

# Advanced Computational Tools for Antenna Placement Studies

**Dr. C. J. Reddy**

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[www.emssusa.com](http://www.emssusa.com)



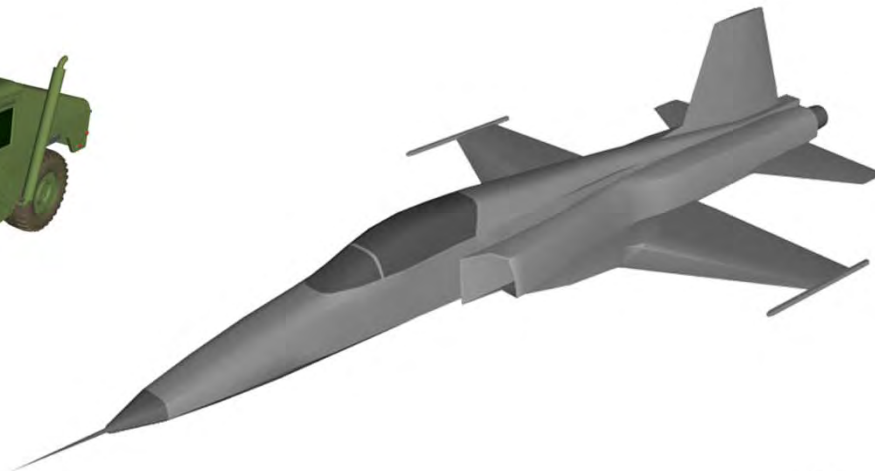
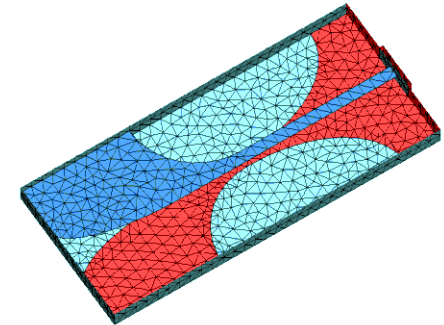
**FEKO**  
Comprehensive Electromagnetic Solutions



# Problem/Issue

## Antenna and Antenna Array Analysis on various platforms

- Antenna Element Design
- Array Design
- In-situ analysis of the array integrated on platforms



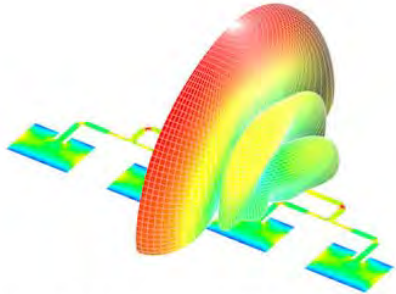
# Problem/Issue



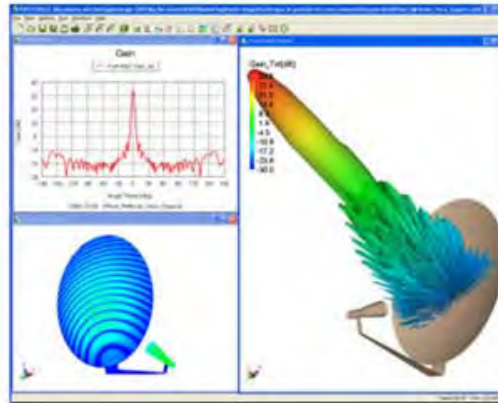
## EM Analysis tools are required for

- Design of antenna elements
- Array analysis
- *In-situ* performance analysis of antenna arrays mounted on aircraft, shipboard and ground platforms

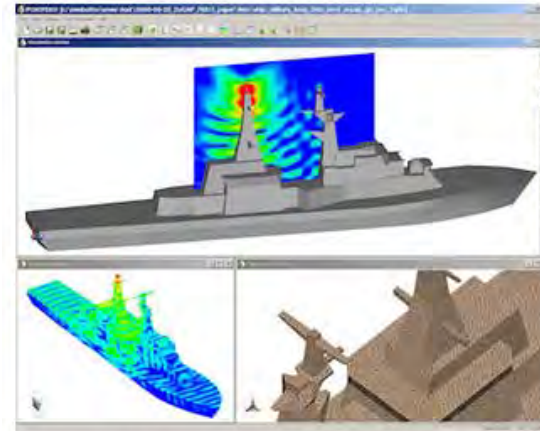
# Problem/Issue



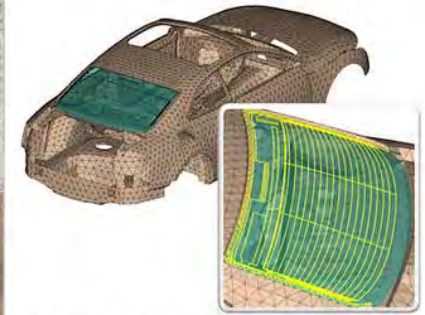
Radiation pattern and surface currents of a microstrip patch array with corporate feed.



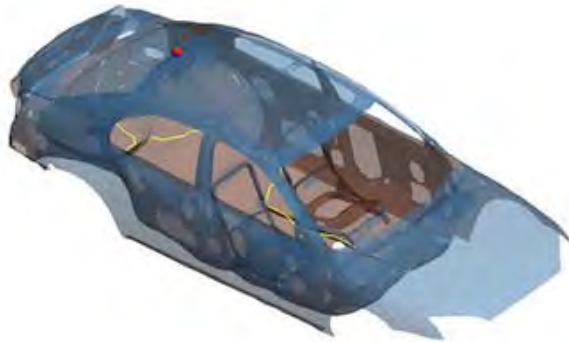
Characterisation of a horn-fed reflector antenna.



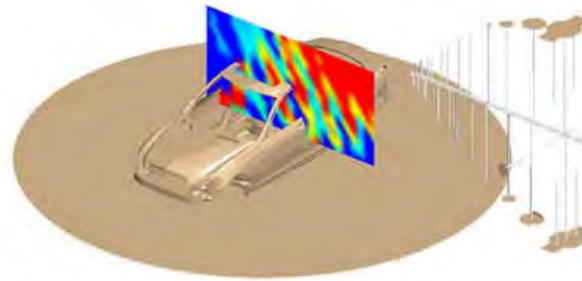
Studying VHF antenna placement on a naval platform.



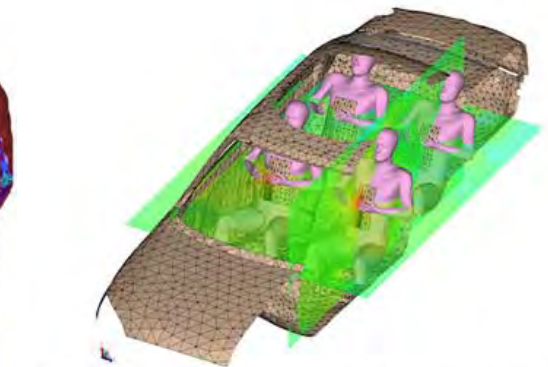
Computational model of a car with integrated windscreen antenna.



Specification of cable paths for automotive cable coupling analysis.



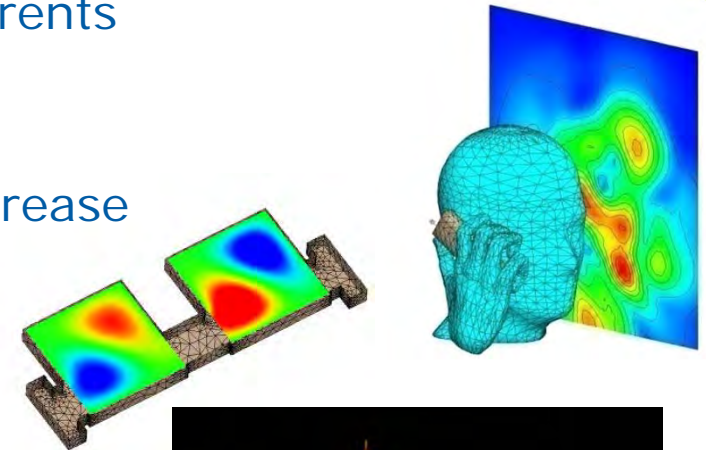
Automotive EMC measurement installation with log periodic source.



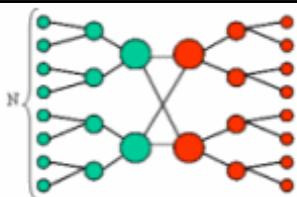
Radiation hazard analysis for personal radios carried inside an automobile. The human phantoms are modelled with the FEM, while the antennas and bodywork are modelled with the MoM.

# Computational Electromagnetics

- Numerical solution based on approximation of currents and/or fields
- Desirable properties of CEM methods:
  - Approximation may be reduced in order to increase accuracy, approaching the analytical result
  - Computational cost (CPU time & memory) must be as low as possible



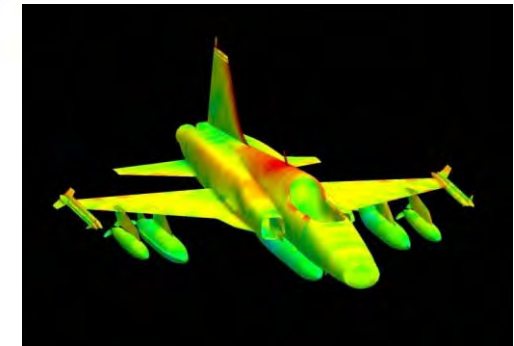
CEM tool



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{A}$$



Computer modeling

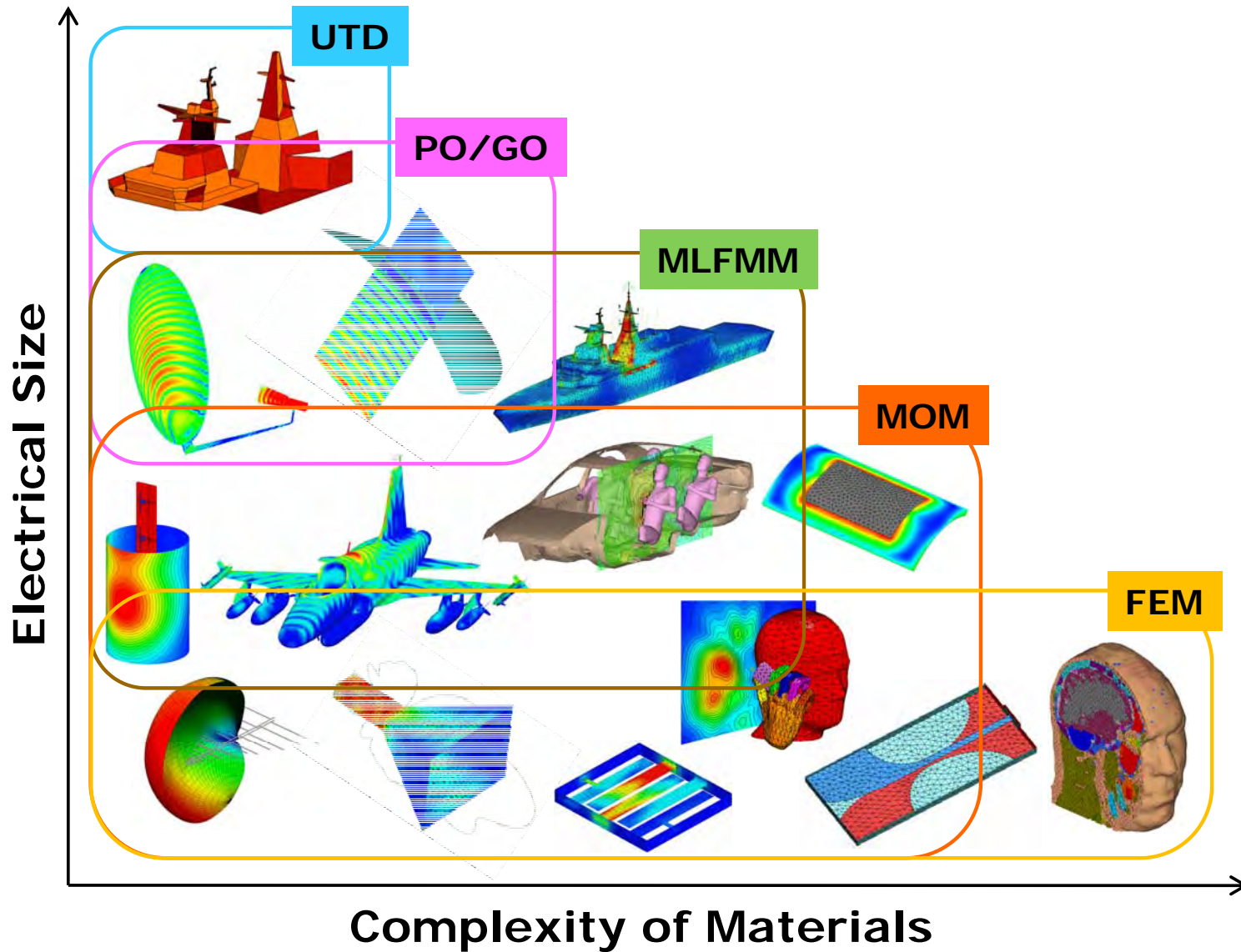
Numerical analysis

# Numerical Techniques

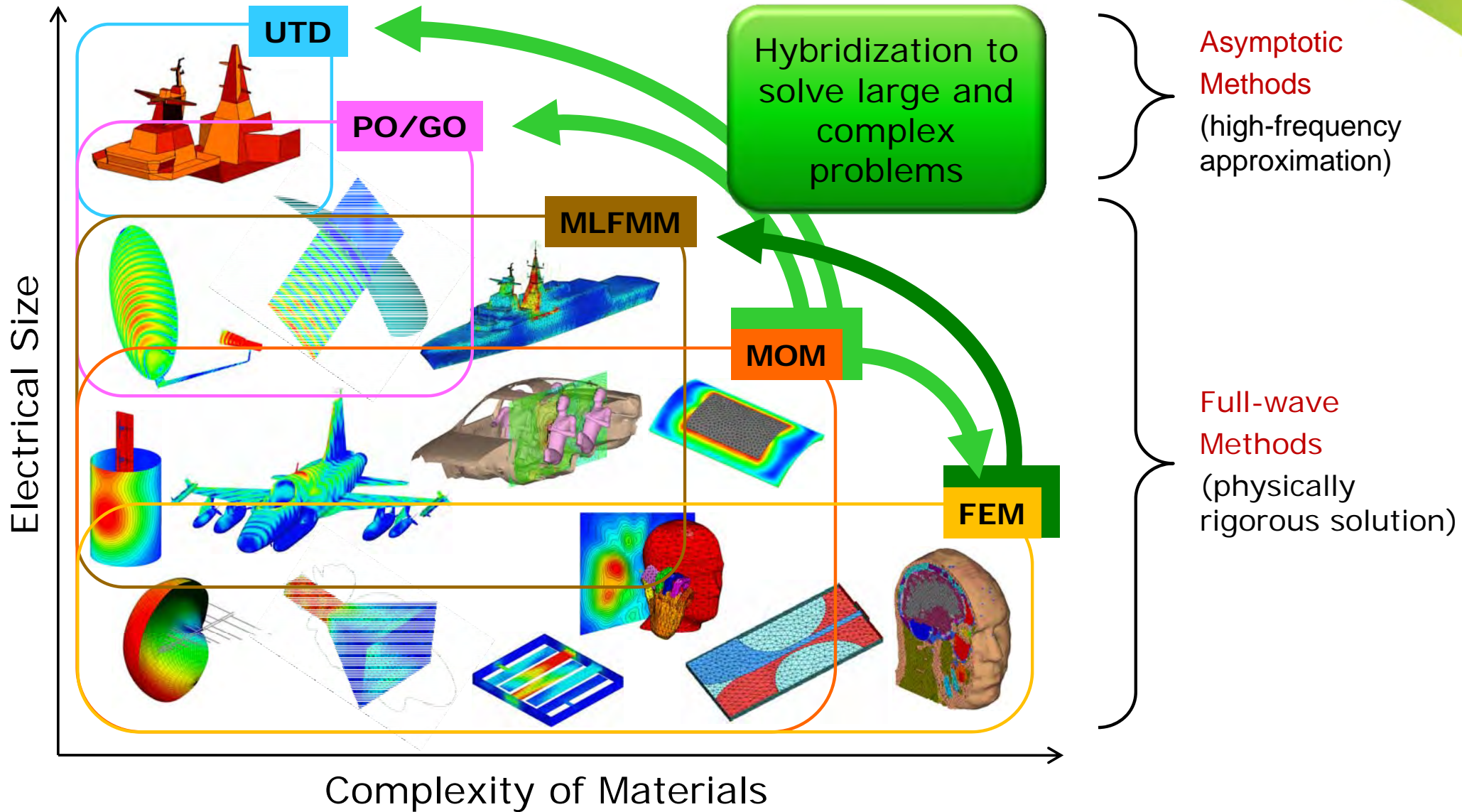


- Method of Moments
  - Antenna Element Design
- Multilevel Fast Multipole Method (MLFMM)
  - Antenna/Array Design/Platform Analysis
- Hybrid MoM/Physical Optics (PO)
  - Platform Analysis
- Hybrid MoM/Uniform Theory of Diffraction (UTD)
  - Platform Analysis

# EM Simulation Map: CEM Methods

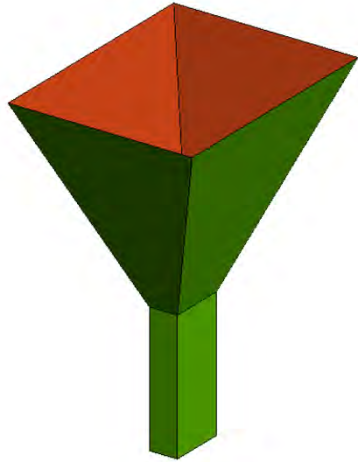


# EM Simulation Map: CEM Methods



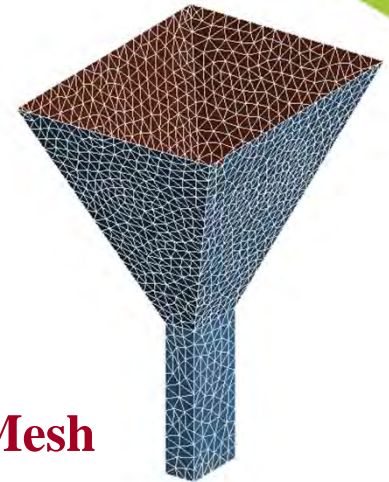


# Method of Moments

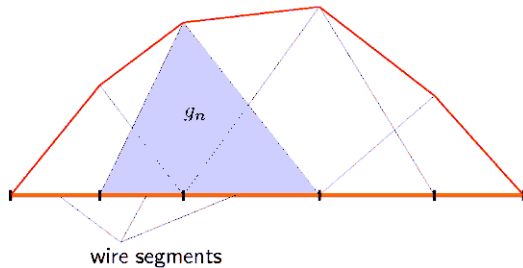


**Geometry**

- Create CAD Model of the geometry
- Create surface mesh – triangles
- Applying the equivalence principle electric or magnetic currents assumed to be unknowns
- RWG basis functions are used
- A set of linear equations are formed



**Mesh**



$$\mathbf{Z} \mathbf{I} = \mathbf{V}$$

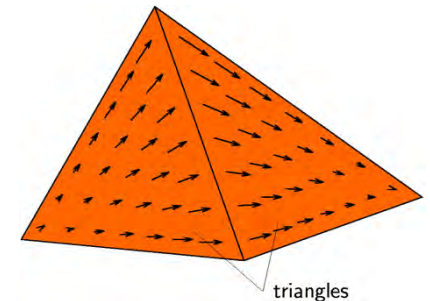
$\mathbf{Z} = \text{NXN complex matrix}$

$\mathbf{I} = \text{Unknown current vector}$

$\mathbf{V} = \text{Known Excitation vector}$

**Linear Basis Functions on wire segments**

- Solving this equation, unknown currents on each triangle is found

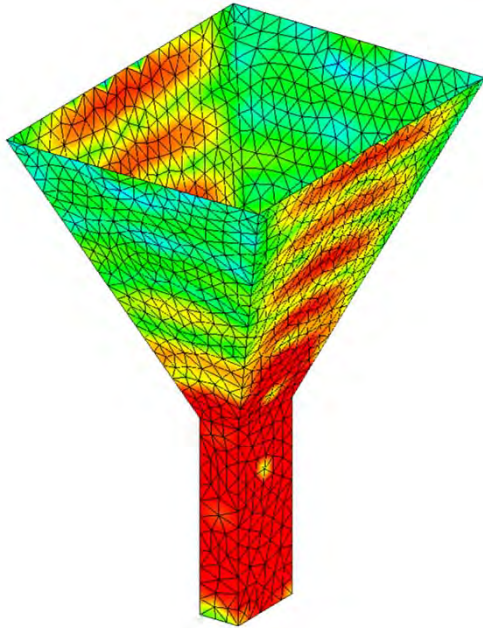


**RWG Basis Functions on triangles**

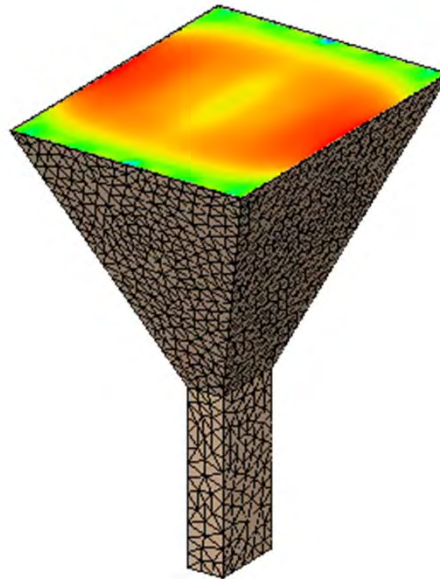
# Method of Moments



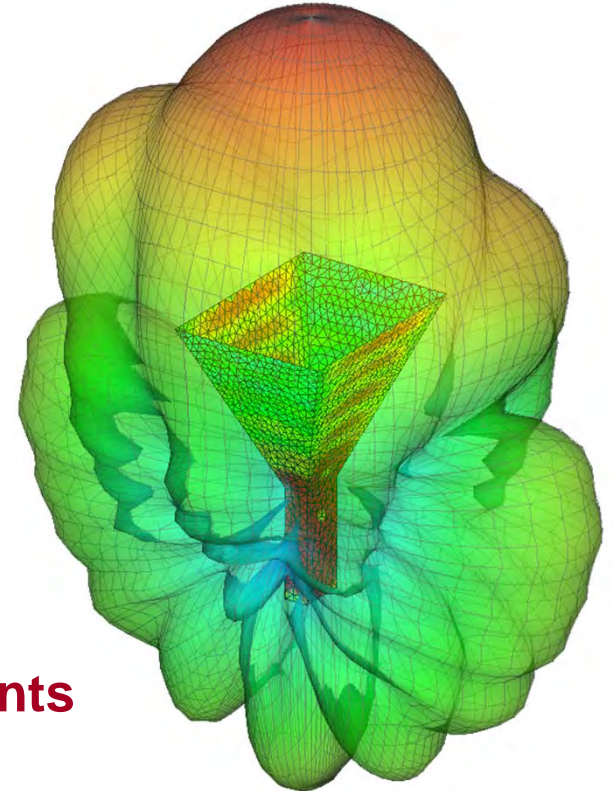
**Surface Currents**



**Near Fields**



**Radiation Patterns**

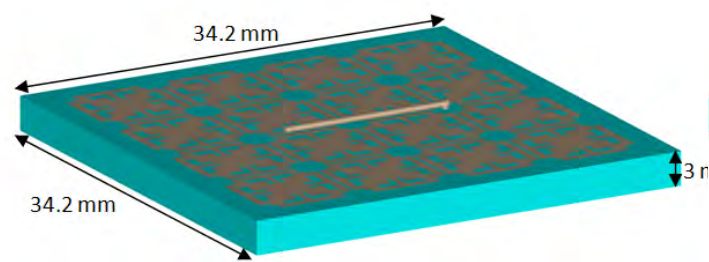


**Antenna Characteristics can be found from the currents calculated:**

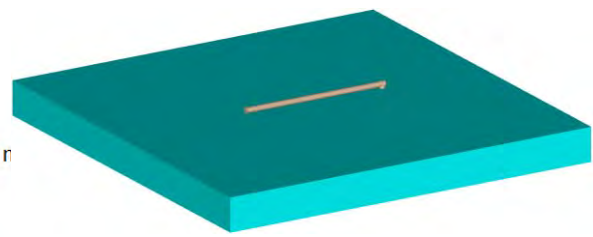
- **Near- or Far-fields**
- **Input impedances**
- **S-parameters etc**



# Very Thin Low-Profile Antenna Using Novel High Impedance Surface (HIS) Ground Plane



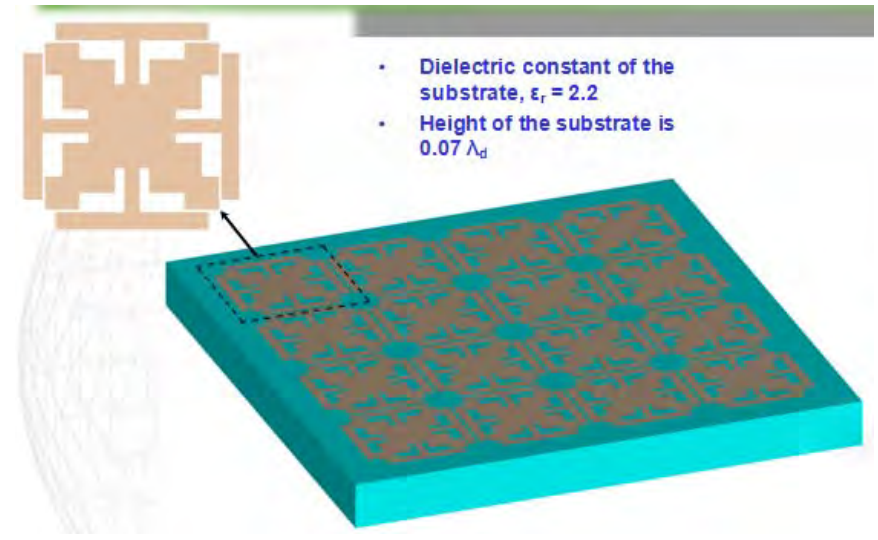
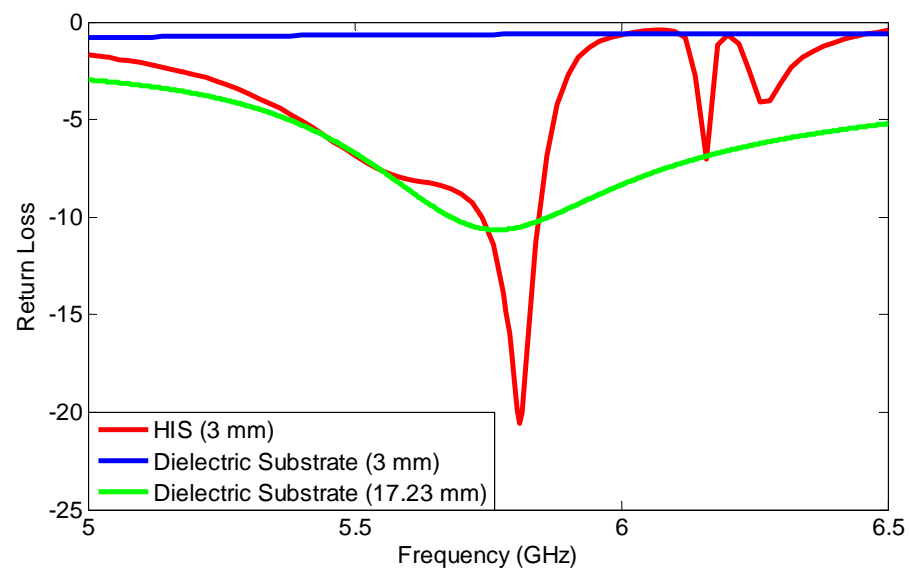
HIS with 3mm thickness



Dielectric 3mm thickness



Dielectric 17mm thickness



Gopinath Gampala, Rohit Sammeta and C.J.Reddy, "A thin, low profile antenna using a novel high impedance surface", Microwave Journal, Technical Feature, July 2010.

# Three Dual-Band Miniaturized Inverted F Antennas Integrated in a PDA for MIMO Applications



2006 IEEE APS Symposium

Wireless LAN (WLAN)

2.45GHz and 5.2GHz

Prototype

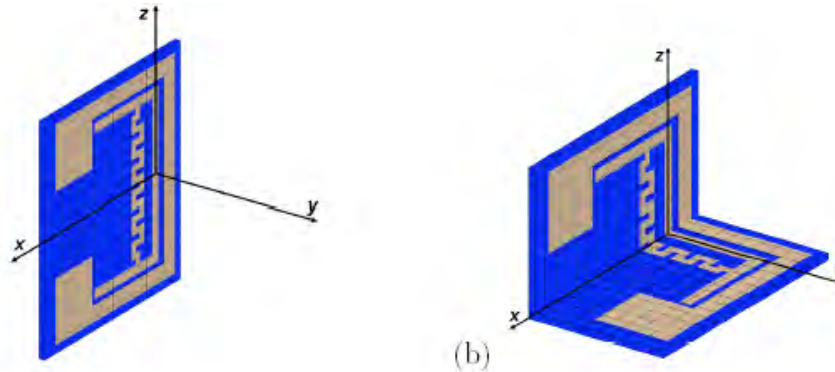
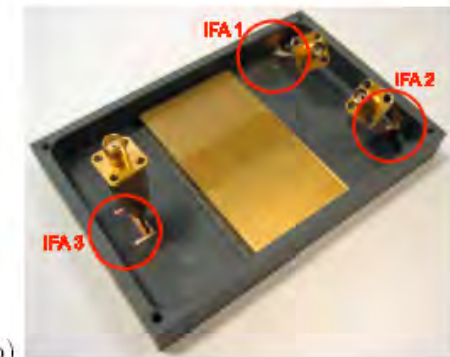


Figure 1: Simulation models for planar (a) and buckled (b) antennas

FEKO Model



(a)

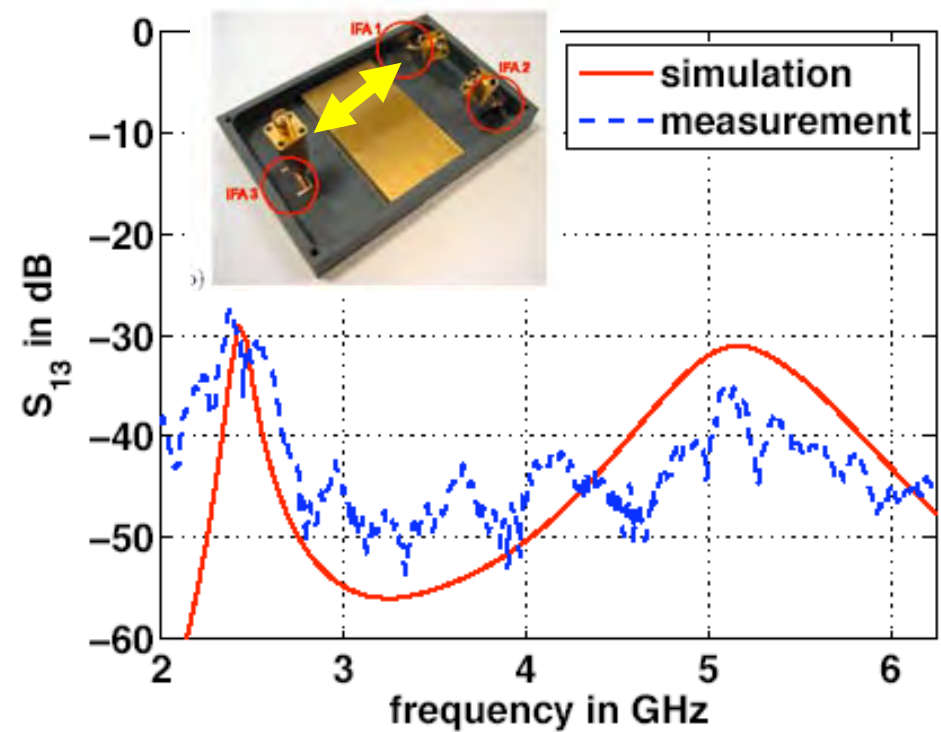
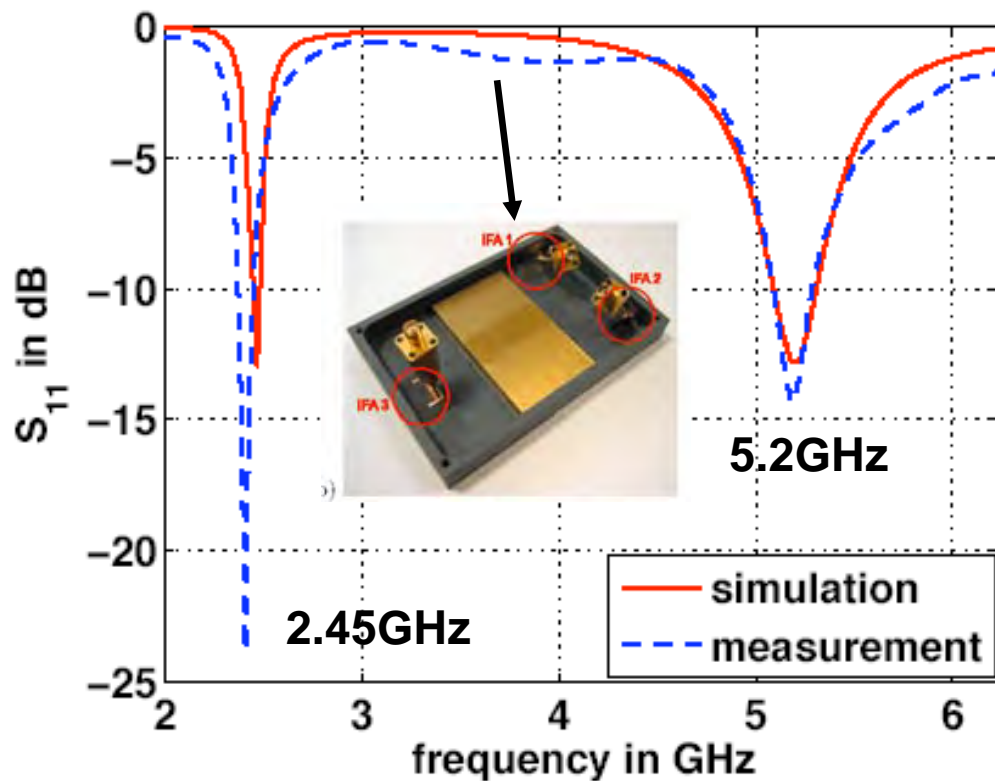


(b)

Stephan Schulteis, Christiane Kuhnert and Werner Wiesbeck, "Three Dual-Band Miniaturized Inverted F Antennas Integrated in a PDA for MIMO Applications", 2006 IEEE APS Symposium, Albuquerque, July 2006

# Three Dual-Band Miniaturized Inverted F Antennas Integrated in a PDA for MIMO Applications

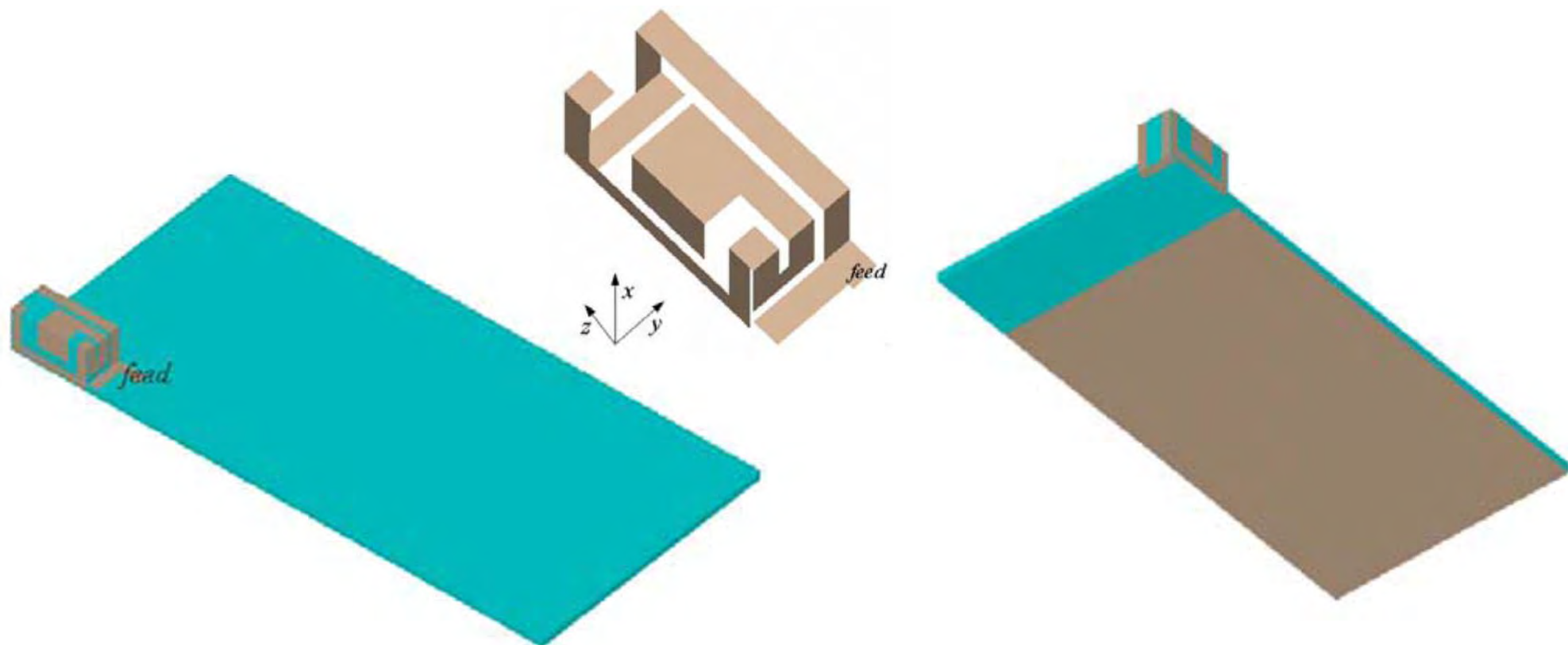
2006 IEEE APS Symposium



Stephan Schulteis, Christiane Kuhnert and Werner Wiesbeck, "Three Dual-Band Miniaturized Inverted F Antennas Integrated in a PDA for MIMO Applications", 2006 IEEE APS Symposium, Albuquerque, July 2006

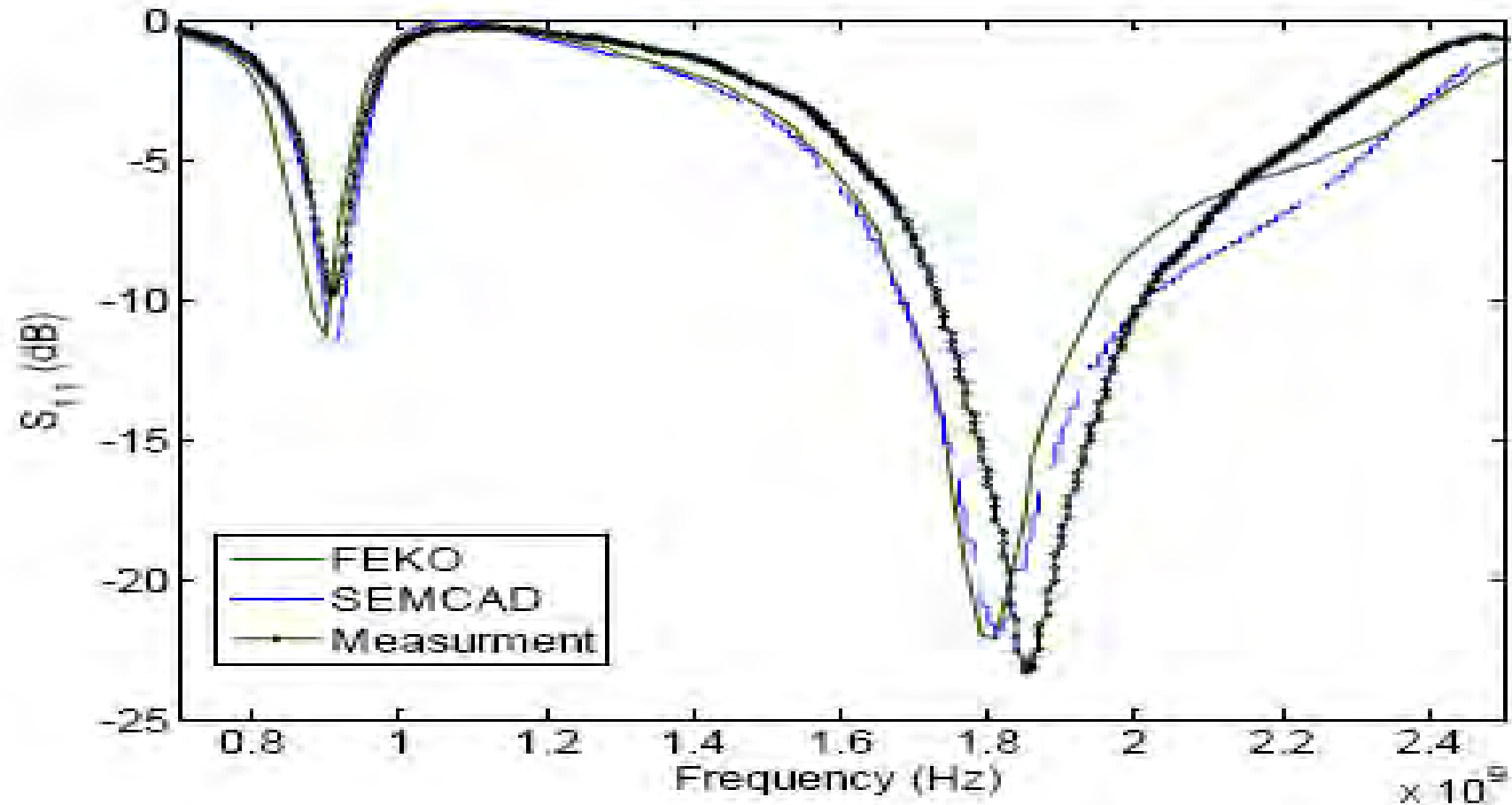
# Design and Analysis of a Novel Pent-band Antenna for Handheld Applications

GSM 800/900/1800/1900 and UMTS 2200



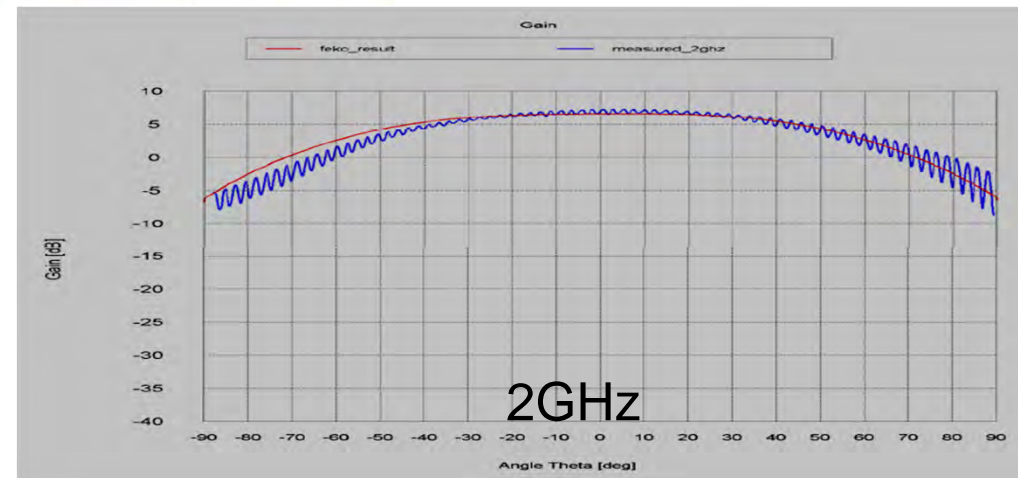
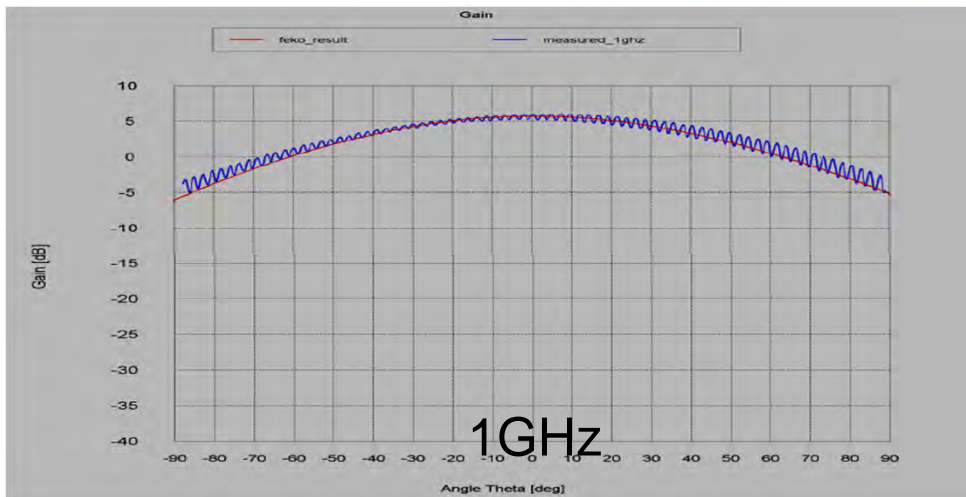
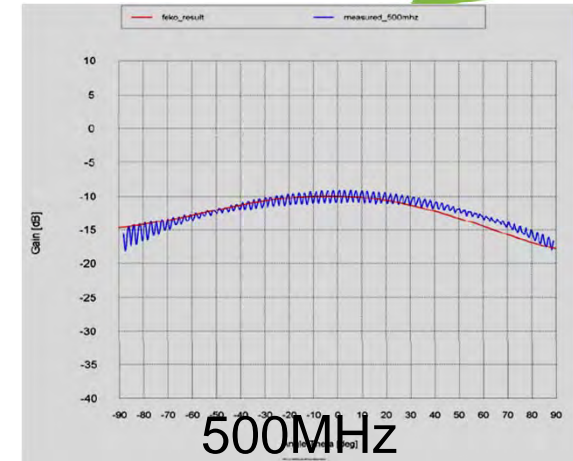
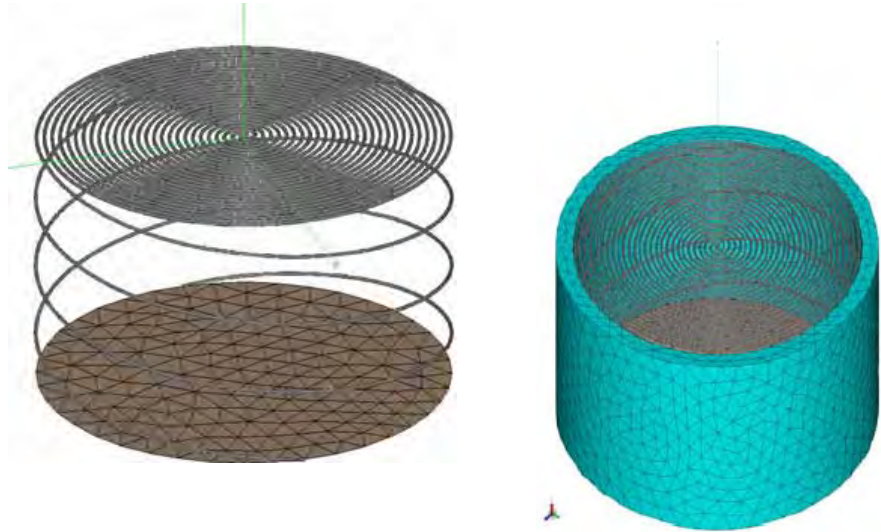
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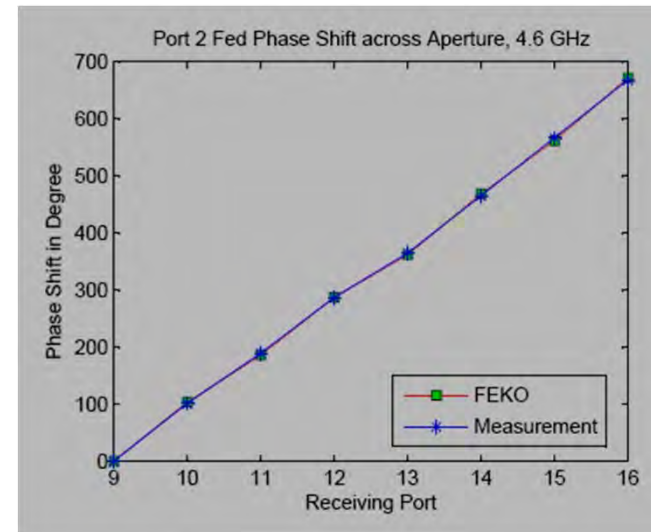
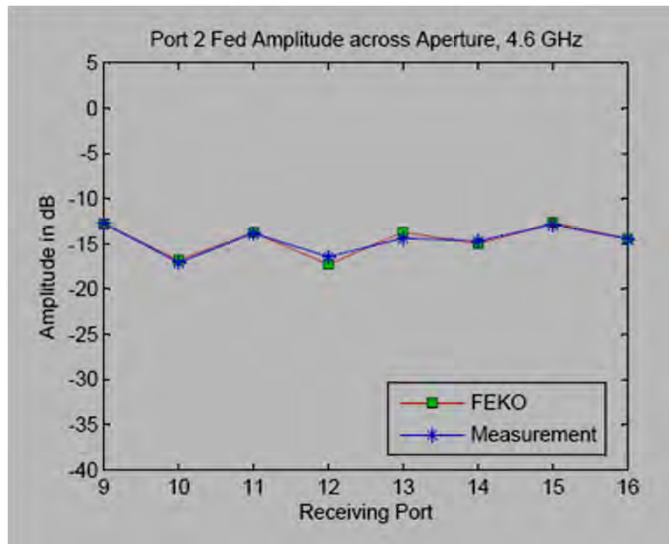
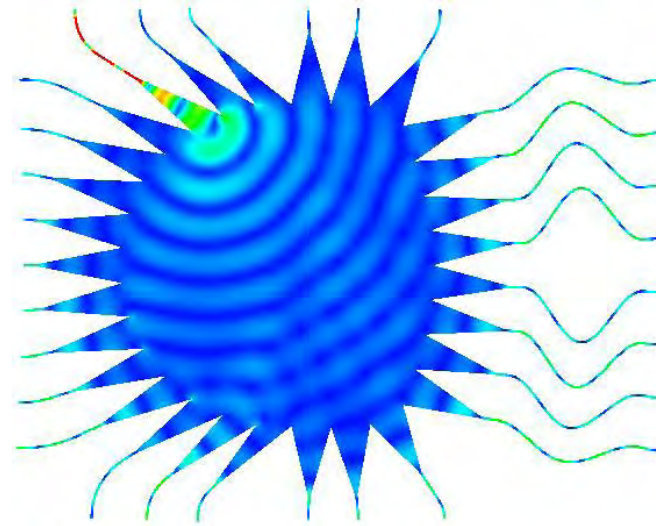
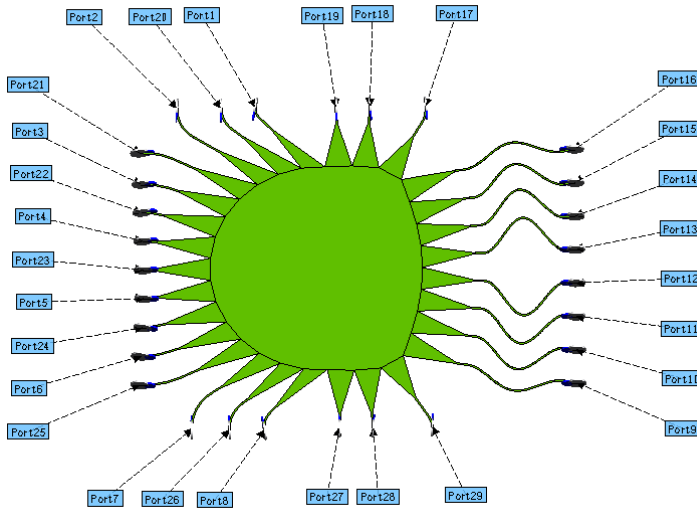
# TWO ARM ARCHIMEDEAN SPIRAL HELICAL ANTENNA WITH WRAP AROUND ABSORBER



Sandeep Palreddy, and Rudolf Cheung, "Two Arm Archimedean Spiral Helical Antenna With Wrap Around Absorber", 2009 Applied Computational Electromagnetic Symposium, Monterey Bay, March 2009



# Accurate Simulation of Rotman Lens



Junwei Dong, Amir I Zaghloul, Rensheng Sun, C.J. Reddy and Steven Weiss, "Accurate Simulation of Rotman Lens using FEKO", 2009 Applied Computational Electromagnetic Symposium, Monterey Bay, March 2009

# Antenna Placement on Electrically Platform

## ATR-42 (Avions de Transport Regional)

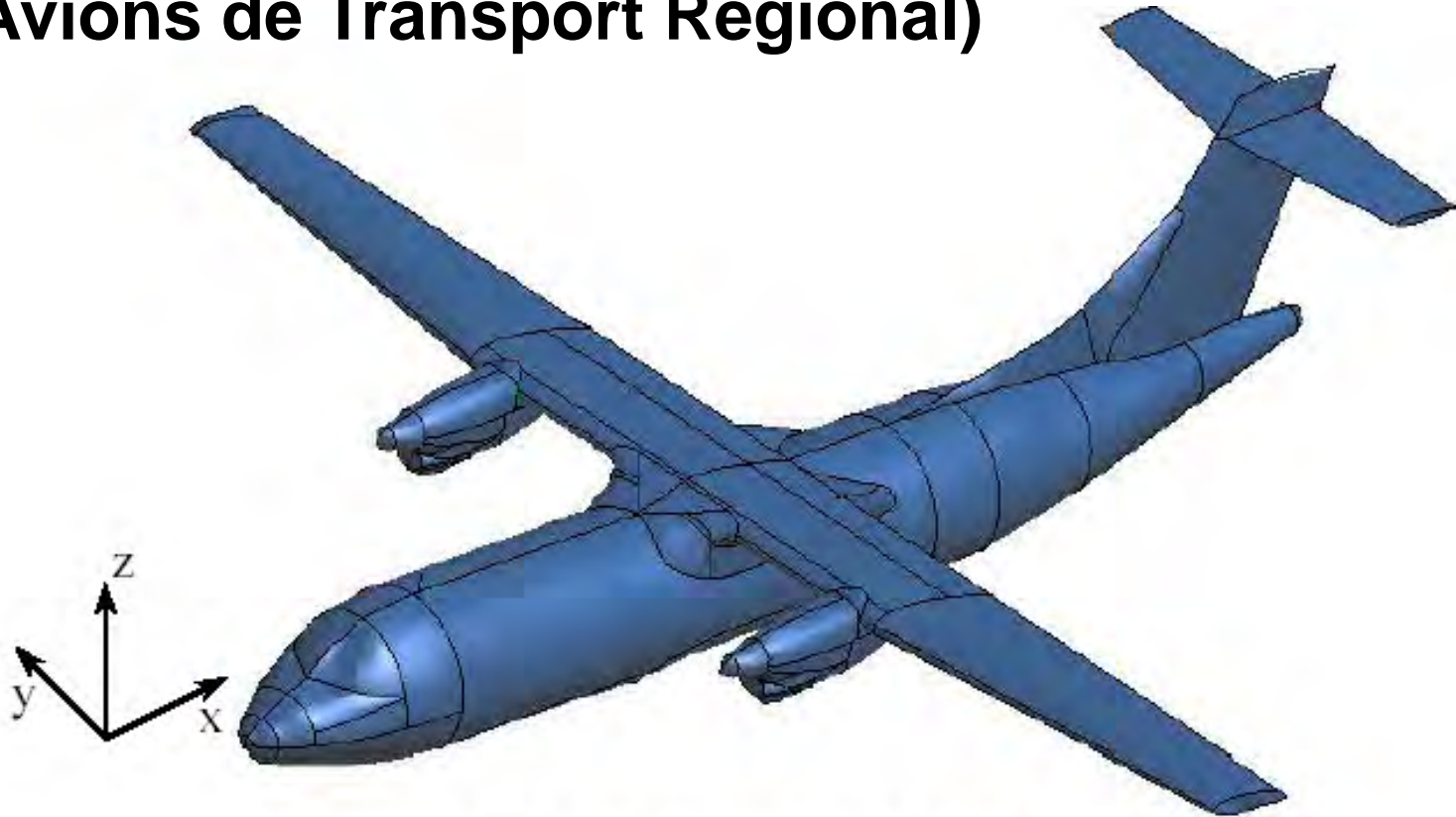


Fig. 1. CAD model of ATR42 aircraft.

M. V. T. Heckler\* and A. Dreher, Analysis of Monopoles Installed on Airframes, 2005 IEEE APS Symposium, Washington DC, July 2005

# Antenna Placement on Electrically Platform

VHF Antennas – 126MHz

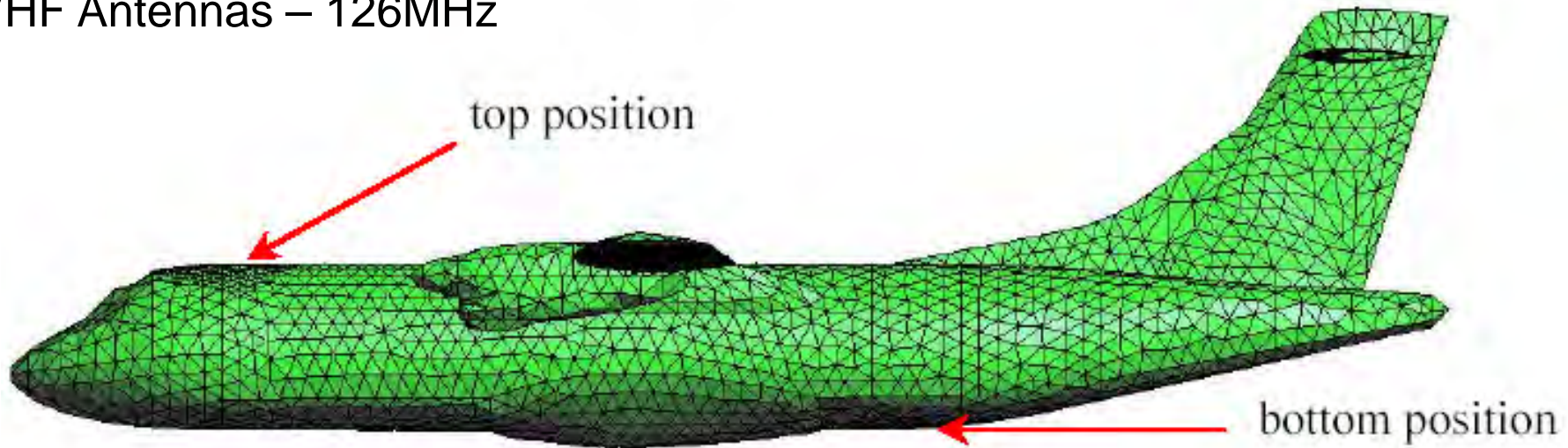


Fig. 2. Indication of the positions of VHF antennas.

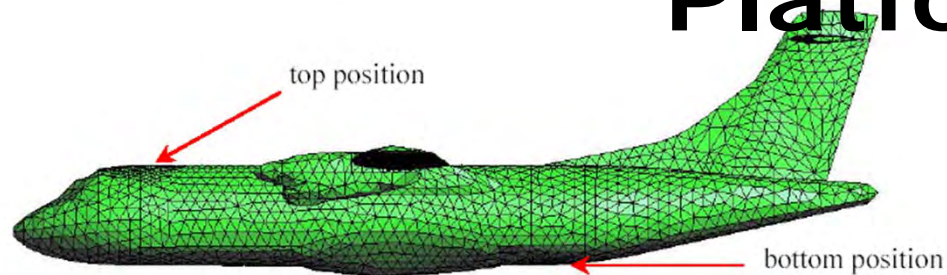
$\lambda/8$  mesh, 4,245 triangles

Calculations with MoM using Magnetic Symmetry  
MLFMM – no symmetry is used

Measurements with 1/12<sup>th</sup> scale model

M. V. T. Heckler\* and A. Dreher, Analysis of Monopoles Installed on Airframes, 2005 IEEE APS Symposium, Washington DC, July 2005

# Antenna Placement on Electrically Platform



## VHF Com Antennas – 126MHz Top Antenna Patterns

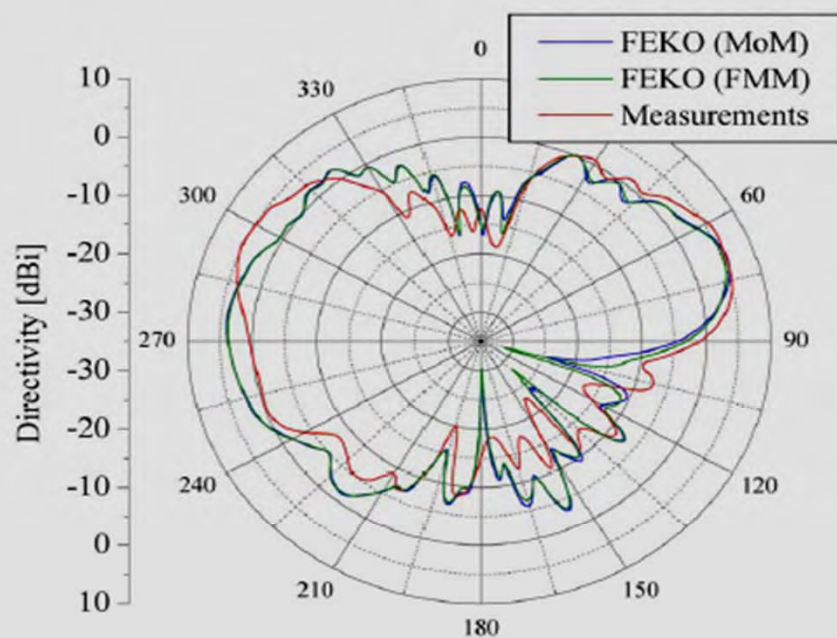


Fig. 3. Directivity in the  $\phi = 0^\circ$  (top position).

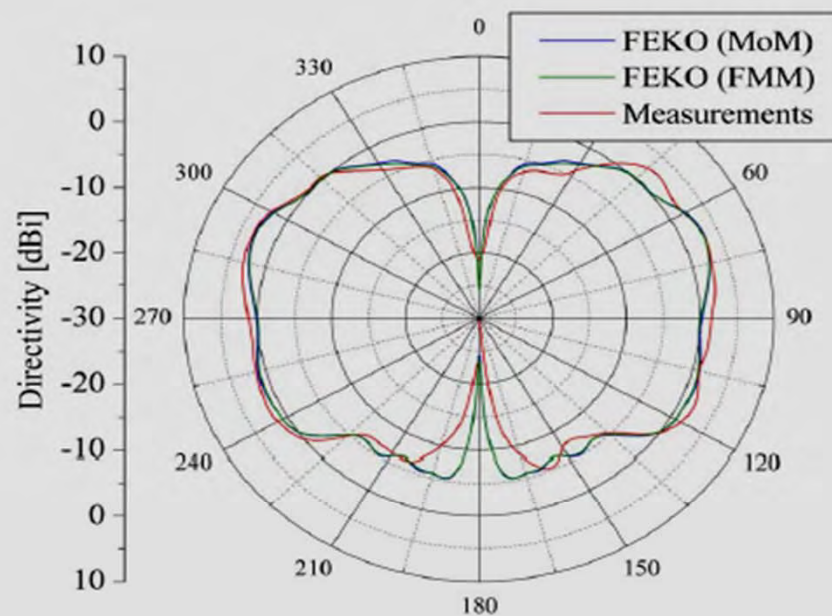
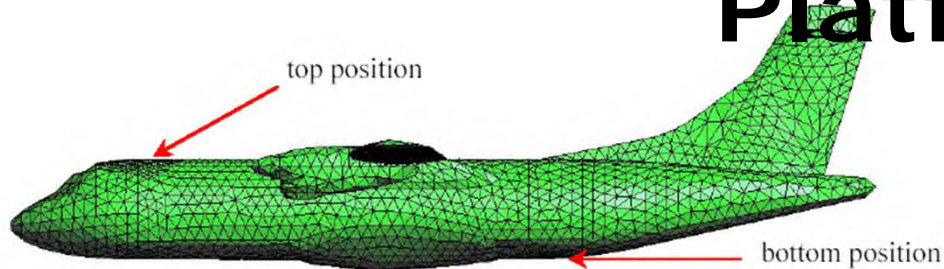


Fig. 4. Directivity in the  $\phi = 90^\circ$  (top position).

M. V. T. Heckler\* and A. Dreher, Analysis of Monopoles Installed on Airframes, 2005 IEEE APS Symposium, Washington DC, July 2005

# Antenna Placement on Electrically Platform



## VHF Com Antennas – 126MHz Bottom Antenna Patterns

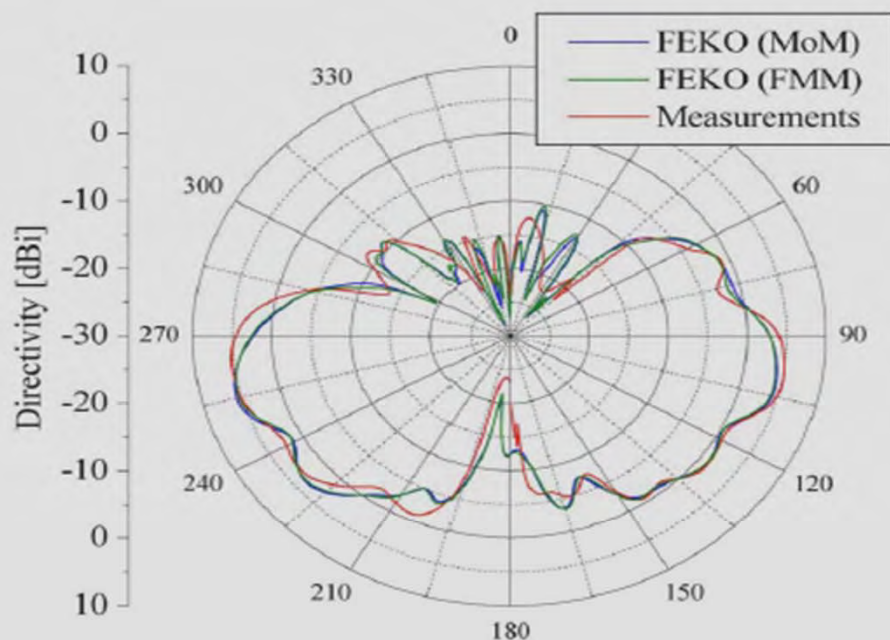


Fig. 5. Directivity in the  $\phi = 0^\circ$  (bottom position).

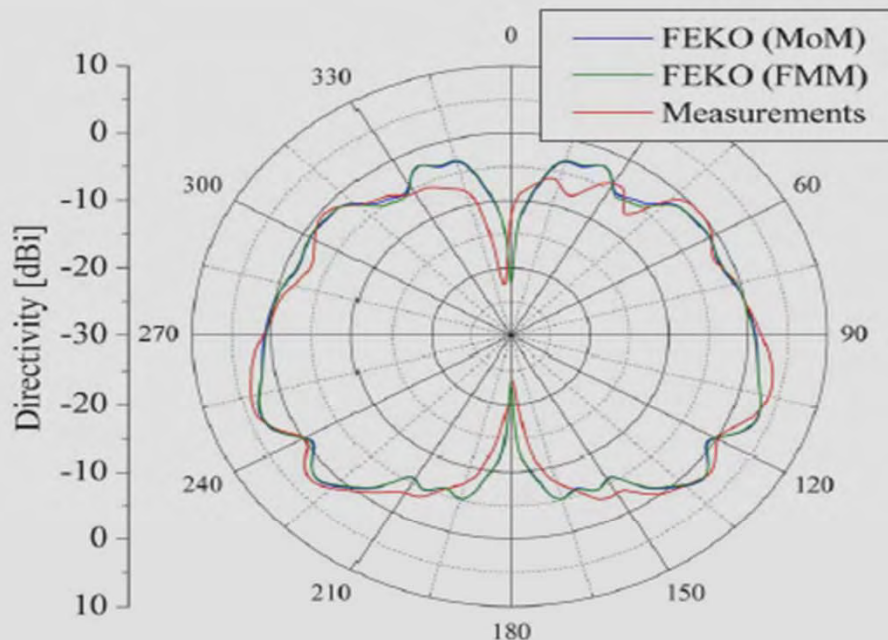
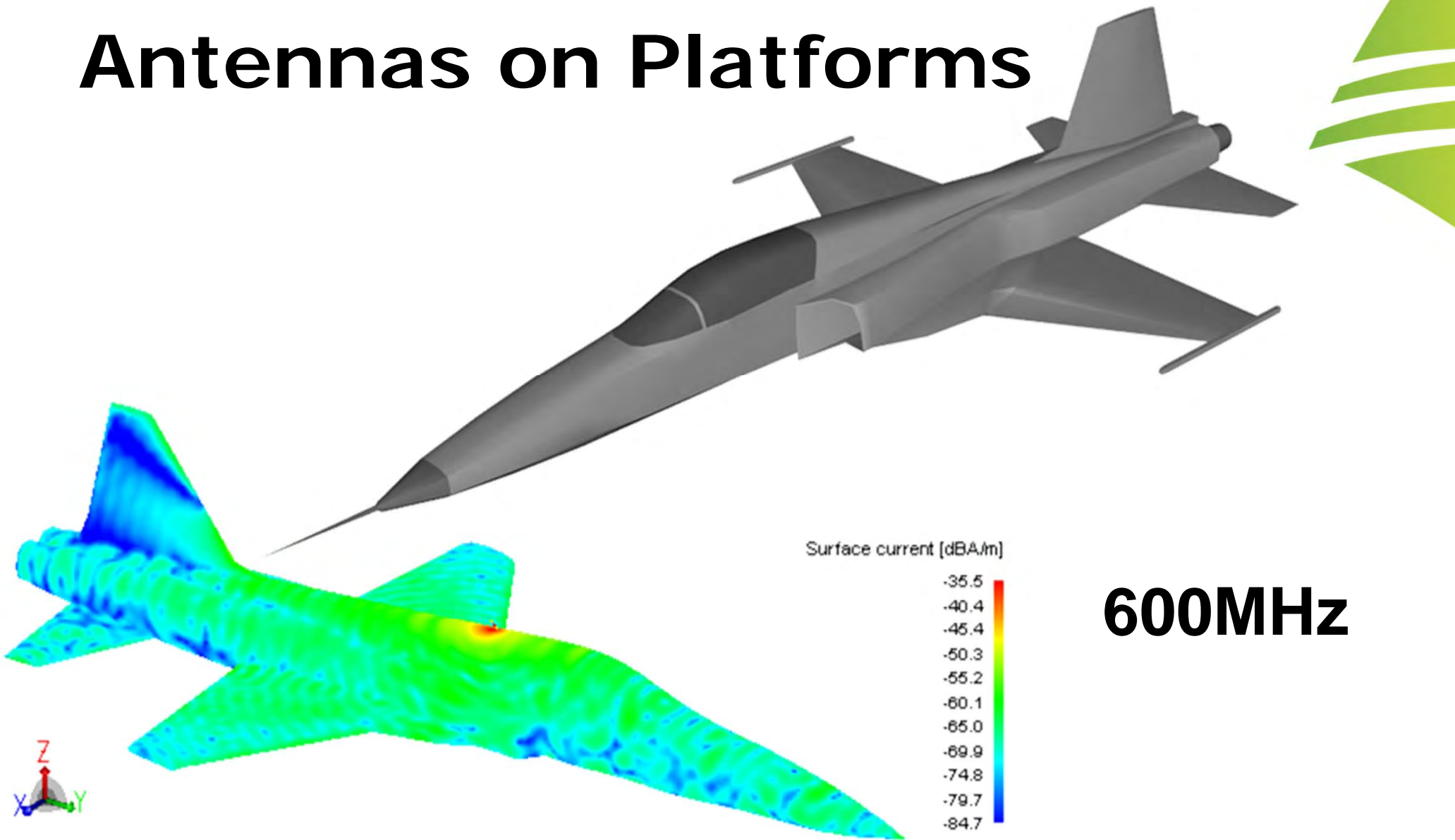


Fig. 6. Directivity in the  $\phi = 90^\circ$  (bottom position).

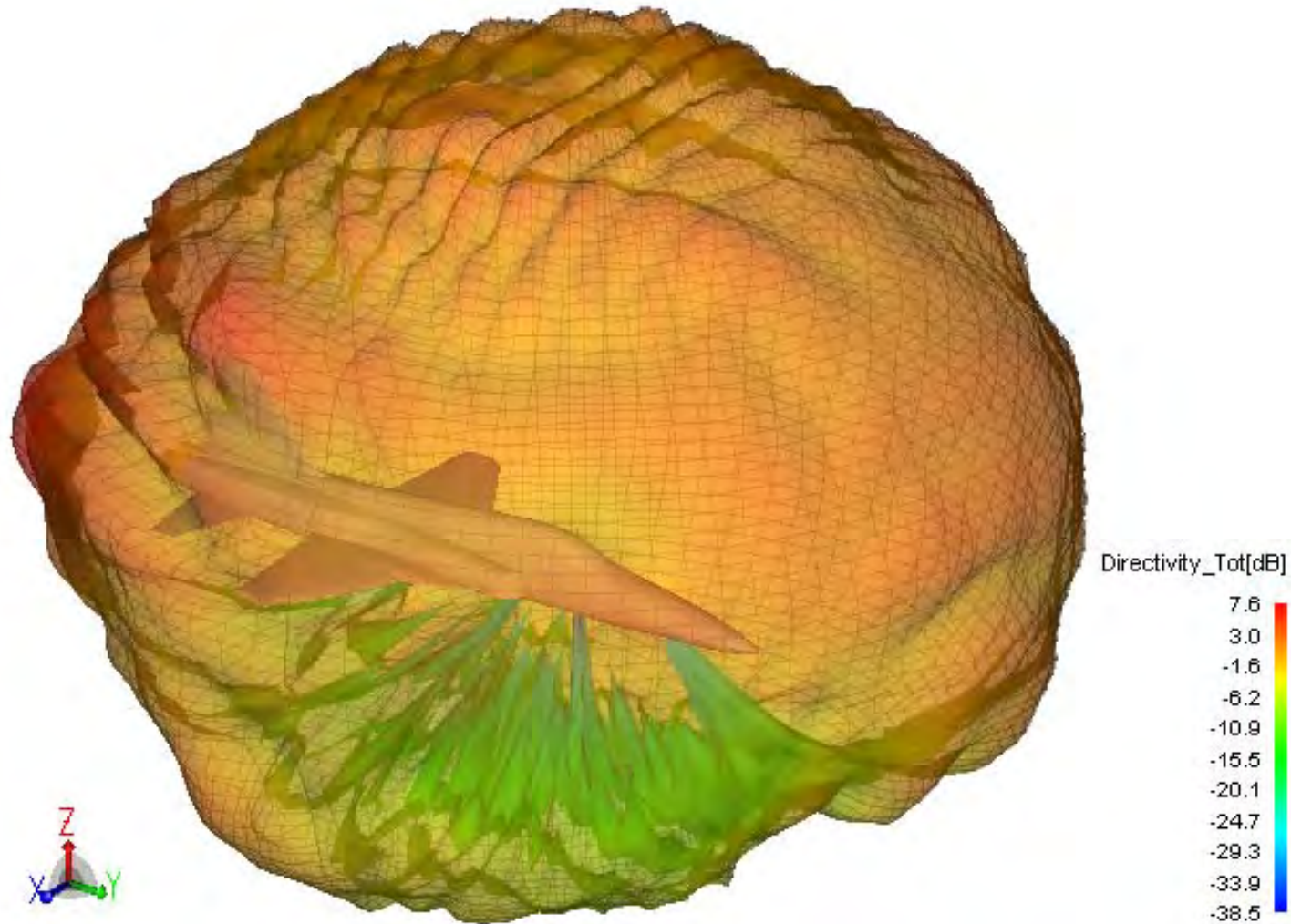
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# Antennas on Platforms



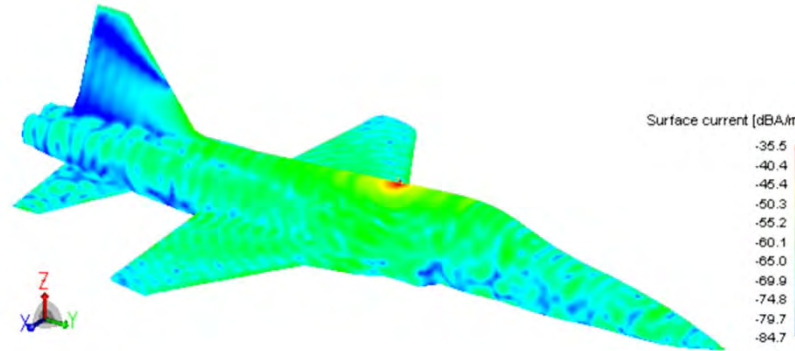
Rensheng Sun and C.J.Reddy, "Benchmark Study on Computational Resources for Numerical Methods in Electromagnetics ", 2008 IEEE AP-S USNC/URSI, San Diego, July 2009

# 3D Radiation Pattern at 600 MHz



Rensheng Sun and C.J.Reddy, "Benchmark Study on Computational Resources for Numerical Methods in Electromagnetics ", 2008 IEEE AP-S USNC/URSI, San Diego, July 2009

# Memory/CPU requirement for the MoM



**600 MHz**

Frequency	Electrical Size of Aircraft		Unknowns	Peak Memory	CPU-time
	length	wingspan			
200 MHz	9.7 $\lambda$	5.3 $\lambda$	5,874	542.6 MB	3.9 min
300 MHz	14.5 $\lambda$	8 $\lambda$	14,189	3.0 GB	29.0 min
450 MHz	21.75 $\lambda$	12 $\lambda$	29,389	13.0 GB	3.0 hours
<b>600 MHz</b>	<b>29 <math>\lambda</math></b>	<b>16 <math>\lambda</math></b>	<b>44,627</b>	<b>30.0 GB</b>	<b>9.1 hours</b>

Rensheng Sun and C.J.Reddy, "Benchmark Study on Computational Resources for Numerical Methods in Electromagnetics ", 2008 IEEE AP-S USNC/URSI, San Diego, July 2009



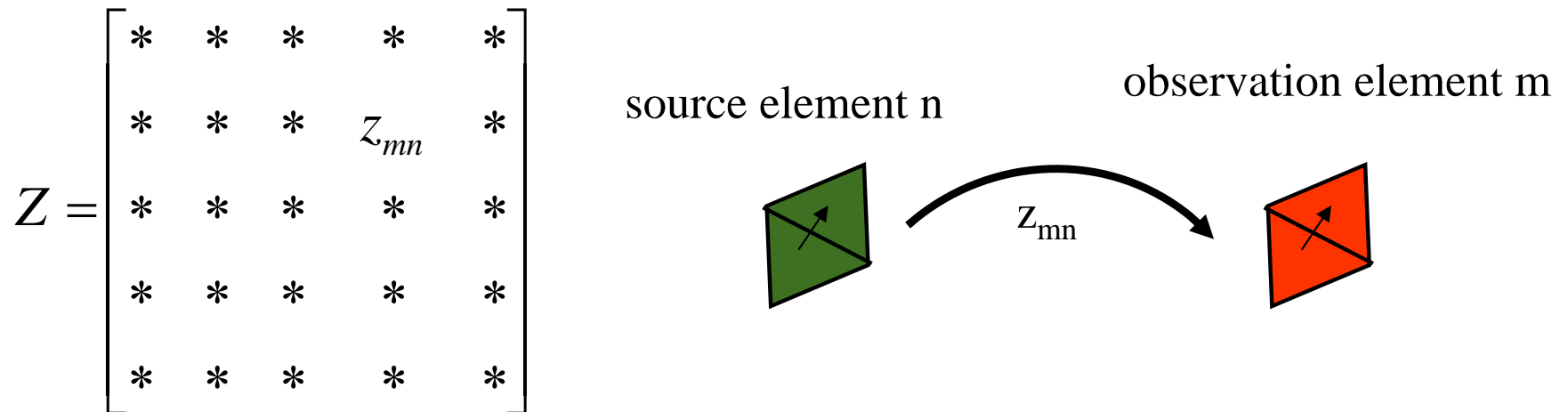
# Multilevel Fast Multipole Method (MLFMM)



MoM based on the solution of a system of linear equations

$$\mathbf{Z}\mathbf{I} = \mathbf{V} \quad \longrightarrow \quad \mathbf{I} = \mathbf{Z}^{-1} \mathbf{V}$$

Impedance matrix  $\mathbf{Z}$  describes interaction of n.th element with m.th element



→ LU-decomposition requires  $O(N^3)$  operations and  $O(N^2)$  memory

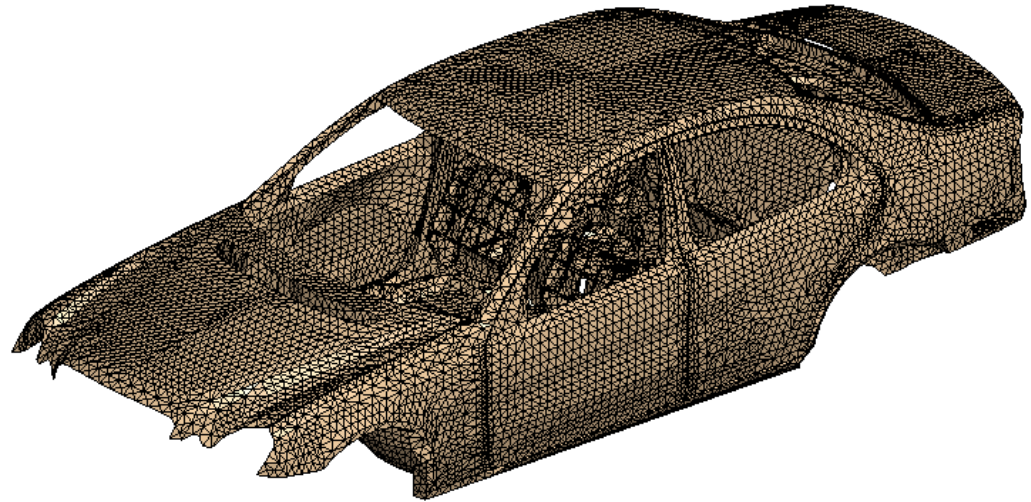
# Resource Requirement

Example:

Automotive simulation  
at 2 GHz instead of 1 GHz:

$$f \longrightarrow 2f$$

$$N \longrightarrow 4N$$

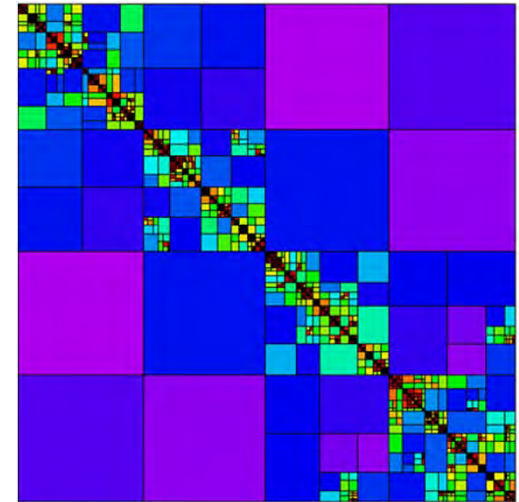


Complexity	Factor
$O(N^3)$	64
$O(N^2)$	16
$O(N)$	4
$O(N \log N)$	$4 \cdot \left(1 + \frac{\log 4}{\log N}\right) < 5$
$O(N \log^2 N)$	$4 \cdot \left(1 + \frac{2 \log 4}{\log N} + \frac{\log^2 4}{\log^2 N}\right) < 6$

# MoM Solution Acceleration

## Adaptive Cross-Approximation:

- Effective for:
  - Low-frequency problems (sub-wavelength geometric detail)
  - Planar Greens function problems
- Large savings in memory and runtime over traditional implementation



ACA hierarchical matrix decomposition

## Hardware Acceleration:

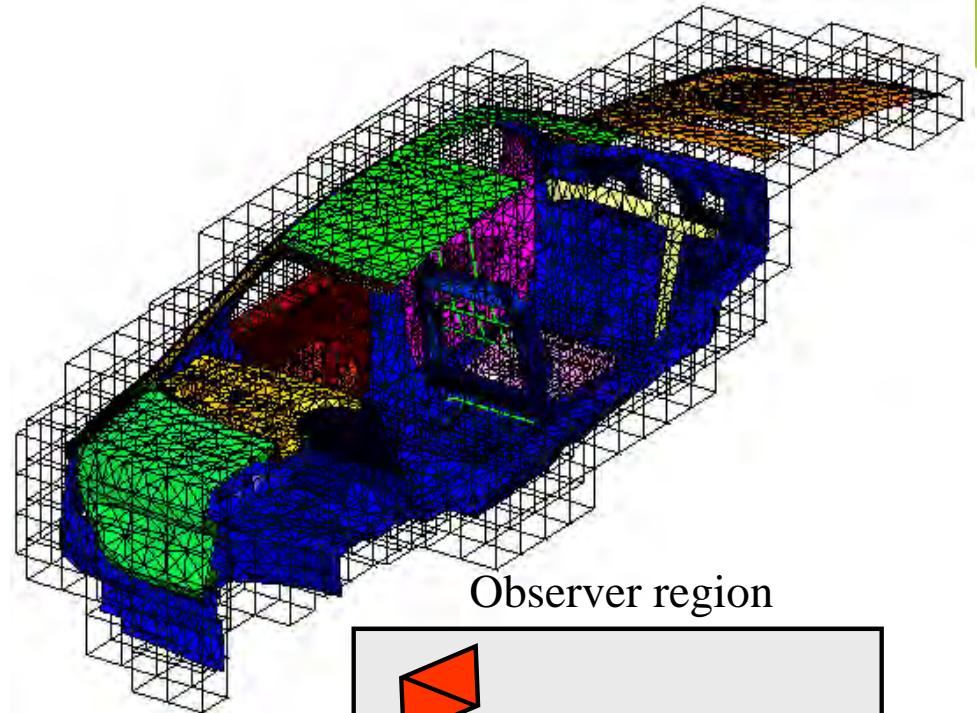
- **Parallelization**
  - Multi-core
  - Multi-CPU
  - Cluster
- **Graphical Processing Unit (GPU)**
  - NVIDIA CUDA enabled



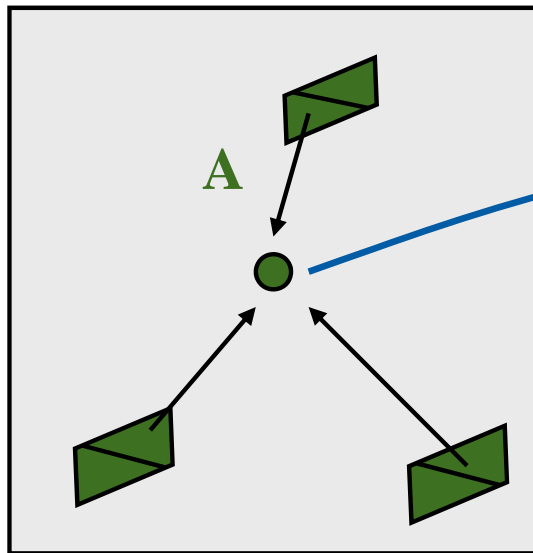
# Multilevel Fast Multipole Method (MLFMM)

- **Multilevel implementation:**

- Divide space into boxes
- Aggregation (A)
- Translation (T)
- Disaggregation (D)

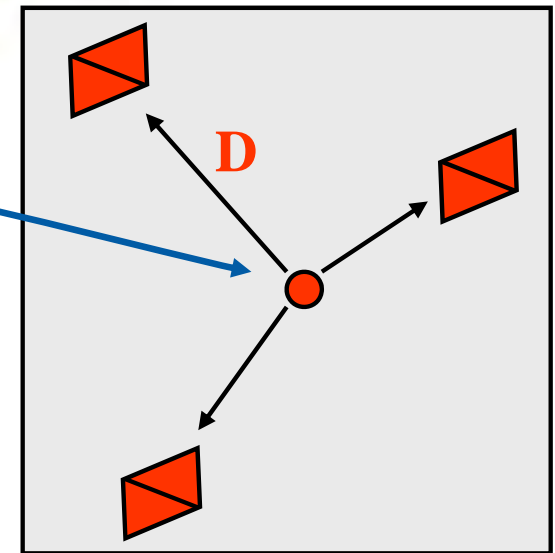


Source region



T

Observer region



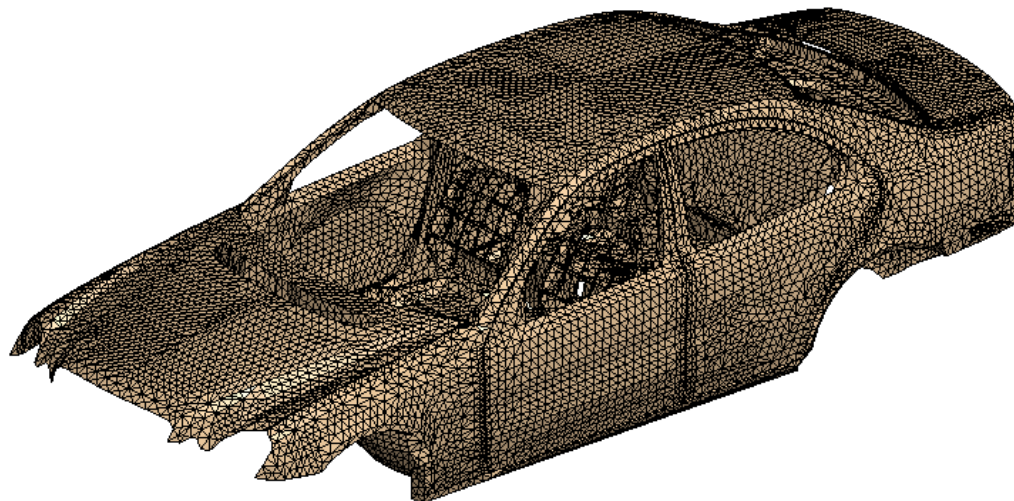
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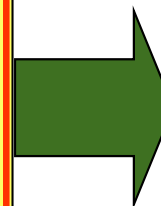
Automotive simulation  
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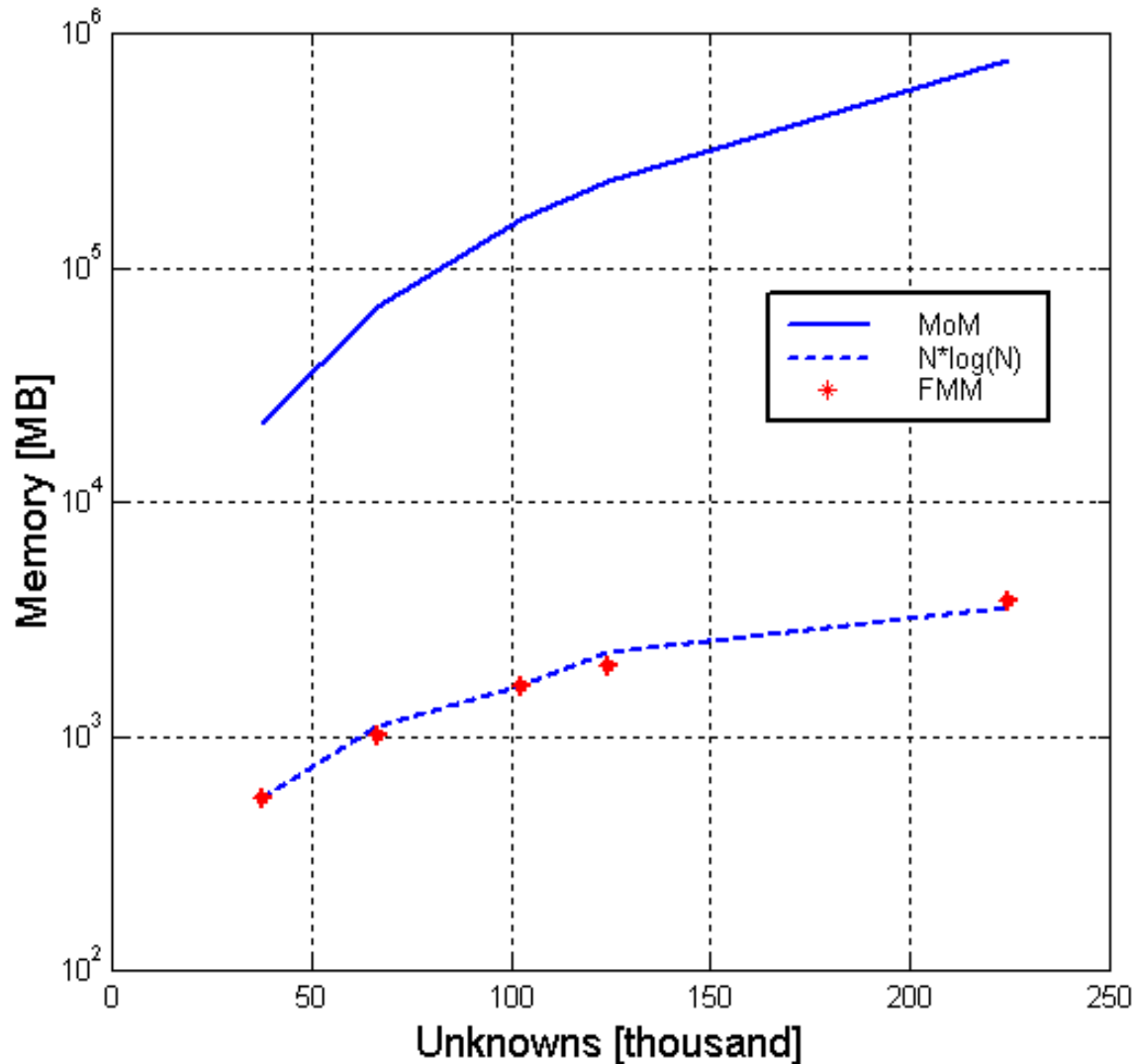


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**MLFMM**

# Memory requirement for the MLFMM



General scaling of the MLFMM:

Memory  
 $N \log(N)$

CPU time  
 $N \log^2(N)$

→ Confirmed by examples

# Possible with MLFMM



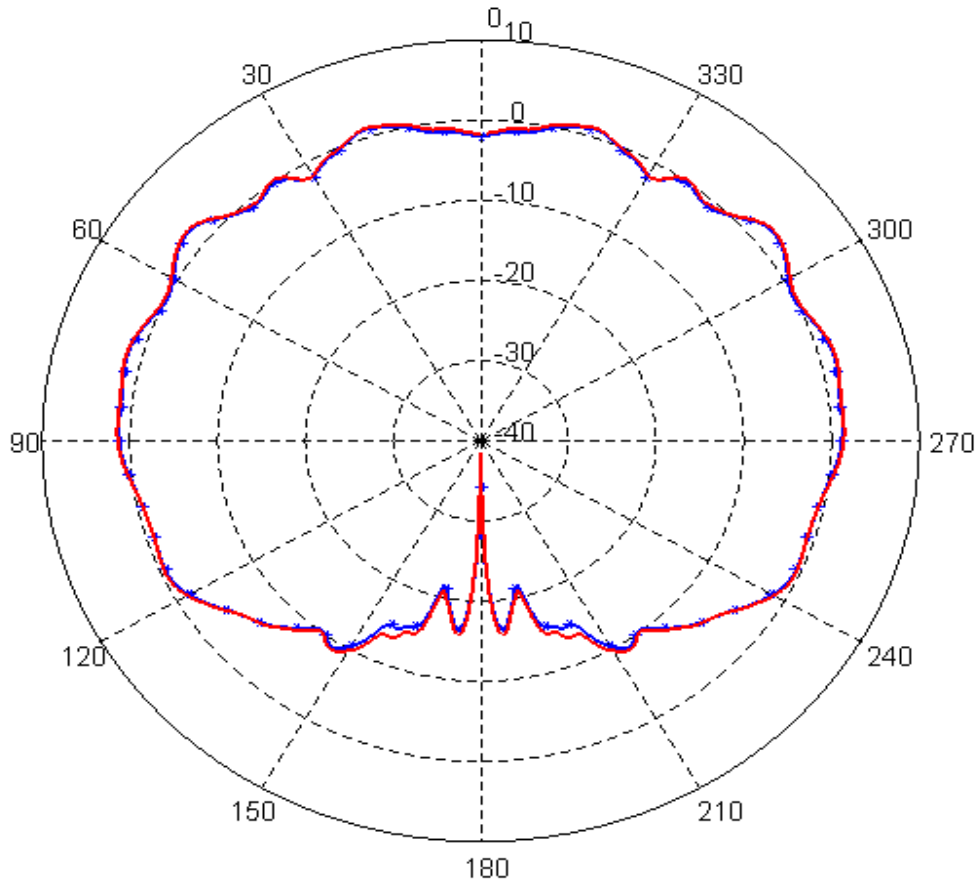
Asymptotic predictions of memory usage for the MoM with and without MLFMM.

N	MoM	MLFMM	Application
100 000	75 GByte	1 GByte	<ul style="list-style-type: none"><li>• Military aircraft at 690 MHz</li><li>• <b>Ship (115 m x 14 m) at 107 MHz</b></li><li>• Reflector antenna with aperture size <math>19\lambda</math></li></ul>
200 000	300 GByte	2 GByte	<ul style="list-style-type: none"><li>• Military aircraft at 960 MHz</li><li>• <b>Ship (115 m x 14 m) at 150 MHz</b></li><li>• Reflector antenna with aperture size <math>27\lambda</math></li></ul>
400 000	1.2 TByte	4.5 GByte	<ul style="list-style-type: none"><li>• Military aircraft at 1.37 GHz</li><li>• <b>Ship (115 m x 14 m) at 214 MHz</b></li><li>• Reflector antenna with aperture size <math>38\lambda</math></li></ul>
1 000 000	7.5 TByte	12 GByte	<ul style="list-style-type: none"><li>• Military aircraft at 2.2 GHz</li><li>• <b>Ship (115 m x 14 m) at 340 MHz</b></li><li>• Reflector antenna with aperture size <math>60\lambda</math></li></ul>

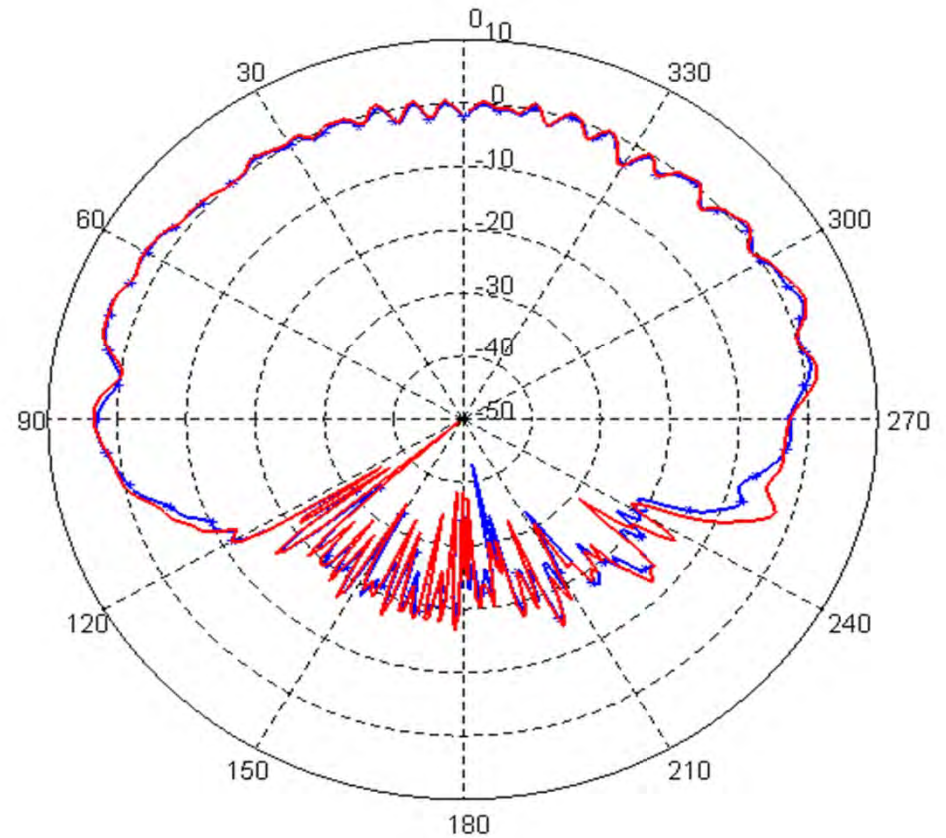
# Comparison between MoM and MLFMM at 600 MHz



Directivity (Phi = 0 deg)



Directivity (Phi = 90 deg)



Rensheng Sun and C.J.Reddy, "Benchmark Study on Computational Resources for Numerical Methods in Electromagnetics ", 2008 IEEE AP-S USNC/URSI, San Diego, July 2009



# Memory/CPU requirement for the MLFMM

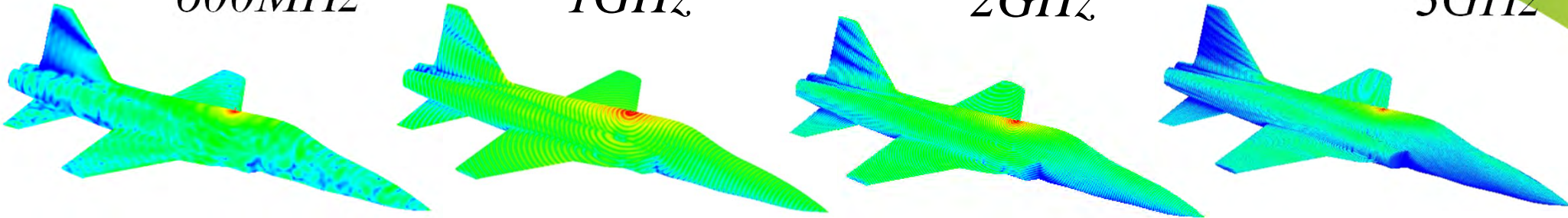


600MHz

1GHz

2GHz

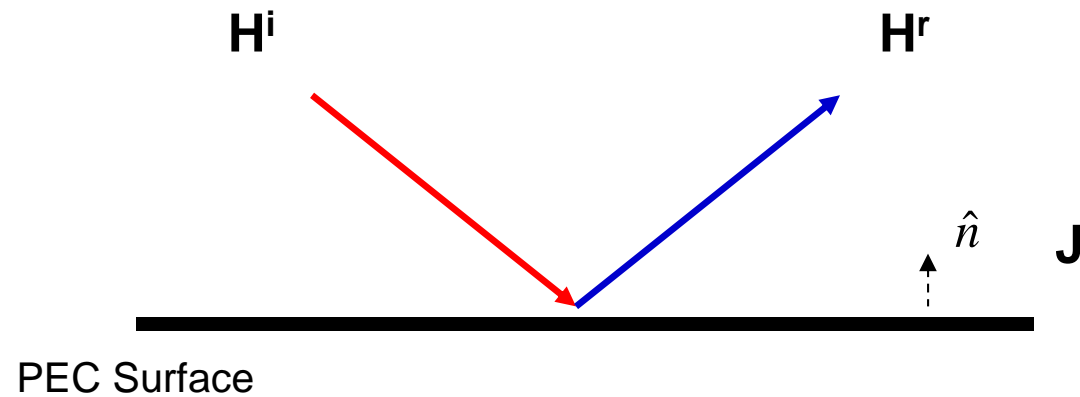
3GHz



Frequency	Electrical Size of Aircraft		Unknowns	Peak Memory	CPU-time
	length	wingspan			
600 MHz	29 $\lambda$	16 $\lambda$	89,866	1.3 GB	20.0 min
1 GHz	48.3 $\lambda$	26.7 $\lambda$	407,648	3.7 GB	1.1 hours
2 GHz	96.6 $\lambda$	53.3 $\lambda$	1,320,430	10.5 GB	3 hours
3 GHz	145 $\lambda$	80 $\lambda$	2,736,834	21.4 GB	7 hours

Rensheng Sun and C.J.Reddy, "Benchmark Study on Computational Resources for Numerical Methods in Electromagnetics ", 2008 IEEE AP-S USNC/URSI, San Diego, July 2009

# Physical Optics



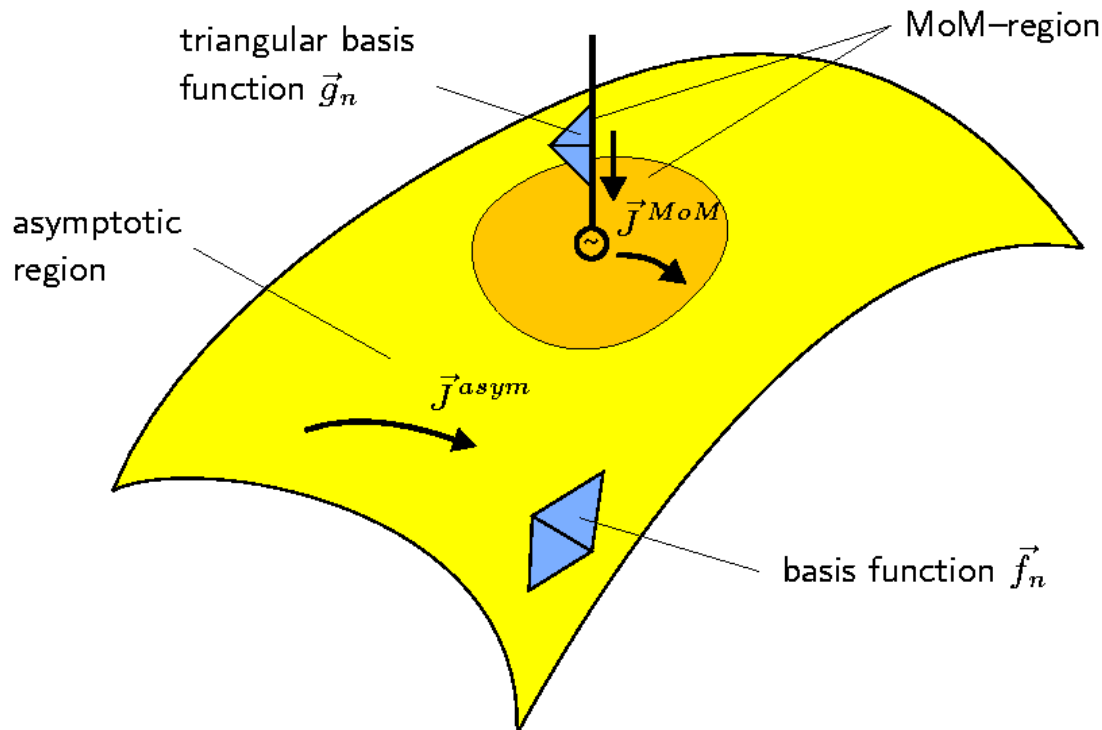
$$\mathbf{J}(\mathbf{r}) = \hat{n} \times \mathbf{H}(\mathbf{r}) = \hat{n} \times [\mathbf{H}^i(\mathbf{r}) + \mathbf{H}^r(\mathbf{r})]$$

$$\mathbf{J}(\mathbf{r}) = 2\hat{n} \times \mathbf{H}^i(\mathbf{r})$$

# Hybrid MoM/Physical Optics (PO) Technique



## Decomposition of domain into MoM and asymptotic region



Two types of coupling:

- $\vec{J}^{MoM}$  radiates H causing asymptotic currents
- $\vec{J}^{asym}$  radiates E which must be considered in the MoM integral equation

$$\vec{\mathcal{E}} \left\{ \vec{J}^{MoM} \right\}_{tan} + \vec{\mathcal{E}} \left\{ \vec{J}^{asym} \right\}_{tan} = -\vec{E}_{i,tan}$$

# Validation of FEKO for Electrically Large Problem



**Air Traffic Control Antenna at 1GHz**

$\lambda/5$  Mesh with 193,212 triangles  
MoM/PO Hybrid Technique  
Measurements – scale model

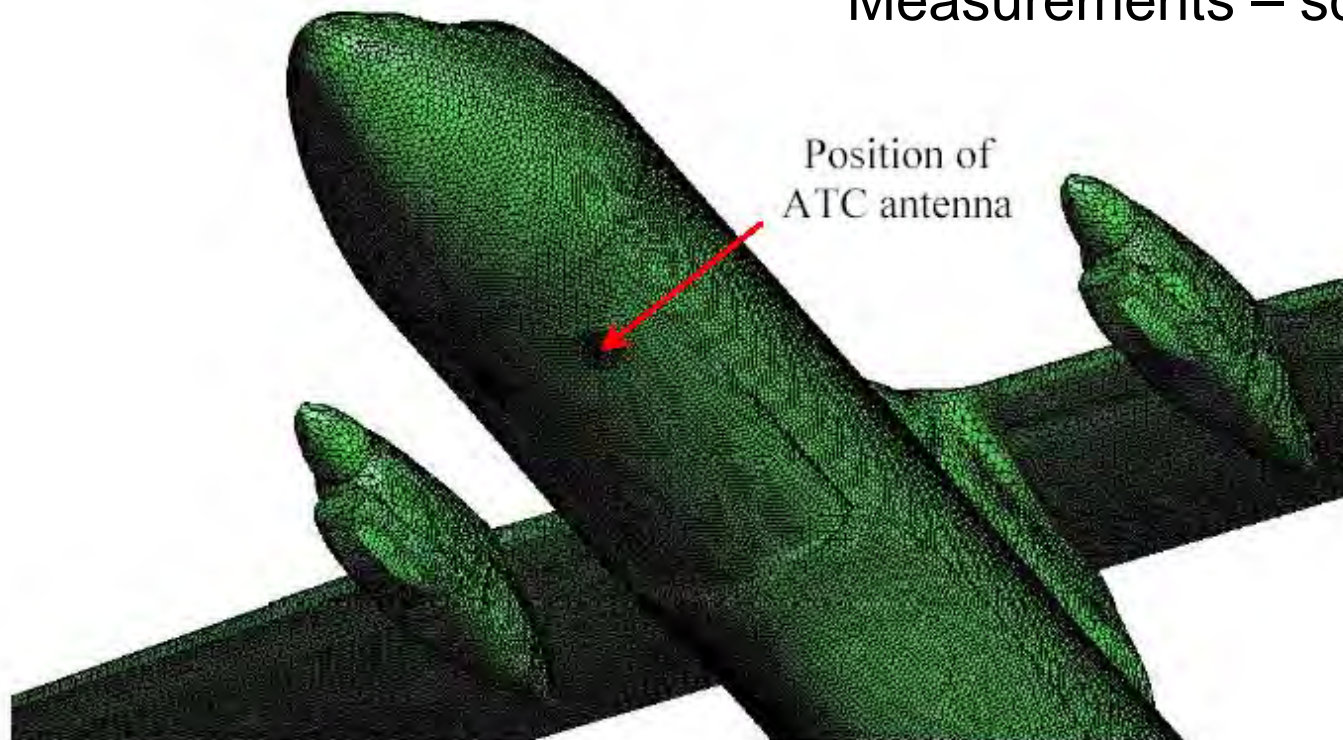


Fig. 7. Indication of the ATC antenna position.

M. V. T. Heckler\* and A. Dreher, Analysis of Monopoles Installed on Airframes, 2005 IEEE APS Symposium, Washington DC, July 2005

# Validation of FEKO for Electrically Large Problem

## Air Traffic Control Antenna at 1GHz

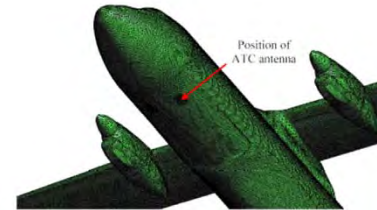


Fig. 7. Indication of the ATC antenna position.

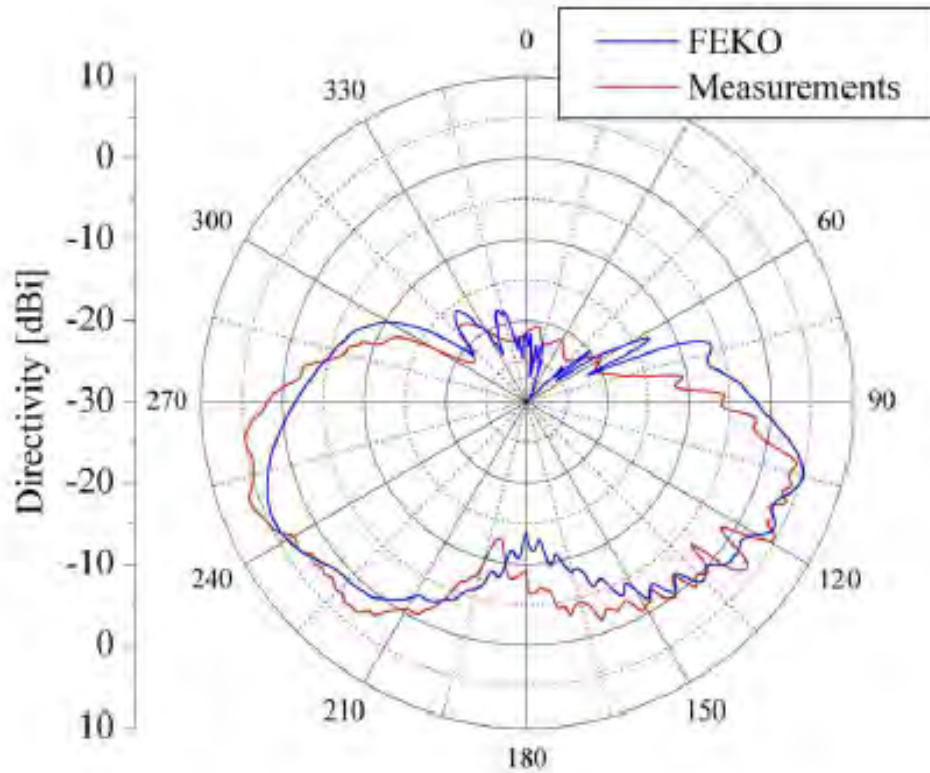


Fig. 8. Directivity in the  $\phi = 0^\circ$ .

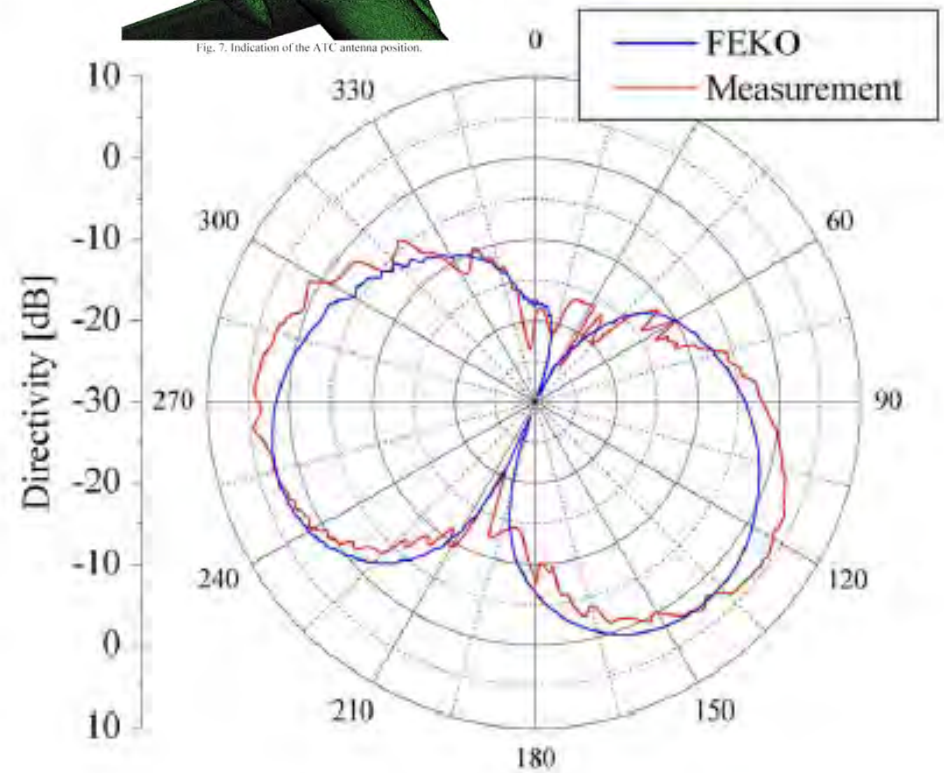
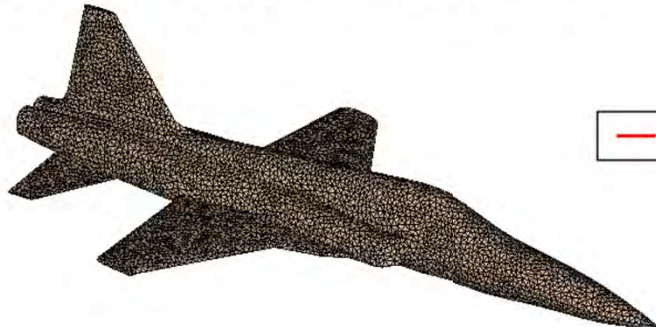


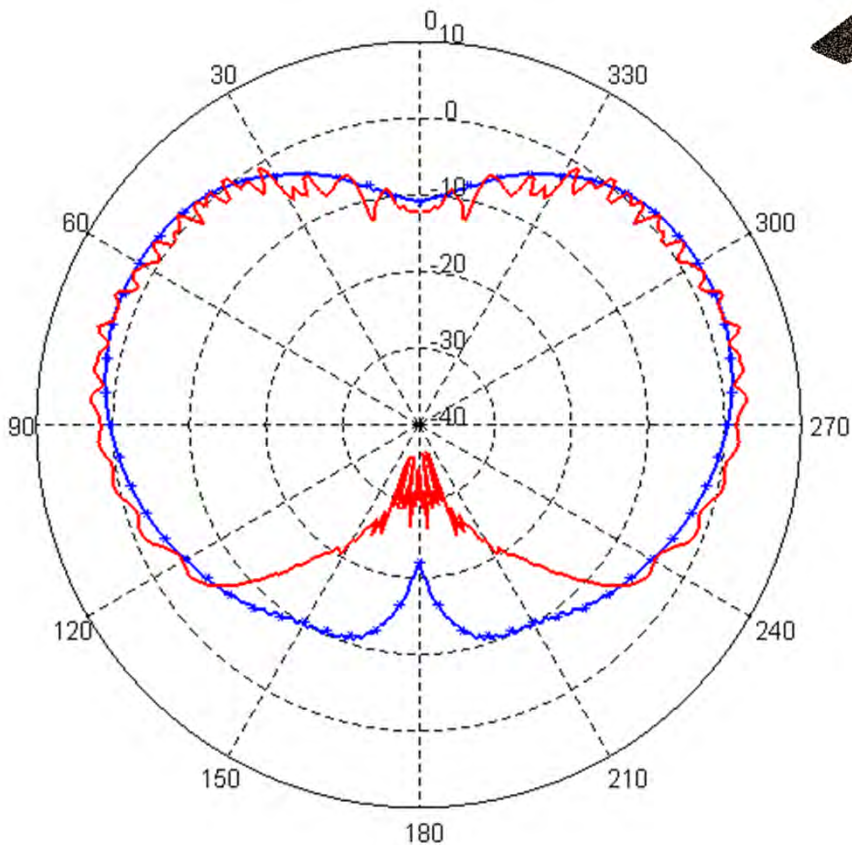
Fig. 9. Directivity in the  $\phi = 90^\circ$ .

# Comparison between MLFMM and MoM/PO at 3 GHz



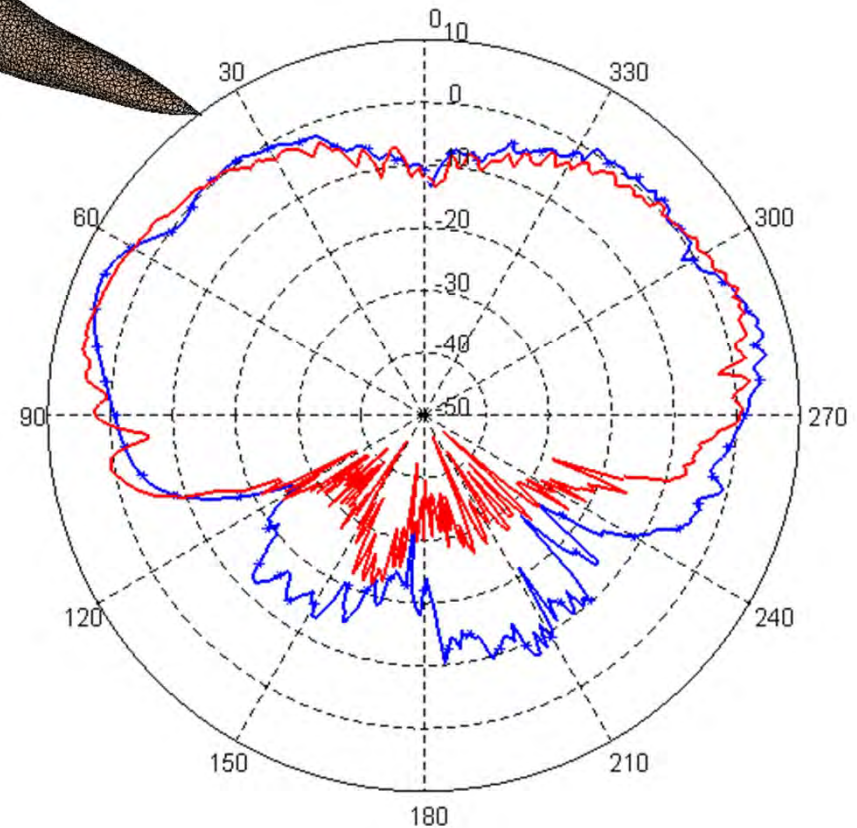
Directivity (Phi = 0 deg)

— MLFMM      — Hybrid MoM/PO



Directivity (Phi = 90 deg)

— MLFMM      — Hybrid MoM/PO



Rensheng Sun and C.J.Reddy, "Benchmark Study on Computational Resources for Numerical Methods in Electromagnetics", 2008 IEEE AP-S USNC/URSI, San Diego, July 2009

# Memory/CPU requirement for the MoM/PO



Frequency	Electrical Size of Aircraft		Unknowns	Peak Memory	CPU-time
	length	wingspan			
3 GHz	145 $\lambda$	80 $\lambda$	808,336	570.0 MB	8.0 min
8 GHz	387 $\lambda$	213 $\lambda$	5,075,060	4.0 GB	1.4 hours
15 GHz	725 $\lambda$	400 $\lambda$	17,271,199	16.0 GB	8.9 hours
20 GHz	967 $\lambda$	533 $\lambda$	30,437,010	30.9 GB	20.1 hours

Rensheng Sun and C.J.Reddy, "Benchmark Study on Computational Resources for Numerical Methods in Electromagnetics ", 2008 IEEE AP-S USNC/URSI, San Diego, July 2009

# Summary of MoM, MLFMM, and MoM/PO

	frequency	Electrical Size		# of Unknowns	Peak Memory	CPU-time
		length	wingspan			
MoM symmetry (mesh size -- $\lambda/8$ )	200 MHz	9.7 $\lambda$	5.3 $\lambda$	5,874	542.6 MB	3.9 min
	300 MHz	14.5 $\lambda$	8 $\lambda$	14,189	3.0 GB	29.0 min
	450 MHz	21.75 $\lambda$	12 $\lambda$	29,389	13.0 GB	3.0 hours
	<b>600 MHz</b>	<b>29 <math>\lambda</math></b>	<b>16 <math>\lambda</math></b>	<b>44,627</b>	<b>30.0 GB</b>	<b>9.1 hours</b>
MLFMM (mesh size -- $\lambda/8$ )	<b>600 MHz</b>	<b>29 <math>\lambda</math></b>	<b>16 <math>\lambda</math></b>	<b>89,866</b>	<b>1.3 GB</b>	<b>20.0 min</b>
	1 GHz	48.3 $\lambda$	26.7 $\lambda$	407,648	3.7 GB	1.1 hours
	2 GHz	96.6 $\lambda$	53.3 $\lambda$	1,320,430	10.5 GB	3.0 hours
	<b>3 GHz</b>	<b>145 <math>\lambda</math></b>	<b>80 <math>\lambda</math></b>	<b>2,736,834</b>	<b>21.4 GB</b>	<b>7.0 hours</b>
MoM/PO symmetry (mesh size -- $\lambda/6$ )	<b>3 GHz</b>	<b>145 <math>\lambda</math></b>	<b>80 <math>\lambda</math></b>	<b>808,336</b>	<b>570.0 MB</b>	<b>8.0 min</b>
	8 GHz	387 $\lambda$	213 $\lambda$	5,075,060	4.0 GB	1.4 hours
	15 GHz	725 $\lambda$	400 $\lambda$	17,271,199	16.0 GB	8.9 hours
	20 GHz	967 $\lambda$	533 $\lambda$	30,437,010	30.9 GB	20.1 hours



# Modern High Performance Computing



**Under \$20,000**



## 2-Node Cluster

Intel Xeon E5-2650 2.0GHz 8-core 20MB cache

Each Node: 2 CPUs (16 cores), 256GB RAM

**Total: 4 CPUs (32 cores), 512 GB RAM**

**Infiniband Interconnect**

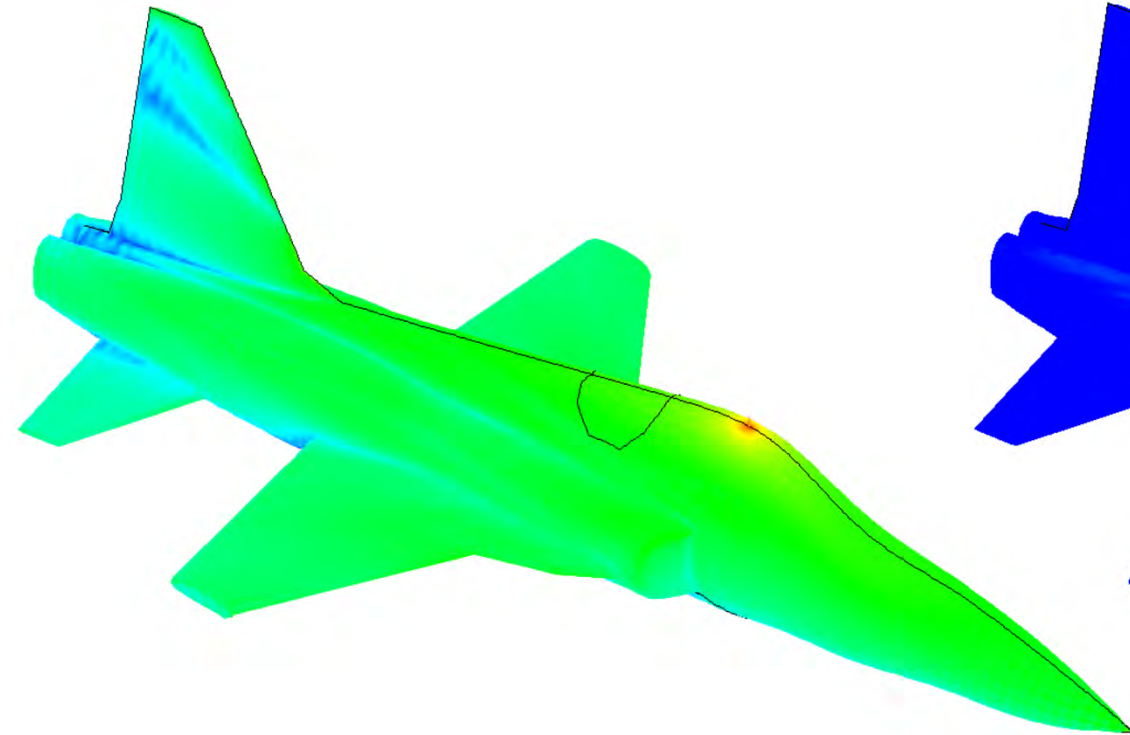
# Resource Requirements



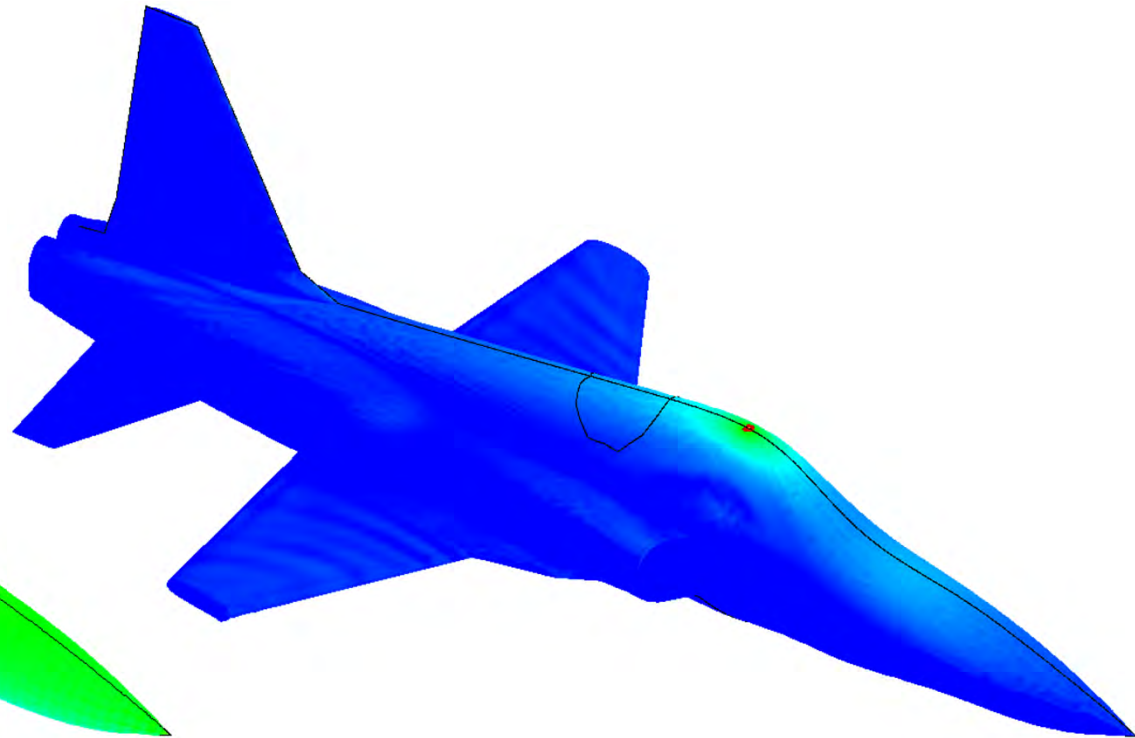
Method	Frequency	Triangles	Unknowns	Memory	Time
MOM	950MHz	162 524	121 537	222.1 GB	4.7 hr
MLFMM	950MHz	162 524	243 726	11.6 GB	0.1 hr
MLFMM	7.00 GHz	8 193 402	12 289 643	220.8 GB	2.6 hr
PO	7.00 GHz	5 119 950	3 838 847	36.9 GB	0.5 hr

Method	Frequency	Electrical Size		Unknowns	Memory	Time
PO	3 GHz	145 $\lambda$	80 $\lambda$	808 336	.57 GB	0.13 hr
PO	8 GHz	387 $\lambda$	213 $\lambda$	5 075 060	4.0 GB	1.40 hr
PO	15 GHz	725 $\lambda$	400 $\lambda$	17 271 199	16.0 GB	8.90 hr
PO	20 GHz	967 $\lambda$	533 $\lambda$	30 437 010	31.0 GB	20.10 hr
PO	45 GHz	2175 $\lambda$	1200 $\lambda$	108 435 565	128.0 GB	93.50 hr
PO	60 GHz	2901 $\lambda$	1599 $\lambda$	191 095 846	248.0 GB	210.00 hr

# Currents

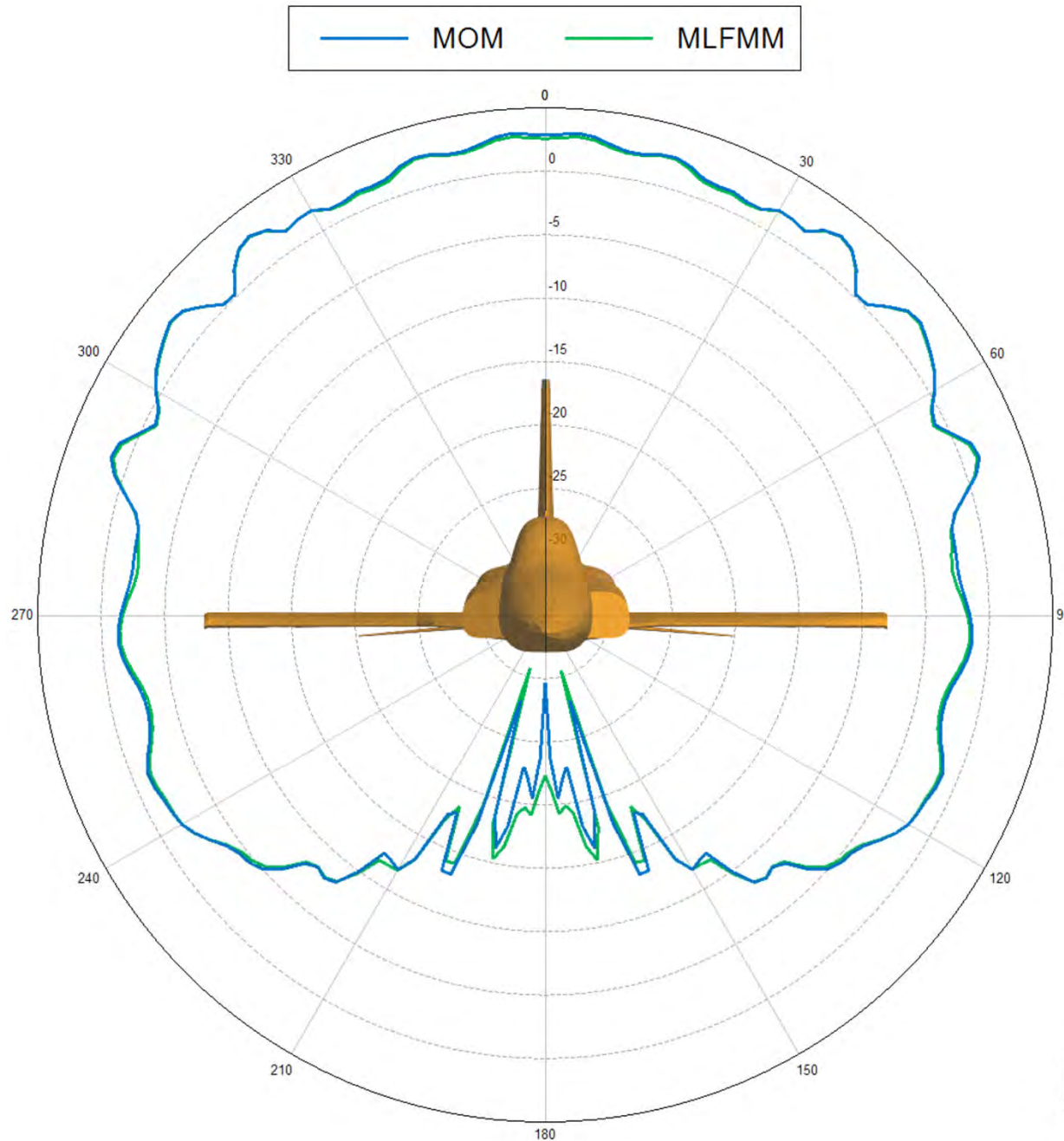


**950 MHz**

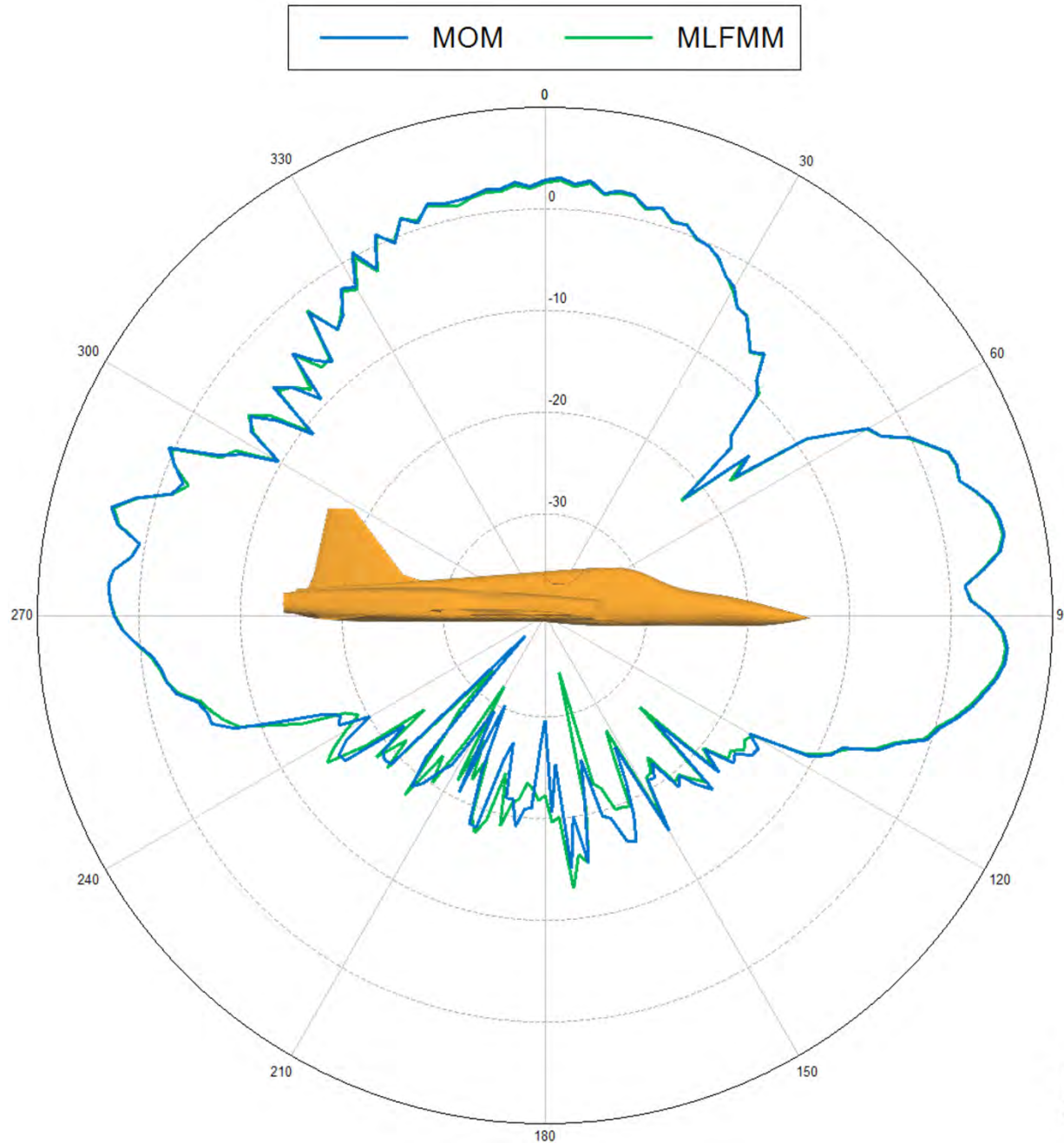


**7 GHz**

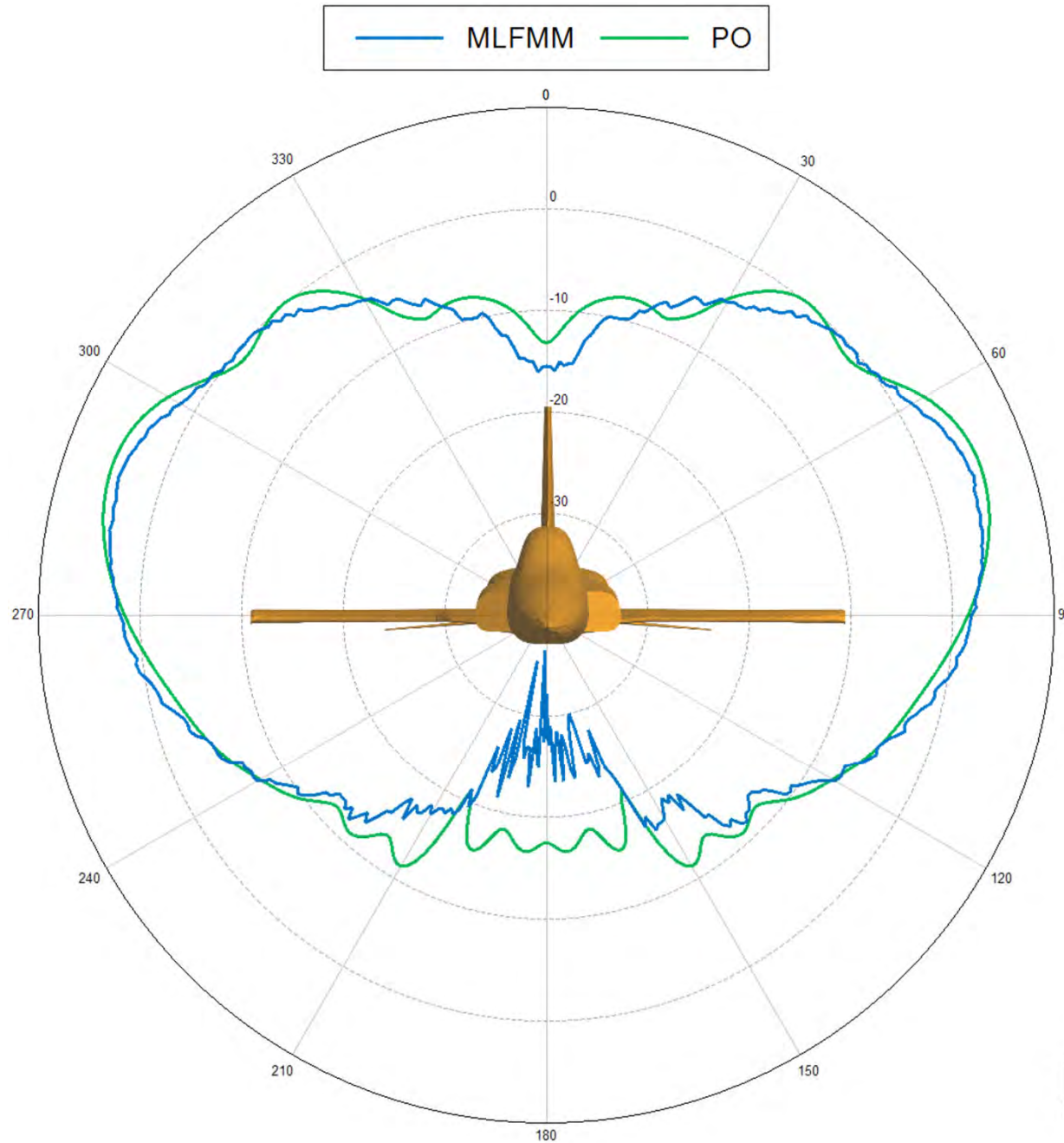
# 950 MHz: $\phi = 0^\circ$



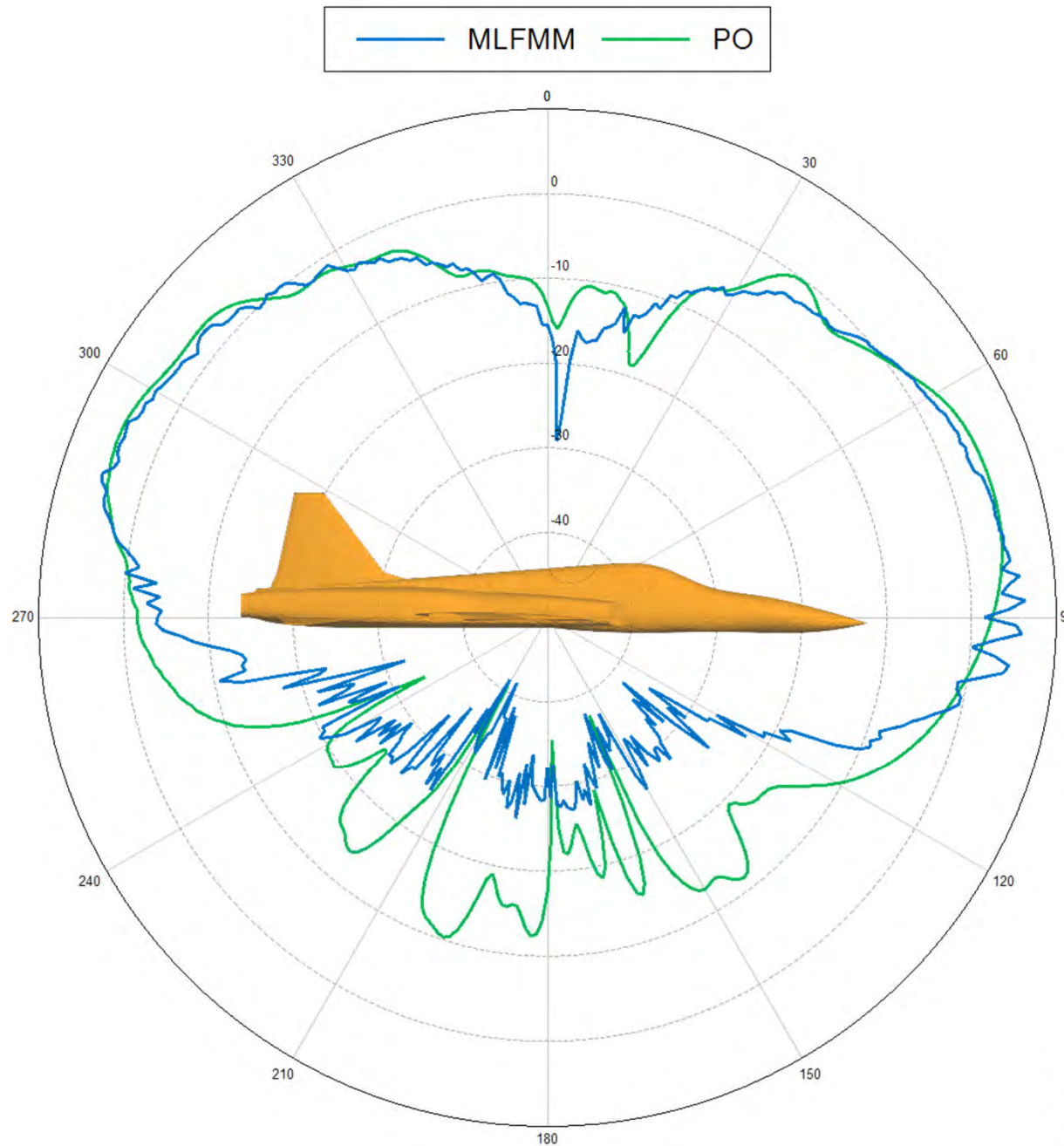
# 950 MHz: $\phi = 90^\circ$



# 7 GHz: $\phi = 0^\circ$



# 7 GHz: $\phi = 90^\circ$



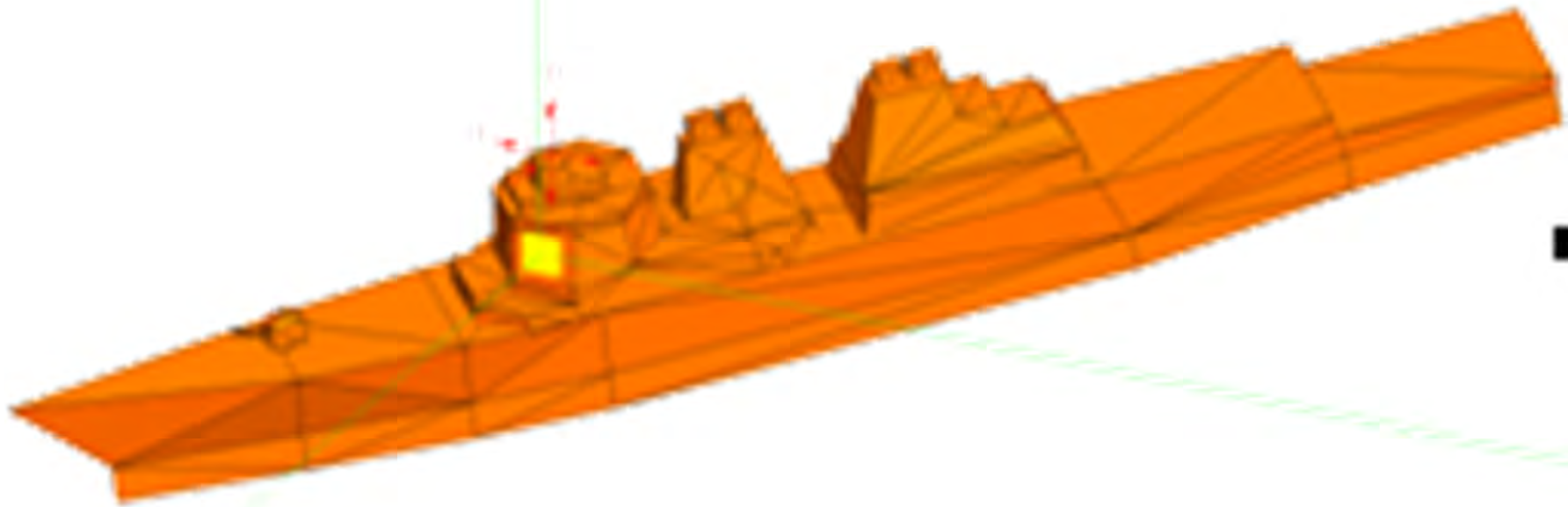
# Ray based hybrid method (MoM / UTD)



PO too expensive for very large  
(in terms of wavelengths) objects

→ FEKO offers the UTD  
(Uniform Theory of Diffraction)

method	formulation	CPU-time	memory
MoM	current-based	$f^{4...6}$	$f^4$
PO	current-based	$f^2$	$f^0$
UTD	ray-based	$f^0$	$f^0$

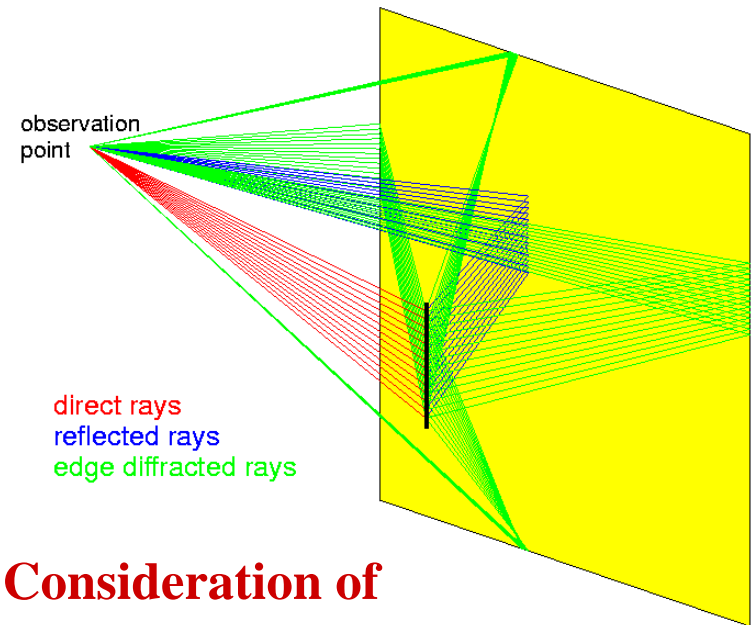
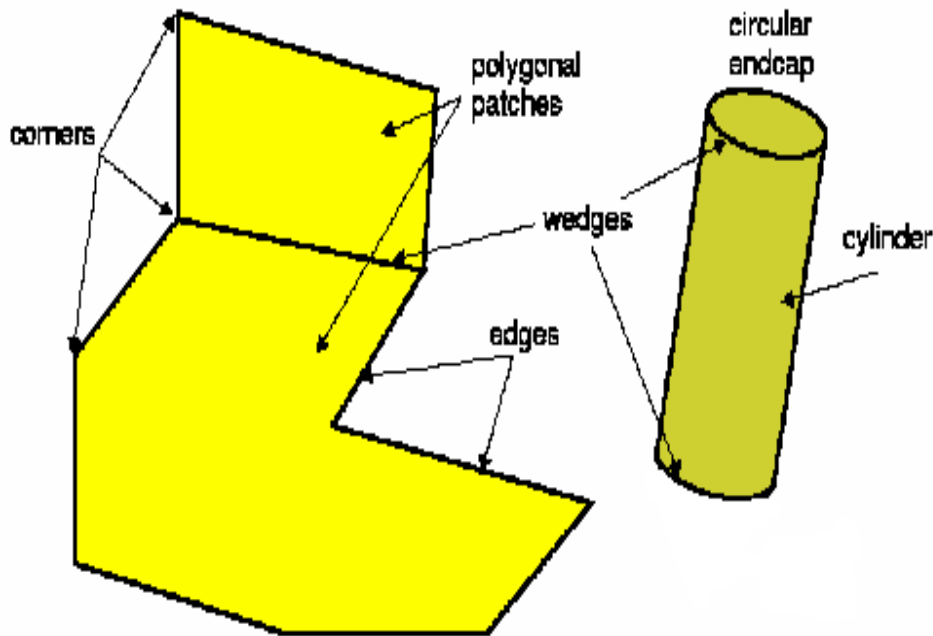




# Ray based hybrid method (MoM / UTD)



UTD applied to polygonal plates or a cylinder

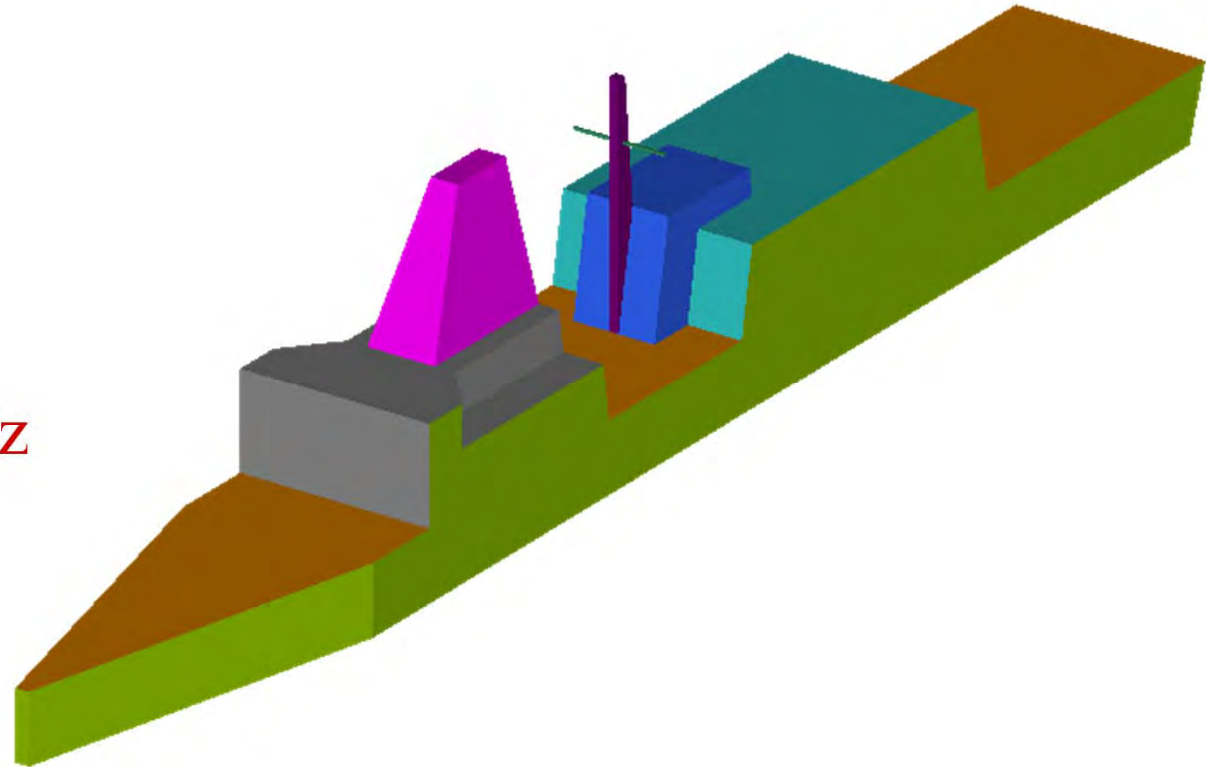


## Consideration of

- direct ray
- reflected rays (also multiple)
- edge diffracted rays
- corner diffracted rays
- combinations of reflections and diffractions
- multiple diffractions
- creeping rays

# 25 by 25 Array on Shipboard

- Ship
  - Length: 115 m
  - Width: 14 m
  - Height: 22 m
- Electrical size at 1.77 GHz
  - Length:  $679 \lambda$
  - Width:  $82 \lambda$
  - Height:  $159 \lambda$



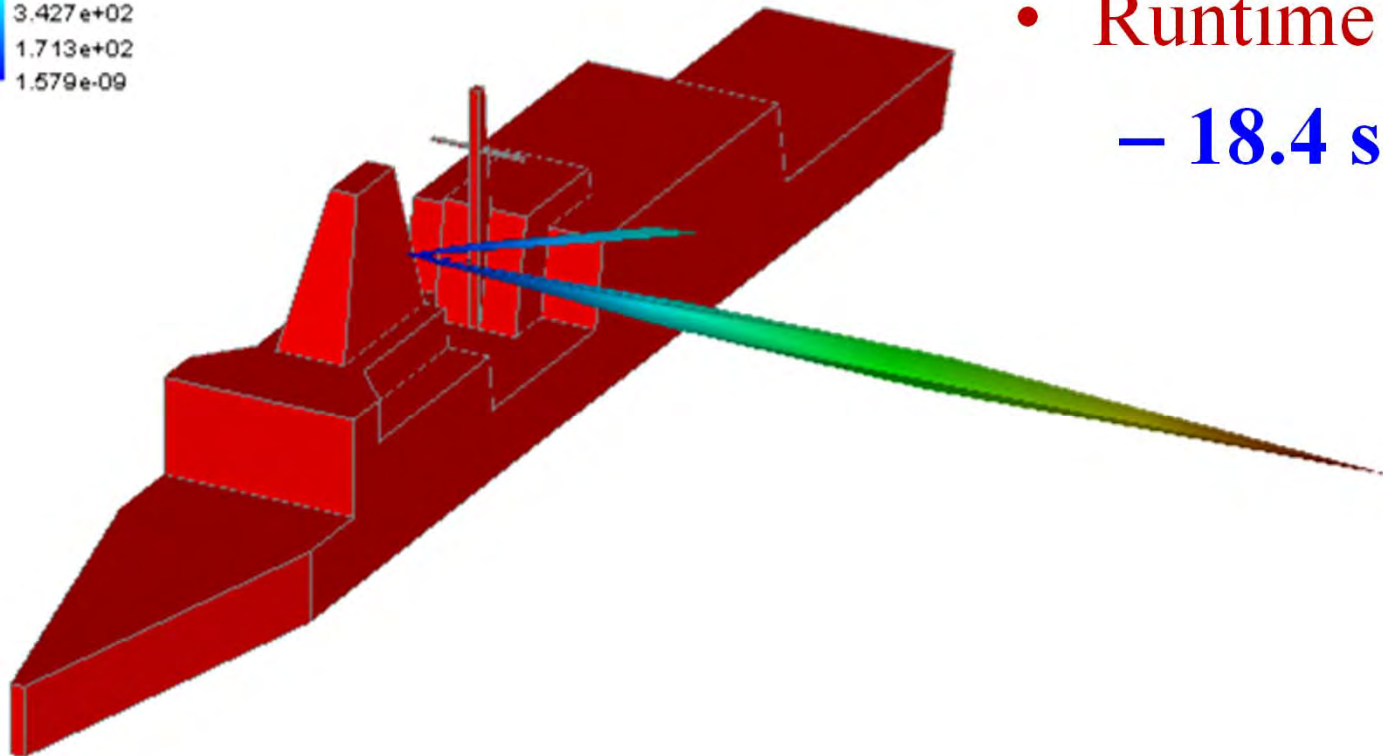
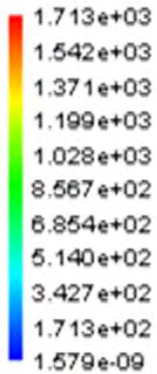
**MoM Unknowns: 32 millions**

# 25 by 25 Array on Shipboard



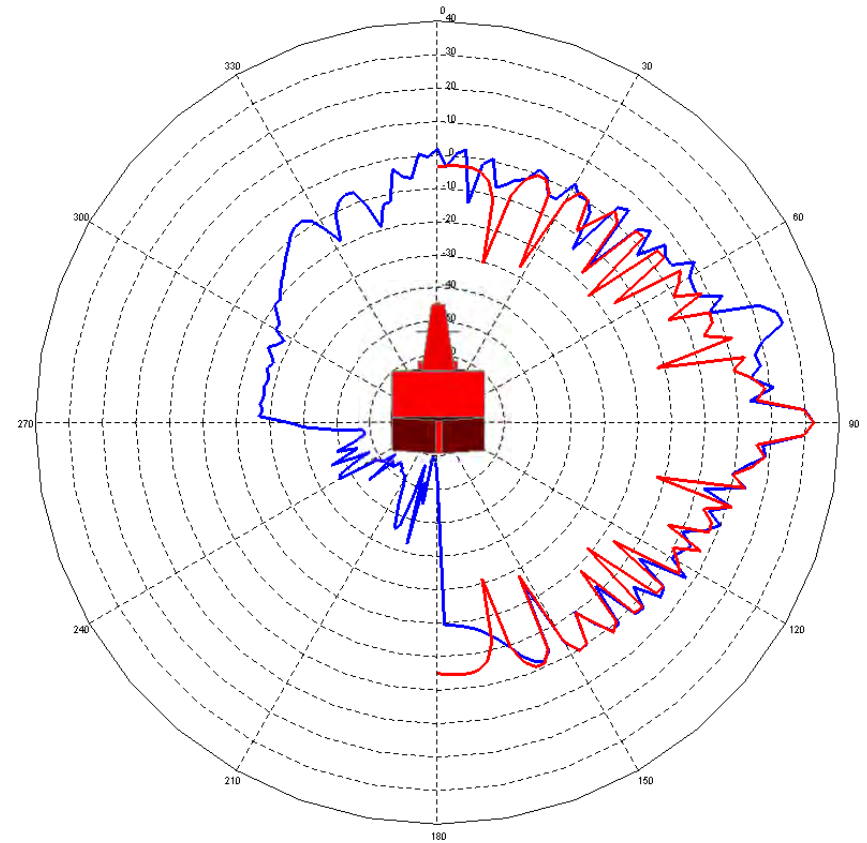
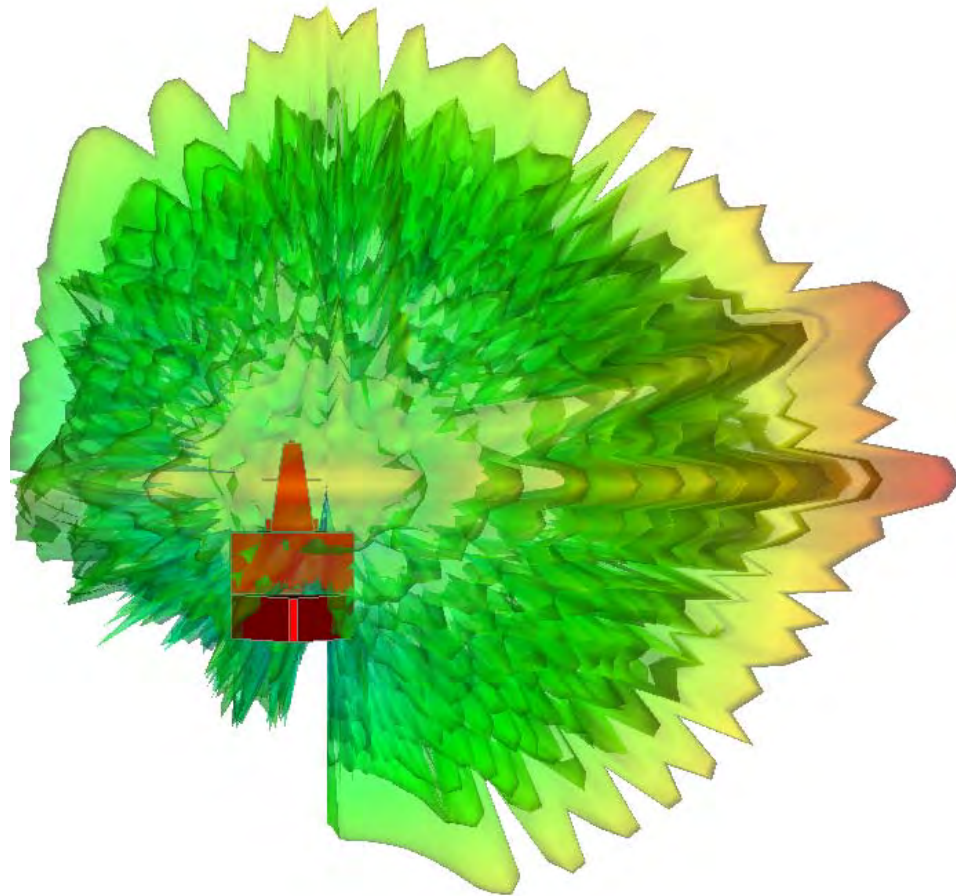
## UTD Solution

Directivity\_Tot



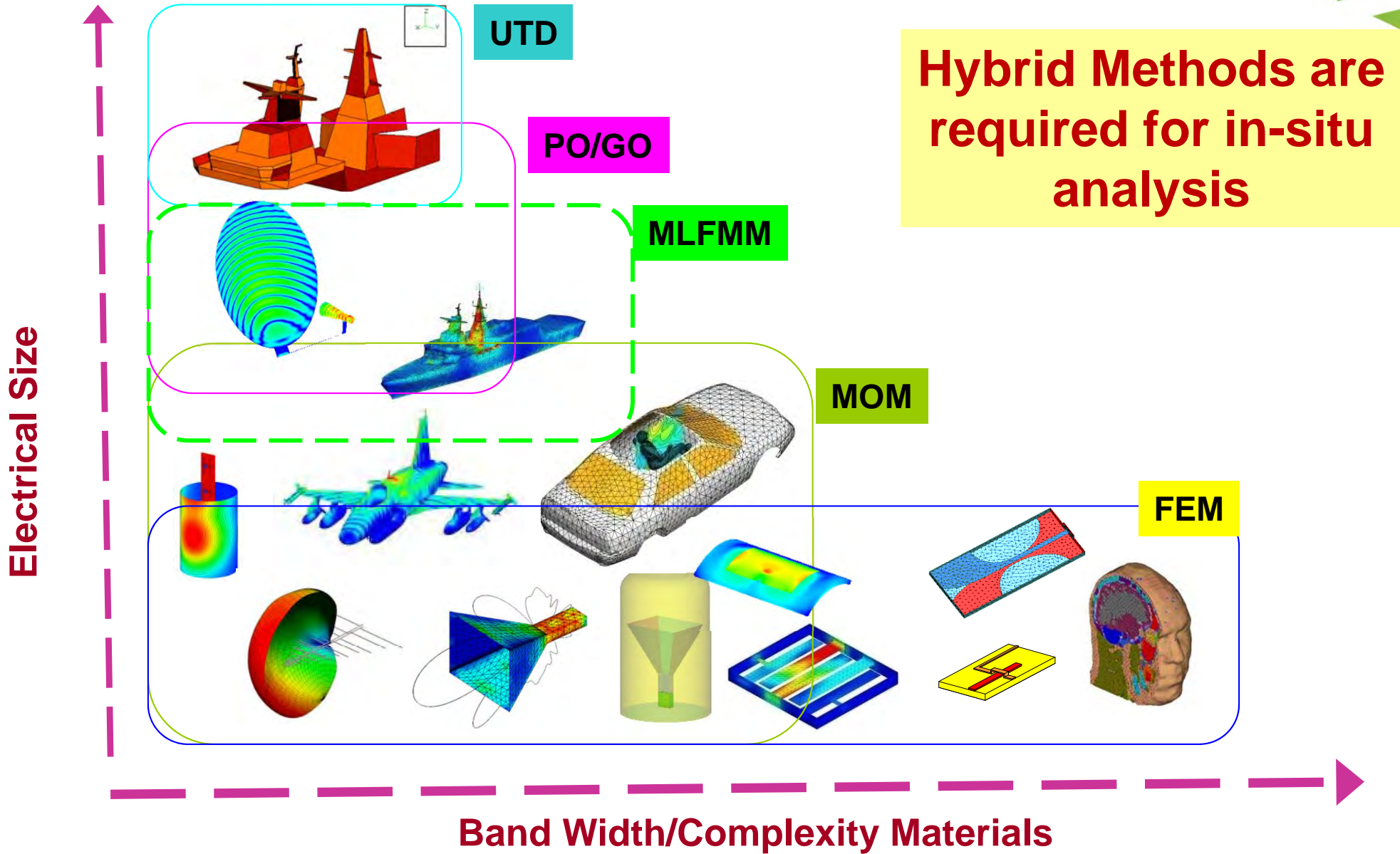
- Instead, UTD is used
  - Peak memory: **8.4MB**
- Runtime for each angle
  - **18.4 seconds**

# 25 by 25 Array on Shipboard



— Ship board array  
— Array in free space

# Summary



# Summary

- **Successful demonstration of antenna design, array analysis and placement of arrays on electrically large structures**
- **Use of various electromagnetic analysis techniques demonstrated**
- **It is possible to design antenna elements/array and carry out in-situ analysis of antennas/array on platforms using commercial EM analysis tools.**

**Simulations in this presentation are done using**

