# A Form Based Control Algorithm for Reducing the Complexity of an Attitude Control System

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***Abstract*— A dynamic decoupling method is presented for precisely controlling the orientation of a satellite while requiring minimal plant information. Using the form of the problem, the complex and unknown system dynamics can be approximated and compensated for in real time within a portion of the control effort. This technique will increase the robustness of the control algorithm and minimize the system level constraints. The solution is demonstrated by controlling the orientation of a three degree of freedom spacecraft simulation. The orientation of the spacecraft is controlled by varying the angular velocity of four reaction wheels mounted in a tetrahedron configuration.**

***Keywords-ADRC, ESO, MIMO, Decoupling, Disturbance Rejection, Form***

## I. INTRODUCTION

One of the main technical constraints when designing a spacecraft is the amount of attitude control that is required for the specific mission. Spacecrafts can be designed: as a tumbler, with a gravity gradient stabilizer, based on magnetic torqers, with thrusters, or by using reaction wheels. Each method provides more or less control over the orientation of the spacecraft, however the design complexity of each method is drastically different. The complexity is seen in the mechanical configuration of the spacecraft and the software that is required to achieve the desired response.

The focus of this paper is to demonstrate that a well designed control algorithm can dramatically reduce: the hardware constraints, the time to test and debug, and the cost to develop a system. This claim shall be demonstrated by developing an attitude control simulation to orient the spacecraft. The roll, pitch, and yaw axes of the spacecraft shall be controllable via four reaction wheels mounted in a tetrahedron configuration. This architecture will not require the wheels to be mounted in the roll, pitch, or yaw axes of the spacecraft, which significantly reduces the hardware complexity by: allowing the reaction wheels to be configured in the best arrangement for the entire system and by reducing the mounting tolerances. If the wheels are not mounted in line with the major axes of the spacecraft, then controlling its orientation becomes more difficult because of the cross coupling terms. Rotating one reaction wheel causes the spacecraft to rotate in multiple axes. Therefore the control algorithm will need to compensate for the system dynamics and the cross coupling terms. Decoupling control is typically difficult and very problem specific. However, using the form of the problem, a control algorithm is developed which will easily account for the cross coupling and provide precise tracking.

The form based control solution only requires a minimal amount of plant information. By using the dominant order and the high frequency gain, an approximation of the dynamics can be used as a portion of the control effort to compensate for the unknown and complex dynamics. This method is less dependent on a trial and error approach, and it is a very systematic way of controlling the system. This approach dramatically improves the performance and reduces the time spent tuning the system.

## II. SPACECRAFT MODEL

The objective of the design is to control the attitude of the spacecraft while relaxing many of the constraints such as: specific alignment of the control mechanisms, mounting tolerances, and complexity of the control algorithm. The roll, pitch, and yaw of the spacecraft are controlled by four reaction wheels configured in a tetrahedron configuration, as illustrated in Figure 1.

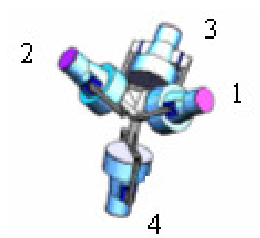


Figure 1 – Tetrahedron Configuration of Reaction Wheels

The alignments of the reaction wheels with respect to the axes of the spacecraft dictate the complexity of the control algorithm. If there is a reaction wheel directly in line with each of the spacecraft axes then the control problem becomes much easier. However, designing the entire spacecraft becomes more difficult. If it is not required that the reaction wheels be mounted inline with the axes of the spacecraft, then it is easier to design the entire spacecraft but it is more difficult to control. The objective of this paper is to demonstrate that a systematic control algorithm can give the system architect flexibility in arranging the hardware without dramatically increasing the complexity of the control algorithm. Therefore, the reaction wheel will not be mounted in line with the axes of the spacecraft; however they will be mounted in a manor that is most effective for the entire system.

The tetrahedron configuration of the reaction wheels shown in Figure 1 is designed to be housed within a satellite structure. The natural alignment of the satellite is such that wheel four is in line with the bottom of the satellite, pointing to earth, which is also the x-axis of the spacecraft. The remaining reaction wheels do not line up with any of the other spacecrafts' axes. Wheel one is the closest to one of the spacecrafts' axes, as its projection is in line with the spacecrafts' y-axis. Wheels two and three are arranged with respect to wheels one and four respectively, to form a tetrahedron.

Formulating the equations to describe the orientation of the satellite, based on the given configuration, begins by examining the total kinetic energy (KE) of the system. The systems KE is dependent on the KE of the satellite platform, and the KE of each reaction wheel.

$$KE = T = T_p + T_1 + T_2 + T_3 + T_4 \tag{1}$$

The KE of the satellite platform is dependent on the angular velocity of the spacecraft and the moment of inertia of the satellite.

$$T_p = \frac{1}{2}\{w_p\}[I_p]\{w_p\}^T \tag{2}$$

$$\{w_p\} = \left\{\dot{\vartheta}_x \quad \dot{\vartheta}_y \quad \dot{\vartheta}_z\right\} \tag{3}$$

$$I_p = \begin{bmatrix} Ixx & Ixy & Ixz \\ Ixy & Iyy & Iyz \\ Ixz & Iyz & Izz \end{bmatrix} \tag{4}$$

The KE of the reaction wheels are dependent on the angular velocity of the reaction wheels *w* in the spacecrafts' frame of reference, where Ω represents the actual angular velocity of the reaction wheels, and the moment of inertia of the reaction wheels.

$$T_n = \frac{1}{2}\{w_n\}[I_n]\{w_n\}^T \tag{5}$$

$$\{w_1\} = \left\{\dot{\vartheta}_x - \frac{\sqrt{2}}{2}\Omega_1 \quad \dot{\vartheta}_y + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}\Omega_1 \quad \dot{\vartheta}_z + \frac{\sqrt{2}}{4}\Omega_1\right\} \tag{6}$$

$$\{w_2\} = \left\{\dot{\vartheta}_x - \frac{\sqrt{2}}{2}\Omega_2 \quad \dot{\vartheta}_y \quad \dot{\vartheta}_z + \frac{\sqrt{2}}{2}\Omega_2\right\} \tag{7}$$

$$\{w_3\} = \left\{\dot{\vartheta}_x - \frac{\sqrt{2}}{2}\Omega_3 \quad \dot{\vartheta}_y - \frac{\sqrt{6}}{4}\Omega_3 \quad \dot{\vartheta}_z + \frac{\sqrt{2}}{4}\Omega_3\right\} \tag{8}$$

$$\{w_4\} = \left\{\dot{\vartheta}_x + \Omega_4 \quad \dot{\vartheta}_y \quad \dot{\vartheta}_z\right\} \tag{9}$$

$$I_n = \begin{bmatrix} A_n & 0 & 0 \\ 0 & B_n & 0 \\ 0 & 0 & C_n \end{bmatrix}; n = 1,2,3,4 \tag{10}$$

Finally, the differential equation that describes the position of the spacecraft can be derived by differentiating the KE of the entire system with respect to the angular velocity in the x, y, and z axes.

$$\frac{dT}{d\dot{\vartheta}_x} = 0 \tag{11}$$

$$\frac{dT}{d\dot{\vartheta}_y} = 0 \tag{12}$$

$$\frac{dT}{d\dot{\vartheta}_z} = 0 \tag{13}$$

$$\dot{\vartheta}_x = -\frac{Ixy}{A^*}\dot{\vartheta}_y - \frac{Ixz}{A^*}\dot{\vartheta}_z + \frac{\sqrt{2}}{2}\frac{A_1}{A^*}\Omega_1 + \frac{\sqrt{2}}{2}\frac{A_2}{A^*}\Omega_2 + \frac{\sqrt{2}}{2}\frac{A_3}{A^*}\Omega_3 - \frac{A_4}{A^*}\Omega_4 \tag{14}$$

$$\dot{\vartheta}_y = -\frac{Ixy}{B^*}\dot{\vartheta}_x - \frac{Ixz}{B^*}\dot{\vartheta}_z - \frac{\sqrt{6}}{2}\frac{B_1}{B^*}\Omega_1 + \frac{\sqrt{6}}{4}\frac{B_3}{B^*}\Omega_3 \tag{15}$$

$$\dot{\vartheta}_z = -\frac{Ixz}{C^*}\dot{\vartheta}_x - \frac{Iyz}{C^*}\dot{\vartheta}_y - \frac{\sqrt{2}}{2}\frac{C_1}{C^*}\Omega_1 - \frac{\sqrt{2}}{2}\frac{C_2}{C^*}\Omega_2 + \frac{\sqrt{2}}{4}\frac{C_3}{C^*}\Omega_3 \tag{16}$$

It is important to notice that the complexity of the system dramatically increases by not aligning the reaction wheels with the spacecrafts' frame of reference. The complexity can be seen by the coupling between the inputs and outputs of the entire system.

## III. FORM BASED CONTROL

The ability to systematically design a control algorithm to account for complex system dynamics will dramatically speed up the development time and reduce the number of hardware constraints. By approximating the system dynamics the algorithm is capable of compensating for unknown and time-varying parameters, nonlinearities, and the cross coupling terms in MIMO systems [1-3]. The means of approximating the system dynamics determines the accuracy and simplicity of the design. Using the form of the problem the system dynamics can be systematically and accurately approximated. A typical system can be represented by the following differential equation.

$$y^{(n)} = f(t, y, y^{(1)}, ..., y^{(n-1)}, d) + bu(t) \tag{17}$$

The differential equation is dependent on the $n^{th}$ and dominant order of the output *y*, the high frequency gain *b*, the input *u*, and the true system dynamics *f* that is also dependent on the states of the system and an unknown disturbance *d*. By monitoring the input and output to the system the dynamics can be approximated

$$f(t) = y^{(n)} - bu(t) \tag{18}$$

by subtracting the scaled input from the nth derivative of the output. The $n^{th}$ derivative is not typically available, therefore an Extended State Observer (ESO) can be used to approximate the system dynamics. The explicit form of the controller is dependent on the dominant order of the system. The given application requires a controller to precisely regulate the speed of the reaction wheels, and a controller to set the speed for each reaction wheel to correctly orient the spacecraft. Each of these controllers is based on a dominant first order system therefore the form for a first order control algorithm will be presented. The ESO

$$\dot{z} = Az + Bu + L(y - z) \tag{19}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b \\ 0 \end{bmatrix}, L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \tag{20}$$

monitors the input and output of the system, and the outputs of the observer are the filtered output and the approximation of the system dynamics. The observer gains are the high frequency gain $b$, and the observer gains $l_1$ and $l_2$. The observer gains are parameterized

$$l_1 = 2\omega_o, l_2 = \omega_o^2 \tag{21}$$

for tuning simplicity by placing the eigenvalues at one location. Once an approximation of the system dynamics is obtained it is used as a portion of the control effort

$$u = \frac{-z_2 + u_0}{b} \tag{22}$$

to compensate for the system dynamics. The inner loop of the control signal creates a unity gain system by dividing by the high frequency gain, and then subtracting the approximation of the dynamics. The result of the inner loop is a system which appears as a cascaded integrator. A front end

$$u_0 = (R - y)k_p \tag{23}$$

controller is then added for command following. The structure of the front end controller is chosen such that the algorithm can be parameterized to be critically damped [4-6]. However, the first order case does not require any additional parameterization.

## IV. SIMULATION RESULTS

The results of the simulation are very promising. The ESO was able to completely decouple the system, as described in section V, and allowed the system to be controlled with multiple SISO controllers. There are three tuning parameters for the specific controllers. The proportional front end controller had a gain of 10, the observer gain was set to 100, and the approximate high frequency gain for each system was assumed to be double the actual high frequency gain, in order to show the robustness of the design. More precise methods can be used to determine this parameter; however the objective of this paper was to demonstrate the robustness of the design. The graphs in Figure 3 show the desired profile vs. the actual angle about the x, y and z axis. The graphs in Figure 4 show the error between the actual and desired angles of the spacecraft. The graphs in Figure 5 show the four reaction wheels angular velocities.

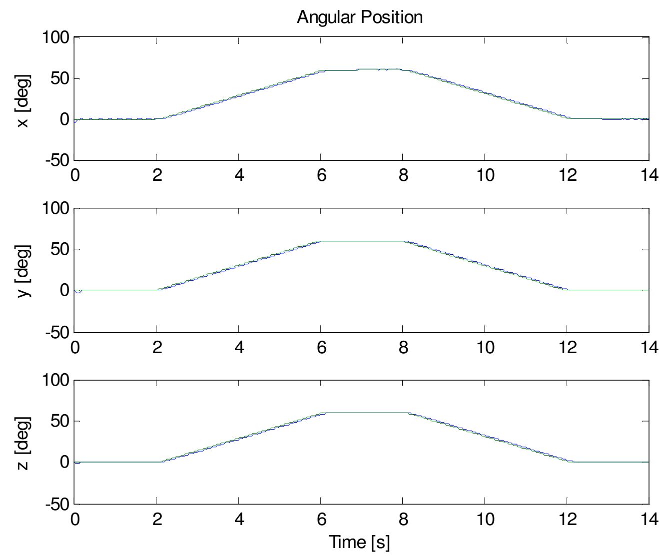


Figure 2 – Orientation of the Spacecraft

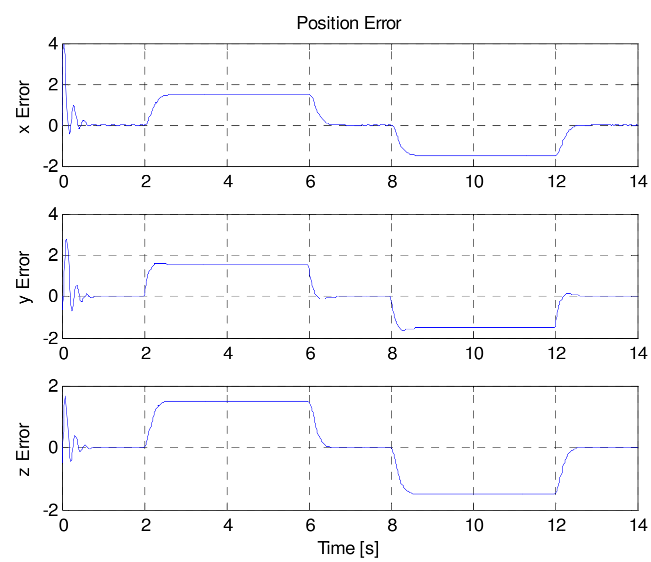


Figure 3 – Orientation Error of the Spacecraft

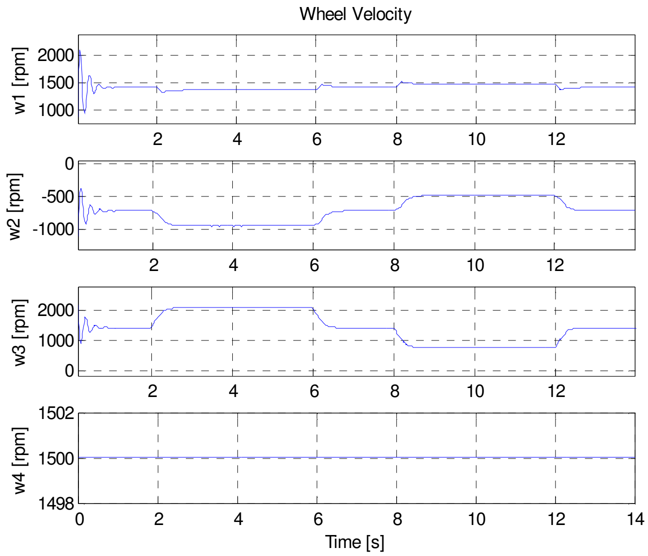


Figure 4 – Angular Velocity of the Reaction Wheels

## V. CONCLUSIONS

This paper has demonstrated that designing the control algorithm around the form of the problem can dramatically simplify the design. The control structure provides an ability to systematically compensate for difficult system dynamics which includes unknown parameters, states, and coupling between axes in a MIMO system.

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