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A Framework for the Simultaneous Clearing of Multiple Markets within a Common Transmission System

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***Abstract*—The possibility for market participants to place their bids in all the markets of an interconnection irrespective of where they are physically connected is investigated in this paper. A mechanism is proposed for managing the resulting congestion. It consists in iteratively sharing the transmission line capacities between the different market operators based each time on their present schedules. The algorithm is applied on a small test system. The results are assessed in terms of their property of being Nash equilibria and of their distance of the set of Pareto optimal operating points.**

***Index Terms*—overlapping markets, congestion management, decomposed optimization, pareto efficiency**

I. Introduction

In recent years, significant discussions and engineering effort have been concentrated on the need and the development of tools to couple individual areas of an interconnection to approach operation as a single electrical market. A driving force behind this integration is enhancing the global economic efficiency for the participants. Specially in the case of Day-Ahead Markets, which is addressed in this paper, efficient coupling of the interconnected markets can furthermore assure adequate market liquidity to each region [1], [2], [3].

For interconnections like those in Europe and North America, merging into a single large market seems for the moment practically impossible. Hence, developing methods of inter-regional coordination that allows separate control, but a seamless market from the participant's perspective, becomes a critical aspect in market design [4], [5]. In this respect, for instance, ETSO (European Transmission System Operators) and EuroPEX (Association of European Power Exchanges) have joined their efforts trying to set up a framework and come up with some applicable algorithms towards creating the Internal Electricity Market in Europe [1], [2], [3].

This paper investigates an approach so that market participants (generators, large consumers) are allowed to participate into the market of their choice, irrespective of where they are geographically located in an interconnection. This is similar to the situation in California, where market participants may choose the scheduling coordinator who will dispatch them [6].

Having different electrical markets operating within an interconnection brings up interaction issues between them, related either to the dispatching of the available energy resources or to the managing of transmission lines congestion. In this paper, an entity named Market Operator (MO) is considered to be responsible for clearing one particular market (i.e. dispatching the generators and loads that bid in this market). Furthermore, the superposition of all the MOs dispatches, that is the power injections on all buses of the interconnection, should result in line flows that do not violate any line limit. The set of line constraints is considered to bring a global set of constraints; MOs are not responsible for a specific set of lines each (i.e. there is no geographical jurisdiction for the MOs in terms of transmission infrastructure).

The paper is organized as follows. In Section II a general framework for approaching the problem is discussed. Section III presents the proposed algorithm, which then, in Section IV, is applied and assessed to a test system. Finally, the work is summarized in the Conclusion.

II. Optimization by multiple players obeying common constraints

A. Problem statement

Typically, the market clearing problem is formulated as an optimization problem where the objective is to minimize the social cost (i.e. maximize the social welfare), with each generator and load announcing a bid corresponding to their marginal cost and benefit, respectively [7]. The optimization problem considered by the i-th MO $(i = 1, \ldots, n)$ takes on the form:

$$\min_{\mathbf{u}_i} f_i(\mathbf{u}_i) \tag{1a}$$

$$\text{subject to} \quad h_i(\mathbf{u}_i) = 0 \tag{1b}$$

$$\mathbf{u}_i^{min} \leq \mathbf{u}_i \leq \mathbf{u}_i^{max} \tag{1c}$$

where the control variables $\mathbf{u}_i$ are the generator active power productions and the load active power consumptions. The equality constraint denotes the power balance within the market.

A set of constraints should be respected at the system operating point resulting from the combined clearings of the several markets. Calling $\mathbf{u}$ the vector made up of all the control

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vectors $\mathbf{u}_i$, we assume that those constraints can be expressed as a set of linear inequalities

$$\mathbf{A}\mathbf{u} - \mathbf{p}^{max} \leq \mathbf{0} \tag{2}$$

where each individual constraint corresponds to a transmission line active power flow limit ($\mathbf{p}^{max}$ is the vector containing all lines limits [1]). In this , a DC model is used to represent the entire network. The matrix $\mathbf{A}$ contains Power Transfer Distribution Factors (PTDF) linking bus power injections with the resulting line power flows [16].

Each MO's objective depends only on its allocated participating generators and loads, not on the allocation of the other MOs. In the long-term, the various MOs may compete for attracting the market actors to bid into their dispatch. In the short-term however, each market participant "belongs" to an MO, i.e. its production or consumption is a control variable of the MO.

Yet, the MOs cannot use their controls as they wish, irrespective of each other, because the results of their decisions interact through the power flow equations. Managing this congestion problem indirectly makes the value of each objective f_i dependent on the values of the other control variables $\mathbf{u}_{i^-}$, where $\mathbf{u}_{i^-}$ contains the generator productions and the load consumptions dispatched by all MOs but the i-th one.

The fact that market participants are allowed, in our approach, to place their bids in whichever MO's market makes the congestion management problem (2) even more complex.

B. Nash equilibrium

The above situation can be generally described as one where n different "players" operate a common system, and compete for the allocation of some available resources. Each player has its own strategy but the resulting overall decisions should obey some common constraints corresponding to admissible operation of components, system security, etc. [8]. Clearly, in our case, the common system is the interconnected transmission network and the constraints relate at least to power flows in individual lines or in aggregated sets of lines.

A solution defined by a control vector $\mathbf{u}^\star$, is a *Nash equilibrium* if all constraints (2) are satisfied, and no player can further improve its objective by modifying its own controls, given the control vectors of the other players [9], [10]. Thus, $\mathbf{u}^\star$ results in a Nash equilibrium if :

$$\begin{aligned} \forall i \in \{1, 2, \ldots, n\}: \qquad & \mathbf{u}_i^\star = \arg\min_{\mathbf{u}_i} f_i(\mathbf{u}_i) \\ \text{subject to} \quad & \mathbf{A}_i\mathbf{u}_i + \mathbf{A}_{i^-}\mathbf{u}_{i^-}^\star - \mathbf{p}^{max} \leq \mathbf{0} \\ & \text{and constraints} \quad (1b), (1c) \end{aligned} \tag{3}$$

where $\mathbf{u}_{i^-}^\star$ denotes the sub-vector of $\mathbf{u}^\star$ containing the controls of all players but the i-th one, and $\mathbf{A}_i$ and $\mathbf{A}_{i^-}$ are the corresponding sub-matrices of $\mathbf{A}$.

[1]Constraint (2) includes both maximum and minimum flow limits with suitable adjustment of matrix $\mathbf{A}$ and vector $\mathbf{p}^{max}$. Minimum flow limits are included to take into account bidirectional power flows.

C. System-wide single multi-objective approach

A well-known, but different approach to market integration consists in combining the various players' objectives into a single multi-objective function $F(f_1(\mathbf{u}_1), \ldots, f_n(\mathbf{u}_n))$. Minimizing simultaneously all the functions involved in $F(\cdot)$ subject to the set of constraints (1b) and (2) is referred to here as the system-wide multi-objective problem.

A solution $\bar{\mathbf{u}}$ of this problem is said to be *Pareto optimal* if it is feasible and is not dominated by any other feasible solution. This means that there is no other solution $\mathbf{u}'$ yielding at least one better objective function $f_i(\mathbf{u}')$ (i.e. $f_i(\mathbf{u}') < f_i(\bar{\mathbf{u}})$) without worsening any of the rest (i.e. $f_{i^-}(\mathbf{u}') \leq f_{i^-}(\bar{\mathbf{u}})$) [8]. Solving the multi-objective problem can give only Pareto optimal solutions. A common way to find such Pareto optimal points is the weighted sum method [11], [12], where a positively weighted convex sum of all the objective functions is minimized for different values of the weighting factors.

It is interesting to point out that a Nash equilibrium $\mathbf{u}^\star$, defined by (3), is generally not a Pareto optimal solution of the system-wide multi-objective problem. This means that there exist other settings for the controls, different from $\mathbf{u}^\star$, that improve at least some of the players' objectives without deteriorating none of the rest, making them preferable operating points than the Nash equilibrium [8].

Quite often F is taken as a linear combination of the individual objectives, thus yielding the single multi-objective optimization problem:

$$\min_{\mathbf{u}} \sum_i w_i f_i(\mathbf{u}_i) \quad \text{subject to} \quad \mathbf{A}\mathbf{u} - \mathbf{p}^{max} \leq \mathbf{0} \tag{4}$$

where the w_i's are weighting factors. This optimization problem can be solved in the following two ways.

1) Centralized scheme: The problem is solved by a central entity, applying some commonly agreed rules regarding the allocation of the common resources. Besides the high dimensionality issue, this approach has the drawback of not respecting possible confidentiality restrictions that each player wants to preserve regarding individual data and strategy.

2) Decentralized scheme: To deal with the above dimensionality and confidentiality issues, decentralized algorithms have been proposed such as those reported in [13], [14], [15]. Simply stated, the interconnected system being decomposed into separate sub-systems, each controlled by a player, the aim is to process the information of each sub-system locally, while at the same time solving the system-wide problem (4). To this purpose, a coordination entity is in charge of passing information between players and possibly performing some upper-level computation.

One practical issue when dealing with (4) is the choice of the weighting factors w_i. Indeed, the various players may question the priorities assigned to their respective objectives through these weighting factors. One option is to try different weighting factors, but this may become computationally intractable.

Normally, as far as market is of concern, all objectives correspond to costs (i.e. they are expressed in the same unit) and hence, a natural choice is to set all w_i's to 1, i.e. consider the objective:

$$F(\mathbf{u}) = \sum_i f_i(\mathbf{u}_i) \tag{5}$$

This leads to optimizing the total "social welfare of all participants" within the interconnection. A decentralized Optimal Power Flow (OPF) has been solved with this objective in [14], [15].

While this seems desirable from a global system perspective, a market operator could argue that it would have better market opportunities (higher social welfare for the market it clears) if it was not incorporated into the overall optimization. Even more, it goes with the freedom and independence of each market to be cleared separately from the others, incorporating maybe its particular rules and operating strategies. The above justify the existence of several markets, instead of a single integrated one.

D. Independent optimizations with a Coordinator

In the previous approach, a central entity is in charge of either solving the system-wide multi-objective optimization or coordinating the decentralized computations aimed at solving that problem. Alternatively, a central entity may be responsible for monitoring and correcting multiple independent optimizations performed by the n players according to certain rules. These rules will reflect a pre-defined *policy* to share the available resources among the players.

Contrary to the single system-wide optimization approach previously considered, the idea is to preserve the operational independence of the players. The players are not constrained to adopt a common objective. On the contrary, they may formulate their operational strategies in different ways. Thus, the players' independence is preserved, but with additional rules applied by the coordinator to reconcile the players' decisions.

This approach is developed in the remaining of this paper. The method consists in decoupling the optimization problems tackled by the different players by dividing the constraints among the players in such a way that each one of them respecting its part of the constraints will result in the whole, original set of constraints being satisfied. Formally, the i-th player will solve an optimization problem of the type:

$$\min_{\mathbf{u}_i} f_i(\mathbf{u}_i) \tag{6a}$$

$$\text{subject to} \quad \mathbf{A}_i\mathbf{u}_i - \mathbf{p}_i^{max} \leq \mathbf{0} \tag{6b}$$

where $\mathbf{A}_i$ is the sub-matrix of $\mathbf{A}$ containing the columns corresponding to the sub-vector $\mathbf{u}_i$ of $\mathbf{u}$, while new $\mathbf{p}_i^{max}$ limits have to be found so that:

$$\mathbf{A}_i\mathbf{u}_i - \mathbf{p}_i^{max} \leq \mathbf{0},\ \forall i \in \{1, \dots, n\} \ \Rightarrow\ \mathbf{A}\mathbf{u} - \mathbf{p}^{max} \leq \mathbf{0} \tag{7}$$

The vectors $\mathbf{p}_i^{max}$ should be adjusted by the coordinator in such a way that a well defined and transparent policy is followed to share the available resources, allowing the players to check the coordinator decisions.

These vectors could be assigned *ex ante* by the coordinator, to have the players perform n completely independent optimizations. Another option, however, is to construct "dynamically" the vectors $\mathbf{p}_i^{max}$ while observing the evolution of the successive optimizations performed by the players, allowing in some sense the coordinator to intervene in its evolution. This second option has been used since it combines flexibility of the coordination policy with an as large as possible operational freedom for the players. In this spirit, a procedure is presented in this paper where after a number of iterations between the players and the coordinator, the whole original set of constraints is satisfied by the final solution of the individual optimization problems.

III. Application to market clearing

The above discussed general idea is now applied to the congestion management problem of the overlapping markets.

A. Problem model

1) Market clearing problem: The i-th MO clears its market by solving the following economic dispatch problem:

$$\min_{\mathbf{g}_i,\mathbf{d}_i} \mathbf{c}_i^T\mathbf{g}_i - \mathbf{b}_i^T\mathbf{d}_i \tag{8a}$$

$$\text{s. t.} \quad \mathbf{1}^T\mathbf{g}_i = \mathbf{1}^T\mathbf{d}_i \tag{8b}$$

$$\mathbf{0} \leq \mathbf{g}_i \leq \mathbf{g}_i^{max} \tag{8c}$$

$$\mathbf{0} \leq \mathbf{d}_i \leq \mathbf{d}_i^{max} \tag{8d}$$

where $\mathbf{c}_i$, $\mathbf{g}_i^{max}$ and respectively $\mathbf{b}_i$, $\mathbf{d}_i^{max}$ are the vectors of bids and quantities placed in market i by respectively the generators and consumers, and $\mathbf{1}$ denotes a unit vector.

Equation (8a) is the social cost of the participants in market i, while (8b) preserves the power balance. Constraints (8c) and (8d) make sure that no generator will be asked to produce more than the capacity it declared and no load to consume more than it asked for.

2) Linear network model: In the linear network model (2), matrix $\mathbf{A}$ is obtained from the bus-to-bus PTDFs [16] by using a slack bus and expressing all bus-to-bus transactions as a pair of two: one towards and another from the slack bus. To this purpose, let us denote by $PTDF_{l,mn}$ the fraction of a 1 MW transaction from bus m to bus n that flows over a line l [2]. Furthermore, let us consider the matrix $\mathbf{A}'$ such that $A'_{l,m} = PTDF_{l,mn}$, where n is the slack bus. Multiplying $\mathbf{A}'$ with the vector of bus power injections $\mathbf{g} - \mathbf{d}$ gives the vector of line power flows $\mathbf{p}$. Matrix $\mathbf{A}$ is nothing but a suitable permutation of $\mathbf{A}'$ in order to be consistent with the order of the elements in vector $\mathbf{u}$ and to account for both the positive and negative direction of flow in a line. In other words, the vector of line active power flows produced by the allocated generation and loads (contained in the overall vector $\mathbf{u}$) is given by:

$$\mathbf{p} = \mathbf{A}\mathbf{u} \tag{9}$$

[2]This PTDF coefficient can be computed as:

$$PTDF_{l,mn} = \frac{X_{im} - X_{jm} - X_{in} + X_{jn}}{x_l}$$

where i is the origin of line l, j its extremity, x_l its reactance and X_{im} the entry in the i-th row and m-th column of the bus reactance matrix $\mathbf{X}$.

and must obey the inequality $\mathbf{p} \leq \mathbf{p}^{max}$ (this inequality considers both positive and negative flows, as already commented), which we rewrite as:

$$\mathbf{A}\mathbf{u} - \mathbf{p}^{max} \leq \mathbf{0} \tag{10}$$

In Section III-B the decomposition of the line flow constraints (10) is presented, while in Section III-C the way the coordinator adjusts the so decomposed constraints among the MOs is developed. Then, in Section III-D a specific congestion management policy is proposed.

B. Constraint decomposition

For the sake of presentation simplicity, we refer here to a case with two MOs, denoted MO1 and MO2 respectively. The generalization to more MOs is straightforward.

After partitioning the control vector, (10) is rewritten as:

$$\mathbf{A}_1\mathbf{u}_1 + \mathbf{A}_2\mathbf{u}_2 - \mathbf{p}^{max} \leq \mathbf{0} \tag{11}$$

It is easily seen that if the following constraints are satisfied:

$$\begin{aligned} &\text{by MO1:} && \mathbf{A}_1\mathbf{u}_1 - \mathbf{p}_1^{max} \leq \mathbf{0} && \text{(12a)}\\ &\text{by MO2:} && \mathbf{A}_2\mathbf{u}_2 - \mathbf{p}_2^{max} \leq \mathbf{0} && \text{(12b)}\\ &\text{where:} && \mathbf{p}_1^{max} + \mathbf{p}_2^{max} = \mathbf{p}^{max} && \text{(12c)} \end{aligned}$$

then the overall constraints (11) are also satisfied. The constraints (12a) and (12b) are of the type (7).

Consider now the j-th constraint in (11), with the corresponding components $p_{1j}^{max}, p_{2j}^{max}$ and p_j^{max} of the $\mathbf{p}_1^{max}, \mathbf{p}_2^{max}$ and $\mathbf{p}^{max}$ vectors, respectively. Clearly, $p_{1j}^{max} + p_{2j}^{max} = p_j^{max}$. It can be guessed that the values of p_{1j}^{max} and p_{2j}^{max} determine how much of the resource (transmission line capacity) is being allocated to MO1 and MO2 respectively. For instance, for a higher value of p_{1j}^{max}, MO1 may be less constrained and a higher control effort will be put on MO2 to satisfy the j-th constraint, and conversely. Thus, the coordinator may implement the agreed congestion management policy by suitably choosing the values p_{ij}^{max} for a congested line j. Furthermore, the coordinator should share the limited resource in a transparent way, that is, its choice should be justified by information that can be made public to all involved MOs.

Note that a solution $(\mathbf{u}_1, \mathbf{u}_2)$ which satisfies (12) will satisfy the original constraints (11). However, the converse is not true: it is possible to find controls $\mathbf{u}_1$ and $\mathbf{u}_2$ satisfying (11) but not both (12a) and (12b). Thus the use of (12) somewhat reduces the feasible space of the original optimization problem. This is a price to pay for the convenience of the decomposition into independent optimizations.

This reduction of the feasible space, however, should be as low as possible. To this purpose, a procedure is proposed that iteratively adjusts the values of $\mathbf{p}_1^{max}$ and $\mathbf{p}_2^{max}$, while converging towards a solution satisfying (10).

C. Adjustment of constraints by the coordinator

Assume that, in a first step, the two MOs optimize their objective functions without taking care of the constraints; let $\widehat{\mathbf{u}}_1$ and $\widehat{\mathbf{u}}_2$ be the corresponding controls. Assume furthermore that the j-th constraint in (10) is violated by these settings, i.e.

$$\mathbf{a}_{1j}\widehat{\mathbf{u}}_1 + \mathbf{a}_{2j}\widehat{\mathbf{u}}_2 - p_j^{max} - \delta_j = 0 \tag{13}$$

where $\mathbf{a}_{1j}$ and $\mathbf{a}_{2j}$ are the j-th rows of matrices $\mathbf{A}_1$ and $\mathbf{A}_2$, respectively, and $\delta_j > 0$ is the amount by which line j is overloaded. New controls $\mathbf{u}_1$ and $\mathbf{u}_2$ are sought, such that:

$$\mathbf{a}_{1j}\mathbf{u}_1 + \mathbf{a}_{2j}\mathbf{u}_2 - p_j^{max} \leq 0 \tag{14}$$

Subtracting (13) from (14) gives:

$$\mathbf{a}_{1j}(\mathbf{u}_1 - \widehat{\mathbf{u}}_1) + \mathbf{a}_{2j}(\mathbf{u}_2 - \widehat{\mathbf{u}}_2) + \delta_j \leq 0 \tag{15}$$

Let the amount δ_j be shared over the two MOs according to:

$$\delta_j = \alpha_1\delta_j + \alpha_2\delta_j \qquad \text{with } \alpha_1 + \alpha_2 = 1 \tag{16}$$

where the choice of the α_1 and α_2 coefficients reflects the coordinator's policy regarding the treatment of the constraint. Introducing (16) into (15) yields:

$$\mathbf{a}_{1j}\mathbf{u}_1 + \alpha_1\delta_j - \mathbf{a}_{1j}\widehat{\mathbf{u}}_1 + \mathbf{a}_{2j}\mathbf{u}_2 + \alpha_2\delta_j - \mathbf{a}_{2j}\widehat{\mathbf{u}}_2 \leq 0 \tag{17}$$

This inequality suggests the following decomposition of the j-th constraint in accordance with (12):

$$\begin{aligned} &\text{for MO1:} && \mathbf{a}_{1j}\mathbf{u}_1 + \alpha_1\delta_j - \mathbf{a}_{1j}\widehat{\mathbf{u}}_1 \leq 0 && \text{(18a)}\\ &\text{for MO2:} && \mathbf{a}_{2j}\mathbf{u}_2 + \alpha_2\delta_j - \mathbf{a}_{2j}\widehat{\mathbf{u}}_2 \leq 0 && \text{(18b)} \end{aligned}$$

This is equivalent to setting:

$$\begin{aligned} p_{1j}^{max} &= \mathbf{a}_{1j}\widehat{\mathbf{u}}_1 - \alpha_1\delta_j && \text{(19a)}\\ p_{2j}^{max} &= \mathbf{a}_{2j}\widehat{\mathbf{u}}_2 - \alpha_2\delta_j && \text{(19b)} \end{aligned}$$

It is easily checked that $p_{1j}^{max} + p_{2j}^{max} = p_j^{max}$.

Generalizing, irrespective of the number of MOs, for each overloaded line corresponding to a constraint j, the coordinator should choose the coefficients α_{ij}, with $\sum_i \alpha_{ij} = 1$. As a result, the line capacity will be shared among the MOs, the i-th one receiving a modified bound p_{ij}^{max}, with $\sum_i p_{ij}^{max} = p_j^{max}$.

Now, if the i-th MO solves its market clearing problem (8) with the additional constraint:

$$\mathbf{a}_{ij}\mathbf{u}_i - p_{ij}^{max} \leq 0 \tag{20}$$

then, the new overall solution $\widehat{\mathbf{u}}$ will be such that the j-th constraint will be satisfied. Now, other constraints may be found violated by the new solution. If so, the coordinator will in the same way share their transmission capacities among the MOs which, in their turn, will clear their markets incorporating the new constraints. In order not to get violated again in the remaining of the procedure, each constraint j found violated once should remain in the set of constraints decomposed by the coordinator and incorporated into the MOs' clearings at subsequent iterations. If a constraint is no longer violated, δ_j will obviously be negative (or equal to zero) but this does not affect the validity of the formula used for sharing the transmission capacity.

In summary, at every iteration k of the algorithm, the coordinator collects the MOs' control vectors $\mathbf{u}_i^{(k)}$, identifies the resulting line overloads needing corrections $\delta_j^{(k)}$ and the remaining available capacities of lines that have been overloaded in a previous iteration, and, decides the coefficients

$\alpha_i^{(k)}$ that define the next share of the line capacities by the various MOs.

Note that as long as a line does not get overloaded, the coordinator does not impose any constraint to the MOs.

As already mentioned, the above decoupling method reduces the space searched for possible solutions of the system-wide problem. On the other hand, the decoupling does not only seek to separate the constraints in a consistent way that allows independent market clearings, but also implements a specific policy for the sharing of the available transmission capacity among the MOs. The iterative procedure sequentially enforces constraints based on the congestion management policy.

D. Congestion management policy

The policy proposed and tested in this paper consists in sharing the effort needed to alleviate a line overload in proportion to the responsibility that each MO has for the existence of the overload.

Due to the linear model used for the representation of the network, this responsibility is easily derived from the power flow equation (9) after decomposing the vector $\mathbf{u}$. Indeed (9) can be rewritten as:

$$\mathbf{p} = \sum_i \mathbf{A}_i \mathbf{u}_i \tag{21}$$

which gives the power flows caused by the i-th MO:

$$\mathbf{p}_i = \mathbf{A}_i \mathbf{u}_i \tag{22}$$

So, if at some step of the procedure, the present solution $\widehat{\mathbf{u}}$ causes a flow $\widehat{p}_j > p_j^{max}$ (and hence $\delta_j = \widehat{p}_j - p_j^{max} > 0$), then the i-th MO, which creates a flow $\widehat{p}_{ij}$ according to (22), can considered responsible for the fraction $\widehat{p}_{ij}/\widehat{p}_j$ of the overload.

Thus, the coefficient that the coordinator will use for the i-th MO when decomposing the constraint will be:

$$\alpha_{ij} = \frac{\widehat{p}_{ij}}{\widehat{p}j} \tag{23}$$

Substituting (23) into (19) gives:

$$p_{ij}^{max} = \widehat{p}_{ij} - \frac{\widehat{p}_{ij}}{\widehat{p}_j}(\widehat{p}_j - p_j^{max}) \tag{24}$$

$$\text{or} \quad p_{ij}^{max} = \frac{\widehat{p}_{ij}}{\widehat{p}_j} p_j^{max} \tag{25}$$

This means that the policy of attributing the correction effort according to the responsibility of the MO is equivalent to sharing the line capacity proportionally to the use each MO is making of the line.

E. Graphical representation

In this subsection, the described decomposition of the set of linear constraints, as well as the iterative adjustment of the decomposed constraints are illustrated in a graphical way. To this purpose, a two-MO case with one control variable per MO (u_1 and u_2 respectively) is assumed. Each constraint j is a linear combination of the two controls: $a_{1j}u_1 + a_{2j}u_2 + b_j \leq 0$.

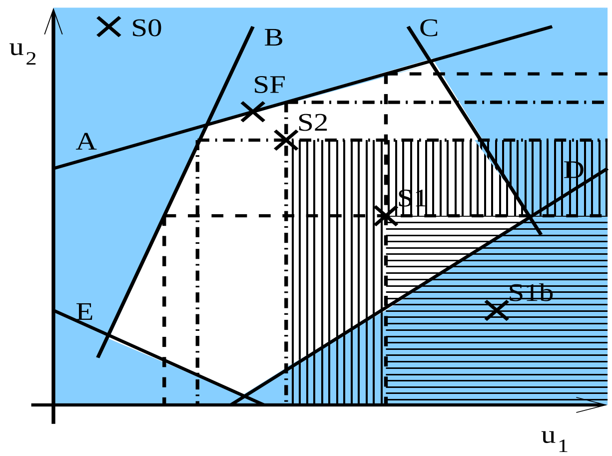

Fig. 1: Graphical illustration

In Fig. 1 the feasible region corresponding to five such constraints is presented (non colored area). It is not possible to construct the same region by constraints that involve either only u_1 or only u_2.

Let us assume that the solution resulting from the independent market clearings violates two of the constraints (point S0 and constraints A and B in Fig. 1). This infeasibility initiates the iterative algorithm and each of the two constraints is decomposed following the congestion management policy. This results into two constraints being communicated to each MO, one for each overloaded line: $a_{1A}u_1 + b_{1A} \leq 0$ to MO1 and $a_{2A}u_2 + b_{2A} \leq 0$ to MO2 for line A (vertical and respectively horizontal dashed lines starting from a point on A), $a_{1B}u_1 + b_{1B} \leq 0$ to MO1 and $a_{2B}u_2 + b_{2B} \leq 0$ to MO2 for line B (similarly, dashed lines starting on B). Each pair of these constraints guarantees that at the next iteration the original constraint will be satisfied while they share the corresponding available transmission capacity between the two MOs. Note that the non violated constraints remain "invisible" to the MOs; the searched space is not reduced unless a constraint violation is encountered. In fact, the searched space for the next solution is the intersection of the above decomposed constraints and is highlighted with horizontal lines in the figure.

Let the point S1 in Fig. 1 be the new solution that results from the next iteration. As this solution happens to be feasible, it could be chosen to be actually implemented and the procedure could stop here. However, in order to give the MOs the opportunity to improve their schedules, constraints A and B are once more decomposed, based on the present operating point (S1), again according to the congestion management policy. The dashed-dotted lines in Fig. 1 indicate this new decomposition. One can see that the searched space for the new solution has now been enlarged by the area shown with vertical lines in the figure. This results in a new solution (point S2). The procedure continues like this, finally converging to the point SF where one of the two initially violated constraints is active.

It is noteworthy that the coordinator has not as objective to guarantee the feasibility of the next iteration's solution. It just checks for convergence and shares the capacity of the already overloaded lines. If at any step of the algorithm a new line gets overloaded, the corresponding constraint will be also subsequently decomposed among the MOs. Coming back to the example of Fig. 1, if S1b had been the solution after iteration 1, then constraint D would have been also decomposed and communicated to the MOs, obliging them to provide solutions above (for MO-2) and on the left (for MO-1) of the two new decomposed constraints, making the searched space be a rectangle.

F. Iterating between market operators and coordinator

The procedure by which the coordinator iteratively enforces constraints to the MOs, based on its congestion management policy, can be summarized as follows:

1) the MOs solve their optimization problems without taking into account any common constraints and come up with their individual controls;
2) the resulting flows are computed by the coordinator;
3) for those line flows exceeding their limits, decomposed constraints are constructed according to (20), (25);
4) the MOs clear again their individual markets, incorporating the constraints received from the coordinator;
5) when new constraints are violated, new decomposed constraints are added to the MO problems, while all previously violated constraints are decomposed again, each time based on the present solution;
6) the procedure terminates when an equilibrium is reached.

G. Nash equilibrium property of the solution

It is important for the algorithm to provide solutions that are Nash equilibria of the original uncoordinated problem, defined by each MO clearing independently its market as in (3). The reason is that this makes the final point acceptable by everybody, since nobody has the power to modify it (for its own profit) by its own means only.

This can be visualized in Fig. 1, where point SF denotes the final solution of the algorithm. No MO can, modifying its control, improve its objective (assuming that MO-1 tries to decrease u_1 and MO-2 t increase u_2 as suggested by the example) without violating the problem's original constraints (in particular constraint A). This makes SF a Nash equilibrium.

This is, indeed, a general property of the algorithm as will be briefly explained in the sequel.

Let us recall that even if no line is overloaded at a given iteration (no $\delta_j > 0$) the procedure continues, sharing to the MOs the remaining capacities of the previously overloaded lines according to the congestion policy, until no change in flows is encountered between two subsequent iterations. Hence, at the final solution, all lines fall into one of the three categories: 1. they have never been overloaded; 2. their capacity is totally used ($\delta_j = 0$); or 3. they have been overloaded but, finally, their capacity is not fully used ($\delta_j < 0$). The third case may happen if a line flow is limited as a side effect of the effort to unload another line.

For the fully used lines, it can be shown using (13) and (19) that the corresponding inequality constraint in (3) is the same as the constraint (20) at the equilibrium of the proposed coordinated algorithm. The other constraints in (3) do not affect the solution obtained at the last iteration of the proposed algorithm, since they are not binding. So, they should not affect the solution of problem (3) either. As a result, the solution obtained by each MO when solving (3) with the other controls fixed to the values of the final solution of the algorithm, is to keep itself the same control settings. This by definition makes this solution a Nash equilibrium of the original uncoordinated problem.

IV. Results and discussion

A. Test system

The iterative algorithm has been tested on a small 15-bus system, divided into 3 areas-markets, each of which is supposed to be cleared by a different MO (see Fig. 2). The costs-bids of the generators have been adapted to create areas with cheap and others with more expensive generation. The load of each area is assumed inelastic and, thus, the MOs compete for the allocation of the most interesting generators. All areas have the same amount of load to serve (600 MW).

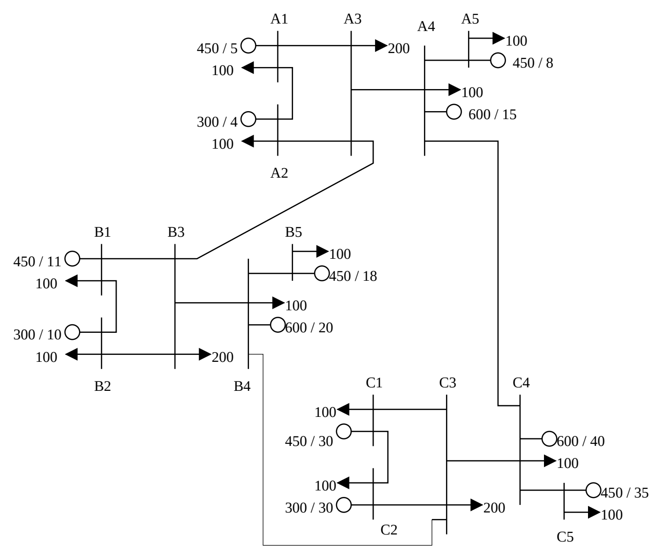

Fig. 2: 3-area test system

In Fig. 2, each area is denoted by a letter (A, B and C). Next to each generator, its maximum production capacity (in MW) as well as its bid (in euros/MWh) are shown. Each generator capacity has been divided by three, i.e. each generator bids one third of its capacity to every MO. For the sake of clarity, the same bid per generator has been placed to all the MOs. Generally, it is the choice of each generator how much of its capacity it offers to every market and at what price (a generator may bid differently to different markets).

B. Schedules by system-wide and proposed algorithm

It took 9 iterations for the algorithm to converge to the final solution. This is presented in Tables I and II, where the generation schedules per MO and the resulting flows in the congested and tie-lines are shown, respectively. At the final solution no line is overloaded, but the capacities of some of

TABLE I: Final point; generation chosen by each MO

Gen	bid	MO-A	MO-B	MO-C	Total
gA1	5	134	99	17	250
gA2	4	96	59	95	250
gA4	15	94	0	0	94
gA5	8	150	80	0	230
gB1	11	26	100	124	250
gB2	10	100	100	50	250
gB4	20	0	12	0	12
gB5	18	0	150	114	264
gC1	30	0	0	28	28
gC2	30	0	0	100	100
gC4	40	0	0	0	0
gC5	35	0	0	72	72

TABLE II: Final point; resulting flows

Line	p_A	p_B	p_C	p	p^{max}
A1A3	21	85	44	150	150
A2A3	9	73	68	150	150
B1B3	52	0	98	150	150
B2B3	74	0	76	150	150
A3B3	-112	178	-42	24	200
A4C4	-14	60	154	200	200
B4C3	14	-60	246	200	200

TABLE III: System-wide market clearing

Gen	bid	MO-A	MO-B	MO-C	Total
gA1	5	150	50	50	250
gA2	4	100	100	50	250
gA4	15	0	0	0	0
gA5	8	150	150	0	300
gB1	11	0	100	150	250
gB2	10	50	100	100	250
gB4	20	0	0	0	0
gB5	18	150	0	150	300
gC1	30	0	0	0	0
gC2	30	0	100	100	200
gC4	40	0	0	0	0
gC5	35	0	0	0	0

TABLE IV: System-wide market clearing; resulting flows

Line	p_A	p_B	p_C	p	p^{max}
A1A3	33	67	50	150	150
A2A3	17	83	50	150	150
B1B3	17	0	133	150	150
B2B3	33	0	117	150	150
A3B3	-133	225	-92	0	200
A4C4	67	-175	308	200	200
B4C3	-67	75	192	200	200

TABLE V: Costs comparison

Cost:	MO-A	MO-B	MO-C	Total
system-wide clearing	5550	6950	8800	21300
proposed algorithm	4950	6412	10740	22102

them are fully used. In Table II, the participation of each MO's schedule to the line flows is also detailed (columns 2-4).

In Tables III and IV, the schedules and the flows resulting from a system-wide common market clearing are presented. It has been obtained by optimizing the social cost (5) of the entire interconnection, subject to the constraints (2), as well as three equality constraints of the type (8b), each of them ensuring the power balance in the corresponding MO's market.

The congestion management policy is highlighted by comparing Tables I, II with Tables III, IV. During the iterations, MO-C is forced to allocate more expensive generation into its own area in order to alleviate, proportionally to its responsibility (Eq. (25)), the congestion appearing in the tie-lines A4C4 and B4C3 (both importing into area C). On the contrary, when the problem is solved as a single optimization, the allocation of generators is made in such a way that, by properly creating some counterflows, the use of more expensive generators in area C is decreased. Table V shows the impact on the costs of the MOs. In the case of the system-wide clearing, the cost of MO-C is smaller than its cost at the final solution of the proposed algorithm, while the costs of MO-A and MO-B are larger. This however results in an overall smaller total cost.

C. Assessing the final solution

It is of interest to examine the properties of the operating point finally reached by the algorithm, in particular whether it is a desirable one to operate at.

In subsection III-G, this operating point was shown to be Nash equilibrium of the original uncoordinated problem. In order for the participants to adhere to such an algorithm, they have to be convinced that the final result will be fair and will exploit in the best possible way the transfer capacity of the electric network. Thus, let us here examine the quality of this Nash equilibrium.

To this purpose, the Pareto efficiency of the final point has been checked. Given an operating point defined by the generation schedules $\mathbf{g}_A, \mathbf{g}_B, \mathbf{g}_C$, with resulting costs C_A, C_B, C_C, a way to check whether this is Pareto optimal is to solve the system-wide market clearing problem described in the previous subsection, with the following three additional constraints:

$$\mathbf{c}_i^T \mathbf{g}_i \leq C_i \ , \ \ i \in \{A, B, C\} \tag{26}$$

Let us call this the Pareto Efficiency Optimization Problem (PEOP).

The proposed method provides a feasible solution, where:

$$\mathbf{c}_i^T \mathbf{g}_i = C_i \ , \ \ i \in \{A, B, C\} \tag{27}$$

So, if the outcome of PEOP satisfies Eq. (27) (this may happen even with a schedule different than $(\mathbf{g}_A, \mathbf{g}_B, \mathbf{g}_C)$), then the equilibrium point of the proposed algorithm is a Pareto optimal one. Otherwise, at the solution $\overline{\mathbf{g}}$ of PEOP, at least one of the inequalities in (26) is a strict one ($\mathbf{c}_i^T \overline{\mathbf{g}}_i < C_i$), which means that there exists (at least) one solution that decreases at least one of the cost functions without increasing any of the others; so the equilibrium point is not a Pareto optimal one.

Figure 3 illustrates this discussion, in a two-dimensional example. A solution inside the colored area dominates the Nash equilibrium, since both objectives are better off there. On the contrary, a solution outside that area cannot be considered "better" than the Nash equilibrium, since there one of the involved MOs is worse off than at the Nash solution.

It turned out that the final solution of the iterative procedure is not a Pareto optimal point. In Table VI the resulting costs are compared. Obviously, if that PEOP solution could be

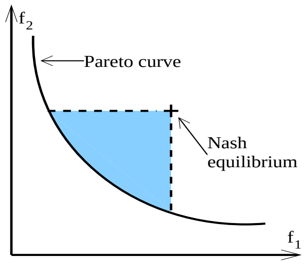

Fig. 3: Nash equilibrium compared to the Pareto set

TABLE VI: Cost comparison with a Pareto point

Cost:	MO-A	MO-B	MO-C	Total
PEOP	4900	6348	10052	21300
proposed algorithm	4950	6412	10740	22102

TABLE VII: Pareto points

w_A	w_B	w_C	$\mathbf{c}_A^T\mathbf{g}_A$	$\mathbf{c}_B^T\mathbf{g}_B$	$\mathbf{c}_C^T\mathbf{g}_C$	Total Cost
1.0	0.0	0.0	4450	6348	10633	21431
0.4	0.3	0.3	4450	6348	10502	21300
0.3	0.4	0.3	4900	5767	10633	21300
0.0	1.0	0.0	4900	5767	10633	21300
0.3	0.3	0.4	4900	6348	10052	21300
0.0	0.0	1.0	4900	6348	10052	21300

implemented, it would be for the profit of all MOs, since it dominates the solution of the proposed algorithm. However, finding this point has been made possible only after assembling together, into a single problem, all the private information of the MOs, which would not preserve the independence of the different markets.

The system-wide market clearing solution (see Table V) is also Pareto optimal. However, it cannot be judged "better" than the outcome of the proposed algorithm because it is not a simultaneous improvement of all the MOs' social costs.

Finally, using (4) as an objective instead of (5) and varying the factors w_i, gave more points dominating the equilibrium solution. However, the one presented in Table VI turned out to be the only one where all three MO social costs are simultaneously decreased. In order to find more generation schedules that improve all three objectives, Eq. (26) has been modified to the following:

$$\mathbf{c}_i^T\mathbf{g}_i \leq \alpha C_i, \quad \text{with} \quad \alpha < 1 \tag{28}$$

For $\alpha < 0.99$ the optimization problem turned out to be infeasible. This shows how close to the Pareto set is the solution of the proposed algorithm. In Table VII some results for $\alpha = 0.99$ are presented for different weighting factors w_i. A minimum reduction of 1% is guaranteed for all costs in all cases, while, depending on the relative values of the weighting factors, some costs may be further decreased.

V. Conclusion

The possibility for market participants to place their bids into the market of their choice, irrespective of where they are geographically located in an interconnection, has been investigated in this paper. Different approaches to face the situation have been presented, while an iterative algorithm has been proposed to deal with the resulting congestion management problem, keeping at the same time the operational independence of the different markets. The algorithm implements a specific policy for managing congestions, according to which the involved MOs are asked to participate to the overload alleviation in proportion to their participation on the line loading.

The outcome of the procedure has been illustrated on a small system. The resulting solution has been assessed in two ways. First, its property of being a Nash equilibrium has been shown, and, second, its proximity to the set of Pareto optimal solutions has been checked with satisfactory results, since it turned out that, even by assembling all the originally private information together and solving a single optimization problem, the MOs social costs can be improved simultaneously by only 1%.

Further research is ongoing, addressing the possibility of replacing the DC model with a full AC model of the interconnection and including $N-1$ security considerations in the algorithm. Alternative congestion management policies could also be envisaged and tested. Finally, the possibility of shifting unused generators from one MO to another during the execution of the algorithm could be contemplated.

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