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Why is Maxwell's Theory so hard to understand?

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Modesty is not always a virtue

In the year 1865, James Clerk Maxwell published his paper "A dynamical theory of the electromagnetic field" in the Philosophical Transactions of the Royal Society. He was then thirty-four years old. We, with the advantage of hindsight, can see clearly that Maxwell's paper was the most important event of the nineteenth century in the history of the physical sciences. If we include the biological sciences as well as the physical sciences, Maxwell's paper was second only to Darwin's "Origin of Species". But the importance of Maxwell's work was not obvious to his contemporaries. For more than twenty years, his theory of electromagnetism was largely ignored. Physicists found it hard to understand because the equations were complicated. Mathematicians found it hard to understand because Maxwell used physical language to explain it. It was regarded as an obscure speculation without much experimental evidence to support it. The physicist Michael Pupin in his autobiography "From Immigrant to Inventor" describes how he travelled from America to Europe in 1883 in search of somebody who understood Maxwell. He set out to learn the Maxwell theory like a knight in quest of the Holy Grail.

Pupin went first to Cambridge and enrolled as a student, hoping to learn the theory from Maxwell himself. He did not know that Maxwell had died four years earlier. After learning that Maxwell was dead, he stayed on in Cambridge and was assigned to a college tutor. But his tutor knew less about the Maxwell theory than he did, and was only interested in training him to solve mathematical tripos problems. He was amazed to discover, as he says, "how few were the physicists who had caught the meaning of the theory, even twenty years after it was stated by Maxwell in 1865". Finally he escaped from Cambridge to Berlin and enrolled as a student with Hermann von Helmholtz. Helmholtz understood the theory and taught Pupin what he knew. Pupin returned to New York, became a professor at Columbia University, and taught the successive

generations of students who subsequently spread the gospel of Maxwell all over America.

How did it happen that Maxwell's theory was so widely ignored? After all, Maxwell was not like his contemporary Gregor Mendel, a monk working in an obscure monastery garden in Bohemia. Maxwell was a famous professor, director of the Cavendish Laboratory in Cambridge, a leading figure in the British scientific community. As an indication of his high standing, he was president of Section A (mathematical and physical sciences) of the British Association for the Advancement of Science when the association held its annual meeting at Liverpool in 1870. He gave a presidential address in Liverpool which was published in volume 2 of the recently founded journal "Nature". The style of his address shows us why his theory was not taken seriously. One might have expected that he would take the opportunity provided by the presidential platform to proclaim to the world the importance of the discoveries that he had made five years earlier. He did nothing of the kind. He was absurdly and infuriatingly modest.

Maxwell first announced the subject of his address, a survey of the recent advances that had been made on the frontier between mathematics and physics. He then talked with great enthusiasm about the vortex theory of molecules that had been recently proposed by Sir William Thompson (who later became Lord Kelvin). "A theory which Sir William has founded on Helmholtz's splendid hydro-dynamical theorems, seeks for the properties of molecules in the ring vortices of a uniform frictionless incompressible fluid. Helmholtz has shown that in a perfect fluid such a whirling ring, if once generated, would go on whirling for ever, would always consist of the very same portion of the fluid which was first set whirling, and could never be cut in two by any natural cause. These ring-vortices are capable of such varied connections, and knotted self-involutions, that the properties of differently knotted vortices must be as different as those of different kinds of molecules can be." And so it goes on. Maxwell explained how the ancient theory that matter is composed of atoms ran into a logical paradox. On the one hand, atoms were supposed to be hard, impenetrable and indestructible. On the other hand, the evidence of spectroscopy and chemistry showed that atoms have internal structure and are influenced by outside forces. This paradox had for many years blocked progress in the understanding of the nature of matter. Now finally the vortex theory of molecules resolved the paradox. Vortices in the aether are soft and have internal structure, and nevertheless, according to Helmholtz, they are individual and indestructible. The only remaining task was to deduce the facts of spectroscopy and chemistry from the laws of interaction of vortices predicted by the hydrodynamics of a perfect fluid. Maxwell held up this vortex theory of matter as a wonderful example of the fruitful interaction between mathematics and physics.

It is not clear whether Maxwell seriously believed what he was saying about the vortex theory. Perhaps he intended his talk to entertain his listeners rather than to enlighten them. He had a sly sense of humour and it is possible that he was praising the vortex

theory with tongue in cheek knowing that the more discerning members of the audience would understand that the theory was a joke. Only at the end of his talk did Maxwell briefly mention his theory of electromagnetism. He mentioned it in a half-hearted manner, saying only: "Another theory of electricity which I prefer denies action at a distance and attributes electric action to tensions and pressures in an all-pervading medium, these stresses being the same in kind with those familiar to engineers, and the medium being identical with that in which light is supposed to be propagated." The phrase "Another theory of electricity which I prefer" seems deliberately intended to obscure the fact that this was his own theory. It was no wonder that his listeners were more impressed by Kelvin's vortices than by Maxwell's equations.

The moral of this story is that modesty is not always a virtue. Maxwell and Mendel were both excessively modest. Mendel's modesty setback the progress of biology by fifty years. Maxwell's modesty setback the progress of physics by twenty years. It is better for the progress of science if people who make great discoveries are not too modest to blow their own trumpets. If Maxwell had had an ego like Galileo or Newton, he would have made sure that his work was not ignored. Maxwell was as great a scientist as Newton and a far more agreeable character. But it was unfortunate that he did not begin the presidential address in Liverpool with words like those that Newton used to introduce the third volume of his Principia Mathematica, "It remains that, from the same principles, I now demonstrate the frame of the system of the world". Newton did not refer to his law of universal gravitation as "another theory of gravitation which I prefer".

Maxwell's Theory and Quantum Mechanics

There were other reasons, besides Maxwell's modesty, why his theory was hard to understand. He replaced the Newtonian universe of tangible objects interacting with one another at a distance by a universe of fields extending through space and only interacting locally with tangible objects. The notion of a field was hard to grasp because fields are intangible. The scientists of that time, including Maxwell himself, tried to picture fields as mechanical structures composed of a multitude of little wheels and vortices extending throughout space. These structures were supposed to carry the mechanical stresses that electric and magnetic fields transmitted between electric charges and currents. To make the fields satisfy Maxwell's equations, the system of wheels and vortices had to be extremely complicated. If you try to visualise the Maxwell theory with such mechanical models, it looks like a throwback to Ptolemaic astronomy with planets riding on cycles and epicycles in the sky. It does not look like the elegant astronomy of Newton. Maxwell's equations, written in the clumsy notations that Maxwell used, were forbiddingly complicated, and his mechanical models were even worse. To his contemporaries, Maxwell's theory was only one of many theories of electricity and magnetism. It was difficult to visualise, and it did not have any clear advantage over other theories that described electric and magnetic forces in Newtonian style as direct

action at a distance between charges and magnets. It is no wonder that few of Maxwell's contemporaries made the effort to learn it.

Maxwell's theory becomes simple and intelligible only when you give up thinking in terms of mechanical models. Instead of thinking of mechanical objects as primary and electromagnetic stresses as secondary consequences, you must think of the electromagnetic field as primary and mechanical forces as secondary. The idea that the primary constituents of the universe are fields did not come easily to the physicists of Maxwell's generation. Fields are an abstract concept, far removed from the familiar world of things and forces. The field-equations of Maxwell are partial differential equations. They cannot be expressed in simple words like Newton's law of motion, force equals mass times acceleration. Maxwell's theory had to wait for the next generation of physicists, Hertz and Lorentz and Einstein, to reveal its power and clarify its concepts. The next generation grew up with Maxwell's equations and was at home in a universe built out of fields. The primacy of fields was as natural to Einstein as the primacy of mechanical structures had been to Maxwell.

The modern view of the world that emerged from Maxwell's theory is a world with two layers. The first layer, the layer of the fundamental constituents of the world, consists of fields satisfying simple linear equations. The second layer, the layer of the things that we can directly touch and measure, consists of mechanical stresses and energies and forces. The two layers are connected, because the quantities in the second layer are quadratic or bilinear combinations of the quantities in the first layer. To calculate energies or stresses, you take the square of the electric field-strength or multiply one component of the field by another. The two-layer structure of the world is the basic reason why Maxwell's theory seemed mysterious and difficult. The objects on the first layer, the objects that are truly fundamental, are abstractions not directly accessible to our senses. The objects that we can feel and touch are on the second layer, and their behaviour is only determined indirectly by the equations that operate on the first layer. The two-layer structure of the world implies that the basic processes of nature are hidden from our view.

We now take it for granted that electric and magnetic fields are abstractions not reducible to mechanical models. To see that this is true, we need only look at the units in which the electric and magnetic fields are supposed to be measured. The conventional unit of electric field-strength is the square-root of a joule per cubic meter. A joule is a unit of energy and a meter is a unit of length, but a square-root of a joule is not a unit of anything tangible. There is no way we can imagine measuring directly the square-root of a joule. The unit of electric field-strength is a mathematical abstraction, chosen so that the square of a field-strength is equal to an energy-density that can be measured with real instruments. The unit of energy-density is a joule per cubic meter, and therefore we say that the unit of field-strength is the square-root of a joule per cubic meter. This does not mean that an electric field-strength can be measured with the square-root of a

calorimeter. It means that an electric field-strength is an abstract quantity, incommensurable with any quantities that we can measure directly.

Sixty years after Maxwell published his theory, Schrödinger and Heisenberg and Dirac invented quantum mechanics. Quantum mechanics was accepted much more rapidly than Maxwell's theory, because it made numerous definite predictions about atomic processes and experiments showed that all the predictions were correct. Within a year or two, everyone believed in quantum mechanics as a practical tool for calculating the basic processes of physics and chemistry. Nature evidently obeyed the rules of quantum mechanics. But the meaning of quantum mechanics remained controversial. Although quantum mechanics was rapidly accepted, it was not rapidly understood. Sharp differences of opinion about the interpretation of quantum mechanics have persisted for seventy years. It took about thirty years after Maxwell before his equations were generally understood. It will take at least twice as long to reach an agreed understanding of quantum mechanics. We still have passionate arguments between believers in various interpretations of quantum mechanics, the Copenhagen interpretation, the many-worlds interpretation, the decoherence interpretation, the hidden-variables interpretation, and many others. The reason for these arguments is that the various interpreters are trying to describe the quantum world in the words of everyday language, and the language is inappropriate for the purpose. Everyday language describes the world as human beings encounter it. Our experience of the world is entirely concerned with macroscopic objects which behave according to the rules of classical physics. All the concepts that appear in our language are classical. Each of the interpretations of quantum mechanics is an attempt to describe quantum mechanics in a language that lacks the appropriate concepts. The battles between the rival interpretations continue unabated and no end is in sight.

It may be helpful for the understanding of quantum mechanics to stress the similarities between quantum mechanics and the Maxwell theory. In two ways, the Maxwell theory may provide a key to the mysteries of quantum mechanics. First, the attempts to understand quantum mechanics in terms of a language based on classical concepts are similar to the attempts to understand the Maxwell theory in terms of mechanical models. The Maxwell theory became elegant and intelligible only after the attempts to represent electromagnetic fields by means of mechanical models were abandoned. Similarly, quantum mechanics becomes elegant and intelligible only after attempts to describe it in words are abandoned. To see the beauty of the Maxwell theory it is necessary to move away from mechanical models and into the abstract world of fields. To see the beauty of quantum mechanics it is necessary to move away from verbal descriptions and into the abstract world of geometry. Mathematics is the language that nature speaks. The language of mathematics makes the world of Maxwell fields and the world of quantum processes equally transparent.

The second connection between Maxwell theory and quantum mechanics is a deep similarity of structure. Like the Maxwell theory, quantum mechanics divides the

universe into two layers. The first layer contains the wave-functions of Schrödinger, the matrices of Heisenberg and the state-vectors of Dirac. Quantities in the first layer obey simple linear equations. Their behaviour can be accurately calculated. But they cannot be directly observed. The second layer contains probabilities of particle collisions and transmutations, intensities and polarisations of radiation, expectation-values of particle energies and spins. Quantities in the second layer can be directly observed but cannot be directly calculated. They do not obey simple equations. They are either squares of first-layer quantities or products of one first-layer quantity by another. In quantum mechanics just as in Maxwell theory, Nature lives in the abstract mathematical world of the first layer, but we humans live in the concrete mechanical world of the second layer. We can describe Nature only in abstract mathematical language, because our verbal language is at home only in the second layer. Just as in the case of the Maxwell theory, the abstract quality of the first-layer quantities is revealed in the units in which they are expressed. For example, the Schrödinger wave-function is expressed in a unit which is the square root of an inverse cubic meter. This fact alone makes clear that the wave-function is an abstraction, forever hidden from our view. Nobody will ever measure directly the square root of a cubic meter.

The ultimate importance of the Maxwell theory is far greater than its immediate achievement in explaining and unifying the phenomena of electricity and magnetism. Its ultimate importance is to be the prototype for all the great triumphs of twentieth-century physics. It is the prototype for Einstein's theories of relativity, for quantum mechanics, for the Yang-Mills theory of generalised gauge invariance, and for the unified theory of fields and particles that is known as the Standard Model of particle physics. All these theories are based on the concept of dynamical fields, introduced by Maxwell in 1865. All of them have the same two-layer structure, separating the world of simple dynamical equations from the world of human observation. All of them embody the same quality of mathematical abstraction that made Maxwell's theory difficult for his contemporaries to grasp. We may hope that a deep understanding of Maxwell's theory will result in dispersal of the fog of misunderstanding that still surrounds the interpretation of quantum mechanics. And we may hope that a deep understanding of Maxwell's theory will help to lead the way toward further triumphs of physics in the twenty-first century.
