Risk-based Var Resource Allocation

Yuan Li
James D. McCalley
Iowa State University

Outline

- Decision making under uncertainty
- How we allocate reactive power
- The effect of Reactive Power
- Modeling of severity
- Benders decomposition
- Problem formulation
- Solution procedure
- Illustration
- Conclusions and future work
- Question

Decision making under uncertainty

- Risk index can facilitate the decision making under uncertainty
- Risk = Pr * Sev
  - Pr: Probability of the contingency
  - Sev: Quantification of contingency severity/consequence
- Modeling of the uncertainty
  - contingency (Covered) / Load level (can be covered)
  - History data
  - Statistic/stochastic model
- Modeling of the severity
  - Capture the consequence the decision maker cares

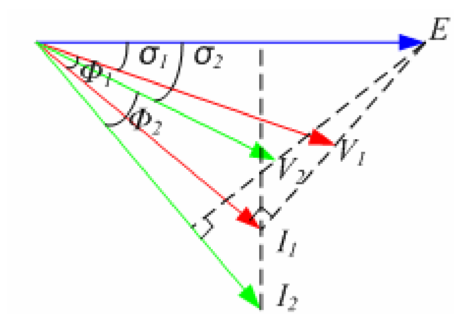
How we allocate reactive power

- An economic-engineering hybrid decision making.
  - Economic: investment
  - Engineering:
    - Voltage profile
    - Loadability
    - Dynamic (not considered in this work)
- Solving method (MINLP in nature)
  - Heuristic method: EA etc.
  - Benders decomposition
- Decomposition approach is best choice
- Decision level: Adequacy/Security/Risk

The effect of Reactive Power I

- It is well known fact that in AC systems dominated by ***reactances*** (as power systems typically are) there is a close link between voltage control and reactive power.
- Real power also affect voltage. But *resistances* are too small.
- The reactive power can be compensated.
- The compensation of real power is called generation/load shedding☺
- Reactive power can be compensated by capacitor, SVC, condenser. Not free.

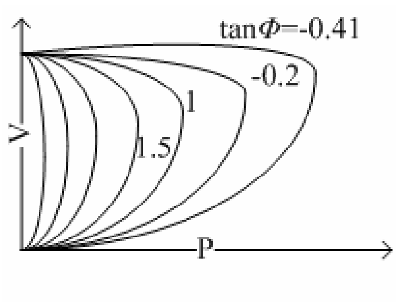
The effects of Reactive Power II



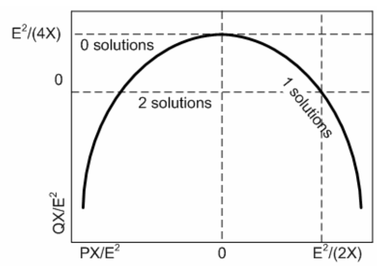
- Enhance voltage level
- Reduce line flow
- Reduce the power angle
- Improve power quality
- Decrease line losses
- Improve stability

The effects of Reactive Power III

The famous PV curves*



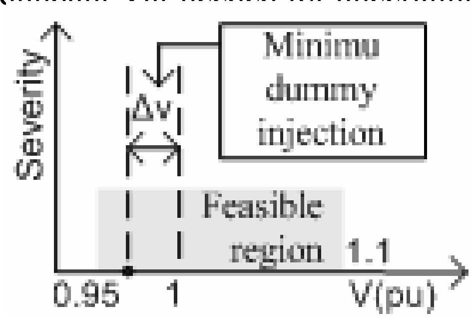
Domain of Existence of a load flow solution*



*T. Van Cutsem and C.Vournas, Voltage stability of electric power system, 1998, Kluwer

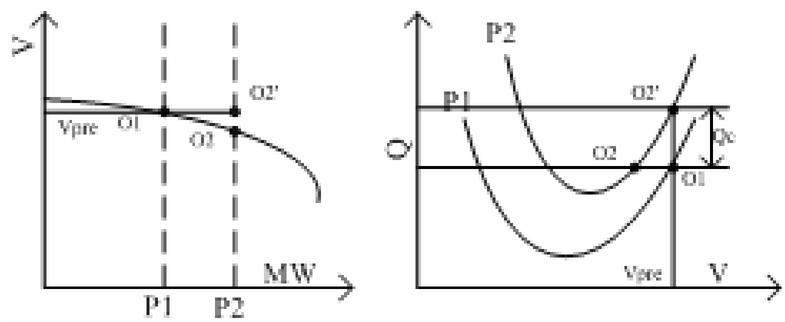
Modeling of severity I

- After the allocation for voltage profile, some buses' voltage will be at the lower bound of the required voltage region.
- Some redundant Var needed for uncertainty.



Modeling of severity II

- Reactive power flow balance equation
  - Q_L-Q_c=-V^2/X + E·V·cosσ/X=Q_I



Benders decomposition I

$$
\begin{array}{llllll}
Min & C^T x & & & & \\
S.t. & Ax & & & \geq & b \\
 & Ex & + & Fy & \geq & h
\end{array}
$$

$$
\begin{array}{llll}
Min & C^T x & & \\
S.t. & Ax & \geq & b
\end{array}
\quad \longrightarrow \quad
\begin{array}{llllllllll}
Min & w(\hat{x}) & = & 1^T s & & & & & & \\
S.t. & Fy & + & s & \geq & h & - & E\hat{x} & & \lambda
\end{array}
$$

$$
w(\hat{x}) + \lambda E(\hat{x} - x) \leq 0
$$

Problem formulation: Adequacy and Security

$$
\begin{array}{lll}
Min & \sum_{k=1}^{N} d_k r_k + c_{rk} q_{rk} + c_{ck} q_{ck} & \\
s/to & MDR & = 0 \\
 & q_r & \leq \bar{q}_r \cdot r \\
 & q_c & \leq \bar{q}_c \cdot r
\end{array}
$$

Whole problem Formulation (Master problem remove the first constraints)

$$
\begin{array}{lll}
MDR = Min & \sum_{k=1}^{N} (y_{1k} + y_{2k}) & \\
s/to & \Delta P & = 0 \\
 & \Delta Q - y_1 + y2 & = 0 \\
 & -q_r + \underline{Q}_{Gk} \leq Q_{Gk} \leq \bar{Q}_{Gk} + q_c & \\
 & \underline{V}_k \leq V_k \leq \bar{V}_k &
\end{array}
$$

Second stage problem: minimum dummy reactive power injection

Benders decomposition II

$$
\begin{array}{llllll}
Min & C^T x & + & D^T y & & \\
S.t. & Ax & & & \geq & b \\
 & Ex & + & Fy & \geq & h
\end{array}
$$

$$
\begin{array}{llll}
Min & C^T x & + & \alpha(x) \\
S.t. & Ax & \geq & b
\end{array}
\longrightarrow
\begin{array}{llllll}
Min & v(\hat{x}) & = & D^T y & & \\
S.t. & Fy & \geq & h & - E\hat{x} & \pi
\end{array}
$$

$$v(\hat{x}) + \lambda E(\hat{x} - x) \leq \alpha(x)$$

Problem formulation: Risk

$$Min \quad \sum_{k=1}^{N}(d_k r_k + c_{rk} q_{rk} + c_{ck} q_{ck}) + \sum_{m=1}^{M}(Pr_m \cdot Sev_m)$$

Whole problem Formulation

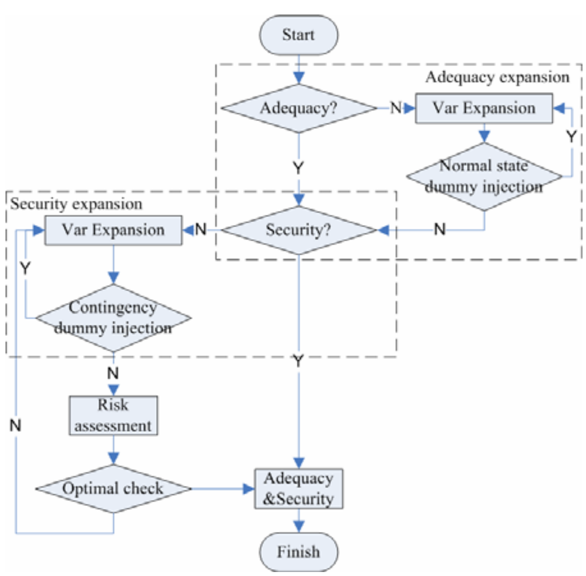
$$\begin{aligned} Min \quad & \textstyle\sum_{k=1}^{N} d_k r_k + c_{rk} q_{rk} + c_{ck} q_{ck} \\ s/to \quad & MDR = 0 \\ & q_r \leq \bar{q}_r \cdot r \\ & q_c \leq \bar{q}_c \cdot r \end{aligned}$$

First stage problem: Severity expansion

$$\begin{aligned} Min \quad & \textstyle\sum_{k=1}^{N} P_k \cdot Sev_k \\ s/to \quad & 1.0 \leq V_k \leq \bar{V}_k \\ & Sev_k = sum(MDR) \end{aligned}$$

Second stage problem: Risk evaluation

Flowchart of the risk-based Var allocation



Illustration

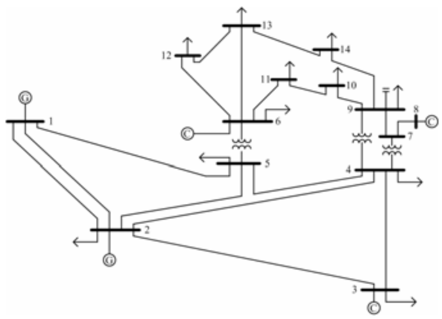


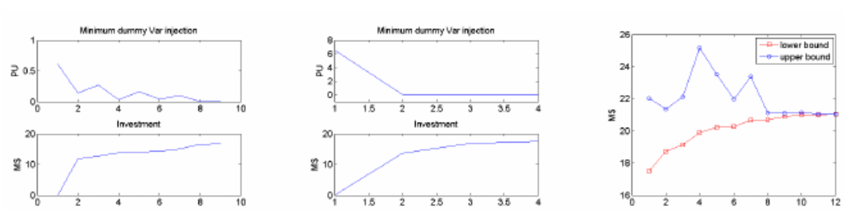
Table 4.5: Line outage probability (1E-3)

Line	Prob	Line	Prob	Line	Prob
(1, 2)	1.0	(4, 5)	2.5	(6, 13)	1.2
(1, 2)	1.0	(4, 7)	0.5	(6, 13)	0.0
(1, 5)	1.5	(4, 9)	1.2	(6, 13)	2.0
(2, 3)	1.1	(3, 5)	1.4	(6, 13)	1.8
(2, 4)	2.0	(5, 6)	2.0	(6, 13)	2.0
(2, 5)	3.0	(6, 11)	2.5	(6, 13)	1.0
(3, 4)	1.1	(6, 12)	0.8	(6, 13)	2.6

Table 4.4: Candidate Var cost information ($M)

Bus	Fixed cost	Variable cost		Maximum	
		Cap	Ind	Cap	Ind
3	4	9	6	0.5	0.4
4	6	8	5	0.45	0.4
5	3	6	7	0.55	0.4
6	7	5	8	0.4	0.3
7	3	9	4	0.46	0.2
8	8	7	9	0.38	0.1
9	6	4	6	0.56	0.3
10	7	7	7	0.35	0.5
11	9	6	5	0.47	0.4
12	6	8	8	0.55	0.24
13	8	7	6	0.48	0.22
14	9	5	7	0.30	0.31

Convergence of different strategies



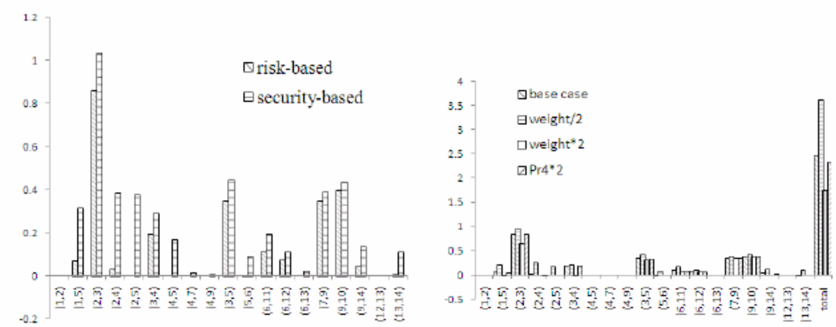
(a) Adequacy expansion (b) Security expansion (c) Risk expansion

Figure 4.11: Convergence of differenct Var allocation strategy

Table 4.6: Cost comparison for different strategies

Adequacy		Security		Risk	
Bus	Size	Bus	Size	Bus	Size
3	0.498	3	0.121	3	0.156
-	-	7	0.131	7	0.204
9	0.56	9	0.56	9	0.56
$16.72M$		$17.51M$		$18.56M$	

Sensitivity analysis



(a) Voltage risk

(b) Sensitivity of risk

Figure 4.12: Voltage risk sensitivity analysis

Conclusions and future work

- Analyzed the role of the reactive power in power system.
- Provide the insight analysis of the relationship of the voltage level and the loadability.
- Provide a Var resource allocation method considering both voltage profile and loadability.
- The approach provide the Var resource allocation strategy under uncertainty.
- The sensitivity analysis is provided to facilitate the decision making.
- Part of the decision making project of our group.

Questions?