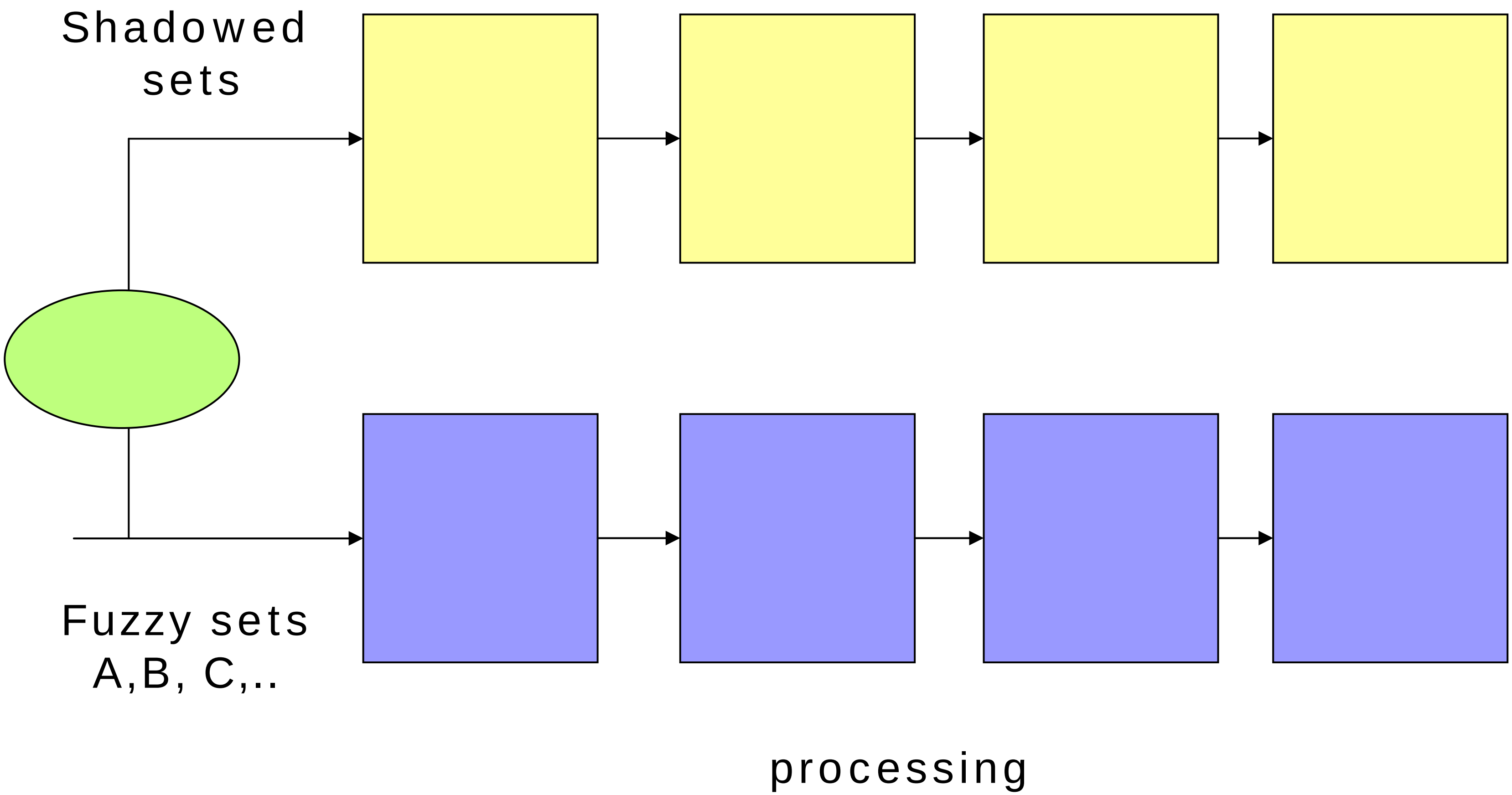
$$\Omega_1 = \sum_{k:u_k \le \beta} u_k$$

$$\Omega_2 = \text{card}\{u_k \mid \beta < u_k < 1-\beta\}$$

$$\Omega_3 = \sum_{k:u_k \ge 1-\beta} (1 - u_k)$$

$$V(\beta) = |\Omega_1 + \Omega_3 - \Omega_2|$$

Environments of fuzzy sets and shadowed sets



Interfaces of Granular Computing

User-centric and user-friendly environment of paramount importance to Granular Computing

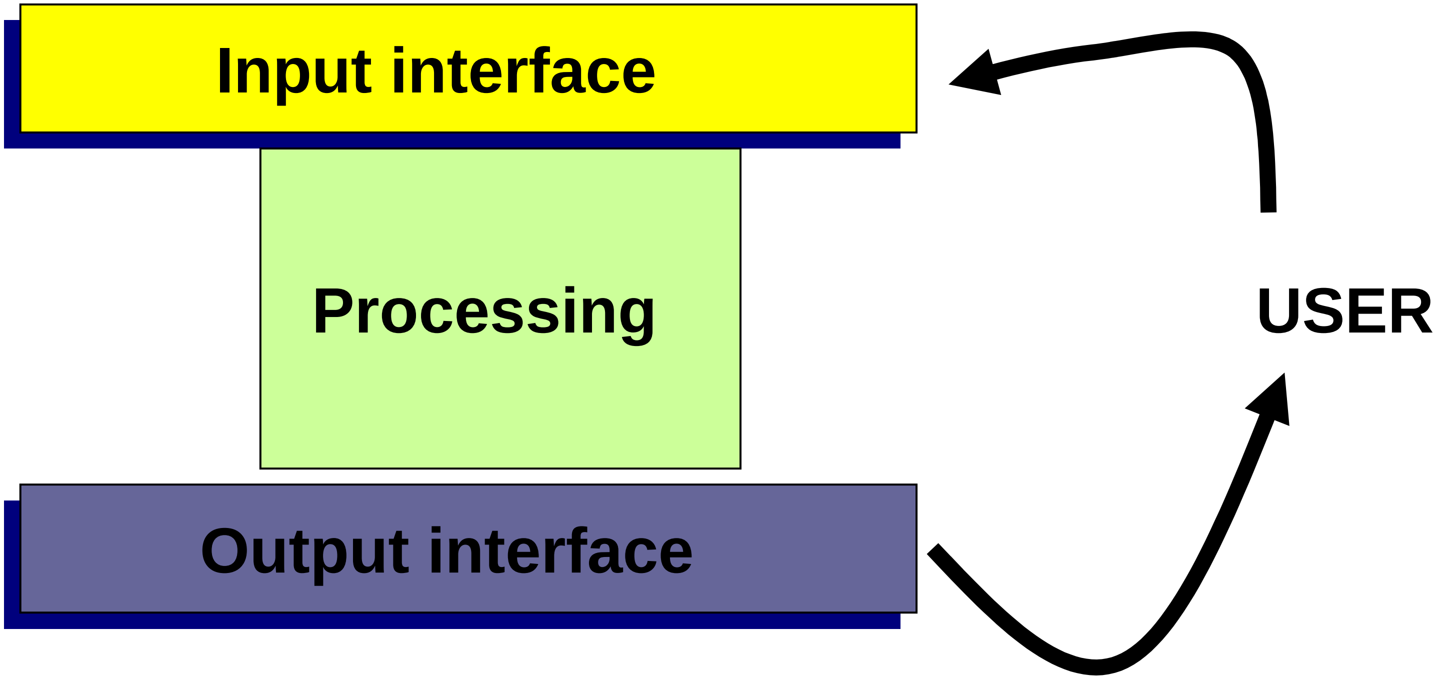
- **User→ system**
- **System → user**

Two categories of interfaces

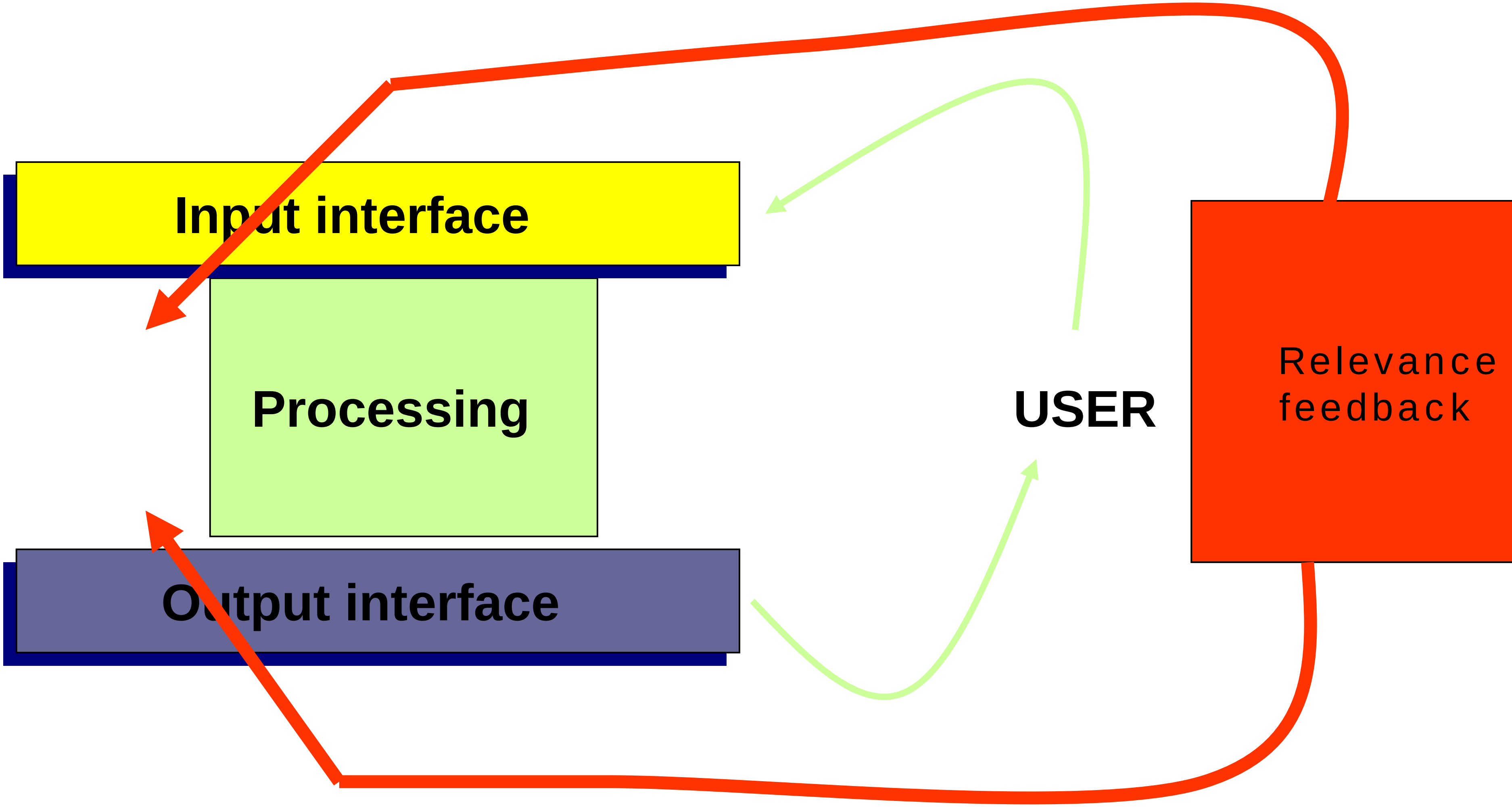
Reflecting the preferences of users

- **Static approach (fixed characteristics)**
- **Dynamic approach (personalization; e.g.relevance feedback)**

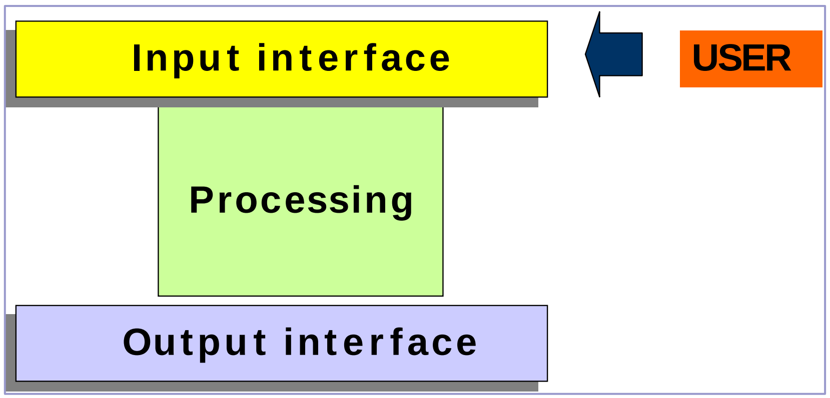
Interfaces: Architectural considerations



Interfaces- personalization



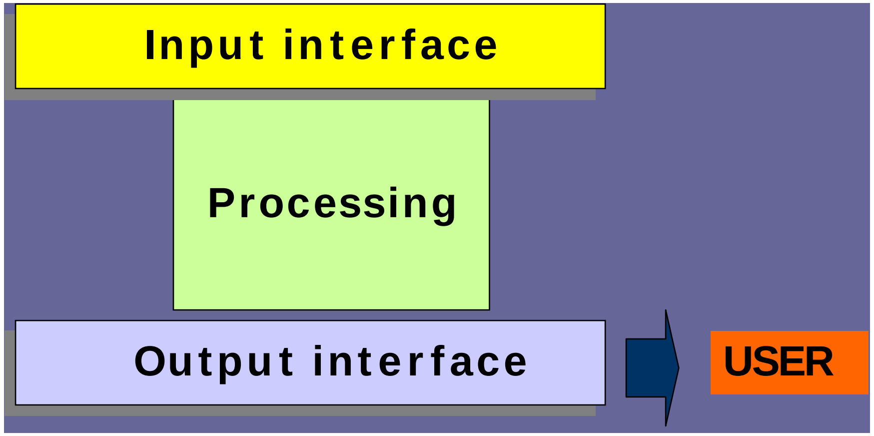
Input Interfaces



Granularity of input information

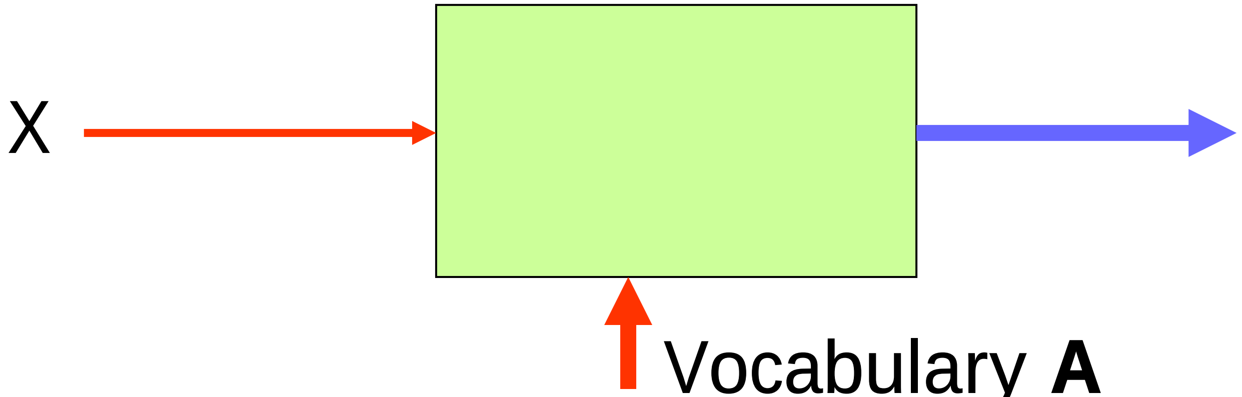
- **Variable level of granularity (modeling level of confidence)**
- **Formal models of granular information**
- **Linguistic data**
- **Computing overhead**
- **Specificity of the processing module**

Output Interfaces



- Preferences of users (level of specificity; summarization)
- Visualization of results
- Numeric condensation of results

Input Interfaces- Design Paradigm

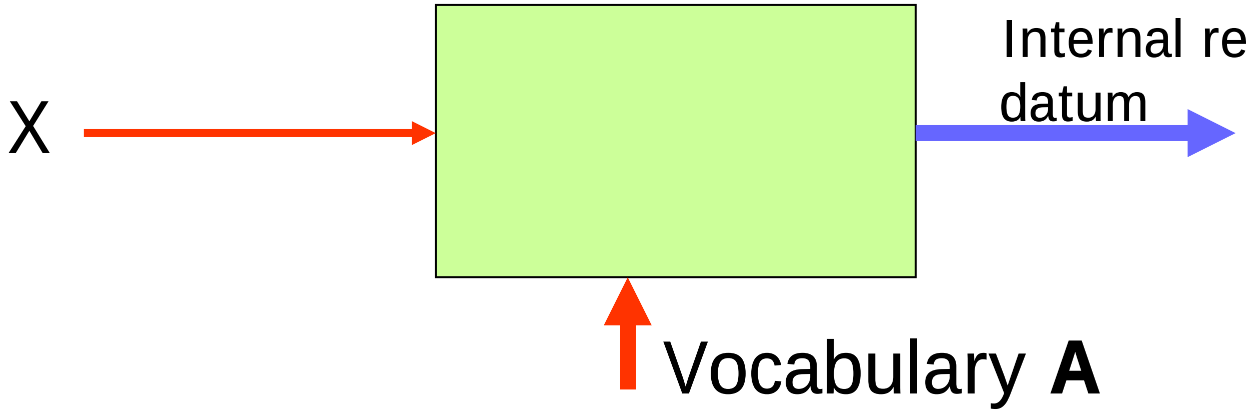


Input datum X

Vocabulary **A** = {A1, A2, ..., Ac}

<u>Problem: expressing X in terms of **A**</u>

Possibility and Necessity Measures

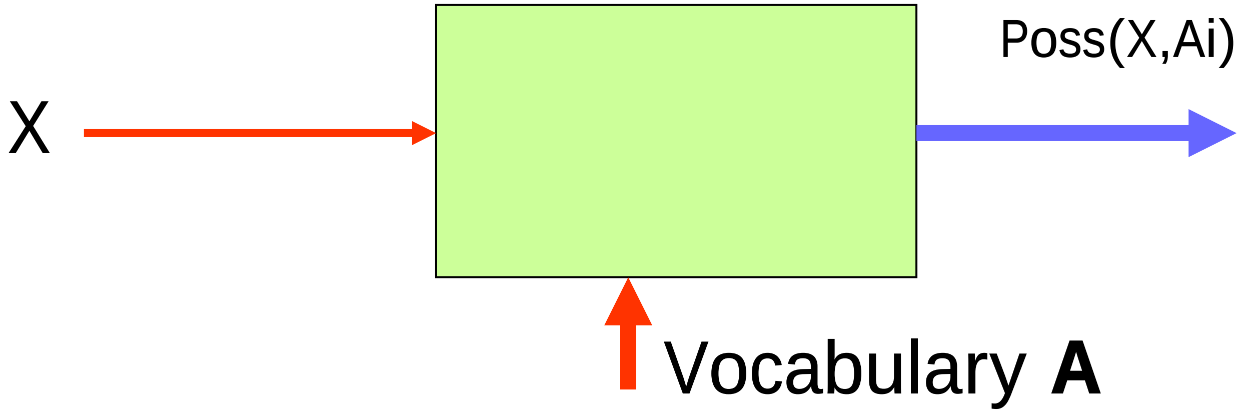


Possibility: Poss(X, Ai)

Necessity: Nec(X, Ai)

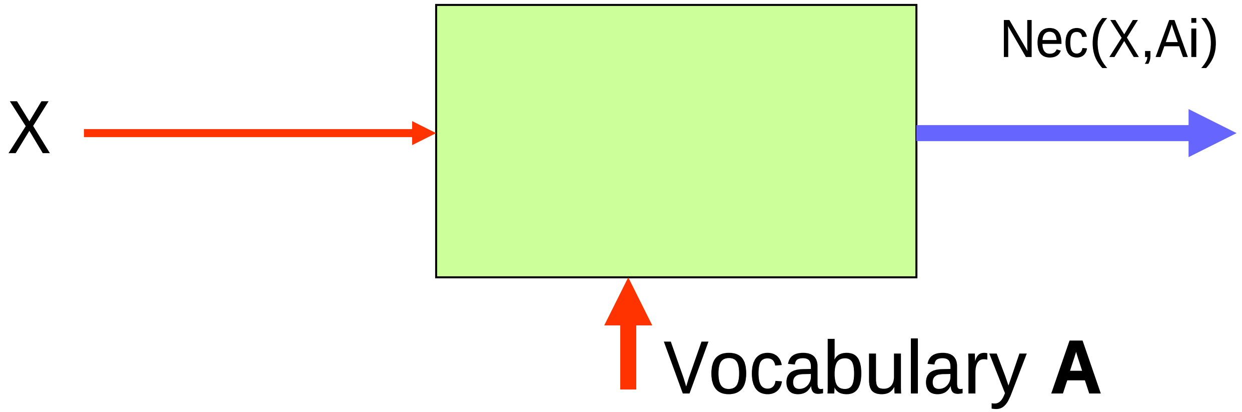
Aggregates of possibility and necessity

Possibility Measure



Poss(X, Ai) -- degree of overlap of X and Ai

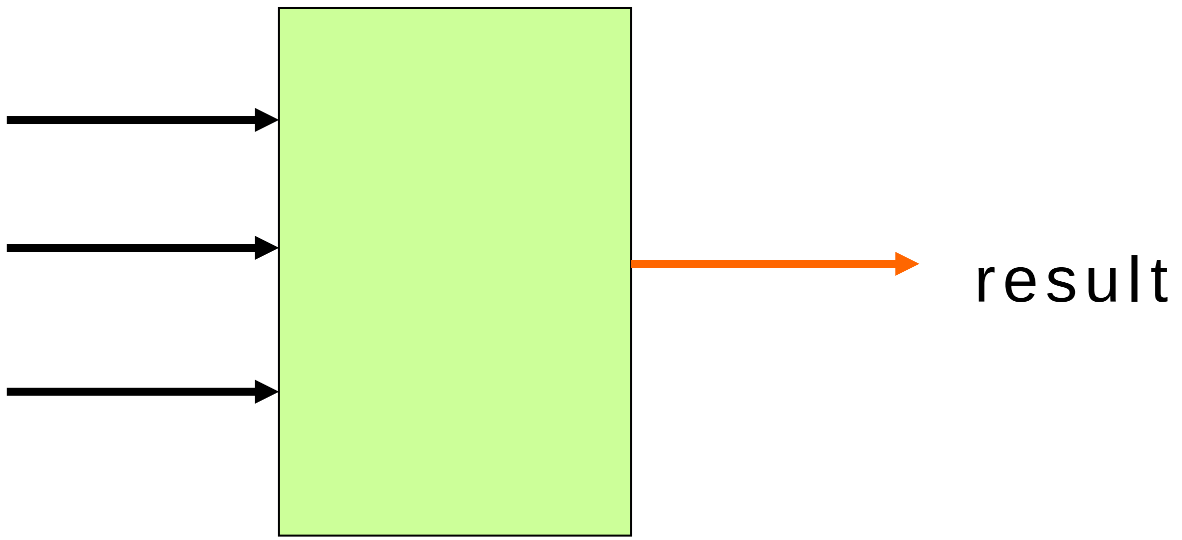
Necessity Measure



Nec(X, Ai) -- degree of inclusion of X in Ai

Output interfaces

communicating results in a meaningful and "readable" manner



Taxonomy of communication modes

Linguistic (granular)

Linguistic approximation

Shadowed set (quantification of uncertainty)

Numeric representation

Taxonomy of communication modes

Linguistic (granular)

Expressing result in terms of the vocabulary of generic linguistic terms

$$\{ A1\ (\lambda 1),\ A2(\lambda 2), \ldots,\ Ac(\lambda c) \}$$

Taxonomy of communication modes

Linguistic approximation

Approximate the result by a single element from the vocabulary

Ai

using eventually linguistic modifiers (τ**;** ***very*****,** ***more or less*****, etc.)**

τ**(Ai)**

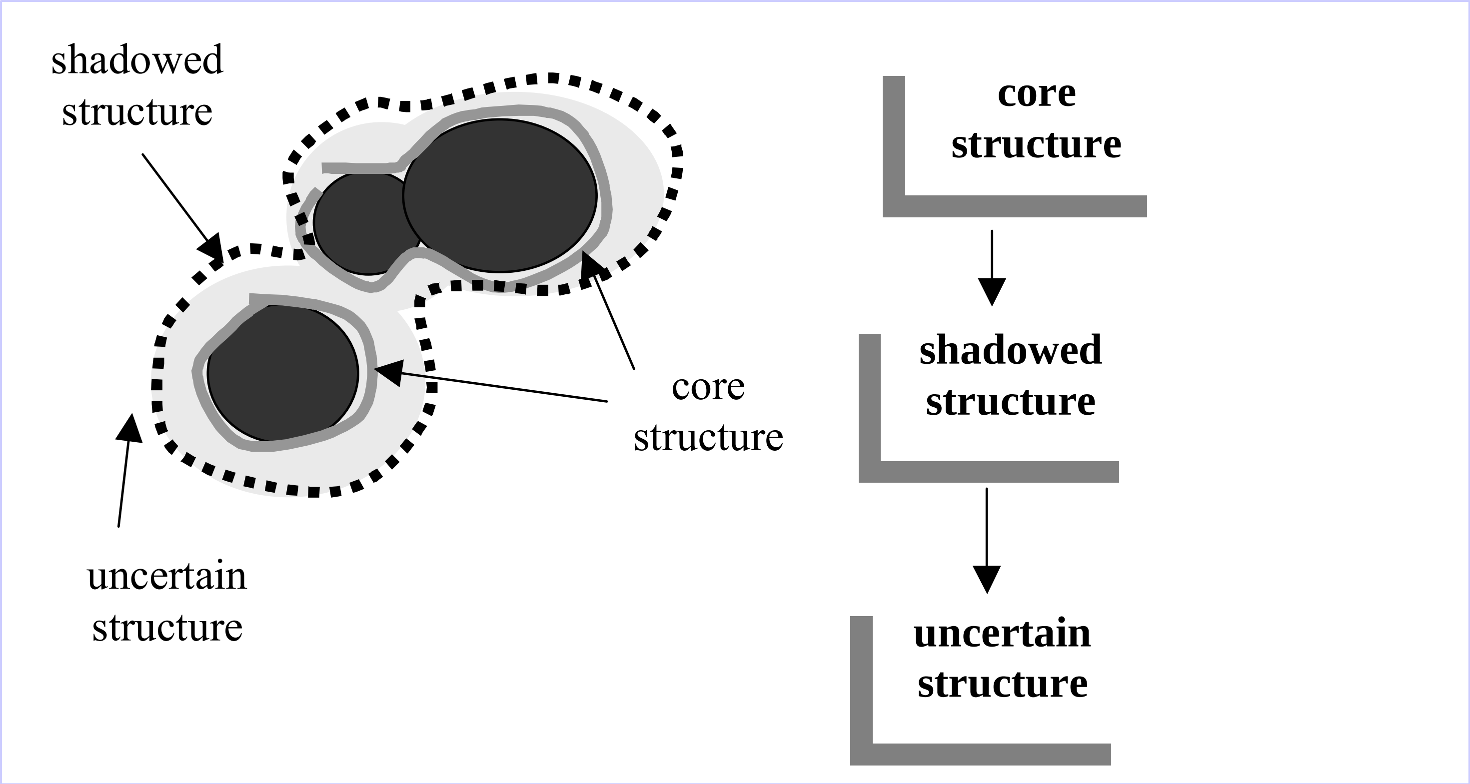
Taxonomy of communication modes

Shadowed set (quantification of uncertainty)

Fuzzy set transformed into a shadowed set which allows for a three-valued quantification

(a) Full membership
(b) “localized” uncertainty
(c) Membership excluded

Shadowed sets: interpretation of data structure and hierarchy of concepts



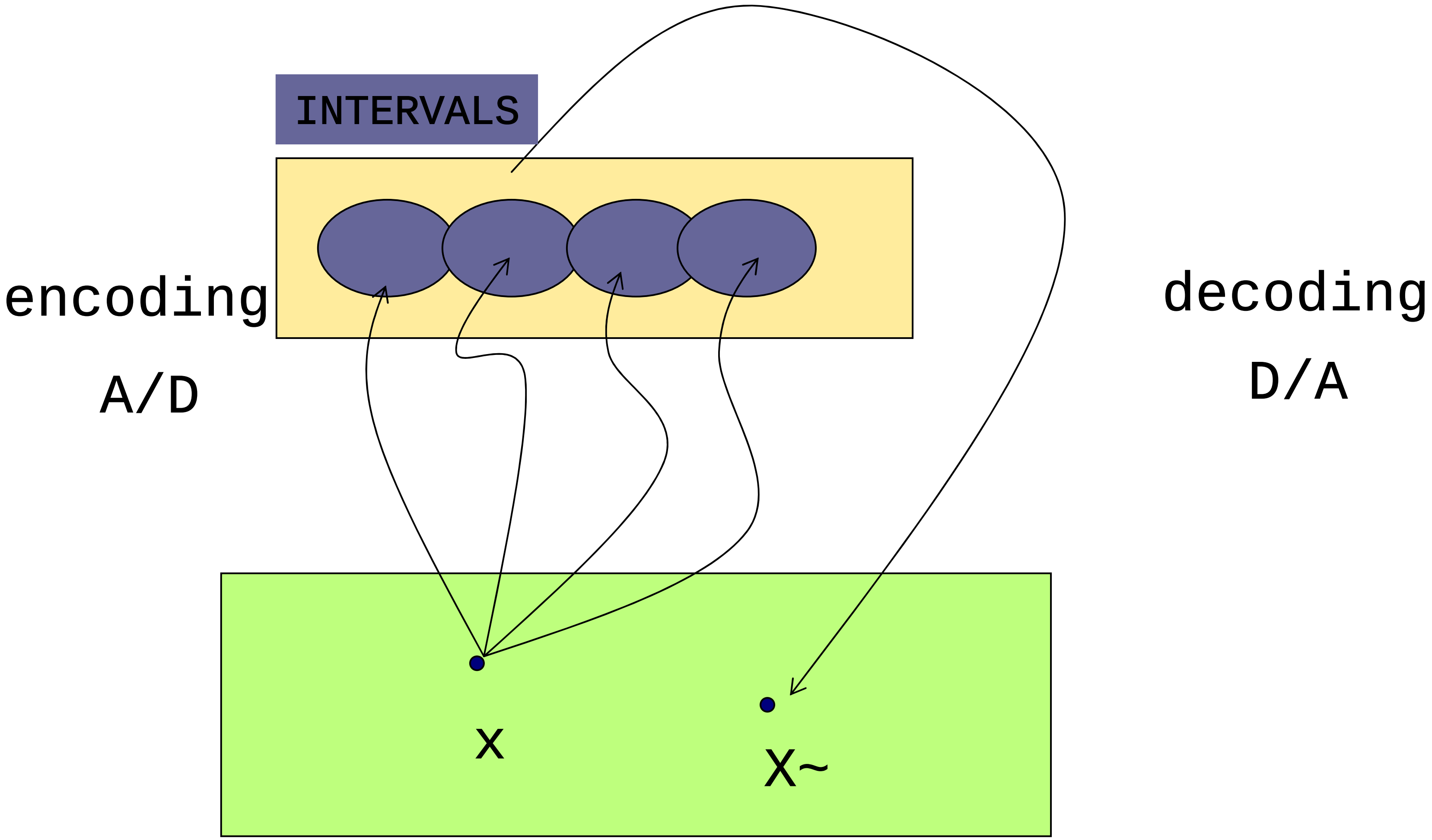
Taxonomy of communication modes

Numeric representation

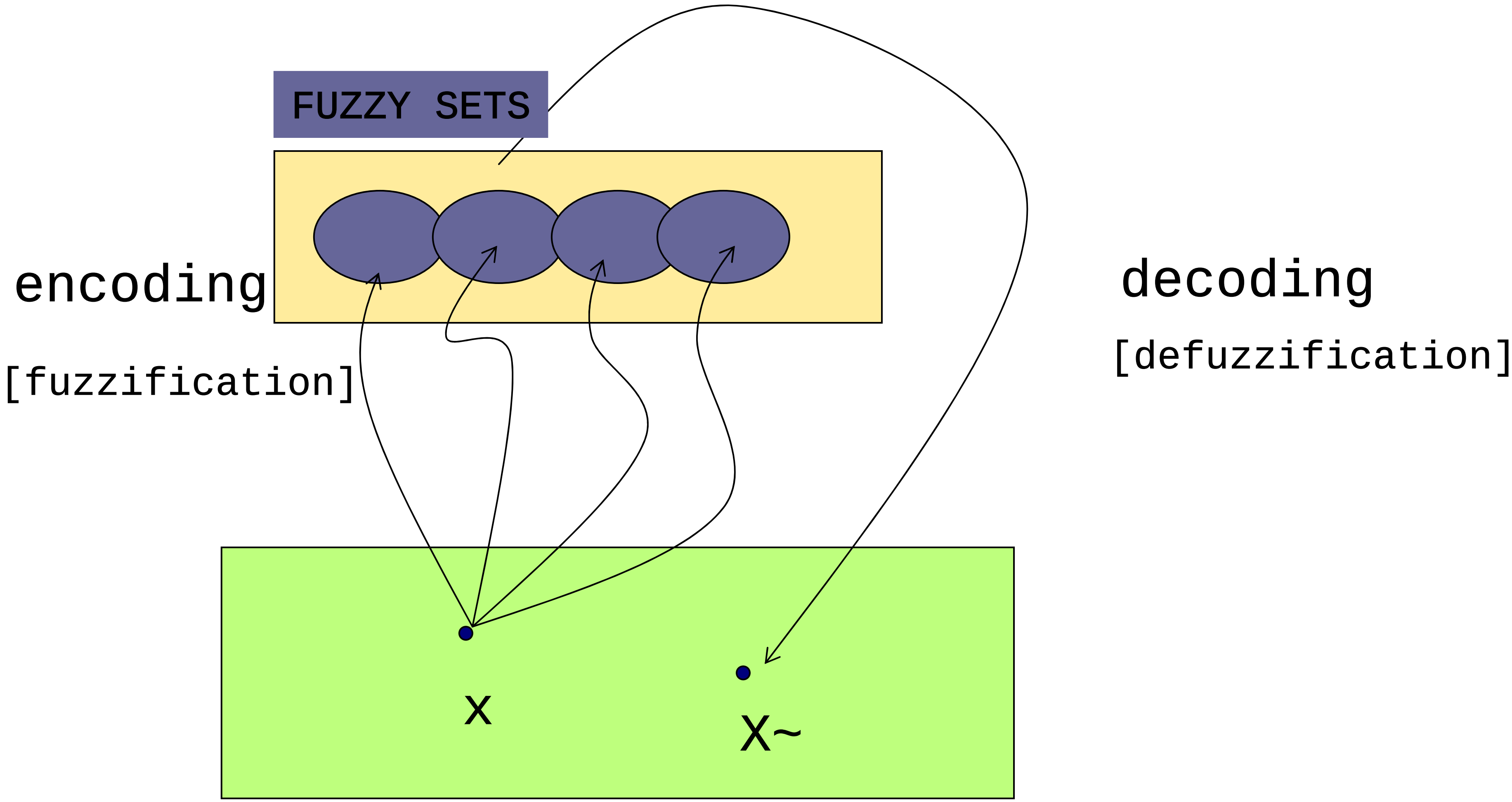
Fuzzy set approximated by a single numeric representative

(a) Very concise but lacks uncertainty quantification
(b) Usually highly nonlinear
(c) Numerous transformations possible (non-unique)

Communication: Numeric data and Intervals [quantization effect]

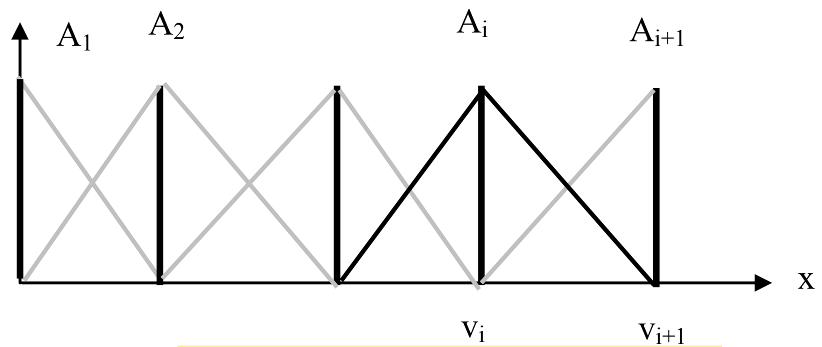


Communication: Numeric data and fuzzy sets [granulation effect]



Decoding : one-dimensional case

codebook-triangular fuzzy sets with ½ overlap



decoding $$\hat{x} = \sum_{i=1}^{c} A_i(x) v_i$$

Codebook produces a zero decoding error $\hat{x} = x$

Numeric representation and associated error

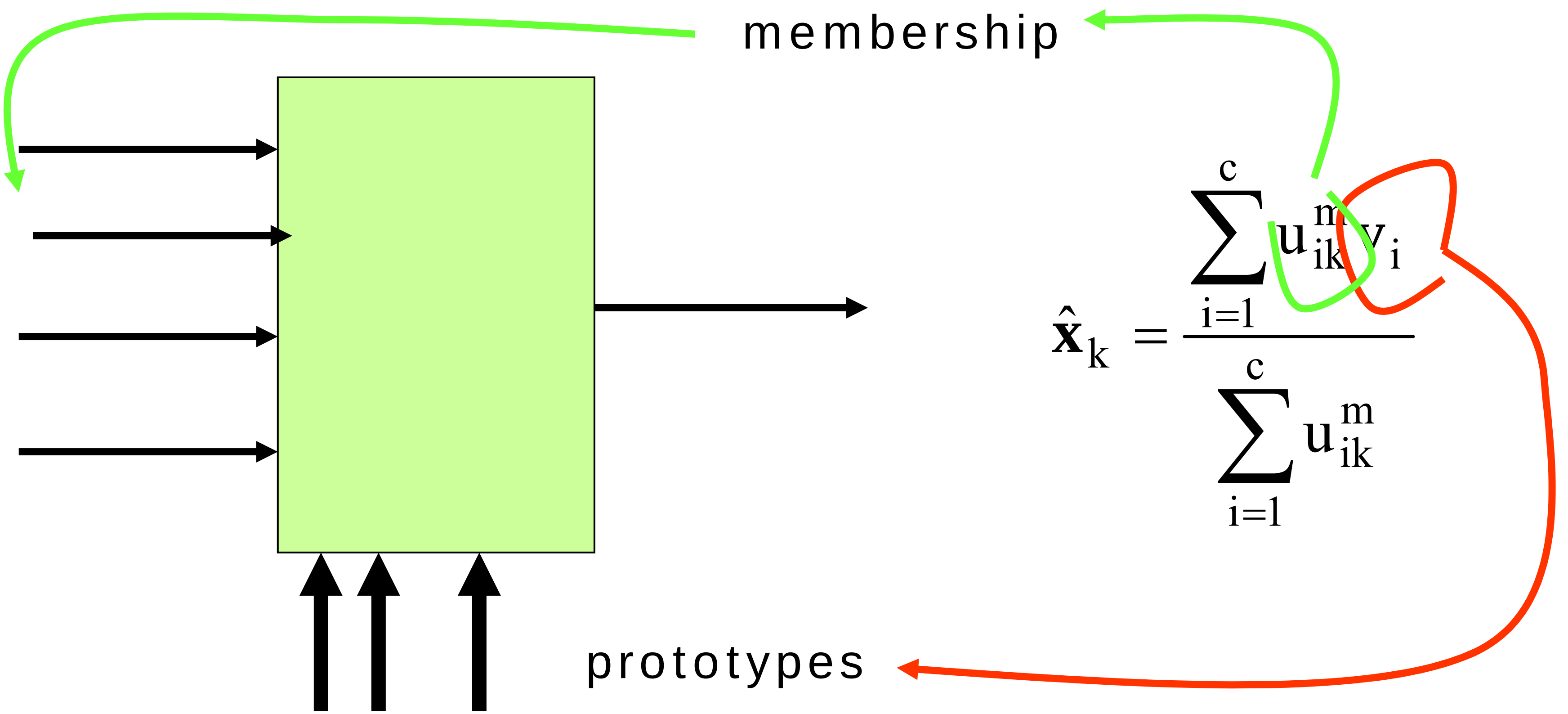
Given the interface formed by clusters (prototypes),

and

current membership values,

determine a numeric representative generated by the interface

Numeric representation and associated error



$$\hat{\mathbf{x}}_k = \frac{\sum_{i=1}^{c} u_{ik}^{m} v_i}{\sum_{i=1}^{c} u_{ik}^{m}}$$

Numeric representation and associated error

Numeric representation

$$\hat{\mathbf{x}}_k = \frac{\sum_{i=1}^{c} u_{ik}^m \mathbf{v}_i}{\sum_{i=1}^{c} u_{ik}^m}$$

u_{ik} – membership in i-th cluster for $\mathbf{x}_k$

Error

$$\sum_{k=1}^{N} \| \mathbf{x}_k - \hat{\mathbf{x}}_k \|^2$$

Concluding note

Analog world → World of Digital Processing:

<u>Interval –based granulation</u>

Human-centric Intelligent World → World of Granular Computing:

<u>(Fuzzy Sets, Rough Sets, …)</u>