CPES

Center for Power Electronics Systems

A National Science Foundation Engineering Research Center

Virginia Tech, University of Wisconsin - Madison, Rensselaer Polytechnic Institute
North Carolina A&T State University, University of Puerto Rico - Mayaguez

Modeling and Control of Three-Phase PWM Converters

Dushan Boroyevich
Virginia Tech, Blacksburg, Virginia, USA

presented at:

PECon 2008

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30 November 2008

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Outline

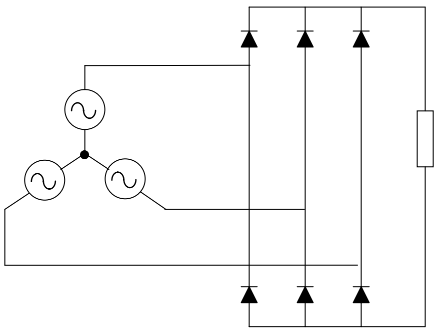
PECon 2008

1. **Introduction**
2. **Switching Modeling and PWM**
3. **Average Modeling**
4. **Small-Signal Modeling**
5. **Closed-Loop Control Design**

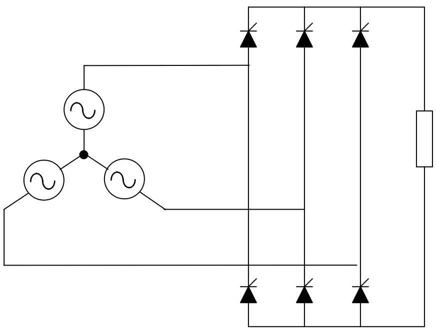
6. **More Complex Converters**

DB-2

First Three-Phase Converters



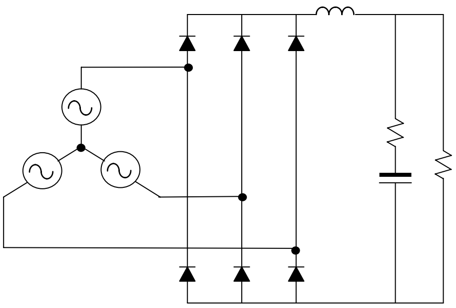
Diode Rectifier

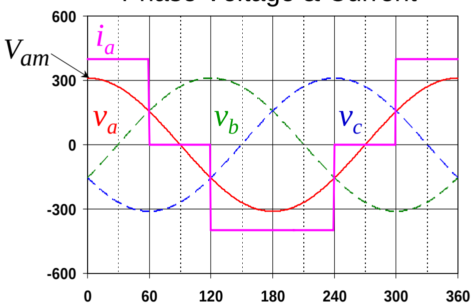


SCR Rectifier

DB-3

Three-Phase Diode Rectifier
with Current Load

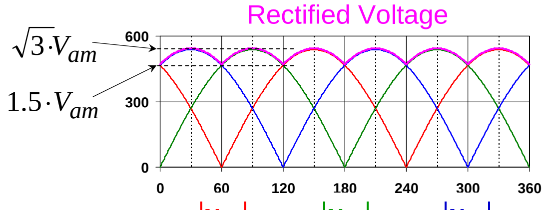




$$v_a = V_{am}\cos(\omega t)$$

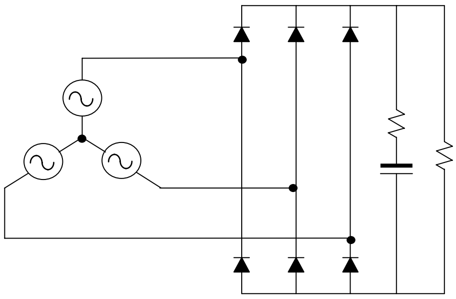
$$v_b = V_{am}\cos(\omega t - \frac{2\pi}{3})$$

$$v_c = V_{am}\cos(\omega t + \frac{2\pi}{3})$$

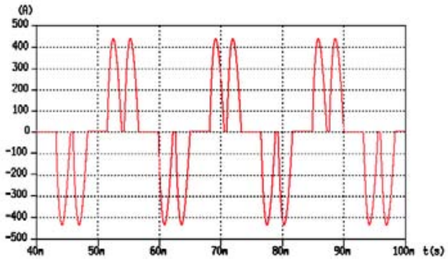


DB-4

Three-Phase Diode Rectifier
with Capacitive Load



Per phase load current



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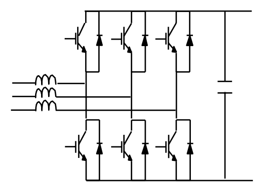
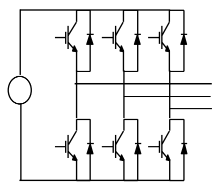
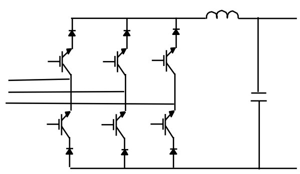
Three-Phase Pulse Width Modulated (PWM) Converters



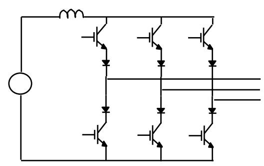
Boost Rectifier



Buck Inverter
Voltage Source Inverter (VSI)



Buck Rectifier



Boost Inverter
Current Source Inverter (CSI)

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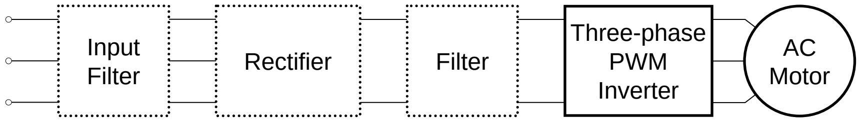
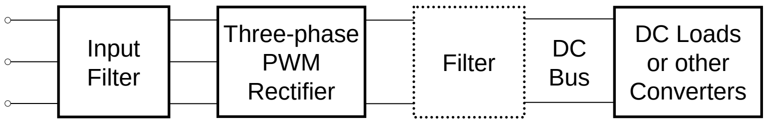
Three-Phase Applications

- AC Motor Drives



- VSI with uncontrolled rectifier or CSI with SCR rectifier
- First and still the most common application
- Regulated output ac voltage or current (amplitude and frequency)
- Usually only unidirectional power flow

- Power Factor Correction



- Adjustable input displacement factor
- Regulated dc bus voltage

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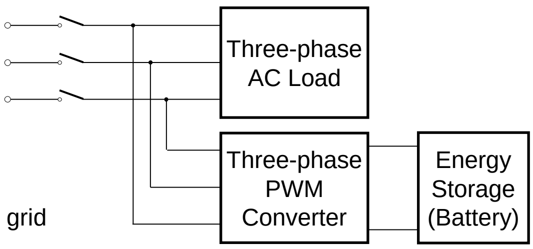
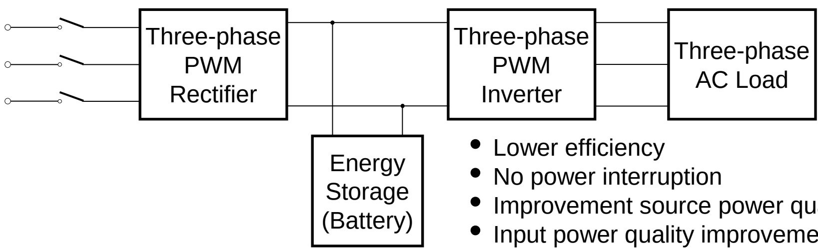
Three-Phase Applications

- Uninterruptible Power Supply (UPS) – Parallel



- High efficiency
- Power interruption
- No power quality improvement to source or grid

- Uninterruptible Power Supply – Series



- Lower efficiency
- No power interruption
- Improvement source power quality
- Input power quality improvement

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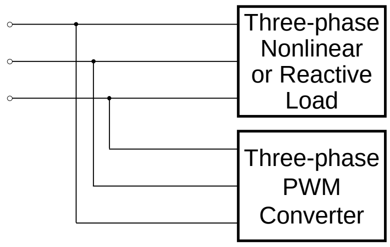
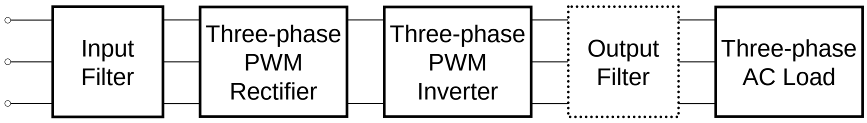
Three-Phase Applications

- Active Filters
  - Power factor control
  - Reduced current distortion
  - Improved damping
  - Utility applications (e.g. STATCOM)



- AC-AC Power Conversion



- Cascade connection of Boost rectifier and VSI or Buck rectifier and CSI
- Adjustable displacement factor at input and output
- Bidirectional power flow
- Utility applications (e.g. UPFC)

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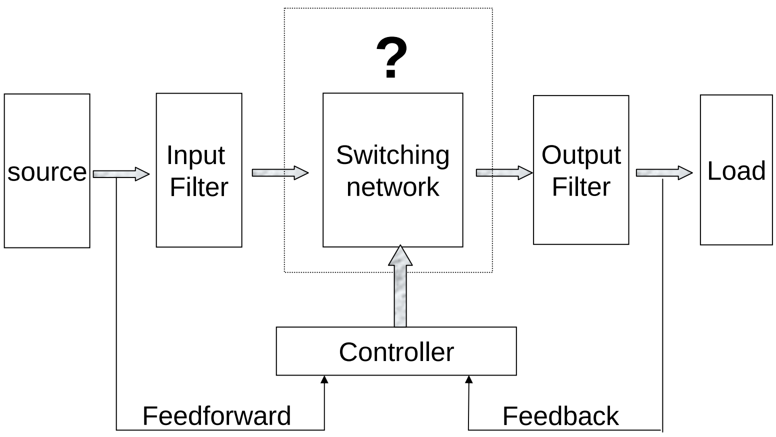
Generalized Structure of A Power Converter



- Switching network is discontinuous and nonlinear

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Motivation

- Three-phase PWM converters below 100 kW operate with relatively high switching frequency (20 kHz - 100 kHz)
  - Elimination of audible noise
  - Reduction of the size of reactive components
  - **Significant improvement in waveform quality and closed-loop perfomance**

1. **MODELING**
2. **CONTROL DESIGN**

Only systems with switching frequency much higher than the line frequency will be studied!

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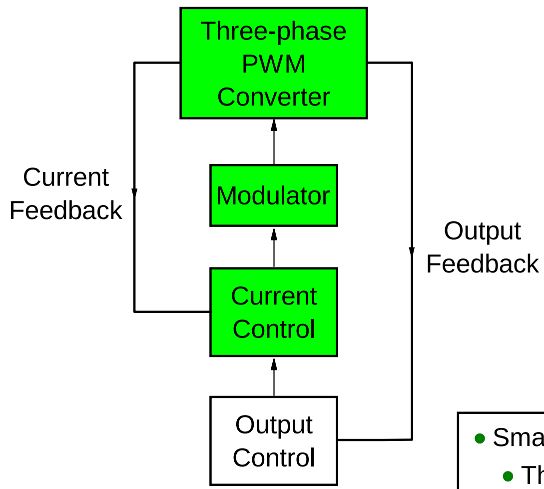
Modeling



- Small-signal modeling of
  - Three-phase PWM converters
  - Three-phase modulator
  - Current controllers

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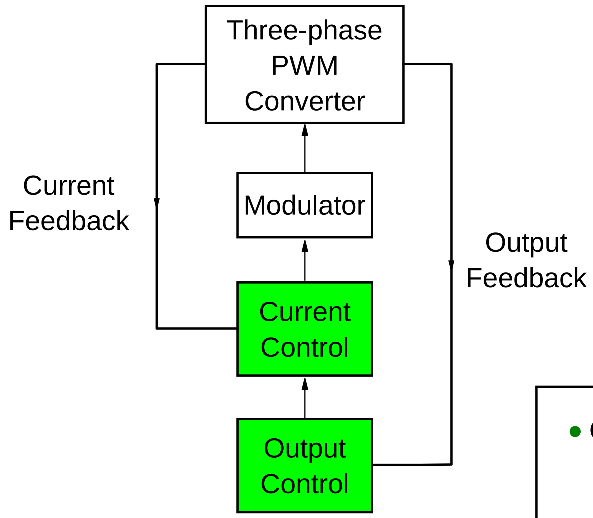
Control Design



- Control design of
  - Current control
  - Outer control loops

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Steps in Modeling of Three-Phase PWM Converters

1. Switching model
   - Time-discontinuous
   - Time-varying
   - Non-linear
2. Average model in stationary coordinates
   - Time-continuous
   - Time-varying
   - Non-linear
3. Average model in rotating (synchronous) coordinates
   - Time-continuous
   - Time-invariant
   - Non-linear
4. Small-signal model
   - Time-continuous
   - Time-invariant
   - Linear

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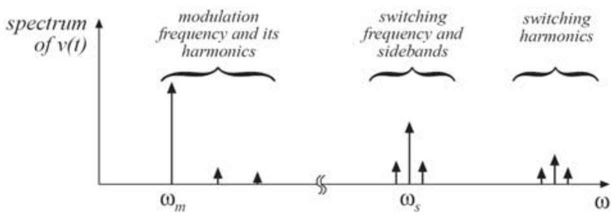
Focus

- Power converter modeling for control design!
- Only converters utilizing high-frequency synthesis:



$\omega_m << \omega_m$

- Minor emphasis on modulation
- Only classical, small-signal control approach
- No power stage design and optimization
- No power device discussion
- No topology evaluation, only control implications
- No application considerations

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Outline

PECon 2008

1. Introduction
   - Vector representation of three-phase variables
2. Switching Modeling and PWM
3. Average Modeling
4. Small-Signal Modeling
5. Closed-Loop Control Design
6. More Complex Converters

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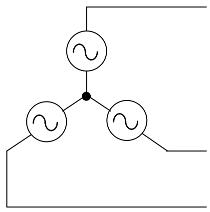
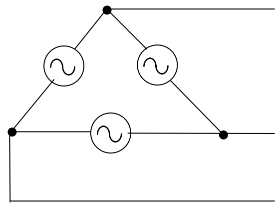
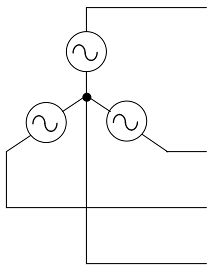
Three-Phase Circuits - Source



Y-connection



Δ-connection



3-phase 4-wire

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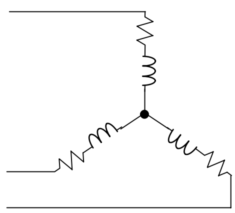
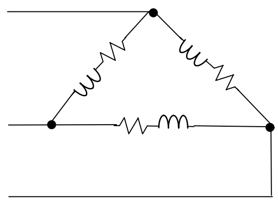
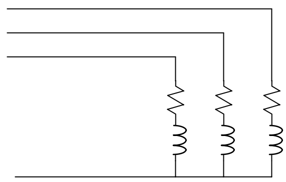
Three-Phase Circuits - Load



Y-connection



Δ-connection



3-phase 4-wire

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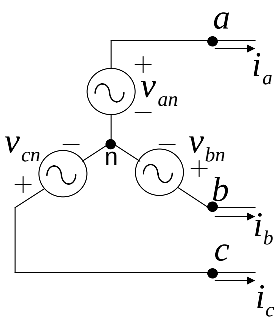
Three-Phase Variables

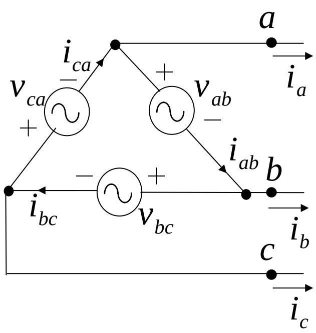
Y-connection



$$v_{ab} = v_a - v_b$$
$$v_{bc} = v_b - v_c$$
$$v_{ca} = v_c - v_a$$

$$i_a = i_{ca} - i_{ab}$$
$$i_b = i_{ab} - i_{bc}$$
$$i_c = i_{bc} - i_{ca}$$

Δ-connection



Y-connection:

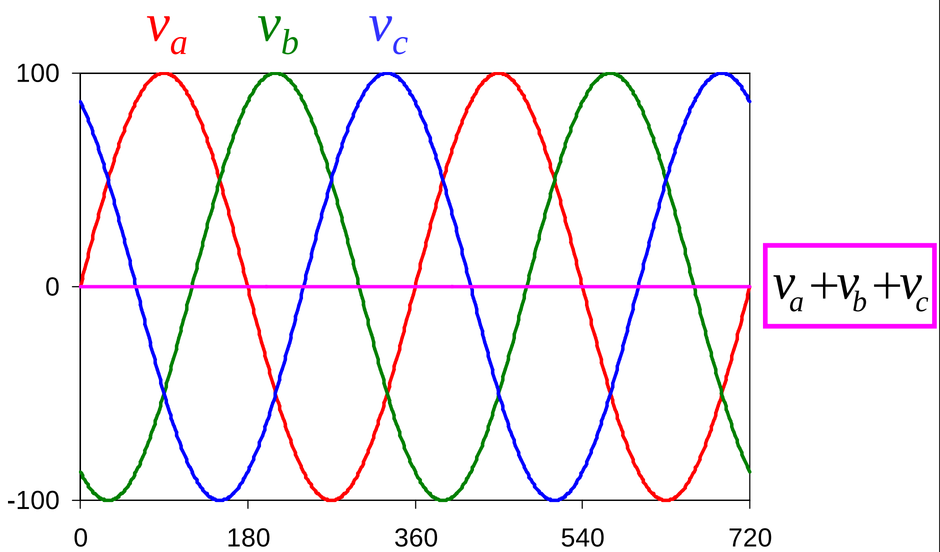
$$i_a + i_b + i_c \equiv 0$$
$$v_{ab} + v_{bc} + v_{ca} \equiv 0$$
$$v_{an} + v_{bn} + v_{cn} \neq 0$$

Δ-connection:

$$i_a + i_b + i_c \equiv 0$$
$$v_{ab} + v_{bc} + v_{ca} \equiv 0$$
$$i_{ab} + i_{bc} + i_{ca} \neq 0$$

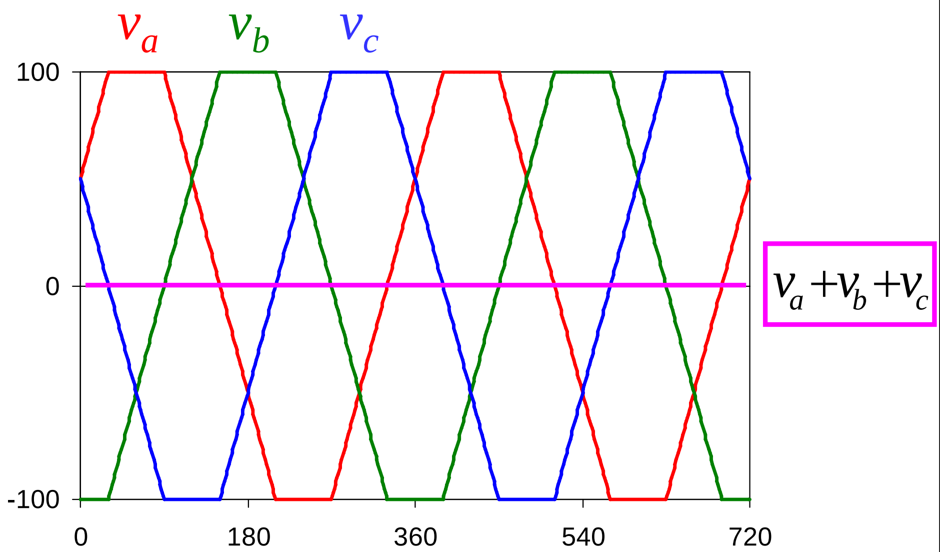
DB-19

Sinusoidal, Balanced, Symmetrical



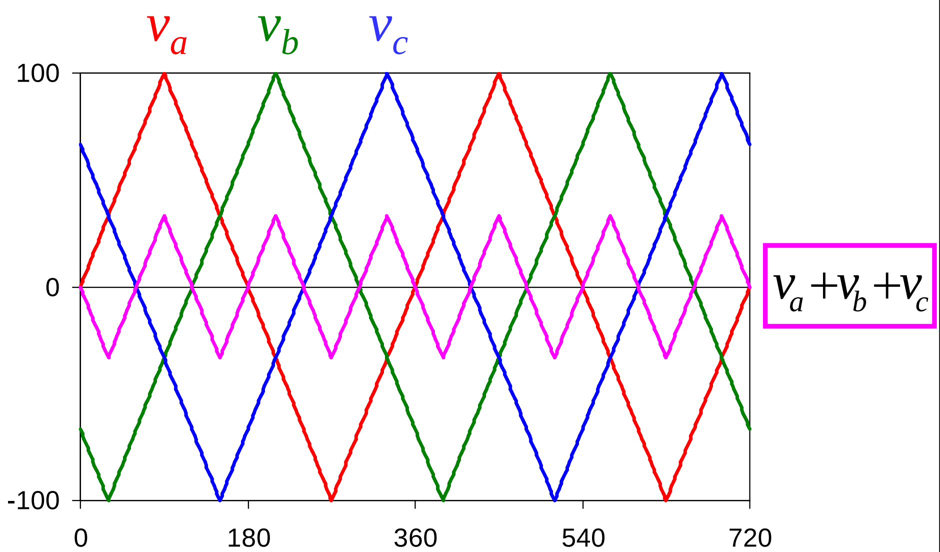
DB-20

Non-sinusoidal, Balanced, Symmetrical



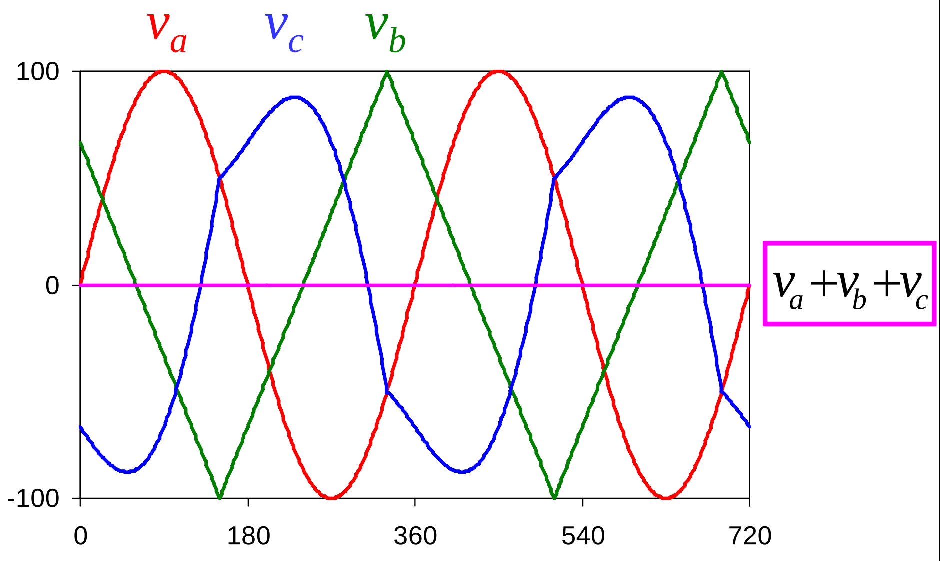
DB-21

Non-sinusoidal, Unbalanced, Symmetrical

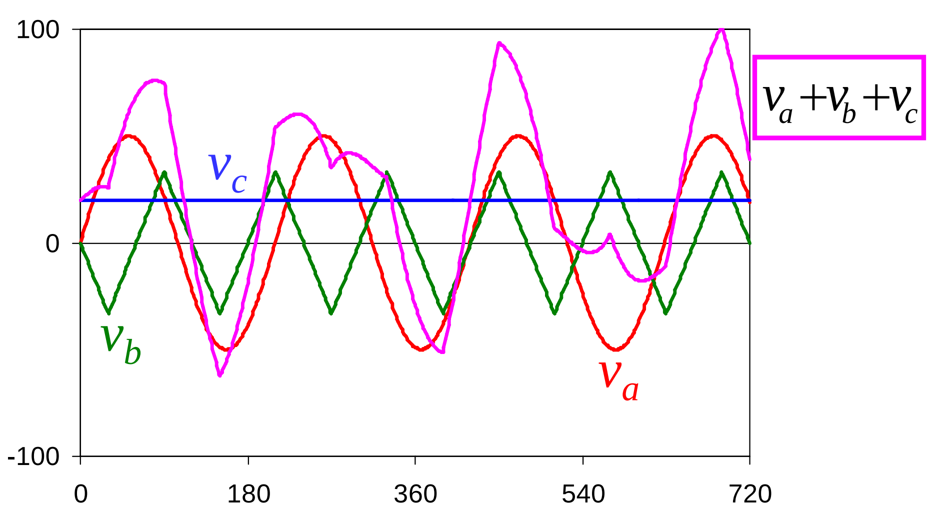


DB-22

Non-sinusoidal, Balanced, Asymmetrical



Non-sinusoidal, Unbalanced, Asymmetrical



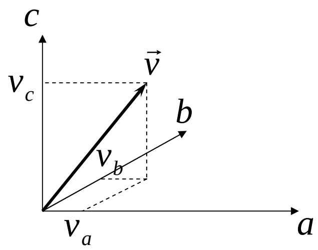
Vector Representations of Three-Phase Variables

Euclid vector representations

$$\vec{v}(t) = \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} \qquad \vec{i}(t) = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

Euclidean Space:

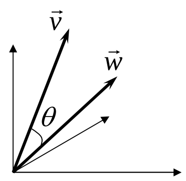
$$\vec{u}_a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{u}_b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \vec{u}_c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



DB-25

Vector Multiplication and Norms in Euclidean Spaces

- Inner product: $<\vec{v},\vec{w}> = \vec{v}^T\vec{w} = \vec{w}^T\vec{v} = \sum_{i=1}^{n} v_i w_i$
- "Dot" product: $<\vec{v},\vec{w}> = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos\theta$
- "Cross" product: $\|\vec{v}\times\vec{w}\| = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \sin\theta$



- Vector norm (length):

$$\|\vec{v}\| = <\vec{v},\vec{v}>^{\frac{1}{2}} = \sqrt{\sum_{i=1}^{n} v_i^2} = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$$

- Norm of a matrix: $\|\mathbf{A}\| = \max_{\forall\vec{x}\neq 0} \frac{\|\mathbf{A}\vec{x}\|}{\|\vec{x}\|}$

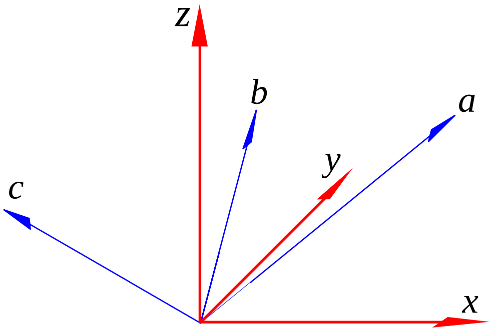
DB-26

Change of Coordinates

- Multiplication of a vector with any nonsingular matrix, $\mathbf{T}$, of the same order:

$$\vec{v}_{xyz} = \mathbf{T} \cdot \vec{v}_{abc}$$

is equivalent to the representation of the same vector in a different coordinate system (*xyz*), whose unit vectors have the following coordinates in the original coordinate system (*abc*):



$$\vec{u}_{x\text{ in }abc} = \mathbf{T}^{-1} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \vec{u}_{y\text{ in }abc} = \mathbf{T}^{-1} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{u}_{z\text{ in }abc} = \mathbf{T}^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- If $\langle \vec{u}_x, \vec{u}_y \rangle = \langle \vec{u}_y, \vec{u}_z \rangle = \langle \vec{u}_z, \vec{u}_x \rangle = 0$, new coordinates are also orthogonal.

DB-27

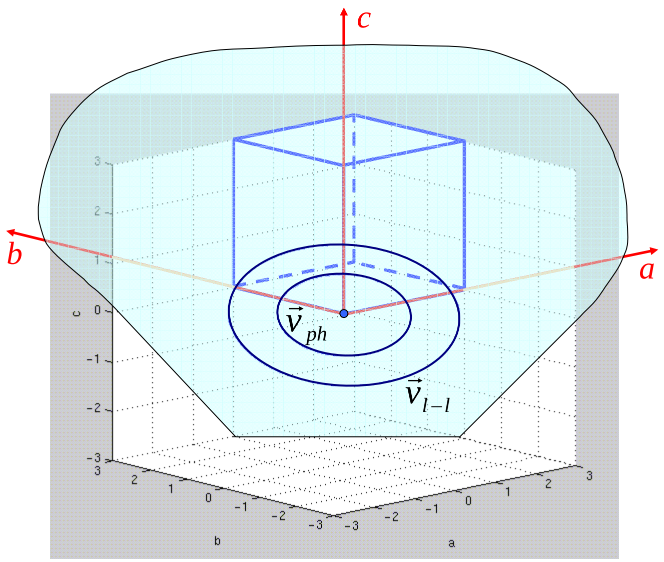
Example: Balanced Three-Phase Voltages in abc Space

$$v_a = \cos t$$

$$v_b = \cos(t - \frac{2\pi}{3})$$

$$v_c = \cos(t + \frac{2\pi}{3})$$

$$\vec{v}_{ph} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$



DB-28

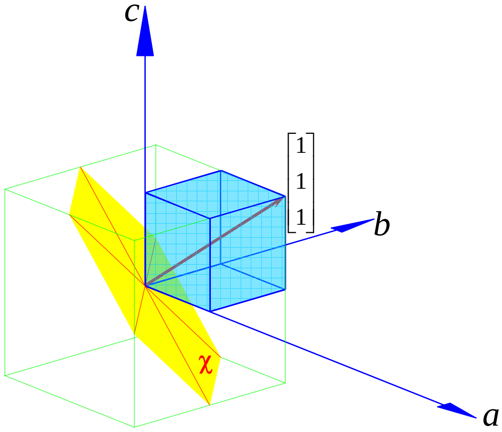
Change of Coordinates (abc to $\alpha\beta\gamma$)

$$i_a + i_b + i_c \equiv 0 \qquad v_{ab} + v_{bc} + v_{ca} \equiv 0$$

This defines a 2-dimensional subspace χ , perpendicular to the vector $[1 \;\; 1 \;\; 1]^T$ in *abc*-space.

$\alpha\beta\gamma$-space is traditionally defined by:

- α-axis is chosen as projection of the *a*-axis onto χ ,
- γ-axis is co-linear with vector $[1 \;\; 1 \;\; 1]^T$
- β- axis is defined by right-hand rule.



DB-29

Transformation Matrix $T_{\alpha\beta\gamma\,/\,abc}$

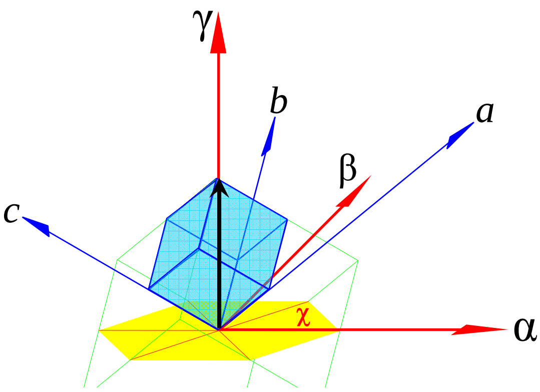
The transformation matrix

$$\| T_{\alpha\beta\gamma\,/\,abc} \| = 1$$

$$T_{\alpha\beta\gamma\,/\,abc} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\vec{v}_{\alpha\beta\gamma} = T_{\alpha\beta\gamma\,/\,abc} \cdot \vec{v}_{abc}$$

$$\vec{i}_{\alpha\beta\gamma} = T_{\alpha\beta\gamma\,/\,abc} \cdot \vec{i}_{abc}$$



Example:

$$\vec{x}_{abc} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \vec{x}_{\alpha\beta\gamma} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{3} \end{bmatrix}$$

DB-30

Transformation Matrix $T_{abc/\alpha\beta\gamma}$

$$T_{abc/\alpha\beta\gamma} = T^{-1}_{\alpha\beta\gamma/abc} = T^{T}_{\alpha\beta\gamma/abc} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\vec{v}_{abc} = T_{abc/\alpha\beta\gamma} \cdot \vec{v}_{\alpha\beta\gamma}$$

$$\vec{i}_{abc} = T_{abc/\alpha\beta\gamma} \cdot \vec{i}_{\alpha\beta\gamma}$$

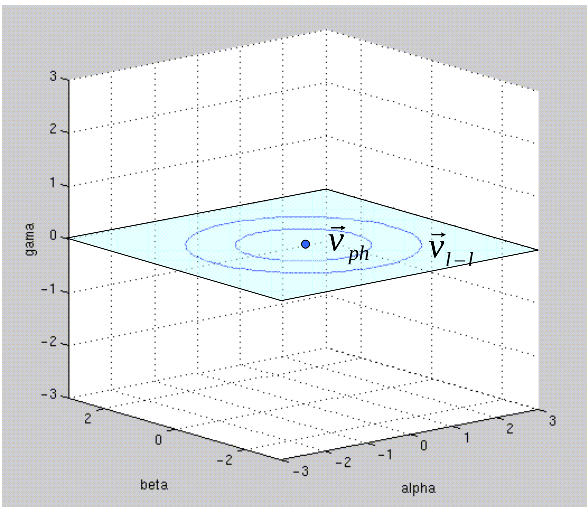
DB-31

Example: Balanced Three-Phase Voltages in $\alpha\beta\gamma$ Space

$$v_a = \cos t \qquad v_b = \cos(t - \frac{2\pi}{3}) \qquad v_c = \cos(t + \frac{2\pi}{3})$$

$$\vec{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

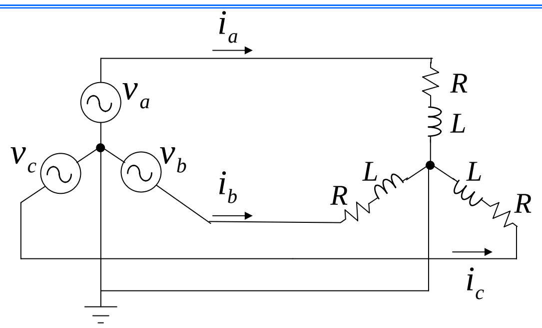
$$\vec{v}_{\alpha\beta\gamma} = T_{\alpha\beta\gamma/abc} \cdot \vec{v}_{abc}$$



DB-32

Example: State-Space Equations

$$\vec{v} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad \vec{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$



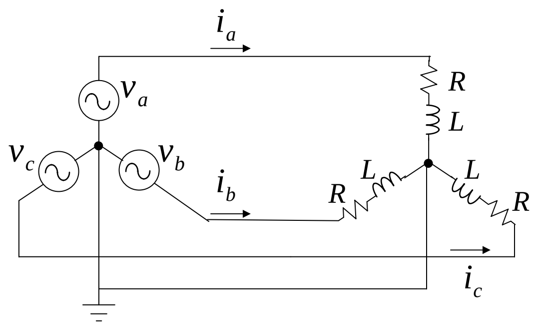
$$\vec{v} = \mathbf{R}\vec{i} + \mathbf{L}\frac{d\vec{i}}{dt} \quad \Leftrightarrow \quad \frac{d\vec{i}}{dt} = \mathbf{L}^{-1}\mathbf{R}\vec{i} + \mathbf{L}^{-1}\vec{v}$$

$\vec{v}$ and $\vec{i}$ are sinusoidal in steady-state! Find a coordinate transformation:

$\vec{v}_x = T_x \cdot \vec{v}$, $\vec{i}_x = T_x \cdot \vec{i}$, such that $\vec{v}_x$ and $\vec{i}_x$ are constant in steady state, and T_x is differentiable and invertible.

DB-33

Example: State-Space Equations



$$\vec{v} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} V_m \cos(\omega t) \\ V_m \cos(\omega t - \frac{2\pi}{3}) \\ V_m \cos(\omega t + \frac{2\pi}{3}) \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}$$

$$\vec{v} = \mathbf{R}\vec{i} + \mathbf{L}\frac{d\vec{i}}{dt} \qquad \vec{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\boxed{\frac{d\vec{i}}{dt} = -\mathbf{L}^{-1}\mathbf{R}\cdot\vec{i} + \mathbf{L}^{-1}\cdot\vec{v}} \qquad \mathbf{L} = \begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \end{bmatrix}$$

DB-34

Example: State-Space Equations – Solution

$$\vec{i}(t) = e^{-\mathbf{L}^{-1}\mathbf{R}\cdot t}\cdot\vec{i}(0) \quad + \quad \int_0^t e^{-\mathbf{L}^{-1}\mathbf{R}\cdot(t-\tau)}\cdot\mathbf{L}^{-1}\cdot\vec{v}(\tau)\cdot d\tau$$

Natural Response Forced Response

$$\vec{i}(t) = \begin{bmatrix} i_a(0) \\ i_b(0) \\ i_c(0) \end{bmatrix}\cdot e^{-\frac{R}{L}\cdot t} + \frac{V_m}{Z}\cdot\begin{bmatrix} \cos(\omega t-\phi) - \frac{R}{Z}\cdot e^{-\frac{R}{L}\cdot t} \\ \cos(\omega t - \frac{2\pi}{3} - \phi) + \frac{R+\sqrt{3}\cdot\omega L}{2Z}\cdot e^{-\frac{R}{L}\cdot t} \\ \cos(\omega t + \frac{2\pi}{3} - \phi) + \frac{R-\sqrt{3}\cdot\omega L}{2Z}\cdot e^{-\frac{R}{L}\cdot t} \end{bmatrix}$$

where per-phase impedance, $\mathbf{Z}$, at the source frequency, ω , is defined as:

$$\mathbf{Z} = R + \mathrm{j}\omega L = Z\cdot e^{\mathrm{j}\phi} \qquad Z = \sqrt{R^2+\omega^2 L^2} \qquad \phi = \arctan\frac{\omega L}{R}$$

DB-35

Example: State-Space Equations – Steady State Solution

$$\lim_{t\to\infty}\left[\vec{i}(t)\right] = I_m\cdot\begin{bmatrix} \cos(\omega t-\phi) \\ \cos(\omega t - \frac{2\pi}{3} - \phi) \\ \cos(\omega t + \frac{2\pi}{3} - \phi) \end{bmatrix} \qquad I_m = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \qquad \phi = \arctan\frac{\omega L}{R}$$

$$\lim_{t\to\infty}\left[\frac{d\vec{i}}{dt}\right] \neq 0$$

Want to find change of variables:

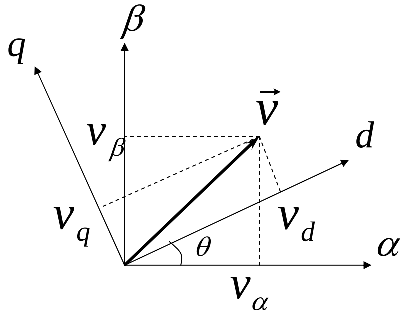
$\vec{v}_x = T_x\cdot\vec{v}$ where: $\vec{v}_x$ and $\vec{i}_x$ are constant in steady state

$\vec{i}_x = T_x\cdot\vec{i}$ T_x : differentiable and invertible

DB-36

Transformation Matrix $T_{dq/\alpha\beta}$

A rotating vector in $\alpha\beta\gamma$ space can be a constant vector in a rotating space



$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}$$

$$\theta = \int_0^t \omega(\tau)d\tau + \theta(0)$$

Where ω is the rotating speed

DB-37

Transformation Matrix $T_{dq0/\alpha\beta\gamma}$

Preserve the same third axis, that is 0-axis is the same as γ-axis

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \\ v_\gamma \end{bmatrix}$$

Therefore

$$\mathrm{T}_{dq0/\alpha\beta\gamma} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\| T_{dq0/\alpha\beta\gamma} \| = 1$$

$$T_{\alpha\beta\gamma/dq0} = T_{dq0/\alpha\beta\gamma}^{-1} = T_{dq0/\alpha\beta\gamma}^{T}$$

DB-38

Transformation Matrix $T_{dq0/abc}$

$$\vec{v}_{dq0} = T_{dq0/\alpha\beta\gamma} \cdot \vec{v}_{\alpha\beta\gamma} = T_{dq0/\alpha\beta\gamma} \cdot T_{\alpha\beta\gamma/abc} \cdot \vec{v}_{abc}$$

Therefore

$$\vec{v}_{dq0} = T_{dq0/abc} \cdot \vec{v}_{abc}$$

where

$$T_{dq0/abc} = T_{dq0/\alpha\beta\gamma} \cdot T_{\alpha\beta\gamma/abc}$$

Park's Transformation

$$= \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\| T_{dq0/abc} \| = 1 \qquad T_{abc/dq0} = T_{dq0/abc}^{-1} = T_{dq0/abc}^{T}$$

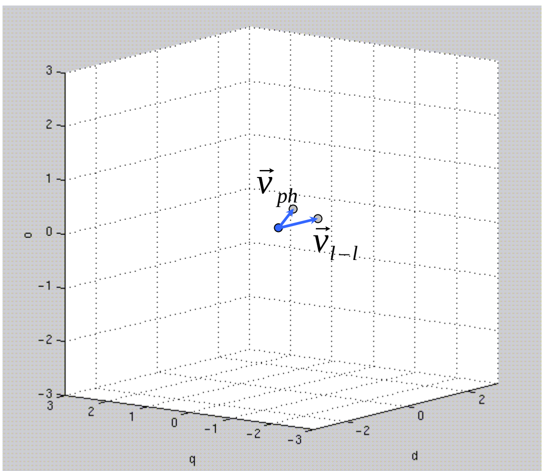
DB-39

Example: Balanced Three-Phase Voltages in dq0 Space

$$v_a = \cos t \qquad v_b = \cos(t - \frac{2\pi}{3}) \qquad v_c = \cos(t + \frac{2\pi}{3})$$

$$\vec{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

$$\vec{v}_{dq0} = T_{dq0/abc} \cdot \vec{v}_{abc}$$



DB-40

Power Definition in Three-Phase Circuits

$$\vec{p} = \vec{v} \cdot \vec{i} = \vec{v}^{\mathrm{T}} \vec{i} = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = v_a i_a + v_b i_b + v_c i_c$$

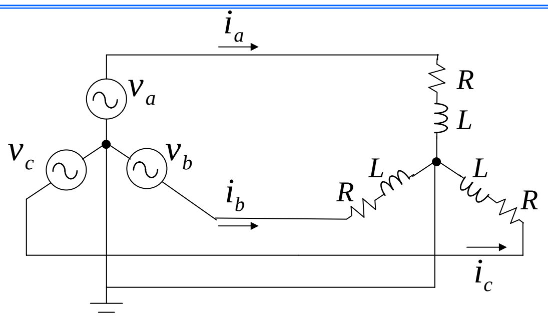
where v & i are corresponding voltages and currents in a three-phase circuit.

It can be easily proved that:

$$v_a i_a + v_b i_b + v_c i_c = v_\alpha i_\alpha + v_\beta i_\beta + v_\gamma i_\gamma = v_d i_d + v_q i_q + v_0 i_0$$

DB-41

Example: Power in Sinusoidal Steady State

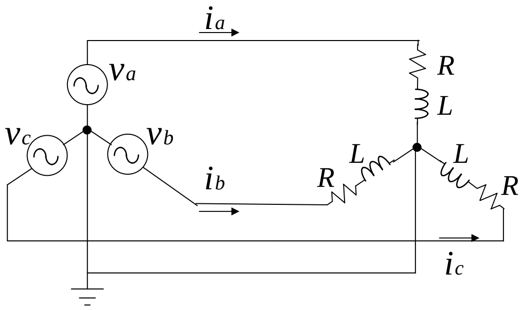


$$\vec{v} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} V_{am} \cos(\omega t) \\ V_{am} \cos(\omega t - \frac{2\pi}{3}) \\ V_{am} \cos(\omega t + \frac{2\pi}{3}) \end{bmatrix} \qquad \vec{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} I_{am} \cos(\omega t - \phi) \\ I_{am} \cos(\omega t - \frac{2\pi}{3} - \phi) \\ I_{am} \cos(\omega t + \frac{2\pi}{3} - \phi) \end{bmatrix}$$

where: $\phi = \arctan \dfrac{\omega L}{R}$

DB-42

Example:
Power in Sinusoidal Steady State



$$P = \| \vec{v} \cdot \vec{i} \| = \| \vec{v} \| \cdot \| \vec{i} \| \cos \phi = \frac{3}{2} V_{am} I_{am} \cos \phi$$

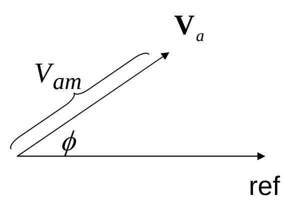
$$Q = \| \vec{v} \times \vec{i} \| = \| \vec{v} \| \cdot \| \vec{i} \| \sin \phi = \frac{3}{2} V_{am} I_{am} \sin \phi$$

DB-43

Phasor Representation

Phasors are defined **ONLY for sinusoidal steady state!**

$$v_a = V_{am} \cos(\omega t + \phi) \Longleftrightarrow \mathbf{V}_a = V_{am} \angle \phi = V_{am} \cdot e^{j\phi}$$



• Vector representation **is NOT** phasor representation!

DB-44

CPES

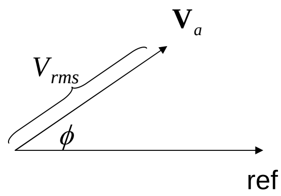
Phasor Representation

Phasors are defined **ONLY for sinusoidal steady state**!

Phasors are very useful for the analysis of linear systems without transients, which are excited by **constant single frequency** (ω) **sinusoidal generators**.

$$v_a = V_m \cos(\omega t + \phi) = \mathrm{Re}\left[V_m \cdot e^{j\phi}\right] = \mathrm{Re}\left[V_m \cos(\omega t + \phi) + \mathrm{j}V_m \sin(\omega t + \phi)\right]$$

$$\mathbf{V}_a = \frac{V_m}{\sqrt{2}} \angle \phi = V_{rms} \cdot e^{j\phi}$$



$$v_a = V_m \cos(\omega t + \phi) = \sqrt{2} \cdot \mathrm{Re}\left(\mathbf{V}_a \cdot e^{j\omega t}\right)$$

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CPES

Outline

PECon 2008

1. **Introduction**
2. **Switching Modeling and PWM**
   - **Switching model of VSI & boost rectifier**
   - **Space vector modulation for VSI & boost rectifier**
   - **Other modulations for VSI & boost rectifier**
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5. **Closed-Loop Control Design**
6. **More Complex Converters**

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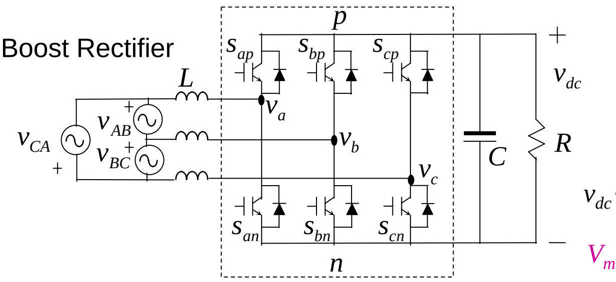
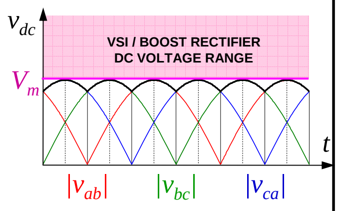
Boost Rectifier / Voltage Source Inverter

- Boost Rectifier



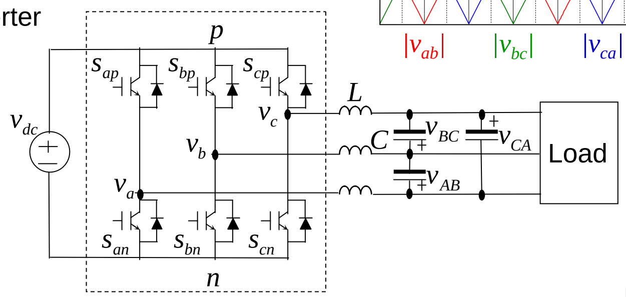
$V_{dc} > V_m$

where V_m is the peak value of the line-to-line input voltage



- Voltage Source Inverter (VSI)

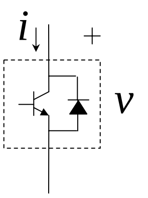
$V_{dc} > V_m$

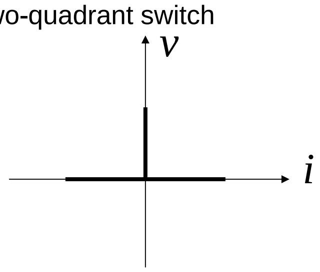


DB-47

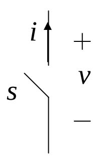
Method of Modeling Switching Network

Current bi-directional two-quadrant switch





Switching function:



$$s = \begin{cases} 1, \ v = 0, \text{ if switch } s \text{ is closed} \\ 0, \ i = 0, \text{ if switch } s \text{ is open} \end{cases}$$

Switching constraints:

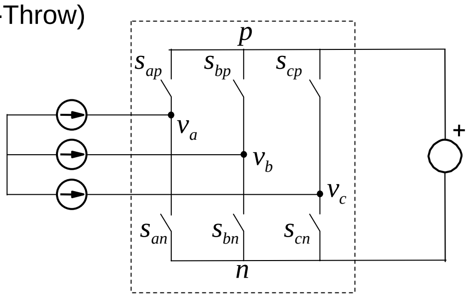
- Voltage source or capacitor cannot be shorted
- Current source or inductor cannot be open

DB-48

DC-Voltage-Unidirectional Three-Phase Switching Network

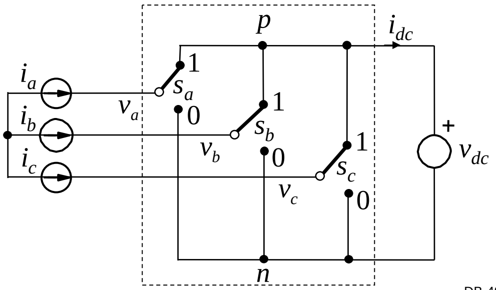
• Three-Switch (Single-Pole-Double-Throw)

Boost Rectifier
Voltage Source Inverter



Allowed switching combinations:

$$s_{ip} + s_{in} = 1; \qquad i \in \{a, b, c\}$$



• Define Voltage-Unidirectional Single-Pole-Double-Throw Switch and switching function

$$s_i = s_{ip} = 1 - s_{in}; \qquad i \in \{a, b, c\}$$

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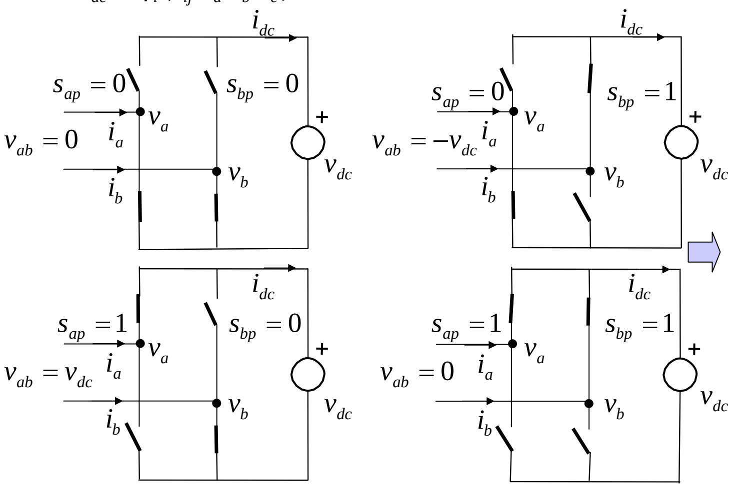
Development of Switching Model

(Boost rectifier / Voltage source inverter)

Find the relationships:

$$v_{ab} = f_{ab}(s_{ab}, v_{dc}) \qquad v_{bc} = f_{bc}(s_{bc}, v_{dc}) \qquad v_{ca} = f_{ca}(s_{ca}, v_{dc})$$

$$i_{dc} = f_i(s_{ij}, i_a, i_b, i_c)$$



$$v_{ab} = v_a - v_b$$
$$= (s_{ap} - s_{bp}) v_{dc}$$
$$= s_{ab} v_{dc}$$

$$i_{dc} = s_{ap} i_a + s_{bp} i_b$$

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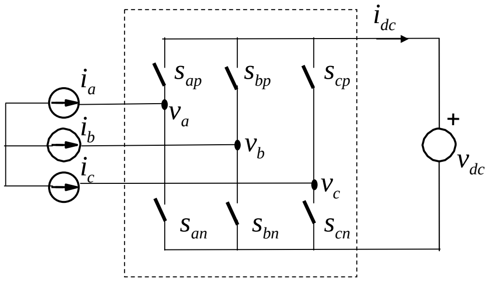
Development of Switching Model
(Boost rectifier / Voltage source inverter)



Define phase-leg switching function

$$s_i = s_{ip} = 1 - s_{in}; \qquad i \in \{a, b, c\}$$

s_a	s_b	s_c	s_a-s_b	s_b-s_c	s_c-s_a	i_{dc}	v_{ab}	v_{bc}	v_{ca}
0	0	0	0	0	0	0	0	0	0
0	0	1	0	-1	1	i_c	0	$-v_{dc}$	v_{dc}
0	1	0	-1	1	0	i_b	$-v_{dc}$	v_{dc}	0
0	1	1	-1	0	1	i_b+i_c	$-v_{dc}$	0	v_{dc}
1	0	0	1	0	-1	i_a	v_{dc}	0	$-v_{dc}$
1	0	1	1	-1	0	i_a+i_c	v_{dc}	$-v_{dc}$	0
1	1	0	0	1	-1	i_a+i_b	0	v_{dc}	$-v_{dc}$
1	1	1	0	0	0	$i_a+i_b+i_c$	0	0	0

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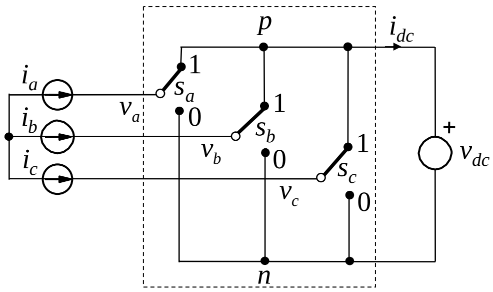
Development of Switching Model
(Boost rectifier / Voltage source inverter)



Instantaneous voltage equation

$$\begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} = \begin{bmatrix} s_a - s_b \\ s_b - s_c \\ s_c - s_a \end{bmatrix} v_{dc} = \begin{bmatrix} s_{ab} \\ s_{bc} \\ s_{ca} \end{bmatrix} v_{dc}$$

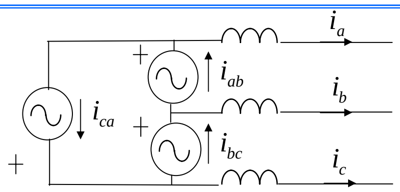
Instantaneous current equation

$$i_{dc} = \begin{bmatrix} s_a & s_b & s_c \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Note that: $s_{ab} = s_a - s_b \quad \dots \qquad v_{ab} = v_a - v_b \quad \dots$

DB-52

Relationship Between Line-to-Line Current and Phase Current



$$i_a = i_{ab} - i_{ca}$$

$$i_b = i_{bc} - i_{ab}$$

$$i_c = i_{ca} - i_{bc}$$

$$\Rightarrow \quad i_a - i_b = i_{ab} - i_{ca} - (i_{bc} - i_{ab}) = 2i_{ab} - (i_{ca} + i_{bc}) = 3i_{ab}$$

Assume $i_{ab} + i_{bc} + i_{ca} = 0$

$$\Rightarrow \quad \boxed{i_{ab} = \frac{1}{3}(i_a - i_b)}$$

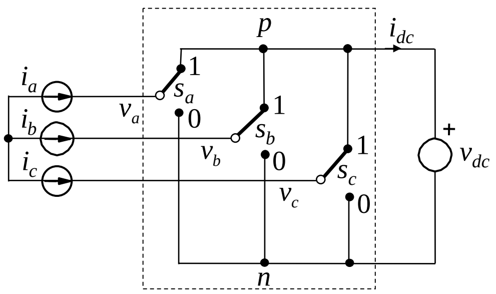
Similarly $\boxed{i_{bc} = \frac{1}{3}(i_b - i_c)}$ $\boxed{i_{ca} = \frac{1}{3}(i_c - i_a)}$

$$i_{dc} = s_a i_a + s_b i_b + s_c i_c = s_a(i_{ab} - i_{ca}) + s_b(i_{bc} - i_{ab}) + s_c(i_{ca} - i_{bc})$$

$$= i_{ab}(s_a - s_b) + i_{bc}(s_b - s_c) + i_{ca}(s_c - s_a) = \begin{bmatrix} s_{ab} & s_{bc} & s_{ca} \end{bmatrix} \cdot \begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix}$$

DB-53

Boost Rectifier / Voltage Source Inverter Switching Model



$$\vec{v}_{l-l} = \vec{s}_{l-l} \cdot v_{dc}$$

$$i_{dc} = \vec{s}_{l-l}^{\,T} \cdot \vec{i}_{l-l}$$

where:

$$\vec{v}_{l-l} = \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} = \begin{bmatrix} v_a - v_b \\ v_b - v_c \\ v_c - v_a \end{bmatrix} \qquad \vec{s}_{l-l} = \begin{bmatrix} s_{ab} \\ s_{bc} \\ s_{ca} \end{bmatrix} = \begin{bmatrix} s_a - s_b \\ s_b - s_c \\ s_c - s_a \end{bmatrix} \qquad \vec{i}_{l-l} = \begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} i_a - i_b \\ i_b - i_c \\ i_c - i_a \end{bmatrix}$$

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PECon 2008

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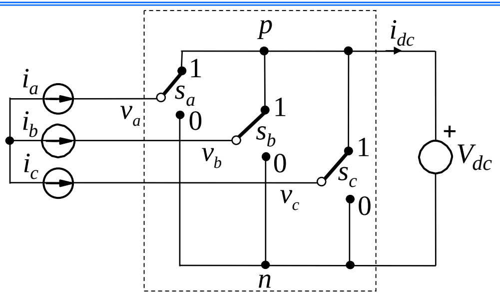
DB-55

Switching States for Boost Rectifier / Voltage Source Inverter

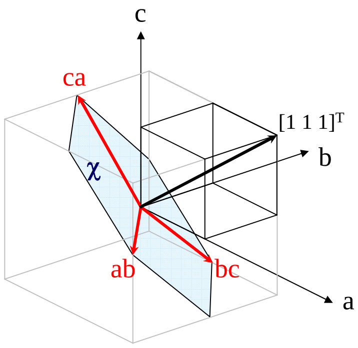


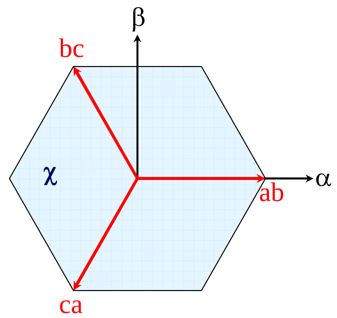
s_a	s_b	s_c	Switching state	i_{dc}	v_{ab}	v_{bc}	v_{ca}
0	0	0	*nnn*	0	0	0	0
0	0	1	*nnp*	i_c	0	$-V_{dc}$	V_{dc}
0	1	0	*npn*	i_b	$-V_{dc}$	V_{dc}	0
0	1	1	*npp*	i_b+i_c	$-V_{dc}$	0	V_{dc}
1	0	0	*pnn*	i_a	V_{dc}	0	$-V_{dc}$
1	0	1	*pnp*	i_a+i_c	V_{dc}	$-V_{dc}$	0
1	1	0	*ppn*	i_a+i_b	0	V_{dc}	$-V_{dc}$
1	1	1	*ppp*	$i_a+i_b+i_c$	0	0	0

DB-56

Vector Space of Line-to-Line Variables

- Phase variables (a, b and c) produce line-to-line variables (ab, bc and ca) in plane-χ
- Line-to-line variables (ab, bc and ca) do not have γ-component in αβγ-coordinate system





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Line-to-Line Voltage Space Vector

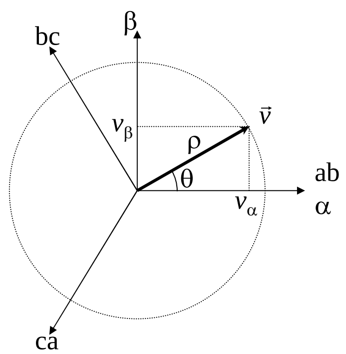
$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = T_{\alpha\beta/abc} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} \quad \textit{where} \quad T_{\alpha\beta/abc} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

- Space vector

$$\vec{v} = \rho \cdot e^{j\theta}$$

$$\rho = \sqrt{v_\alpha^2 + v_\beta^2}$$

$$\theta = \tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right)$$



☞ If V_m is the amplitude of balanced, symmetrical, three-phase line-to-line voltages, then $\rho = \sqrt{\frac{3}{2}} \cdot V_m$

DB-58

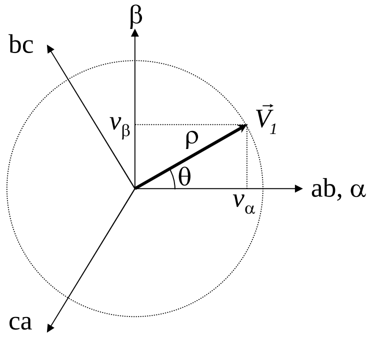
Switching State Vector [pnn]

$$\vec{V}_{pnn} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}_{pnn} = T_{\alpha\beta/abc} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{pnn} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} V_{dc} \\ 0 \\ -V_{dc} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{3}{2}} \cdot V_{dc} \\ \sqrt{\frac{1}{2}} \cdot V_{dc} \end{bmatrix}$$

$$\vec{V}_{pnn} = \vec{V}_1 = \rho \cdot e^{j\theta}$$

$$\rho = \sqrt{2} \cdot V_{dc}$$

$$\theta = \tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right) = 30°$$



DB-59

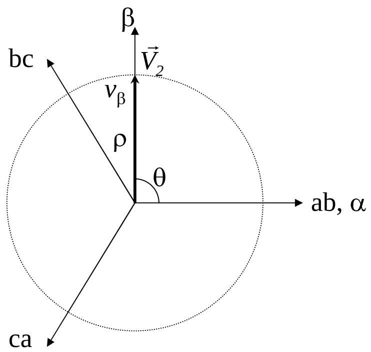
Switching State Vector [ppn]

$$\vec{V}_{ppn} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}_{ppn} = T_{\alpha\beta/abc} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{ppn} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ V_{dc} \\ -V_{dc} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \cdot V_{dc} \end{bmatrix}$$

$$\vec{V}_{ppn} = \vec{V}_2 = \rho \cdot e^{j\theta}$$

$$\rho = \sqrt{2} \cdot V_{dc}$$

$$\theta = \tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right) = 90°$$

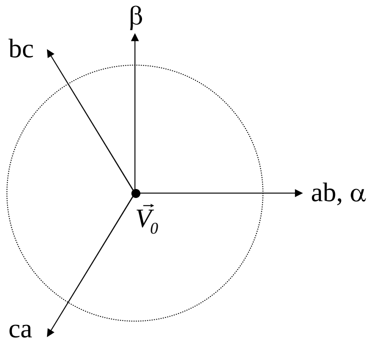


DB-60

Switching State Vector [ppp]

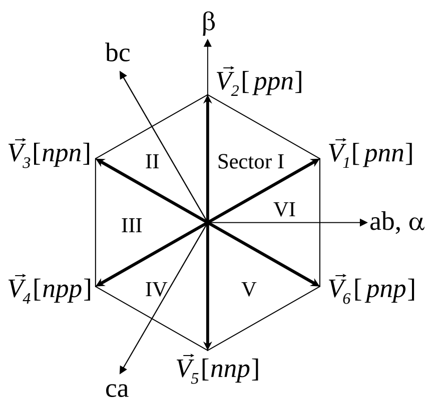
$$\vec{V}_{ppp} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}_{ppp} = T_{\alpha\beta/abc} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{ppp} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{V}_{ppp} = \vec{V}_0 = 0$$



Switching State Vectors

	ρ	θ (°)
$\vec{V}_1[pnn]$	$\sqrt{2} \cdot V_{dc}$	30
$\vec{V}_2[ppn]$		90
$\vec{V}_3[npn]$		150
$\vec{V}_4[npp]$		-150
$\vec{V}_5[nnp]$		-90
$\vec{V}_6[pnp]$		-30
$\vec{V}_0[ppp]$	0	0
$\vec{V}_0[nnn]$		0



$\vec{V}_0 = [ppp] = [nnn]$ at center point

Reference Voltage Vector, Vref

Assume $\begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}_{ref} = \begin{bmatrix} V_m \cdot \cos(\omega t) \\ V_m \cdot \cos(\omega t - 120°) \\ V_m \cdot \cos(\omega t + 120°) \end{bmatrix}$

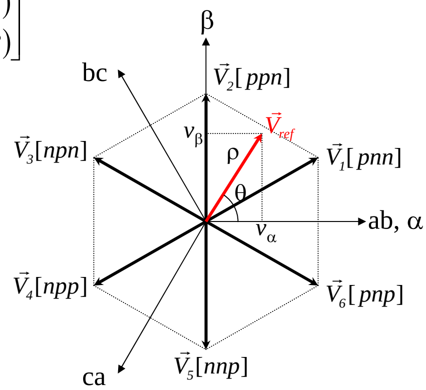
$$\vec{V}_{ref} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}_{ref} = \rho \cdot e^{j\theta}$$

where $\rho = \sqrt{v_\alpha^2 + v_\beta^2} = \sqrt{\frac{3}{2}} \cdot V_m$

$$\theta = \tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right) = \omega t$$

☞ In general,

$$\vec{V}_{ref}(t) = \sqrt{\frac{3}{2}} \cdot V_m(t) \cdot e^{j\theta(t)}$$



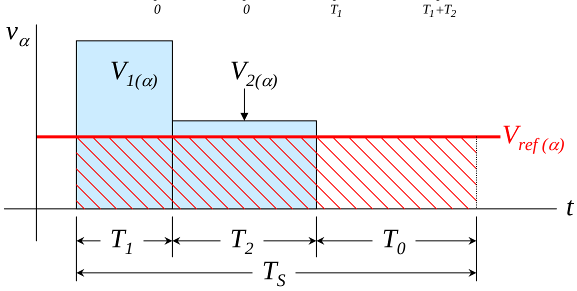
$\vec{V}_0 = [ppp] = [nnn]$ at center point

DB-63

Definition of High Frequency Synthesis

$$\int_0^{T_S} \vec{V}_{ref}\, dt = \sum_i \left(\int_0^{T_i} \vec{V}_i\, dt \right), \qquad \sum_i T_i = T_S$$

☞ For example $\int_0^{T_S} \vec{V}_{ref}\, dt = \int_0^{T_1} \vec{V}_1\, dt + \int_{T_1}^{T_1+T_2} \vec{V}_2\, dt + \int_{T_1+T_2}^{T_S} \vec{V}_0\, dt$



Total area of [blue box] = Area of [red hatched box]

DB-64

Space Vector Modulation

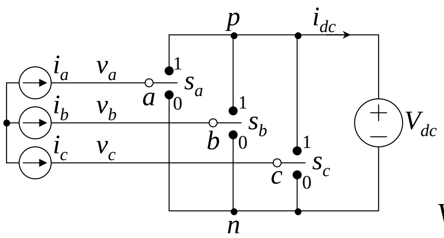
Step 1 : Choose desired switching state vectors to synthesize $\vec{V}_{ref}$

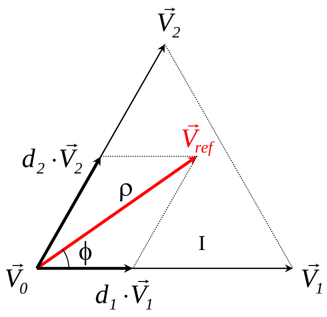
Step 2 : Calculate the duty ratios of chosen switching state vectors

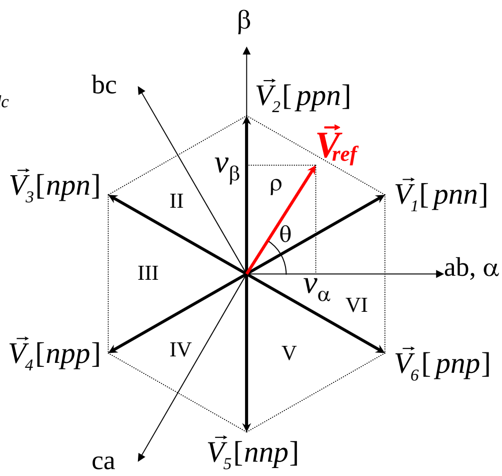
Step 3 : Make the sequence of chosen switching state vectors

DB-65

Switching State Vectors







$$\vec{V}_0 = [ppp] = [nnn]$$

DB-66

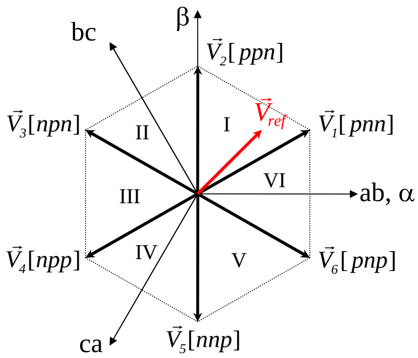
Step 1 : Choice of Switching State Vectors

- Minimize the number of switching
- Minimize the harmonic distortion

Choose minimum number of switching state vectors adjacent to $\vec{V}_{ref}$.

$\vec{V}_{ref}$ location	Chosen vectors
Sector I	$\vec{V}_1$ and $\vec{V}_2$
Sector II	$\vec{V}_2$ and $\vec{V}_3$
Sector III	$\vec{V}_3$ and $\vec{V}_4$
Sector IV	$\vec{V}_4$ and $\vec{V}_5$
Sector V	$\vec{V}_5$ and $\vec{V}_6$
Sector VI	$\vec{V}_6$ and $\vec{V}_1$

and $\vec{V}_0$



$\vec{V}_0 = [ppp] = [nnn]$ at center point

DB-67

Step 2 : Duty Ratio of Switching State Vectors at Sector I

From HF synthesis definition, $\int_0^{T_S} \vec{V}_{ref}\,dt = \int_0^{T_1} \vec{V}_1\,dt + \int_{T_1}^{T_1+T_2} \vec{V}_2\,dt + \int_{T_1+T_2}^{T_S} \vec{V}_0\,dt$

Assume $\vec{V}_{ref}$ is constant in T_S, $\vec{V}_{ref}\cdot T_S = \vec{V}_1\cdot T_1 + \vec{V}_2\cdot T_2$

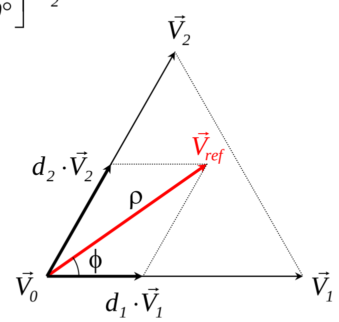
$$\rho\cdot\begin{bmatrix}\cos\phi\\ \sin\phi\end{bmatrix}\cdot T_S = \|V_1\|\cdot\begin{bmatrix}1\\ 0\end{bmatrix}\cdot T_1 + \|V_2\|\cdot\begin{bmatrix}\cos 60°\\ \sin 60°\end{bmatrix}\cdot T_2$$

where $\phi = \theta - 30°$

$$\frac{T_1}{T_S} = d_1 = \frac{2}{\sqrt{3}}\cdot\frac{\rho}{\|V_1\|}\cdot\sin(60° - \phi)$$

$$\frac{T_2}{T_S} = d_2 = \frac{2}{\sqrt{3}}\cdot\frac{\rho}{\|V_2\|}\cdot\sin\phi$$

$$d_0 = 1 - d_1 - d_2$$



DB-68

Duty Ratio of Switching State Vectors

☞ Other sectors have the same results of duty ratio.

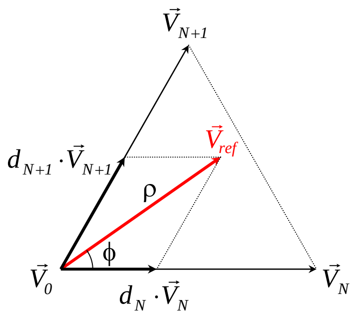
$$\frac{T_N}{T_S} = d_N = \frac{2}{\sqrt{3}} \cdot \frac{\rho}{\|V_N\|} \cdot \sin(60° - \phi)$$

$$\frac{T_{N+1}}{T_S} = d_{N+1} = \frac{2}{\sqrt{3}} \cdot \frac{\rho}{\|V_{N+1}\|} \cdot \sin\phi$$

$$d_0 = 1 - d_N - d_{N+1}$$

where $\phi = \theta - (N-1) \cdot 60° - 30°$

N : sector number (1 ~ 6)



DB-69

Modulation Index

For all the switching state vectors, $\|V_N\| = \sqrt{2} \cdot V_{dc}$ and $\rho = \sqrt{\frac{3}{2}} \cdot V_m$

$$d_N = \frac{V_m}{V_{dc}} \cdot \sin(60° - \phi)$$

$$d_{N+1} = \frac{V_m}{V_{dc}} \cdot \sin\phi$$

$$d_0 = 1 - d_N - d_{N+1}$$

☞ Define the modulation index $M = \frac{V_m}{V_{dc}}$

$$d_N = M \cdot \sin(60° - \phi)$$

$$d_{N+1} = M \cdot \sin\phi$$

$$d_0 = 1 - d_N - d_{N+1}$$

DB-70

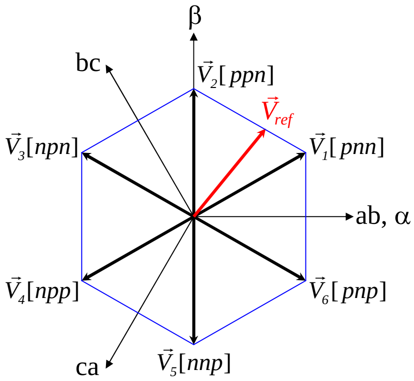
Maximum Amplitude of V_{ref}

Assume $d_0 = 0$, then $d_N + d_{N+1} = 1$

$$d_N + d_{N+1} = M \cdot (\sin(60° - \phi) + \sin\phi)$$
$$= M \cdot \cos(30° - \phi)$$
$$= \frac{V_m}{V_{dc}} \cdot \cos(30° - \phi)$$
$$= 1$$

$$\therefore V_m = \frac{V_{dc}}{\cos(30° - \phi)}$$

☞ The trajectory of $\vec{V}_{ref}$ makes a hexagon.



$\vec{V}_0 = [ppp] = [nnn]$ at center point

DB-71

Maximum Amplitude of V_{ref}

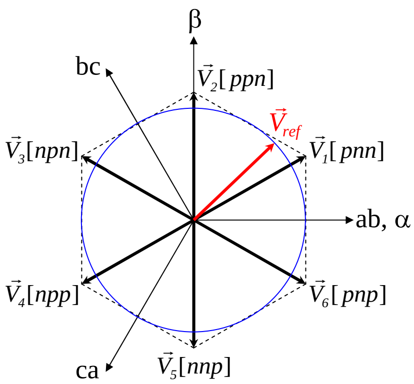
Assume $M = 1$, then $\frac{V_m}{V_{dc}} = 1$

$$\therefore V_m = V_{dc}$$

$$\vec{V}_{ref} = \sqrt{\frac{3}{2}} \cdot V_{dc} \cdot e^{j\theta}$$

☞ The trajectory of $\vec{V}_{ref}$ makes a circle whose radius is $\sqrt{\frac{3}{2}} \cdot V_{dc}$

☞ This trajectory of $\vec{V}_{ref}$ represents the largest 3-phase sinusoidal voltage that can be synthesized.

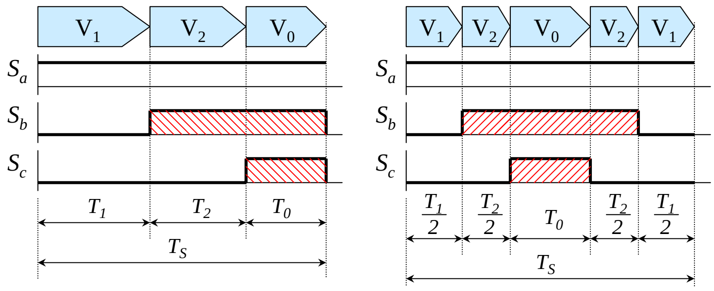


$\vec{V}_0 = [ppp] = [nnn]$ at center point

DB-72

Step 3 : Sequence of Switching State Vectors

Asymmetrical sequence	2-phase commutation	Feed forward
Symmetrical sequence	3-phase commutation	Feedback

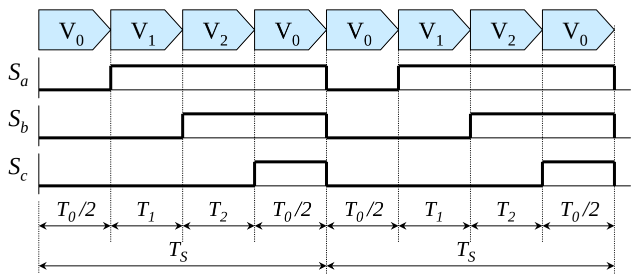


$$= \int_0^{T_S} \vec{V}_{ref}\, dt = \sum_{i=1} \left(\int_0^{T_i} \vec{V}_i\, dt \right) =$$

DB-73

Sequence of SSVs – SVM 1 (Three-Phase – Right Aligned: 3Φ-RA)

- Use both zero switching state vectors
- Asymmetrical sequence
- Six commutations per switching cycle



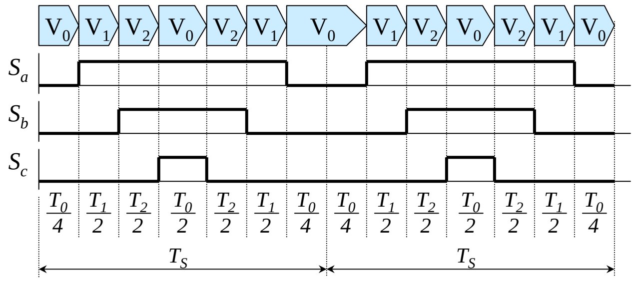
< Example in sector I >

- 3Φ-LA has same characteristics

DB-74

Sequence of SSVs – SVM 2
(Three-Phase – Centered: 3Φ-C)

- Use both zero switching state vectors
- Symmetrical sequence ⇨ Low THD
- Six commutations per switching cycle

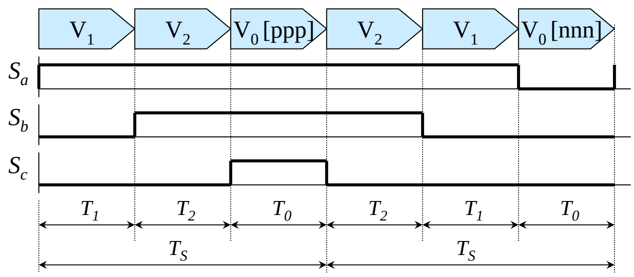


< Example in sector I >

DB-75

Sequence of SSVs – SVM 3
(Three-Phase – Double-Period: 3Φ-2T)

- Use zero vectors alternatively in adjacent switching cycle
- Asymmetrical sequence in T_S, but symmetrical in $2 \cdot T_S$
- Three commutations ⇨ 50 % switching loss reduction
- Introduction of harmonics around $\frac{1}{2 \cdot T_S}$

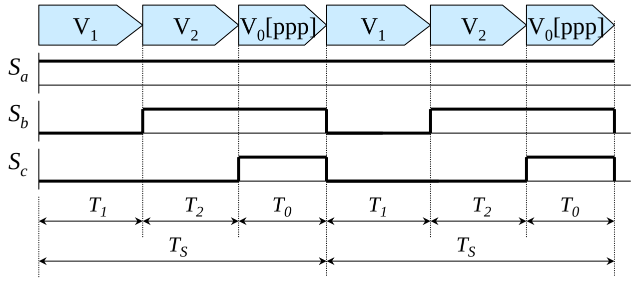


< Example in sector I >

DB-76

Sequence of SSVs – SVM 4
(Two-Phase – Right Aligned: 2Φ-RA)

- Use a zero vector in one switching cycle { Sector I, III, V : [ppp]; Sector II, IV, VI : [nnn] }
- Asymmetrical sequence
- Four commutations ⇒ Reduced switching losses



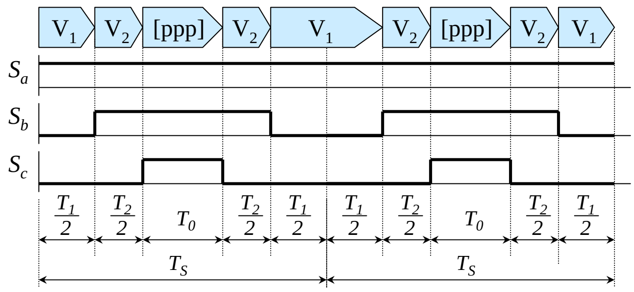
< Example in sector I >

- 2Φ-LA has same characteristics

DB-77

Sequence of SSVs – SVM 5
(Two-Phase – Centered: 2Φ-C)

- Use a zero vector in one switching cycle { Sector I, III, V : [ppp]; Sector II, IV, VI : [nnn] }
- Symmetrical sequence ⇒ Low THD
- Four commutations ⇒ Reduced switching losses



< Example in sector I >

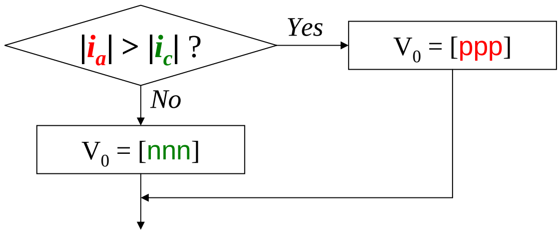
DB-78

Sequence of SSVs – SVM 6 (Minimum-Loss SVM)

(Two-Phase – Right Aligned – **m**inimum Loss: **2Φ-RA-mL**)

- Choose a zero switching state vector to avoid switching the phase with the highest instantaneous current
- Reduce the switching losses up to 50% compared to 3Φ modulations, assuming that switching losses are proportional to the current

Choice of zero vector in sector I (pnn, ppn, and ppp or nnn)

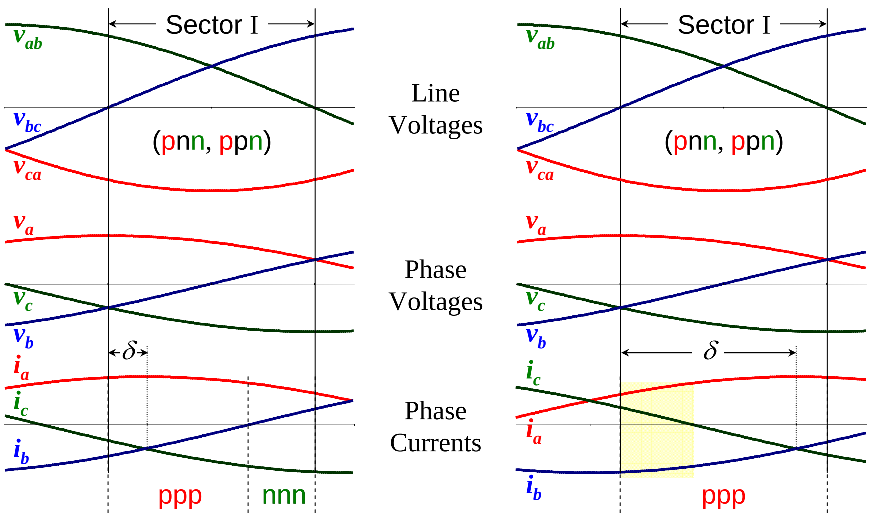


- Possible sequences: 2Φ-RA-mL, 2Φ-LA-mL, or 2Φ-C-mL

DB-79

Example of 2Φ- *x* -mL SVM in Sector I

for balanced, symmetrical, sinusoidal, steady-state case



- $|\delta| \le 30°$
- Switching loss reduction = 50%

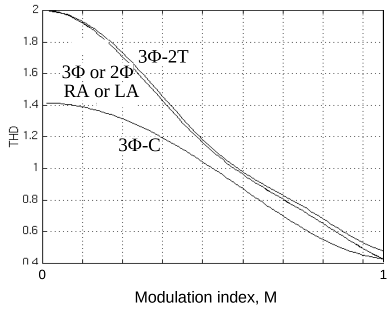
- $|\delta| > 30°$
- Switching loss reduction < 50%

DB-80

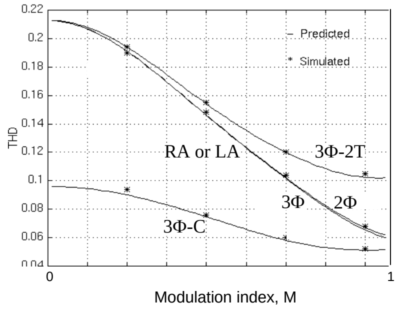
Comparison: Total Harmonic Distortion (THD)

With the Fourier series $V = \sum_{n=1}^{\infty} V_n \cdot e^{jn\omega t}$, $THD = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1}$

Calculated and switching-model simulation results for $f_S > 100 \cdot f_{line}$.



THD of line-to-line voltage



THD of phase current with $L + V_{ABC}$ load/source

DB-81

Comparison: Peak-to-Peak Current Ripple

For RA or LA modulations:

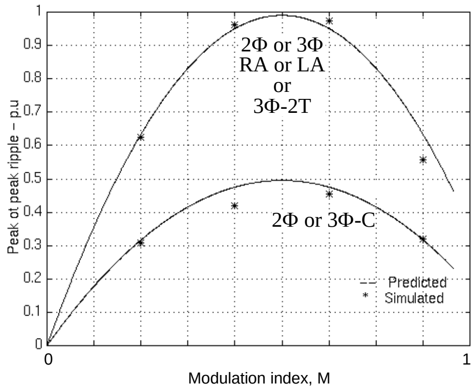
$$I_{pp} = \frac{2 \cdot V_{dc}}{3 \cdot L} \cdot (1 - M) \cdot M \cdot T_S$$

For centered modulations:

$$I_{pp} = \frac{V_{dc}}{3 \cdot L} \cdot (1 - M) \cdot M \cdot T_P$$

where $T_P = T_S$, except
$T_P = 2T_S$, for 3Φ-2T.

M : modulation index
L : load inductance



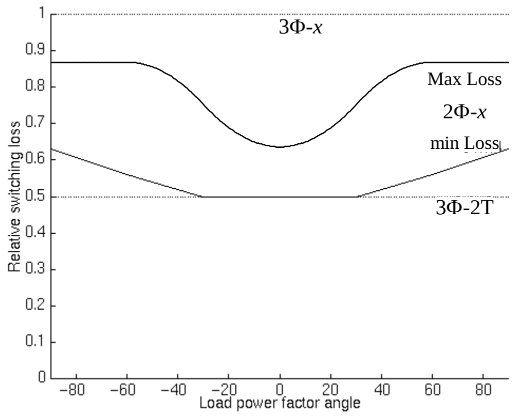
Relative peak-to-peak current ripple

DB-82

Comparison: Switching Losses

- Number of commutations per switching cycle:

3Φ-*x*	: 6
3Φ-2T	: 3
2Φ-*x*	: 4



☞ 2Φ- *x* -mL avoids switching the phase with highest current between -30° and 30°.

DB-83

Outline

PECon 2008

1. **Introduction**
2. **Switching Modeling and PWM**
   - **Switching model of VSI & boost rectifier**
   - **Space vector modulation for VSI & boost rectifier**
   - **Other modulations for VSI & boost rectifier**
   - Switching model and modulation for CSI & buck rectifier
3. **Average Modeling**
4. **Small-Signal Modeling**
5. **Closed-Loop Control Design**
6. **More Complex Converters**

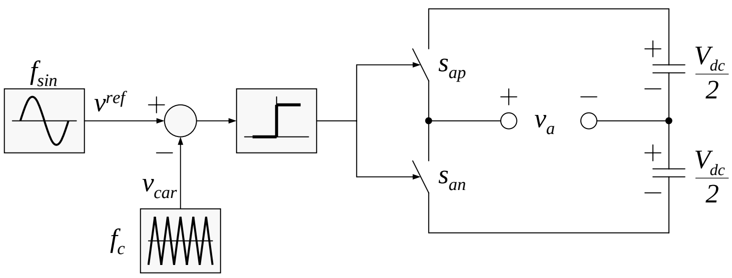
DB-84

Modulation Methods

- Pulse Width Modulation
  - Feedforward scheme
    - Sinusoidal PWM
    - Third-harmonic injection PWM
    - Space vector modulations (SVM1, 2, 3, 4 and 5)
    - ⋮
  - Feedback scheme
    - Hysteresis current control
    - Space vector modulations (SVM 6)
    - ⋮
- Pulse Amplitude Modulation

DB-85

Sinusoidal Pulse Width Modulation (SPWM)



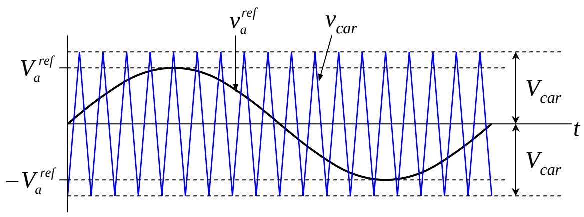
- Determine the switching state by magnitude of v^{ref} and v_{car}

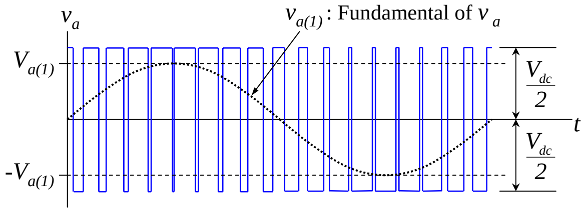
  If $|v^{ref}| > |v_{car}|$, then s_{ap} on,
  If $|v^{ref}| < |v_{car}|$, then s_{an} on

- Switching frequency is the same as carrier wave *frequency*, f_c

DB-86

Waveforms of Single-Phase SPWM

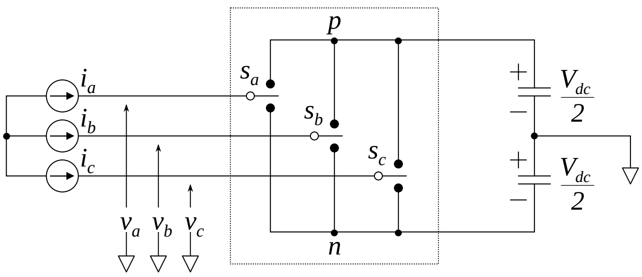


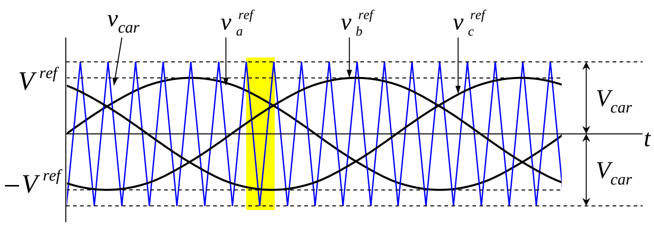


$$\frac{V_a^{ref}}{V_{car}} = \frac{V_{a(1)}}{\left(\frac{V_{dc}}{2}\right)} = \frac{2 \cdot V_{a(1)}}{V_{dc}} \quad \Rightarrow \quad V_{a(1)} = \frac{V_{dc}}{2} \cdot \frac{V_a^{ref}}{V_{car}}$$

DB-87

Three-Phase SPWM





DB-88

Modulation Example of SPWM in Sector I

Assume v_a^{ref}, v_b^{ref} and v_c^{ref} are constant in a switching cycle.

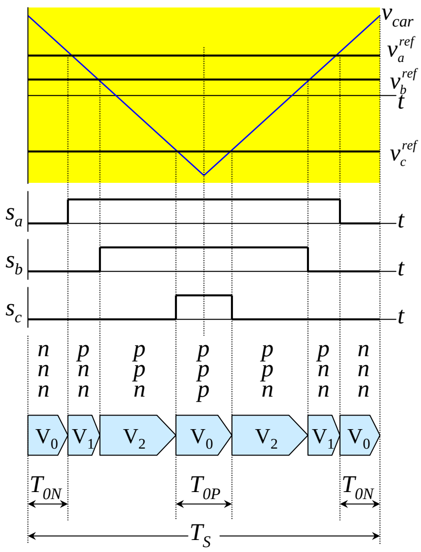
☞ Symmetrical (Center-based) Three-phase commutation

⇕

SVM 2

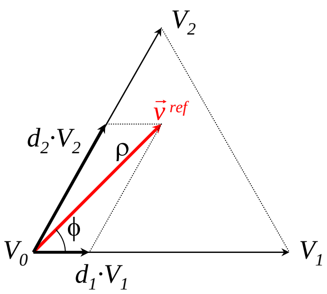
☞ $$T_{0N} = -\frac{T_S}{4 \cdot V_{car}} \cdot (v_a^{ref} - V_{car})$$

$$\frac{T_{0P}}{2} = \frac{T_S}{2} + \frac{T_S}{4 \cdot V_{car}} \cdot (v_c^{ref} - V_{car}) = \frac{T_S}{4 \cdot V_{car}} \cdot (v_c^{ref} + V_{car})$$



DB-89

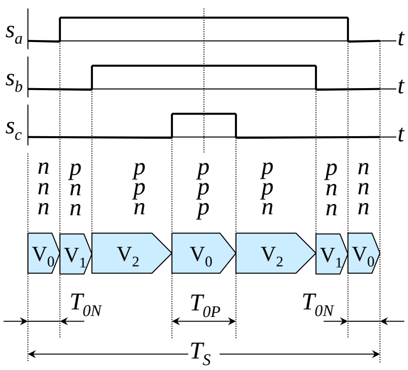
Modulation Example of SVM2 in Sector I



$$\frac{T_1}{T_S} = d_1 = \sqrt{2} \cdot \frac{V_m}{\|V_1\|} \cdot \sin(60° - \phi)$$

$$\frac{T_2}{T_S} = d_2 = \sqrt{2} \cdot \frac{V_m}{\|V_2\|} \cdot \sin\phi$$

$$d_0 = 1 - d_1 - d_2$$



$$\frac{T_{0P}}{2} = T_{0N} = \frac{T_0}{4} = \frac{T_S}{4} - \frac{V_m}{V_{dc}} \cdot \frac{T_S}{4} \cdot \cos(30° - \phi)$$

DB-90

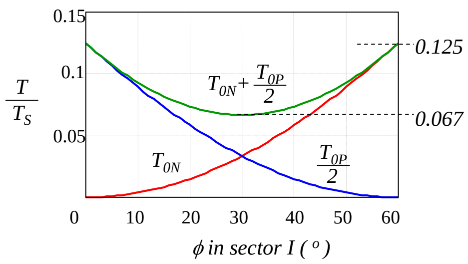
Zero Vector Timings in Sector I

- In SPWM, assume $V^{ref} = V_{car}$

$$T_{0N} = -\frac{T_S}{4 \cdot V^{ref}} \cdot (v_a^{ref} - V^{ref})$$

$$\frac{T_{0P}}{2} = \frac{T_S}{4 \cdot V^{ref}} \cdot (v_c^{ref} + V^{ref})$$

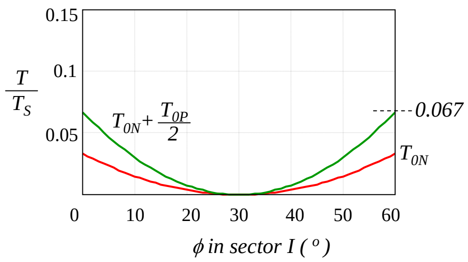
where $v_a^{ref} = V^{ref} \cdot \cos\phi$
$v_c^{ref} = V^{ref} \cdot \cos(\phi + 120°)$



- In SVM2, assume $V_m = V_{dc}$

$$\frac{T_{0P}}{2} = T_{0N} = \frac{T_0}{4}$$

$$= \frac{T_S}{4} - \frac{T_S}{4} \cdot \cos(30° - \phi)$$



DB-91

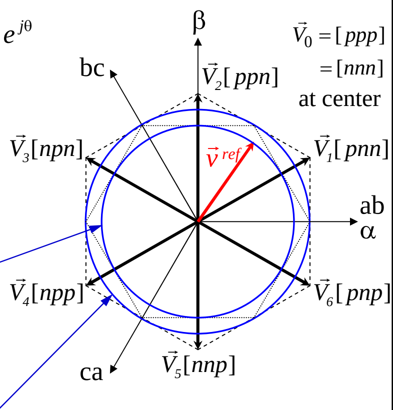
Maximum AC Voltage of SPWM

The $\vec{v}^{ref}$ trajectory in SPWM: $\vec{v}^{ref} = \frac{3}{2\sqrt{2}} \cdot V_{dc} \cdot e^{j\theta}$

By definition: $\vec{v}^{ref} = \sqrt{\frac{3}{2}} \cdot V_m \cdot e^{j\theta}$

The maximum AC voltage: $V_m = \frac{\sqrt{3}}{2} \cdot V_{dc}$

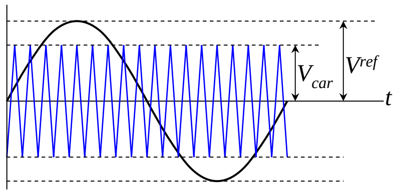
The maximum AC voltage of SVM: $V_m = V_{dc}$



☞ $\frac{V_m|_{SVM}}{V_m|_{SPWM}} = \frac{V_{dc}}{\left(\frac{\sqrt{3} \cdot V_{dc}}{2}\right)} = 1.155$ ⇨ SVM produces 15.5% higher maximum output than SPWM !

DB-92

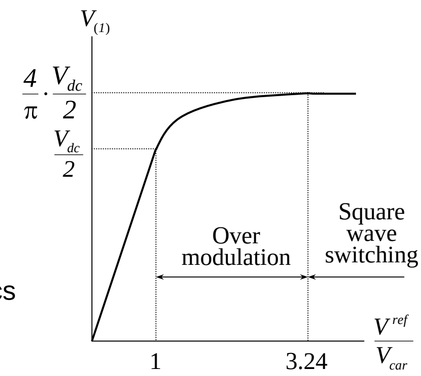
Over Modulation of SPWM



- $V^{ref} \le V_{car}$: Linear modulation
- $V^{ref} > V_{car}$: Over modulation

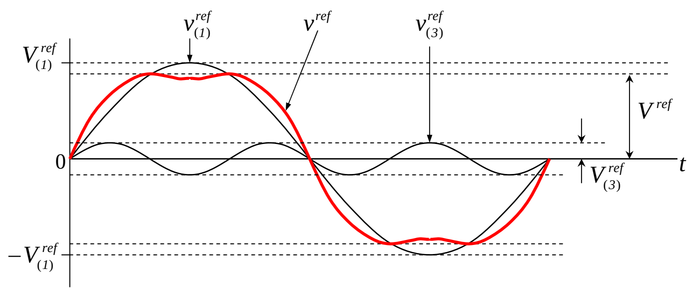
☞ Over modulation region is non linear with more harmonics

☞ $$\frac{V_{(1)}^{max}\big|_{SPWM}}{V_{(1)}^{max}\big|_{Square}} = \frac{\left(\frac{V_{dc}}{2}\right)}{\left(\frac{4}{\pi}\cdot\frac{V_{dc}}{2}\right)} = \frac{\pi}{4} = 0.785$$



DB-93

Third-Harmonic Injection PWM



$$v^{ref} = V_{(1)}^{ref}\cdot\sin\omega t + V_{(3)}^{ref}\cdot\sin 3\omega t$$

$$V^{ref} = V_{(1)}^{ref} + V_{(3)}^{ref}$$

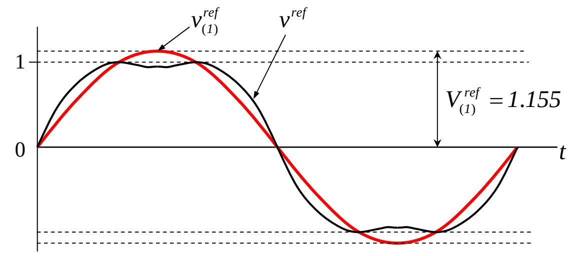
DB-94

Maximum AC Voltage of Third-Harmonic Injection PWM

In general, $v^{ref} = V_{(1)}^{ref} \cdot \left(\sin\theta + \frac{1}{6} \cdot \sin 3\theta \right)$

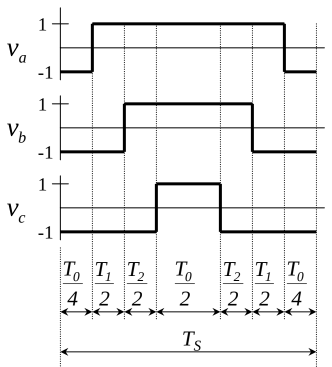
Assume $V^{ref} = 1$,

then $V_{(1)}^{ref} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1.155$ ⇨ The maximum AC voltage is 15.5 % more than SPWM



DB-95

Average Values of Phase-to Neutral Voltage for SVM 2 (3Φ-C) in Sector I



$$\frac{\bar{v}_a}{V_{dc}} = -\frac{d_0}{2} + d_1 + d_2 + \frac{d_0}{2}$$
$$= \sin(60° - \phi) + \sin\phi$$
$$= \sin(60° + \phi)$$

$$\frac{\bar{v}_b}{V_{dc}} = -\frac{d_0}{2} - d_1 + d_2 + \frac{d_0}{2}$$
$$= -\sin(60° - \phi) + \sin\phi$$
$$= \sqrt{3} \cdot \sin(\phi - 30°)$$

$$\frac{\bar{v}_c}{V_{dc}} = -\frac{d_0}{2} - d_1 - d_2 + \frac{d_0}{2}$$
$$= -\sin(60° + \phi)$$
$$= -\bar{v}_a$$

DB-96

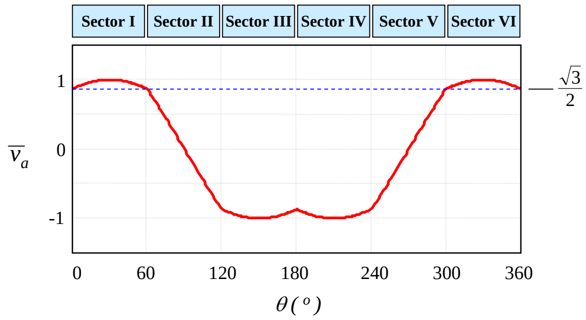
Average Values of Phase-to Neutral Voltage for SVM 2 (3Φ-C)

- In sector II,

$$\frac{\bar{v}_a}{V_{dc}} = -\frac{d_0}{2} + d_1 - d_2 + \frac{d_0}{2}$$
$$= \sin(60° - \phi) - \sin\phi$$
$$= \sqrt{3}\cdot\sin(30° - \phi)$$
$$= \sqrt{3}\cdot\sin(90° - \theta)$$

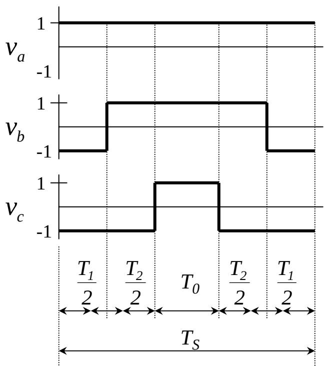
- In sector III,

$$\frac{\bar{v}_a}{V_{dc}} = -\frac{d_0}{2} - d_1 - d_2 + \frac{d_0}{2}$$
$$= -\sin(60° - \phi) - \sin\phi$$
$$= -\sin(60° + \phi)$$
$$= -\sin(\phi - 60°)$$



DB-97

Average Values of Phase-to Neutral Voltage for SVM 6 (2Φ-C-mL) in Sector I



- If i_a is the largest current,

$$\bar{v}_a = 1$$
$$\bar{v}_b = -d_1 + d_2 + d_0$$
$$= 1 - 2\cdot d_1$$
$$= 1 - 2\cdot\sin(60° - \phi)$$
$$\bar{v}_c = -d_1 - d_2 + d_0$$
$$= 1 - 2\cdot(d_1 + d_2)$$
$$= 1 - 2\cdot\sin(60° + \phi)$$

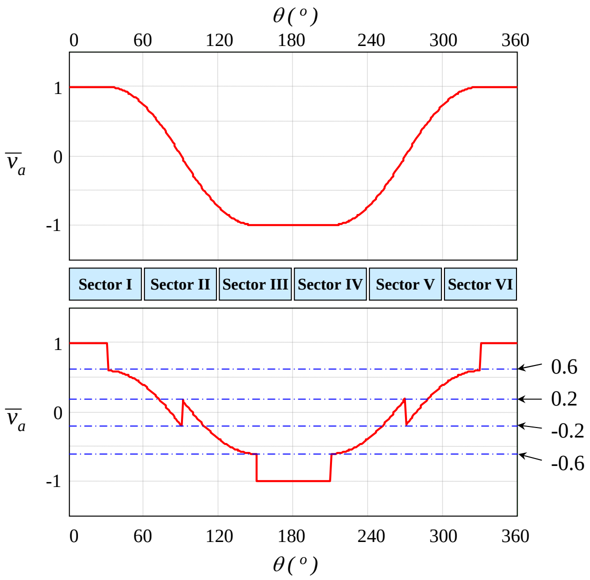
- If i_c is the largest current,

$$\bar{v}_a = -d_0 + d_1 + d_2$$
$$= 2\cdot\sin(60° + \phi) - 1$$
$$= 2\cdot\sin(60° + \theta) - 1$$

DB-98

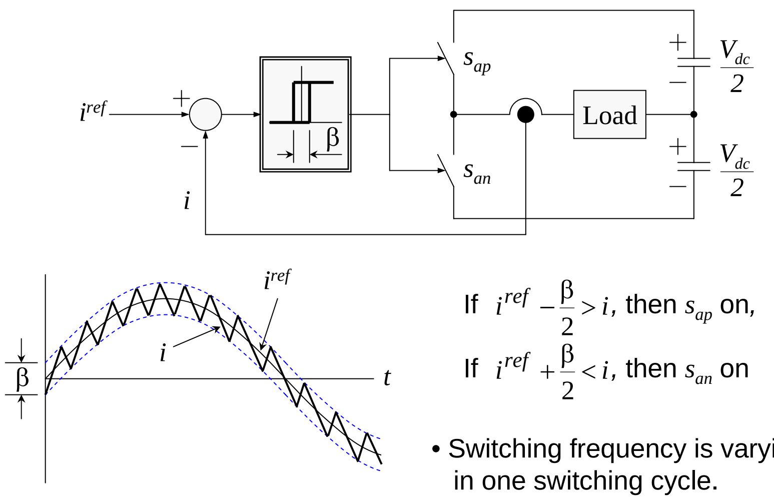
Average Values of Phase-to Neutral Voltage for SVM 6 (2Φ-C-mL)

- M = 1
- M = 0.8



DB-99

Hysteresis Current Control



If $i^{ref} - \frac{\beta}{2} > i$, then s_{ap} on,

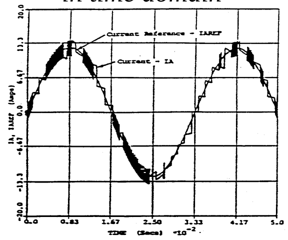
If $i^{ref} + \frac{\beta}{2} < i$, then s_{an} on

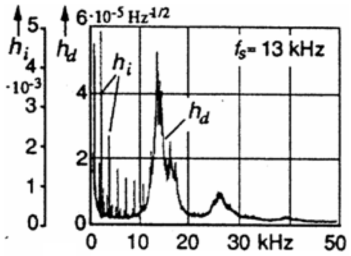
- Switching frequency is varying in one switching cycle.

DB-100

Pros and Cons of Hysteresis Current Control

in time domain





in frequency domain

- ***Pros:***

• Simple to implement
• Excellent dynamic performance

- ***Cons:***

• Strong harmonics lower than the switching frequency (Subharmonics)
• No intercommunication between the individual hysteresis controllers
  ➡ Increase the switching frequency at lower modulation index
• A tendency at lower speed to lock into limit cycle of high-frequency switching
• Not strictly limit the current error

DB-101

Outline

PECon 2008

1. **Introduction**
2. **Switching Modeling and PWM**
   - **Switching model of VSI & boost rectifier**
   - **Space vector modulation for VSI & boost rectifier**
   - **Other modulations for VSI & boost rectifier**
   - Switching model and modulation for CSI & buck rectifier
3. **Average Modeling**
4. **Small-Signal Modeling**
5. **Closed-Loop Control Design**

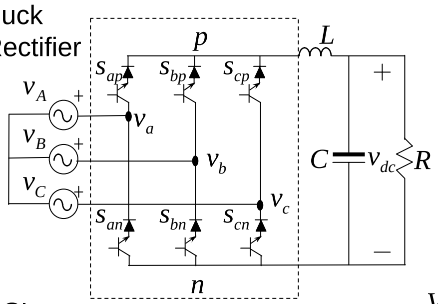
6. **More Complex Converters**

DB-102

Buck Rectifier / Current Source Inverter

– similar approach but different results –

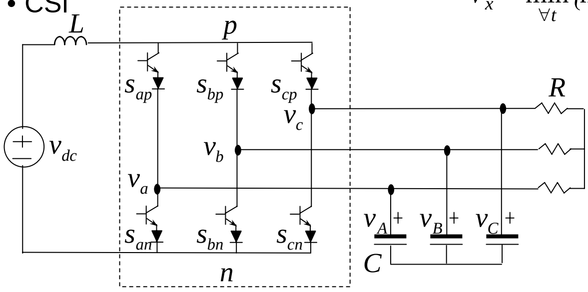
- Buck Rectifier

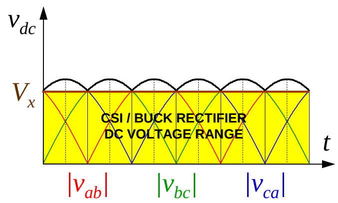


- Unidirectional DC current
- Bi-directional DC voltage
- $V_{dc} < V_x = \left(\frac{\sqrt{3}}{2} V_m\right)$

$$V_x = \min_{\forall t}\{\max\{|v_{ab}(t)|, |v_{bc}(t)|, |v_{ca}(t)|\}\}$$

- CSI

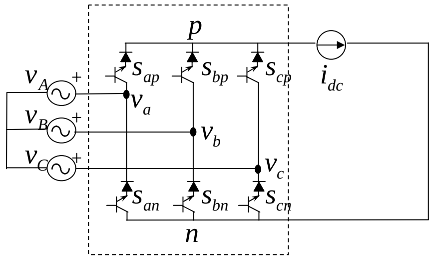




DB-103

DC-Current-Unidirectional Three-Phase Switching Network

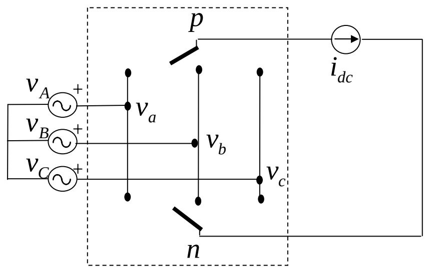
- Topology:
  - Three-phase terminals are voltage controlled
  - DC port is current controlled
  - Six current-unidirectional, voltage-bi-directional, switches



- Allowed switching combinations:

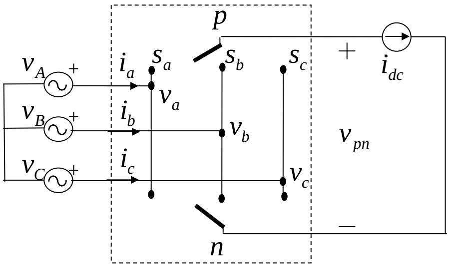
$$s_{ak} + s_{bk} + s_{ck} = 1; \qquad k \in \{p, n\}$$

- Two single-pole-triple-throw (SPTT) current-unidirectional switches



DB-104

Buck Rectifier / Current Source Inverter Switching Model



$$\vec{i}_{abc} = \vec{s}_{abc} \cdot i_{dc}$$

$$v_{pn} = \vec{s}_{abc}^{\,T} \cdot \vec{v}_{abc}$$

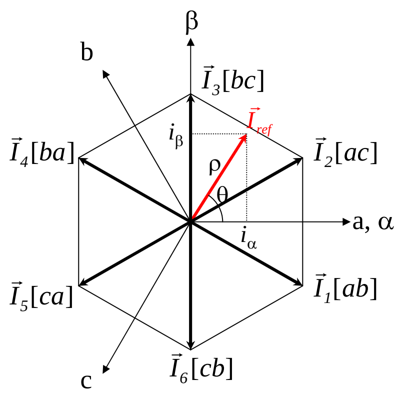
where:

$$\vec{i}_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad \vec{s}_{abc} = \begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix} = \begin{bmatrix} s_{ap} - s_{an} \\ s_{bp} - s_{bn} \\ s_{cp} - s_{cn} \end{bmatrix} \qquad \vec{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

DB-105

Switching State Vectors

	ρ	θ (°)
$\vec{I}_1[ab]$	$\sqrt{2} \cdot I_{dc}$	-30
$\vec{I}_2[ac]$		30
$\vec{I}_3[bc]$		90
$\vec{I}_4[ba]$		150
$\vec{I}_5[ca]$		-150
$\vec{I}_6[cb]$		-90
$\vec{I}_0[aa]$	0	0
$\vec{I}_0[bb]$		
$\vec{I}_0[cc]$		



$\vec{I}_0 = [aa] = [bb] = [cc]$ at center point

DB-106

Outline

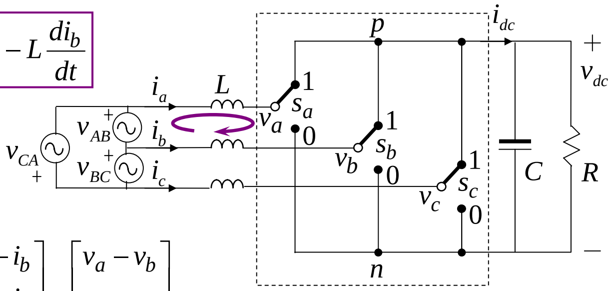
PECon 2008

1. Introduction
2. Switching Modeling and PWM
3. Average Modeling
    - Average model of boost rectifier
    - Average model of VSI
    - Average models in rotating coordinates
4. Small-Signal Modeling
5. Closed-Loop Control Design
6. More Complex Converters

DB-107

Boost Rectifier Switching Model State-Space Equations

$$v_{AB} = L\frac{di_a}{dt} + v_{ab} - L\frac{di_b}{dt}$$



$$\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} = L\frac{d}{dt}\begin{bmatrix} i_a - i_b \\ i_b - i_c \\ i_c - i_a \end{bmatrix} + \begin{bmatrix} v_a - v_b \\ v_b - v_c \\ v_c - v_a \end{bmatrix}$$

$$= 3L\frac{d}{dt}\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} + \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}$$

$$i_{dc} = C\frac{dv_{dc}}{dt} + \frac{v_{dc}}{R}$$

$\Rightarrow$

$$\frac{d}{dt}\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} = \frac{1}{3L}\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} - \frac{1}{3L}\begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix}$$

$$\frac{dv_{dc}}{dt} = \frac{1}{C}i_{dc} - \frac{v_{dc}}{RC}$$

DB-108

Boost Rectifier Switching Model State-Space Equations

$$\vec{v}_{L-L} = \begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} \quad \vec{v}_{l-l} = \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} \quad \vec{i}_{l-l} = \begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} \quad \vec{s}_{l-l} = \begin{bmatrix} s_{ab} \\ s_{bc} \\ s_{ca} \end{bmatrix}$$

$$\frac{d\vec{i}_{l-l}}{dt} = \frac{1}{3L}\vec{v}_{L-L} - \frac{1}{3L}\vec{v}_{l-l} \qquad \vec{v}_{l-l} = \vec{s}_{l-l} \cdot v_{dc}$$

$$\frac{dv_{dc}}{dt} = \frac{1}{C} i_{dc} - \frac{v_{dc}}{RC} \qquad i_{dc} = \vec{s}_{l-l}^{T} \cdot \vec{i}_{l-l}$$

$$\Rightarrow \quad \frac{d\vec{i}_{l-l}}{dt} = \frac{1}{3L}\vec{v}_{L-L} - \frac{1}{3L}\vec{s}_{l-l} \cdot v_{dc}$$

$$\frac{dv_{dc}}{dt} = \frac{1}{C}\vec{s}_{l-l}^{T} \cdot \vec{i}_{l-l} - \frac{v_{dc}}{RC}$$

DB-109

Average Circuit Modeling

Applying an average operator to switching model $\bar{x}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)d\tau$

- Switch duty cycle $d_{ap} = \bar{s}_{ap}(t) = \frac{1}{T}\int_{t-T}^{t} s_{ap}(\tau)d\tau$
- Phase-leg duty cycle $d_a = d_{ap} = 1 - d_{an}$
- Line-to-line duty cycle $d_{ab} = \bar{s}_{ab}(t) = \frac{1}{T}\int_{t-T}^{t} s_{ab}(\tau)d\tau = d_a - d_b$
- KVL and KCL $\Sigma\bar{v} = 0 \qquad \Sigma\bar{i} = 0$
- Linear components $\bar{v}_R = R\bar{i}_R \qquad \bar{v}_L = L\frac{d\bar{i}_L}{dt} \qquad \bar{i}_C = C\frac{d\bar{v}_C}{dt}$

DB-110

CPES

Averaging of Quadratic Terms

$$v_{ab} = s_{ab} \cdot v_{dc}$$

$$\bar{v}_{ab} = \frac{1}{T} \int_{t-T}^{t} s_{ab}(\tau) \cdot v_{dc}(\tau) d\tau \approx \bar{s}_{ab} \cdot \bar{v}_{dc} = d_{ab} \cdot \bar{v}_{dc}$$

if maximum-frequency components of $v_{dc}(t)$ are $<< 1/2T$.

$$\Rightarrow \quad \overline{\vec{s}_{l-l} \cdot v_{dc}} \approx \bar{\vec{s}}_{l-l} \cdot \bar{v}_{dc} = \vec{d}_{l-l} \cdot \bar{v}_{dc}$$

$$\overline{\vec{s}_{l-l}^{\,T} \cdot \vec{i}_{l-l}} \approx \overline{\vec{s}_{l-l}^{\,T}} \cdot \bar{\vec{i}}_{l-l} = \vec{d}_{l-l}^{\,T} \cdot \bar{\vec{i}}_{l-l}$$

DB-111

CPES

Development of Boost Rectifier Average Model

$$\begin{cases} \frac{d\vec{i}_{l-l}}{dt} = \frac{1}{3L}\vec{v}_{L-L} - \frac{1}{3L}\vec{s}_{l-l} \cdot v_{dc} \\ \frac{dv_{dc}}{dt} = \frac{1}{C}\vec{s}_{l-l}^{\,T} \cdot \vec{i}_{l-l} - \frac{v_{dc}}{RC} \end{cases}$$

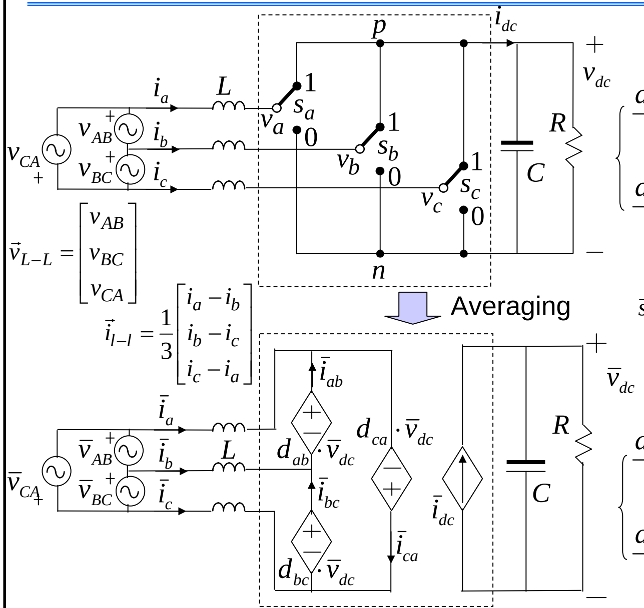
Applying average operator $\Rightarrow$

$$\begin{cases} \frac{1}{T}\int_{t-T}^{t} \frac{d\vec{i}_{l-l}(\tau)}{dt} d\tau = \frac{1}{T}\int_{t-T}^{t} \left(\frac{1}{3L}\vec{v}_{l-l}(\tau) - \frac{1}{3L}\vec{s}_{l-l}(\tau) \cdot v_{dc}(\tau)\right) d\tau \\ \frac{1}{T}\int_{t-T}^{t} \frac{dv_{dc}(\tau)}{dt} d\tau = \frac{1}{T}\int_{t-T}^{t} \left(\frac{1}{C}\vec{s}_{l-l}^{\,T}(\tau) \cdot \vec{i}_{l-l}(\tau) - \frac{v_{dc}(\tau)}{RC}\right) d\tau \end{cases}$$

$$\Rightarrow \quad \begin{cases} \frac{d\bar{\vec{i}}_{l-l}}{dt} = \frac{1}{3L}\bar{\vec{v}}_{L-L} - \frac{1}{3L}\vec{d}_{l-l} \cdot \bar{v}_{dc} \\ \frac{d\bar{v}_{dc}}{dt} = \frac{1}{C}\vec{d}_{l-l}^{\,T} \cdot \bar{\vec{i}}_{l-l} - \frac{\bar{v}_{dc}}{RC} \end{cases} \qquad \vec{d}_{l-l} = \begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix}$$

DB-112

Average Model of Three-Phase Boost Rectifier



$$\vec{v}_{L-L} = \begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} \qquad \vec{i}_{l-l} = \frac{1}{3}\begin{bmatrix} i_a - i_b \\ i_b - i_c \\ i_c - i_a \end{bmatrix}$$

Switching Model

$$\begin{cases} \dfrac{d\vec{i}_{l-l}}{dt} = \dfrac{1}{3L}\vec{v}_{L-L} - \dfrac{1}{3L}\vec{s}_{l-l}\cdot v_{dc} \\ \dfrac{dv_{dc}}{dt} = \dfrac{1}{C}\vec{s}_{l-l}^{\,T}\cdot\vec{i}_{l-l} - \dfrac{v_{dc}}{RC} \end{cases}$$

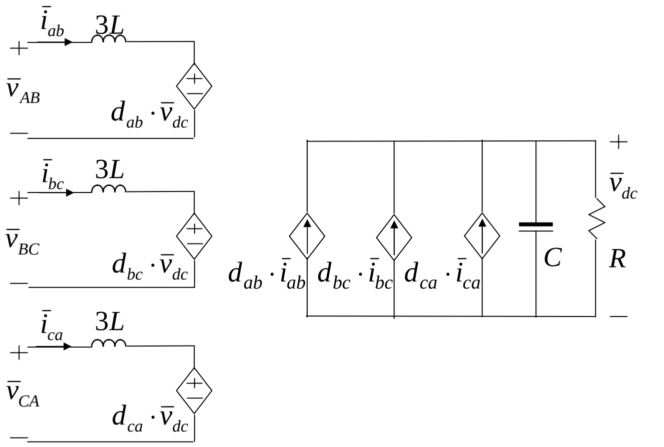
$$\vec{s}_{l-l} = \begin{bmatrix} s_a - s_b \\ s_b - s_c \\ s_c - s_a \end{bmatrix} \qquad \vec{d}_{l-l} = \begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix}$$

Average Model

$$\begin{cases} \dfrac{d\vec{\bar{i}}_{l-l}}{dt} = \dfrac{1}{3L}\vec{\bar{v}}_{L-L} - \dfrac{1}{3L}\vec{d}_{l-l}\cdot\bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\vec{d}_{l-l}^{\,T}\cdot\vec{\bar{i}}_{l-l} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

DB-113

Another Equivalent Circuit for Boost Rectifier Average Model



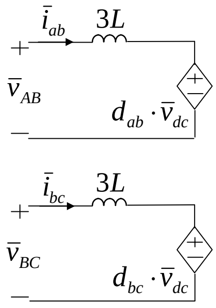
- Third order system due to degeneration

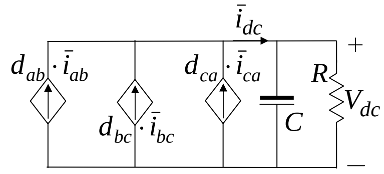
DB-114

Steady-State Operation under Balanced Sinusoidal Excitation

Given:

$$\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} = \begin{bmatrix} V_m \cos(\omega t) \\ V_m \cos(\omega t - 2\pi/3) \\ V_m \cos(\omega t + 2\pi/3) \end{bmatrix}$$





Goal:

$$\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} = \begin{bmatrix} I_m \cos(\omega t) \\ I_m \cos(\omega t - 2\pi/3) \\ I_m \cos(\omega t + 2\pi/3) \end{bmatrix}$$

Assume:

$$\begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix} = \begin{bmatrix} D_m \cos(\omega t - \theta) \\ D_m \cos(\omega t - 2\pi/3 - \theta) \\ D_m \cos(\omega t + 2\pi/3 - \theta) \end{bmatrix}$$

$$\bar{i}_{dc} = D_m I_m \left[\cos\omega t \cos(\omega t - \theta) + \cos(\omega t - \frac{2\pi}{3}) \cos(\omega t - \frac{2\pi}{3} - \theta) + \cos(\omega t + \frac{2\pi}{3}) \cos(\omega t + \frac{2\pi}{3} - \theta) \right]$$

$$\bar{i}_{dc} = \frac{D_m I_m}{2} \left[\cos\theta + \cos(2\omega t - \theta) + \cos\theta + \cos(2\omega t - \frac{4\pi}{3} - \theta) + \cos\theta + \cos(2\omega t + \frac{4\pi}{3} - \theta) \right]$$

$$\bar{i}_{dc} = \frac{3}{2} D_m I_m \cos\theta = \text{const.} \quad \Rightarrow \quad V_{dc} = \frac{3}{2} R D_m I_m \cos\theta = \text{const.}$$

DB-115

Steady-State Operation under Balanced Sinusoidal Excitation

$$\bar{v}_{dc} = \text{const.} = V_{dc}, \quad \Rightarrow \quad d_{ab} \cdot \bar{v}_{dc} = D_m \cdot V_{dc} \cos(\omega t - \theta)$$

Can use positive sequence phasors for steady state:

$$\mathbf{V}_{AB} = \frac{V_m}{\sqrt{2}} \qquad \mathbf{I}_{AB} = \frac{I_m}{\sqrt{2}}$$

$$\mathbf{D}_{ab} \cdot V_{dc} = \frac{D_m \cdot V_{dc}}{\sqrt{2}} \cos\theta - \mathrm{j} \cdot \frac{D_m \cdot V_{dc}}{\sqrt{2}} \sin\theta$$

$$\mathbf{V}_{AB} = \mathrm{j}3\omega L \cdot \mathbf{I}_{AB} + \mathbf{D}_{ab} \cdot V_{dc}$$

$$\theta = \frac{1}{2} \sin^{-1}\left(\frac{4\omega L}{R D_m^2} \right), \quad \boxed{V_{dc} = \frac{V_m}{D_m \cos\theta}}, \quad I_m = \frac{2}{3} \cdot \frac{V_m}{R D_m^2 \cos^2\theta}$$

$$d_{ab} = d_a - d_b \equiv d_{ap} - d_{bp}, \quad d_{ap}, d_{bp} \in [0,1] \quad \Rightarrow \quad -1 \le d_{ab} \le 1$$

$$D_m \le 1 \quad \Rightarrow \quad \boxed{V_{dc} \ge V_m}$$

DB-116

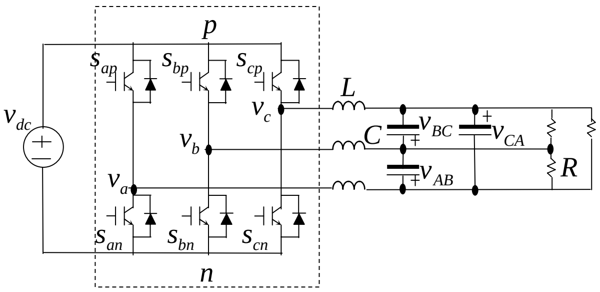
PECon 2008

Outline

1. **Introduction**
2. **Switching Modeling and PWM**
3. **Average Modeling**
   - **Average model of boost rectifier**
   - **Average model of VSI**
   - **Average models in rotating coordinates**
4. **Small-Signal Modeling**
5. **Closed-Loop Control Design**
6. **More Complex Converters**

DB-117

Development of VSI Average Model



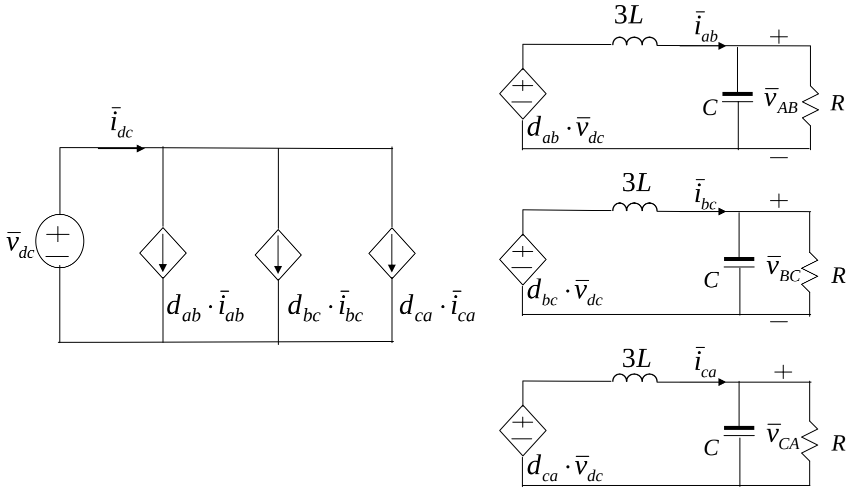
$$\begin{cases} \dfrac{d\vec{i}_{l-l}}{dt} = \dfrac{1}{3L}\vec{s}_{l-l}\cdot v_{dc} - \dfrac{1}{3L}\vec{v}_{L-L} \\ \dfrac{d\vec{v}_{L-L}}{dt} = \dfrac{1}{C}\vec{i}_{l-l} - \dfrac{1}{RC}\vec{v}_{L-L} \\ i_{dc} = \vec{s}^{\,T}_{l-l}\cdot\vec{i}_{l-l} \end{cases}$$

Averaging

$$\begin{cases} \dfrac{d\overline{\vec{i}}_{l-l}}{dt} = \dfrac{1}{3L}\vec{d}_{l-l}\cdot\overline{v}_{dc} - \dfrac{1}{3L}\overline{\vec{v}}_{L-L} \\ \dfrac{d\overline{\vec{v}}_{L-L}}{dt} = \dfrac{1}{C}\overline{\vec{i}}_{l-l} - \dfrac{1}{RC}\overline{\vec{v}}_{L-L} \\ i_{dc} = \vec{d}^{\,T}_{l-l}\cdot\overline{\vec{i}}_{l-l} \end{cases}$$

DB-118

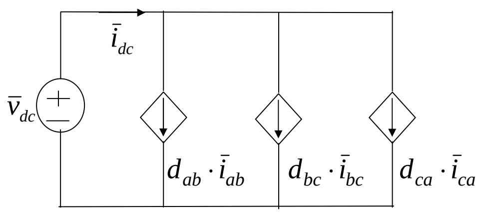
Equivalent Circuit for VSI Average Model

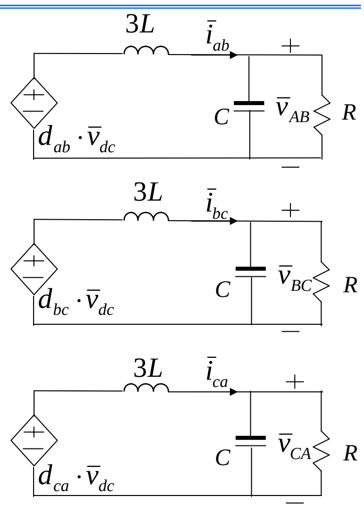


- Fourth order system due to degeneration

DB-119

Steady-State Operation under DC Input and Balanced Sinusoidal Duty-Cycles





For: $V_{dc} = \text{const.}$

and $\begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix} = \begin{bmatrix} D_m \cos(\omega t) \\ D_m \cos(\omega t - 2\pi/3) \\ D_m \cos(\omega t + 2\pi/3) \end{bmatrix} \Rightarrow$

$$\mathbf{V}_{AB} = \frac{\dfrac{R}{1+\mathrm{j}\omega CR}}{\mathrm{j}3\omega L + \dfrac{R}{1+\mathrm{j}\omega CR}} \cdot \mathbf{D}_{ab} \cdot V_{dc} \quad \Rightarrow \quad \boxed{|\mathbf{V}_{AB}| \le V_{dc}}$$

- Steady-state ac voltages and currents are sinusoidal if
- Difficult to define small-signal model. Operating point ?

DB-120

CPES

Outline

PECon 2008

1. Introduction
2. Switching Modeling and PWM
3. Average Modeling
   - Average model of boost rectifier
   - Average model of VSI
   - Average models in rotating coordinates
4. Small-Signal Modeling
5. Closed-Loop Control Design
6. More Complex Converters

DB-121

CPES

Coordinate Transformation

Choose

$$T_{dq0/abc} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\omega t & \cos(\omega t - \frac{2\pi}{3}) & \cos(\omega t + \frac{2\pi}{3}) \\ -\sin\omega t & -\sin(\omega t - \frac{2\pi}{3}) & -\sin(\omega t + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

where: $\omega=2\pi f$, f is ac line frequency
(source frequency for boost rectifier;
desired output frequency for VSI)

$$X_{dq0} = T \cdot X_{abc} \qquad X_{abc} = T^{-1} \cdot X_{dq0}$$

$$(T = T_{dq0/abc})$$

DB-122

Coordinate Transformation – Boost Rectifier –

$$\begin{cases} \dfrac{d\vec{\vec{i}}_{l-l}}{dt} = \dfrac{1}{3L}\vec{\vec{v}}_{L-L} - \dfrac{1}{3L}\vec{d}_{l-l}\cdot \bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\vec{d}^{T}_{l-l}\cdot\vec{\vec{i}}_{l-l} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

$X_{abc} = T^{-1}\cdot X_{dq0}$ ⇨

$$\begin{cases} \dfrac{d(T^{-1}\cdot\vec{\vec{i}}_{dq0})}{dt} = \dfrac{1}{3L}T^{-1}\cdot\vec{\vec{v}}_{dq0} - \dfrac{1}{3L}T^{-1}\cdot\vec{d}_{dq0}\cdot\bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\vec{d}^{T}_{l-l}\cdot T^{-1}T\cdot\vec{\vec{i}}_{l-l} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

⇨

$$\begin{cases} \dfrac{dT^{-1}}{dt}\cdot\vec{\vec{i}}_{dq0} + T^{-1}\cdot\dfrac{d\vec{\vec{i}}_{dq0}}{dt} = T^{-1}\cdot\dfrac{1}{3L}\vec{\vec{v}}_{dq0} - T^{-1}\cdot\dfrac{1}{3L}\vec{d}_{dq0}\cdot\bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\cdot(T\cdot\vec{d}_{l-l})^{T}\cdot T\cdot\vec{\vec{i}}_{l-l} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

$T\times$ ⇨

DB-123

Coordinate Transformation – Boost Rectifier –

⇨

$$\begin{cases} T\dfrac{dT^{-1}}{dt}\cdot\vec{\vec{i}}_{dq0} + \dfrac{d\vec{\vec{i}}_{dq0}}{dt} = \dfrac{1}{3L}\vec{\vec{v}}_{dq0} - \dfrac{1}{3L}\vec{d}_{dq0}\cdot\bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\cdot(T\cdot\vec{d}_{l-l})^{T}\cdot T\cdot\vec{\vec{i}}_{l-l} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

$T\cdot\vec{d}_{l-l} = \vec{d}_{dq0}$ $\qquad$ $T\cdot\vec{\vec{i}}_{l-l} = \vec{\vec{i}}_{dq0}$

⇨

$$\begin{cases} T\dfrac{dT^{-1}}{dt}\cdot\vec{\vec{i}}_{dq0} + \dfrac{d\vec{\vec{i}}_{dq0}}{dt} = \dfrac{1}{3L}\vec{\vec{v}}_{dq0} - \dfrac{1}{3L}\vec{d}_{dq0}\cdot\bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\vec{d}^{T}_{dq0}\cdot\vec{\vec{i}}_{dq0} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

DB-124

Coordinate Transformation

$$T\cdot\frac{dT^{-1}}{dt}=T\cdot\frac{dT^{T}}{dt}=T\cdot\frac{d}{dt}\left(\sqrt{\frac{2}{3}}\begin{bmatrix}\cos\omega t & -\sin\omega t & \frac{1}{\sqrt{2}}\\ \cos(\omega t-\frac{2\pi}{3}) & -\sin(\omega t-\frac{2\pi}{3}) & \frac{1}{\sqrt{2}}\\ \cos(\omega t+\frac{2\pi}{3}) & -\sin(\omega t+\frac{2\pi}{3}) & \frac{1}{\sqrt{2}}\end{bmatrix}\right)$$

$$=T\cdot\sqrt{\frac{2}{3}}\begin{bmatrix}-\omega\sin\omega t & -\omega\cos\omega t & 0\\ -\omega\sin(\omega t-\frac{2\pi}{3}) & -\omega\cos(\omega t-\frac{2\pi}{3}) & 0\\ -\omega\sin(\omega t+\frac{2\pi}{3}) & -\omega\cos(\omega t+\frac{2\pi}{3}) & 0\end{bmatrix}$$

$$=\sqrt{\frac{2}{3}}\begin{bmatrix}\cos\omega t & \cos(\omega t-\frac{2\pi}{3}) & \cos(\omega t+\frac{2\pi}{3})\\ -\sin\omega t & -\sin(\omega t-\frac{2\pi}{3}) & -\sin(\omega t+\frac{2\pi}{3})\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\cdot\sqrt{\frac{2}{3}}\begin{bmatrix}-\omega\sin\omega t & -\omega\cos\omega t & 0\\ -\omega\sin(\omega t-\frac{2\pi}{3}) & -\omega\cos(\omega t-\frac{2\pi}{3}) & 0\\ -\omega\sin(\omega t+\frac{2\pi}{3}) & -\omega\cos(\omega t+\frac{2\pi}{3}) & 0\end{bmatrix}$$

DB-125

Coordinate Transformation

Using the following trigonometric relationships

$$\cos^2 x+\cos^2(x-\frac{2\pi}{3})+\cos^2(x+\frac{2\pi}{3})=\frac{3}{2}$$

$$\sin^2 x+\sin^2(x-\frac{2\pi}{3})+\sin^2(x+\frac{2\pi}{3})=\frac{3}{2}$$

$$\sin x\cdot\cos x+\sin(x-\frac{2\pi}{3})\cdot\cos(x-\frac{2\pi}{3})+\sin(x+\frac{2\pi}{3})\cdot\cos(x+\frac{2\pi}{3})=0$$

$$\cos x+\cos(x-\frac{2\pi}{3})+\cos(x+\frac{2\pi}{3})=0$$

$$\sin x+\sin(x-\frac{2\pi}{3})+\sin(x+\frac{2\pi}{3})=0$$

DB-126

Coordinate Transformation – Boost Rectifier –

$$T \cdot \frac{dT^{-1}}{dt} = \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore:

$$\begin{cases} T\dfrac{dT^{-1}}{dt} \cdot \vec{\bar{i}}_{dq0} + \dfrac{d\vec{\bar{i}}_{dq0}}{dt} = \dfrac{1}{3L}\vec{\bar{v}}_{dq0} - \dfrac{1}{3L}\vec{d}_{dq0} \cdot \bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\vec{d}_{dq0}^{\,T} \cdot \vec{\bar{i}}_{dq0} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{d\vec{\bar{i}}_{dq0}}{dt} = \dfrac{1}{3L}\vec{\bar{v}}_{dq0} - \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \vec{\bar{i}}_{dq0} - \dfrac{1}{3L}\vec{d}_{dq0} \cdot \bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\vec{d}_{dq0}^{\,T} \cdot \vec{\bar{i}}_{dq0} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

DB-127

State-Space Equations – Boost Rectifier –

$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix} \bar{i}_{ab} \\ \bar{i}_{bc} \\ \bar{i}_{ca} \end{bmatrix} = \dfrac{1}{3L}\begin{bmatrix} \bar{v}_{AB} \\ \bar{v}_{BC} \\ \bar{v}_{CA} \end{bmatrix} - \dfrac{1}{3L}\begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix} \cdot \bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\begin{bmatrix} d_{ab} & d_{bc} & d_{ca} \end{bmatrix} \cdot \begin{bmatrix} \bar{i}_{ab} \\ \bar{i}_{bc} \\ \bar{i}_{ca} \end{bmatrix} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

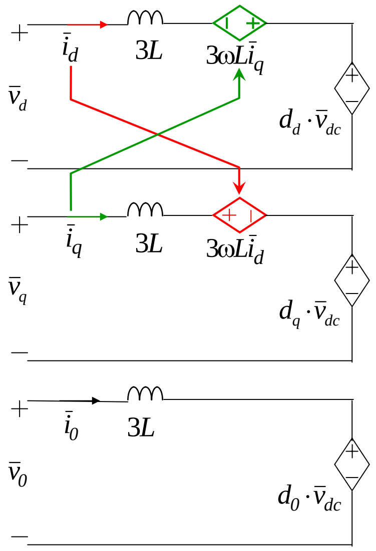
(abc coordinates)

$$\Rightarrow \begin{cases} \dfrac{d}{dt}\begin{bmatrix} \bar{i}_{d} \\ \bar{i}_{q} \\ \bar{i}_{0} \end{bmatrix} = \dfrac{1}{3L}\begin{bmatrix} \bar{v}_{d} \\ \bar{v}_{q} \\ \bar{v}_{0} \end{bmatrix} - \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \bar{i}_{d} \\ \bar{i}_{q} \\ \bar{i}_{0} \end{bmatrix} - \dfrac{1}{3L}\begin{bmatrix} d_{d} \\ d_{q} \\ d_{0} \end{bmatrix} \cdot \bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\begin{bmatrix} d_{d} & d_{q} & d_{0} \end{bmatrix} \cdot \begin{bmatrix} \bar{i}_{d} \\ \bar{i}_{q} \\ \bar{i}_{0} \end{bmatrix} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

(dq0 coordinates)

DB-128

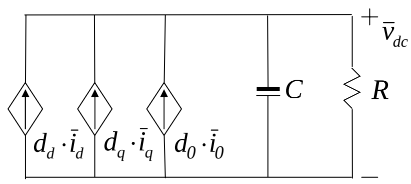
Equivalent Circuit in dq0 Coordinates – Boost Rectifier –



The cross-coupling terms,

$3\omega L\bar{i}_q$ and $3\omega L\bar{i}_d$,

in dc coordinates (dq0), account for the voltage drops across inductances at line frequency in ac coordinates, $j\cdot 3\omega L\bar{i}_{ab}$, $j\cdot 3\omega L\bar{i}_{bc}$, $j\cdot 3\omega L\bar{i}_{ca}$.



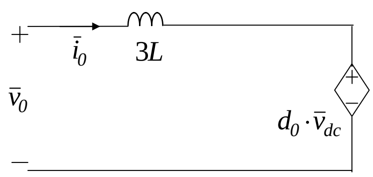
DB-129

0-Channel

Since

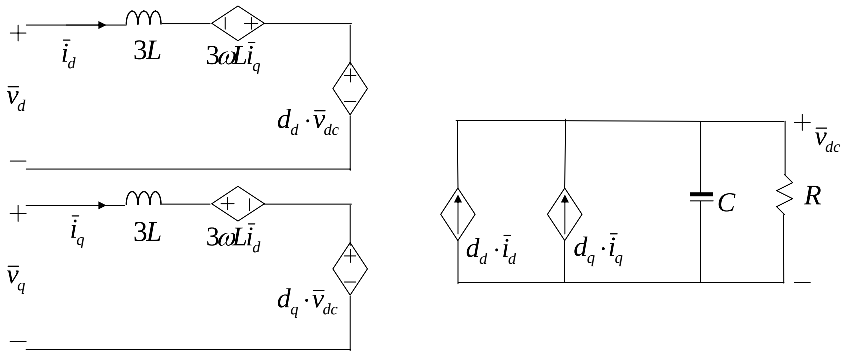
$$\begin{cases} \bar{v}_{AB} + \bar{v}_{BC} + \bar{v}_{CA} \equiv 0 \\ \bar{i}_{ab} + \bar{i}_{bc} + \bar{i}_{ca} \equiv 0 \\ d_{ab} + d_{bc} + d_{ca} \equiv 0 \end{cases} \Rightarrow \begin{matrix} \bar{v}_0 \equiv 0 \\ \bar{i}_0 \equiv 0 \\ d_0 \equiv 0 \end{matrix}$$

0-channel can be omitted



DB-130

Equivalent Circuit in dq0 Coordinates – Boost Rectifier –



$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix}\bar{i}_d \\ \bar{i}_q\end{bmatrix} = \dfrac{1}{3L}\begin{bmatrix}\bar{v}_d \\ \bar{v}_q\end{bmatrix} - \begin{bmatrix}0 & -\omega \\ \omega & 0\end{bmatrix}\cdot\begin{bmatrix}\bar{i}_d \\ \bar{i}_q\end{bmatrix} - \dfrac{1}{3L}\begin{bmatrix}d_d \\ d_q\end{bmatrix}\cdot\bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\begin{bmatrix}d_d & d_q\end{bmatrix}\cdot\begin{bmatrix}\bar{i}_d \\ \bar{i}_q\end{bmatrix} - \dfrac{\bar{v}_{dc}}{RC} \end{cases}$$

DB-131

State-Space Equations – Voltage Source Inverter –

$$\begin{cases} \dfrac{d\vec{\bar{i}}_{l-l}}{dt} = \dfrac{1}{3L}\vec{d}_{l-l}\cdot\bar{v}_{dc} - \dfrac{1}{3L}\vec{\bar{v}}_{L-L} \\ \dfrac{d\vec{\bar{v}}_{L-L}}{dt} = \dfrac{1}{C}\vec{\bar{i}}_{l-l} - \dfrac{1}{RC}\vec{\bar{v}}_{L-L} \\ \bar{i}_{dc} = \vec{d}^{T}_{l-l}\cdot\vec{\bar{i}}_{l-l} \end{cases}$$

(abc coordinates)

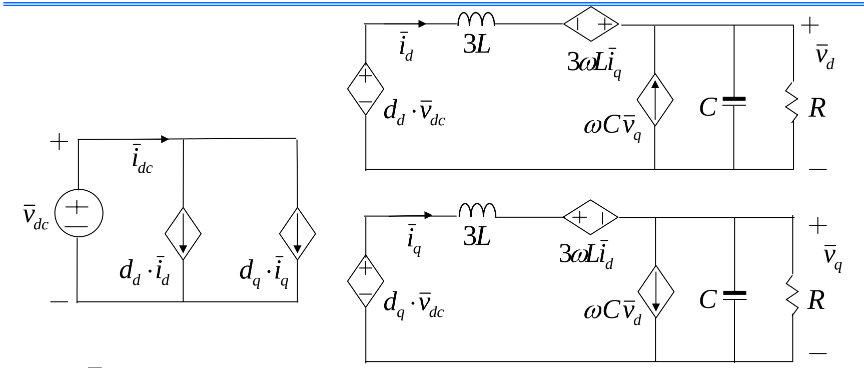
Transformation

$$\begin{cases} \dfrac{d\vec{\bar{i}}_{dq0}}{dt} = \dfrac{1}{3L}\vec{d}_{dq0}\cdot\bar{v}_{dc} - \begin{bmatrix}0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}\cdot\vec{\bar{i}}_{dq0} - \dfrac{1}{3L}\vec{\bar{v}}_{dq0} \\ \dfrac{d\vec{\bar{v}}_{dq0}}{dt} = \dfrac{1}{C}\vec{\bar{i}}_{dq0} - \begin{bmatrix}0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}\cdot\vec{\bar{v}}_{dq0} - \dfrac{1}{RC}\vec{\bar{v}}_{dq0} \\ \bar{i}_{dc} = \vec{d}^{T}_{dq0}\cdot\vec{\bar{i}}_{dq0} \end{cases}$$

(dq0 coordinates)

DB-132

Equivalent Circuit in dq0 Coordinates – Voltage Source Inverter –



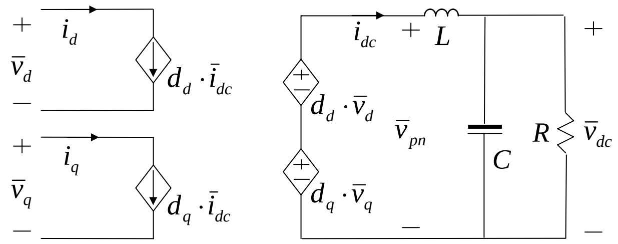
$$\begin{cases} \dfrac{d\bar{\vec{i}}_{dq}}{dt} = \dfrac{1}{3L}\vec{d}_{dq}\cdot\bar{v}_{dc} - \begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}\cdot\bar{\vec{i}}_{dq} - \dfrac{1}{3L}\bar{\vec{v}}_{dq} \\ \dfrac{d\bar{\vec{v}}_{dq}}{dt} = \dfrac{1}{C}\bar{\vec{i}}_{dq} - \begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}\cdot\bar{\vec{v}}_{dq} - \dfrac{1}{RC}\bar{\vec{v}}_{dq} \\ \bar{i}_{dc} = \vec{d}_{dq}^{\,T}\cdot\bar{\vec{i}}_{dq} \end{cases}$$

DB-133

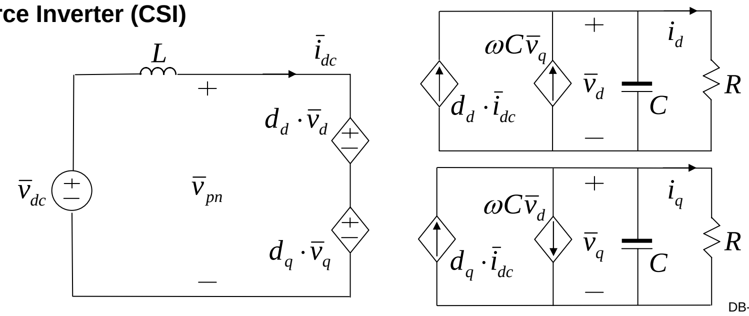
Buck Rectifier / CSI dq0 Model

– similar approach but different results –

Buck Rectifier



Current Source Inverter (CSI)



DB-134

Outline

PECon 2008

1. Introduction
2. Switching Modeling and PWM
3. Average Modeling
4. Small-Signal Modeling
   - Small-signal model of boost rectifier
   - Small-signal model of VSI
   - Three-phase modulator modeling
5. Closed-Loop Control Design
6. More Complex Converters

DB-135

Linearization

Autonomous dynamic system: $\frac{d\vec{x}}{dt} = \vec{f}(\vec{x},\vec{u})$

If $\vec{f}$ is analytic it can be expressed as Taylor series:

$$\vec{f}(\vec{x},\vec{u}) = \vec{f}(\vec{x}_0,\vec{u}_0) + \frac{\partial\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{x}}\cdot(\vec{x}-\vec{x}_0) + \frac{\partial\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{u}}\cdot(\vec{u}-\vec{u}_0) +$$

$$+\frac{1}{2!}\left[\frac{\partial^2\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{x}^2}(\vec{x}-\vec{x}_0)^2 + \frac{\partial^2\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{x}\partial\vec{u}}(\vec{x}-\vec{x}_0)(\vec{u}-\vec{u}_0) + \frac{\partial^2\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{u}^2}(\vec{u}-\vec{u}_0)^2\right]$$

$$+ \ \ldots$$

Retaining the first 3 terms results in linear approximation of $\vec{f}$:

$$\vec{f}(\vec{x},\vec{u}) \cong \vec{f}(\vec{x}_0,\vec{u}_0) + \frac{\partial\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{x}}\cdot(\vec{x}-\vec{x}_0) + \frac{\partial\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{u}}\cdot(\vec{u}-\vec{u}_0)$$

But the dynamic system is **NOT** linear because:

$$\underbrace{\frac{d\vec{x}}{dt}}_{\dot{\vec{x}}} \cong \underbrace{\frac{\partial\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{x}}}_{\mathbf{A}}\cdot\underbrace{\vec{x}}_{\vec{x}} + \underbrace{\frac{\partial\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{u}}}_{\mathbf{B}}\cdot\underbrace{\vec{u}}_{\vec{u}} + \underbrace{\vec{f}(\vec{x}_0,\vec{u}_0) - \frac{\partial\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{x}}\cdot\vec{x}_0 - \frac{\partial\vec{f}(\vec{x}_0,\vec{u}_0)}{\partial\vec{u}}\cdot\vec{u}_0}_{\vec{g}\neq 0}$$

DB-136

Linearization

$$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x},\vec{u}) \cong \vec{f}(\vec{x}_0,\vec{u}_0) + \frac{\partial \vec{f}(\vec{x}_0,\vec{u}_0)}{\partial \vec{x}}\cdot(\vec{x}-\vec{x}_0) + \frac{\partial \vec{f}(\vec{x}_0,\vec{u}_0)}{\partial \vec{u}}\cdot(\vec{u}-\vec{u}_0)$$

If $(\vec{x}_0,\vec{u}_0)$ is an equilibrium point $(\vec{X},\vec{U})$, and $(\tilde{\vec{x}},\tilde{\vec{u}})$ is perturbation around it:

⇨ $\vec{f}(\vec{X},\vec{U}) \equiv 0 \qquad \vec{x} = \vec{X} + \tilde{\vec{x}} \qquad \vec{u} = \vec{U} + \tilde{\vec{u}}$

⇨ $\frac{d\vec{X}}{dt} = 0$ ⇨ $\frac{d\vec{x}}{dt} = \frac{d\vec{X}}{dt} + \frac{d\tilde{\vec{x}}}{dt} = \frac{d\tilde{\vec{x}}}{dt}$

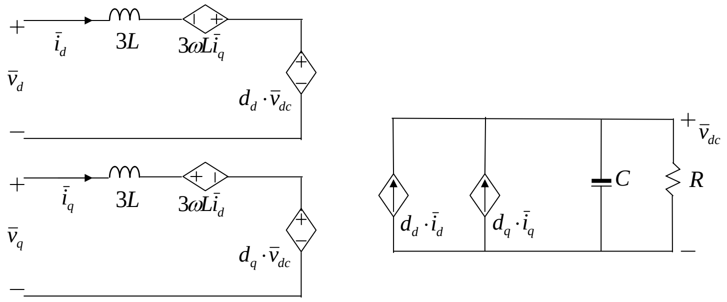
⇨ $$\frac{d\tilde{\vec{x}}}{dt} \cong \left.\frac{\partial \vec{f}(\vec{x},\vec{u})}{\partial \vec{x}}\right|_{(\vec{X},\vec{U})} \cdot \tilde{\vec{x}} + \left.\frac{\partial \vec{f}(\vec{x},\vec{u})}{\partial \vec{u}}\right|_{(\vec{X},\vec{U})} \cdot \tilde{\vec{u}}$$

⇨ $$\dot{\tilde{\vec{x}}} = \mathbf{A}\cdot\tilde{\vec{x}} + \mathbf{B}\cdot\tilde{\vec{u}}$$

$$\mathbf{A} = \left.\frac{\partial \vec{f}(\vec{x},\vec{u})}{\partial \vec{x}}\right|_{(\vec{X},\vec{U})} \qquad \mathbf{B} = \left.\frac{\partial \vec{f}(\vec{x},\vec{u})}{\partial \vec{u}}\right|_{(\vec{X},\vec{U})}$$

DB-137

Average Large-Signal Model Boost Rectifier



A steady-state operating point:

$V_d = \sqrt{\frac{3}{2}}\cdot V_m$ (V_m: Max line-to-line voltage) $\qquad D_d = \frac{V_d}{V_{dc}} \qquad I_d = \frac{V_{dc}}{R\cdot D_d}$

$V_q = 0 \qquad D_q = -\frac{3\omega L I_d}{V_{dc}} \qquad I_q = 0$

DB-138

Linearization – Boost Rectifier

$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix}\bar{i}_d\\ \bar{i}_q\end{bmatrix} = \dfrac{1}{3L}\begin{bmatrix}\bar{v}_d\\ \bar{v}_q\end{bmatrix} - \begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}\cdot\begin{bmatrix}\bar{i}_d\\ \bar{i}_q\end{bmatrix} - \dfrac{1}{3L}\begin{bmatrix}d_d\\ d_q\end{bmatrix}\cdot\bar{v}_{dc} \\ \dfrac{d\bar{v}_{dc}}{dt} = \dfrac{1}{C}\begin{bmatrix}d_d & d_q\end{bmatrix}\cdot\begin{bmatrix}\bar{i}_d\\ \bar{i}_q\end{bmatrix} - \dfrac{\bar{v}_{dc}}{RC}\end{cases}$$

Linearization →

$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix}\tilde{i}_d\\ \tilde{i}_q\end{bmatrix} = \dfrac{1}{3L}\begin{bmatrix}\tilde{v}_d\\ \tilde{v}_q\end{bmatrix} - \begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}\cdot\begin{bmatrix}\tilde{i}_d\\ \tilde{i}_q\end{bmatrix} - \dfrac{1}{3L}\begin{bmatrix}\tilde{d}_d\\ \tilde{d}_q\end{bmatrix}\cdot V_{dc} - \dfrac{1}{3L}\begin{bmatrix}D_d\\ D_q\end{bmatrix}\cdot\tilde{v}_{dc} \\ \dfrac{d\tilde{v}_{dc}}{dt} = \dfrac{1}{C}\begin{bmatrix}\tilde{d}_d & \tilde{d}_q\end{bmatrix}\cdot\begin{bmatrix}I_d\\ I_q\end{bmatrix} + \dfrac{1}{C}\begin{bmatrix}D_d & D_q\end{bmatrix}\cdot\begin{bmatrix}\tilde{i}_d\\ \tilde{i}_q\end{bmatrix} - \dfrac{\tilde{v}_{dc}}{RC}\end{cases}$$

DB-139

Small-Signal Model – Boost Rectifier

$$\underbrace{\underbrace{\frac{d}{dt}\begin{bmatrix}\tilde{i}_d\\ \tilde{i}_q\\ \tilde{v}_{dc}\end{bmatrix}}_{\dot{\vec{x}}} = \underbrace{\begin{bmatrix}0 & \omega & -\frac{D_d}{3L}\\ -\omega & 0 & -\frac{D_q}{3L}\\ \frac{D_d}{C} & \frac{D_q}{C} & -\frac{1}{RC}\end{bmatrix}}_{\mathbf{A}}\cdot\underbrace{\begin{bmatrix}\tilde{i}_d\\ \tilde{i}_q\\ \tilde{v}_{dc}\end{bmatrix}}_{\vec{x}}}_{\text{Intrinsic System Dynamics}} + \underbrace{\underbrace{\begin{bmatrix}-\frac{V_{dc}}{3L} & 0\\ 0 & -\frac{V_{dc}}{3L}\\ \frac{I_d}{C} & \frac{I_q}{C}\end{bmatrix}}_{\mathbf{B}}\cdot\underbrace{\begin{bmatrix}\tilde{d}_d\\ \tilde{d}_q\end{bmatrix}}_{\vec{u}}}_{\text{Control Input}} + \underbrace{\underbrace{\begin{bmatrix}\frac{1}{3L} & 0\\ 0 & \frac{1}{3L}\\ 0 & 0\end{bmatrix}}_{\mathbf{D}}\cdot\underbrace{\begin{bmatrix}\tilde{v}_d\\ \tilde{v}_q\end{bmatrix}}_{\vec{v}}}_{\text{Disturbance Input}}$$

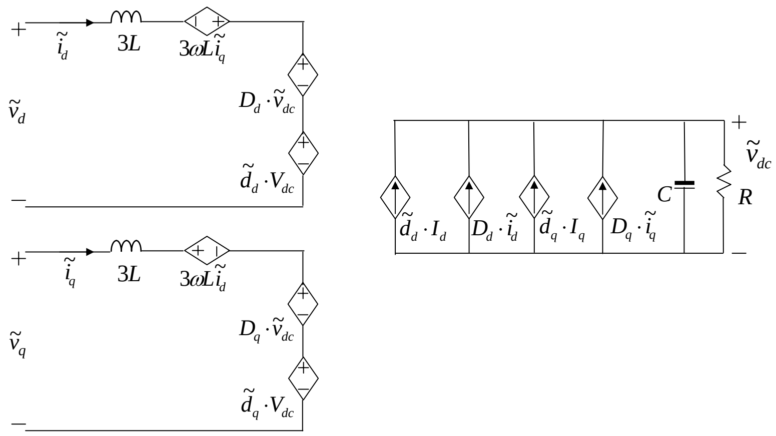
Intrinsic
System Dynamics

Control
Input

Disturbance
Input

DB-140

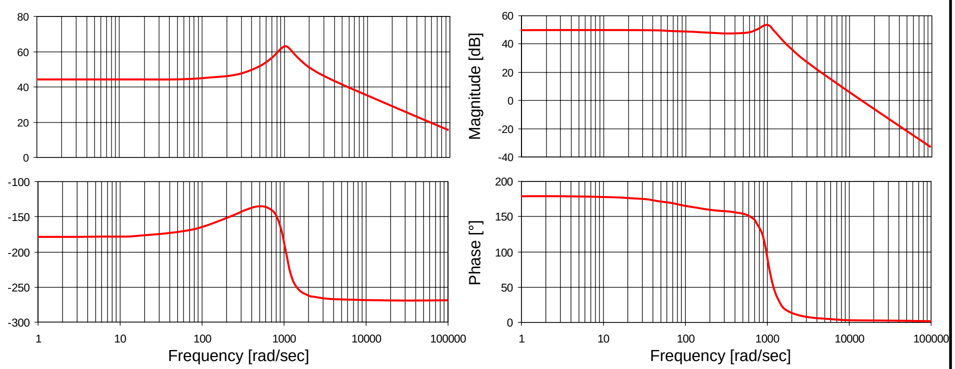
Small-Signal Model – Boost Rectifier



DB-141

Open-Loop Transfer Functions

$$\frac{\tilde{i}_d}{\tilde{d}_d} = \frac{K_{iddd} \cdot (s + z_{iddd\,1}) \cdot (s + z_{iddd\,2})}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)}$$



$$\frac{\tilde{i}_d}{\tilde{d}_q} = \frac{K_{iddq} \cdot (s + z_{iddq})}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)}$$

DB-142

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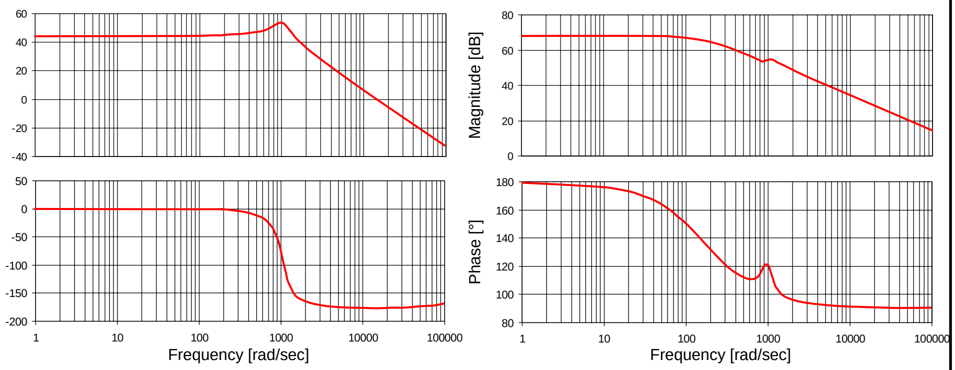
Open-Loop Transfer Functions

$$\frac{\tilde{i}_q}{\tilde{d}_d} = \frac{K_{iqdd} \cdot (s + z_{iqdd\,1}) \cdot (s + z_{iqdd\,2})}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)}$$



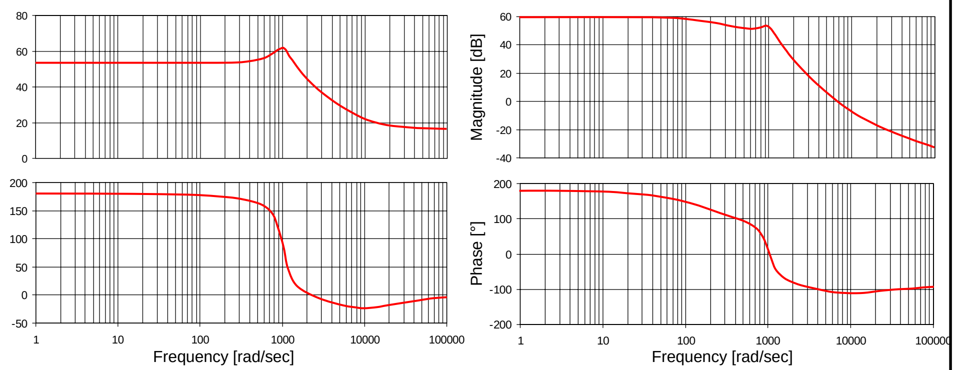
$$\frac{\tilde{i}_q}{\tilde{d}_q} = \frac{K_{iqdq} \cdot (s + z_{iqdq}) \cdot (s + z_{iqdq}^*)}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)}$$

DB-143

CPES

Open-Loop Transfer Functions

$$\frac{\tilde{v}_{dc}}{\tilde{d}_d} = \frac{K_{vdcdd} \cdot (s + z_{vdcdd\,1}) \cdot (s + z_{vdcdd\,2}) \cdot (s - z_{RHP})}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)}$$



$$\frac{\tilde{v}_{dc}}{\tilde{d}_q} = \frac{K_{vdcdq} \cdot (s + z_{vdcdq\,1}) \cdot (s - z_{RHP})}{(s + p_1) \cdot (s + p_2) \cdot (s + p_2^*)}$$

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CPES PECon 2008

Outline

1. **Introduction**
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   - **Small-signal model of VSI**
   - **Three-phase modulator modeling**
5. **Closed-Loop Control Design**
6. **More Complex Converters**

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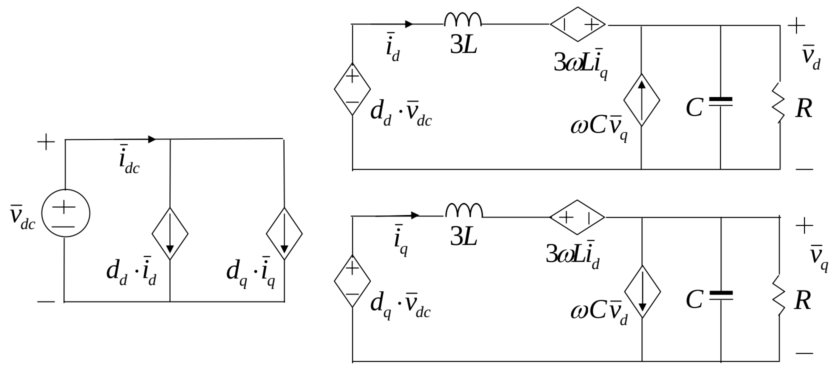
Average Large-Signal Model – VSI



A steady-state operating point:

$$I_d = \frac{V_d}{R} - \omega C V_q \qquad D_d = \frac{V_d - 3\omega L I_q}{V_{dc}}$$

$$I_q = \frac{V_q}{R} + \omega C V_d \qquad D_q = \frac{V_q + 3\omega L I_d}{V_{dc}}$$

DB-146

Linearization – VSI

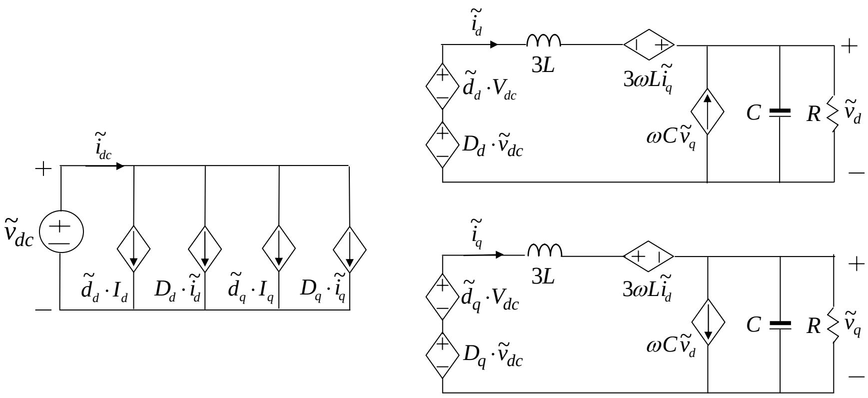
$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix}\bar{i}_d \\ \bar{i}_q\end{bmatrix} = \dfrac{1}{3L}\begin{bmatrix}d_d \\ d_q\end{bmatrix}\cdot \bar{v}_{dc} - \begin{bmatrix}0 & -\omega \\ \omega & 0\end{bmatrix}\cdot\begin{bmatrix}\bar{i}_d \\ \bar{i}_q\end{bmatrix} - \dfrac{1}{3L}\begin{bmatrix}\bar{v}_d \\ \bar{v}_q\end{bmatrix} \\ \dfrac{d}{dt}\begin{bmatrix}\bar{v}_d \\ \bar{v}_q\end{bmatrix} = \dfrac{1}{C}\begin{bmatrix}\bar{i}_d \\ \bar{i}_q\end{bmatrix} - \begin{bmatrix}0 & -\omega \\ \omega & 0\end{bmatrix}\cdot\begin{bmatrix}\bar{v}_d \\ \bar{v}_q\end{bmatrix} - \dfrac{1}{RC}\begin{bmatrix}\bar{v}_d \\ \bar{v}_q\end{bmatrix} \\ i_{dc} = \begin{bmatrix}d_d & d_q\end{bmatrix}\cdot\begin{bmatrix}\bar{i}_d \\ \bar{i}_q\end{bmatrix} \end{cases}$$

Linearization

$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix}\tilde{i}_d \\ \tilde{i}_q\end{bmatrix} = \dfrac{1}{3L}\begin{bmatrix}\tilde{d}_d \\ \tilde{d}_q\end{bmatrix}\cdot V_{dc} + \dfrac{1}{3L}\begin{bmatrix}D_d \\ D_q\end{bmatrix}\cdot \tilde{v}_{dc} - \begin{bmatrix}0 & -\omega \\ \omega & 0\end{bmatrix}\cdot\begin{bmatrix}\tilde{i}_d \\ \tilde{i}_q\end{bmatrix} - \dfrac{1}{3L}\begin{bmatrix}\tilde{v}_d \\ \tilde{v}_q\end{bmatrix} \\ \dfrac{d}{dt}\begin{bmatrix}\tilde{v}_d \\ \tilde{v}_q\end{bmatrix} = \dfrac{1}{C}\begin{bmatrix}\tilde{i}_d \\ \tilde{i}_q\end{bmatrix} - \begin{bmatrix}0 & -\omega \\ \omega & 0\end{bmatrix}\cdot\begin{bmatrix}\tilde{v}_d \\ \tilde{v}_q\end{bmatrix} - \dfrac{1}{RC}\begin{bmatrix}\tilde{v}_d \\ \tilde{v}_q\end{bmatrix} \\ \tilde{i}_{dc} = \begin{bmatrix}D_d & D_q\end{bmatrix}\cdot\begin{bmatrix}\tilde{i}_d \\ \tilde{i}_q\end{bmatrix} + \begin{bmatrix}\tilde{d}_d & \tilde{d}_q\end{bmatrix}\cdot\begin{bmatrix}I_d \\ I_q\end{bmatrix} \end{cases}$$

DB-147

Small-Signal Circuit Model – VSI



DB-148

Small-Signal State-Space Model – VSI

$$\underbrace{\frac{d}{dt}\begin{bmatrix}\tilde{i}_d\\ \tilde{i}_q\\ \tilde{v}_d\\ \tilde{v}_q\end{bmatrix}}_{\dot{\vec{x}}} = \underbrace{\begin{bmatrix}0 & \omega & -\frac{1}{3L} & 0\\ -\omega & 0 & 0 & -\frac{1}{3L}\\ \frac{1}{C} & 0 & -\frac{1}{RC} & \omega\\ 0 & \frac{1}{C} & -\omega & -\frac{1}{RC}\end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix}\tilde{i}_d\\ \tilde{i}_q\\ \tilde{v}_d\\ \tilde{v}_q\end{bmatrix}}_{\vec{x}} + \underbrace{\begin{bmatrix}\frac{V_{dc}}{3L} & 0\\ 0 & \frac{V_{dc}}{3L}\\ 0 & 0\\ 0 & 0\end{bmatrix}}_{\mathbf{B}} \cdot \underbrace{\begin{bmatrix}\tilde{d}_d\\ \tilde{d}_q\end{bmatrix}}_{\vec{u}} + \underbrace{\begin{bmatrix}\frac{D_d}{3L}\\ \frac{D_q}{3L}\\ 0\\ 0\end{bmatrix}}_{\mathbf{D}} \cdot \underbrace{\tilde{v}_{dc}}_{\vec{v}}$$

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Outline

PECon 2008

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   - **Three-phase modulator modeling**
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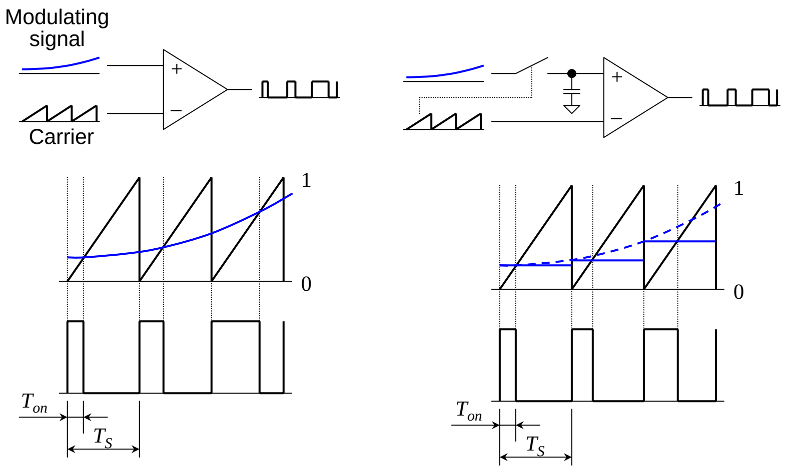
DB-150

Natural and Uniform Sampling

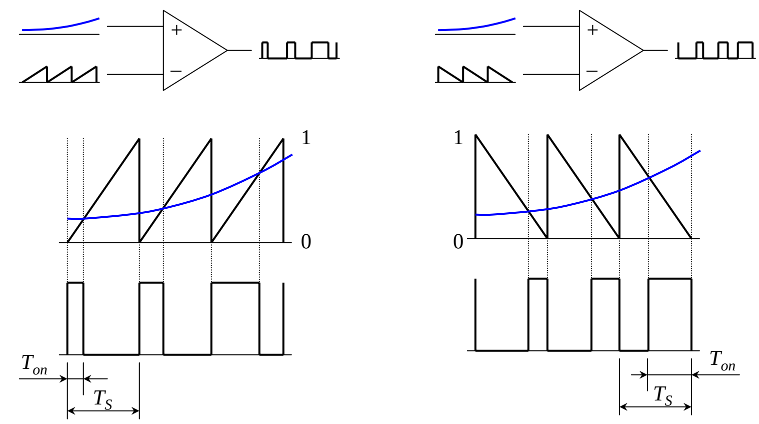
Natural Sampling | **Uniform Sampling**



DB-151

Trailing- and Leading-Edge Modulation

Trailing-Edge Modulation | **Leading-Edge Modulation**



☞ Both are the natural sampling

DB-152

Review of Modulator Modeling for DC-DC Converters

- Natural sampling
  - Unity gain
  - No delay
- Uniform sampling
  1. Trailing-edge modulation
     - Unity gain
     - Phase delay at a modulation frequency

$$D \cdot T_s \cdot \omega_m$$

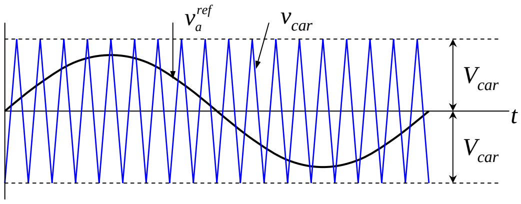
  2. Leading-edge modulation
     - Unity gain
     - Phase delay at a modulation frequency

$$(1 - D) \cdot T_s \cdot \omega_m$$

DB-153

Three-Phase Modulator Modeling

- All naturally sampled modulators can be modeled by the constant gain term
  - Modulators associated with analog controllers



$$F_m = \frac{1}{V_{car}}$$

- Small-signal models **have to be derived for uniformly sampled** three-phase modulators
  - Modulators associated with digital controllers

DB-154

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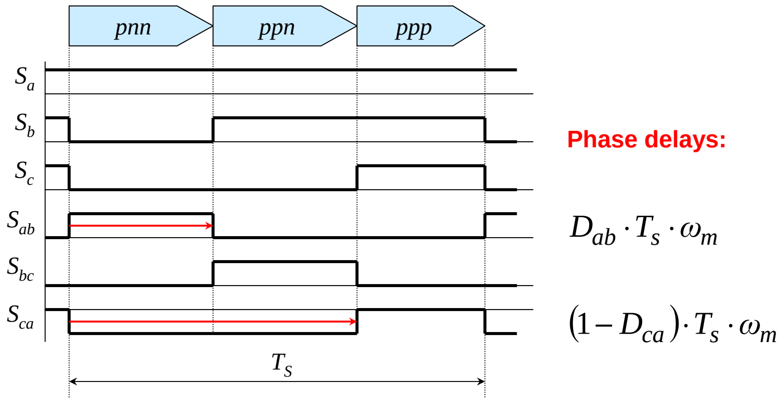
Example – for Boost Rectifier and VSI

- Trailing-edge modulation (**2Φ-RA**)



- Two out of three switching functions always have pulses synchronized to the beginning of the switching period

DB-155

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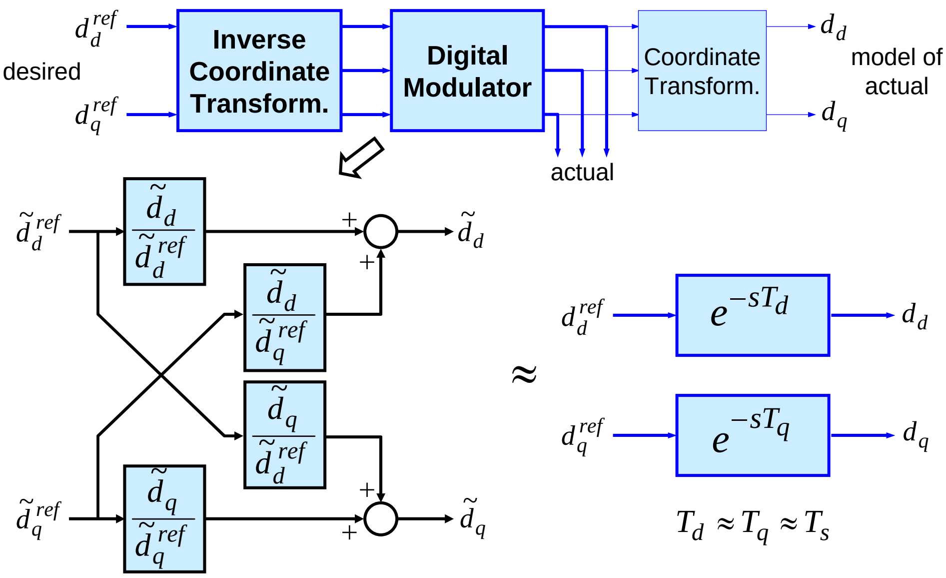
Approximate Average Model of Digital Modulator



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   - Limiting
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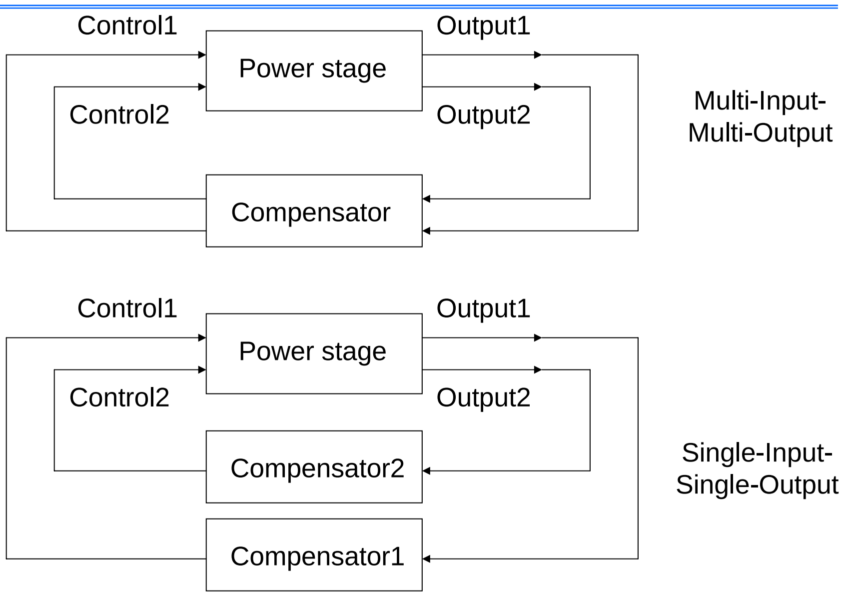
Classical Linear Control Design Approach



DB-158

Closed-Loop Control Design

- Based on small-signal models

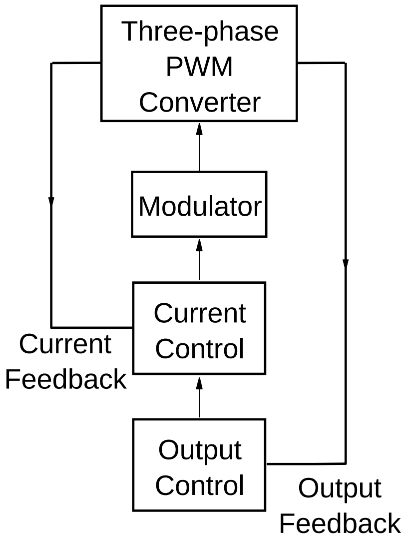
Advantage
- Classical control design methods can be used (Bode plots, root loci, etc.)

Disadvantages
- Design is valid only at a certain operating point
- The approach does not guarantee large-signal stability

DB-159

Control Structure: Cascade Control

1. Inner current control
   a. Bang-bang current control (and PWM) in stationary coordinates
   b. Current control in stationary coordinates with two independent or three dependent current controllers (P, PI, or resonant regulators)
   c. Current control in rotating coordinates with two independent current controllers (P or PI regulators)
2. Outer output voltage or torque / flux / speed / position control (PI regulators)



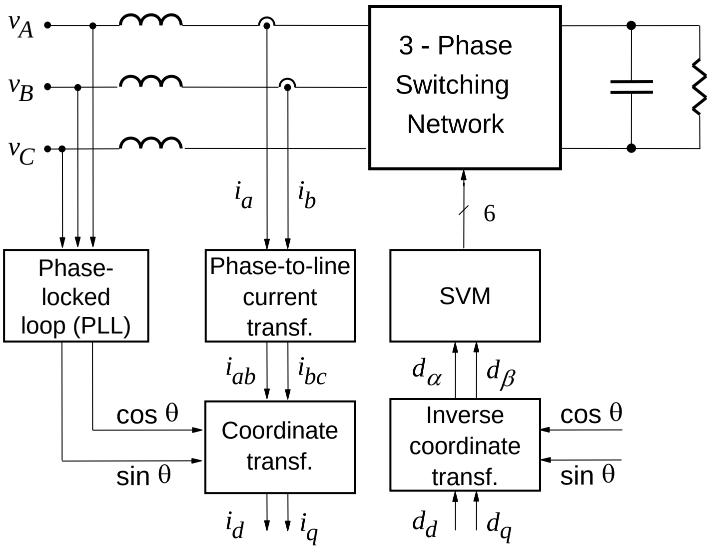
DB-160

Control Realization

1. Completely analog control
   Complete analog control is rarely used because of hardware complexity
2. Combination of analog and digital control
   Analog current control in stationary and digital output voltage or speed and flux control
3. Completely digital control
   Used in most new designs

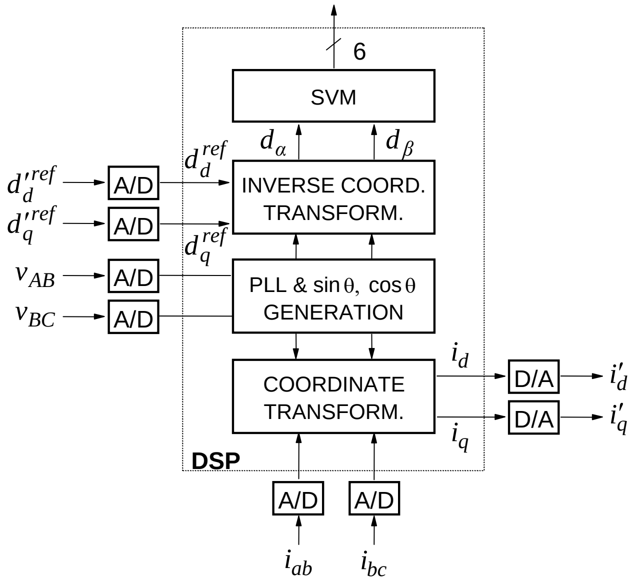
DB-161

Open-loop Boost Rectifier Control



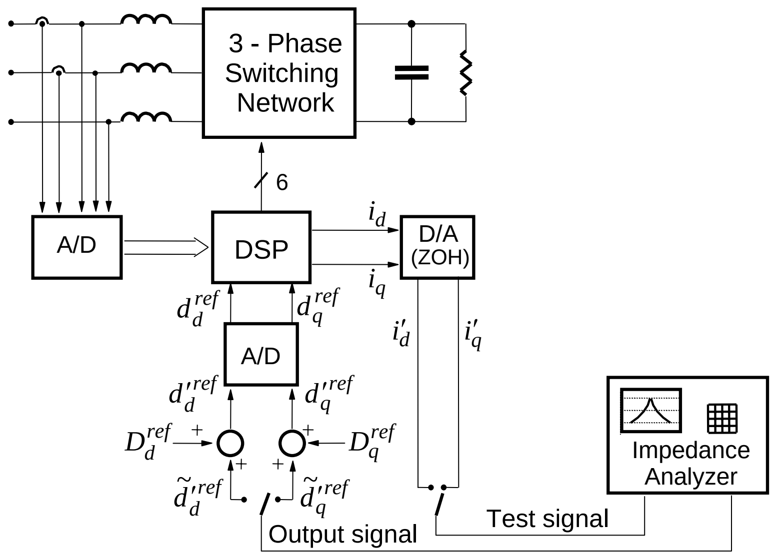
DB-162

Open-loop Boost Rectifier Control
Digital Signal Processor (DSP) Implementation



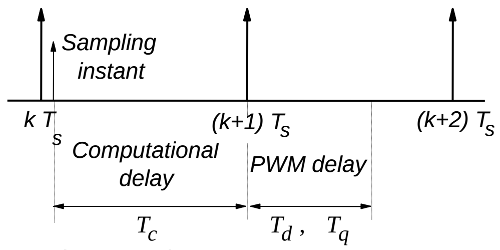
DB-163

Transfer Function Measurement Set-up

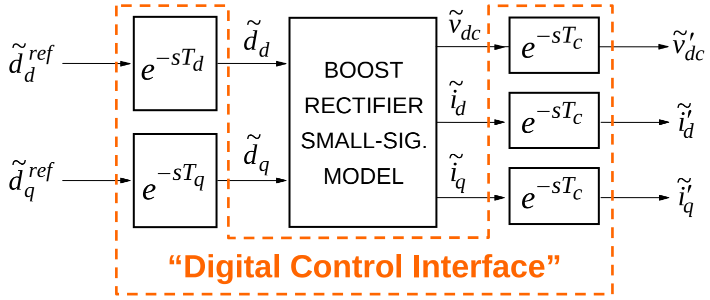


DB-164

Digital Delay

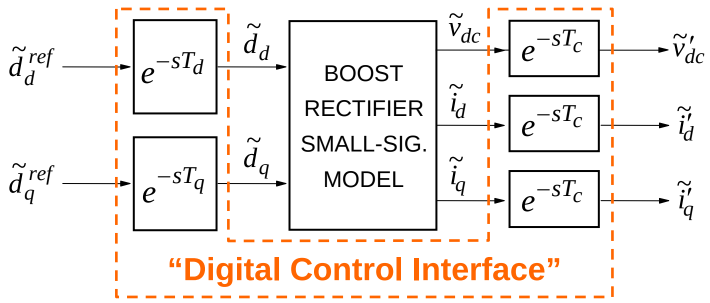


- Modified small-signal model



DB-165

Digital Delay



- Simplified approximations in continuous time domain:

$$e^{-sT_{del}} \approx \frac{1 - \frac{sT_{del}}{2} + \frac{(sT_{del})^2}{12}}{1 + \frac{sT_{del}}{2} + \frac{(sT_{del})^2}{12}}$$

- Often choose: $T_{del} \approx T_c \approx T_d \approx T_q \approx T_{sampling} = T_{switching}$

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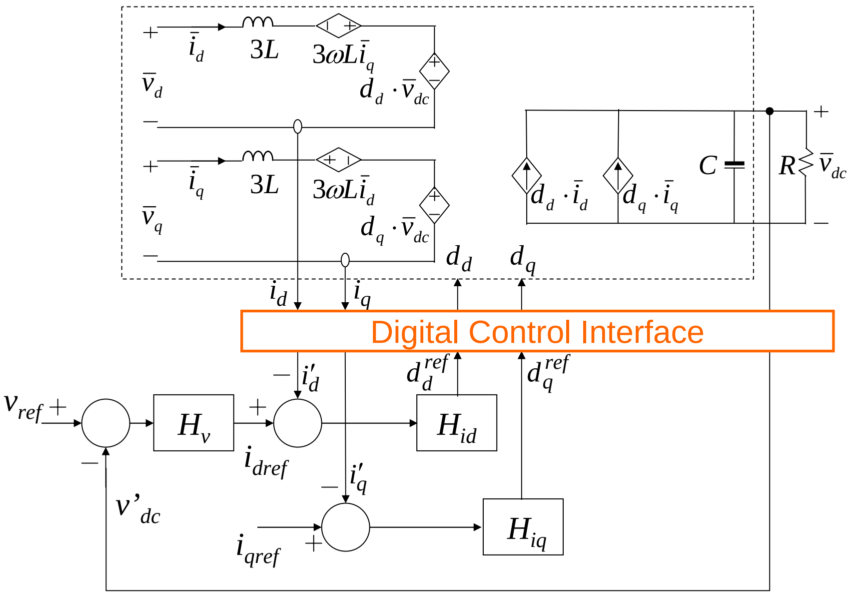
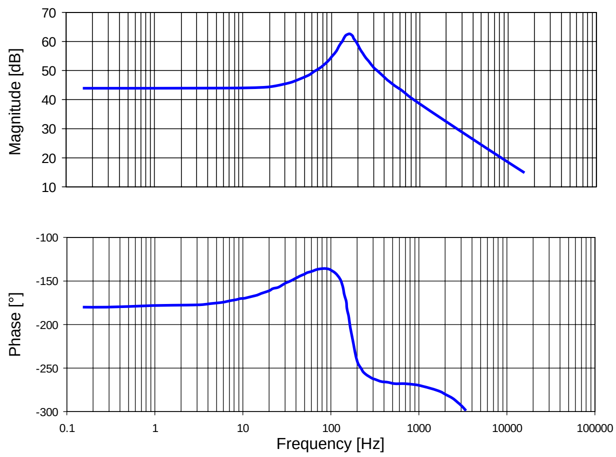
Current Loop Design



DB-168

Control-to-Current Transfer Function

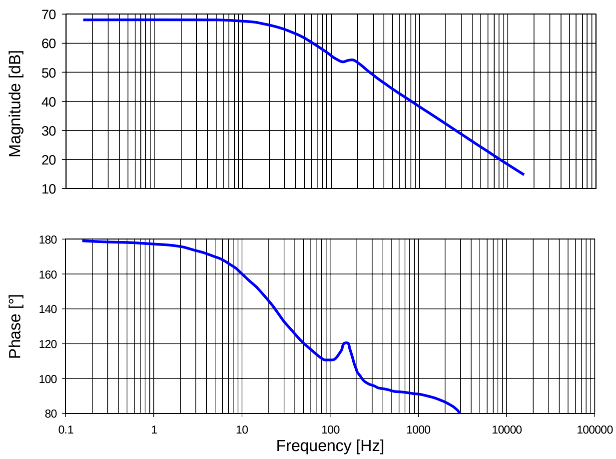


$$\frac{\tilde{i}_d}{\tilde{d}_d} = \frac{K_{iddd}\cdot(s+z_{iddd\,1})\cdot(s+z_{iddd\,2})}{(s+p_1)\cdot(s+p_2)\cdot(s+p_2^*)}$$

$$H_{id} = K_p + \frac{K_i}{s}$$

DB-169

Control-to-Current Transfer Function

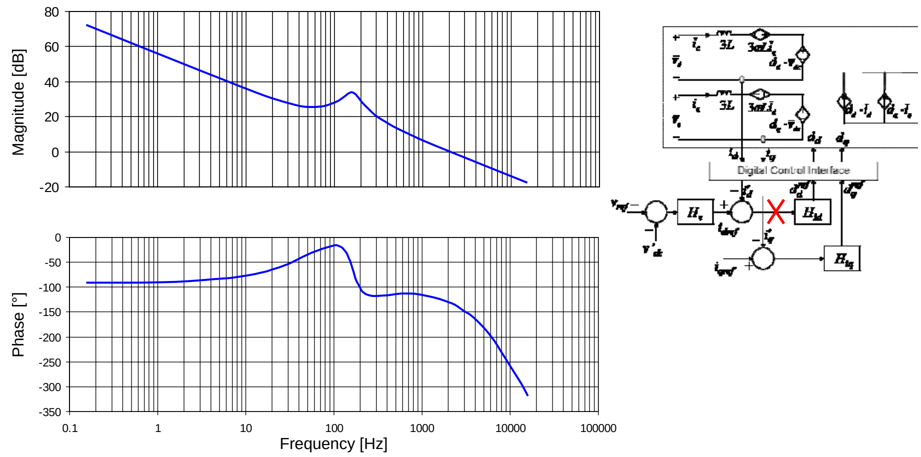


$$\frac{\tilde{i}_q}{\tilde{d}_q} = \frac{K_{iqdq}\cdot(s+z_{iqdq})\cdot(s+z_{iqdq}^*)}{(s+p_1)\cdot(s+p_2)\cdot(s+p_2^*)}$$

$$H_{iq} = K_p + \frac{K_i}{s}$$

DB-170

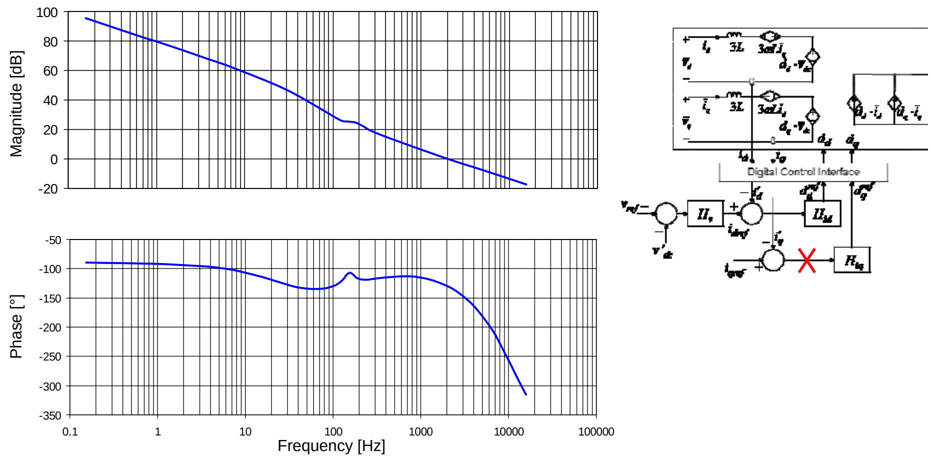
Current Loop-Gain



- D channel loop-gain T_d
- Bandwidth is limited by delay (f_{sw}=20kHz)

DB-171

Current Loop-Gain

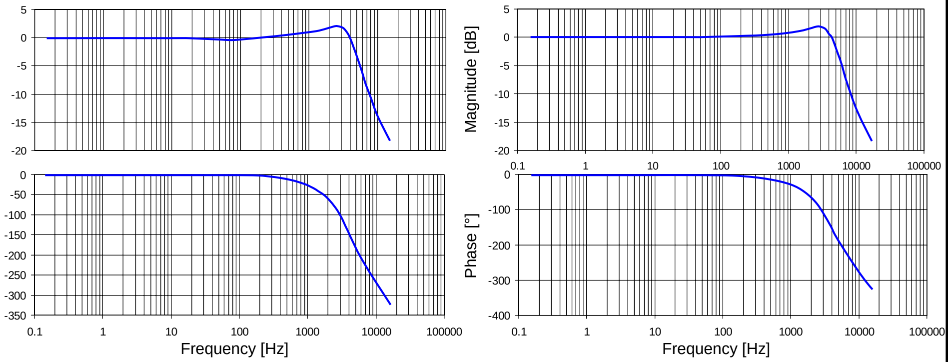


- Q channel loop-gain T_q
- Bandwidth is limited by delay (f_{sw}=20kHz)

DB-172

Current Regulation

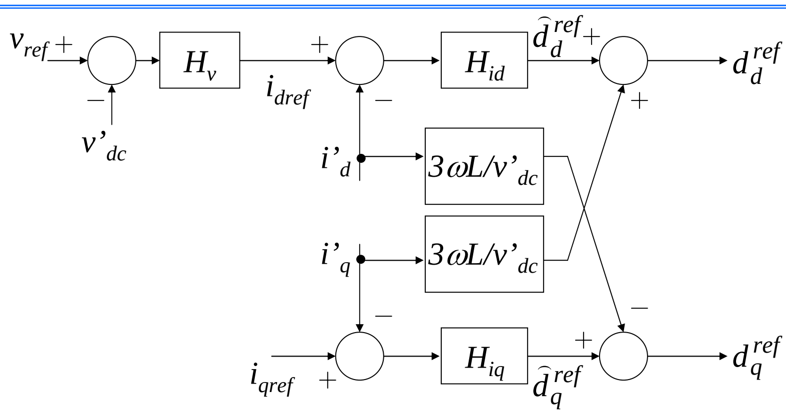
• Peak is more pronounced when gain increases



$$\frac{\tilde{i}'_d}{\tilde{i}_{dref}} \qquad \frac{\tilde{i}'_q}{\tilde{i}_{qref}}$$

DB-173

Current Loop with D and Q Decoupling



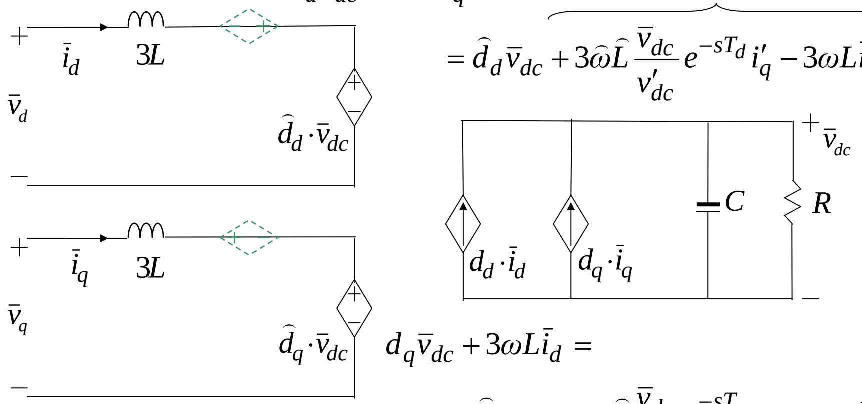
$$d_d = d_d^{ref}\cdot e^{-sT_d} = (\hat{d}_d^{ref} + 3\omega L i'_q / v'_{dc})\cdot e^{-sT_d} = \hat{d}_q + 3\omega L\frac{1}{v'_{dc}}\cdot e^{-sT_d}\cdot i'_q$$

$$d_q = d_q^{ref}\cdot e^{-sT_q} = (\hat{d}_q^{ref} - 3\omega L i'_d / v'_{dc})\cdot e^{-sT_q} = \hat{d}_q - 3\omega L\frac{1}{v'_{dc}}\cdot e^{-sT_q}\cdot i'_d$$

DB-174

Decoupled D and Q Channels

$$d_d \bar{v}_{dc} - 3\omega L \bar{i}_q = \hat{d}_d \bar{v}_{dc} + \overbrace{3\hat{\omega}\hat{L}\frac{\bar{v}_{dc}}{v'_{dc}} e^{-sT_d} i'_q - 3\omega L \bar{i}_q}^{\approx 0}$$



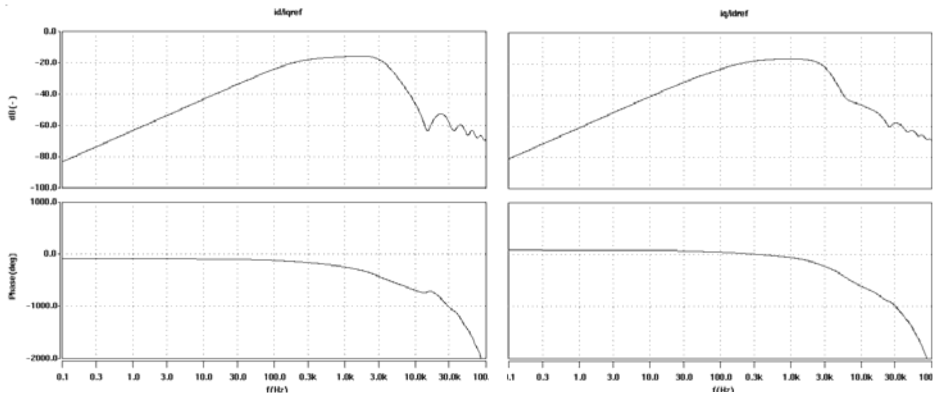
$$d_q \bar{v}_{dc} + 3\omega L \bar{i}_d = \hat{d}_q \bar{v}_{dc} \underbrace{- 3\hat{\omega}\hat{L}\frac{\bar{v}_{dc}}{v'_{dc}} e^{-sT_q} i'_d + 3\omega L \bar{i}_d}_{\approx 0}$$

- Similar to two parallel dc-dc boost converters after d and q decoupled

DB-175

Cross-Coupling Effect After Decoupling

$$\frac{\tilde{i}'_d}{\tilde{i}_{qref}} \qquad \frac{\tilde{i}'_q}{\tilde{i}_{dref}}$$



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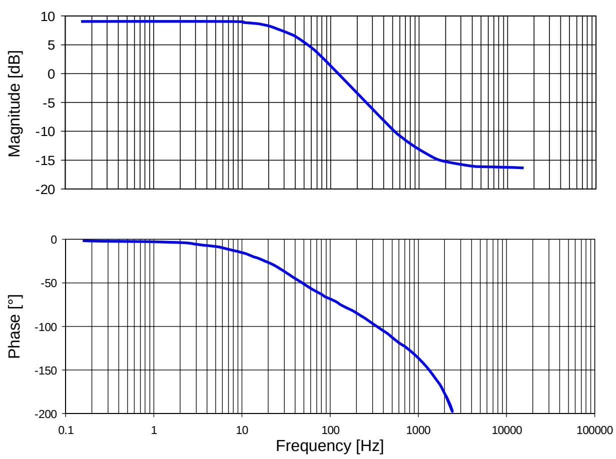
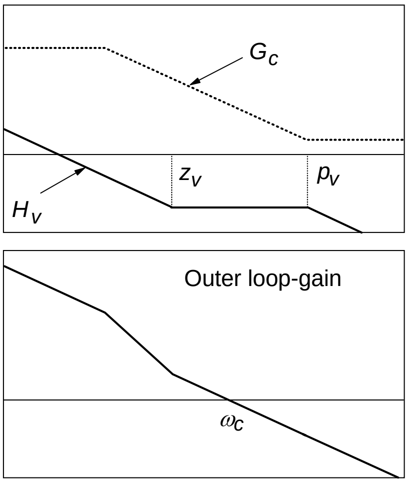
Output Voltage Loop Design



$$G_c = \frac{\tilde{v}_{dc}}{\tilde{i}_{dref}} = \frac{K \cdot (s - z_{RHP})}{(s + p_L) \cdot (s + p_H)}$$

DB-178

COMPENSATOR DESIGN



- Voltage compensator

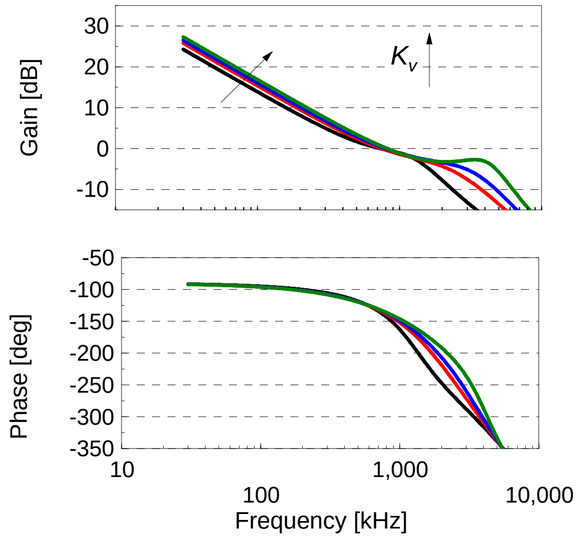
$$H_v = \frac{K_v(1 + s/z_v)}{s(1 + s/p_v)}$$

- Place z_v as high as possible for required phase margin.
- Place p_v for loop-gain attenuation.
- Attainable voltage-loop bandwidth:

$$\omega_c < 1/4\, z_{RHP}$$

DB-179

Attainable Voltage Loop Bandwidth with Delay



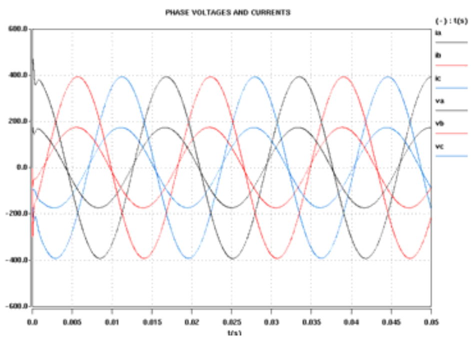
- Voltage compensator

$$H_v = \frac{K_v(1 + s/z_v)}{s(1 + s/p_v)}$$

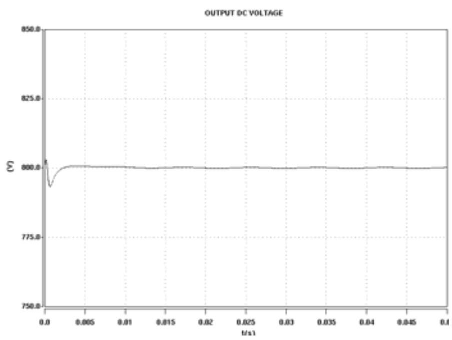
- Zero is placed close to the crossover to improve the phase margin

DB-180

Time Domain Simulation Results



Phase voltages and currents
(PFC operation)



Output voltage

DB-181

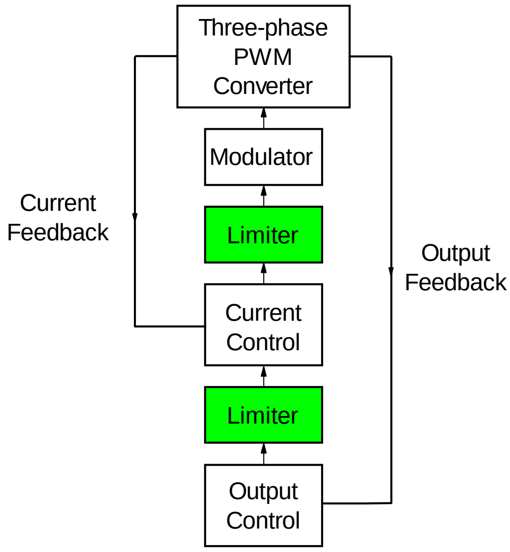
Outline

PECon 2008

1. **Introduction**
2. **Switching Modeling and PWM**
3. **Average Modeling**
4. **Small-Signal Modeling**
5. **Closed-Loop Control Design**
    - **Control approach**
    - **Current-loop design**
    - **Voltage-loop design**
    - **Limiting**
6. **More Complex Converters**

DB-182

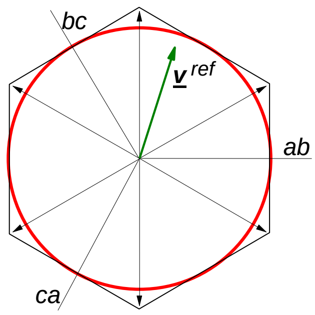
LIMITING IN THREE-PHASE CONVERTERS



DB-183

Duty Cycle Limiting

- Boost Rectifier/VSI Switching State Hexagon



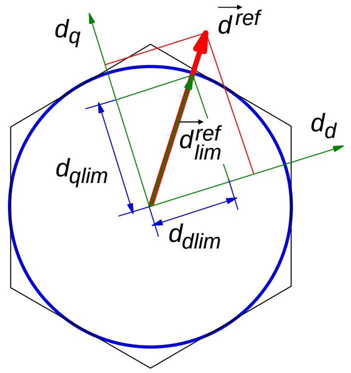
$$\vec{v}^{\,ref} = \vec{d}^{\,ref} \cdot \sqrt{\frac{3}{2}} \cdot V_{dc}$$

- Maximal attainable voltage vector lies on the hexagon
- For sinusoidal average output voltages the voltage vector must be inside the inscribed circle

$$\left|\vec{v}^{\,ref}\right| \le \sqrt{\frac{3}{2}} \cdot V_{dc} \quad \Rightarrow \quad \left|\vec{d}^{\,ref}\right| \le 1 \quad \Rightarrow \quad d_d^2 + d_q^2 \le 1$$

DB-184

Duty Cycle Limiting



If $d_d^2 + d_q^2 > 1$

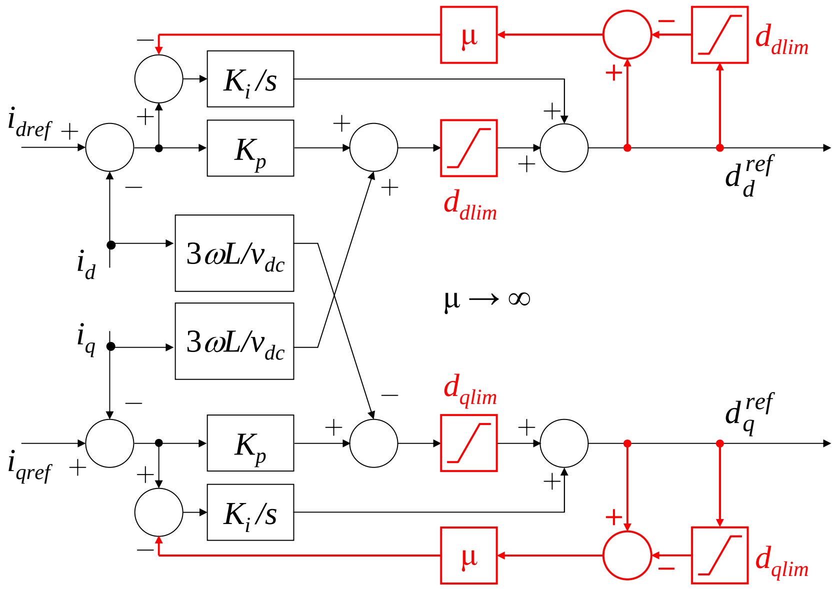
$$d_{d\,\lim} = \frac{d_d}{\sqrt{d_d^2 + d_q^2}}$$

$$d_{q\,\lim} = \frac{d_q}{\sqrt{d_d^2 + d_q^2}}$$

- Vector angle is kept constant
- $(d_d)_{lim}$ and $(d_q)_{lim}$ should be fed back to the anti-windup current controllers
- If output of each current controller is limited separately it can result in unattainable voltage vector

DB-185

Variable Limit PI ("Smart Anti-windup") Current Controllers



DB-186

Current Limiting

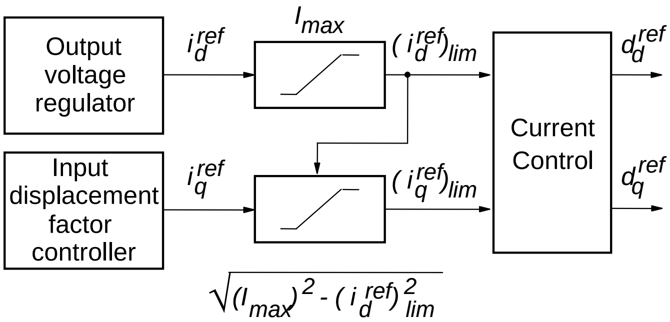
- Maximal current magnitude is limited

$$\sqrt{(i_d^{ref})^2 + (i_q^{ref})^2} \leq I_{max}$$

- Limiting of each reference current component is determined by the control algorithm

EXAMPLE

- Boost rectifier control



DB-187

Outline

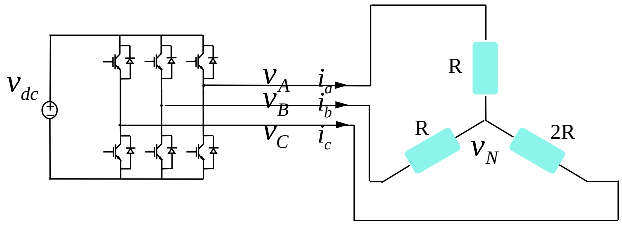
PECon 2008

1. **Introduction**
2. **Switching Modeling and PWM**
3. **Average Modeling**
4. **Small-Signal Modeling**
5. **Closed-Loop Control Design**

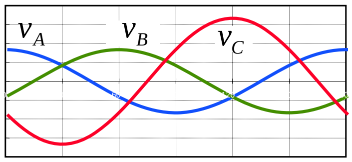
6. **More Complex Converters**
   - **Three-phase four-wire (four-phase) converter**
   - **Multilevel converters**
   - **Parallel converters**

DB-188

Limitations of Three-Leg Converters



Output Voltages



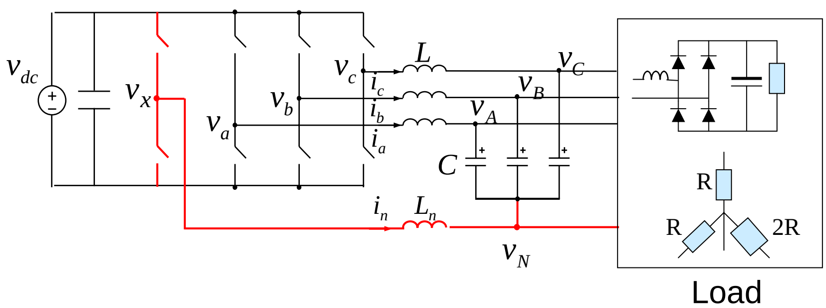
- There is no path for the neutral current.
- Output voltages are unbalanced.

Conventional inverter is not good for highly unbalanced load.

DB-189

Applications

- Provide three-phase four-wire
- Deal with unbalanced and nonlinear load

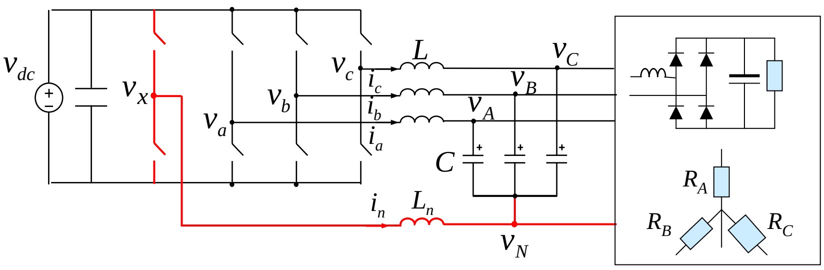


Advantages

- The fourth leg provides a neutral current path
- Compared to generating v_n with capacitive divider:
  - Much smaller dc-link capacitance is needed
  - Full utilization of dc-link voltage (15% higher) with SVM

DB-190

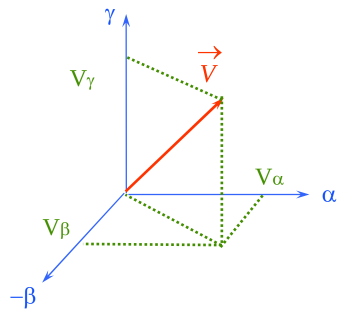
Three-Dimensional Space Vector for Four-Leg Converter



a-b-c coordinates ⇨ α–β–γ coordinates

$$V_\gamma = V_{an} + V_{bn} + V_{cn} \neq 0$$

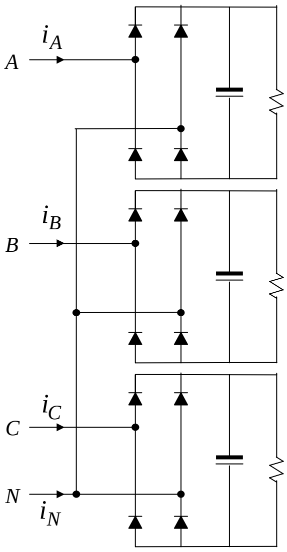
- Vγ is the zero-sequence component
- Vγ is related to the neutral current



DB-191

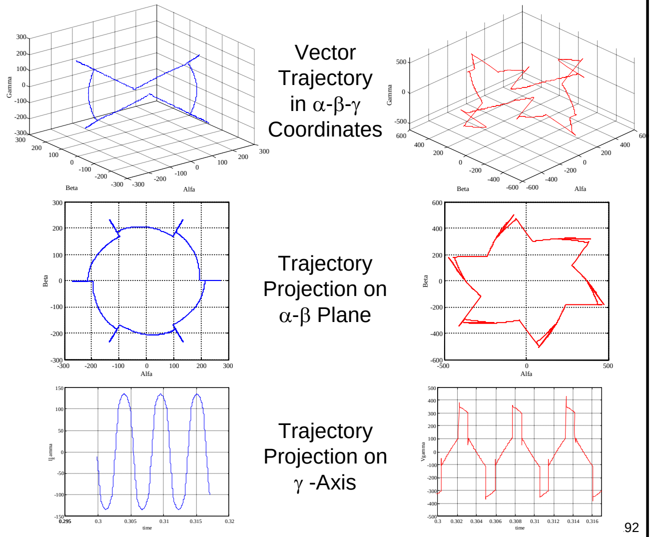
A Nonlinear Load

Three Single-Phase Diode Bridge Rectifiers with C Filter



Load current $[i_A \ \ i_B \ \ i_C]^T$

Desired voltage $[v_{ax} \ \ v_{bx} \ \ v_{cx}]^T$ for sinusoidal $[v_{AN} \ \ v_{BN} \ \ v_{CN}]^T$



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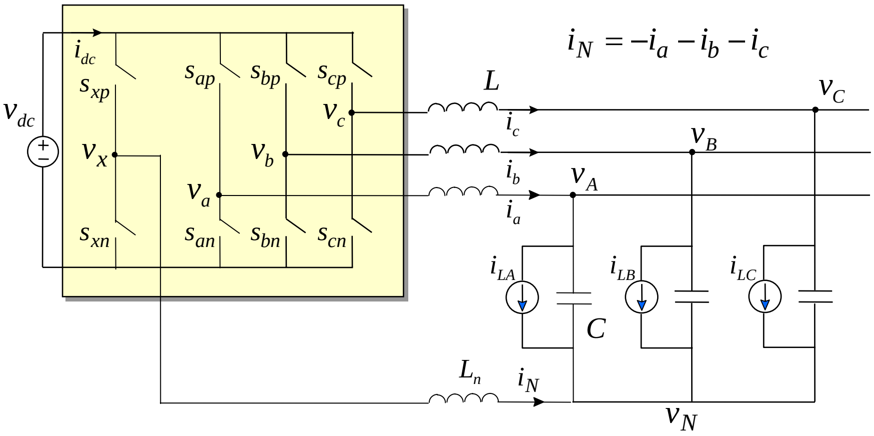
Switching Model

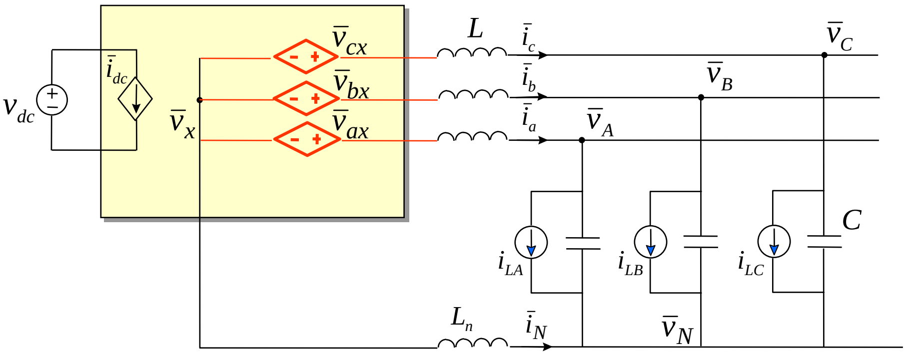


$$i_N = -i_a - i_b - i_c$$

$$\begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix} = \begin{bmatrix} s_{ap} - s_{xp} \\ s_{bp} - s_{xp} \\ s_{cp} - s_{xp} \end{bmatrix} \qquad i_{dc} = \begin{bmatrix} s_a & s_b & s_c \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad \begin{bmatrix} v_{ax} \\ v_{bx} \\ v_{cx} \end{bmatrix} = \begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix} \cdot v_{dc}$$

DB-193

Average Model in Stationary Coordinates

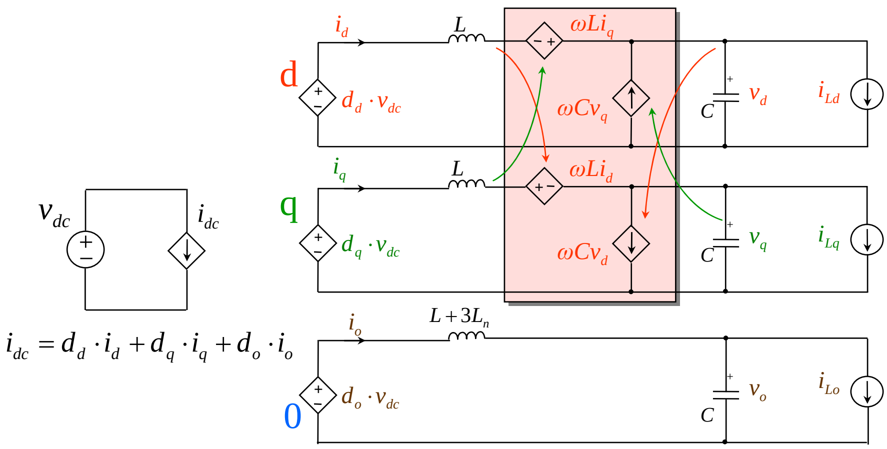


$$\bar{i}_{dc} = \begin{bmatrix} d_a & d_b & d_c \end{bmatrix} \cdot \begin{bmatrix} \bar{i}_a \\ \bar{i}_b \\ \bar{i}_c \end{bmatrix} \qquad \begin{bmatrix} \bar{v}_{ax} \\ \bar{v}_{bx} \\ \bar{v}_{cx} \end{bmatrix} = \begin{bmatrix} d_a \\ d_b \\ d_c \end{bmatrix} \cdot v_{dc} \qquad \begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix}_{ref} = \begin{bmatrix} V_m \cos(\omega t) \\ V_m \cos(\omega t - 2\pi/3) \\ V_m \cos(\omega t + 2\pi/3) \end{bmatrix}$$

- Controlled voltage sources are balanced for balanced load
- Controlled voltage sources are unbalanced for unbalanced load
- Difficult to describe the circuit operation in abc coordinates

DB-194

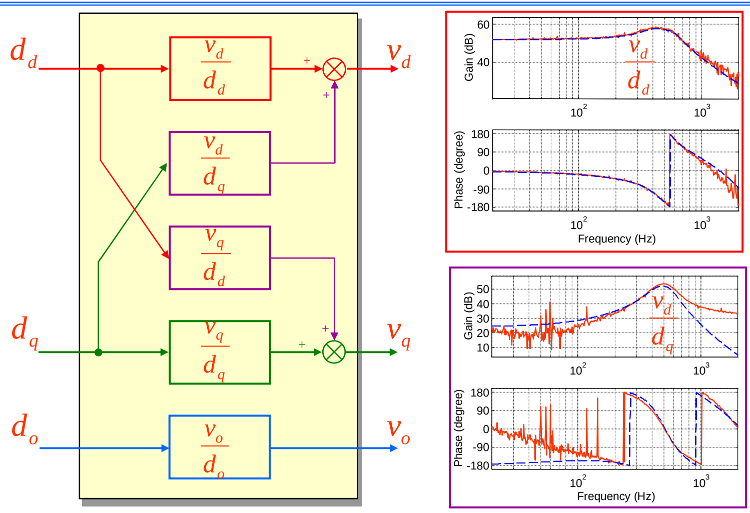
Average Model in Rotating Coordinates



- 2 decoupled subsystems with reduced system orders
- Transfer functions can be derived based on the dc operating point

DB-195

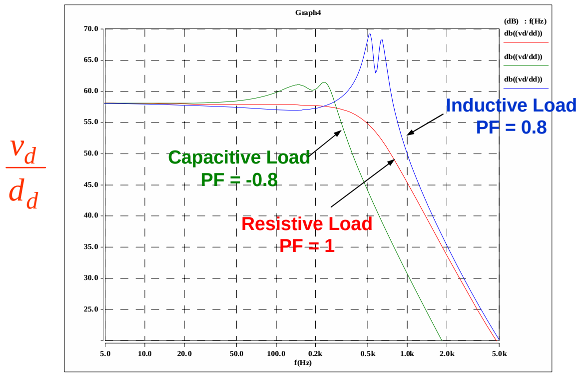
Transfer Functions



- Sampling delay and PWM delay are incorporated in the model
- The derived model agrees with the measurement very well

DB-196

Impact of Load Power Factor on Control-to-Output Voltage TFs

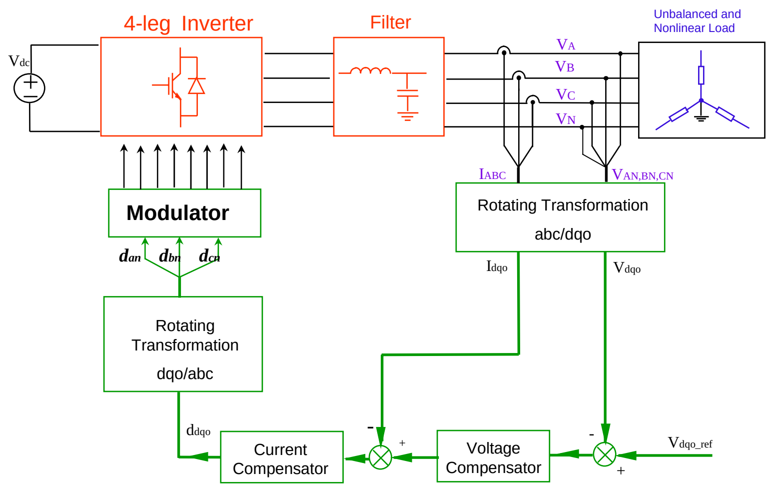


Resistive, capacitive (PF=-0.8), and inductive (PF=0.8) are all at 150 kW

- Capacitive load shift the resonant frequency to lower frequency
- Inductive load leads to a higher system order and higher resonant peaking
- Both capacitive and inductive loads are worse loads than resistive load

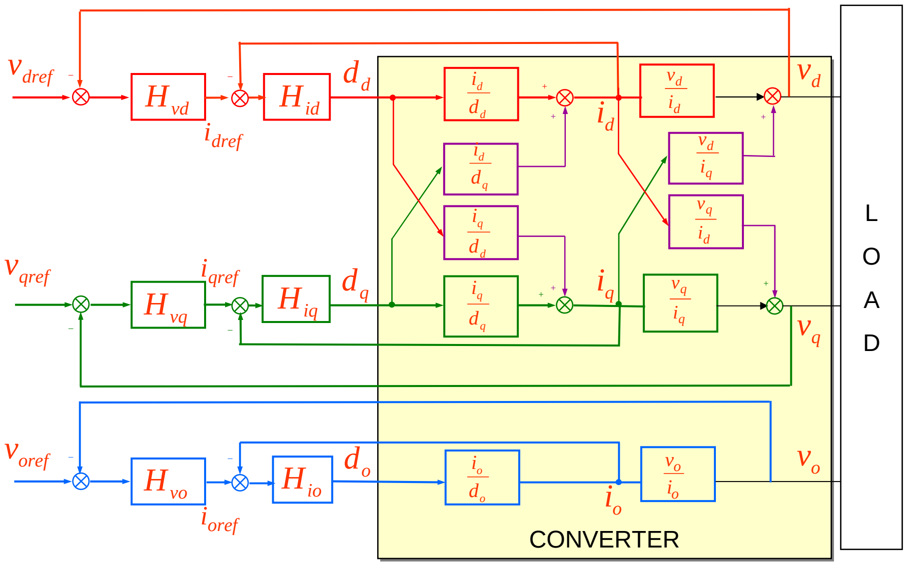
DB-197

Control Block Diagram



DB-198

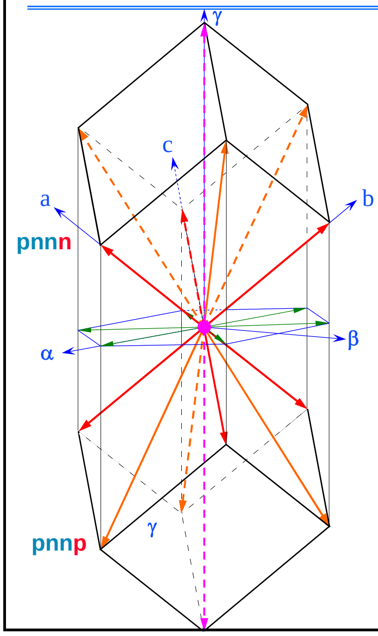
Double-Loop Design



- Decoupling is not included

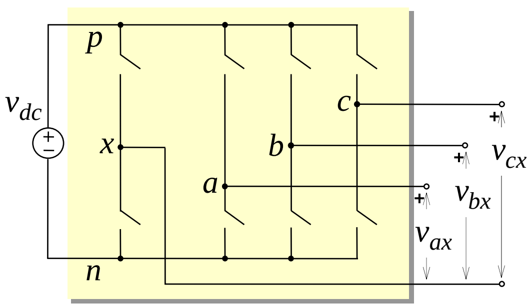
DB-199

Switching State Vectors in Three-Dimensional Space



- 16 Switching State Vectors

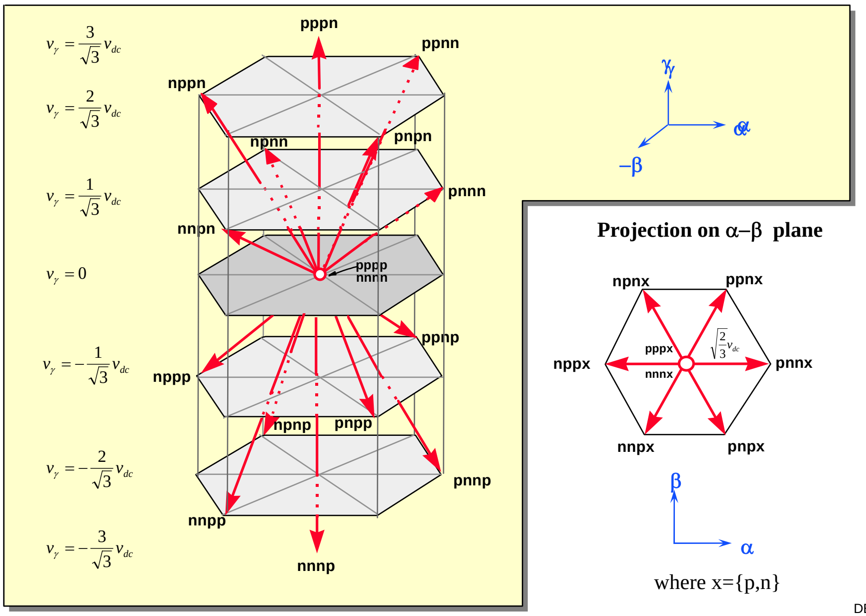
abcx



- 2×6 vectors with non-zero α & β
- 2 “zero vectors” with non-zero γ
- 2 zero vectors

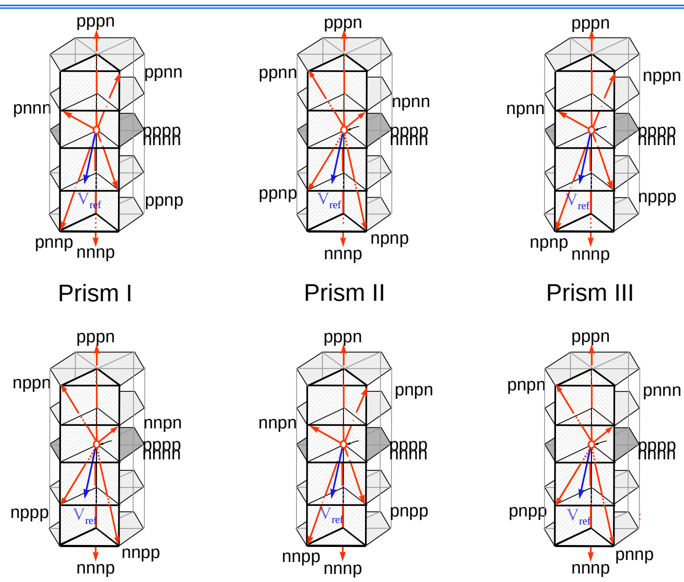
DB-200

Three-Dimensional Switching Vectors



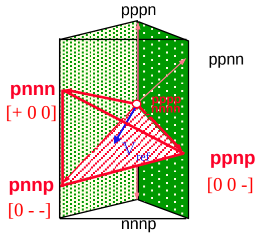
DB-201

Three-Dimensional SVM
Step 1: Prism Identification

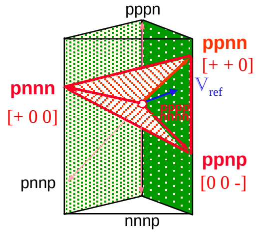


DB-202

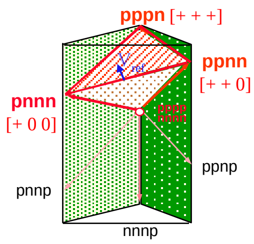
Three-Dimensional SVM
Step 2: Tetrahedron Identification



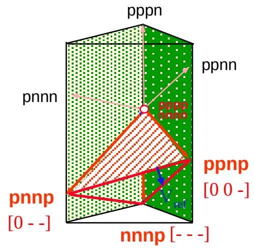
Tetrahedron 1



Tetrahedron 2



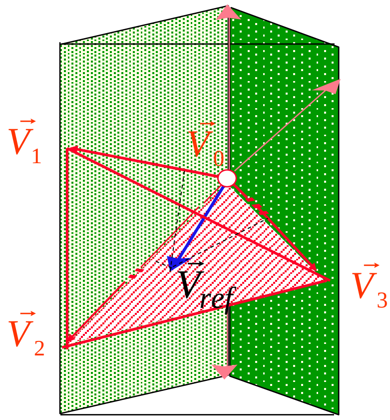
Tetrahedron 3



Tetrahedron 4

DB-203

Three-Dimensional SVM:
Step 3: Duty-Cycle Calculation

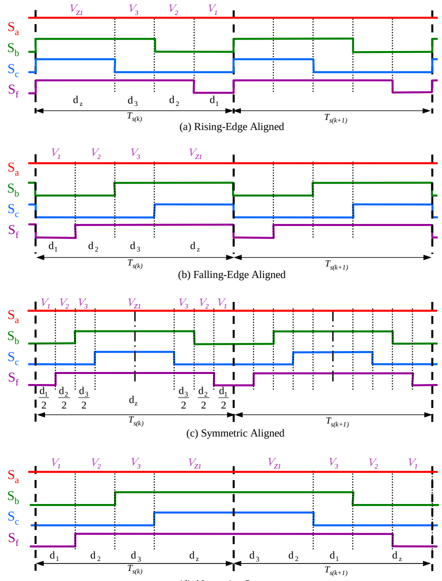


$$\vec{V}_{ref} = d_1\vec{V}_1 + d_2\vec{V}_2 + d_3\vec{V}_3$$

$$d_0 = 1 - d_1 - d_2 - d_3$$

DB-204

Three-Dimensional SVM: Step 4: Switching Sequencing (Minimum Loss)



(a) Rising-Edge Aligned

(b) Falling-Edge Aligned

(c) Symmetric Aligned

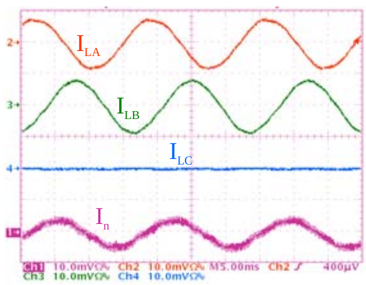
(d) Alternative Sequence

DB-205

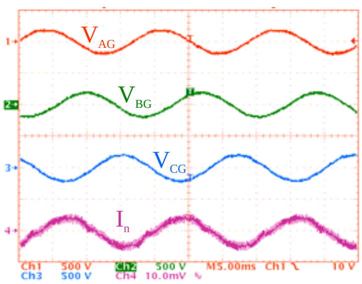
Experimental Results with Only Voltage Loops Closed

Unbalanced Load

Load current and neutral current



Output voltage and neutral current



DB-206

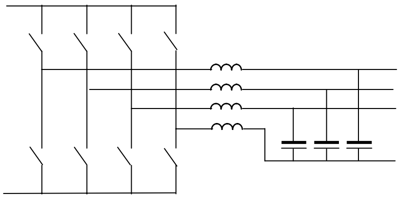
A Four-Leg Inverter with Common-Mode Filter Function

- Common-Mode Noise
- A 4-Leg Inverter for Common-Mode Noise Elimination
- A New Modulation and Control Scheme for Inverter Power Supplies with Unbalanced/Nonlinear Load

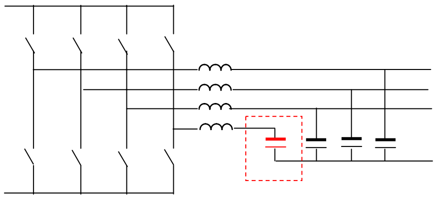
DB-207

A Four-Leg Inverter with Common-Mode Filter Function

4-leg inverter for unbalanced / nonlinear load



4-leg inverter for common-mode reduction



If $v_a + v_b + v_c + v_x \equiv 0$, there is NO common-mode noise!

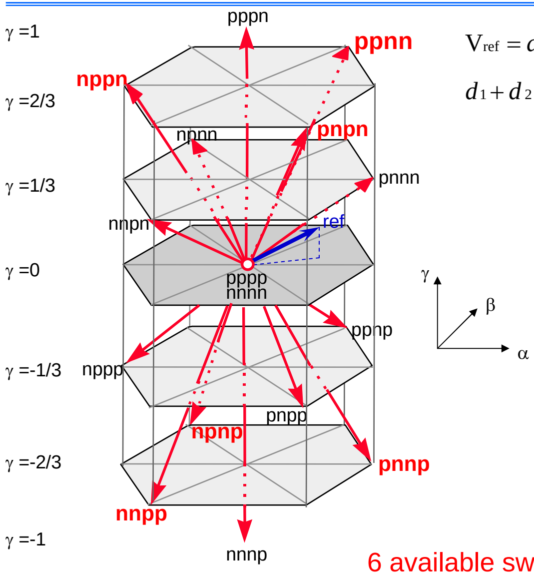
In order to have both functions, we need

⇨ A new switching modulation strategy

⇨ A new control design because of the series capacitor in neutral leg

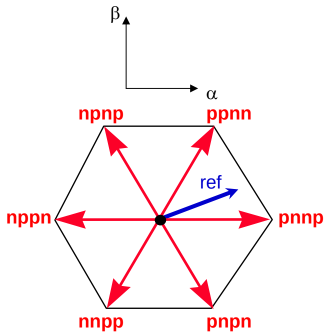
DB-208

Use Only Switching Vectors with 2 p's and 2 n's



$$V_{ref} = d_1 \cdot V_{pnpn} + d_2 \cdot V_{pnnp} + d_3 \cdot V_{ppnn} + d_4 \cdot V_{npnp}$$

$$d_1 + d_2 + d_3 + d_4 = 1$$

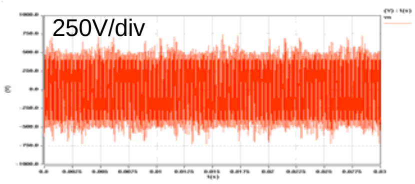


6 available switching vectors out of 2^4 vectors

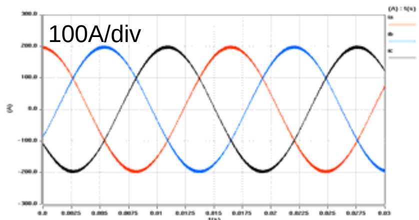
DB-209

Common-Mode Noise Reduction and Trade-Offs

Conventional 4-leg inverter

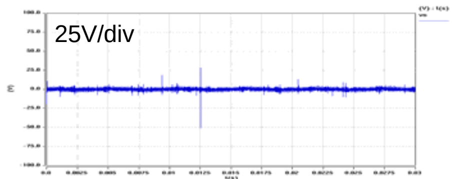


Common-mode voltage

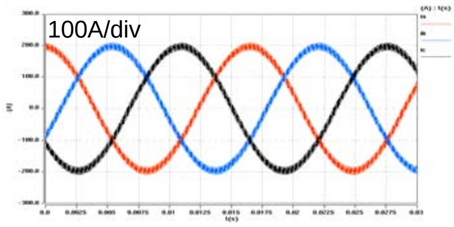


Three-phase inductor current

4-leg inverter with CM reduction



Common-mode voltage



Three-phase inductor current

- **DM ripple is increased because of reduced vector choice**
- **15% smaller maximum modulation index**

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Outline

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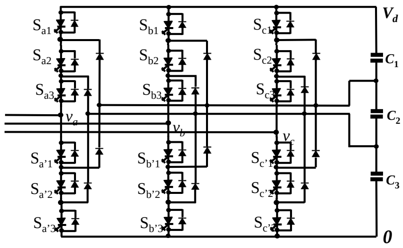
5. Closed-Loop Control Design
6. More Complex Converters
   - Three-phase four-wire (four-phase) converter
   - Multilevel converters
   - Parallel converters

DB-211

Major Multilevel Three-Phase Topologies

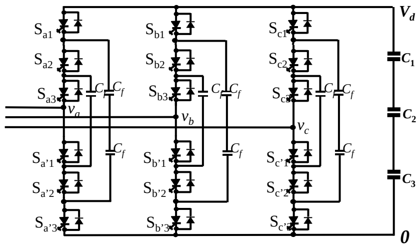
Diode clamped

Nabae, 1980; Choe, 1991; Carpita, 1991

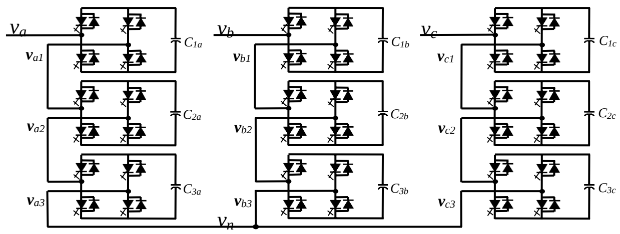


Flying capacitor

Meynard, 1992



Cascaded H-bridge

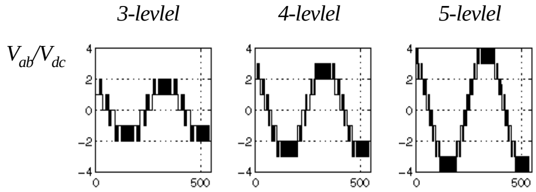


DB-212

Merits of Multilevel Converter Technologies

Advantages

- Easy voltage sharing among devices
- Improved spectral performance of output waveforms
- Reduced dv/dt resulting in reduced reflections and damage to insulation
- Reduced switching and conduction losses

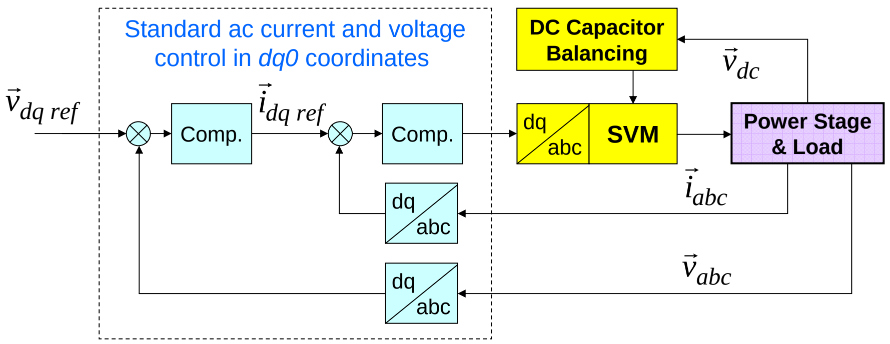


Disadvantages

- Increased number of devices
- Increased control complexity
- Depending on topology:
  - Issues with capacitors balancing
  - Issues with bi-directional power flow

DB-213

Multilevel Converter Control

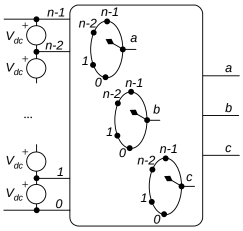


- Average and small-signal models are the same as for two-level VSI or boost rectifier, except for multiple dc voltages
- Closed-loop control of ac quantities in rotating *dq* coordinates is the same as for two-level VSI or boost rectifier
- **Additional controller for voltage balancing on dc-link capacitors**
- **Modulator is significantly more complex**

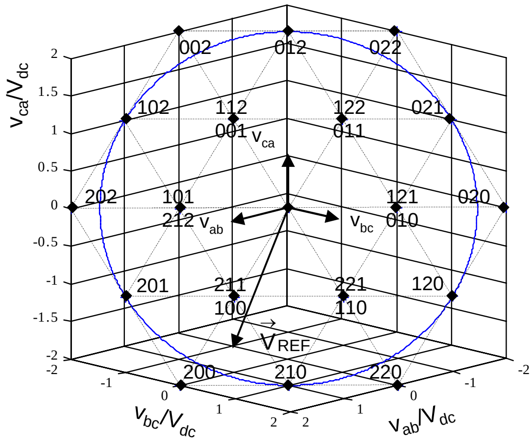
DB-214

Space-Vector Representation of Three-Level Converters

- Switching model of n-level converter: Three single-pole n-throw switches



$$\vec{v}_{ijk} = V_{dc} \cdot \begin{bmatrix} i-j \\ j-k \\ k-i \end{bmatrix}$$

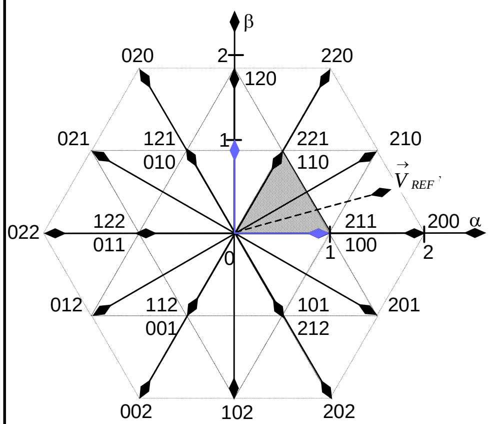


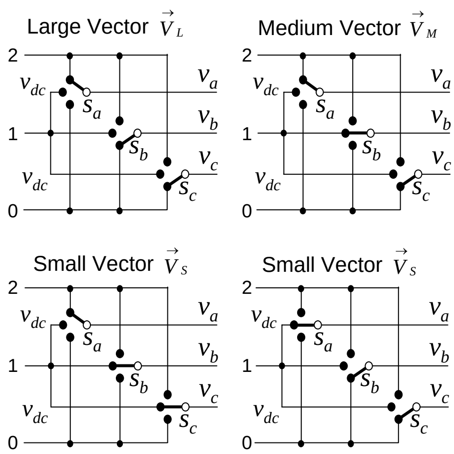
- Voltage space-vectors can be represented in a three dimensional line-to-line coordinate system

DB-215

Voltage Space-Vectors of Three-Level Converters

$$KVL: \quad v_{ab} + v_{bc} + v_{ca} = 0$$





- All the voltage space-vectors of a three-phase converter are located on a plane
- Voltage space-vectors have different length: small, medium and large.
- Every small vector has two corresponding switching states—redundancy.

DB-216

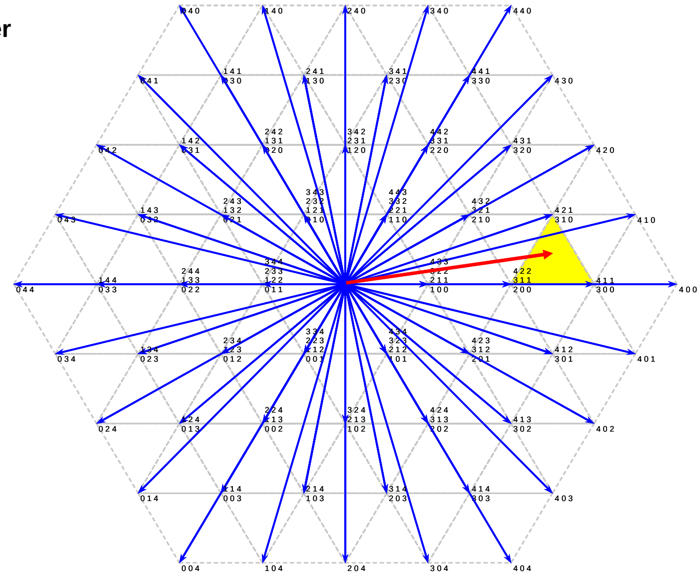
SVM for Multilevel Three-Phase Converters?

Five-level converter

- Switching states: $n^3 = 125$
- Switching vectors: $3n(n-1)+1 = 61$
- Triangular areas: $6(n-1)^2 = 96$

How to do SVM?

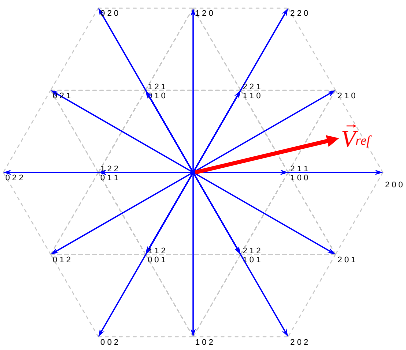
Complexity?
Speed?

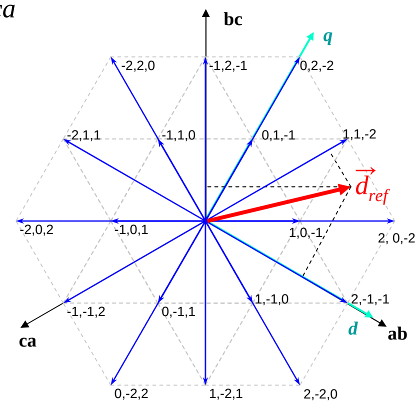


DB-217

Coordinates and Transformation

- Choose line-to-line coordinates: *ab*, *bc* and *ca*





- Transform $\vec{V}_{ref} / V_{dc}$ from *dq* to *abc*:

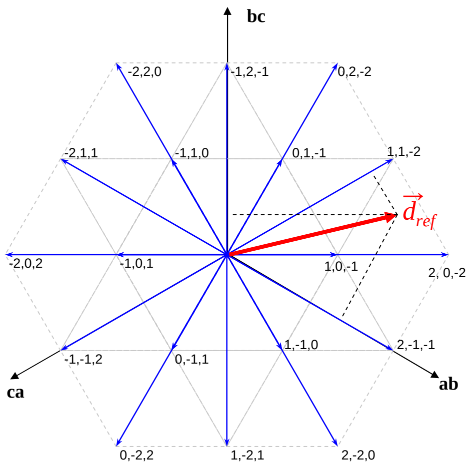
$$\vec{d}_{ref} = \begin{bmatrix} d_{ab} \\ d_{bc} \\ d_{ca} \end{bmatrix} = \frac{1}{V_{dc}} \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & -\sin\theta \\ \cos(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{2\pi}{3}) \\ \cos(\theta + \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} v_{d\,ref} \\ v_{q\,ref} \end{bmatrix}$$

DB-218

Identify the Vectors and Calculate their Duty Cycles

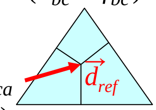
- Calculate the floor and ceiling of the projections:

$$f_{ab} \le d_{ab} \le c_{ab}\,,\quad f_{bc} \le d_{bc} \le c_{bc}\,,\quad f_{ca} \le d_{ca} \le c_{ca}$$



- If $f_{ab} + f_{bc} + f_{ca} = -1$ then $\vec{d}_{ref}$ falls in a triangle as follows:

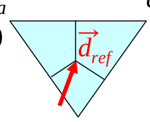
f_{ab}, c_{bc}, f_{ca} $(d_{bc} - f_{bc})$



f_{ab}, f_{bc}, c_{ca} $(d_{ca} - f_{ca})$ c_{ab}, f_{bc}, f_{ca} $(d_{ab} - f_{ab})$

- If $f_{ab} + f_{bc} + f_{ca} = -2$ then $\vec{d}_{ref}$ falls in a triangle as follows:

f_{ab}, c_{bc}, c_{ca} $(c_{ab} - d_{ab})$ c_{ab}, c_{bc}, f_{ca} $(c_{ca} - d_{ca})$

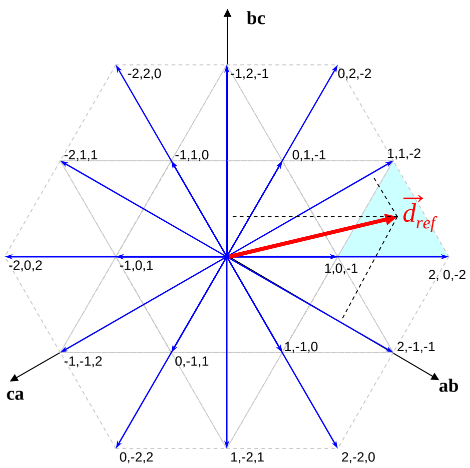


c_{ab}, f_{bc}, c_{ca} $(c_{bc} - d_{bc})$

DB-219

An Example

$\vec{d}_{ref}$ with a length of 1.7 and an angle of 45º from *ab* axis



1. The projections are

$d_{ab} = 1.202 \quad d_{bc} = 0.440 \quad d_{ca} = -1.642$

2. The floors and ceilings are

$f_{ab} = 1 \quad f_{bc} = 0 \quad f_{ca} = -2$

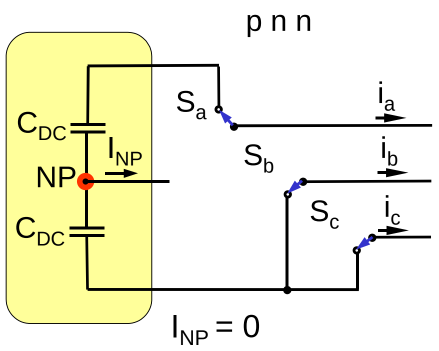
$c_{ab} = 2 \quad c_{bc} = 1 \quad c_{ca} = -1$

3. Since $f_{ab} + f_{bc} + f_{ca} = -1$, the nearest three vectors and their duty cycles are:

2,0,–2	$d_{2,0,-2} = d_{ab} - f_{ab} = 0.202$
1,1,–2	$d_{1,1,-2} = d_{bc} - f_{bc} = 0.440$
1,0,–1	$d_{1,0,-1} = d_{ca} - f_{ca} = 0.358$

DB-220

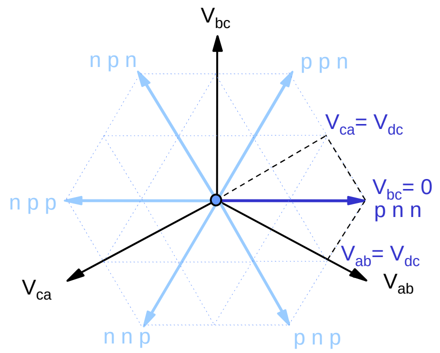
Capacitor Balancing Problem in Three-Level Neutral-Point (NP) Clamped Converter



$V_{ab} = V_{dc}$
$V_{bc} = 0$
$V_{ca} = -V_{dc}$

Amplitude $= \frac{2}{\sqrt{3}} \cdot V_{dc}$

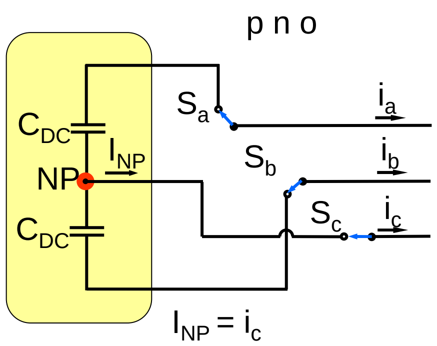
Phase $= 0^{\circ}$



Large vectors do not affect NP balance

DB-221

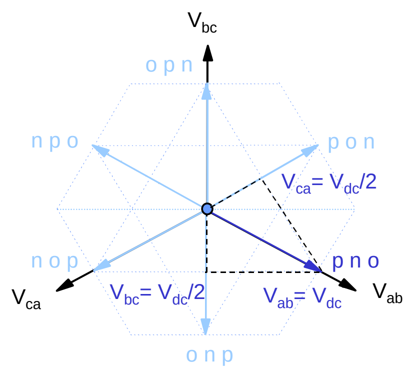
Medium vectors affect the charge balance in the neutral point causing the low frequency ripple



$V_{ab} = V_{dc}$
$V_{bc} = -V_{dc}/2$
$V_{ca} = -V_{dc}/2$

Amplitude $= V_{dc}$

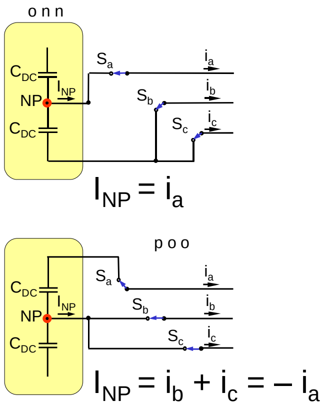
Phase $= -30^{\circ}$



Full load current is connected to NP for the duration of the medium vector duty cycle

DB-222

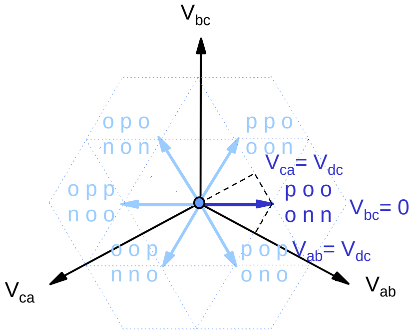
Small vectors also affect the capacitor charge balance and their effect can be controlled



$V_{ab} = V_{dc}/2$ Amplitude $= \frac{1}{\sqrt{3}} \cdot V_{dc}$

$V_{bc} = 0$

$V_{ca} = -V_{dc}/2$ Phase $= 0^\circ$



Small vectors come in pairs

Freedom to select either vector in a pair helps balance NP

DB-223

Modified SVM algorithm can effectively compensate the voltage ripple in the neutral point

- Compute the location of voltage space vectors in every switching cycle

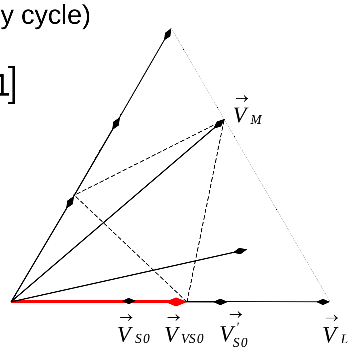
$$\vec{V} = \frac{2}{3}(v_{ab} + v_{bc} \cdot e^{j\gamma} + v_{ca} e^{j2\gamma}) \qquad \gamma = \frac{2\pi}{3}$$

- Based on the commanded balancing effort, virtual small vector
- can be computed (need to use both vectors every cycle)

$$\vec{V}_{VS0} = c \cdot \vec{V}_{S0} + (1-c) \cdot \vec{V}'_{S0} \qquad c \in [0,1]$$

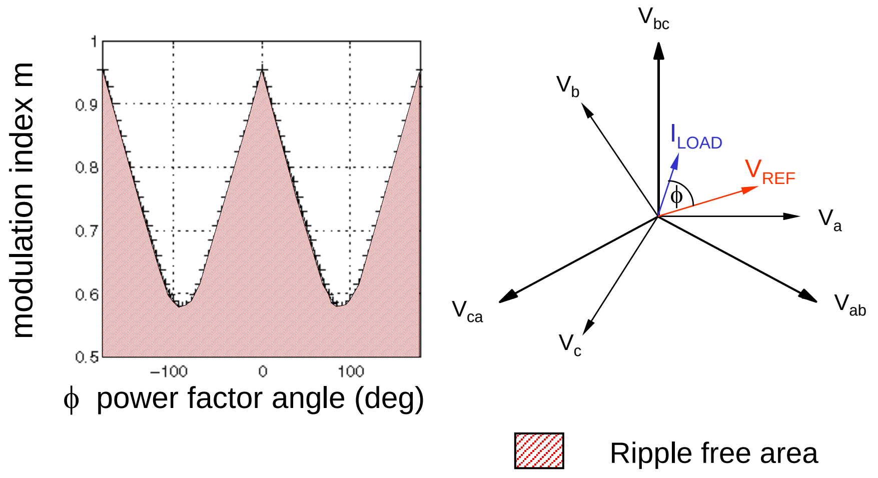
- Determine the sector location of the current $\mathbf{V_{REF}}$

- Compute the duty cycles



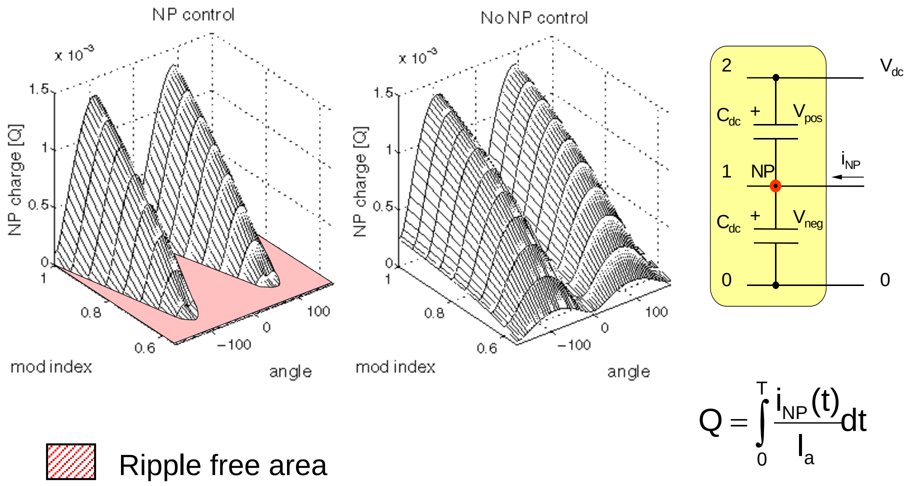
DB-224

Neutral point can be balanced in every switching cycle only in the part of the operating region



DB-225

Finding the normalized charge ripple for all operating conditions helps design dc-link capacitors



$$Q = \int_0^T \frac{i_{NP}(t)}{I_a} dt$$

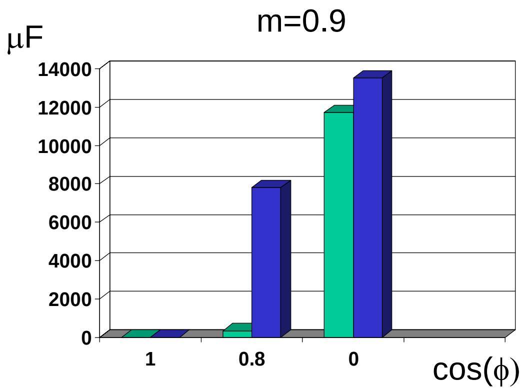
DB-226

As expected, the effect of NP charge control diminishes for the decreasing power factor angle

System Parameters
- 1800 V DC bus
- 1% NP ripple
- 200 A Peak current

With NP control

Without NP control



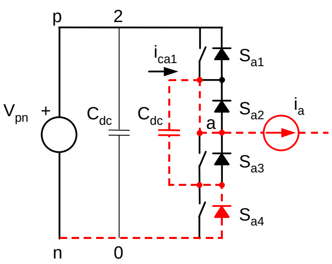
$$2 \cdot C_{DC} = \frac{K \cdot I_{max}}{\Delta U_{DC\_max}}$$

K - normalized amplitude of NP charge

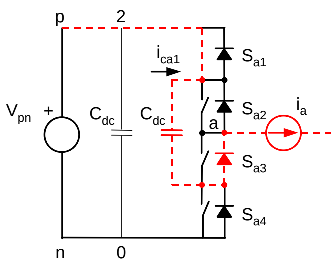
DB-227

Balancing of clamping capacitor for flying capacitor converter is load-independent

$V_{an} = V_{pn}/2$



$i_{ca1} = i_a$



$i_{ca1} = - i_a$

- The inner capacitor handles only switching frequency ripple current
- There is no clamping diode reverse recovery problem
- The modulation and higher level control remains the same

- Flying capacitor converters with any number of levels can be balanced.
- Diode-clamped converters with number of levels above three CANNOT!

DB-228

Outline

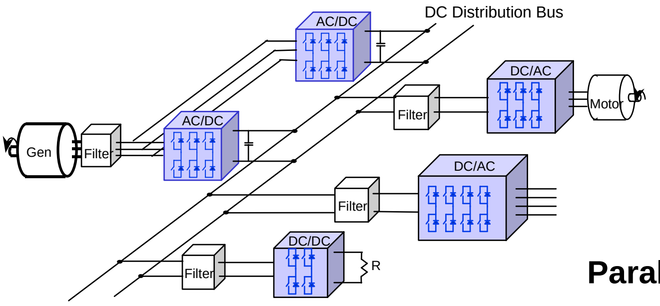
PECon 2008

1. Introduction
2. Switching Modeling and PWM
3. Average Modeling
4. Small-Signal Modeling
5. Closed-Loop Control Design

6. More Complex Converters
   - Three-phase four-wire (four-phase) converter
   - Multilevel converters
   - Parallel converters

DB-229

Motivation



A Distributed Power System

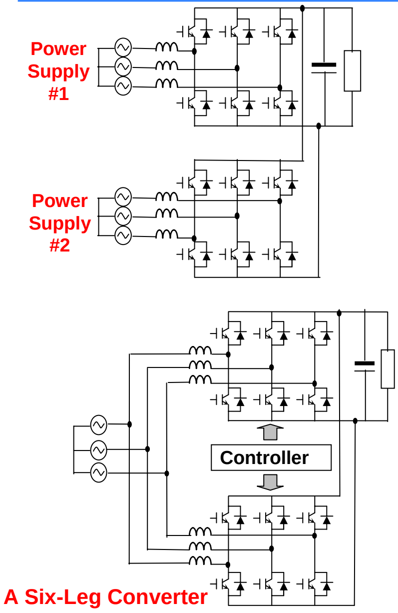
Parallelability

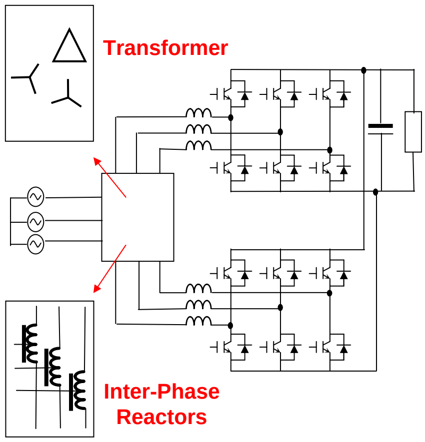
- Modular design
- N+1 redundancy (reliability)
- No limit for current level
- Maintainability
- Availability
- Reduced cost, size and weight
- Improved performance

DB-230

Possible Solutions

- Separate power supplies
- Transformer isolation
- Inter-phase reactors
- Six-leg converter





DB-231

Issues – Modeling

Challenges

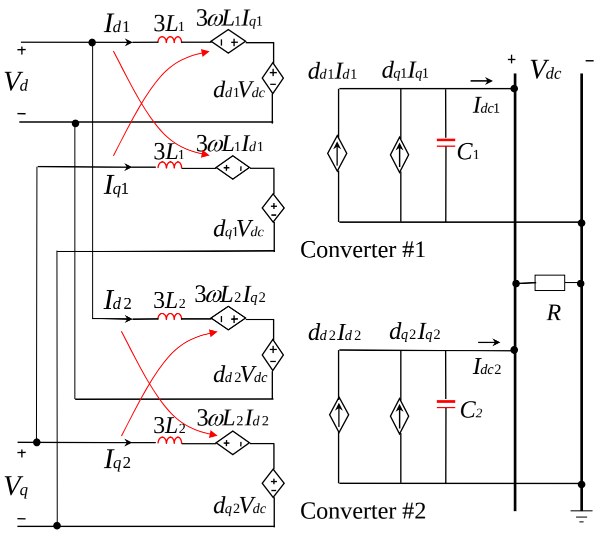
- High-order system
- Coupling

Existing results

- High-order multi-input-multi-output modeling[1]
- Reduced order modeling[2]

[1]Matakas, 1993

[2]Mao, 1994

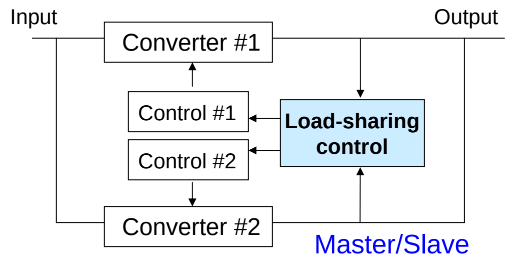


DB-232

Issues – Load Sharing

Challenge

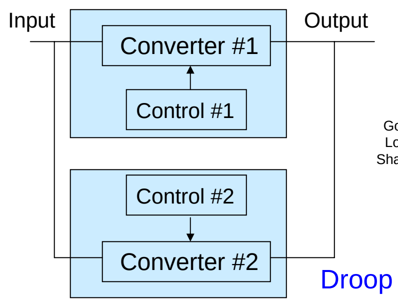
- Load sharing
- Voltage regulation
- Modular design

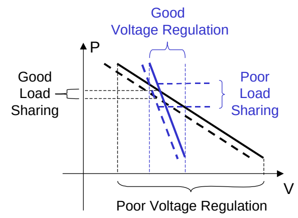


Existing results

- Master/Slave[1]
- Droop[2]

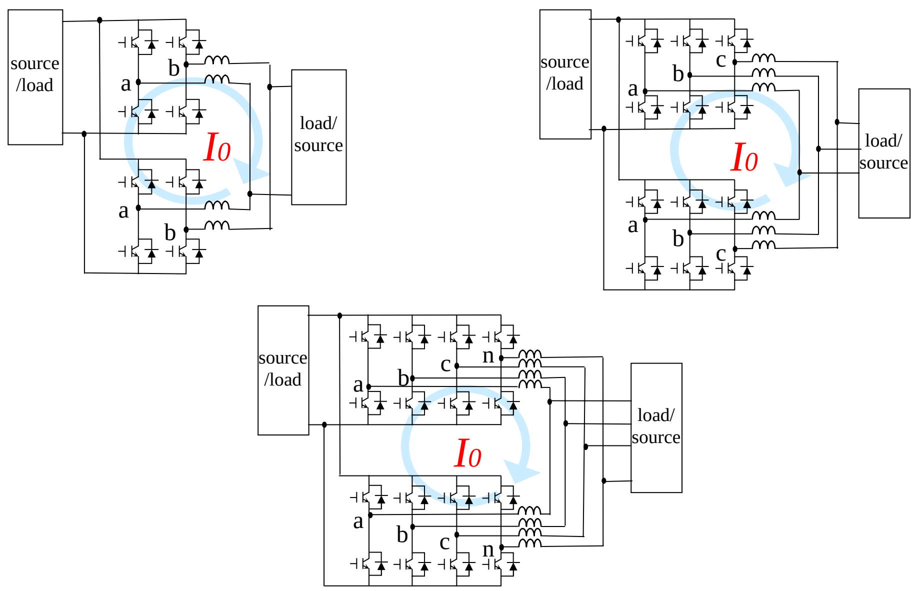
[1]Siri, 1992
[2]Jamerson, 1994





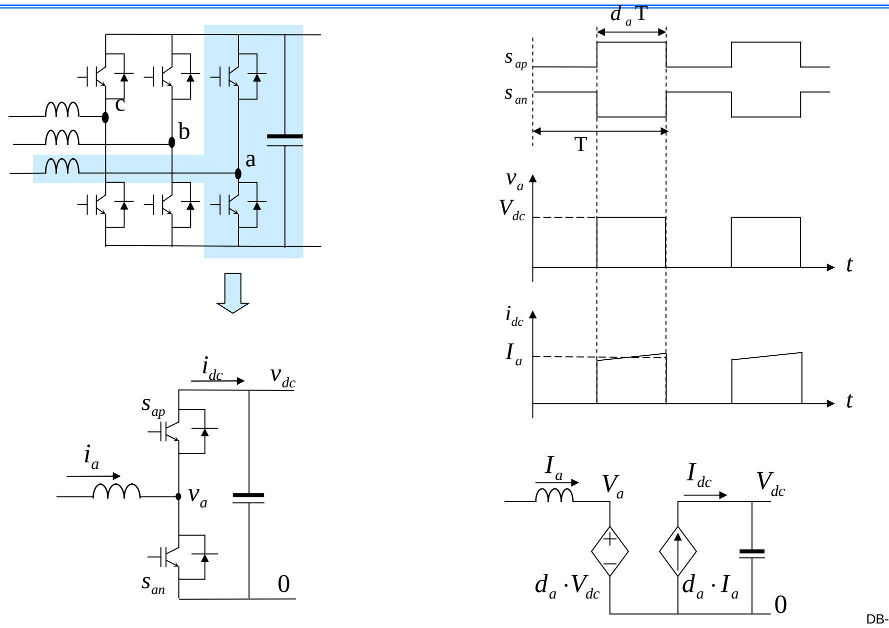
DB-233

Issues – Zero-Sequence Current



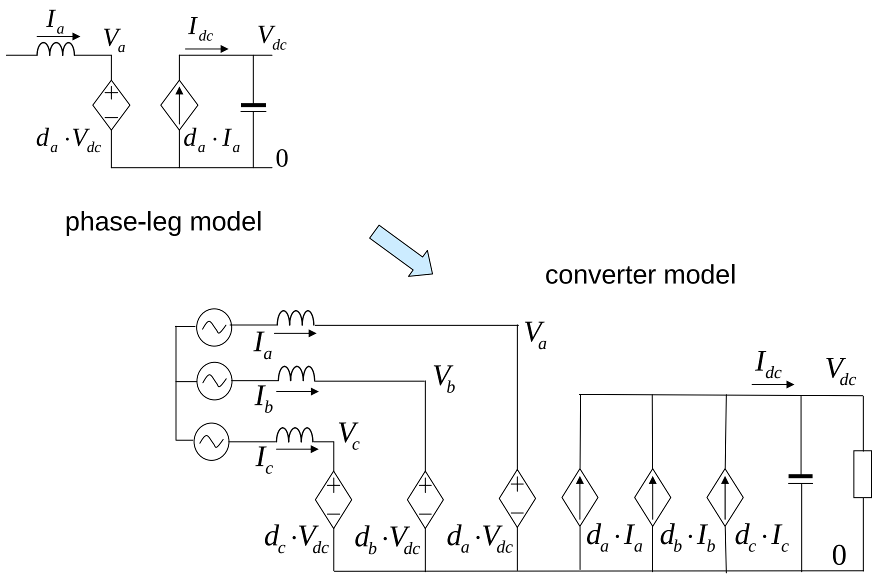
DB-234

Phase-Leg Averaging



DB-235

An Averaged Three-Phase Converter Model



DB-236

Extraction of Zero-Sequence Components

A zero-sequence component ≡ the sum of all phases components

e.g. $I_0 = I_a + I_b + I_c$

Define: $d_z = d_a + d_b + d_c$

$$\Rightarrow (d_a - d_z/3) + (d_b - d_z/3) + (d_c - d_z/3) = 0$$

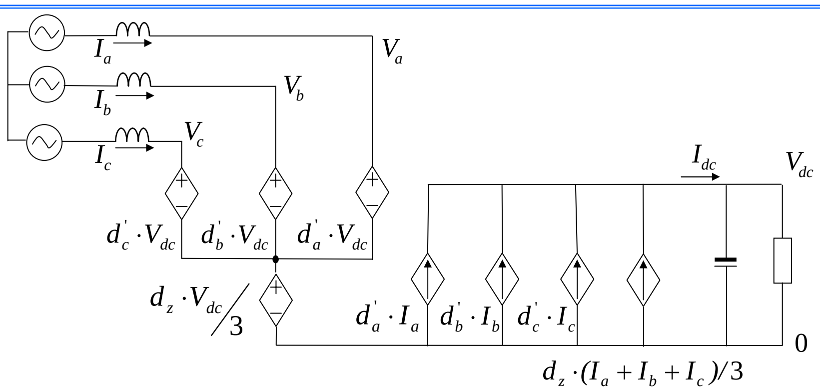
$$\Rightarrow d_a' + d_b' + d_c' = 0$$

Where: $d_a' = d_a - d_z/3 \qquad d_b' = d_b - d_z/3 \qquad d_c' = d_c - d_z/3$

$$\Rightarrow \boxed{d_a = d_a' + d_z/3 \qquad d_b = d_b' + d_z/3 \qquad d_c = d_c' + d_z/3}$$

DB-237

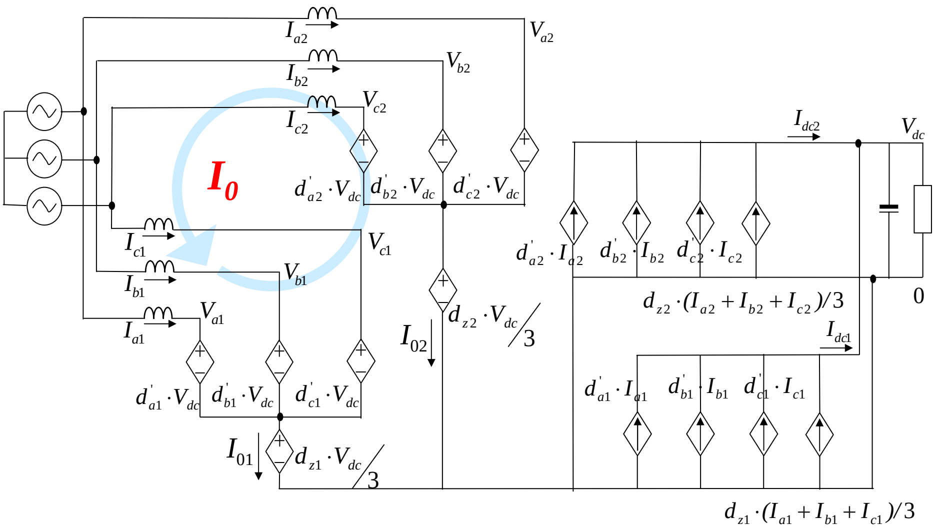
The Model with Zero-Sequence Components



- For a single converter, there is a zero-sequence voltage but no zero-sequence current because $I_a + I_b + I_c \equiv 0$

DB-238

Zero-Sequence Current in Parallel Converters



DB-239

An Averaged Model of Zero-Sequence Dynamic

In AC side, there are three-loops forming three equations:

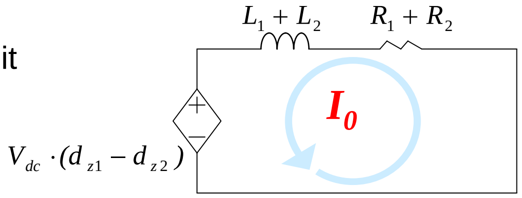
$$d_{z1} \cdot V_{dc}/3 + d'_{a1} \cdot V_{dc} - L_1 \frac{dI_{a1}}{dt} - R_1 \cdot I_{a1} = d_{z2} \cdot V_{dc}/3 + d'_{a2} \cdot V_{dc} - L_2 \frac{dI_{a2}}{dt} - R_2 \cdot I_{a2}$$

$$d_{z1} \cdot V_{dc}/3 + d'_{b1} \cdot V_{dc} - L_1 \frac{dI_{b1}}{dt} - R_1 \cdot I_{b1} = d_{z2} \cdot V_{dc}/3 + d'_{b2} \cdot V_{dc} - L_2 \frac{dI_{b2}}{dt} - R_2 \cdot I_{b2}$$

$$d_{z1} \cdot V_{dc}/3 + d'_{c1} \cdot V_{dc} - L_1 \frac{dI_{c1}}{dt} - R_1 \cdot I_{c1} = d_{z2} \cdot V_{dc}/3 + d'_{c2} \cdot V_{dc} - L_2 \frac{dI_{c2}}{dt} - R_2 \cdot I_{c2}$$

$+ \Rightarrow$ $$V_{dc} \cdot (d_{z1} - d_{z2}) = (L_1 + L_2)\frac{dI_0}{dt} + (R_1 + R_2) \cdot I_0$$

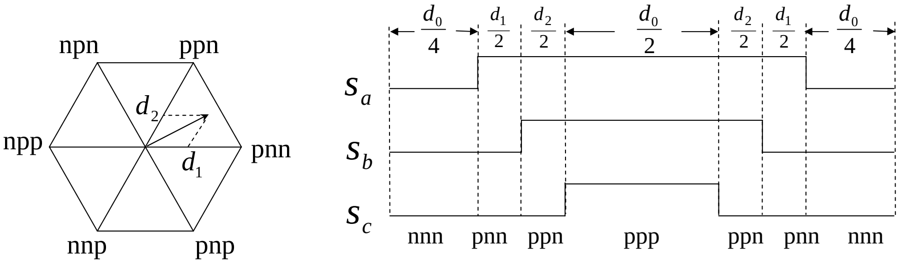
Equivalent circuit



DB-240

Zero-Sequence Duty Cycle

For example:

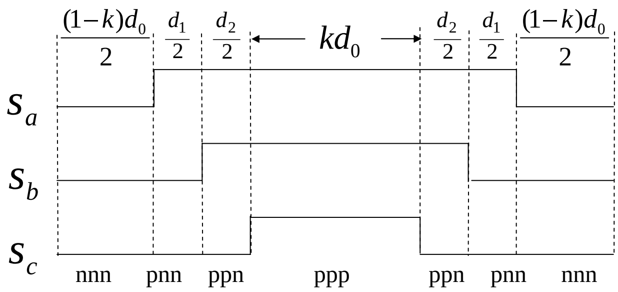


$$d_z = d_a + d_b + d_c = (d_1 + d_2 + 0.5d_0) + (d_2 + 0.5d_0) + 0.5d_0$$
$$= d_1 + 2d_2 + 1.5d_0$$

DB-241

A New Zero-Sequence Control Variable *k*

$k \equiv d_{ppp}$ i.e. the duration of zero-vector ppp



Therefore:

$$d_z = d_a + d_b + d_c = (d_1 + d_2 + kd_0) + (d_2 + kd_0) + kd_0$$
$$= d_1 + 2d_2 + 3kd_0$$

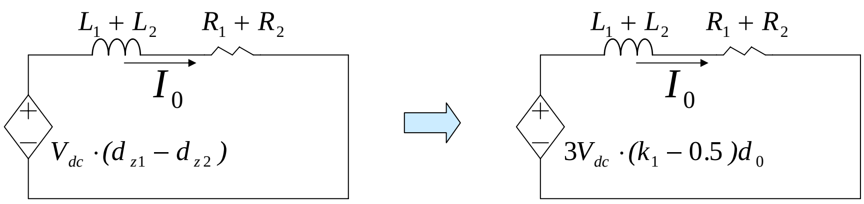
- k is not definable in minimum loss SVM schemes

DB-242

A New Zero-Sequence Model with the New Control Variable *k*

$$d_{z1} - d_{z2} = d_1 + 2d_2 + 3k_1 d_0 - (d_1 + 2d_2 + 3k_2 d_0)$$
$$= 3(k_1\text{-}0.5)d_0$$

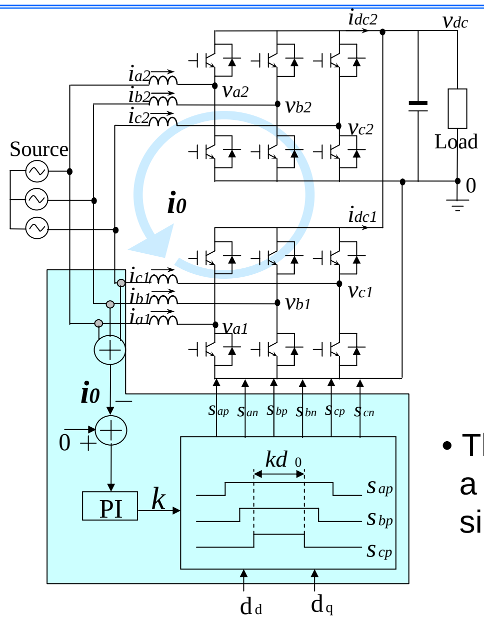
where k_2 is set to 0.5



- **I_0 can be controlled by controlling k_1 dynamically**
- **High control bandwidth can be achieved because it is a first-order system**

DB-243

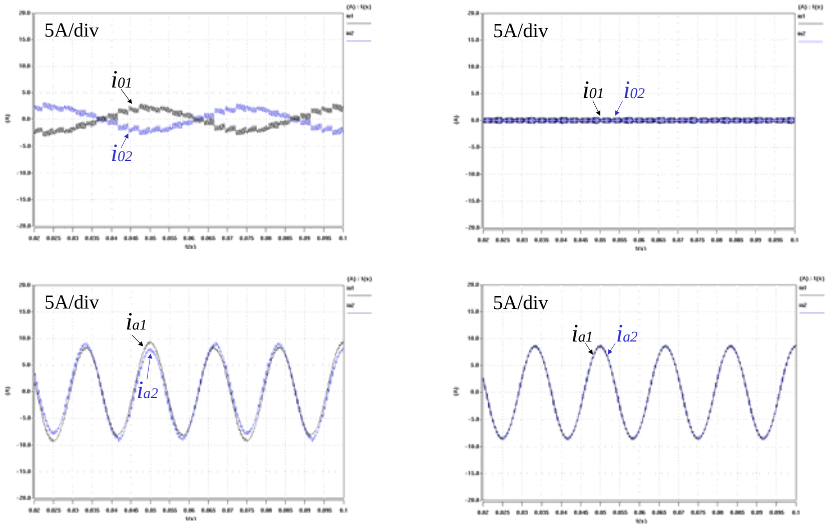
Implementation



- This part is added onto a regular controller for a single converter.

DB-244

Simulation Results

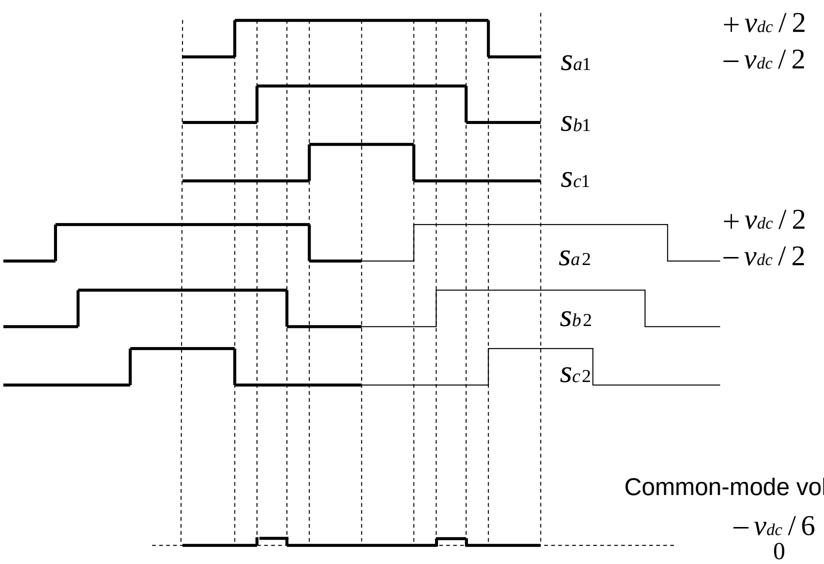


Without Zero-sequence control　　With Zero-sequence control

(Two converters with different switching frequencies operate independently)

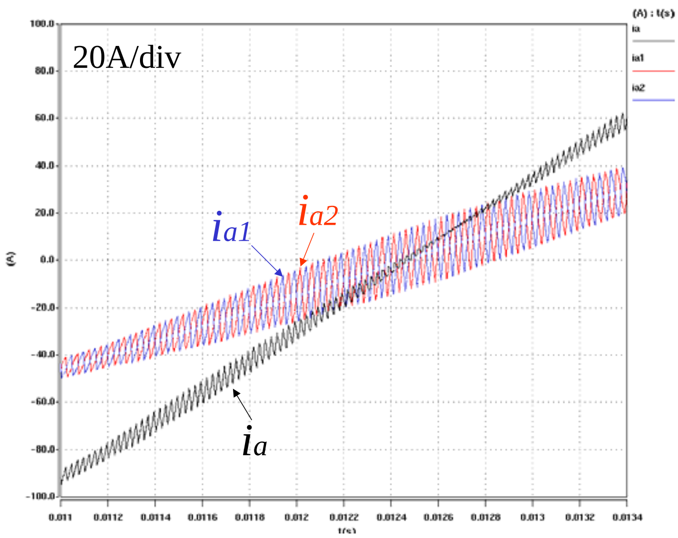
DB-245

Clock Phase Shifting Common-Mode Voltage Reduction



DB-246

Clock Phase Shifting
Differential-Mode Ripple Reduction



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Thank You

PECon 2008

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Joint PELS / IAS / IES Chapter of IEEE Malaysia Section

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