Capacity Benefit Margin Assessment Based on Multi-area Reliability Exponential Analytic Model

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Capacity Benefit Margin

- In the environment of the power market, available transfer capability (ATC) is a measure of transfer capability as the technical and commercial indices.
- CBM is a transmission capability margin reserved to meet the generation reliability requirements, the calculation of this margin has some relationships with reliability levels of all areas in the system
- multi-area generation reliability exponential analytic model is put forward to avoid optimization iteration

Current Solutions

- Definitive method: generally CBM is related with the maximum generation output unit of this area, or the firm percentage of TTC. This method has less precision and definitive method can not reflect the effect of system state change on the margin;
- Reliability index method: this method introduced the conception of multi-area reliability
  - Ou. Yan and C. Singh calculated CBM through generation reliability index LOLE (Loss of Load Expected). Areas that don't meet the standard are combined together for simplification.
  - Othman used evolution programming to determine CBM, for a large size power system with many areas, every change in generation during the optimization procedure needs to compute reliability indexes repeatedly, so it is complicated and computationally time consuming.

Method

- How to optimize CBM value among areas for an *economical* and *reliable* system?

- To solve the optimization problem, we must re-calculate and check reliability indexes at every step of optimization.

- Is there a mathematical relation between the risk and the capacity (or transfer capability) change?

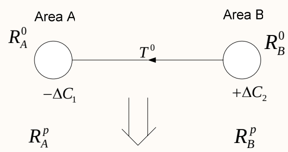
- The multi-area generation reliability exponential analytic model illustrates multi-area generation reliability as the function of generation installed capacity and intertie capacity between areas, thus the optimization procedure can be solved regarding reliability indexes as inequality constraints.

Exponential Analytic Model

- Garver suggested an exponential approximation for system risk as a function of capacity:

$$R = A \cdot e^{-C/M}$$

- two-area system:



$$\begin{bmatrix} \ln \frac{R_A^p}{R_A^0} \\ \ln \frac{R_B^p}{R_B^0} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & t_{11} \\ d_{21} & d_{22} & t_{21} \end{bmatrix} \cdot \begin{bmatrix} \Delta C_1 \\ \Delta C_2 \\ \Delta Tie_1 \end{bmatrix}$$

Theory Proof d_{22}

- Cumulative outage probability after *ΔC* in area B:

$$P'_{equ,B}(X) = (1 - F_{OR}) \cdot P_{equ,B}(X) + F_{OR} \cdot (X - \Delta C)$$

- Reliability indices before and after addition:

$$R_B^0 = P_{equ,B}(A_0 + B_0) = K_B \cdot \exp(-(A_0 + B_0)/M_B)$$

$$R_B = P'_{equ,B}(A_0 + B_0 + \Delta C) = (1 - F_{OR}) \cdot K_B \cdot \exp(-(A_0 + B_0 + \Delta C)/M_B) + F_{OR} \cdot K_B \cdot \exp(-(A_0 + B_0)/M_B)$$

- The element d_{22} solution of matrix:

$$d_{22} = -\frac{1}{\Delta C} \cdot \ln \frac{R_B}{R_B^0} \approx M_B$$

Theory Proof d_{12}

- Cumulative outage probability of the *Equivalent Sending Unit* (ESU: power support from area B to A)

$$P_{equ}(X) = K_{equ} \cdot \exp(-X / M_{equ})$$

- Generation reliability indices of area A using convolution:

$$R_A^0 = \sum_{i=0}^{n_a} p_A(i \cdot \Delta X) \cdot P_{equ}(A_0 + B_0 - i \cdot \Delta X) = \sum_{i=0}^{n_a} p_A(i \cdot \Delta X) \cdot K_{equ} \cdot \exp(-(A_0 + B_0 - i \cdot \Delta X) / M_{equ})$$

$$= K_{equ} \cdot \exp(-(A_0 + B_0) / M_{equ}) \cdot \sum_{i=0}^{n_a} p_A(i \cdot \Delta X) \cdot \exp(i \cdot \Delta X / M_{equ})$$

$$R_A = \sum_{i=0}^{n_a} p_A(i \cdot \Delta X) \cdot P_{equ}(A_0 + B_0 + \Delta C - i \cdot \Delta X) = \sum_{i=0}^{n_a} p_A(i \cdot \Delta X) \cdot K_{equ} \cdot \exp(-(A_0 + B_0 + \Delta C - i \cdot \Delta X) / M_{equ})$$

$$= K_{equ} \cdot \exp(-(A_0 + B_0 + \Delta C) / M_{equ}) \cdot \sum_{i=0}^{n_a} p_A(i \cdot \Delta X) \cdot \exp(i \cdot \Delta X / M_{equ})$$

- The element d_{12} solution of matrix:

$$d_{12} = -\frac{1}{\Delta C} \cdot \ln \frac{R_A}{R_A^0} \approx M_{equ}$$

Multi-Area Generation Reliability

- Suppose that transfer capabilities in interfaces are abundant enough with no transmission constraints, reliability levels of all areas are generation-dominant problems. (*n* areas, *m* tie lines system)

$$\begin{cases} \boldsymbol{R} = diag(R_1^0, R_2^0, \ldots, R_n^0) \cdot [e^{-\boldsymbol{D_1} \cdot \boldsymbol{\Delta C}}, e^{-\boldsymbol{D_2} \cdot \boldsymbol{\Delta C}} \cdots e^{-\boldsymbol{D_n} \cdot \boldsymbol{\Delta C}}]^T \\ \boldsymbol{R} = [R_1, R_2, \ldots\ldots R_n]^T; \\ \boldsymbol{R^0} = [R_1^0, R_2^0, \ldots\ldots R_n^0]^T; \\ \boldsymbol{\Delta C} = [\Delta C_1, \Delta C_2, \ldots\ldots \Delta C_n]^T \end{cases}$$

$$\boldsymbol{D}_{n \times n} = \begin{bmatrix} d_{11} \cdots d_{1j} \cdots d_{1n} \\ \vdots \quad \vdots \quad \vdots \\ d_{i1} \cdots d_{ij} \cdots d_{in} \\ \vdots \quad \vdots \quad \vdots \\ d_{n1} \cdots d_{nj} \cdots d_{nn} \end{bmatrix}$$

- The element d_{ij} means the influence factor of generation installed capacity change in area *j* on the reliability level of area *i*, R_i^0means reliability level of area *i* at the base case, □C_i means installed capacity change in area *i*.

Multi-Area Generation Reliability

- Suppose that there are abundant total generation installed capacity and transmission constraints influence power interchange, then reliability levels of all areas become transmission-dominant problems.

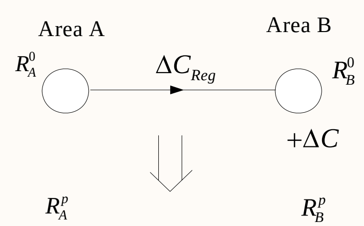
$$\begin{cases} \boldsymbol{R} = diag(R_1^0, R_2^0, \ldots, R_n^0) \cdot [e^{-T_1 \cdot \Delta Tie}, e^{-T_2 \cdot \Delta Tie} \cdots e^{-T_n \cdot \Delta Tie}]^T \\ \boldsymbol{R} = [R_1, R_2, \ldots\ldots R_n]^T; \\ \boldsymbol{R^0} = [R_1^0, R_2^0, \ldots\ldots R_n^0]^T; \\ \boldsymbol{\Delta Tie} = [\Delta Tie_1, \Delta Tie_2, \ldots\ldots \Delta Tie_m]^T \end{cases}$$

$$\boldsymbol{T}_{n\times m} = \begin{bmatrix} t_{11} \cdots t_{1j} \cdots t_{1m} \\ \vdots \quad \vdots \quad \vdots \\ t_{i1} \cdots t_{ij} \cdots t_{im} \\ \vdots \quad \vdots \quad \vdots \\ t_{n1} \cdots t_{nj} \cdots t_{nm} \end{bmatrix}$$

- The element t_{ij} means the influence factor of transmission capacity change in interface j on the reliability level of area i, □Tie_j means transmission capacity change in interface j

CBM Definition

For a simple two-area system, A is a source area, B is a sink area, at the base case of power flow, area B does not meet the reliability level. After power support increment ΔC_{reg} from area A to area B, if area B just meets the reliability requirement, then ΔC_{reg} could be regarded as capacity benefit margin.



Optimization Models

- A system with *n* areas and *m* interfaces
- Case A: Generation-Dominant Model: if only one area (area) does not meet the reliability requirement due to installed capacity shortage, other areas provide power support to area on condition that their own reliability requirements are still satisfied after support.
- Case B: Transmission-Dominant Model: if there are *l* areas do not meet the reliability requirement due to transmission constraints, the other (*n*-*l*) areas support the deficient areas.

Generation-Dominant Model

Case A: CBMs are optimized as the object of total power interchanges □$Tie_j(j=1,2,\ldots m)$ through the *m* interfaces are minimized, i.e., the transfer capability □Tie_j set aside for reliability requirement equals to CBM in interface *j*.

$$Min \quad \sum_{j=1}^{m} \left|\Delta Tie_j\right|$$

$$s.t. \quad \boldsymbol{R} = diag(R_1^0, R_2^0, \ldots, R_n^0) \cdot [e^{-D_1 \cdot \boldsymbol{\Delta C}}, e^{-D_2 \cdot \boldsymbol{\Delta C}} \cdots e^{-D_n \cdot \boldsymbol{\Delta C}}]^T$$

$$\sum_{i=1}^{n} \Delta C_i = 0$$

$$A_{n\times m} \cdot \boldsymbol{\Delta Tie} = \boldsymbol{\Delta C}$$

$$R_i = R^{thres}$$

$$R_k \le R^{thres} \quad k = 1, 2, \ldots n, \; k \ne i$$

Transmission-Dominant Model

Case B: CBM are optimized as the object of minimal transmission addition of total power interchanges ΔTie_j $(j=1,2,\ldots m)$ through the m interfaces, then the incremental transfer capability ΔTie_j for reliability requirement equals to CBM in interface j.

$$\begin{aligned}
&Min \quad \sum_{j=1}^{m} \left|\Delta Tie_j\right| \\
&s.t. \quad \boldsymbol{R} = diag(R_1^0, R_2^0, \ldots, R_n^0) \cdot [e^{-T_1 \cdot \boldsymbol{\Delta Tie}}, e^{-T_2 \cdot \boldsymbol{\Delta Tie}} \cdots e^{-T_n \cdot \boldsymbol{\Delta Tie}}]^T \\
&\qquad R_i = R^{thres} \quad i = 1,2,\ldots r \\
&\qquad R_k \le R^{thres} \quad k = r+1, r+2, \ldots n
\end{aligned}$$

Sequential Quadratic Programming

- Sequential quadratic programming is an effective method to solve constrained non-linear programming.

$$\begin{aligned}
&Max \quad f(x) \\
&s.t. \quad g(x) = 0 \\
&\qquad h(x) \le 0
\end{aligned}$$

Computation Process of SQP

- To build related Lagrange function:

$$L(x,\lambda) = f(x) + \lambda_g g(x) + \lambda_h h(x)$$

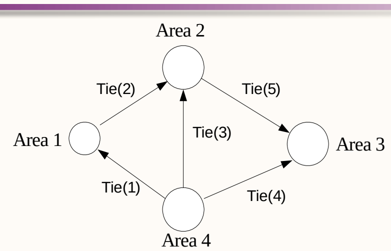
- $f(x)$ is approximated by the Taylor expansion equation to the second order, $g(x)$ and $h(x)$ are linearly approximated:

$$\min \nabla f^T(x^k)s^k + 0.5(s^k)^T H_k s^k$$
$$s.t. \quad g(x^k) + \nabla g^T(x^k)s^k = 0$$
$$h(x^k) + \nabla h^T(x^k)s^k \le 0$$

- the optimal step length α^k is obtained through one-dimension linear search:

$$x^{k+1} = x^k + \alpha^k s^k$$
$$\min \ f(x^{k+1}) + \lambda_g g(x^{k+1}) + \lambda_h \max[0, h(x^{k+1})]$$

Case Study



Generation installed capacity and load level (MW)

Area	Installed Capacity	Load Level
1	2200	1780
2	5800	4871
3	3000	4714
4	8800	5036

Power flow and transmission capacity of ties

Tie	Base Flow	Case A	Case B
Tie(1)	320.43	9999	500
Tie(2)	236.22	9999	500
Tie(3)	367.4	9999	500
Tie(4)	2087.4	9999	2000
Tie(5)	737.4	9999	500

Generation-Dominant Model

$$\mathbf{D} = \begin{bmatrix} 4.43153 & 2.48200 & 0.00000 & 4.55300 \\ 2.51700 & 3.49440 & 0.00000 & 3.84566 \\ 4.61208 & 1.87289 & 2.46768 & 3.01467 \\ 3.78866 & 2.35674 & 0.00000 & 5.22609 \end{bmatrix} \times 10^{-3}$$

- All elements in this matrix are 0, this means installed capacity increase in each area will be helpful to improve reliability level of all areas;
- Area 3 is a sink area, installed capacity increase in this area will improve its own reliability level obviously and do not have significant influence on other areas.

Generation-Dominant Model

Optimization results with respect to variable LOLP of area 3

LOLP (Hours/yr)	Total CBM	Tie(2) Margin(MW)	Tie(4) Margin(MW)	Tie(5) Margin(MW)
8.0000	55.5630	0.8562	30.8392	24.7244
7.0000	56.6591	5.3268	32.3795	24.2796
6.0000	57.9220	10.4840	34.1659	23.7561
5.0000	59.4200	16.5921	36.2610	23.1590
4.0000	61.2508	24.0630	38.8351	22.4157
3.0000	63.6113	33.6947	42.1537	21.4576
2.0000	63.7526	34.2715	42.3524	21.4002

- Power support from area 4 do have great impact on the reliability level of area 3 (d_{34}=3.01467);
- Area 4 can export some amount of power as support through tie (4) when its own reliability level still meets the requirement.
- When LOLP requirement becomes strict (<5.0 Hours/yr), more power flows through tie (2) and tie (5), when LOLP requirement <4.0 Hours/yr, area 2 even absorbs some part of power through tie (2) because margin in tie (2) is greater than margin in tie (5).

Transmission-Dominant Model

$$\mathbf{T} = \begin{bmatrix} +1.8205 & -4.1086 & -0.7478 & +0.0000 & -2.8059 \\ -1.9240 & -0.7110 & +1.3237 & +0.0000 & -3.6407 \\ +0.0000 & -1.1341 & -1.1341 & +0.4371 & +0.9512 \\ -4.6653 & -1.8756 & -2.0749 & -3.6072 & +0.0000 \end{bmatrix} \times 10^{-3}$$

- Area 4 is a source area, transmission capacity decrement of interfaces will improve reliability level in this area. So all elements in the 4-*th* row are 0;
- Area 3 is a sink area, transmission capacity decrement of tie (4) and tie (5) will deteriorate reliability level of area 3.

Environmental Constraints

Optimization results with respect to variable LOLP of area 3

LOLP (Hours/yr)	Total CBM	Tie(1) Margin(MW)	Tie(4) Margin(MW)	Tie(5) Margin(MW)
4.8	6.6378	0.0000	2.0899	4.5479
4.5	20.2088	3.5866	6.9739	9.6483
4.2	36.2018	9.6099	13.0357	13.5562
4.0	49.7948	14.7293	18.1878	16.8777
3.8	61.3422	19.0782	22.5646	19.6994
3.5	81.8430	26.1974	31.3272	24.3184
3.0	117.1095	26.1974	66.5937	24.3184

- From Table IV, when LOLP requirement is not so strict, the main support to area 3 is from area 2 and area 4 through tie (5) and tie (4) respectively, this is because t_{34}, t_{35} are positive, which means increment of interface capacity will improve reliability level of area 3. When LOLP requirement becomes strict, support from tie (5) maintains about 24 MW; while support from tie (4) increases rapidly.

Conclusions

- Multi-area generation reliability exponential analytic model takes the coupling influence between areas into account.
- This model can deal with the scenario of several areas not meeting reliability requirements simultaneously, no need to iterate and verify reliability levels because reliability requirements are expressed as in-equality constraints in this model through reliability exponential analytic expressions.
- From testing results of Case A and Case B, two scenarios of generation shortage and transmission constraint problems are illustrated. According to the influences of generation installed capacity (or intertie capacity between areas) on multi-area generation reliability, the optimal capacity is set aside as CBM.

Thanks