

# On The Concept of Vector (Polarization) Electromagnetic Inverse Boundary Conditions for the Perfectly and Imperfectly Conducting Cases and its Applications: Why is renewed interest in EM-IBC forthcoming?

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**Abstract:** The inverse problem of electromagnetic scattering pertains to the problem of recovering the size, shape, material surface and interior constituents of an unknown scatterer, given the incident and the scattered fields everywhere, together with the laws governing the interaction. In the direct problems of scattering and diffraction, the electric size, the shape and the material surface and interior constituents are known *a priori* together with the pre-specified incident field in terms of a computational coordinate system so that their unknown vector electric and vector magnetic fields can be derived from Maxwell's equations by incorporating the known parameters into the boundary conditions which are well established. In contrary, in the inverse problem such local boundary conditions must be sought which *in particular* do not depend on either the size, shape or the surface normal and material properties of the scattering body and its enclosing surface, but must allow to specify these parameters uniquely from the recovered near fields.

Here two separate cases are considered, a perfectly and an imperfectly conducting closed-shaped smooth body for which the total complex vector electric fields  $\mathbf{E} = \mathbf{E}^{inc} + \mathbf{E}^{scat}$  and  $\mathbf{H} = \mathbf{H}^{inc} + \mathbf{H}^{scat}$  are assumed to be given everywhere and expressed in terms of a computational coordinate system, where for a

1. Perfectly conducting closed-shaped, convex and smooth body, the following inverse boundary conditions hold

$$\mathbf{E} \cdot \mathbf{H} = 0 \quad \text{and} \quad |\mathbf{E}^{inc}| = |\mathbf{E}^{scat}| \quad \text{necessary and sufficient} \quad (1)$$

$$\mathbf{E} \times \mathbf{E}^* = 0 \quad \text{necessary and not locally sufficient} \quad (2)$$

2. Imperfectly conducting closed-shaped, convex and smooth body, for which the following inverse boundary conditions are derived from the Leontovich direct boundary condition

$$\mathbf{E} \times \hat{\mathbf{n}} = \eta \hat{\mathbf{n}} \times (\mathbf{H} \times \hat{\mathbf{n}}) \quad (3)$$

Where  $\hat{\mathbf{n}}$  denotes outward local surface normal,  $\eta$  defines the local normal Leontovich surface impedance

By inverting this equation (3), two characteristic orthogonal vector quantities,  $\mathbf{A}$  and  $\mathbf{B}$ , are obtained which are tangential to the local surface and independent of its local normal

$$\mathbf{A} = \mathbf{E} \times \mathbf{E}^* - \eta \eta^* \mathbf{H} \times \mathbf{H}^* \quad \text{and} \quad \mathbf{B} = \eta \mathbf{E}^* \times \mathbf{H} - \eta^* \mathbf{E} \times \mathbf{H}^* \quad (4)$$

Satisfying the following three necessary but not locally yet globally sufficient conditions

$$\mathbf{A} \cdot \mathbf{B} = 0 \quad \text{orthogonality condition} \quad (5a)$$

$$A^2 = B^2 \quad \text{normality condition} \quad (5b)$$

$$\hat{n} \cdot \mathbf{A} = 0 \quad \text{and} \quad \hat{n} \cdot \mathbf{B} = 0 \quad \text{tangentiality condition} \quad (5c)$$

These two sets of boundary conditions are tested for the case of cylinders and spheres of different radii and for two slightly different frequencies for verifying that these conditions are indeed satisfied for (i) the perfectly conducting and (ii) the imperfectly conducting cases explaining why these conditions are exact, and why all of these conditions are necessary but some of the conditions are not locally but only globally sufficient.

Although attempts had been made soon after these vector inverse electromagnetic boundary conditions were first discovered forty years ago, verifying them with experimental data was not possible at that time due to the lack of measurement capabilities and accuracy. However, within the past decade this has changed and sufficiently accurate measurement data for near and far field measurements are now becoming available for testing them which is of considerable relevance to non-destructive material testing and also for advancing vector electromagnetic inverse scattering theory and techniques.

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