

## Biography of Fangxing (Fran) Li

- **B.S. (1994) and M.S. (1997) degrees from Southeast University, China**
- **Ph.D. (2001) degree from Virginia Tech, USA**
  
- **Senior Engineer, 02/2001 ~ 03/2005, ABB Consulting, Raleigh, NC, USA**
- **Principal Engineer, 04/2005 ~ 08/2005, ABB Consulting, Raleigh, NC, USA**
- **Assistant Professor, 08/2005 ~ present, The University of Tennessee, Knoxville, TN, USA**
  
- **Registered Professional Engineer in North Carolina (NC)**
- **IEEE Senior Member, 2005**
- **UT Eta Kappa Nu Outstanding Teacher Award, 2006**



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## DCOPF-Based LMP Simulation: Algorithm, Comparison with ACOPF, and Sensitivity

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## Outlines

- Introduction
- LMP simulation with DC based model
- Comparison with AC based model
- Sensitivity of LMP
- Conclusions



## Introduction



## LMP simulation with DC based model (DCOPF)

- Locational Marginal Pricing is the dominant pricing method in many US electricity markets
  - PJM, NYISO, ISO-New England, California ISO, ERCOT, & MISO.
- LMP simulation based on DC model
  - Based on production cost minimization model (economic dispatch)
  - Fast and efficient for chronological simulation for market-based planning
  - Employed by market simulators from leading vendors like ABB, GE, etc.



## LMP simulation based on DC model



## DCOPF-based LMP simulation: lossless

### ➤ Lossless model

$$\begin{aligned} & \text{Min } \sum_{i=1}^N c_i \times G_i \\ & \text{s.t. } \sum_{i=1}^N G_i = \sum_{i=1}^N D_i \\ & \sum_{i=1}^N GSF_{k-i} \times (G_i - D_i) \leq \text{Limit}_k, \text{ for } k=1, 2, \dots, M \\ & G_i^{\min} \leq G_i \leq G_i^{\max}, \text{ for } i=1, 2, \dots, N \end{aligned}$$

$GSF_{k-i}$  = generation shift factor to line  $k$  from Bus  $i$ .

$$LMP_B = \lambda + \left( \sum_{k=1}^M \mu_k \times GSF_{k-B} \right)$$



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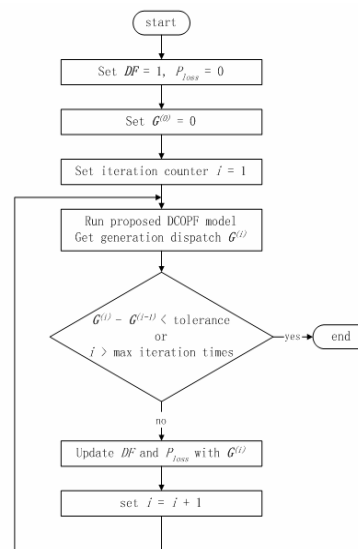
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## DCOPF-based LMP simulation: with losses

### • With Loss Model

$$\begin{aligned} & \text{Min } \sum_{i=1}^N c_i \times G_i \\ & \text{s.t. } \sum_{i=1}^N DF_i^{est} \times G_i - \sum_{i=1}^N DF_i^{est} \times D_i + P_{loss}^{est} = 0 \\ & \sum_{i=1}^N GSF_{k-i} \times (G_i - D_i) \leq \text{Limit}_k, \text{ for } k \in \{\text{all lines}\} \\ & G_i^{\min} \leq G_i \leq G_i^{\max}, \text{ for } i \in \{\text{all generators}\} \end{aligned}$$

$$DF_i = 1 - LF_i = 1 - \frac{\partial P_{Loss}}{\partial P_i}$$



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## DCOPF-based LMP simulation: about DF

- Discussion on delivery factors (DF)

$$DF_i = 1 - LF_i = 1 - \frac{\partial P_{Loss}}{\partial P_i}$$

$$\frac{\partial P_{Loss}}{\partial P_i} = \frac{\partial}{\partial P_i} \left( \sum_{k=1}^M F_k^2 \times R_k \right)$$

$$F_k = \sum_{j=1}^N GSF_{k-j} \times (G_j - D_j) = \sum_{j=1}^N GSF_{k-j} \times P_j$$

$$\frac{\partial P_{Loss}}{\partial P_i} = \sum_{k=1}^M \frac{\partial}{\partial P_i} (F_k^2 \times R_k) = \sum_{k=1}^M R_k \times 2F_k \times \frac{\partial F_k}{\partial P_i} = \sum_{k=1}^M 2 \times R_k \times GSF_{k-i} \times \left( \sum_{j=1}^N GSF_{k-j} \times P_j \right)$$



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## LMP Decomposition

- From augmented Lagrangian formulation

$$\psi = \left( \sum_{i=1}^N c_i \cdot G_i \right) - \lambda \left( \sum_{i=1}^N DF_i \cdot G_i - \sum_{i=1}^N DF_i \cdot D_i + P_{loss} \right) - \sum_{k=1}^M \mu_k \left( \sum_{i=1}^N GSF_{k-i} \times (G_i - D_i) - Limit_k \right)$$

$$LMP_B = \frac{\partial \psi}{\partial D_B} = \lambda \cdot DF_B + \left( \sum_{k=1}^M \mu_k \times GSF_{k-B} \right)$$

$$= \lambda + \left( \sum_{k=1}^M \mu_k \times GSF_{k-B} \right) + \lambda (DF_B - 1)$$

Energy  
Price

Congestion  
Price

Loss  
Price



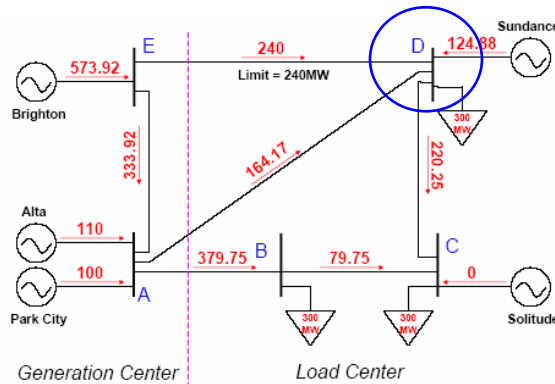
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## A little problem here ...

- All losses absorbed by the reference bus.

$$\sum_{i=1}^N GSF_{k-i} \times (G_i - D_i) \leq Limit_k$$



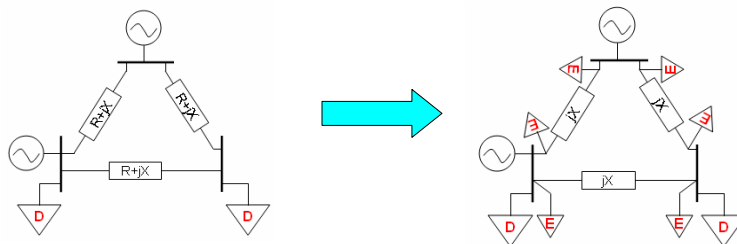
Imbalance  
=8.8 MW  
=total (G-D)

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## FND model

- Fictitious Nodal Demand (FND) model



$$E_i = \sum_{k=1}^{M_i} \frac{1}{2} \times F_k^2 \times R_k$$

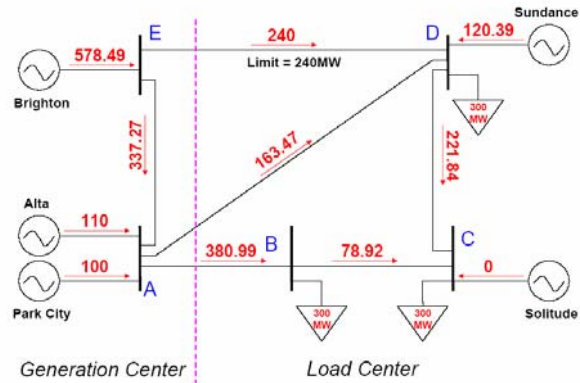
$$F_k = \sum_{j=1}^N GSF_{k-j} \times (G_j - D_j - E_j^{est}) \leq Limit_k$$

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## A brief result with FND model

- Losses are now distributed into each line



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## Comparison with AC based model



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# ACOPF

LMPs are the Lagrange multipliers

$$\text{Min } \sum c_{Gi} \times P_{Gi}$$

Subject to:

$$P_{Gi} - P_{Li} - P(V, \theta) = 0 \text{ (Real power balance)}$$

$$Q_{Gi} - Q_{Li} - Q(V, \theta) = 0 \text{ (Reactive power balance)}$$

$$F_k \leq F_k^{\text{max}} \text{ (Line flow MVA limits)}$$

$$P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}} \text{ (Gen. real power limits)}$$

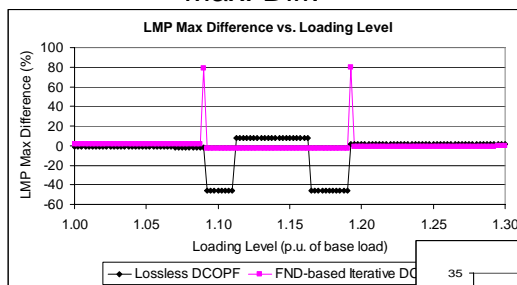
$$Q_{Gi}^{\text{min}} \leq Q_{Gi} \leq Q_{Gi}^{\text{max}} \text{ (Gen. reactive power limits)}$$

$$V_i^{\text{min}} \leq V_i \leq V_i^{\text{max}} \text{ (Bus voltage limits)}$$

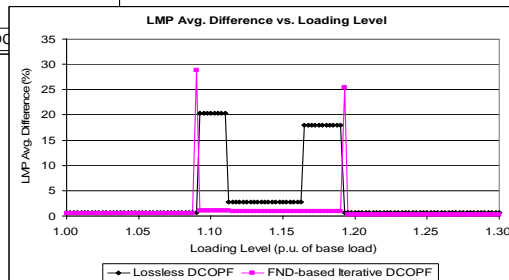


# Test on the PJM 5-bus system (1)

Max. Diff.

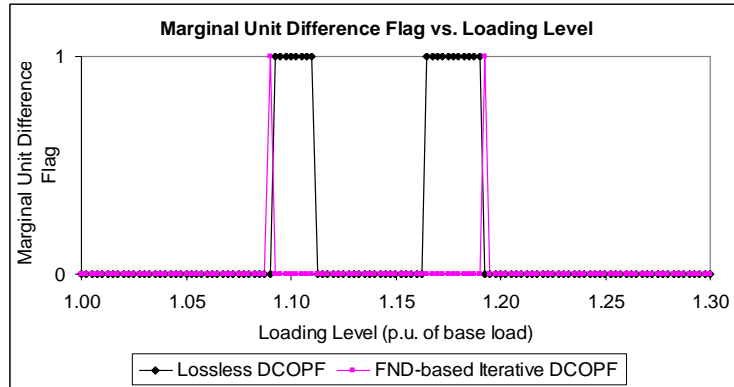


Avg. Diff.





## Test on the PJM 5-bus system (2)



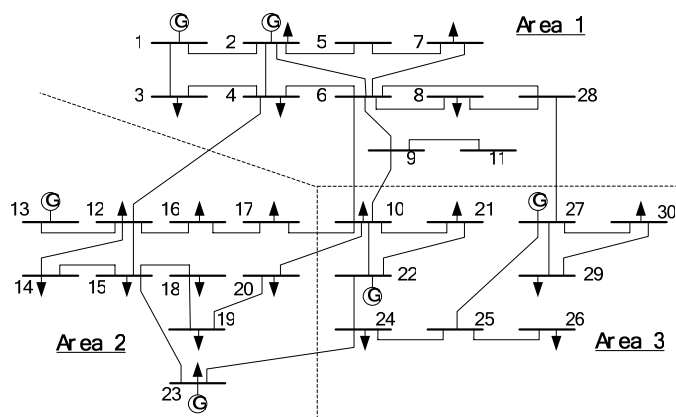
- 1) LMP difference is caused by the different set of marginal units (MU).
- 2) A better approximation algorithm should have a narrower range of incorrect identification of MU.



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## Test on the IEEE 30-bus system (1)

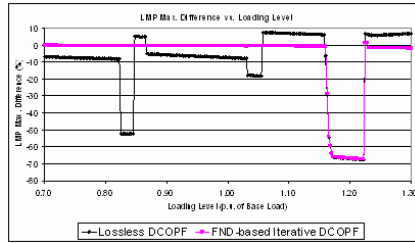


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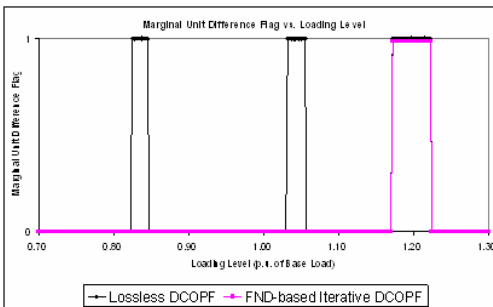
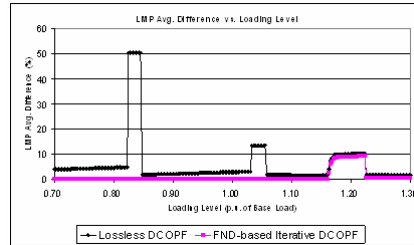
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## Test on the IEEE 30-bus system (2)

Max. Diff.



Avg. Diff.



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## Sensitivity Analysis

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## Sensitivity to load variation

- **No loss**

$$\frac{\Delta LMP_i^{no\_loss}}{\Delta D_j} = \frac{\Delta \lambda}{\Delta D_j} + \sum_{k=1}^M \frac{\Delta \mu_k}{\Delta D_j} \times GSF_{k-i} = 0$$

- **With losses**

$$\frac{\Delta LMP_i}{\Delta D_j} = \frac{\Delta \left( DF_i \cdot \lambda + \sum_{k=1}^M \mu_k \cdot GSF_{k-i} \right)}{\Delta D_j}$$

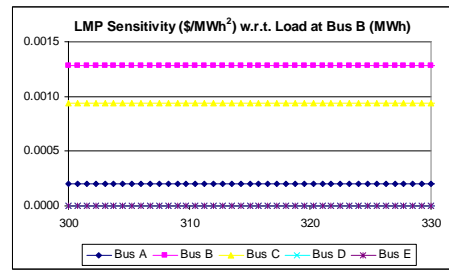
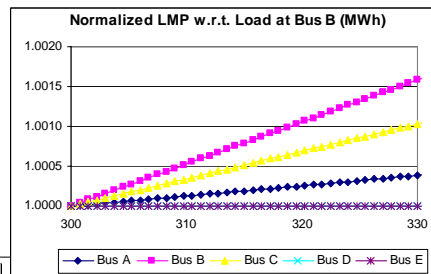
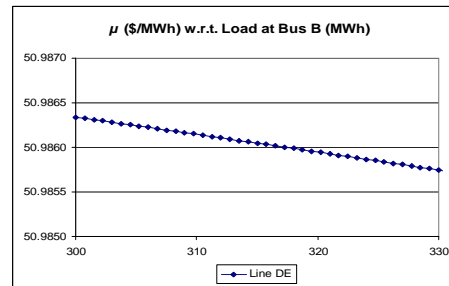
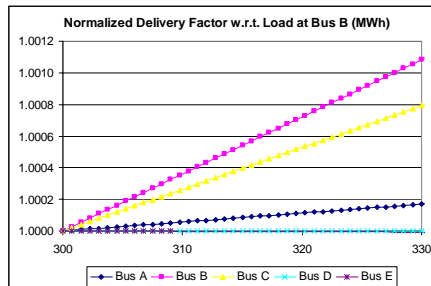
$$\frac{\Delta LMP_i}{\Delta D_j} = \frac{\Delta DF_i}{\Delta D_j} \cdot \lambda + \frac{\Delta \lambda}{\Delta D_j} \cdot DF_i + \sum_{k=1}^M \frac{\Delta \mu_k}{\Delta D_j} \cdot GSF_{k-i}$$



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## Test on the PJM 5-bus system



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## Verification

$$\text{Actual } \frac{\Delta DF_j}{\Delta D_j} \cdot \lambda = 3.647 \times 10^{-5} \times 35 = 1.277 \times 10^{-3} (\$/MWh^2)$$

$$\frac{\Delta \lambda}{\Delta D_j} \cdot DF_j = 0 (\$/MWh^2)$$

$$\frac{\Delta \mu_{DE}}{\Delta D_j} \cdot GSF_{DE-i} = \frac{(50.9857 - 50.9863)}{30} \times (-0.2176) \\ = 0.004 \times 10^{-3} (\$/MWh^2)$$

$$\frac{\Delta DF_j}{\Delta D_j} \cdot \lambda + \frac{\Delta \lambda}{\Delta D_j} \cdot DF_j + \sum_{k=1}^M \frac{\Delta \mu_k}{\Delta D_j} \cdot GSF_{k-i} = (1.277 + 0 + 0.004) \times 10^{-3} = 1.281 \times 10^{-3} (\$/MWh^2)$$

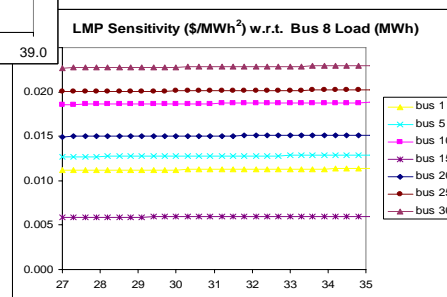
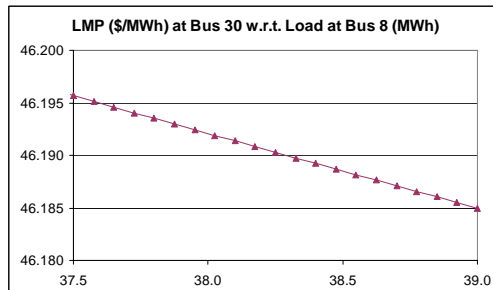
$$\text{Actual } \frac{\Delta LMP}{\Delta D_j} = 24.30 \times \left( \frac{0.001581}{30} \right) = 1.281 \times 10^{-3} (\$/MWh^2)$$



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## Test on the IEEE 30-bus system



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## Verification using the IEEE 30-bus system

$$\frac{\Delta DF_j}{\Delta D_j} \cdot \lambda = (-0.0007781/0.075) \times 26.96535 = -0.27976 (\$/MWh^2)$$

$$\frac{\Delta \lambda}{\Delta D_j} \cdot DF_j = (-0.012238/0.075) \times 1.109955 = -0.18112 (\$/MWh^2)$$

$$\frac{\Delta \mu_{Line 8}}{\Delta D_j} \cdot GSF_{Line 8-30} = (-0.26436/0.075) \times (-0.12866) = 0.45350 (\$/MWh^2)$$

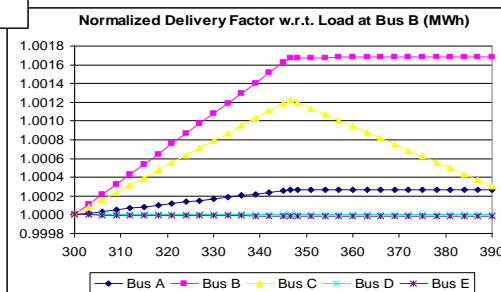
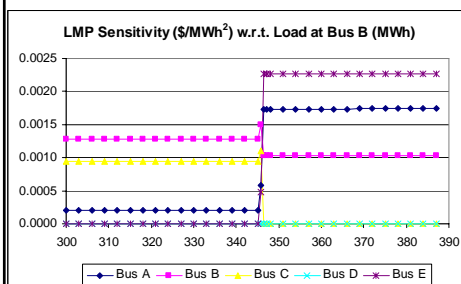
$$\frac{\Delta DF_j}{\Delta D_j} \cdot \lambda + \frac{\Delta \lambda}{\Delta D_j} \cdot DF_j + \sum_{i=1}^M \frac{\Delta \mu_i}{\Delta D_j} \cdot GSF_{i-j} = -0.007383 (\$/MWh^2)$$

Also, we have

$$\frac{\Delta LMP_j}{\Delta D_j} = -0.00054264/0.075 = -0.007235 (\$/MWh^2)$$



## Challenges and future works



## Conclusions



## Conclusions

- **LMP simulation based on DC model**
- **Comparison with ACOPF shows that DCOPF is acceptable at most cases**
  - LMP difference is caused by the different set of marginal units (MU).
- **LMP sensitivity is related to the loss component and linearly related to the sensitivity of delivery factors.**
  - There is a step change of LMP (infinite sensitivity) when there is a change of MU
- **Future works**
  - It is a challenge to calculate or estimate when (under which load level) the next congestion will occur.
  - A better approximation algorithm should have a narrower range of incorrect identification of MU.



Thank you!

Questions and Answers?



## LMP Simulation: Doubled loss

- On the energy balance equations

$$\sum_{i=1}^N DF_i^{est} \times G_i - \sum_{i=1}^N DF_i^{est} \times D_i + P_{loss}^{est} = 0$$

