Biography of Fangxing (Fran) Li

- **B.S. (1994) and M.S. (1997) degrees from Southeast University, China**
- **Ph.D. (2001) degree from Virginia Tech, USA**

- **Senior Engineer, 02/2001 ~ 03/2005, ABB Consulting, Raleigh, NC, USA**
- **Principal Engineer, 04/2005 ~ 08/2005, ABB Consulting, Raleigh, NC, USA**
- **Assistant Professor, 08/2005 ~ present, The University of Tennessee, Knoxville, TN, USA**

- **Registered Professional Engineer in North Carolina (NC)**
- **IEEE Senior Member, 2005**
- **UT Eta Kappa Nu Outstanding Teacher Award, 2006**

DCOPF-Based LMP Simulation: Algorithm, Comparison with ACOPF, and Sensitivity

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Outlines

- Introduction
- LMP simulation with DC based model
- Comparison with AC based model
- Sensitivity of LMP
- Conclusions

Introduction

LMP simulation with DC based model (DCOPF)

- Locational Marginal Pricing is the dominant pricing method in many US electricity markets
  - PJM, NYISO, ISO-New England, California ISO, ERCOT, & MISO.
- LMP simulation based on DC model
  - Based on production cost minimization model (economic dispatch)
  - Fast and efficient for chronological simulation for market-based planning
  - Employed by market simulators from leading vendors like ABB, GE, etc.

LMP simulation based on DC model

DCOPF-based LMP simulation: lossless

- Lossless model

$$\text{Min} \sum_{i=1}^{N} c_i \times G_i$$

$$\text{s.t.} \quad \sum_{i=1}^{N} G_i = \sum_{i=1}^{N} D_i$$

$$\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) \le Limit_k \text{ , for } k = 1, 2, \ldots, M$$

$$G_i^{\min} \le G_i \le G_i^{\max} \text{, for } i = 1, 2, \ldots, N$$

GSF_{k-i} = generation shift factor to line k from Bus i.

$$LMP_B = \lambda + \left(\sum_{k=1}^{M} \mu_k \times GSF_{k-B} \right)$$

DCOPF-based LMP simulation: with losses

- **With Loss Model**

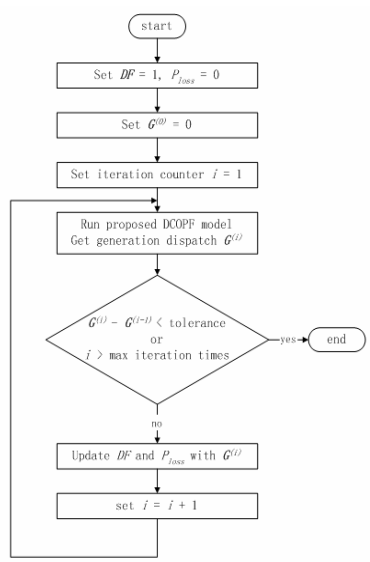
$$\text{Min} \sum_{i=1}^{N} c_i \times G_i$$

$$\text{s.t.} \quad \sum_{i=1}^{N} DF_i^{est} \times G_i - \sum_{i=1}^{N} DF_i^{est} \times D_i + P_{loss}^{est} = 0$$

$$\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) \le Limit_k \text{ , for } k \in \{\text{all lines}\}$$

$$G_i^{\min} \le G_i \le G_i^{\max} \text{, for } i \in \{\text{all generators}\}$$

$$DF_i = 1 - LF_i = 1 - \frac{\partial P_{Loss}}{\partial P_i}$$



DCOPF-based LMP simulation: about DF

- **Discussion on delivery factors (DF)**

$$DF_i = 1 - LF_i = 1 - \frac{\partial P_{Loss}}{\partial P_i}$$

$$\frac{\partial P_{Loss}}{\partial P_i} = \frac{\partial}{\partial P_i}\left(\sum_{k=1}^{M} F_k^2 \times R_k\right)$$

$$F_k = \sum_{j=1}^{N} GSF_{k-j} \times \left(G_j - D_j\right) = \sum_{j=1}^{N} GSF_{k-j} \times P_j$$

$$\frac{\partial P_{Loss}}{\partial P_i} = \sum_{k=1}^{M} \frac{\partial}{\partial P_i}\left(F_k^2 \times R_k\right) = \sum_{k=1}^{M} R_k \times 2F_k \times \frac{\partial F_k}{\partial P_i} = \sum_{k=1}^{M} 2 \times R_k \times GSF_{k-i} \times \left(\sum_{j=1}^{N} GSF_{k-j} \times P_j\right)$$

LMP Decomposition

- **From augmented Lagrangian formulation**

$$\psi = \left(\sum_{i=1}^{N} c_i \cdot G_i\right) - \lambda\left(\sum_{i=1}^{N} DF_i \cdot G_i - \sum_{i=1}^{N} DF_i \cdot D_i + P_{loss}\right) - \sum_{k=1}^{M} \mu_k \left(\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) - Limit_k\right)$$

$$LMP_B = \frac{\partial \psi}{\partial D_B} = \lambda \cdot DF_B + \left(\sum_{k=1}^{M} \mu_k \times GSF_{k-B}\right)$$

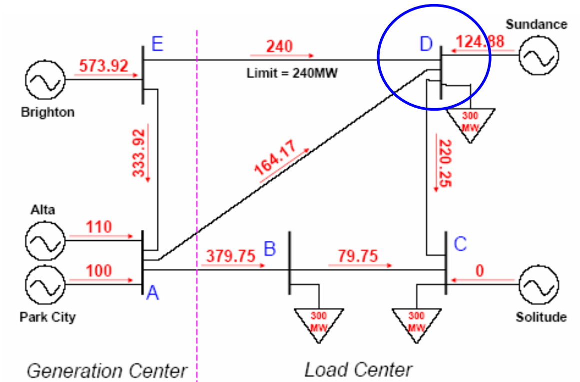
$$= \lambda + \left(\sum_{k=1}^{M} \mu_k \times GSF_{k-B}\right) + \lambda\left(DF_B - 1\right)$$

Energy Price | Congestion Price | Loss Price

A little problem here …

- **All losses absorbed by the reference bus.**

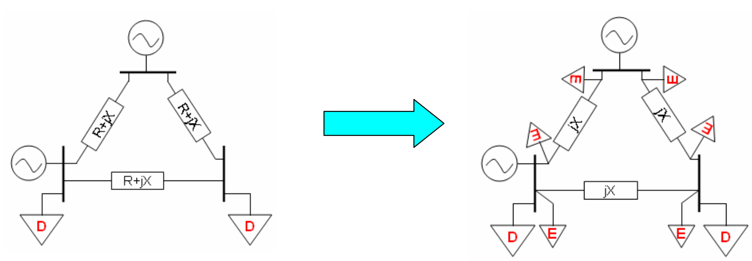
$$\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) \le Limit_k$$



Imbalance
=8.8 MW
=total (G-D)

FND model

- **Fictitious Nodal Demand (FND) model**

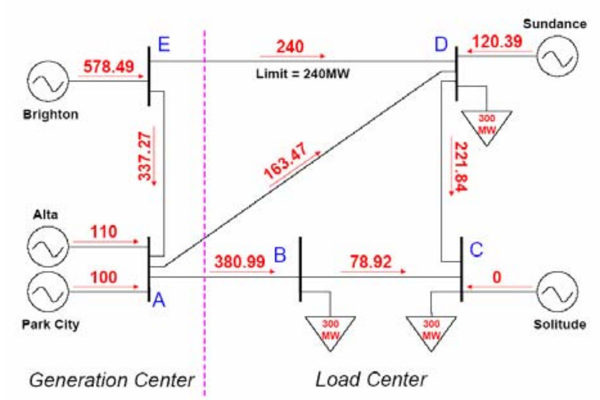


$$E_i = \sum_{k=1}^{M_i} \tfrac{1}{2} \times F_k^2 \times R_k$$

$$F_k = \sum_{j=1}^{N} GSF_{k-j} \times (G_j - D_j - E_j^{est}) \le Limit_k$$

A brief result with FND model

- **Losses are now distributed into each line**



Comparison with AC based model

ACOPF

LMPs are the Lagrange multipliers

$$\text{Min} \sum c_{Gi} \times P_{Gi}$$

Subject to:

$$P_{Gi} - P_{Li} - P(V,\theta) = 0 \quad \text{(Real power balance)}$$

$$Q_{Gi} - Q_{Li} - Q(V,\theta) = 0 \quad \text{(Reactive power balance}$$

$$F_k \le F_k^{\max} \quad \text{(Line flow MVA limits)}$$

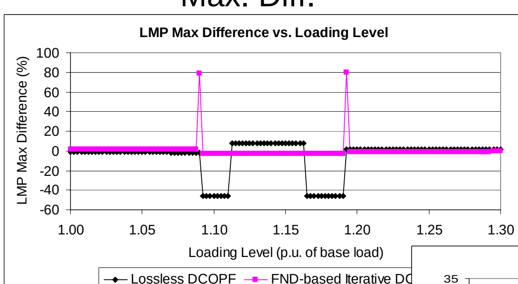
$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \quad \text{(Gen. real power limits)}$$

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \quad \text{(Gen. reactive power limits)}$$

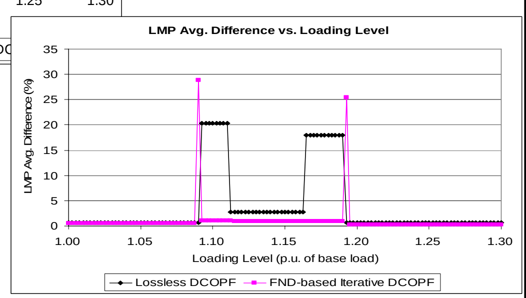
$$V_i^{\min} \le V_i \le V_i^{\max} \quad \text{(Bus voltage limits)}$$

Test on the PJM 5-bus system (1)

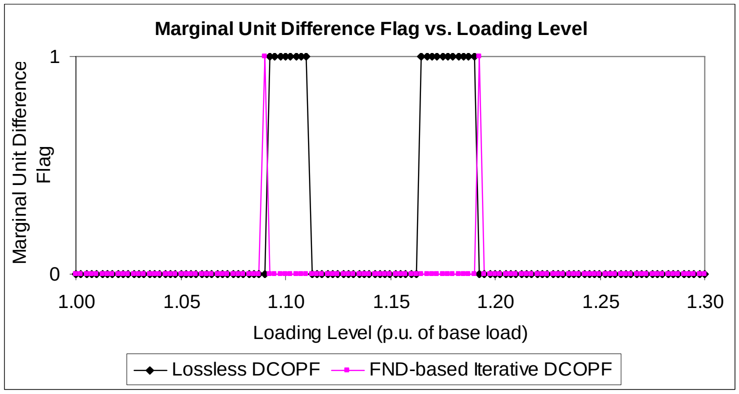
Max. Diff.



Avg. Diff.



Test on the PJM 5-bus system (2)



1) LMP difference is caused by the different set of marginal units (MU).
2) A better approximation algorithm should have a narrower range of incorrect identification of MU.

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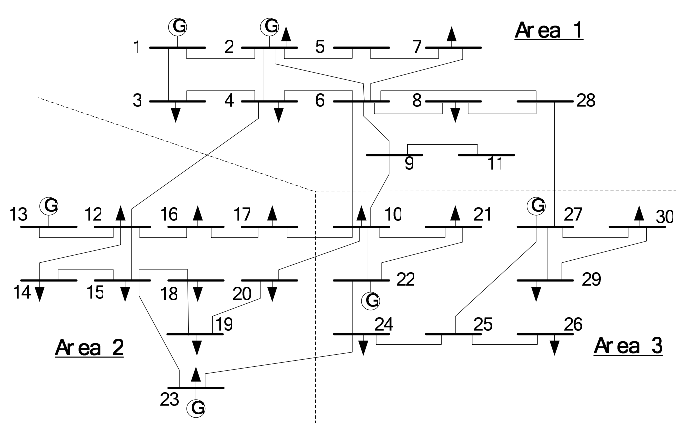
Test on the IEEE 30-bus system (1)



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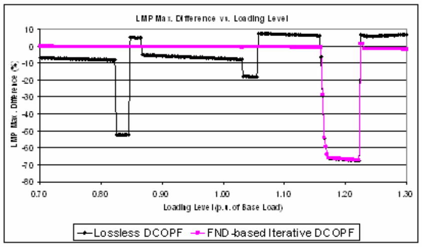
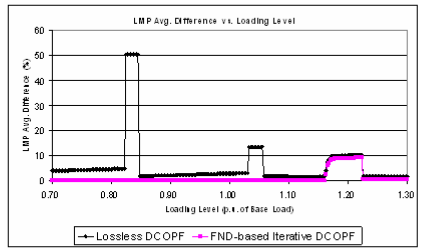
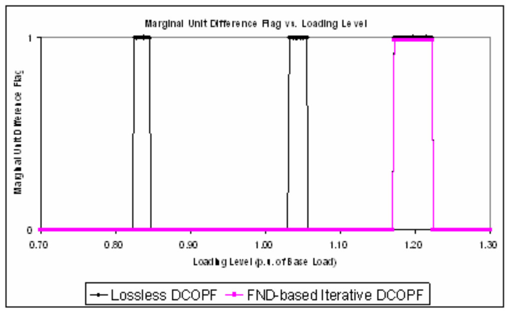
Test on the IEEE 30-bus system (2)

Max. Diff.



Avg. Diff.





Sensitivity Analysis

Sensitivity to load variation

- **No loss**

$$\frac{\Delta LMP_i^{no\_loss}}{\Delta D_j} = \frac{\Delta \lambda}{\Delta D_j} + \sum_{k=1}^{M} \frac{\Delta \mu_k}{\Delta D_j} \times GSF_{k-i} = 0$$

- **With losses**

$$\frac{\Delta LMP_i}{\Delta D_j} = \frac{\Delta\left(DF_i \cdot \lambda + \sum_{k=1}^{M} \mu_k \cdot GSF_{k-i} \right)}{\Delta D_j}$$

$$\frac{\Delta LMP_i}{\Delta D_j} = \frac{\Delta DF_i}{\Delta D_j} \cdot \lambda + \frac{\Delta \lambda}{\Delta D_j} \cdot DF_i + \sum_{k=1}^{M} \frac{\Delta \mu_k}{\Delta D_j} \cdot GSF_{k-i}$$

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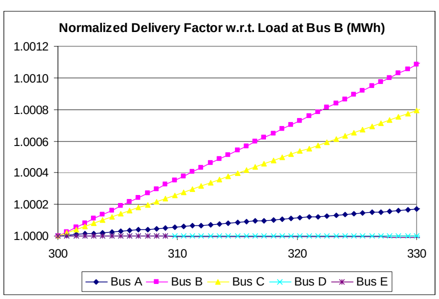
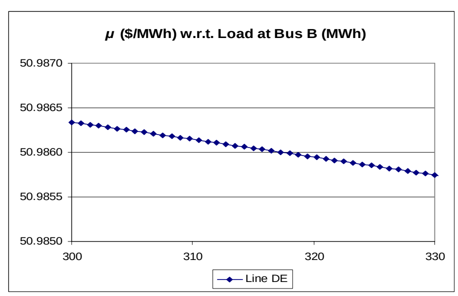
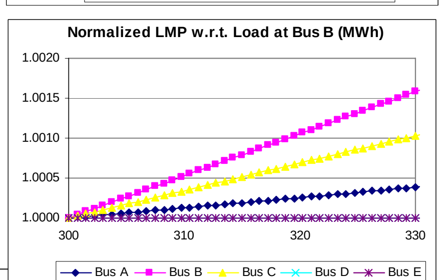
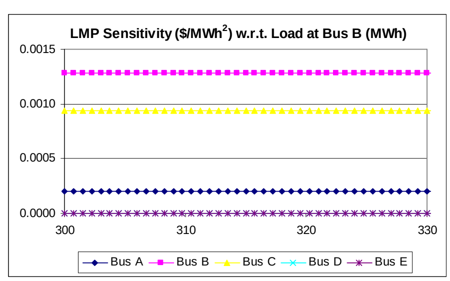
Test on the PJM 5-bus system









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Verification

$$\text{Actual } \frac{\Delta DF_i}{\Delta D_j}\cdot\lambda = 3.647\times10^{-5}\times35 = 1.277\times10^{-3}\ (\$/\text{MWh}^2)$$

$$\frac{\Delta\lambda}{\Delta D_j}\cdot DF_i = 0\ (\$/\text{MWh}^2)$$

$$\frac{\Delta\mu_{DE}}{\Delta D_j}\cdot GSF_{DE-i} = \frac{(50.9857 - 50.9863)}{30}\times(-0.2176)$$

$$=0.004\times10^{-3}\ (\$/\text{MWh}^2)$$

$$\frac{\Delta DF_i}{\Delta D_j}\cdot\lambda + \frac{\Delta\lambda}{\Delta D_j}\cdot DF_i + \sum_{k=1}^{M}\frac{\Delta\mu_k}{\Delta D_j}\cdot GSF_{k-i} = (1.277+0+0.004)\times10^{-3} = 1.281\times10^{-3}\ (\$/\text{MWh}^2)$$

$$\text{Actual } \frac{\Delta LMP_i}{\Delta D_j} = 24.30\times\left(\frac{0.001581}{30}\right) = 1.281\times10^{-3}\ (\$/\text{MWh}^2)$$

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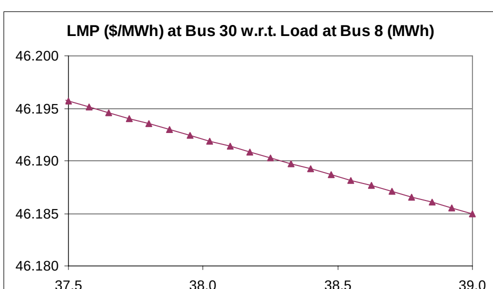
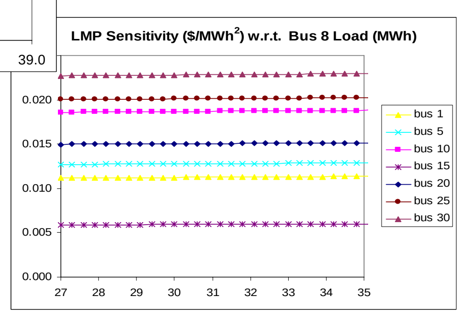
Test on the IEEE 30-bus system





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Verification using the IEEE 30-bus system

$$\frac{\Delta DF_i}{\Delta D_j}\cdot\lambda = (-0.0007781/0.075)\times 26.96535 = -0.27976(\$/MWh^2)$$

$$\frac{\Delta\lambda}{\Delta D_j}\cdot DF_i = (-0.012238/0.075)\times 1.109955 = -0.18112\ (\$/MWh^2)$$

$$\frac{\Delta\mu_{Line6,8}}{\Delta D_j}\cdot GSF_{Line6,8-30} = (-0.\ 26436/0.075)\times(-0.12866)$$

$$= 0.45350(\$/MWh^2)$$

$$\frac{\Delta DF_i}{\Delta D_j}\cdot\lambda + \frac{\Delta\lambda}{\Delta D_j}\cdot DF_i + \sum_{k=1}^{M}\frac{\Delta\mu_k}{\Delta D_j}\cdot GSF_{k-i} = -0.007383(\$/MWh^2)$$

Also, we have

$$\frac{\Delta LMP_i}{\Delta D_j} = -0.00054264/0.075 = -0.007235\ (\$/MWh^2)$$

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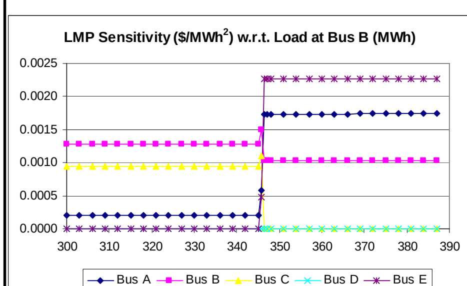
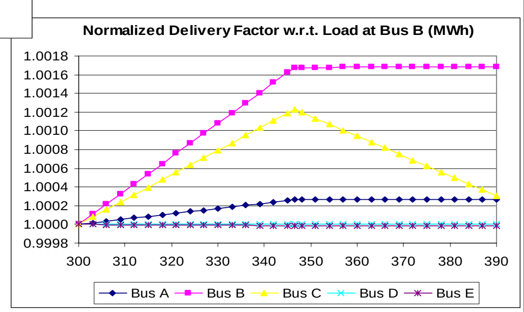
Challenges and future works





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Conclusions

Conclusions

- **LMP simulation based on DC model**
- **Comparison with ACOPF shows that DCOPF is acceptable at most cases**
  - **LMP difference is caused by the different set of marginal units (MU).**
- **LMP sensitivity is related to the loss component and linearly related to the sensitivity of delivery factors.**
  - **There is a step change of LMP (infinite sensitivity) when there is a change of MU**
- **Future works**
  - **It is a challenge to calculate or estimate when (under which load level) the next congestion will occur.**
  - **A better approximation algorithm should have a narrower range of incorrect identification of MU.**

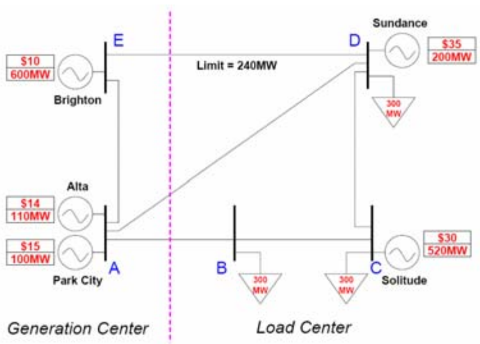
Thank you!

Questions and Answers?

LMP Simulation: Doubled loss

- **On the energy balance equations**

$$\sum_{i=1}^{N} DF_i^{est} \times G_i - \sum_{i=1}^{N} DF_i^{est} \times D_i + P_{loss}^{est} = 0$$



Total Load	Without P_{loss}^{est} in Eq. (11)			With P_{loss}^{est} in Eq. (11)		
	Scheduled Losses	Actual Losses	Error	Scheduled Losses	Actual Losses	Error
900	17.61	8.81	100%	8.81	8.81	0%
945	18.71	9.35	100%	9.35	9.35	0%
990	19.73	9.86	100%	9.93	9.93	0%
1035	19.02	9.51	100%	9.77	9.77	0%
1080	18.35	9.18	100%	9.42	9.42	0%
1125	17.73	8.86	100%	9.09	9.09	0%
1170	17.14	8.57	100%	8.78	8.78	0%
1215	16.60	8.30	100%	8.49	8.49	0%
1260	16.10	8.05	100%	8.22	8.22	0%
1305	15.64	7.82	100%	7.98	7.98	0%
1350	15.22	7.61	100%	7.76	7.76	0%