

## Performance Limits of FEC and Modulation Formats in Optical Fiber Communications

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### I. INTRODUCTION

In optical fiber communication system designs, reducing the required optical signal-to-noise ratio (OSNR) and, thus lower the channel power and/or extend the distance between optical amplifiers, is an effective way to cut the system cost. In recent years, the required OSNR for reliable optical fiber communications has been dramatically reduced with forward error correction (FEC) and modulation techniques [1–5]. A turbo product code (TPC) has been implemented at 10-Gb/s information rate providing a 10.1-dB net coding gain [3]. The differential binary-phase-shift-keying (DBPSK) format has been successfully applied in a 10-Gb/s 13-Mm field trial [4]. If employing both the TPC and DBPSK, the required OSNR at  $10^{-15}$  BER can be reduced by  $\sim 11.4$  dB compared to an uncoded on-off keying system. With so much progress already made, it is important to understand how much further the required OSNR can be reduced with the combination of sophisticated modulation and FEC.

The fundamental limit on performance of communication systems was derived by Shannon [6] in his theorem for channel capacity, which defines the maximum information rate that a noisy channel allows for an arbitrarily low BER. In optical fiber communications, the channel capacity gives a lower bound on the required OSNR at a target information rate. Our focus in this paper is to approach the lower OSNR bound, which is important especially for long-haul transmissions.

A channel model assuming additive white Gaussian noise (AWGN) is commonly used in channel capacity studies. For optical fiber communications, however, efforts have been made to evaluate the channel capacities taking into account some particular system features such as non-Gaussian noise statistics and fiber nonlinearities [7–14]. A comprehensive comparison on spectral efficiencies of several modulation schemes considering one polarization transmission can be found in [14].

Implementations of coherent detection based on high-speed digital signal processing have been recently proposed [15–18]. These techniques may enable further reduction of required OSNR of which, however, the evaluation involves two polarizations and has not been thoroughly exploited. In this paper we evaluate the channel capacity of single-mode fiber systems considering two polarizations. We assume systems perturbed by linear amplified spontaneous emission noise (ASEN), namely, ASEN channels. We also assume an ideal coherent receiver that detects a channel's full optical field. We show that the system required OSNR can still be significantly reduced with combined multidimensional modulation and large-overhead FEC [19].

### II. CAPACITY OF AWGN AND ASEN CHANNELS

A channel in which any particular signal function can be specified by  $n$  real numbers is said to have  $n$  dimensions. According to Shannon's theory, a  $T$ -second transmission ( $T \rightarrow \infty$ ) in an AWGN channel of band  $W$  can be specified by  $2WT$  numbers, and the channel capacity is  $2WT \times 0.5 \log_2(1+P/N)$  bits/transmission, where  $P$  and  $N$  are average signal and noise power, respectively, including all channel dimensions. In other words, the AWGN channel has  $2WT$  dimensions each having a capacity of  $0.5 \log_2(1+P/N)$  bits/transmission.

A single-mode fiber channel can support two orthogonal polarizations, and the polarization of light provides another information carrier. Assuming no polarization coupling, noise samples in orthogonal polarizations are independent Gaussian random variables (RV). Also, ASEN is additive to signal in parallel polarization and separable from signal in orthogonal polarization. Hence, given  $W$ , the ASEN channel has twice as much channel dimensions as the AWGN channel and, thus, a channel capacity  $C_{\text{ASEN}} = 2C_{\text{AWGN}} = 2W \log_2(1+P/N)$  bits/s, where  $P$  and  $N$  include two polarizations in the evaluation of  $C_{\text{ASEN}}$ . In terms of signal-to-noise ratio (SNR) per bit, i.e.  $(P/N)/(B/W)$ , where  $B$  is the information rate, the ASEN channel requires half as much SNR per bit when providing twice as much channel capacity as the AWGN channel of the same bandwidth.

OSNR is a commonly used measure in optical fiber communications, defined as the ratio of total optical signal power to the power of unpolarized ASEN in a resolution bandwidth  $W_0$ . For a fair comparison of OSNR measured at different information rate  $B$ , the ratio of  $B/W_0$  should be kept a constant. Substitute  $\text{OSNR}W_0/W$  for  $P/N$ , a bandwidth normalized channel capacity is given by  $C_{\text{ASEN}}/W = 2 \log_2(1+\text{OSNR}W_0/W)$  bit/s/Hz. It can be proved when  $C_{\text{ASEN}}/W \rightarrow 0$ ,  $\text{OSNR} \rightarrow 0.5 \ln 2 (B/W_0)$ , which is the Shannon limit on ASEN channels, below which information can never be transmitted error free.

### III. APPROACH THE LIMIT WITH FEC AND MODULATION TECHNIQUES

In an ideal ASEN channel with a carrier frequency  $f_c$  and signal frequencies within  $[-W/2-f_c, -f_c+W/2] \cup [-W/2+f_c, f_c+W/2]$ ,  $f_c \gg W$ , a  $T$ -second transmission ( $T \rightarrow \infty$ ) can be divided into  $WT$  4-dimensional (4-D) symbols with their centers spaced  $1/W$  second apart. The 4 symbol dimensions physically correspond to the 4 combinations of 2 polarizations with 2 quadrature dimensions of the carrier. The projection of the  $k$ th symbol on a plane defined by a symbol dimension and time ( $t$ ) is a sinc function weighted by an information-bearing RV, i.e.  $X_k \text{sinc}(Wt-k)$ . The polarization and phase of the carrier,  $W$ , and  $T$  together define a  $4WT$ -D signal space. In principle the ASEN channel capacity can only be achieved when signal power is evenly distributed in all  $4WT$  dimensions, and all  $4WT$  information-bearing RVs are independent identical Gaussian RVs.

In practical systems, however, it is impossible to use sinc pulses of infinite time duration and/or information-bearing RVs having an infinite alphabet. Nevertheless, it is practical to generate band-limited optical pulses of duration  $2/W$  and divide a  $T$ -second transmission into  $WT/2$  time slots. In addition, quadrature amplitude/phase modulation and polarization multiplexing can help to generate 4-D symbols. These techniques together, referred to as multidimensional modulations in this paper, can generate  $2WT$ -D signals. The information-bearing RVs can be practically generated by binary or multilevel signaling in each dimension. In this paper, binary or multilevel signaling refers to the signaling in one signal dimension or mathematically on a real line, to be distinguished from the multidimensional modulations.

Figure 1 plots the spectral efficiency ( $R = C_{\text{ASEN}}/W$ ) as a function of OSNR ( $B/W_0 = 0.8$ ) of several phase-shift-keying (PSK) and quadrature amplitude modulations (QAM) assuming ideal FEC. Depending on whether a system concentrates signal power in 1 polarization or distributes signal power in two polarizations, it is labeled with “1pol.” or “2pol.”. All formats except that labeled with “RZ” (return-to-zero) assume sinc pulses. The 2pol. RZ-QPSK assumes RZ pulses achieving half the spectral efficiency of sinc pulses, which makes it overlap with the 1pol. QPSK and 2pol. BPSK curves. There are mainly two features that differentiate the curves: 1) the asymptotic limit ( $L$ ) at high OSNR, 2) the slope at low OSNR. When combined with a modulation, FEC overhead =  $L/R - 1$ , which implies that the FEC overhead can be enlarged with a higher level modulation at a desired spectral efficiency. For example, the 2pol. QPSK, 1pol. 16QAM, and 1pol. 16PSK all have  $L = 4$  bits/s/Hz and can support a 400% FEC overhead at  $R = 0.8$  bit/s/Hz. Either a higher-level multidimensional modulation or a higher-level multilevel signaling can enlarge FEC overhead, but the former is more effective for reducing required OSNR due to the steeper slope of the corresponding spectral efficiency curve.

It can be proved that, when OSNR is low, the slope of a spectral efficiency curve is linearly proportional to the number of signal dimensions, but it is insensitive to multilevel signaling. For example, from 1pol. BPSK to 1pol. QPSK and to 2pol. QPSK, the number of signal dimensions is doubled twice, and so is the slope of the spectral efficiency curves. On the other hand, the 2pol. QPSK and the ultimate ASEN channel limit show little difference for  $\text{OSNR} < -4$  dB, although in each dimension the former does only symmetric binary signaling and the later assumes infinite-level signaling. Hence, the most effective way of lowering required OSNR without losing spectral efficiency is to enlarge FEC overhead with multidimensional modulations and do just binary signaling in each dimension.

Figure 1 also plots a point corresponding to a linear RZ-DBPSK system using the TPC described in [3], from where the required OSNR can be reduced by about 8 dB before touching the limit at 0.8 bit/s/Hz. To realize the potential improvement, it requires 2pol. QPSK + 400%-overhead ideal FEC + sinc pulse. But even with practical RZ pulses, the combined 2pol. RZ-QPSK and a 150%-overhead ideal FEC can still potentially lower the required OSNR by about 7.2 dB. Without an enlarged overhead from multidimensional modulations, however, an ideal 25%-overhead FEC alone can only further reduce the required OSNR by about 1.4 dB in the RZ-DBPSK system.

#### IV. CONCLUSION

Efficient communications in AWGN channels require efficient distribution of power in time, whereas in ASEN channels it involves also power distribution in 2 polarizations. Using polarization of light as an additional information carrier, an ASEN channel requires half as much SNR per bit when providing twice as much channel capacity as an AWGN channel of the same bandwidth. The required OSNR in current optical fiber communications can still be significantly reduced with combined modulation and FEC without losing spectral efficiency. The most effective way to do so is to enlarge FEC overhead with multidimensional modulations and do symmetric binary signaling in each dimension. How to efficiently design coding and modulation as one entity, a concept first introduced by Ungerboeck [20], is an interesting area to exploit in optical fiber communications research.

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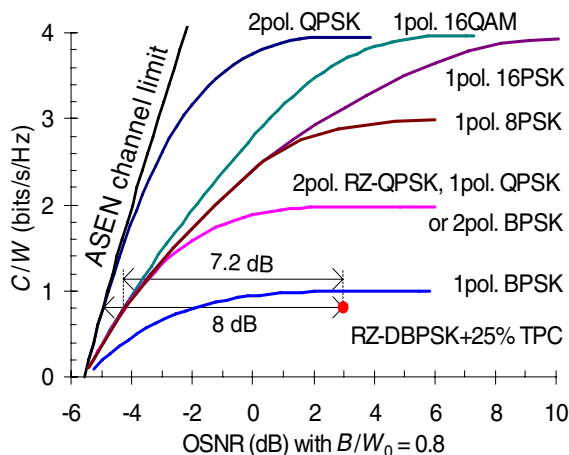


Figure 1: ASEN channel spectral efficiencies