Modeling and Control of
Three-Phase PWM Converters

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Outline

1. Introduction
2. Switching Modeling and PWM
3. Average Modeling
4. Small-Signal Modeling
5. Closed-Loop Control Design
6. More Complex Converters
First Three-Phase Converters

Diode Rectifier

SCR Rectifier

Three-Phase Diode Rectifier with Current Load

\[ v_a = V_{am} \cos(\omega t) \]

\[ v_b = V_{am} \cos(\omega t - \frac{2\pi}{3}) \]

\[ v_c = V_{am} \cos(\omega t + \frac{2\pi}{3}) \]
Three-Phase Diode Rectifier with Capacitive Load

Per phase load current

Three-Phase Pulse Width Modulated (PWM) Converters

Boost Rectifier

Buck Inverter

Voltage Source Inverter (VSI)

Buck Rectifier

Boost Inverter

Current Source Inverter (CSI)
Three-Phase Applications

- AC Motor Drives

  ![Diagram of AC Motor Drive System]

  - VSI with uncontrolled rectifier or CSI with SCR rectifier
  - First and still the most common application
  - Regulated output ac voltage or current (amplitude and frequency)
  - Usually only unidirectional power flow

- Power Factor Correction

  ![Diagram of Power Factor Correction System]

  - Adjustable input displacement factor
  - Regulated dc bus voltage

Three-Phase Applications

- Uninterruptible Power Supply (UPS) – Parallel

  ![Diagram of UPS Parallel System]

  - High efficiency
  - Power interruption
  - No power quality improvement to source or grid

- Uninterruptible Power Supply – Series

  ![Diagram of UPS Series System]

  - Lower efficiency
  - No power interruption
  - Improvement source power quality
  - Input power quality improvement
Three-Phase Applications

- Active Filters
  - Power factor control
  - Reduced current distortion
  - Improved damping
  - Utility applications (e.g. STATCOM)

- AC-AC Power Conversion
  - Cascade connection of Boost rectifier and VSI or Buck rectifier and CSI
  - Adjustable displacement factor at input and output
  - Bidirectional power flow
  - Utility applications (e.g. UPFC)

Generalized Structure of A Power Converter

- Switching network is discontinuous and nonlinear

Dushan Boroyevich: Modeling and Control of Three-Phase PWM Converters
Tutorial at PECon 2008, Johor Bahru, Malaysia, 30 November 2008
Three-phase PWM converters below 100 kW operate with relatively high switching frequency (20 kHz - 100 kHz)
- Elimination of audible noise
- Reduction of the size of reactive components
- Significant improvement in waveform quality and closed-loop performance

Motivation

1. Modeling
2. Control Design

Only systems with switching frequency much higher than the line frequency will be studied!
Steps in Modeling of Three-Phase PWM Converters

1. Switching model
   - Time-discontinuous
   - Time-varying
   - Non-linear

2. Average model in stationary coordinates
   - Time-continuous
   - Time-varying
   - Non-linear

3. Average model in rotating (synchronous) coordinates
   - Time-continuous
   - Time-invariant
   - Non-linear

4. Small-signal model
   - Time-continuous
   - Time-invariant
   - Linear
Focus

- Power converter modeling for control design!
- Only converters utilizing high-frequency synthesis:

![Spectrum of v(t)](image)

\[ \omega_m \ll \omega_m \]

- Minor emphasis on modulation
- Only classical, small-signal control approach
- No power stage design and optimization
- No power device discussion
- No topology evaluation, only control implications
- No application considerations

Outline

1. Introduction
   - Vector representation of three-phase variables
2. Switching Modeling and PWM
3. Average Modeling
4. Small-Signal Modeling
5. Closed-Loop Control Design
6. More Complex Converters
Three-Phase Circuits - Source

Y-connection

Δ-connection

3-phase 4-wire

Three-Phase Circuits - Load

Y-connection

Δ-connection

3-phase 4-wire
Three-Phase Variables

Y-connection

\[ i_a + i_b + i_c = 0 \]
\[ v_{ab} + v_{bc} + v_{ca} = 0 \]
\[ v_{an} + v_{bn} + v_{cn} \neq 0 \]

\[ v_{ab} = v_a - v_b \]
\[ v_{bc} = v_b - v_c \]
\[ v_{ca} = v_c - v_a \]

\[ i_a = i_{ca} - i_{ab} \]
\[ i_b = i_{ab} - i_{bc} \]
\[ i_c = i_{bc} - i_{ca} \]

\[ a \]
\[ b \]
\[ c \]

\[ i_a \]
\[ i_b \]
\[ i_c \]

\[ a \]
\[ b \]
\[ c \]

\[ i_a \]
\[ i_b \]
\[ i_c \]

\[ v_a \]
\[ v_b \]
\[ v_c \]

\[ v_a + v_b + v_c \]

DB-19

Sinusoidal, Balanced, Symmetrical

DB-20
Non-sinusoidal, Balanced, Symmetrical

Non-sinusoidal, Unbalanced, Symmetrical
Non-sinusoidal, Balanced, Asymmetrical

Non-sinusoidal, Unbalanced, Asymmetrical
Vector Representations of Three-Phase Variables

Euclid vector representations

$$\vec{v}(t) = \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} \quad \vec{i}(t) = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

Euclidean Space:

$$\ddot{u}_a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \ddot{u}_b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \ddot{u}_c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Vector Multiplication and Norms in Euclidean Spaces

- Inner product: $\langle \vec{v}, \vec{w} \rangle = \vec{v}^T \vec{w} = \sum_{i=1}^{n} v_i w_i$
- "Dot" product: $\langle \vec{v}, \vec{w} \rangle = ||\vec{v}|| \cdot ||\vec{w}|| \cdot \cos \theta$
- "Cross" product: $||\vec{v} \times \vec{w}|| = ||\vec{v}|| \cdot ||\vec{w}|| \cdot \sin \theta$
- Vector norm (length):
  $$||\vec{v}|| = \sqrt{\sum_{i=1}^{n} v_i^2} = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$$
- Norm of a matrix:
  $$\|A\| = \max_{\forall \vec{x} \neq 0} \frac{||A\vec{x}||}{||\vec{x}||}$$
Change of Coordinates

- Multiplication of a vector with any nonsingular matrix, $\mathbf{T}$, of the same order:

\[ \vec{v}_{xyz} = \mathbf{T} \cdot \vec{v}_{abc} \]

is equivalent to the representation of the same vector in a different coordinate system $(xyz)$, whose unit vectors have the following coordinates in the original coordinate system $(abc)$:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\vec{u}_x \\
\vec{u}_y \\
\vec{u}_z
\end{bmatrix}
\]

- If $\langle \vec{u}_x, \vec{u}_y \rangle = \langle \vec{u}_y, \vec{u}_z \rangle = \langle \vec{u}_z, \vec{u}_x \rangle = 0$, new coordinates are also orthogonal.

Example: Balanced Three-Phase Voltages in $abc$ Space

\[ v_a = \cos t \]
\[ v_b = \cos(t - \frac{2\pi}{3}) \]
\[ v_c = \cos(t + \frac{2\pi}{3}) \]

\[ \vec{v}_{ph} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \]
Change of Coordinates \((abc \text{ to } \alpha\beta\gamma)\)

\[ i_a + i_b + i_c = 0 \quad v_{ab} + v_{bc} + v_{ca} = 0 \]

This defines a 2-dimensional subspace \(\chi\), perpendicular to the vector \([1 \ 1 \ 1]^T\) in \(abc\)-space.

\(\alpha\beta\gamma\)-space is traditionally defined by:
- \(\alpha\)-axis is chosen as projection of the \(a\)-axis onto \(\chi\),
- \(\gamma\)-axis is co-linear with vector \([1 \ 1 \ 1]^T\)
- \(\beta\)-axis is defined by right-hand rule.

\[
\begin{bmatrix}
1 & \frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

Transformation Matrix \(T_{\alpha\beta\gamma} / abc\)

The transformation matrix

\[ \| T_{\alpha\beta\gamma} / abc \| = 1 \]

Example:

\[ \tilde{x}_{abc} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \implies \tilde{x}_{\alpha\beta\gamma} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{3} \end{bmatrix} \]
Transformation Matrix $T_{abc/αβγ}$

$$T_{abc/αβγ} = T_{αβγ/abc}^{-1} = T_{αβγ/abc}^T = \begin{bmatrix}
1 & 0 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}$$

$$\vec{v}_{abc} = T_{abc/αβγ} \cdot \vec{v}_{αβγ}$$

$$\vec{i}_{abc} = T_{abc/αβγ} \cdot \vec{i}_{αβγ}$$

Example: Balanced Three-Phase Voltages in $αβγ$ Space

$$v_a = \cos t$$
$$v_b = \cos(t - \frac{2π}{3})$$
$$v_c = \cos(t + \frac{2π}{3})$$

$$\vec{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

$$\vec{v}_{αβγ} = T_{αβγ/abc} \cdot \vec{v}_{abc}$$
Example: State-Space Equations

\[
\vec{v} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}
\]

\[
\vec{v} = R\vec{i} + L \frac{d\vec{i}}{dt} \iff \frac{d\vec{i}}{dt} = L^{-1}R\vec{i} + L^{-1}\vec{v}
\]

\(\vec{v}\) and \(\vec{i}\) are sinusoidal in steady-state! Find a coordinate transformation:

\[
\vec{v}_x = T_x \cdot \vec{v}, \quad \vec{i}_x = T_x \cdot \vec{i},
\]

such that \(\vec{v}_x\) and \(\vec{i}_x\) are constant

in steady state, and \(T_x\) is differentiable and invertible.

Example: State-Space Equations

\[
\vec{v} = \begin{bmatrix} V_m \cos(\omega t) \\ V_m \cos(\omega t - \frac{2\pi}{3}) \\ V_m \cos(\omega t + \frac{2\pi}{3}) \end{bmatrix} = \begin{bmatrix} \vec{v}_a \\ \vec{v}_b \\ \vec{v}_c \end{bmatrix}
\]

\[
\begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}
\]

\[
\begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \end{bmatrix}
\]
Example: State-Space Equations – Solution

\[ i(t) = e^{-t \frac{R}{L} \cdot i(0)} + \int_{0}^{t} e^{-\frac{R}{L} \cdot i(\tau)} \cdot \mathbf{L}^{-1} \cdot \mathbf{v}(\tau) \cdot d\tau \]

Natural Response

\[
\begin{bmatrix}
i_a(0) \\
i_b(0) \\
i_c(0)
\end{bmatrix} \cdot \frac{R}{L} + \frac{V_m}{Z}
\]

Forced Response

\[
\begin{bmatrix}
\cos(\omega t - \phi) - \frac{R}{Z} \cdot e^{-\frac{R}{L} t} \\
\cos(\omega t - \frac{2\pi}{3} - \phi) + \frac{R + \sqrt{3} \cdot \omega L}{2Z} \cdot e^{-\frac{R}{L} t} \\
\cos(\omega t + \frac{2\pi}{3} - \phi) + \frac{R - \sqrt{3} \cdot \omega L}{2Z} \cdot e^{-\frac{R}{L} t}
\end{bmatrix}
\]

where per-phase impedance, \( Z \), at the source frequency, \( \omega \), is defined as:

\[ Z = R + j\omega L = Z \cdot e^{j\phi} \]

\[ Z = \sqrt{R^2 + \omega^2 L^2} \]

\[ \phi = \arctan \frac{\omega L}{R} \]

Example: State-Space Equations – Steady State Solution

\[
\lim_{t \to \infty} [\mathbf{\ddot{v}}(t)] = I_m \cdot \begin{bmatrix}
\cos(\omega t - \phi) \\
\cos(\omega t - \frac{2\pi}{3} - \phi) \\
\cos(\omega t + \frac{2\pi}{3} - \phi)
\end{bmatrix}
\]

\[ I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \]

\[ \phi = \arctan \frac{\omega L}{R} \]

\[
\lim_{t \to \infty} \left[ \frac{d\mathbf{\ddot{i}}}{dt} \right] \neq 0
\]

Want to find change of variables:

\[ \mathbf{\ddot{v}}_x = \mathbf{T}_x \cdot \mathbf{\ddot{v}} \]

where: \( \mathbf{\ddot{v}}_x \) and \( \mathbf{\ddot{i}}_x \) are constant in steady state

\[ \mathbf{\ddot{i}}_x = \mathbf{T}_x \cdot \mathbf{\ddot{i}} \]

\( \mathbf{T}_x \): differentiable and invertible
A rotating vector in $\alpha\beta\gamma$ space can be a constant vector in a rotating space.

\[
\begin{bmatrix}
v_d \\
v_q \\
v_\theta \\
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
v_\alpha \\
v_\beta \\
v_\gamma \\
\end{bmatrix}
\]

\[
\theta = \int_0^\tau \omega(\tau)d\tau + \theta(0)
\]

Where $\omega$ is the rotating speed.

---

Preserve the same third axis, that is 0-axis is the same as $\gamma$-axis.

\[
\begin{bmatrix}
v_d \\
v_q \\
v_\theta \\
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
v_\alpha \\
v_\beta \\
v_\gamma \\
\end{bmatrix}
\]

Therefore

\[
T_{dq0/\alpha\beta\gamma} = \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

\[
\| T_{dq0/\alpha\beta\gamma} \| = 1
\]

\[
T_{\alpha\beta\gamma/dq0} = T_{dq0/\alpha\beta\gamma}^{-1} = T_{dq0/\alpha\beta\gamma}^T
\]
Transformation Matrix \( T_{dq0/abc} \)

\[
\vec{v}_{dq0} = T_{dq0/abc} \cdot \vec{v}_{abc}
\]

Therefore

\[
\vec{v}_{dq0} = T_{dq0/abc} \cdot \vec{v}_{abc}
\]

where

\[
T_{dq0/abc} = T_{dq0/abcdef} \cdot T_{abcdef/abc}
\]

Park’s Transformation

\[
\begin{bmatrix}
\cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
-\sin \theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
-\sin \theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
\end{bmatrix}
\]

Example: Balanced Three-Phase Voltages in \( dq0 \) Space

\[
v_a = \cos t \quad v_b = \cos(t - \frac{2\pi}{3}) \quad v_c = \cos(t + \frac{2\pi}{3})
\]

\[
\vec{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}
\]

\[
\vec{v}_{dq0} = T_{dq0/abc} \cdot \vec{v}_{abc}
\]
Power Definition in Three-Phase Circuits

\[ \mathbf{p} = \mathbf{v} \cdot \mathbf{i} = \mathbf{v}^T \mathbf{i} = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = v_a i_a + v_b i_b + v_c i_c \]

where \( v \) & \( i \) are corresponding voltages and currents in a three-phase circuit.

It can be easily proved that:

\[ v_a i_a + v_b i_b + v_c i_c = v_a i_a + v_\beta i_\beta + v_\gamma i_\gamma = v_d i_d + v_q i_q + v_0 i_0 \]

Example:

Power in Sinusoidal Steady State

\[ \mathbf{v} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} V_{am} \cos(\omega t) \\ V_{am} \cos(\omega t - \frac{2\pi}{3}) \\ V_{am} \cos(\omega t + \frac{2\pi}{3}) \end{bmatrix}, \quad \mathbf{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} I_{am} \cos(\omega t - \phi) \\ I_{am} \cos(\omega t - \frac{2\pi}{3} - \phi) \\ I_{am} \cos(\omega t + \frac{2\pi}{3} - \phi) \end{bmatrix} \]

where: \( \phi = \arctan \frac{\omega L}{R} \)
Example:
Power in Sinusoidal Steady State

\[ P = \| \vec{v} \cdot \vec{i} \| = \| \vec{v} \| \cdot \| \vec{i} \| \cos \phi = \frac{3}{2} V_{am} I_{am} \cos \phi \]

\[ Q = \| \vec{v} \times \vec{i} \| = \| \vec{v} \| \cdot \| \vec{i} \| \sin \phi = \frac{3}{2} V_{am} I_{am} \sin \phi \]

Phasor Representation

Phasors are defined **ONLY for sinusoidal steady state**!

\[ v_a = V_{am} \cos(\omega t + \phi) \quad \rightarrow \quad V_a = V_{am} e^{j\phi} = V_{am} \angle \phi \]

• Vector representation is **NOT** phasor representation!
Phasor Representation

Phasors are defined **ONLY for sinusoidal steady state**!

Phasors are very useful for the analysis of linear systems without transients, which are excited by **constant single frequency** \((\omega)\) **sinusoidal generators**.

\[
v_a = V_m \cos(\omega t + \phi) = \text{Re}\left[V_m \cdot e^{j\phi}\right] = \text{Re}\left[V_m \cos(\omega t + \phi) + j V_m \sin(\omega t + \phi)\right]
\]

\[
V_a = \frac{V_m}{\sqrt{2}} \angle \phi = V_{\text{rms}} \cdot e^{j\phi}
\]

\[
v_a = V_m \cos(\omega t + \phi) = \sqrt{2} \cdot \text{Re}\left(V_a \cdot e^{j\omega t}\right)
\]
Boost Rectifier / Voltage Source Inverter

- Boost Rectifier
- Voltage Source Inverter (VSI)

Switching function:
\[ s = \begin{cases} 1, & v = 0, \text{if switch } s \text{ is closed} \\ 0, & i = 0, \text{if switch } s \text{ is open} \end{cases} \]

Switching constraints:
- Voltage source or capacitor cannot be shorted
- Current source or inductor cannot be open

Method of Modeling Switching Network

Current bi-directional two-quadrant switch

VSI / BOOST RECTIFIER DC VOLTAGE RANGE

\[ V_{dc} > V_m \]

where \( V_m \) is the peak value of the line-to-line input voltage

Load
Three-Switch (Single-Pole-Double-Throw) Boost Rectifier Voltage Source Inverter

Allowed switching combinations:

\[ s_p + s_a = 1; \quad i \in \{a, b, c\} \]

Define Voltage-Unidirectional Single-Pole-Double-Throw Switch and switching function

\[ s_i = s_p = 1 - s_a; \quad i \in \{a, b, c\} \]

Find the relationships:

\[ v_{ab} = f_{ab}(s_{ab}, v_{dc}) \quad v_{bc} = f_{bc}(s_{bc}, v_{dc}) \quad v_{ca} = f_{ca}(s_{ca}, v_{dc}) \]

\[ i_{dc} = f_i(s_{j}, i_x, i_y, i_z) \]

\[ s_{sp} = 0; \quad v_{ab} = 0 \]

\[ s_{sp} = 1; \quad v_{ab} = v_{dc} \]

\[ s_{sp} = 0; \quad v_{ab} = -v_{dc} i_a \]

Development of Switching Model (Boost rectifier / Voltage source inverter)
Development of Switching Model
(Boost rectifier / Voltage source inverter)

Define phase-leg switching function
\[ s_i = s_m = 1 - s_i; \quad i \in \{a, b, c\} \]

<table>
<thead>
<tr>
<th>(s_a)</th>
<th>(s_b)</th>
<th>(s_c)</th>
<th>(s_a-s_b)</th>
<th>(s_b-s_c)</th>
<th>(s_c-s_a)</th>
<th>(i_{dc})</th>
<th>(v_{ab})</th>
<th>(v_{bc})</th>
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<td>(i_a+i_b+i_c)</td>
<td>0</td>
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</tr>
</tbody>
</table>

Note that:
\[ s_{ab} = s_a - s_b \] ...

\[ v_{ab} = v_a - v_b \] ...

Instantaneous voltage equation
\[
\begin{bmatrix}
    v_{ab} \\
v_{bc} \\
v_{ca}
\end{bmatrix}
= \begin{bmatrix}
    s_a - s_b \\
s_b - s_c \\
s_c - s_a
\end{bmatrix}
\begin{bmatrix}
v_{dc}
\end{bmatrix}
\]

Instantaneous current equation
\[
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
= \begin{bmatrix}
    s_a & s_b & s_c
\end{bmatrix}
\begin{bmatrix}
i_{dc}
\end{bmatrix}
\]

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Relationship Between Line-to-Line Current and Phase Current

\[ i_a = i_{ab} - i_{ca} \]
\[ i_b = i_{bc} - i_{ab} \]
\[ i_c = i_{ca} - i_{bc} \]

\[ i_a - i_b = i_{ab} - i_{ca} - (i_{bc} - i_{ab}) = 2i_{ab} - (i_{ca} + i_{bc}) = 3i_{ab} \]

Assume \( i_{ab} + i_{bc} + i_{ca} = 0 \)

Similarly

\[ i_{ab} = \frac{1}{3} (i_a - i_b) \]
\[ i_{bc} = \frac{1}{3} (i_b - i_c) \]
\[ i_{ca} = \frac{1}{3} (i_c - i_a) \]

\[ i_{dc} = s_a i_a + s_b i_b + s_c i_c = s_a (i_{ab} - i_{ca}) + s_b (i_{bc} - i_{ab}) + s_c (i_{ca} - i_{bc}) \]

\[ = i_{ab} (s_a - s_b) + i_{bc} (s_b - s_c) + i_{ca} (s_c - s_a) = \begin{bmatrix} s_{ab} & s_{bc} & s_{ca} \end{bmatrix} \begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} \]

Boost Rectifier / Voltage Source Inverter Switching Model

\[ \bar{v}_{i-l} = \bar{s}_{i-l} \cdot v_{dc} \]
\[ i_{dc} = \bar{s}_{i-l}^T \cdot \bar{i}_{i-l} \]

where:

\[ \bar{v}_{i-l} = \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} = \begin{bmatrix} v_a - v_b \\ v_b - v_c \\ v_c - v_a \end{bmatrix} \]
\[ \bar{s}_{i-l} = \begin{bmatrix} s_{ab} \\ s_{bc} \\ s_{ca} \end{bmatrix} = \begin{bmatrix} s_a - s_b \\ s_b - s_c \\ s_c - s_a \end{bmatrix} \]
\[ \bar{i}_{i-l} = \begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} = \begin{bmatrix} i_a - i_b \\ i_b - i_c \\ i_c - i_a \end{bmatrix} \]
Outline

1. Introduction
2. Switching Modeling and PWM
   - Switching model of VSI & boost rectifier
   - Space vector modulation for VSI & boost rectifier
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6. More Complex Converters

Switching States for Boost Rectifier / Voltage Source Inverter

<table>
<thead>
<tr>
<th>$s_a$</th>
<th>$s_b$</th>
<th>$s_c$</th>
<th>Switching state</th>
<th>$i_{dc}$</th>
<th>$v_{ab}$</th>
<th>$v_{bc}$</th>
<th>$v_{cd}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$nnn$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$nnp$</td>
<td>$i_s$</td>
<td>0</td>
<td>$-V_{dc}$</td>
<td>$V_{dc}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$nnp$</td>
<td>$i_b$</td>
<td>$-V_{dc}$</td>
<td>$V_{dc}$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$npp$</td>
<td>$i_b+i_s$</td>
<td>$-V_{dc}$</td>
<td>0</td>
<td>$V_{dc}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$pnn$</td>
<td>$i_s$</td>
<td>$V_{dc}$</td>
<td>0</td>
<td>$-V_{dc}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$ppn$</td>
<td>$i_s+i_b$</td>
<td>$V_{dc}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$pnp$</td>
<td>$i_s+i_b$</td>
<td>0</td>
<td>$V_{dc}$</td>
<td>$-V_{dc}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$ppp$</td>
<td>$i_s+i_b+i_c$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Vector Space of Line-to-Line Variables

- Phase variables (a, b and c) produce line-to-line variables (ab, bc and ca) in plane-χ.
- Line-to-line variables (ab, bc and ca) do not have γ-component in αβγ-coordinate system.

Line-to-Line Voltage Space Vector

\[
\begin{bmatrix}
    v_α \\
    v_β \\
    v_γ 
\end{bmatrix} = T_{αβγ/abc} \cdot 
\begin{bmatrix}
    v_{ab} \\
    v_{bc} \\
    v_{ca} 
\end{bmatrix}
\]

where

\[
T_{αβγ/abc} = \frac{2}{\sqrt{3}} \begin{bmatrix}
    1 & -\frac{1}{2} & -\frac{1}{2} \\
    0 & \sqrt{3}/2 & -\sqrt{3}/2 \\
    \sqrt{3}/2 & 0 & -\sqrt{3}/2 
\end{bmatrix}
\]

- Space vector
  \[\vec{v} = \rho \cdot e^{iθ}\]
  \[\rho = \sqrt{v_α^2 + v_β^2}\]
  \[θ = \tan^{-1}\left(\frac{v_β}{v_α}\right)\]

If \(V_n\) is the amplitude of balanced, symmetrical, three-phase line-to-line voltages, then \(\rho = \sqrt{3} \cdot V_n\)
Switching State Vector \([pnn]\)

\[
\vec{V}_{\text{pnn}} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}_{\text{pnn}} = T_{\alpha\beta/abc} \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{\text{pnn}} = \frac{2}{3} \sqrt{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ 0 \\ -V_{dc} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} V_{dc} \\ \frac{1}{2} V_{dc} \end{bmatrix}
\]

\[
\vec{v}_{\text{pnn}} = \vec{V}_1 = \rho \cdot e^{i\phi}
\]

\[
\rho = \sqrt{2} \cdot V_{dc}
\]

\[
\theta = \tan^{-1} \left( \frac{v_b}{v_a} \right) = 30^\circ
\]

Switching State Vector \([ppn]\)

\[
\vec{V}_{\text{ppn}} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}_{\text{ppn}} = T_{\alpha\beta/abc} \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{\text{ppn}} = \frac{2}{3} \sqrt{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ V_{dc} \\ -V_{dc} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \cdot V_{dc} \end{bmatrix}
\]

\[
\vec{v}_{\text{ppn}} = \vec{V}_2 = \rho \cdot e^{i\phi}
\]

\[
\rho = \sqrt{2} \cdot V_{dc}
\]

\[
\theta = \tan^{-1} \left( \frac{v_b}{v_a} \right) = 90^\circ
\]
Switching State Vector $[ppp]$

\[ \bar{V}_{ppp} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}_{ppp} = T_{ab/abc} \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{ppp} = \frac{2}{\sqrt{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

\[ \bar{V}_{ppp} = \bar{V}_{o} = 0 \]

Switching State Vectors

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\theta$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{V}_{[pnn]}$</td>
<td>30</td>
</tr>
<tr>
<td>$\bar{V}_{[ppn]}$</td>
<td>90</td>
</tr>
<tr>
<td>$\bar{V}_{[nnp]}$</td>
<td>$\sqrt{2} \cdot V_{dc}$</td>
</tr>
<tr>
<td>$\bar{V}_{[npp]}$</td>
<td>-150</td>
</tr>
<tr>
<td>$\bar{V}_{[pnp]}$</td>
<td>-90</td>
</tr>
<tr>
<td>$\bar{V}_{[ppp]}$</td>
<td>-30</td>
</tr>
<tr>
<td>$\bar{V}_{[nnn]}$</td>
<td>0</td>
</tr>
</tbody>
</table>

$\bar{V}_{o} = [ppp] = [nnn]$ at center point
Reference Voltage Vector, $V_{ref}$

Assume

$$
\begin{bmatrix}
V_{ab} \\
V_{bc} \\
V_{ca,r_{ref}}
\end{bmatrix} = \begin{bmatrix}
V_m \cdot \cos(\omega t) \\
V_m \cdot \cos(\omega t - 120^\circ) \\
V_m \cdot \cos(\omega t + 120^\circ)
\end{bmatrix}
$$

$$
\vec{V}_{ref} = \begin{bmatrix}
V_{\alpha} \\
V_{\beta}
\end{bmatrix}_{r_{ref}} = \rho \cdot e^{j\theta}
$$

where

$$
\rho = \sqrt{V_{\alpha}^2 + V_{\beta}^2} = \frac{3}{2} V_m
$$

$$
\theta = \tan^{-1}\left(\frac{V_{\beta}}{V_{\alpha}}\right) = \omega t
$$

In general,

$$
\vec{V}_{ref}(t) = \frac{3}{2} V_m(t) \cdot e^{j\theta(t)}
$$

$\bar{V}_a = [ppp] = [nnn]$ at center point

Definition of High Frequency Synthesis

$$
\int_0^{T_s} \bar{V}_{v_{avg}} dt = \sum \int_0^{T_i} \bar{V}_i dt, \quad \sum T_i = T_s
$$

For example

$$
\int_0^{T_s} \bar{V}_{v_{avg}} dt = \int_0^{T_1} \bar{V}_1 dt + \int_{T_1}^{T_1+T_2} \bar{V}_2 dt + \int_{T_1+T_2}^{T_s} \bar{V}_3 dt
$$

Total area of $V_{ref}$ = Area of $V_{1(a)}$ and $V_{2(a)}$
Space Vector Modulation

**Step 1**: Choose desired switching state vectors to synthesize $\bar{v}_{ref}$

**Step 2**: Calculate the duty ratios of chosen switching state vectors

**Step 3**: Make the sequence of chosen switching state vectors

Dushan Boroyevich: Modeling and Control of Three-Phase PWM Converters
Tutorial at PECon 2008, Johor Bahru, Malaysia, 30 November 2008
**Step 1: Choice of Switching State Vectors**

- Minimize the number of switching
- Minimize the harmonic distortion

<table>
<thead>
<tr>
<th>( V_{\text{ref}} ) location</th>
<th>Chosen vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector I</td>
<td>( \vec{V}_1 ) and ( \vec{V}_2 )</td>
</tr>
<tr>
<td>Sector II</td>
<td>( \vec{V}_1 ) and ( \vec{V}_3 )</td>
</tr>
<tr>
<td>Sector III</td>
<td>( \vec{V}_2 ) and ( \vec{V}_3 )</td>
</tr>
<tr>
<td>Sector IV</td>
<td>( \vec{V}_3 ) and ( \vec{V}_4 )</td>
</tr>
<tr>
<td>Sector V</td>
<td>( \vec{V}_4 ) and ( \vec{V}_5 )</td>
</tr>
<tr>
<td>Sector VI</td>
<td>( \vec{V}_5 ) and ( \vec{V}_6 )</td>
</tr>
</tbody>
</table>

Choose minimum number of switching state vectors adjacent to \( \vec{V}_{\text{ref}} \).

**Step 2: Duty Ratio of Switching State Vectors at Sector I**

From HF synthesis definition,

\[
\int_{0}^{T_s} V_{\text{ref}} \, dt = \int_{0}^{T_1} \vec{V}_1 \, dt + \int_{T_1}^{T_1 + T_2} \vec{V}_2 \, dt + \int_{T_1 + T_2}^{T_1 + T_2 + T_s} \vec{V}_3 \, dt
\]

Assume \( V_{\text{ref}} \) is constant in \( T_s \), \( V_{\text{ref}} \cdot T_s = \vec{V}_1 \cdot T_1 + \vec{V}_2 \cdot T_2 \)

\[
\rho \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} \cdot T_s = \begin{bmatrix} I_1 \\ 0 \end{bmatrix} \cdot T_1 + \begin{bmatrix} \cos 60^\circ \\ \sin 60^\circ \end{bmatrix} \cdot T_2
\]

where \( \phi = 0 - 30^\circ \)

\[
\begin{align*}
T_1 &= d_1 = \frac{2}{\sqrt{3}} \frac{\rho}{|\vec{V}_3|} \sin(60^\circ - \phi) \\
T_2 &= d_2 = \frac{2}{\sqrt{3}} \frac{\rho}{|\vec{V}_3|} \sin \phi \\
d_0 &= 1 - d_1 - d_2
\end{align*}
\]
### Duty Ratio of Switching State Vectors

Other sectors have the same results of duty ratio.

\[
\frac{T_N}{T_s} = d_N = \frac{2}{\sqrt{3}} \frac{\rho}{|V_N|} \cdot \sin(60^\circ - \phi)
\]

\[
\frac{T_{N+1}}{T_s} = d_{N+1} = \frac{2}{\sqrt{3}} \frac{\rho}{|V_{N+1}|} \cdot \sin \phi
\]

\[
d_0 = 1 - d_N - d_{N+1}
\]

where \( \phi = 0 - (N - 1) \cdot 60^\circ - 30^\circ \)

\( N \): sector number (1 ~ 6)

### Modulation Index

For all the switching state vectors, \( |V_N| = \sqrt{2} \cdot V_{dc} \) and \( \rho = \frac{\sqrt{3}}{2} \cdot V_w \)

\[
d_N = \frac{V_m}{V_{dc}} \cdot \sin(60^\circ - \phi)
\]

\[
d_{N+1} = \frac{V_m}{V_{dc}} \cdot \sin \phi
\]

\[
d_0 = 1 - d_N - d_{N+1}
\]

Define the modulation index \( M = \frac{V_m}{V_{dc}} \)

\[
d_N = M \cdot \sin(60^\circ - \phi)
\]

\[
d_{N+1} = M \cdot \sin \phi
\]

\[
d_0 = 1 - d_N - d_{N+1}
\]
Assume \( d_0 = 0 \), then \( d_N + d_{N+1} = 1 \)

\[
d_N + d_{N+1} = M \cdot (\sin(60^\circ - \phi) + \sin \phi)
= M \cdot \cos(30^\circ - \phi)
= \frac{V_m}{V_{dc}} \cdot \cos(30^\circ - \phi)
= I
\]

\[\therefore V_m = \frac{V_m}{\cos(30^\circ - \phi)}\]

The trajectory of \( \hat{V}_r \) makes a hexagon.

Assume \( M = 1 \), then \( \frac{V_m}{V_{dc}} = I \)

\[\therefore V_m = V_{dc}\]

\[\hat{V}_{ref} = \frac{\sqrt{3}}{2} V_{dc} \cdot e^{j\theta}\]

The trajectory of \( \hat{V}_{ref} \) makes a circle whose radius is \( \frac{\sqrt{3}}{2} V_{dc} \).

This trajectory of \( \hat{V}_{ref}^2 \) represents the largest 3-phase sinusoidal voltage that can be synthesized.
Step 3: Sequence of Switching State Vectors

Asymmetrical sequence
Symmetrical sequence

2-phase commutation
3-phase commutation

Feed forward
Feedback

\[ \frac{\tau_1}{\tau_0} \int I_{ref}^t dt = \sum_{i=1}^{n} \left( \frac{\tau_i}{\tau_0} \int i_t^t dt \right) = \]

Sequence of SSVs – SVM 1
(Three-Phase – Right Aligned: 3Φ-RA)

- Use both zero switching state vectors
- Asymmetrical sequence
- Six commutations per switching cycle

\[ V_0 \quad V_1 \quad V_2 \quad V_0 \quad V_0 \quad V_1 \quad V_2 \quad V_0 \]

\[ S_a \quad S_b \quad S_c \]

\[ T_0/2 \quad T_1 \quad T_2 \quad T_0/2 \quad T_0/2 \quad T_1 \quad T_2 \quad T_0/2 \]

< Example in sector I >

- 3Φ-LA has same characteristics
Sequence of SSVs – SVM 2
(Three-Phase – Centered: 3Φ-C)

• Use both zero switching state vectors
• Symmetrical sequence \(\Rightarrow\) Low THD
• Six commutations per switching cycle

\[
\begin{array}{cccccccc}
S_a & V_0 & V_1 & V_2 & V_3 & V_4 & V_5 & V_0 \\
S_b & T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 \\
S_c & T_8 & T_9 & T_{10} & T_{11} & T_{12} & T_{13} & T_{14} \\
\end{array}
\]

< Example in sector I >

Sequence of SSVs – SVM 3
(Three-Phase – Double-Period: 3Φ-2T)

• Use zero vectors alternatively in adjacent switching cycle
• Asymmetrical sequence in \(T_3\), but symmetrical in \(2\cdot T_3\)
• Three commutations \(\Rightarrow\) 50 % switching loss reduction
• Introduction of harmonics around \(\frac{I}{2 \cdot T_s}\)

\[
\begin{array}{cccccccc}
S_a & V_1 & V_2 & V_3[ppp] & V_4 & V_1 & V_4[nnn] \\
S_b & T_1 & T_2 & T_3 & T_4 & T_5 & T_6 \\
S_c & T_7 & T_8 & T_9 & T_{10} & T_{11} & T_{12} \\
\end{array}
\]

< Example in sector I >
Sequence of SSVs – SVM 4
(Two-Phase – Right Aligned: $2\Phi$-RA)

- Use a zero vector in one switching cycle
- Asymmetrical sequence
- Four commutations $\Rightarrow$ Reduced switching losses

Sequence of SSVs – SVM 5
(Two-Phase – Centered: $2\Phi$-C)

- Use a zero vector in one switching cycle
- Symmetrical sequence $\Rightarrow$ Low THD
- Four commutations $\Rightarrow$ Reduced switching losses
**Sequence of SSVs – SVM 6**  
(Minimum-Loss SVM)

(Two-Phase – Right Aligned – minimum Loss: 2Φ-RA-mL)

- Choose a zero switching state vector to avoid switching the phase with the highest instantaneous current
- Reduce the switching losses up to 50% compared to 3Φ modulations, assuming that switching losses are proportional to the current

Choice of zero vector in sector I (pnn, ppp, and ppp or nnn)

\[ |i_a| > |i_c| ? \]

- Yes: \( V_o = [ppp] \)
- No: \( V_o = [nnn] \)

- Possible sequences: 2Φ-RA-mL, 2Φ-LA-mL, or 2Φ-C-mL

---

**Example of 2Φ-x-mL SVM in Sector I**  
for balanced, symmetrical, sinusoidal, steady-state case

- \( |\delta| \leq 30^\circ \)
- Switching loss reduction = 50%

- \( |\delta| > 30^\circ \)
- Switching loss reduction < 50%
Comparison:
Total Harmonic Distortion (THD)

With the Fourier series
\[ V = \sum_{n=1}^{\infty} V_n \cdot e^{jn\omega t}, \quad THD = \sqrt{\frac{\sum_{n=2}^{\infty} V_n^2}{V_1}} \]

Calculated and switching-model simulation results for \( f_s > 100 \cdot f_{line} \).

Comparison:
Peak-to-Peak Current Ripple

For RA or LA modulations:
\[ I_{pp} = \frac{2 \cdot V_{dc}}{3 \cdot L} \cdot (1 - M) \cdot M \cdot T_S \]

For centered modulations:
\[ I_{pp} = \frac{V_{dc}}{3 \cdot L} \cdot (1 - M) \cdot M \cdot T_P \]

where \( T_P = T_S \), except \( T_P = 2T_S \) for 3Φ-2T.

\( M \): modulation index
\( L \): load inductance
Comparison: Switching Losses

- Number of commutations per switching cycle:
  - $3\Phi-x : 6$
  - $3\Phi-2T : 3$
  - $2\Phi-x : 4$

$2\Phi-x - mL$ avoids switching the phase with highest current between -30° and 30°.

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Modulation Methods

Pulse Width Modulation
- Sinusoidal PWM
- Third-harmonic injection PWM
- Space vector modulations (SVM1, 2, 3, 4 and 5)

Feedback scheme
- Hysteresis current control
- Space vector modulations (SVM 6)

Pulse Amplitude Modulation

Sinusoidal Pulse Width Modulation (SPWM)

- Determine the switching state by magnitude of \( v_{\text{ref}} \) and \( v_{\text{car}} \)
  
  If \( |v_{\text{ref}}| > |v_{\text{car}}| \), then \( s_{ap} \) on.
  
  If \( |v_{\text{ref}}| < |v_{\text{car}}| \), then \( s_{an} \) on.

- Switching frequency is the same as carrier wave frequency, \( f_c \).
Waveforms of Single-Phase SPWM

\[ \frac{V_{ref}}{V_{car}} = \frac{V_{a(t)}}{\left(\frac{V_{dc}}{2}\right)} \times \frac{2}{V_{dc}} \Rightarrow V_{a(t)} = \frac{V_{dc}}{2} \times \frac{V_{ref}}{V_{car}} \]

Three-Phase SPWM

Dushan Boroyevich: Modeling and Control of Three-Phase PWM Converters
Tutorial at PECon 2008, Johor Bahru, Malaysia, 30 November 2008
Assume \( v_{a}^{ref}, v_{b}^{ref}, \) and \( v_{c}^{ref} \) are constant in a switching cycle.

Symmetrical (Center-based) Three-phase commutation

\[
T_{ON} = -\frac{T_S}{4 \cdot V_{car}} \cdot (v_{a}^{ref} - V_{car})
\]

\[
T_{OFF} = T_{ON} + \frac{T_S}{2} + \frac{T_S}{4 \cdot V_{car}} \cdot (v_{a}^{ref} - V_{car})
\]

\[
= \frac{T_S}{4 \cdot V_{car}} \cdot (v_{c}^{ref} + V_{car})
\]

---

Modulation Example of SVM2 in Sector I

\[
\frac{T_{ON}}{T_S} = d_1 = \sqrt{2} \cdot \frac{V_m}{V_1} \cdot \sin(60^\circ - \phi)
\]

\[
\frac{T_{OFF}}{T_S} = d_2 = \sqrt{2} \cdot \frac{V_m}{V_2} \cdot \sin \phi
\]

\[
d_0 = l - d_1 - d_2
\]

\[
T_{OFF} = T_{ON} = T_0
\]

\[
= \frac{T_S}{4} - \frac{V_{dc}}{V_{in}} \cdot \frac{T_S}{4} \cdot \cos(30^\circ - \phi)
\]
Zero Vector Timings in Sector I

- In SPWM, assume $V_{\text{ref}} = V_{\text{car}}$
  
  \[ T_{0N} = \frac{T_s}{4} \cdot V_{\text{ref}} \cdot (v_{\phi}^{\text{ref}} - V_{\text{ref}}) \]

  \[ T_{0P} \left( \frac{2}{T_s} \right) = \frac{T_s}{4} \cdot V_{\text{ref}} \cdot (v_{\phi}^{\text{ref}} + V_{\text{ref}}) \]

  where \[ v_{\phi}^{\text{ref}} = V_{\text{ref}} \cdot \cos \phi \]

  \[ v_{\phi}^{\text{ref}} = V_{\text{ref}} \cdot \cos(\phi + 120^\circ) \]

- In SVM2, assume $V_m = V_{\text{dc}}$

  \[ T_{0P} = T_{0N} = \frac{T_s}{4} \]

  \[ = \frac{T_s}{4} - \frac{T_s}{4} \cdot \cos(30^\circ - \phi) \]

Maximum AC Voltage of SPWM

The $\bar{v}_{\text{ref}}$ trajectory in SPWM: $\bar{v}_{\text{ref}} = \frac{3}{2\sqrt{2}} \cdot V_{\text{dc}} \cdot e^{j\theta}$

By definition: $\bar{v}_{\text{ref}} = \frac{\sqrt{3}}{2} \cdot V_m \cdot e^{j\theta}$

The maximum AC voltage: $V_m = \sqrt{3} \cdot V_{\text{dc}}$

The maximum AC voltage of SVM: $V_m = V_{\text{dc}}$

\[ \frac{V_m^{\text{SVM}}}{V_m^{\text{SPWM}}} = \frac{V_{\text{dc}}}{\left( \frac{\sqrt{3}}{2} \cdot V_{\text{dc}} \right)} = 1.155 \]

SVM produces 15.5% higher maximum output than SPWM!
Over Modulation of SPWM

- $V_{ref} \leq V_{car}$: Linear modulation
- $V_{ref} > V_{car}$: Over modulation

Over modulation region is nonlinear with more harmonics

$$\frac{V_{max}(\text{SPWM})}{V_{max}(\text{Square})} = \frac{\frac{V_{dc}}{2}}{\frac{4}{\pi} \frac{V_{dc}}{2}} = \frac{\pi}{4} \approx 0.785$$

Third-Harmonic Injection PWM

$$V_{ref} = V_{ref}^{(1)} \sin \omega t + V_{ref}^{(3)} \sin 3 \omega t$$

$$V_{ref} = V_{ref}^{(1)} + V_{ref}^{(3)}$$
Maximum AC Voltage of Third-Harmonic Injection PWM

In general, \( v_{ref} = V_{(i)}^{ref} \left( \sin 0 + \frac{1}{6} \cdot \sin 30 \right) \)

Assume \( V_{ref} = 1 \),

then \( V_{(i)}^{ref} = \left( \frac{1}{\sqrt{3} \cdot 2} \right) = 1.155 \)

The maximum AC voltage is 15.5 % more than SPWM

Average Values of Phase-to Neutral Voltage for SVM 2 (3Φ-C) in Sector I

\[
\begin{align*}
\frac{v_a}{V_{dc}} &= -\frac{d_a}{2} + d_i + d_z + \frac{d_b}{2} \\
&= \sin(60^\circ - \phi) + \sin \phi \\
&= \sin(60^\circ + \phi) \\
\frac{v_b}{V_{dc}} &= -\frac{d_a}{2} + d_i + d_z + \frac{d_b}{2} \\
&= -\sin(60^\circ - \phi) + \sin \phi \\
&= \sqrt{3} \cdot \sin(\phi - 30^\circ) \\
\frac{v_c}{V_{dc}} &= -\frac{d_a}{2} - d_i - d_z + \frac{d_b}{2} \\
&= -\sin(60^\circ + \phi) \\
&= -v_a
\end{align*}
\]
Average Values of Phase-to Neutral Voltage for SVM 2 (3Φ-C)

- In sector II,
  \[
  \frac{V_a}{V_{dc}} = -\frac{d_0}{2} + d_1 - d_2 + \frac{d_0}{2}
  \]
  \[
  = \sin(60^\circ - \phi) - \sin \phi
  \]
  \[
  = \sqrt{3} \cdot \sin(30^\circ - \phi)
  \]
  \[
  = \sqrt{3} \cdot \sin(90^\circ - \theta)
  \]

- In sector III,
  \[
  \frac{V_a}{V_{dc}} = -\frac{d_0}{2} - d_1 - d_2 + \frac{d_0}{2}
  \]
  \[
  = -\sin(60^\circ - \phi) - \sin \phi
  \]
  \[
  = -\sin(60^\circ + \phi)
  \]
  \[
  = -\sin(\phi - 60^\circ)
  \]

Average Values of Phase-to Neutral Voltage for SVM 6 (2Φ-C-mL) in Sector I

- If \(i_a\) is the largest current,
  \[
  V_a = 1
  \]
  \[
  V_b = -d_1 + d_2 + d_0
  = 1 - 2 \cdot d_1
  = 1 - 2 \cdot \sin(60^\circ - \phi)
  \]
  \[
  V_c = -d_1 - d_2 + d_0
  = 1 - 2 \cdot (d_1 + d_2)
  = 1 - 2 \cdot \sin(60^\circ + \phi)
  \]

- If \(i_c\) is the largest current,
  \[
  V_a = -d_0 + d_1 + d_2
  = 2 \cdot \sin(60^\circ + \phi) - 1
  \]
  \[
  = 2 \cdot \sin(60^\circ + 0) - 1
  \]
Average Values of Phase-to Neutral Voltage for SVM 6 (2Φ-C-mL)

- \( M = 1 \)
- \( M = 0.8 \)

\[ V_a \]

\[ 0 \quad 60 \quad 120 \quad 180 \quad 240 \quad 300 \quad 360 \]
\[ \theta (\text{o}) \]

Sector I  Sector II  Sector III  Sector IV  Sector V  Sector VI

Hysteresis Current Control

- Switching frequency is varying in one switching cycle.

If \( i_{ref} - \frac{\beta}{2} > i \), then \( s_{ap} \) on,

If \( i_{ref} + \frac{\beta}{2} < i \), then \( s_{an} \) on.
Pros and Cons of Hysteresis Current Control

Pros:
- Simple to implement
- Excellent dynamic performance

Cons:
- Strong harmonics lower than the switching frequency (Subharmonics)
- No intercommunication between the individual hysteresis controllers
  - Increase the switching frequency at lower modulation index
- A tendency at lower speed to lock into limit cycle of high-frequency switching
- Not strictly limit the current error

Outline

1. Introduction
2. Switching Modeling and PWM
   - Switching model of VSI & boost rectifier
   - Space vector modulation for VSI & boost rectifier
   - Other modulations for VSI & boost rectifier
     - Switching model and modulation for CSI & buck rectifier
3. Average Modeling
4. Small-Signal Modeling
5. Closed-Loop Control Design
6. More Complex Converters
Buck Rectifier / Current Source Inverter

- similar approach but different results -

- Unidirectional DC current
- Bi-directional DC voltage

\[ V_{dc} < V_x = \left( \frac{\sqrt{3}}{2} V_m \right) \]

\[ V_x = \min_{v_{xy}} \{ \max \{ |v_{ab}(t)|, |v_{bc}(t)|, |v_{ca}(t)| \} \} \]

DC-Current-Unidirectional Three-Phase Switching Network

- Topology:
  - Three-phase terminals are voltage controlled
  - DC port is current controlled
  - Six current-unidirectional, voltage-bi-directional, switches

- Allowed switching combinations:
  \[ s_{ak} + s_{bk} + s_{ck} = 1; \quad k \in \{p, n\} \]

- Two single-pole-triple-throw (SPTT) current-unidirectional switches
Buck Rectifier / Current Source Inverter
Switching Model

\[ \vec{i}_{abc} = \vec{s}_{abc} \cdot \vec{i}_{dc} \]

\[ v_{pn} = \vec{s}_{abc}^T \cdot \vec{v}_{abc} \]

where:

\[ \vec{i}_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \quad \vec{s}_{abc} = \begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix}, \quad v_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \]

Switching State Vectors

<table>
<thead>
<tr>
<th>Switching State Vectors</th>
<th>( \rho )</th>
<th>( 0 (\degree) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{i}_1[ab] )</td>
<td></td>
<td>-30</td>
</tr>
<tr>
<td>( \vec{i}_2[ac] )</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>( \vec{i}_3[bc] )</td>
<td></td>
<td>90</td>
</tr>
<tr>
<td>( \vec{i}_4[ba] )</td>
<td></td>
<td>150</td>
</tr>
<tr>
<td>( \vec{i}_5[ca] )</td>
<td></td>
<td>-150</td>
</tr>
<tr>
<td>( \vec{i}_6[cb] )</td>
<td></td>
<td>-90</td>
</tr>
<tr>
<td>( \vec{i}_0[aa] )</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>( \vec{i}_0[bb] )</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>( \vec{i}_0[cc] )</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

\( \vec{i}_0 = [aa] = [bb] = [cc] \) at center point
Outline

1. Introduction
2. Switching Modeling and PWM
3. Average Modeling
   • Average model of boost rectifier
   • Average model of VSI
   • Average models in rotating coordinates
4. Small-Signal Modeling
5. Closed-Loop Control Design
6. More Complex Converters

Boost Rectifier Switching Model
State-Space Equations

\[
\begin{align*}
   v_{AB} &= L \frac{di_a}{dt} + v_{ab} - L \frac{di_b}{dt} \\
   v_{BC} &= \frac{L}{3} \frac{di_b}{dt} + \frac{i_a - i_b}{i_{ca}} + v_{ab} \\
   v_{CA} &= \frac{L}{3} \frac{di_c}{dt} + \frac{i_b - i_c}{i_{ca}} + v_{bc} \\
   \frac{d}{dt} \begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} &= \begin{bmatrix} \frac{1}{3L} & 1 & \frac{1}{3L} \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} \\
   i_{dc} &= \frac{C}{R} \frac{dv_{dc}}{dt} + \frac{v_{dc}}{R} \\
   \frac{dv_{dc}}{dt} &= \frac{1}{C} i_{dc} - \frac{v_{dc}}{RC}
\end{align*}
\]
Boost Rectifier Switching Model
State-Space Equations

\[
\begin{align*}
\tilde{v}_{L-L} &= \begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} \\
\tilde{v}_{l-l} &= \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} \\
\tilde{i}_{l-l} &= \begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} \\
\tilde{s}_{l-l} &= \begin{bmatrix} s_{ab} \\ s_{bc} \\ s_{ca} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\frac{d\tilde{i}_{l-l}}{dt} &= \frac{1}{3L} \tilde{v}_{L-L} - \frac{1}{3L} \tilde{v}_{l-l} \\
\frac{d\tilde{v}_{dc}}{dt} &= \frac{1}{C} \tilde{i}_{dc} - \frac{v_{dc}}{RC}
\end{align*}
\]

\[
\begin{align*}
\tilde{v}_{l-l} &= \tilde{s}_{l-l} \cdot v_{dc} \\
i_{dc} &= \tilde{s}_{l-l}^T \cdot \tilde{i}_{l-l}
\end{align*}
\]

\[
\begin{align*}
\frac{d\tilde{v}_{dc}}{dt} &= \frac{1}{C} \tilde{s}_{l-l}^T \cdot \tilde{i}_{l-l} - \frac{v_{dc}}{RC}
\end{align*}
\]

Average Circuit Modeling

Applying an average operator to switching model
\[
\bar{x}(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau)d\tau
\]

- Switch duty cycle
  \[
d_{ap} = \bar{s}_{ap}(t) = \frac{1}{T} \int_{t-T}^{t} s_{ap}(\tau)d\tau
\]

- Phase-leg duty cycle
  \[
d_a = d_{ap} = 1 - d_{an}
\]

- Line-to-line duty cycle
  \[
d_{ab} = \bar{s}_{ab}(t) = \frac{1}{T} \int_{t-T}^{t} s_{ab}(\tau)d\tau = d_a - d_b
\]

- KVL and KCL
  \[
  \Sigma \bar{v} = 0 \quad \Sigma \bar{i} = 0
\]

- Linear components
  \[
  \bar{v}_R = R\bar{i}_R \quad \bar{v}_L = L \frac{d\bar{i}_L}{dt} \quad \bar{i}_C = C \frac{d\bar{v}_C}{dt}
\]

Dushan Boroyevich: Modeling and Control of Three-Phase PWM Converters
Tutorial at PECon 2008, Johor Bahru, Malaysia, 30 November 2008
Averaging of Quadratic Terms

\[ v_{ab} = s_{ab} \cdot v_{dc} \]

\[ \bar{v}_{ab} = \frac{1}{T} \int_{t-T}^{t} s_{ab}(\tau) \cdot v_{dc}(\tau) d\tau \approx \bar{s}_{ab} \cdot \bar{v}_{dc} = d_{ab} \cdot \bar{v}_{dc} \]

if maximum-frequency components of \( v_{dc}(t) \) are \( << 1/2T \).

\[ \bar{s}_{l-l} \cdot \bar{v}_{dc} \approx \bar{s}_{l-l} \cdot \bar{v}_{dc} = \bar{d}_{l-l} \cdot \bar{v}_{dc} \]

\[ \bar{S}_{l-l} \cdot \bar{i}_{l-l} \approx \bar{S}_{l-l} \cdot \bar{i}_{l-l} = \bar{d}_{l-l} \cdot \bar{i}_{l-l} \]

Development of Boost Rectifier Average Model

\[
\begin{align*}
\frac{d\bar{i}_{l-L}}{dt} &= \frac{1}{3L} \bar{v}_{l-L} - \frac{1}{3L} \bar{s}_{l-l} \cdot \bar{v}_{dc} \\
\frac{dv_{dc}}{dt} &= \frac{1}{C} \bar{s}_{l-l} \cdot \bar{i}_{l-l} - \frac{v_{dc}}{RC} \\
\frac{1}{T} \int_{t-T}^{t} \frac{d\bar{i}_{l-L}(\tau)}{dt} d\tau &= \frac{1}{T} \int_{t-T}^{t} \frac{1}{3L} \bar{v}_{l-L}(\tau) - \frac{1}{3L} \bar{s}_{l-l}(\tau) \cdot v_{dc}(\tau) d\tau \\
\frac{1}{T} \int_{t-T}^{t} \frac{dv_{dc}(\tau)}{dt} d\tau &= \frac{1}{T} \int_{t-T}^{t} \frac{1}{C} \bar{s}_{l-l}(\tau) \cdot \bar{i}_{l-l}(\tau) - \frac{v_{dc}(\tau)}{RC} d\tau
\end{align*}
\]

Applying average operator

\[
\begin{align*}
\frac{d\bar{i}_{l-L}}{dt} &= \frac{1}{3L} \bar{v}_{l-L} - \frac{1}{3L} \bar{d}_{l-l} \cdot \bar{v}_{dc} \\
\frac{dv_{dc}}{dt} &= \frac{1}{C} \bar{d}_{l-l} \cdot \bar{i}_{l-l} - \frac{v_{dc}}{RC}
\end{align*}
\]

\[
\begin{bmatrix}
\bar{d}_{l-L} \\
\bar{d}_{dc} \\
\bar{d}_{in}
\end{bmatrix} = \begin{bmatrix}
d_{ab} \\
d_{dc} \\
d_{in}
\end{bmatrix}
\]
Average Model of Three-Phase Boost Rectifier

\[
\begin{align*}
\frac{d\bar{i}_{1-l}}{dt} &= \frac{1}{3L} \bar{v}_{L-L} - \frac{1}{3L} \bar{s}_{l-l} \cdot \bar{v}_{dc} \\
\frac{d\bar{v}_{dc}}{dt} &= \frac{1}{C} \bar{s}_{l-l} \cdot \bar{i}_{l-l} - \frac{\bar{v}_{dc}}{RC} \\
\frac{d\bar{i}_{l-l}}{dt} &= \begin{bmatrix} s_a - s_b \\ s_b - s_c \\ s_c - s_a \end{bmatrix} \cdot \begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix}
\end{align*}
\]

Another Equivalent Circuit for Boost Rectifier Average Model

- Third order system due to degeneration
Steady-State Operation under Balanced Sinusoidal Excitation

Given:
\[
\begin{align*}
V_{ab} & = V_m \cos(\omega t - 2\pi/3) \\
V_{bc} & = V_m \cos(\omega t + 2\pi/3) \\
V_{ca} & = V_m \cos(\omega t)
\end{align*}
\]

\[
\begin{align*}
\bar{I}_{ab} & = 3L \bar{V}_{ab} \\
\bar{I}_{bc} & = 3L \bar{V}_{bc} \\
\bar{I}_{dc} & = D_m l_m \left[ \cos \omega t \cos(\omega t - \theta) + \cos(\omega t - 2\pi/3) \cos(\omega t - 2\pi/3 - \theta) + \cos(\omega t + 2\pi/3) \cos(\omega t + 2\pi/3 - \theta) \right]
\end{align*}
\]

Goal:
\[
\begin{align*}
I_{ab} & = I_m \cos(\omega t - 2\pi/3) \\
I_{bc} & = I_m \cos(\omega t + 2\pi/3) \\
I_{ca} & = I_m \cos(\omega t)
\end{align*}
\]

Assume:
\[
\begin{align*}
\bar{I}_{ab} & = \frac{D_m l_m}{2} \left[ \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - 2\pi/3 - \theta) + \cos(2\omega t + 2\pi/3 - \theta) \right] \\
\bar{I}_{bc} & = \frac{D_m l_m}{2} \left[ \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - 2\pi/3 - \theta) + \cos(2\omega t + 2\pi/3 - \theta) \right] \\
\bar{I}_{dc} & = \frac{3}{2} D_m l_m \cos \theta = \text{const.} \quad \Rightarrow \quad V_{dc} = \frac{3}{2} R D_m l_m \cos \theta = \text{const.}
\end{align*}
\]

Steady-State Operation under Balanced Sinusoidal Excitation

\[
\bar{V}_{dc} = V_{dc} \quad \Rightarrow \quad \bar{d}_{ab} \cdot \bar{V}_{dc} = D_m \cdot V_{dc} \cos(\omega t - \theta)
\]

Can use positive sequence phasors for steady state:
\[
\begin{align*}
V_{AB} & = \frac{V_m}{\sqrt{2}} \\
I_{AB} & = \frac{I_m}{\sqrt{2}}
\end{align*}
\]

\[
\bar{D}_{ab} \cdot \bar{V}_{dc} = \frac{D_m \cdot V_{dc}}{\sqrt{2}} \cos \theta - j \cdot \frac{D_m \cdot V_{dc}}{\sqrt{2}} \sin \theta
\]

\[
V_{AB} = j3\omega L \cdot I_{AB} + \bar{D}_{ab} \cdot \bar{V}_{dc}
\]

\[
\theta = \frac{1}{2} \sin^{-1} \left( \frac{4\omega L}{RD_m^2} \right), \quad V_{dc} = \frac{V_m}{D_m \cos \theta}, \quad I_m = \frac{2}{3} \cdot \frac{V_m}{RD_m^2 \cos^2 \theta}
\]

\[
\bar{d}_{ab} = d_a - d_b = d_{ap} - d_{bp}, \quad d_{ap}, d_{bp} \in [0, 1] \quad \Rightarrow \quad -1 \leq \bar{d}_{ab} \leq 1
\]

\[
D_m \leq 1 \quad \Rightarrow \quad V_{dc} \geq V_m
\]
Outline

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Development of VSI Average Model

\[
\begin{align*}
\frac{d\bar{i}_{dc}}{dt} &= \frac{1}{3L} \bar{s}_{L-L} \cdot \bar{v}_{dc} - \frac{1}{3L} \bar{v}_{L-L} \\
\frac{d\bar{v}_{L-L}}{dt} &= \frac{1}{C} \bar{i}_{L-L} - \frac{1}{RC} \bar{v}_{L-L} \\
\bar{i}_{dc} &= \bar{s}_{L-L}^{T} \cdot \bar{i}_{L-L}
\end{align*}
\]

Averaging

\[
\begin{align*}
\frac{d\tilde{i}_{dc}}{dt} &= \frac{1}{3L} \tilde{d}_{L-L} \cdot \tilde{v}_{dc} - \frac{1}{3L} \tilde{v}_{L-L} \\
\frac{d\tilde{v}_{L-L}}{dt} &= \frac{1}{C} \tilde{i}_{L-L} - \frac{1}{RC} \tilde{v}_{L-L} \\
\tilde{i}_{dc} &= \tilde{d}_{L-L}^{T} \cdot \tilde{i}_{L-L}
\end{align*}
\]
• Fourth order system due to degeneration

Steady-State Operation under DC Input and Balanced Sinusoidal Duty-Cycles

For: $V_{dc} = \text{const.}$

and

$$\begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix} = \begin{bmatrix} D_m \cos(\omega t) \\ D_m \cos(\omega t - 2\pi/3) \\ D_m \cos(\omega t + 2\pi/3) \end{bmatrix} \Rightarrow$$

$$V_{AB} = \frac{R}{1 + j3\omega CR} \cdot D \cdot V_{dc} \Rightarrow |V_{AB}| \leq V_{dc}$$

• Steady-state ac voltages and currents are sinusoidal if
• Difficult to define small-signal model. Operating point?
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Coordinate Transformation

Choose

\[
T_{dq0/abc} = \sqrt{\frac{2}{3}} \begin{bmatrix}
\cos \omega t & \cos(\omega t - \frac{2\pi}{3}) & \cos(\omega t + \frac{2\pi}{3}) \\
-\sin \omega t & -\sin(\omega t - \frac{2\pi}{3}) & -\sin(\omega t + \frac{2\pi}{3}) \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\]

where: \( \omega = 2\pi f \), \( f \) is ac line frequency
(source frequency for boost rectifier; desired output frequency for VSI)

\[
X_{dq0} = T \cdot X_{abc} \quad X_{abc} = T^{-1} \cdot X_{dq0}
\]

\((T = T_{dq0/abc})\)
Coordinate Transformation
– Boost Rectifier –

\[
\begin{align*}
\frac{d\overline{d}_{d-q}}{dt} &= \frac{1}{3L} \overline{V}_{L-L} - \frac{1}{3L} \overline{d}_{d-q} \cdot \overline{V}_{dc} \\
\frac{d\overline{y}_{dc}}{dt} &= \frac{1}{C} \overline{d}_{d-q} \cdot \overline{y}_{d-q} - \overline{V}_{dc} \cdot \frac{1}{RC} \\
X_{abc} &= T^{-1} \cdot X_{dq0} \\
\frac{d(T^{-1} \cdot \overline{y}_{dq0})}{dt} &= \frac{1}{3L} T^{-1} \cdot \overline{V}_{dq0} - \frac{1}{3L} T^{-1} \cdot \overline{d}_{dq0} \cdot \overline{V}_{dc} \\
\frac{d\overline{y}_{dc}}{dt} &= \frac{1}{C} \overline{d}_{d-q} \cdot T^{-1} \cdot \overline{y}_{d-q} - \overline{V}_{dc} \cdot \frac{1}{RC} \\
\frac{dT^{-1} \cdot \overline{y}_{dq0} + T^{-1} \cdot \overline{d}_{dq0}}{dt} &= T^{-1} \cdot \frac{1}{3L} \overline{V}_{dq0} - T^{-1} \cdot \frac{1}{3L} \overline{d}_{dq0} \cdot \overline{V}_{dc} \\
\frac{d\overline{y}_{dc}}{dt} &= \frac{1}{C} (T \cdot \overline{d}_{d-q}) \cdot T \cdot \overline{y}_{d-q} - \overline{V}_{dc} \cdot \frac{1}{RC} \\
\end{align*}
\]

Coordinate Transformation
– Boost Rectifier –

\[
\begin{align*}
T \cdot \frac{dT^{-1} \cdot \overline{y}_{dq0} + \frac{dT^{-1} \cdot \overline{d}_{dq0}}{dt}}{dt} &= \frac{1}{3L} \overline{V}_{dq0} - \frac{1}{3L} \overline{d}_{dq0} \cdot \overline{V}_{dc} \\
\frac{d\overline{y}_{dc}}{dt} &= \frac{1}{C} \overline{d}_{d-q} \cdot \overline{y}_{d-q} - \overline{V}_{dc} \cdot \frac{1}{RC} \\
T \cdot \overline{d}_{d-q} &= \overline{d}_{dq0} \\
T \cdot \overline{y}_{d-q} &= \overline{y}_{dq0} \\
\end{align*}
\]

\[
\begin{align*}
T \cdot \frac{dT^{-1} \cdot \overline{y}_{dq0} + \frac{dT^{-1} \cdot \overline{d}_{dq0}}{dt}}{dt} &= \frac{1}{3L} \overline{V}_{dq0} - \frac{1}{3L} \overline{d}_{dq0} \cdot \overline{V}_{dc} \\
\frac{d\overline{y}_{dc}}{dt} &= \frac{1}{C} \overline{d}_{d-q} \cdot \overline{y}_{d-q} - \overline{V}_{dc} \cdot \frac{1}{RC} \\
\end{align*}
\]
Coordinate Transformation

\[ T \cdot \frac{dT^{-1}}{dt} = T \cdot \frac{dT}{dt} = T \cdot \frac{d}{dt} \left( \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\omega t - \frac{2\pi}{3}) & -\sin(\omega t - \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\omega t + \frac{2\pi}{3}) & -\sin(\omega t + \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \end{bmatrix} \right) \]

\[ = T \cdot \frac{2}{\sqrt{3}} \begin{bmatrix} -\omega \sin(\omega t) & -\omega \cos(\omega t) & 0 \\ -\omega \sin(\omega t - \frac{2\pi}{3}) & -\omega \cos(\omega t - \frac{2\pi}{3}) & 0 \\ -\omega \sin(\omega t + \frac{2\pi}{3}) & -\omega \cos(\omega t + \frac{2\pi}{3}) & 0 \end{bmatrix} \]

\[ = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\omega t - \frac{2\pi}{3}) & \cos(\omega t + \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ -\sin(\omega t - \frac{2\pi}{3}) & -\sin(\omega t + \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot T \cdot \frac{2}{\sqrt{3}} \begin{bmatrix} -\omega \sin(\omega t - \frac{2\pi}{3}) & -\omega \cos(\omega t - \frac{2\pi}{3}) & 0 \\ -\omega \sin(\omega t + \frac{2\pi}{3}) & -\omega \cos(\omega t + \frac{2\pi}{3}) & 0 \end{bmatrix} \]

Using the following trigonometric relationships

\[ \cos^2 x + \cos^2 (x - \frac{2\pi}{3}) + \cos^2 (x + \frac{2\pi}{3}) = \frac{3}{2} \]

\[ \sin^2 x + \sin^2 (x - \frac{2\pi}{3}) + \sin^2 (x + \frac{2\pi}{3}) = \frac{3}{2} \]

\[ \sin x \cdot \cos x + \sin(x - \frac{2\pi}{3}) \cdot \cos(x - \frac{2\pi}{3}) + \sin(x + \frac{2\pi}{3}) \cdot \cos(x + \frac{2\pi}{3}) = 0 \]

\[ \cos x + \cos(x - \frac{2\pi}{3}) + \cos(x + \frac{2\pi}{3}) = 0 \]

\[ \sin x + \sin(x - \frac{2\pi}{3}) + \sin(x + \frac{2\pi}{3}) = 0 \]
Coordinate Transformation – Boost Rectifier –

\[
T \cdot \frac{dT^{-1}}{dt} = \begin{bmatrix}
0 & -\omega & 0 \\
\omega & 0 & 0 \\
0 & 0 & 0 
\end{bmatrix}
\]

Therefore:

\[
\begin{align*}
T \cdot \frac{dT^{-1}}{dt} \cdot \ddot{i}_{dq0} + \frac{d\ddot{i}_{dq0}}{dt} &= \frac{1}{3L} \ddot{v}_{dq0} - \frac{1}{3L} \dddot{d}_{dq0} \cdot \dddot{v}_{dc} \\
\frac{d\ddot{v}_{dc}}{dt} &= \frac{1}{C} \dddot{d}_{dq0} \cdot \dddot{v}_{dc} \\
\frac{d\dddot{i}_{dq0}}{dt} &= \frac{1}{3L} \ddot{v}_{dq0} \begin{bmatrix}
0 & -\omega & 0 \\
\omega & 0 & 0 \\
0 & 0 & 0 
\end{bmatrix} \cdot \dddot{i}_{dq0} - \frac{1}{3L} \dddot{d}_{dq0} \cdot \dddot{v}_{dc} \\
\frac{d\ddot{v}_{dc}}{dt} &= \frac{1}{C} \dddot{d}_{dq0} \cdot \dddot{v}_{dc} 
\end{align*}
\]

State-Space Equations – Boost Rectifier –

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix}
\ddot{i}_{ab} \\
\ddot{i}_{bc} \\
\ddot{i}_{ca}
\end{bmatrix} &= \frac{1}{3L} \begin{bmatrix}
\ddot{v}_{ab} \\
\ddot{v}_{bc} \\
\ddot{v}_{ca}
\end{bmatrix} - \frac{1}{3L} \begin{bmatrix}
\ddot{d}_{ab} \\
\ddot{d}_{bc} \\
\ddot{d}_{ca}
\end{bmatrix} \cdot \dddot{v}_{dc} \\
\frac{d\ddot{v}_{dc}}{dt} &= \frac{1}{C} \begin{bmatrix}
\ddot{d}_{ab} & \ddot{d}_{bc} & \ddot{d}_{ca}
\end{bmatrix} \begin{bmatrix}
\ddot{i}_{ab} \\
\ddot{i}_{bc} \\
\ddot{i}_{ca}
\end{bmatrix} - \dddot{v}_{dc} 
\end{align*}
\]

(abc coordinates)

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix}
\ddot{i}_d \\
\ddot{i}_q \\
\ddot{i}_0
\end{bmatrix} &= \frac{1}{3L} \begin{bmatrix}
\ddot{v}_d \\
\ddot{v}_q \\
\ddot{v}_0
\end{bmatrix} - \begin{bmatrix}
0 & -\omega & 0 \\
\omega & 0 & 0 \\
0 & 0 & 0 
\end{bmatrix} \begin{bmatrix}
\ddot{i}_d \\
\ddot{i}_q \\
\ddot{i}_0
\end{bmatrix} - \frac{1}{3L} \begin{bmatrix}
\ddot{d}_d \\
\ddot{d}_q \\
\ddot{d}_0
\end{bmatrix} \cdot \dddot{v}_{dc} \\
\frac{d\ddot{v}_{dc}}{dt} &= \frac{1}{C} \begin{bmatrix}
\ddot{d}_d & \ddot{d}_q & \ddot{d}_0
\end{bmatrix} \begin{bmatrix}
\ddot{i}_d \\
\ddot{i}_q \\
\ddot{i}_0
\end{bmatrix} - \dddot{v}_{dc} 
\end{align*}
\]

(dq0 coordinates)
The cross-coupling terms, $3\omega L\bar{q}_d$ and $3\omega L\bar{d}_q$, in dc coordinates (dq0), account for the voltage drops across inductances at line frequency in ac coordinates, $j \cdot 3\omega L\bar{i}_{ab} + j \cdot 3\omega L\bar{i}_{bc} + j \cdot 3\omega L\bar{i}_{ca}$.

Since
\[
\begin{align*}
\bar{v}_{AB} + \bar{v}_{BC} + \bar{v}_{CA} & = 0 \\
\bar{i}_{ab} + \bar{i}_{bc} + \bar{i}_{ca} & = 0 \\
d_{ab} + d_{bc} + d_{ca} & = 0
\end{align*}
\]

0-channel can be omitted
Equivalent Circuit in dq0 Coordinates
– Boost Rectifier –

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} &= \frac{1}{3L} \begin{bmatrix} v_d \\ v_q \end{bmatrix} - \frac{\omega}{3L} \begin{bmatrix} i_d \\ i_q \end{bmatrix} - \frac{1}{3L} \begin{bmatrix} d_d \\ d_q \end{bmatrix} \cdot \bar{v}_{dc} \\
\frac{d\bar{v}_{dc}}{dt} &= \frac{1}{C} \begin{bmatrix} d_d \\ d_q \end{bmatrix} \cdot \frac{i_d}{i_q} - \frac{\bar{v}_{dc}}{RC}
\end{align*}
\]

State-Space Equations
– Voltage Source Inverter –

Transformation

\[
\begin{align*}
\frac{d\bar{v}_{dc}}{dt} &= \frac{1}{C} \frac{\bar{v}_{dq0}}{\bar{i}_{dq0}} \\
\frac{d\bar{v}_{dq0}}{dt} &= \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \omega \\ 0 \end{bmatrix} - \frac{1}{3L} \bar{v}_{dc} - \frac{1}{3L} \bar{v}_{dq0} \\
\bar{i}_{dc} &= \bar{d}_{dq0} \cdot \bar{i}_{dq0}
\end{align*}
\]

(abc coordinates)

(dq0 coordinates)
Equivalent Circuit in dq0 Coordinates
– Voltage Source Inverter –

\[
\frac{di_d}{dt} = \frac{1}{3L} d_{dq} \cdot v_{dc} - \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} i_{dq} - \frac{1}{3L} v_{dq} \\
\frac{dv_{dq}}{dt} = \frac{1}{C} i_{dq} - \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_{dq} - \frac{1}{RC} i_{dq} \\
i_{dc} = d_{dq} \cdot i_{dq}
\]

Buck Rectifier / CSI dq0 Model
– similar approach but different results –

Buck Rectifier

Current Source Inverter (CSI)
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Linearization

Autonomous dynamic system: \( \frac{dx}{dt} = \tilde{f}(x, u) \)

If \( \tilde{f} \) is analytic it can be expressed as Taylor series:

\[
\tilde{f}(\tilde{x}, \tilde{u}) = \tilde{f}(\tilde{x}_0, \tilde{u}_0) + \frac{\partial\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{x}} (\tilde{x} - \tilde{x}_0) + \frac{\partial\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{u}} (\tilde{u} - \tilde{u}_0) + \\
+ \frac{1}{2!}\left[ \frac{\partial^2\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{x}^2} (\tilde{x} - \tilde{x}_0)^2 + \frac{\partial^2\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{x} \partial \tilde{u}} (\tilde{x} - \tilde{x}_0)(\tilde{u} - \tilde{u}_0) + \frac{\partial^2\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{u}^2} (\tilde{u} - \tilde{u}_0)^2 \right] + \ldots
\]

Retaining the first 3 terms results in linear approximation of \( \tilde{f} \):

\[
\tilde{f}(\tilde{x}, \tilde{u}) \approx \tilde{f}(\tilde{x}_0, \tilde{u}_0) + \frac{\partial\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{x}} (\tilde{x} - \tilde{x}_0) + \frac{\partial\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{u}} (\tilde{u} - \tilde{u}_0)
\]

But the dynamic system is NOT linear because:

\[
\frac{dx}{dt} \approx \frac{\partial\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{x}} \cdot \tilde{x} + \frac{\partial\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{u}} \cdot \tilde{u} + \tilde{f}(\tilde{x}_0, \tilde{u}_0) - \frac{\partial\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{x}} \cdot \tilde{x}_0 - \frac{\partial\tilde{f}(\tilde{x}_0, \tilde{u}_0)}{\partial \tilde{u}} \cdot \tilde{u}_0
\]

\[
\dot{x} = A \cdot \dot{x} + B \cdot \tilde{u} + g \neq 0
\]
**Linearization**

\[
\frac{dx}{dt} = \dot{f}(x, u) \approx \dot{f}(\bar{x}, \bar{u}) + \frac{\partial \dot{f}(\bar{x}, \bar{u})}{\partial x} (x - \bar{x}) + \frac{\partial \dot{f}(\bar{x}, \bar{u})}{\partial u} (u - \bar{u})
\]

If \((\bar{x}, \bar{u})\) is an equilibrium point \((\bar{X}, \bar{U})\), and \((\tilde{x}, \tilde{u})\) is perturbation around it:

\[
\begin{align*}
\dot{\tilde{x}} &= \frac{\partial \dot{f}(\bar{x}, \bar{u})}{\partial x} \tilde{x} + \frac{\partial \dot{f}(\bar{x}, \bar{u})}{\partial u} \tilde{u} \\
A &= \frac{\partial \dot{f}(\bar{x}, \bar{u})}{\partial x} \\
B &= \frac{\partial \dot{f}(\bar{x}, \bar{u})}{\partial u}
\end{align*}
\]

**Average Large-Signal Model**

**Boost Rectifier**

A steady-state operating point:

\[
V_d = \sqrt{3} \cdot V_m \quad (V_m: \text{Max line-to-line voltage})
\]

\[
V_q = 0
\]

\[
D_d = \frac{V_d}{V_{dc}} \quad I_d = \frac{V_{dc}}{R \cdot D_d}
\]

\[
D_q = \frac{3 \alpha L_j}{V_{dc}} \quad I_q = 0
\]
**Linearization – Boost Rectifier**

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \ddot{v}_d \\ \ddot{v}_q \\ \ddot{t}_d \\ \ddot{t}_q \end{bmatrix} &= \begin{bmatrix} 1 & 0 & -\omega & 0 \\ 0 & 1 & 0 & -\omega \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v}_d \\ \dot{v}_q \\ \dot{t}_d \\ \dot{t}_q \end{bmatrix} - \frac{1}{3L} \begin{bmatrix} d_d \\ d_q \end{bmatrix} \cdot \ddot{v}_{dc}
\end{align*}
\]

\[
\frac{d}{dt} \begin{bmatrix} \ddot{v}_{dc} \end{bmatrix} = \frac{1}{C} \begin{bmatrix} d_d & d_q \end{bmatrix} \begin{bmatrix} \ddot{t}_d \\ \ddot{t}_q \end{bmatrix} - \frac{\ddot{v}_{dc}}{RC}
\]

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \ddot{i}_d \\ \ddot{i}_q \end{bmatrix} &= \frac{1}{3L} \begin{bmatrix} \ddot{v}_d \\ \ddot{v}_q \end{bmatrix} - \frac{1}{3L} \begin{bmatrix} \ddot{d}_d \\ \ddot{d}_q \end{bmatrix} \cdot V_{dc} - \frac{1}{3L} \begin{bmatrix} D_d \\ D_q \end{bmatrix} \cdot \ddot{v}_{dc}
\end{align*}
\]

\[
\frac{d}{dt} \begin{bmatrix} \ddot{i}_d \ddot{i}_q \end{bmatrix} = \frac{1}{C} \begin{bmatrix} I_d & I_q \end{bmatrix} \begin{bmatrix} \ddot{t}_d \\ \ddot{t}_q \end{bmatrix} + \frac{1}{C} \begin{bmatrix} D_d & D_q \end{bmatrix} \begin{bmatrix} \ddot{i}_d \\ \ddot{i}_q \end{bmatrix} - \frac{\ddot{v}_{dc}}{RC}
\]

**Small-Signal Model – Boost Rectifier**

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \ddot{i}_d \\ \ddot{i}_q \\ \ddot{v}_{dc} \end{bmatrix} &= \begin{bmatrix} 0 & \omega & -D_d & -D_d \\ -\omega & 0 & D_d & D_d \\ D_d & D_q & \ddot{v}_{dc} \end{bmatrix} + \begin{bmatrix} V_{dc} & 0 & 0 \\ 0 & V_{dc} & 0 \\ I_d & I_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{d}_d \\ \ddot{d}_q \\ \ddot{v}_{dc} \end{bmatrix} + \frac{1}{3L} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{v}_d \\ \ddot{v}_q \\ \ddot{v}_{dc} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\dot{\tilde{x}} &= A \begin{bmatrix} \tilde{x} \\ \tilde{u} \\ \tilde{v} \end{bmatrix} \\
\text{Intrinsic} & \text{ System Dynamics} \quad \text{Control} \quad \text{Disturbance} \\
\text{System Dynamics} & \text{Input} \quad \text{Input}
\end{align*}
\]

Dushan Boroyevich: Modeling and Control of Three-Phase PWM Converters
Tutorial at PECon 2008, Johor Bahru, Malaysia, 30 November 2008
Small-Signal Model – Boost Rectifier

\[
\tilde{i}_d = \frac{K_{idd} \cdot (s + z_{idd1}) \cdot (s + z_{idd2})}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)} \cdot \tilde{d}_d \cdot -V_d
\]

Open-Loop Transfer Functions

\[
\tilde{i}_d = \frac{K_{idd} \cdot (s + z_{idd})}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)} \cdot \tilde{d}_q
\]
Open-Loop Transfer Functions

\[
\tilde{i}_q = \frac{K_{tqdd} \cdot (s + z_{tgdd 1}) \cdot (s + z_{tgdd 2})}{(s + p_1) \cdot (s + p_2) \cdot (s + p^*_2)}
\]

\[
\tilde{i}_q = \frac{K_{tqdq} \cdot (s + z_{tgdq}) \cdot (s + z^*_{tgdq})}{(s + p_1) \cdot (s + p_2) \cdot (s + p^*_2)}
\]

Open-Loop Transfer Functions

\[
\tilde{v}_{dc} = \frac{K_{vdcd} \cdot (s + z_{vdcdd 1}) \cdot (s + z_{vdcdd 2}) \cdot (s - z_{RHP})}{(s + p_1) \cdot (s + p_2) \cdot (s + p^*_2)}
\]

\[
\tilde{v}_{dc} = \frac{K_{vdcq} \cdot (s + z_{vdcq}) \cdot (s - z_{RHP})}{(s + p_1) \cdot (s + p_2) \cdot (s + p^*_2)}
\]
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Average Large-Signal Model – VSI

A steady-state operating point:

\[ I_d = \frac{V_d}{R} - \alpha CV_q \]
\[ D_d = \frac{V_d - 3\omega L_d I_d}{V_{dc}} \]
\[ I_q = \frac{V_q}{R} + \alpha CV_d \]
\[ D_q = \frac{V_q + 3\omega L_d I_d}{V_{dc}} \]
Linearization – VSI

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} &= \frac{1}{3L} \begin{bmatrix} d_d \\ d_q \end{bmatrix} \cdot v_{dc} - \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} - \frac{1}{3L} \begin{bmatrix} \bar{v}_d \\ \bar{v}_q \end{bmatrix} \\
\frac{d}{dt} \begin{bmatrix} \bar{v}_d \\ \bar{v}_q \end{bmatrix} &= \frac{1}{C} \begin{bmatrix} \bar{i}_d \\ \bar{i}_q \end{bmatrix} - \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \begin{bmatrix} \bar{v}_d \\ \bar{v}_q \end{bmatrix} - \frac{1}{RC} \begin{bmatrix} \bar{v}_d \\ \bar{v}_q \end{bmatrix} \\
i_{dc} &= \begin{bmatrix} d_d \\ d_q \end{bmatrix} \begin{bmatrix} \bar{i}_d \\ \bar{i}_q \end{bmatrix}
\end{align*}
\]
Small-Signal State-Space Model – VSI

\[
\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \\ \tilde{v}_d \\ \tilde{v}_q \end{bmatrix} = \begin{bmatrix} 0 & \omega & -\frac{1}{3L} & 0 \\ -\omega & 0 & 0 & -\frac{1}{3L} \\ \frac{1}{C} & 0 & -\frac{1}{RC} & \omega \\ \frac{1}{C} & -\omega & -\frac{1}{RC} & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \tilde{v}_d \\ \tilde{v}_q \end{bmatrix} + \begin{bmatrix} \frac{V_{dc}}{3L} & 0 & \frac{\tilde{d}_d}{3L} \\ 0 & \frac{V_{dc}}{3L} & \frac{\tilde{d}_q}{3L} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{v}_d \\ \tilde{v}_q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tilde{v}_{dc}
\]

\[
\dot{x} = Ax + Bu + D\tilde{u} + \tilde{v}.
\]

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Natural and Uniform Sampling

Natural Sampling

Uniform Sampling

Trailing- and Leading-Edge Modulation

Both are the natural sampling
Review of Modulator Modeling for DC-DC Converters

- Natural sampling
  - Unity gain
  - No delay
- Uniform sampling
  1. Trailing-edge modulation
     - Unity gain
     - Phase delay at a modulation frequency
     \[ D \cdot T_s \cdot \omega_m \]
  2. Leading-edge modulation
     - Unity gain
     - Phase delay at a modulation frequency
     \[ (1 - D) \cdot T_s \cdot \omega_m \]

Three-Phase Modulator Modeling

- All naturally sampled modulators can be modeled by the constant gain term
  - Modulators associated with analog controllers

- Small-signal models **have to be derived for uniformly sampled** three-phase modulators
  - Modulators associated with digital controllers
Example – for Boost Rectifier and VSI

- Trailing-edge modulation (2Φ-RA)

Phase delays:

\[ D_{ab} \cdot T_s \cdot \omega_m \]

\[ (1 - D_{ca}) \cdot T_s \cdot \omega_m \]

- Two out of three switching functions always have pulses synchronized to the beginning of the switching period

Approximate Average Model of Digital Modulator

\[ d_d^{ref} \]
\[ d_q^{ref} \]
\[ d_d \]
\[ d_q \]
\[ e^{-sT_d} \]
\[ e^{-sT_q} \]

Inverse Coordinate Transform.

Digital Modulator

Coordinate Transform.
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Classical Linear Control Design Approach

[Diagram of control system showing power stage, control, output, and compensator with single-input single-output and multi-input multi-output configurations]
Closed-Loop Control Design

- Based on small-signal models

Advantage
- Classical control design methods can be used
  (Bode plots, root loci, etc.)

Disadvantages
- Design is valid only at a certain operating point
- The approach does not guarantee large-signal stability

Control Structure: Cascade Control

1. Inner current control
   a. Bang-bang current control (and
      PWM) in stationary coordinates
   b. Current control in stationary
      coordinates with two independent
      or three dependent current
      controllers (P, PI, or resonant
      regulators)
   c. Current control in rotating
      coordinates with two independent
      current controllers (P or PI
      regulators)

2. Outer output voltage or torque /
   flux / speed / position control
   (PI regulators)
Control Realization

1. Completely analog control
   Complete analog control is rarely used because of hardware complexity

2. Combination of analog and digital control
   Analog current control in stationary and digital output voltage or speed and flux control

3. Completely digital control
   Used in most new designs

Open-loop Boost Rectifier Control

Diagram showing the control realization and the components involved in the control system.
Modified small-signal model

Simplified approximations in continuous time domain:

\[ e^{-sT_{del}} \approx 1 - \frac{sT_{del}}{2} + \frac{(sT_{del})^2}{12} \]

Often choose: \( T_{del} \approx T_c \approx T_d \approx T_q \approx T_{sampling} = T_{switching} \)
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Current Loop Design

Digital Control Interface

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Control-to-Current Transfer Function

\[
\frac{\tilde{i}_d}{d_d} = \frac{K_{idd}\cdot (s + z_{idd}) \cdot (s + z_{idd}^2)}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)}
\]

\[H_{id} = K_p + \frac{K_i}{s}\]

Control-to-Current Transfer Function

\[
\frac{\tilde{i}_q}{d_q} = \frac{K_{iqdq} \cdot (s + z_{iqdq})\cdot (s + z_{iqdq}^*)}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)}
\]

\[H_{iq} = K_p + \frac{K_i}{s}\]
• D channel loop-gain $T_d$
• Bandwidth is limited by delay ($f_{sw}=20\text{kHz}$)

• Q channel loop-gain $T_q$
• Bandwidth is limited by delay ($f_{sw}=20\text{kHz}$)
Current Regulation

- Peak is more pronounced when gain increases

\[ \frac{i_d}{i_{d\text{ref}}} \quad \frac{i_q}{i_{q\text{ref}}} \]

Current Loop with D and Q Decoupling

\[ d_d = d_d^{\text{ref}} \cdot e^{-sT_d} = (\hat{d}_d^{\text{ref}} + 3\omega L i'_q / v'_{dc}) \cdot e^{-sT_d} = \hat{d}_q + 3\omega L \frac{1}{v'_{dc}} \cdot e^{-sT_d} \cdot i'_q \]

\[ d_q = d_q^{\text{ref}} \cdot e^{-sT_q} = (\hat{d}_q^{\text{ref}} - 3\omega L i'_q / v'_{dc}) \cdot e^{-sT_q} = \hat{d}_q - 3\omega L \frac{1}{v'_{dc}} \cdot e^{-sT_q} \cdot i'_d \]
Decoupled D and Q Channels

\[
d_d \bar{v}_d - 3\omega L \bar{i}_q = \approx 0
\]

\[
d_d \bar{v}_d + 3\omega L \bar{v}_d e^{-sT_d} \bar{i}_q - 3\omega L \bar{i}_q
\]

\[
d_q \bar{v}_d + 3\omega L \bar{i}_d = \approx 0
\]

- Similar to two parallel dc-dc boost converters after d and q decoupled

Cross-Coupling Effect After Decoupling

\[
\begin{align*}
\frac{\tilde{i}_d}{i_{d\text{ref}}} & \quad \frac{\tilde{i}_q}{i_{q\text{ref}}} \\
\begin{array}{c}
\bar{i}_d \\
\bar{i}_q
\end{array} & \quad \begin{array}{c}
\bar{i}_d \\
\bar{i}_q
\end{array}
\end{align*}
\]

\[
\begin{align*}
\frac{\tilde{i}_d}{i_{d\text{ref}}} & \quad \frac{\tilde{i}_q}{i_{q\text{ref}}}
\end{align*}
\]
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Output Voltage Loop Design

\[ G_e = \frac{\bar{V}_{dc}}{I_{dref}} = \frac{K \cdot (s - z_{RHP})}{(s + p_L) \cdot (s + p_H)} \]

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COMPENSATOR DESIGN

- Voltage compensator
  
  \[ H_v = \frac{K_v (1 + s / z_v)}{s (1 + s / p_v)} \]

- Place \( z_v \) as high as possible for required phase margin.
- Place \( p_v \) for loop-gain attenuation.
- Attainable voltage-loop bandwidth:
  
  \[ \omega_c < \frac{1}{4} z_{RHP} \]

Attainable Voltage Loop Bandwidth with Delay

- Voltage compensator
  
  \[ H_v = \frac{K_v (1 + s / z_v)}{s (1 + s / p_v)} \]

- Zero is placed close to the crossover to improve the phase margin
Time Domain Simulation Results

Phase voltages and currents (PFC operation)  
Output voltage

Outline

1. Introduction
2. Switching Modeling and PWM
3. Average Modeling
4. Small-Signal Modeling
5. Closed-Loop Control Design
   • Control approach
   • Current-loop design
   • Voltage-loop design
   • Limiting
6. More Complex Converters
LIMITING IN THREE-PHASE CONVERTERS

Three-phase PWM Converter
   Modulator
      Limiter
         Current Control
            Limiter
               Output Control

Current Feedback

Output Feedback

Duty Cycle Limiting

- Boost Rectifier/VSI Switching State Hexagon

\[ \vec{v}^\text{ref} = \vec{d}^\text{ref} \cdot \sqrt{\frac{3}{2}} \cdot V_{dc} \]

- Maximal attainable voltage vector lies on the hexagon
- For sinusoidal average output voltages the voltage vector must be inside the inscribed circle

\[ |\vec{v}^\text{ref}| \leq \sqrt{\frac{3}{2}} \cdot V_{dc} \Rightarrow |\vec{d}^\text{ref}| \leq 1 \Rightarrow d_d^2 + d_q^2 \leq 1 \]
Vector angle is kept constant

(\(d_d\))_lim and (\(d_q\))_lim should be fed back to the anti-windup current controllers

If output of each current controller is limited separately it can result in unattainable voltage vector

\[
\frac{d_d}{d_d^2 + d_q^2} > 1
\]

\[
d_d = \frac{d_d}{d_d^2 + d_q^2}
\]

\[
d_q = \frac{d_q}{d_d^2 + d_q^2}
\]
Maximal current magnitude is limited

\[ \sqrt{(i_d^{ref})^2 + (i_q^{ref})^2} \leq I_{max} \]

Limiting of each reference current component is determined by the control algorithm.

**EXAMPLE**

- Boost rectifier control

![Diagram showing output voltage regulator and input displacement factor controller with current limiting equations.]

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   - Three-phase four-wire (four-phase) converter
   - Multilevel converters
   - Parallel converters
Limitations of Three-Leg Converters

- There is no path for the neutral current.
- Output voltages are unbalanced.

Conventional inverter is not good for highly unbalanced load.

Applications

- Provide three-phase four-wire
- Deal with unbalanced and nonlinear load

Advantages

- The fourth leg provides a neutral current path
- Compared to generating \(v_n\) with capacitive divider:
  - Much smaller dc-link capacitance is needed
  - Full utilization of dc-link voltage (15% higher) with SVM
Three-Dimensional Space Vector for Four-Leg Converter

- $V_\gamma$ is the zero-sequence component
- $V_\gamma$ is related to the neutral current

A Nonlinear Load

- Three Single-Phase Diode Bridge Rectifiers with C Filter
- Load current
  $$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$
- Desired voltage for sinusoidal
  $$\begin{bmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix}$$
Switching Model

\[ i_N = -i_a - i_b - i_c \]

\[
\begin{bmatrix}
  s_a \\
  s_b \\
  s_c
\end{bmatrix} = \begin{bmatrix}
  s_{ap} - s_{xp} \\
  s_{bp} - s_{xp} \\
  s_{cp} - s_{xp}
\end{bmatrix}
\]

\[ i_{dc} = \begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
\]

\[ \begin{bmatrix}
  v_{ax} \\
  v_{bx} \\
  v_{cx}
\end{bmatrix} = \begin{bmatrix}
  s_a \\
  s_b \\
  s_c
\end{bmatrix} \cdot v_{dc} \]

\[ \frac{\pi}{2} \cos(\omega t) \]

\[ \frac{3}{2} \cos(\omega t - \frac{2\pi}{3}) \]

\[ \frac{3}{2} \cos(\omega t + \frac{2\pi}{3}) \]

Average Model in Stationary Coordinates

- Controlled voltage sources are balanced for balanced load
- Controlled voltage sources are unbalanced for unbalanced load
- Difficult to describe the circuit operation in abc coordinates

\[ i_{dc} = \begin{bmatrix}
  d_a \\
  d_b \\
  d_c
\end{bmatrix}
\]

\[ \frac{v_{an}}{v_{an}} \]

\[ \frac{v_{bn}}{v_{bn}} \]

\[ \frac{v_{cn}}{v_{cn}} \]

\[ V_m \cos(\omega t) \]

\[ V_m \cos(\omega t - 2\pi/3) \]

\[ V_m \cos(\omega t + 2\pi/3) \]
Average Model in Rotating Coordinates

\[ i_d = d_d \cdot i_d + d_q \cdot i_q + d_o \cdot i_o \]

- 2 decoupled subsystems with reduced system orders
- Transfer functions can be derived based on the dc operating point

Transfer Functions

- Sampling delay and PWM delay are incorporated in the model
- The derived model agrees with the measurement very well
Impact of Load Power Factor on Control-to-Output Voltage TFs

Resistive, capacitive (PF=-0.8), and inductive (PF=0.8) are all at 150 kW

- Capacitive load shift the resonant frequency to lower frequency
- Inductive load leads to a higher system order and higher resonant peaking
- Both capacitive and inductive loads are worse loads than resistive load

Control Block Diagram
Double-Loop Design

- Decoupling is not included

Switching State Vectors in Three-Dimensional Space

- 16 Switching State Vectors
  abc

- 2x6 vectors with non-zero $\alpha$ & $\beta$
- 2 "zero vectors" with non-zero $\gamma$
- 2 zero vectors
Three-Dimensional Switching Vectors

\[ v_y = \frac{1}{\sqrt{3}} v_x \]
\[ v_y = \frac{2}{\sqrt{3}} v_x \]
\[ v_y = 0 \]
\[ v_y = -\frac{1}{\sqrt{3}} v_x \]
\[ v_y = -\frac{2}{\sqrt{3}} v_x \]
\[ v_y = -\frac{3}{\sqrt{3}} v_x \]

Projection on \( \alpha-\beta \) plane

where \( x = \{p,n\} \)

Three-Dimensional SVM
Step 1: Prism Identification

Prism I
Prism II
Prism III
Prism IV
Prism V
Prism VI
Three-Dimensional SVM
Step 2: Tetrahedron Identification

![Tetrahedron Diagrams](image)

Three-Dimensional SVM:
Step 3: Duty-Cycle Calculation

\[ \vec{V}_{ref} = d_1 \vec{V}_1 + d_2 \vec{V}_2 + d_3 \vec{V}_3 \]

\[ d_0 = 1 - d_1 - d_2 - d_3 \]
Three-Dimensional SVM:

Step 4: Switching Sequencing (Minimum Loss)

(a) Rising-Edge Aligned

(b) Falling-Edge Aligned

(c) Symmetric Aligned

(d) Alternative Sequence

Experimental Results with Only Voltage Loops Closed

Unbalanced Load

Load current and neutral current

Output voltage and neutral current
A Four-Leg Inverter with Common-Mode Filter Function

- Common-Mode Noise
- A 4-Leg Inverter for Common-Mode Noise Elimination
- A New Modulation and Control Scheme for Inverter Power Supplies with Unbalanced/Nonlinear Load

In order to have both functions, we need

- A new switching modulation strategy
- A new control design because of the series capacitor in neutral leg

If \( v_a + v_b + v_c + v_x = 0 \), there is NO common-mode noise!
Use Only Switching Vectors with 2 p’s and 2 n’s

\[ V_{cm} = d_1 \cdot V_{ppp} + d_2 \cdot V_{pnn} + d_3 \cdot V_{npn} + d_4 \cdot V_{nnp} \]
\[ d_1 + d_2 + d_3 + d_4 = 1 \]

6 available switching vectors out of 2^4 vectors

Common-Mode Noise Reduction and Trade-Offs

- DM ripple is increased because of reduced vector choice
- 15% smaller maximum modulation index
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Major Multilevel Three-Phase Topologies

Diode clamped
Nabae, 1980; Choe, 1991; Carpita, 1991

Flying capacitor
Meynard, 1992

Cascaded H-bridge

Dushan Boroyevich: Modeling and Control of Three-Phase PWM Converters
Tutorial at PECon 2008, Johor Bahru, Malaysia, 30 November 2008
Merits of Multilevel Converter Technologies

Advantages
- Easy voltage sharing among devices
- Improved spectral performance of output waveforms
- Reduced $dv/dt$ resulting in reduced reflections and damage to insulation
- Reduced switching and conduction losses

Disadvantages
- Increased number of devices
- Increased control complexity
- Depending on topology:
  - Issues with capacitors balancing
  - Issues with bi-directional power flow

Multilevel Converter Control

- Average and small-signal models are the same as for two-level VSI or boost rectifier, except for multiple dc voltages
- Closed-loop control of ac quantities in rotating $dq$ coordinates is the same as for two-level VSI or boost rectifier
- Additional controller for voltage balancing on dc-link capacitors
- Modulator is significantly more complex
Space-Vector Representation of Three-Level Converters

- Switching model of n-level converter: Three single-pole n-throw switches

\[ \vec{v}_{ijk} = V_{dc} \cdot \begin{bmatrix} i - j \\ j - k \\ k - i \end{bmatrix} \]

- Voltage space-vectors can be represented in a three dimensional line-to-line coordinate system

Voltage Space-Vectors of Three-Level Converters

- All the voltage space-vectors of a three-phase converter are located on a plane

- Voltage space-vectors have different length: small, medium and large.
- Every small vector has two corresponding switching states—redundancy.
SVM for Multilevel Three-Phase Converters?

Five-level converter
- Switching states: \( n^3 = 125 \)
- Switching vectors: \( 3n(n-1)+1 = 61 \)
- Triangular areas: \( 6(n-1)^2 = 96 \)

How to do SVM?

Complexity? Speed?

Coordinates and Transformation

- Choose line-to-line coordinates: \( ab, bc \) and \( ca \)

- Transform \( \vec{v}_{\text{ref}} / V_{dc} \)
  \[
  \vec{d}_{\text{ref}} = \begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix} = \frac{1}{V_{dc}} \begin{bmatrix} \cos \theta \\ \cos(\theta - \frac{2\pi}{3}) \\ \cos(\theta + \frac{2\pi}{3}) \end{bmatrix} - \sin \theta \begin{bmatrix} \sin \theta \\ \sin(\theta - \frac{2\pi}{3}) \\ \sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} V_{d\text{ref}} \\ V_{q\text{ref}} \end{bmatrix}
  \]

Dushan Boroyevich: Modeling and Control of Three-Phase PWM Converters
Tutorial at PECon 2008, Johor Bahru, Malaysia, 30 November 2008
Identify the Vectors and Calculate their Duty Cycles

- Calculate the floor and ceiling of the projections:
  \[ f_{ab} \leq d_{ab} \leq c_{ab}, \]
  \[ f_{bc} \leq d_{bc} \leq c_{bc}, \]
  \[ f_{ca} \leq d_{ca} \leq c_{ca}. \]

- If \( f_{ab} + f_{bc} + f_{ca} = -1 \) then \( \vec{d}_{ref} \) falls in a triangle as follows:
  \[ f_{ab}, c_{bc}, f_{ca} \]
  \[ (d_{bc} - f_{bc}) \]

- If \( f_{ab} + f_{bc} + f_{ca} = -2 \) then \( \vec{d}_{ref} \) falls in a triangle as follows:
  \[ f_{ab}, c_{bc}, c_{ca} \]
  \[ (c_{ab} - d_{ab}) \]

An Example

- \( \vec{d}_{ref} \) with a length of 1.7 and an angle of 45° from \( ab \) axis

1. The projections are
   \[ d_{ab} = 1.202 \quad d_{bc} = 0.440 \quad d_{ca} = -1.642 \]
2. The floors and ceilings are
   \[ f_{ab} = 1 \quad f_{bc} = 0 \quad f_{ca} = -2 \]
   \[ c_{ab} = 2 \quad c_{bc} = 1 \quad c_{ca} = -1 \]
3. Since \( f_{ab} + f_{bc} + f_{ca} = -1 \), the nearest three vectors and their duty cycles are:
   \[ 2,0,-2 \quad d_{2,0,-2} = d_{ab} - f_{ab} = 0.202 \]
   \[ 1,1,-2 \quad d_{1,1,-2} = d_{bc} - f_{bc} = 0.440 \]
   \[ 1,0,-1 \quad d_{1,0,-1} = d_{ca} - f_{ca} = 0.358 \]
Capacitor Balancing Problem in Three-Level Neutral-Point (NP) Clamped Converter

Large vectors do not affect NP balance

Medium vectors affect the charge balance in the neutral point causing the low frequency ripple

Full load current is connected to NP for the duration of the medium vector duty cycle
Small vectors also affect the capacitor charge balance and their effect can be controlled.

Small vectors come in pairs

Freedom to select either vector in a pair helps balance NP

\[
V_{ab} = V_{dc}/2, \quad V_{bc} = 0, \quad V_{ca} = -V_{dc}/2
\]

Amplitude: \( \frac{1}{\sqrt{3}} V_{dc} \)

Phase: \( 0^\circ \)

\[ V_{ab} = \frac{V_{dc}}{2}, \quad V_{bc} = 0, \quad V_{ca} = -\frac{V_{dc}}{2} \]

\[ V_{bc} = \text{Amplitude} = \frac{1}{\sqrt{3}} V_{dc} \]

\[ \text{Phase} = 0^\circ \]

Modified SVM algorithm can effectively compensate the voltage ripple in the neutral point

- Compute the location of voltage space vectors in every switching cycle

\[
\bar{V} = \frac{2}{3} \left( v_{ab} + v_{bc} \cdot e^{i\gamma} + v_{ca} \cdot e^{i2\gamma} \right) \quad \gamma = \frac{2\pi}{3}
\]

- Based on the commanded balancing effort, virtual small vector
- can be computed (need to use both vectors every cycle)

\[
V_{VSO} = c \cdot V_{S0} + (1 - c) \cdot V_{\bar{S}0} \quad c \in [0, 1]
\]

- Determine the sector location of the current \( V_{REF} \)
- Compute the duty cycles
Neutral point can be balanced in every switching cycle only in the part of the operating region.

Finding the normalized charge ripple for all operating conditions helps design dc-link capacitors.

\[ Q = \int_{0}^{T} i_{NP}(t) \, dt \]
As expected, the effect of NP charge control diminishes for the decreasing power factor angle

System Parameters
- 1800 V DC bus
- 1% NP ripple
- 200 A Peak current

\[
2 \cdot C_{DC} = \frac{K \cdot I_{\text{max}}}{\Delta U_{\text{DC, max}}}
\]

K - normalized amplitude of NP charge

Balancing of clamping capacitor for flying capacitor converter is load-independent

- The inner capacitor handles only switching frequency ripple current
- There is no clamping diode reverse recovery problem
- The modulation and higher level control remains the same
- Flying capacitor converters with any number of levels can be balanced.
- Diode-clamped converters with number of levels above three CANNOT!
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Motivation

A Distributed Power System

• Modular design
• N+1 redundancy (reliability)
• No limit for current level
• Maintainability
• Availability
• Reduced cost, size and weight
• Improved performance
Possible Solutions

- Separate power supplies
- Transformer isolation
- Inter-phase reactors
- Six-leg converter

A Six-Leg Converter

Issues – Modeling

Challenges

- High-order system
- Coupling

Existing results

- High-order multi-input-multi-output modeling\(^1\)
- Reduced order modeling\(^2\)

\(^1\)Matakas, 1993
\(^2\)Mao, 1994
**Issues – Load Sharing**

**Challenge**
- Load sharing
- Voltage regulation
- Modular design

**Existing results**
- Master/Slave
- Droop

1Siri, 1992
2Jamerson, 1994

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**Issues – Zero-Sequence Current**
Phase-Leg Averaging

An Averaged Three-Phase Converter Model
Extraction of Zero-Sequence Components

A zero-sequence component = the sum of all phases components

e.g. $I_0 = I_a + I_b + I_c$

Define: $d_z = d_a + d_b + d_c$

$\rightarrow (d_a - d_z/3) + (d_b - d_z/3) + (d_c - d_z/3) = 0$

$\rightarrow d'_a + d'_b + d'_c = 0$

Where: $d'_a = d_a - d_z/3$, $d'_b = d_b - d_z/3$, $d'_c = d_c - d_z/3$

$\rightarrow d_a = d'_a + d_z/3$, $d_b = d'_b + d_z/3$, $d_c = d'_c + d_z/3$

The Model with Zero-Sequence Components

• For a single converter, there is a zero-sequence voltage but no zero-sequence current because $I_a + I_b + I_c \equiv 0$
Zero-Sequence Current in Parallel Converters

An Averaged Model of Zero-Sequence Dynamic

In AC side, there are three-loops forming three equations:

\[
\begin{align*}
    & \frac{d}{dt} V_{dc}^1 + \frac{d}{dt} V_{dc}^2 - L_1 \frac{d}{dt} I_{a1} = \frac{d}{dt} V_{dc}^3 + \frac{d}{dt} V_{dc}^4 - L_2 \frac{d}{dt} I_{a2} \\
    & \frac{d}{dt} V_{dc}^2 + \frac{d}{dt} V_{dc}^3 - L_1 \frac{d}{dt} I_{b1} = \frac{d}{dt} V_{dc}^4 + \frac{d}{dt} V_{dc}^5 - L_2 \frac{d}{dt} I_{b2} \\
    & \frac{d}{dt} V_{dc}^1 + \frac{d}{dt} V_{dc}^3 - L_1 \frac{d}{dt} I_{c1} = \frac{d}{dt} V_{dc}^4 + \frac{d}{dt} V_{dc}^6 - L_2 \frac{d}{dt} I_{c2}
\end{align*}
\]

\[
V_{dc} \cdot (d_{z1} - d_{z2}) = (L_1 + L_2) \frac{d}{dt} I_0 + (R_1 + R_2) \cdot I_0
\]

Equivalent circuit
Zero-Sequence Duty Cycle

For example:

\[
d_z = d_a + d_b + d_c = (d_1 + d_2 + 0.5d_0) + (d_2 + 0.5d_0) + 0.5d_0
= d_1 + 2d_2 + 1.5d_0
\]

A New Zero-Sequence Control Variable \( k \)

\[
k \equiv d_{ppp} \quad \text{i.e. the duration of zero-vector ppp}
\]

Therefore:

\[
d_z = d_a + d_b + d_c = (d_1 + d_2 + kd_0) + (d_2 + kd_0) + kd_0
= d_1 + 2d_2 + 3kd_0
\]

\( k \) is not definable in minimum loss SVM schemes
A New Zero-Sequence Model with the New Control Variable $k$

$$d_{z1} - d_{z2} = d_1 + 2d_2 + 3k_2d_0 - (d_1 + 2d_2 + 3k_2d_0)$$

$$= 3(k_1 - 0.5)d_0$$

where $k_2$ is set to 0.5

- $I_0$ can be controlled by controlling $k_1$ dynamically
- High control bandwidth can be achieved because it is a first-order system

Implementation

- This part is added onto a regular controller for a single converter.
Simulation Results

Without Zero-sequence control
With Zero-sequence control
(Two converters with different switching frequencies operate independently)

Clock Phase Shifting
Common-Mode Voltage Reduction

Common-mode voltage
$-\frac{V_{dc}}{6}$
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