Sensitivity-based Security-constraint OPF Market Clearing Model

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Who I am

- Federico Milano received from the University of Genoa, Italy, the Electrical Engineering degree and the Ph.D. degree in 1999 and 2003, respectively.
- From 2001 to 2002 he worked at the University of Waterloo, Canada, as a Visiting Scholar.
- He is currently an assistant Professor at the University of Castilla-La Mancha, Ciudad Real, Spain.
- His research interests include voltage stability, electricity markets and computer-based power system analysis and control.

Summary

- Outlines of Optimal Power Flow
- Security Constrained Optimal Power Flow
- Sensitivity-based Security-constrained OPF
- Example

Optimal Power Flow

OPF - Objective Function

- The Optimal Power Flow (OPF) is a nonlinear programming problem.
- In the market problem the objective function is maximizing the social welfare:

$$\text{Max.} \quad G = \sum_{j \in \mathcal{J}} c_{D_j}(P_{D_j}) - \sum_{i \in \mathcal{I}} c_{S_i}(P_{S_i})$$

OPF - Network Contraints (I)

- Power flow equations (network side):

$$
\begin{aligned}
P_h = {} & V_h^2(g_h + g_{h0}) \\
& - V_h \sum_{\ell \neq h}^{n_\ell} V_\ell (g_{h\ell} \cos(\delta_h - \delta_\ell) + b_{h\ell} \sin(\delta_h - \delta_\ell)) \\
& \forall h \in \mathcal{B} \\
Q_h = {} & -V_h^2(b_h + b_{h0}) \\
& + V_h \sum_{\ell \neq h}^{n_\ell} V_\ell (g_{h\ell} \sin(\delta_h - \delta_\ell) - b_{h\ell} \cos(\delta_h - \delta_\ell)) \\
& \forall h \in \mathcal{B}
\end{aligned}
$$

OPF - Network Constraints (II)

- Power flow equations (participant side):

$$P_h = \sum_{i \in \mathcal{I}_h} (P_{G_{i0}} + P_{S_i}) - \sum_{j \in \mathcal{J}_h} (P_{L_{0j}} + P_{D_{0j}})$$
$$\forall h \in \mathcal{B}$$

$$Q_h = \sum_{i \in \mathcal{I}_h} Q_{G_i} - \sum_{j \in \mathcal{J}_h} (P_{L_{0j}} + P_{D_{0j}}) \tan(\phi_{D_i})$$
$$\forall h \in \mathcal{B}$$

OPF - Network Constraints (III)

- V and δ are bus voltages.
- Loads are modeled as constant powers.
- P_{G_0} and P_{L_0} are fixed (inelastic) powers such as:
  - Must-run generators.
  - Uninterruptible loads.
  - Bilateral contracts.

OPF - Network Constraints (IV)

- Advantages of using the complete power flow equations:
  - Both active and reactive power balances are respected.
  - Losses are taken into account.

OPF - Bid Blocks

- Supply and demand bid blocks:

$$P_{S_{\min_i}} \leq P_{S_i} \leq P_{S_{\max_i}} \quad \forall i \in \mathcal{I}$$

$$P_{D_{\min_j}} \leq P_{D_j} \leq P_{D_{\max_j}} \quad \forall j \in \mathcal{J}$$

OPF - Technical Constraints

- Reactive power and voltage limits:

$$Q_{G_{\min_i}} \leq Q_{G_i} \leq Q_{G_{\max_i}} \quad \forall i \in \mathcal{I}$$

$$V_{\min_h} \leq V_h \leq V_{\max_h} \quad \forall h \in \mathcal{B}$$

OPF - Thermal Limits

- Transmission line thermal limits:

$$I_{hk}(\delta, V) \leq I_{hk_{\max}} \quad \forall (h,k) \in \mathcal{N}$$
$$I_{kh}(\delta, V) \leq I_{kh_{\max}}$$

- I_{hk} and I_{kh} are the transmission line currents.

OPF - Standard Security Limits

- Standard security limits are generally modeled as active power limits on transmission lines:

$$P_{hk}(\delta, V) \leq P_{hk_{\max}} \quad \forall (h,k) \in \mathcal{N}$$
$$P_{kh}(\delta, V) \leq P_{kh_{\max}}$$

- $P_{hk_{\max}}$ y $P_{hk_{\max}}$ are typically computed off-line.

- We are proposing a different way of including security limits.

Security Constrained OPF

Security Constraints - Desired Features

- Security constraints should be computed **on-line**.
- Contingencies should be taken into account.
- It should be possible to take into account a variety of security and stability phenomena.

Security Constraints - Exisiting Models (I)

- Multiobjective SC-OPF.
  - "All-in-one" phylosophy. The social welfare is maximized along with the security margin.
  - It is possible to take into account contingencies (ranking).
  - Hardly accepted by practicioners.

Multiobiective SC-OPF (I)

Min. $G = -\omega_1(C_D^T P_D - C_S^T P_S) - \omega_2\lambda_c$

s.t. $f(\delta, V, Q_G, P_S, P_D) = 0$ $\longrightarrow$ Power Flow

$f(\delta_c, V_c, Q_{G_c}, \lambda_c, P_S, P_D) = 0$ $\longrightarrow$ "Critical" Power Flow

$\lambda_{c_{\min}} \leq \lambda_c \leq \lambda_{c_{\max}}$ $\longrightarrow$ Security Margin

Multiobjective SC-OPF (II)

- (*follows*)

$$0 \leq P_S \leq P_{S_{\max}} \quad \longrightarrow \text{Supply Bids}$$

$$0 \leq P_D \leq P_{D_{\max}} \quad \longrightarrow \text{Demand Bids}$$

$$I_{ij}(\delta, V) \leq I_{ij_{\max}} \quad \longrightarrow \text{Thermal Limits}$$

$$I_{ji}(\delta, V) \leq I_{ji_{\max}}$$

$$I_{ij}(\delta_c, V_c) \leq I_{ij_{\max}}$$

$$I_{ji}(\delta_c, V_c) \leq I_{ji_{\max}}$$

Multiobjective SC-OPF (III)

- (*follows*)

$$Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}} \quad \longrightarrow \text{ Reactive Power Limits}$$

$$Q_{G_{\min}} \leq Q_{G_c} \leq Q_{G_{\max}}$$

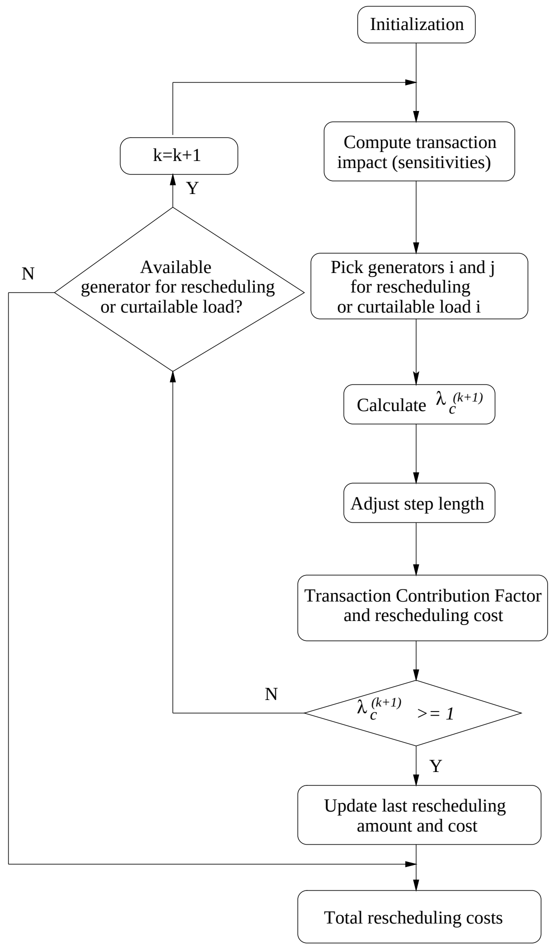
$$V_{\min} \leq V \leq V_{\max} \quad \longrightarrow \text{ Voltage Limits}$$

$$V_{\min} \leq V_c \leq V_{\max}$$

Security Constraints - Exisiting Models (II)

- Simple auction with security constraints.
  - Iterative method.
  - Linear market clearing model and CPF analysis to take into account voltage stability.
  - A sensitivity analysis is used to link the market clearing model and the CPF analysis.
  - The market clearing problem is clearly separated from the stability analysis.

Simple Auction with Security Constraints



Security Constraints - References

- **F. Milano**, C. A. Cañizares, and M. Invernizzi, "Multiobjective Optimization for Pricing System Security in Electricity Markets", *IEEE Transactions on Power Systems*, vol. 18, no. 2, pp. 596-604, May 2003.
- **F. Milano**, C. A. Cañizares, and M. Invernizzi, "Voltage Stability Constrained OPF Market Models Considering N-1 Contingency Criteria", *Electric Power System Research*, no. 74, pp. 27-36, March 2005.
- C. A. Cañizares, H. Chen, **F. Milano**, and A. Singh, "Transmission Congestion Management and Pricing in Simple Auction Electricity Markets", *International Journal of Emerging Electric Power Systems*, vol. 1, issue 1, article 1, 28 pages, 2004.

Proposed Method

Sensitivity-based Security-constrained OPF (I)

- Iterative method.
- Three steps:
  - Market solution (OPF).
  - Sensitivity analysis based on OPF solution (Lagrangian multipliers).
  - Stability analysis (CPF).

Sensitivity-based Security-constrained OPF (II)

- First step:
  - Firstly, one solves a OPF with security constraints (similar to the multiobjective OPF but with fixed security margin).
  - The objective function is the social welfare. At this step, the security margin is not optimized.

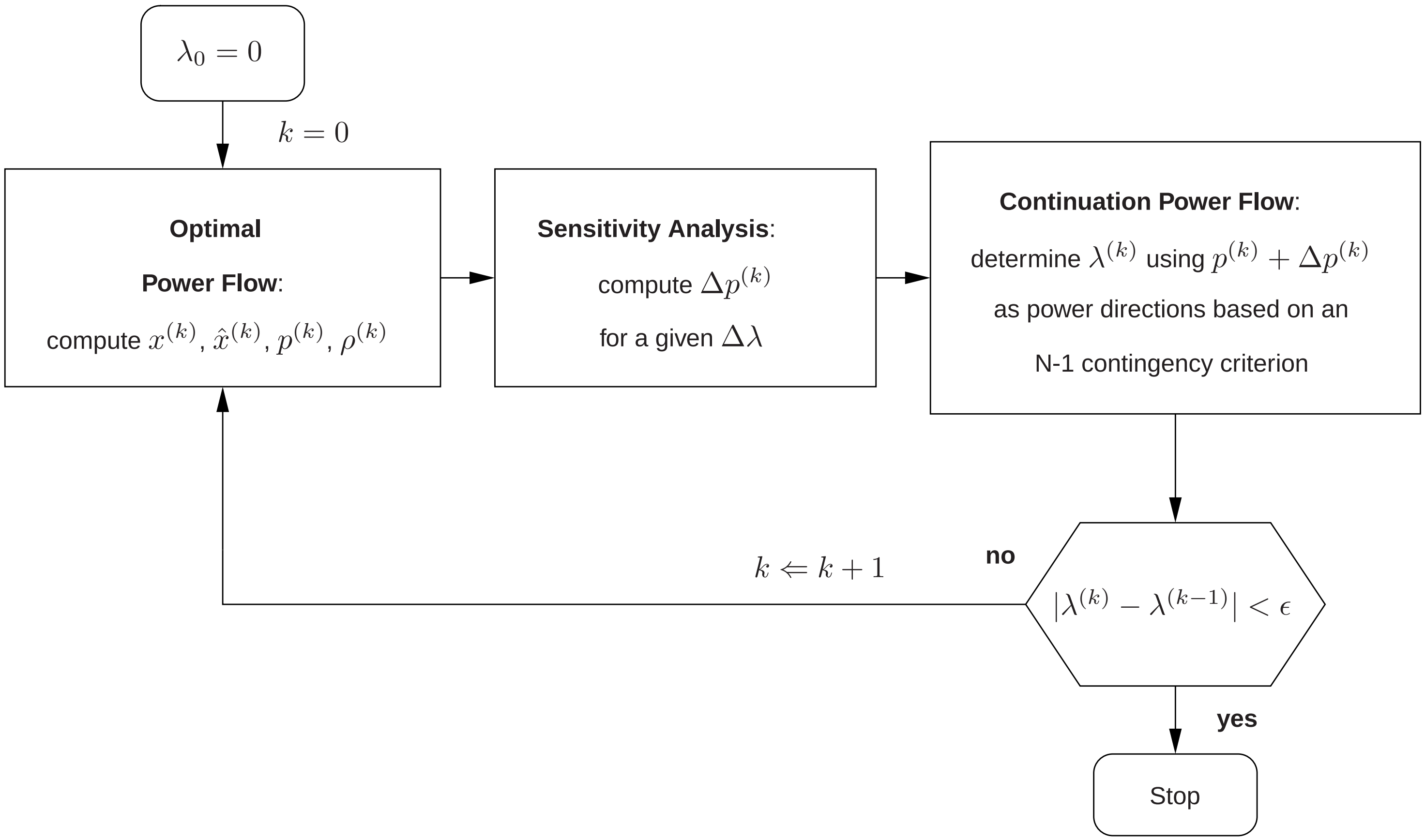
Sensitivity-based Security-constrained OPF(III)

- Second step:
  - The OPF solution is used in the sensitivity analysis step.
  - From the sensitivity analysis one obtains the power increments that are used as the input of the stability analysis.

Sensitivity-based Security-constrained OPF (IV)

- Third step:
  - The power increments are the inputs of the CPF analysis with inclusion of contingencies that leads to the new value of the security margin.
  - The OPF is repeated using the new value of the security margin.
  - The process stops when the security margin cannot be further increased.

Flowchart


$\lambda_0 = 0$
$k = 0$
Optimal
Power Flow:
compute $x^{(k)}$, $\hat{x}^{(k)}$, $p^{(k)}$, $\rho^{(k)}$
Sensitivity Analysis:
compute $\Delta p^{(k)}$
for a given $\Delta\lambda$
Continuation Power Flow:
determine $\lambda^{(k)}$ using $p^{(k)} + \Delta p^{(k)}$
as power directions based on an
N-1 contingency criterion
$|\lambda^{(k)} - \lambda^{(k-1)}| < \epsilon$
no
$k \Leftarrow k + 1$
yes
Stop

Proposed OPF Model OPF (I)

Min. $f \;=\; -\left(C_D^T P_D - C_S^T P_S\right)$

s.t. $g(\delta, V, Q_G, P_S, P_D) = 0$ $\longrightarrow$ Power flow

$\hat{g}(\hat{\delta}, \hat{V}, \hat{Q}_G, \lambda, P_S, P_D) = 0$ $\longrightarrow$ Critical power flow

$\lambda = \hat{\lambda}$ $\longrightarrow$ Security margin

Proposed OPF Model OPF (II)

- (*follows*)

$$0 \leq P_S \leq P_{S_{\max}} \quad \longrightarrow \text{Supply limits}$$

$$0 \leq P_D \leq P_{D_{\max}} \quad \longrightarrow \text{Demand limits}$$

$$I_{ij}(\delta, V) \leq I_{ij_{\max}} \quad \longrightarrow \text{Thermal limits}$$

$$I_{ji}(\delta, V) \leq I_{ji_{\max}}$$

$$I_{ij}(\hat{\delta}, \hat{V}) \leq I_{ij_{\max}}$$

$$I_{ji}(\hat{\delta}, \hat{V}) \leq I_{ji_{\max}}$$

Proposed OPF Model OPF (III)

- (*follows*)

$$Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}} \longrightarrow \text{Reactive power limits}$$

$$Q_{G_{\min}} \leq \hat{Q}_G \leq Q_{G_{\max}}$$

$$V_{\min} \leq V \leq V_{\max} \longrightarrow \text{Voltage limits}$$

$$V_{\min} \leq \hat{V} \leq V_{\max}$$

Sensitivity Formulas (I)

$$\left.\frac{\mathrm{d}f}{\mathrm{d}\lambda}\right|_* = \frac{df}{d\lambda^*} = -\rho_\lambda$$

$$\left.\frac{\mathrm{d}f}{\mathrm{d}\hat{g}_{pi}}\right|_* = \frac{df}{d\hat{g}^*_{pi}} = -\rho_{\hat{g}_{pi}}$$

Sensitivity Formulas (II)

- Hence:

$$\left.\frac{\mathrm{d}f}{\mathrm{d}p_i}\right|_* = \left.\frac{\mathrm{d}f}{\mathrm{d}\hat{g}_{pi}}\right|_* \left.\frac{\mathrm{d}\hat{g}_{pi}}{\mathrm{d}p_i}\right|_* = -\rho_{\hat{g}_{pi}} \nabla_{p_i} \hat{g}_{pi}$$

- and, finally:

$$\left.\frac{\mathrm{d}p_i}{\mathrm{d}\lambda}\right|_* = \left.\frac{\mathrm{d}p_i}{\mathrm{d}f}\right|_* \left.\frac{\mathrm{d}f}{\mathrm{d}\lambda}\right|_* = \frac{\rho_\lambda}{\rho_{\hat{g}_{pi}} \nabla_{p_i} \hat{g}_{pi}}$$

Sensitivity Formulas (III)

- In scalar form:

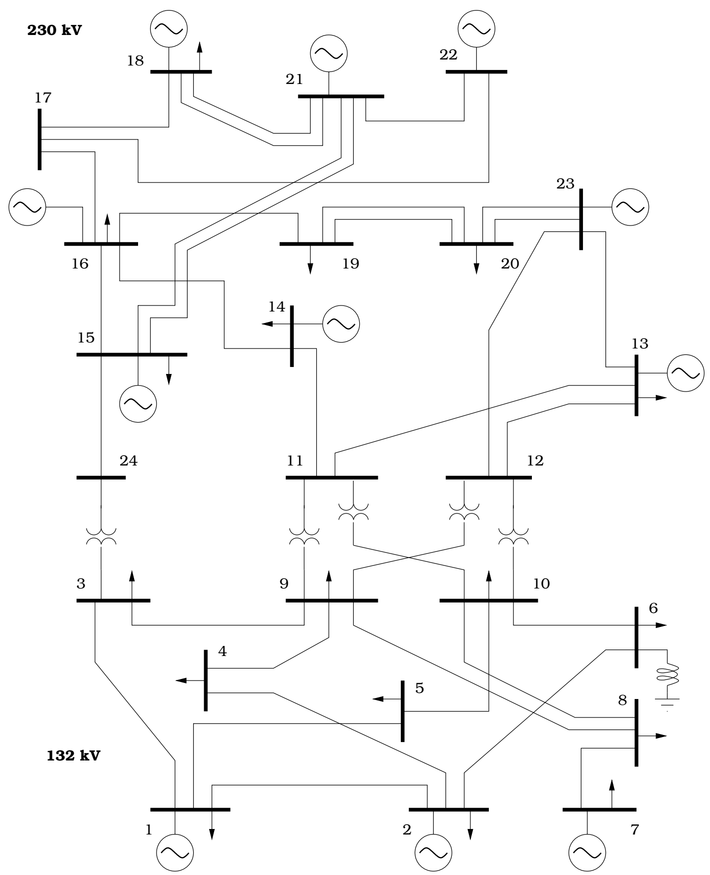
$$\left.\frac{\mathrm{d}P_{S_i}}{\mathrm{d}\lambda}\right|_* = \frac{1}{(1+\lambda^* + \hat{k}_G^*)}\frac{\rho_\lambda}{\rho_{\hat{g}_{P_i}}}$$

$$\left.\frac{\mathrm{d}P_{D_i}}{\mathrm{d}\lambda}\right|_* = -\frac{1}{(1+\lambda^*)}\frac{\rho_\lambda}{\rho_{\hat{g}_{P_i}}}$$

Example

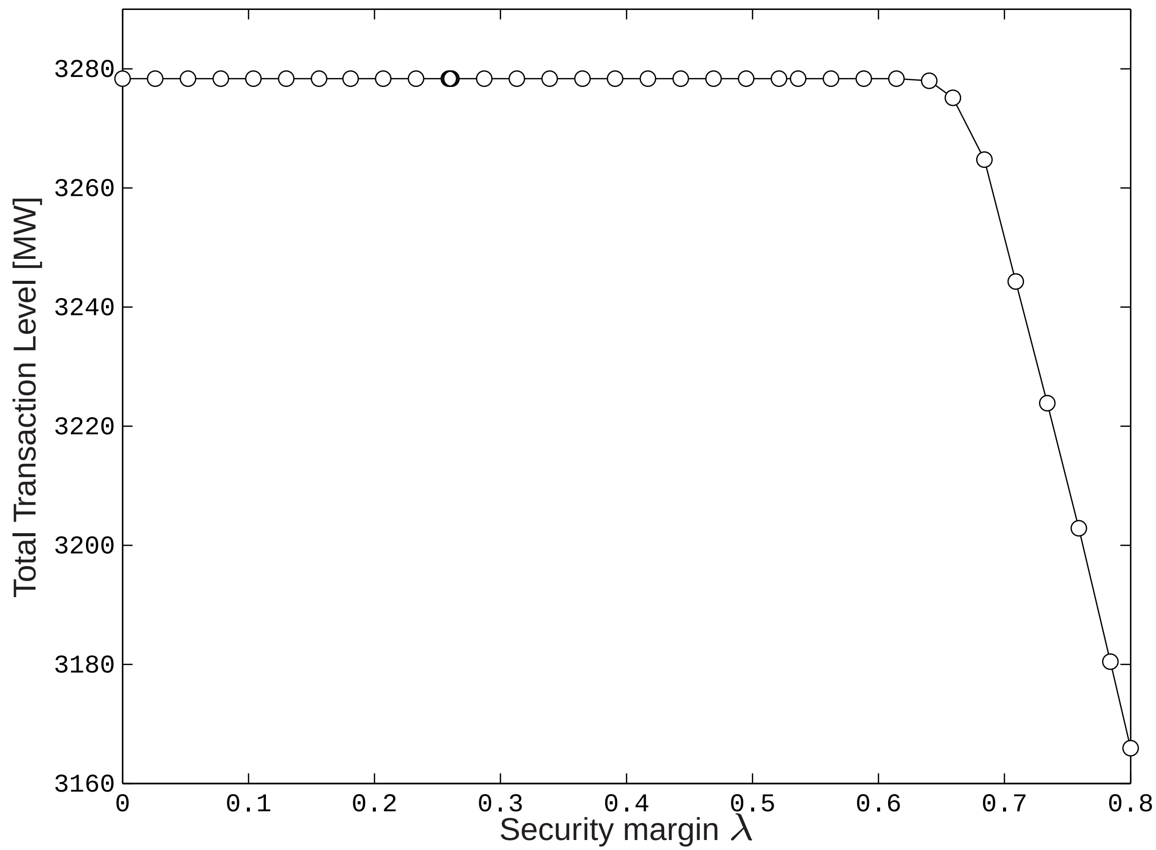
Example (I)

- 24-bus network (RTS 96):



Example (II)

- Total demand (elastic demand, line 3-24 contingency):



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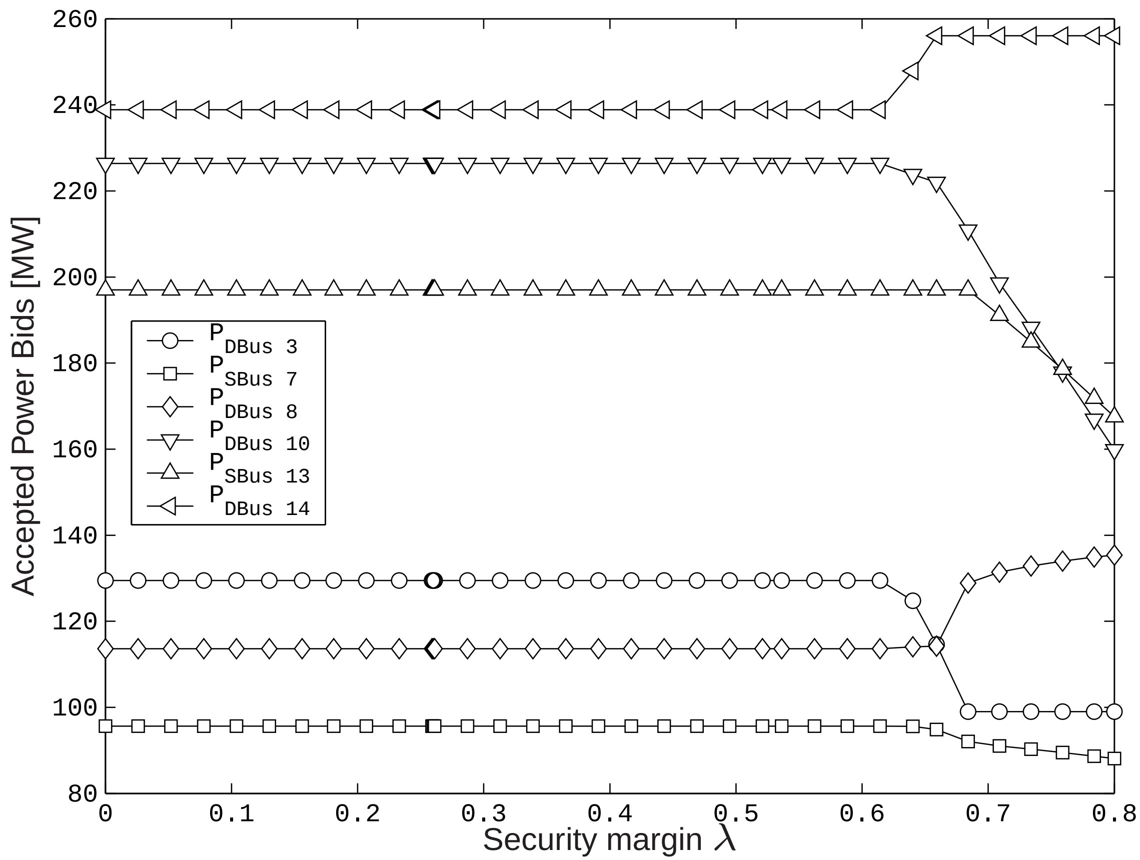
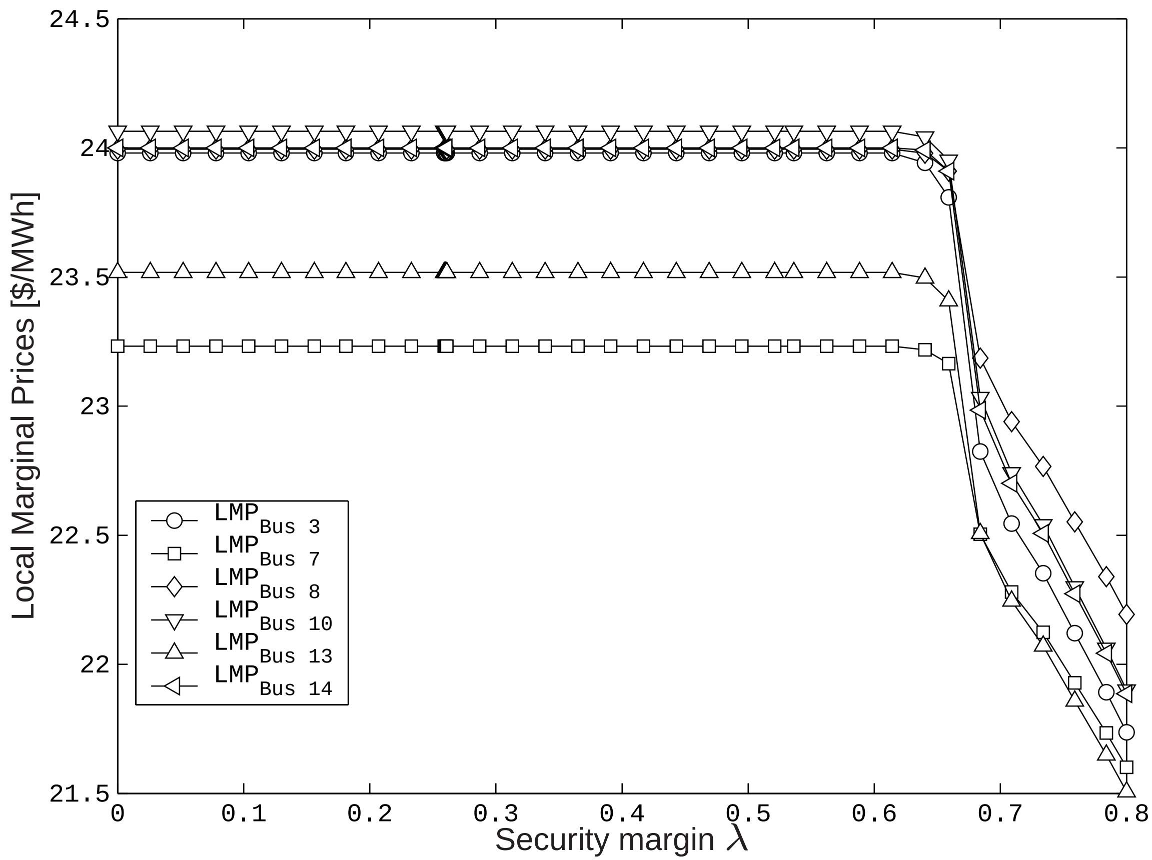
Example (III)

- Relevant accepted bids (elastic demand, line 3-24 contingency):



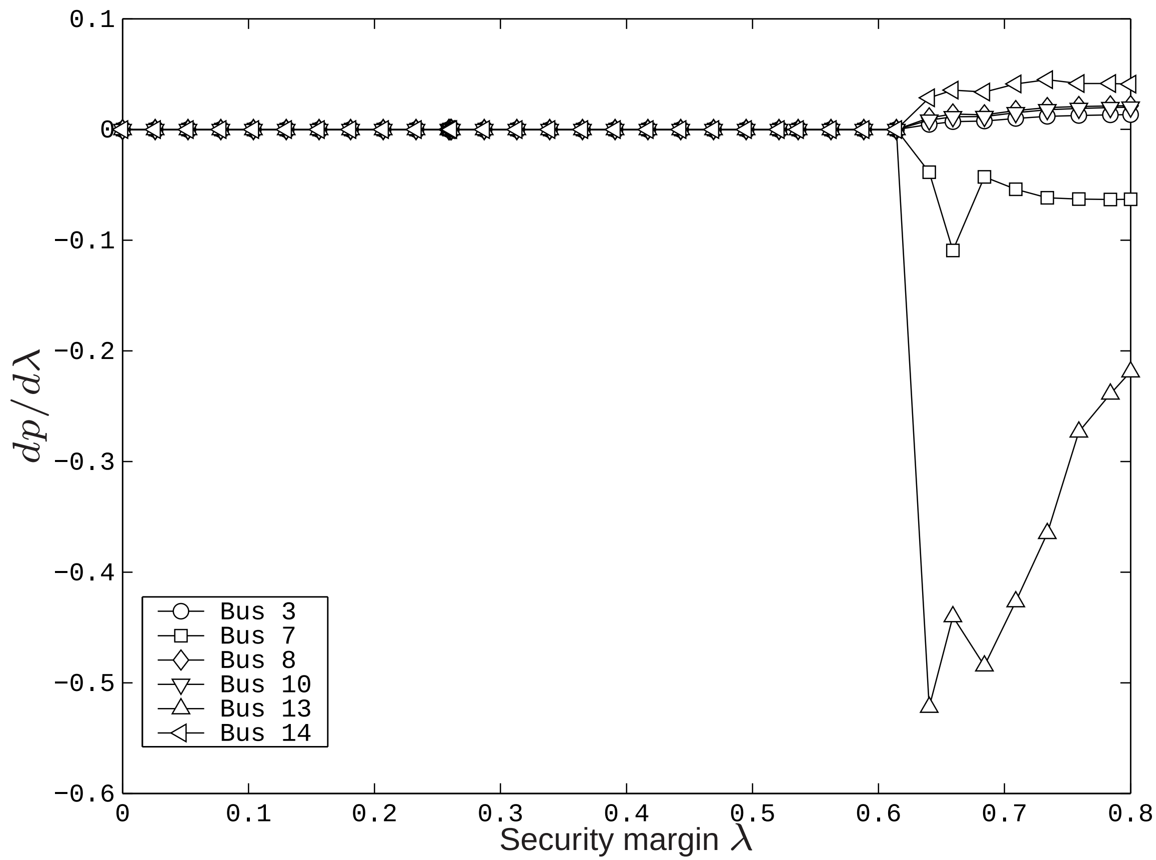
Example (IV)

- Locational marginal prices (elestic demand, line 3-24 contingency):



Example (V)

- Sensitivities (elestic demand, line 3-24 contingency):



Conclusions

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Conclusions

- The proposed method allows taking into account accurate security limits in the market clearing.
- It allows taking into account contingencies.
- The market solution is separated from the security analysis.

Future Works

- It is possible to improve the proposed model in several ways.
- Modify the OPF model (e.g. mixed linear integer programming, unit commitment, generator ramping, etc.).
- Add dynamic models in the security analysis step (Hopf bifurcatons, angle stability, etc.).

Reference

- F. Milano, C. A. Cañizares, and A. J. Conejo, “Sensitivity-based Security-constrained OPF Market Clearing Model”, *IEEE Transactions on Power Systems*, Vol. 20, No. 4, November 2005, pages 2051-2061.

Thanks for your attention!

Questions?