MIMO Coding and Relaying

元　永　亮
Prof Guan, Yong Liang

Head – Communication Engineering Division

Director - Positioning and Wireless Technology Centre

School of Electrical and Electronic Engineering

NANYANG TECHNOLOGICAL UNIVERSITY

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Outline

- Group-Decodable STBC with Rate ≤ 1
- Fast-Group-Decodable STBC with Rate > 1
- Shift-Orthogonal STBC
- 2-Step 2-Way MIMO Relaying

Group-Decodable (Quasi-Orthogonal) STBC

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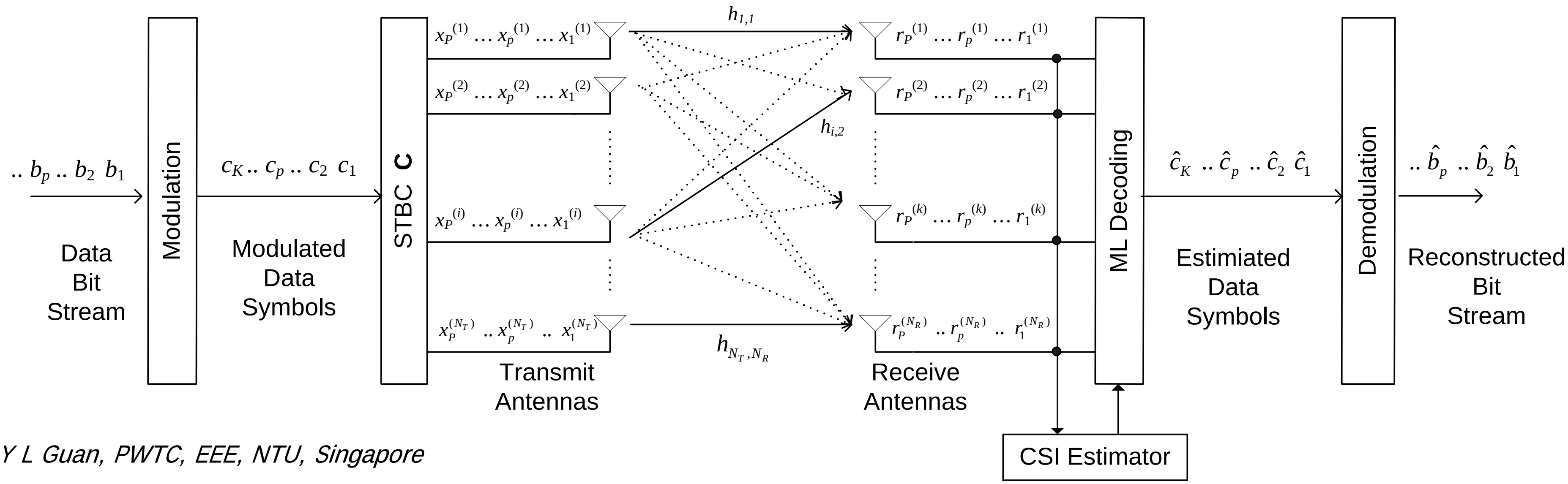
Introduction: MIMO Communications

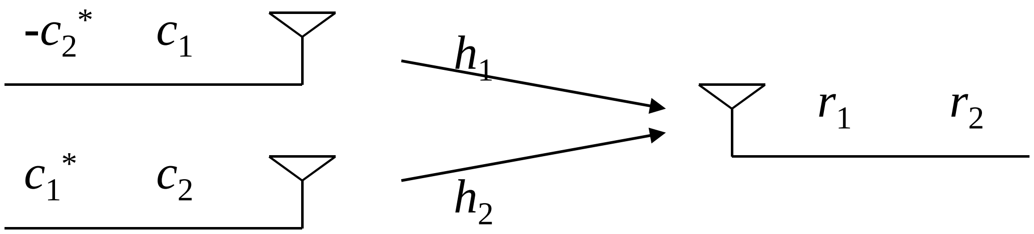
- MIMO: Multiple-Input Multiple-Output
- Multi-antenna transmitter/receiver, space time processing.
- Benefits - Increased data rate (spatial multiplexing gain)
  - Improved link reliability (transmit diversity gain)
- Bell Lab Layered Space Time (BLAST) code (1998), Alamouti's code (1998), Golden code (2005) …



Introduction: Transmit Diversity

- *Slow flat fading* cause severe reception problems in wireless radio communication
  - FEC decoding fails due to long error bursts
  - Typically mitigated by Rx antenna diversity
  - But Rx diversity may not be effective in mobile phones due to insufficient antenna separation and cost concern.
- *Transmit diversity* offers an attractive solution because
  - Base stations can afford more antenna separation
  - Cost of additional antennas at BS is amortized over all users

Introduction: Transmit Diversity



$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2^* \end{bmatrix}$$

$\mathbf{H_c}$

$$\begin{bmatrix} \hat{c}_1 \\ \hat{c}_2 \end{bmatrix} = \mathbf{H}_\mathbf{C}^\mathrm{H} \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix}$$

$$= \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} h_1^* \eta_1 + h_2 \eta_2^* \\ h_2^* \eta_1 - h_1 \eta_2^* \end{bmatrix}$$

$\mathbf{H_c}^\mathrm{H}\mathbf{H_c}$

1. c_1 and c_2 decoupled
2. MRC effect achieved using multiple tx antennas

Orthogonal STBC

- 2-antenna tx diversity code on last page

  = Alamouti's STBC

  = an orthogonal space-time block code (O-STBC)

- Strength of O-STBC
  - Provide full tx diversity (diversity order = no. of tx ant)
  - Linear symbol-by-symbol ML decoding

    → single symbol decodable

- **O-STBC code is unitary:** $\mathbf{H}_{\mathbf{C}}^{\mathrm{H}}\mathbf{H}_{\mathbf{C}} = \alpha \mathbf{I}_{N_{\mathrm{T}}}$

Orthogonal STBC

But O-STBC has low code rates

- Complex O-STBC (MPSK, QAM) have code rate < 1 for diversity level > 2
  - Diversity level 3-4:
    → max rate = ¾
  - Diversity level 5-8
    → max rate = ½ (for sq O-STBC)

→ Requires rate adaptation

→ Complicates design of cooperative relay network etc.

$$\mathbf{GS4} = \begin{bmatrix} c_1 & 0 & c_2 & -c_3 \\ 0 & c_1 & c_3^* & c_2^* \\ -c_2^* & -c_3 & c_1^* & 0 \\ c_3^* & -c_2 & 0 & c_1^* \end{bmatrix}$$

$$\mathbf{C8} = \begin{bmatrix} c_1 & c_1 & c_2^* & -c_2^* & c_3^* & -c_3^* & c_4^* & -c_4^* \\ c_1^* & -c_1^* & c_2 & c_2 & c_3 & c_3 & c_4 & c_4 \\ -c_2^* & c_2^* & c_1 & c_1 & c_4 & -c_4 & -c_3 & c_3 \\ -c_2 & -c_2 & c_1^* & -c_1^* & c_4^* & c_4^* & -c_3^* & -c_3^* \\ -c_3^* & c_3^* & -c_4 & c_4 & c_1 & c_1 & c_2 & -c_2 \\ -c_3 & -c_3 & -c_4^* & -c_4^* & c_1^* & -c_1^* & c_2^* & c_2^* \\ -c_4^* & c_4^* & c_3 & -c_3 & -c_2 & c_2 & c_1 & c_1 \\ -c_4 & -c_4 & c_3^* & c_3^* & -c_2^* & -c_2^* & c_1^* & -c_1^* \end{bmatrix}$$

Quasi-Orthogonal STBC

$$\mathbf{C} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2^* & c_1^* & -c_4^* & c_3^* \\ -c_3^* & -c_4^* & c_1^* & c_2^* \\ c_4 & -c_3 & -c_2 & c_1 \end{bmatrix}$$

$$\mathbf{r} = \mathbf{H}_\mathbf{c}\mathbf{c} + \boldsymbol{\eta} \qquad \begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2^* \\ \eta_3^* \\ \eta_4 \end{bmatrix}$$

$$\hat{\mathbf{c}} = \mathbf{H}_\mathbf{c}^\mathrm{H}\mathbf{r} \qquad \begin{bmatrix} \hat{c}_1 \\ \hat{c}_2 \\ \hat{c}_3 \\ \hat{c}_4 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & \beta & 0 \\ 0 & \beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \begin{bmatrix} \eta_1' \\ \eta_2' \\ \eta_3' \\ \eta_4' \end{bmatrix}$$

Quasi-Orthogonal STBC

$$\begin{bmatrix}\hat{c}_1\\\hat{c}_2\\\hat{c}_3\\\hat{c}_4\end{bmatrix}=\begin{bmatrix}\alpha & 0 & 0 & \beta\\0 & \alpha & \beta & 0\\0 & \beta & \alpha & 0\\\beta & 0 & 0 & \alpha\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\\c_4\end{bmatrix}+\begin{bmatrix}\eta'_1\\\eta'_2\\\eta'_3\\\eta'_4\end{bmatrix}$$

$$\begin{bmatrix}\hat{c}_1\\\hat{c}_4\end{bmatrix}=\begin{bmatrix}\alpha & \beta\\\beta & \alpha\end{bmatrix}\begin{bmatrix}c_1\\c_4\end{bmatrix}+\begin{bmatrix}\eta'_1\\\eta'_4\end{bmatrix}$$

$$\begin{bmatrix}\hat{c}_2\\\hat{c}_3\end{bmatrix}=\begin{bmatrix}\alpha & \beta\\\beta & \alpha\end{bmatrix}\begin{bmatrix}c_2\\c_3\end{bmatrix}+\begin{bmatrix}\eta'_2\\\eta'_3\end{bmatrix}$$

QO-STBC: $H_c^H H_c$ is block diagonal

- Quasi-orthogonal → group-orthogonal → group-decodable
- Symbols within a group are not orthogonal
  → must be jointly detected → higher decoding complexity

QO-STBC with Minimum Decoding Complexity

- MDC QO-STBC
  - Joint detection of 2 real symbols ≡ single (complex) symbol decodable
  - **Smallest group size for any QO-STBC**

    [Yuen, Guan, Tjhung, IEEE Trans WCOM, Sep 2005], [X G Xia et al], [S Rajan et al] .
- Can be systematically constructed using smaller O-STBC:

$$\text{Rule \#1: } \mathbf{A}_q = \begin{bmatrix} \underline{\mathbf{A}}_q & \mathbf{0} \\ \mathbf{0} & \underline{\mathbf{A}}_q \end{bmatrix}; \quad \text{Rule \#3: } \mathbf{A}_{K+q} = \begin{bmatrix} j\underline{\mathbf{B}}_q & \mathbf{0} \\ \mathbf{0} & j\underline{\mathbf{B}}_q \end{bmatrix};$$

$$1 \le q \le K$$

$$\text{Rule \#2: } \mathbf{B}_q = \begin{bmatrix} \mathbf{0} & j\underline{\mathbf{A}}_q \\ j\underline{\mathbf{A}}_q & \mathbf{0} \end{bmatrix}; \quad \text{Rule \#4: } \mathbf{B}_{K+q} = \begin{bmatrix} \mathbf{0} & \underline{\mathbf{B}}_q \\ \underline{\mathbf{B}}_q & \mathbf{0} \end{bmatrix}.$$

MDC QO-STBC

$$
C_4 = \begin{bmatrix}
c_1^R + jc_3^I & c_2^R + jc_4^I & -c_3^R - jc_1^I & -c_4^R - jc_2^I \\
-c_2^R + jc_4^I & c_1^R - jc_3^I & -c_4^R + jc_2^I & c_3^R - jc_1^I \\
c_3^R + jc_1^I & c_4^R + jc_2^I & c_1^R + jc_3^I & c_2^R + jc_4^I \\
c_4^R - jc_2^I & -c_3^R + jc_1^I & -c_2^R + jc_4^I & c_1^R - jc_3^I
\end{bmatrix}
$$

← **rate = 4 / 4 = 1**

rate = 6 / 8 = 3 / 4 ↙

$$
C_8 = \begin{bmatrix}
c_1^R + jc_4^I & c_2^R + jc_5^I & c_3^R + jc_6^I & 0 & c_4^R + jc_1^I & c_5^R + jc_2^I & c_6^R + jc_3^I & 0 \\
-c_2^R + jc_5^I & c_1^R - jc_4^I & 0 & -c_3^R - jc_6^I & c_5^R - jc_2^I & -c_4^R + jc_1^I & 0 & c_6^R + jc_3^I \\
c_3^R - jc_6^I & 0 & -c_1^R + jc_4^I & -c_2^R - jc_5^I & -c_6^R + jc_3^I & 0 & c_4^R - jc_1^I & c_5^R + jc_2^I \\
0 & -c_3^R + jc_6^I & c_2^R - jc_5^I & -c_1^R - jc_4^I & 0 & c_6^R - jc_3^I & -c_5^R + jc_2^I & c_4^R + jc_1^I \\
-c_4^R - jc_1^I & -c_5^R - jc_2^I & -c_6^R - jc_3^I & 0 & c_1^R + jc_4^I & c_2^R + jc_5^I & c_3^R + jc_6^I & 0 \\
-c_5^R + jc_2^I & c_4^R - jc_1^I & 0 & c_6^R + jc_3^I & -c_2^R + jc_5^I & c_1^R - jc_4^I & 0 & c_3^R + jc_6^I \\
c_6^R - jc_3^I & 0 & -c_4^R + jc_1^I & c_5^R + jc_2^I & c_3^R - jc_6^I & 0 & -c_1^R + jc_4^I & c_2^R + jc_5^I \\
0 & c_6^R - jc_3^I & -c_5^R + jc_2^I & -c_4^R - jc_1^I & 0 & c_3^R - jc_6^I & -c_2^R + jc_5^I & -c_1^R - jc_4^I
\end{bmatrix}
$$

MDC QO-STBC: Single Symbol Decodable

Decoding metrics for C_4

$$f_1(c_1)=\sum_{k=1}^{N_R}\left[\left(\sum_{i=1}^{4}\left|h_{i,k}\right|^2\right)\left(\left|c_1^R\right|^2+\left|c_1^I\right|^2\right)+2\,\mathrm{Re}\left\{c_1^R(\alpha)-c_1^I(\beta)-c_1^Rc_1^I(\gamma)\right\}\right]$$

$$f_2(c_2)=\sum_{k=1}^{N_R}\left[\left(\sum_{i=1}^{4}\left|h_{i,k}\right|^2\right)\left(\left|c_2^R\right|^2+\left|c_2^I\right|^2\right)+2\,\mathrm{Re}\left\{c_2^R(\chi)-c_2^I(\delta)-c_2^Rc_2^I(\varphi)\right\}\right]$$

$$f_3(c_3)=\sum_{k=1}^{N_R}\left[\left(\sum_{i=1}^{4}\left|h_{i,k}\right|^2\right)\left(\left|c_3^R\right|^2+\left|c_3^I\right|^2\right)+2\,\mathrm{Re}\left\{jc_3^R(\alpha)+jc_3^I(\beta)+c_3^Rc_3^I(\gamma)\right\}\right]$$

$$f_4(c_4)=\sum_{k=1}^{N_R}\left[\left(\sum_{i=1}^{4}\left|h_{i,k}\right|^2\right)\left(\left|c_4^R\right|^2+\left|c_4^I\right|^2\right)+2\,\mathrm{Re}\left\{jc_4^R(\chi)+jc_4^I(\delta)+c_4^Rc_4^I(\varphi)\right\}\right]$$

$$\alpha=-h_{1,k}r_1^{(k)*}-h_{2,k}^*r_2^{(k)}-h_{3,k}r_3^{(k)*}-h_{4,k}^*r_4^{(k)},\qquad \chi=-h_{2,k}r_1^{(k)*}+h_{1,k}^*r_2^{(k)}-h_{4,k}r_3^{(k)*}+h_{3,k}^*r_4^{(k)},$$

$$\beta=-h_{3,k}r_1^{(k)*}-h_{4,k}^*r_2^{(k)}-h_{1,k}r_3^{(k)*}-h_{2,k}^*r_4^{(k)},\qquad \delta=-h_{4,k}r_1^{(k)*}+h_{3,k}^*r_2^{(k)}-h_{2,k}r_3^{(k)*}+h_{1,k}^*r_4^{(k)},$$

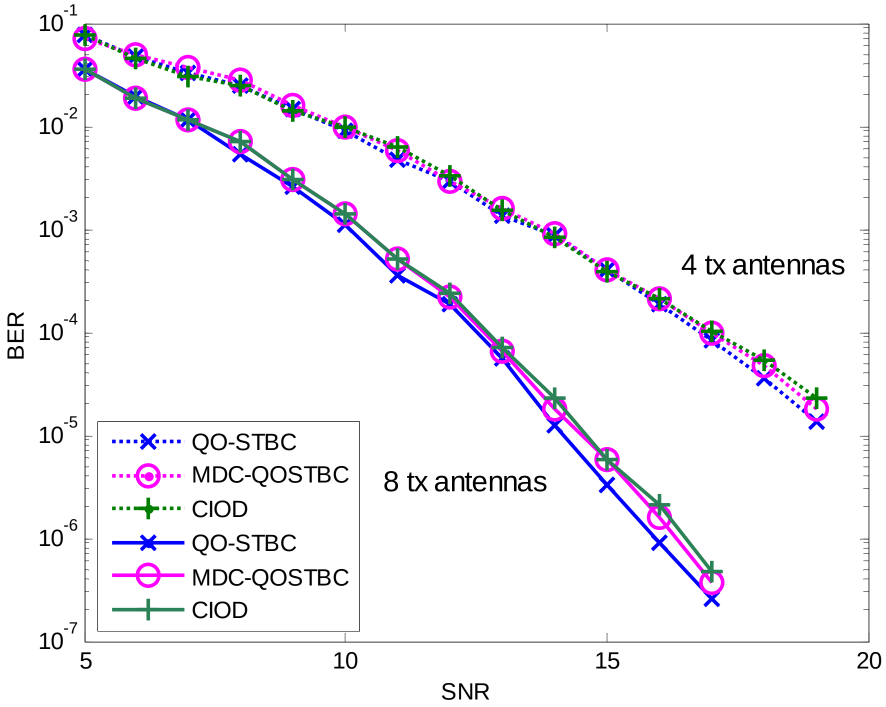
$$\gamma=2\,\mathrm{Re}(h_{1,k}h_{3,k}^*+h_{2,k}h_{4,k}^*),\qquad \varphi=2\,\mathrm{Re}(h_{1,k}h_{3,k}^*+h_{2,k}h_{4,k}^*)$$

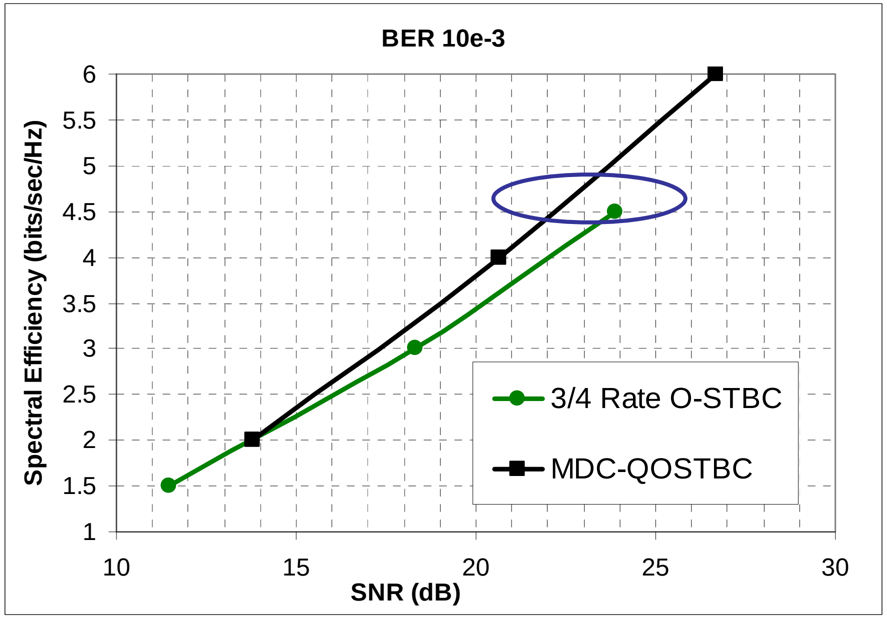
MDC QO-STBC: Benefits

- vs. Square O-STBC
  - **Rate increased by 1/4**
    - → smaller QAM size
    - → larger Euclidean distance
    - → lower BER/SER
- vs. other QO-STBC
  - **Min. decoding complexity**
  - little/no performance loss

	Code Rate		Decoding Complexity	Origin
	3-4 tx ants	**5-8 tx ants**		
O-STBC	3/4	1/2	Symbol-by-symbol detection	AT&T, Nokia, UU
QO-STBC	1	3/4	Joint detection of 2 **complex** symbols	AT&T, Nokia, Bell Lab
MDC-QOSTBC	**1**	**3/4**	**Joint detection of 2 real symbols**	**NTU**

MDC QO-STBC: Performance





< 0.5 dB loss from *double*-symbol-decodable QO-STBC

gain over rate-3/4 *O-STBC* increases with spectral efficiency

Non-Coherent Tx Diversity

- Non-coherent tx diversity can be achieved using Differential Space Time Modulation (DSTM)
  - Start with a known seed codeword $\mathbf{X}_o$
  - Transmit $\mathbf{X}_p = \mathbf{X}_{p-1}\mathbf{U}_p$
    where U_p = **unitary** info-carrying STBC
  - Receive $\mathbf{R}_p = \mathbf{H}_p\mathbf{X}_p + \mathbf{N}_p$
    $\cong \mathbf{R}_{p-1}\mathbf{U}_p + \tilde{\mathbf{N}}_p$ ← assume $\mathrm{H}_p \cong \mathrm{H}_{p-1}$
  - Decode $\hat{\mathbf{U}}_p = \arg \underset{\mathbf{U}_p \in \boldsymbol{U}}{\mathrm{Max}} \,\mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{R}_p^{\mathrm{H}}\mathbf{R}_{p-1}\mathbf{U}_p\right)\right\}$

    ↑ no need to know H_p

Non-Coherent Tx Diversity

QO-STBC is not unitary → cannot be used for DSTM

$$\mathbf{C}\mathbf{C}^{\mathrm{H}} = \frac{\alpha}{K}\begin{bmatrix} \mathbf{I}_{N_t/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_t/2} \end{bmatrix} + \frac{\beta}{K}\begin{bmatrix} \mathbf{0} & \mathbf{I}_{N_t/2} \\ \mathbf{I}_{N_t/2} & \mathbf{0} \end{bmatrix}$$

$$\alpha = \sum_{i=1}^{K} |c_i|^2 \qquad \beta = 2\sum_{i=1}^{K/2} -c_i^{\mathrm{R}} c_i^{\mathrm{I}} + c_{K/2+i}^{\mathrm{R}} c_{K/2+i}^{\mathrm{I}}$$

But MDC-QOSTBC can be rendered quasi-unitary if its symbol constellation meets following conditions:

$\mathrm{E}(\alpha)$ = constant $\qquad \beta = 0$

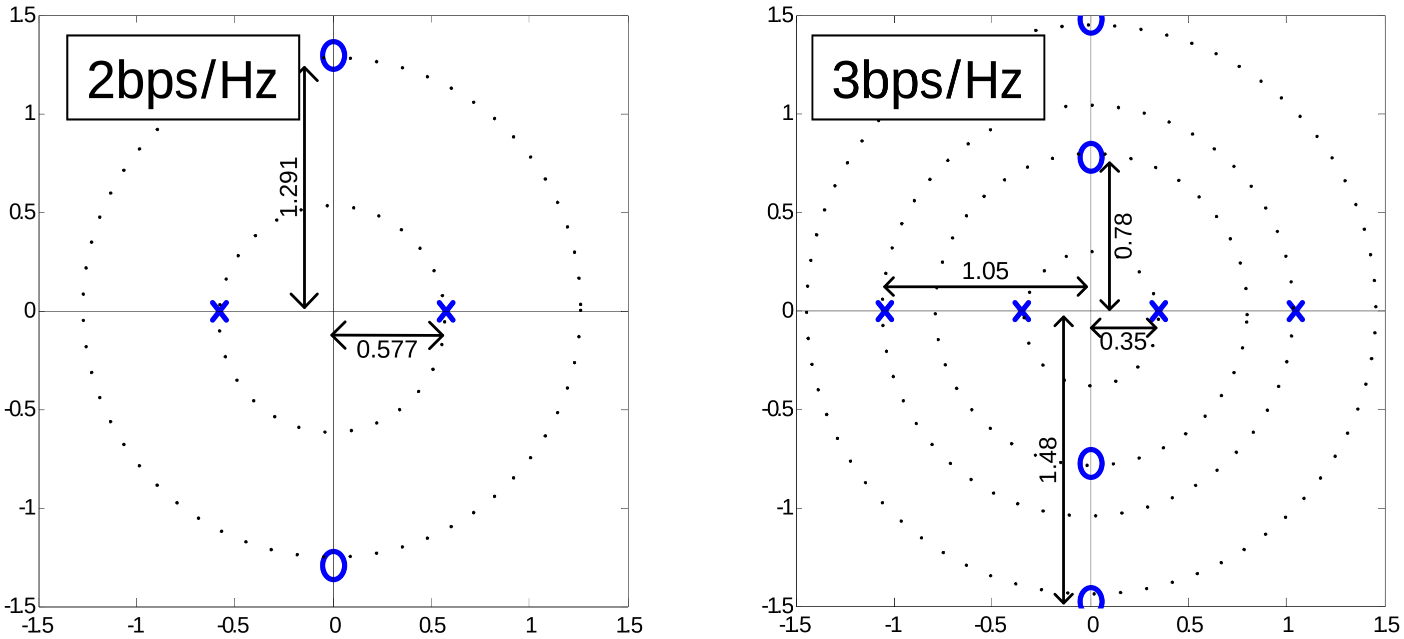
Single-Symbol-Decodable Differential ST Modulation

- Differential MDC QO-STBC
  - Single-symbol-decodable DSTM
  - **Unitary criterion met by special constellation design**

    [Yuen, Guan, Tjhung, IEEE Trans WCOM, Oct 2006]
  - Same code rate and benefits as coherent MDC QO-STBC

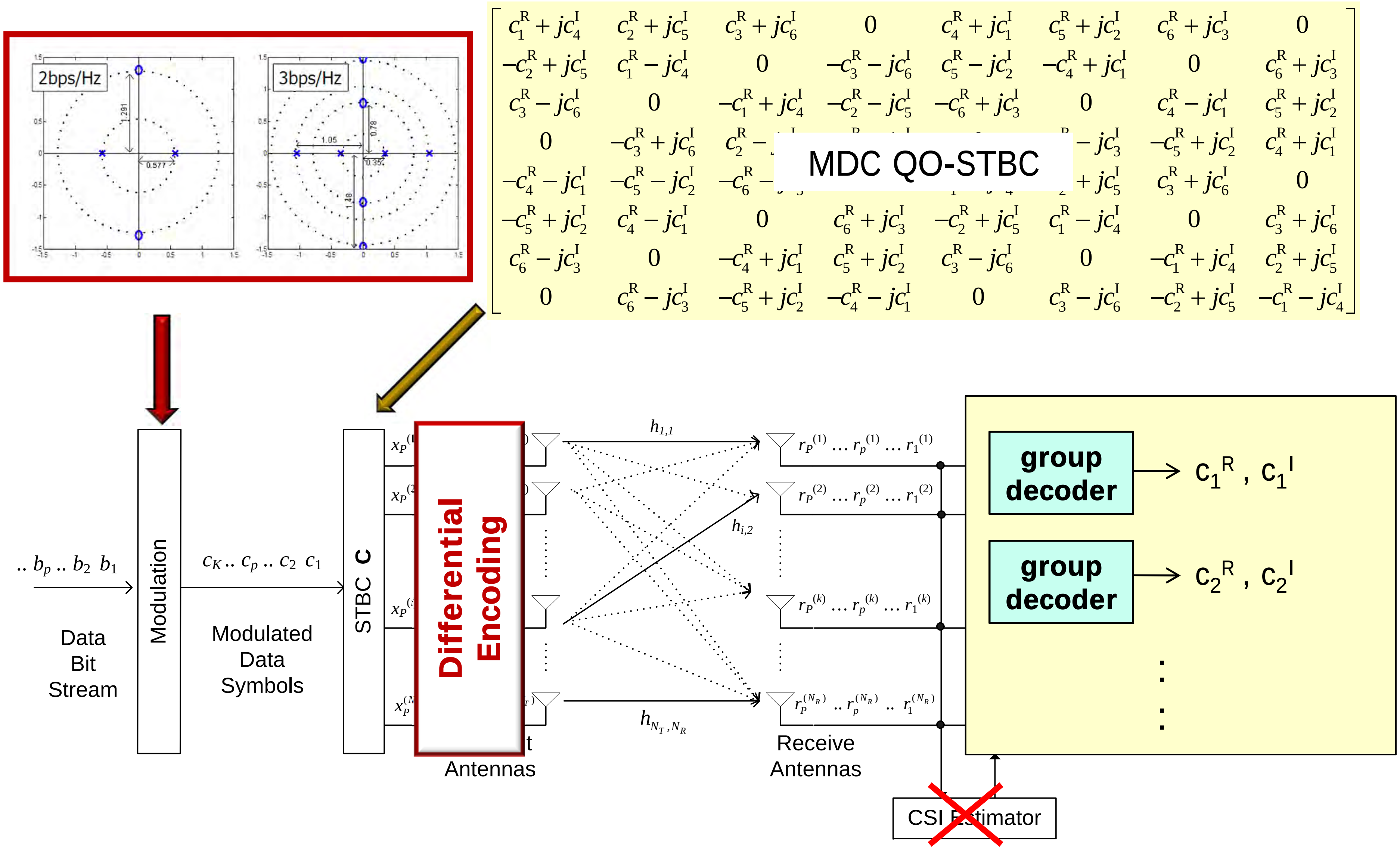
Single-Symbol-Decodable Differential ST Modulation

Single-Symbol-Decodable DSTM constellation design for 4 antennas



spectral efficiency = $R \log M$ bps/Hz

where R = MDC-QOSTBC code rate, M = constellation size

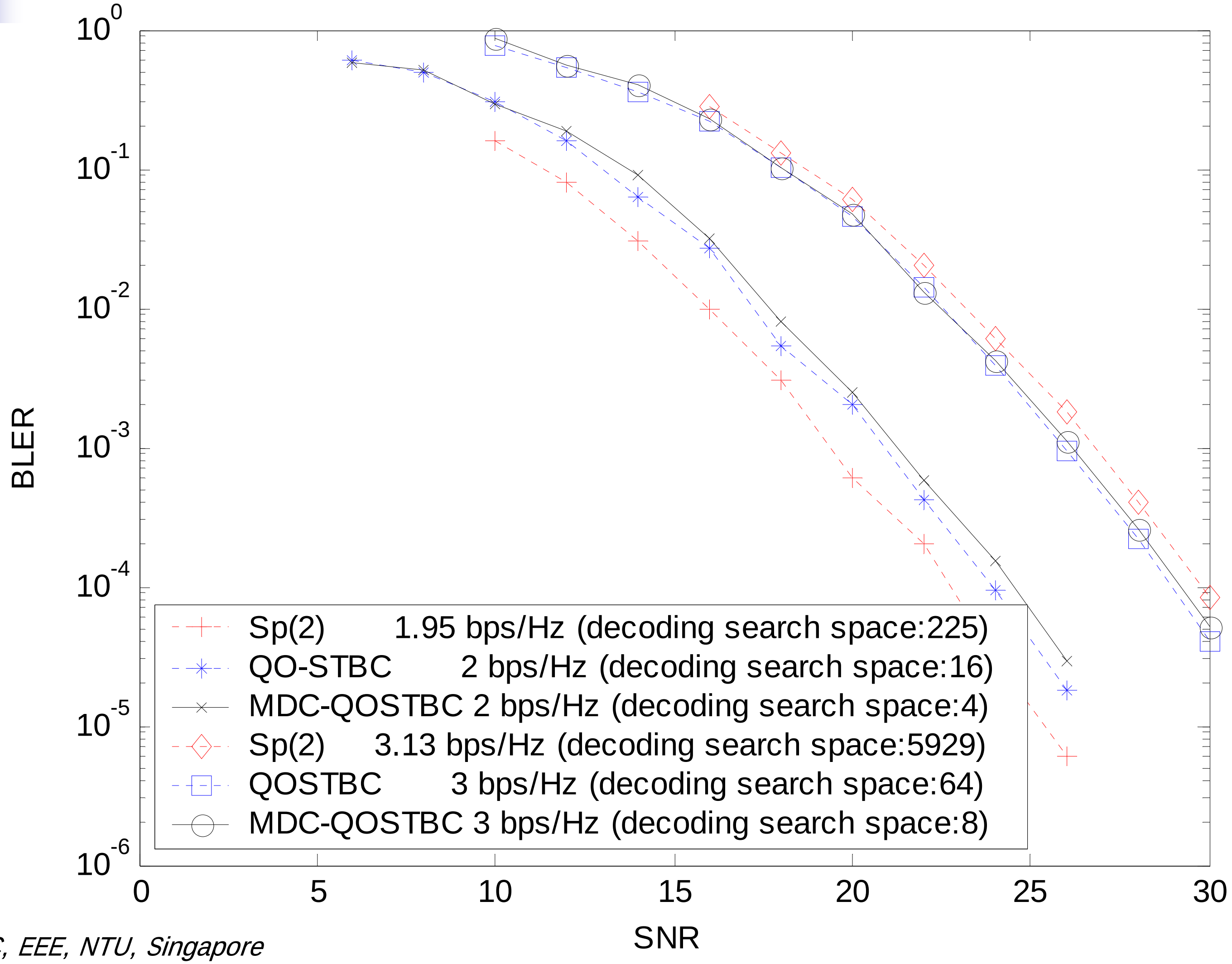


2bps/Hz
3bps/Hz
MDC QO-STBC
Modulation
STBC C
Differential Encoding
Data Bit Stream
Modulated Data Symbols
Antennas
Receive Antennas
group decoder
group decoder
CSI Estimator

Single-Symbol-Decodable DSTM: Decoding Complexity

bps/Hz	*DSTM Scheme*	*Decoding Search Space Dimension*
1.95	Sp2 Code	225
2	Group Code	256
2	Zhu & Jafarkhani WCNC 2004	16
2	**Single-Symbol-Decodable DSTM (proposed)**	**4**
3.13	Sp2 Code	5929
3	Zhu & Jafarkhani WCNC 2004	64
3	**Single-Symbol-Decodable DSTM (proposed)**	**8**

Single-Symbol-Decodable DSTM: Performance



References

- (monograph) C Yuen, Y L Guan, T T Tjhung, *Quasi-Orthogonal Space Time Block Code*, Imperial College Press, Dec 2007.
- C. Yuen; Y. L. Guan; T. T. Tjhung, "Quasi-Orthogonal STBC with Minimum Decoding Complexity", *IEEE Trans. Wireless Comms.*, vol. 4, Sept. 2005.
- C. Yuen; Y. L. Guan; T. T. Tjhung, "Single-Symbol-Decodable Differential Space-Time Modulation Based on QO-STBC", *IEEE Trans Wireless Comm*, October 2006.

Outline

- Group-Decodable STBC with Rate ≤ 1
- Fast-Group-Decodable STBC with Rate > 1
- Shift-Orthogonal STBC
- 2-Step 2-Way MIMO Relaying

Group-Decodable STBC with Rate > 1

Fast-Group-Decodable STBC with Rate > 1

T. P. Ren

National University of Defense Technology, China

Y. L. Guan

Nanyang Technological University, Singapore

C. Yuen

Instititute for Infocomm Research, Singapore

Motivation

- **To have spatial multiplexing gain, STBC needs code rate > 1 (e.g. Golden code, Perfect code)**
- But decoding complexity of high-rate STBC is very high, unless the code has group-decodable structure.
- Only two group-decodable high-rate STBC reported:
  - 2-group-decodable SMC of rate 1.25 for 4 transmit antennas obtained by computer search [C. Yuen, Y. L. Guan, and T. T. Tjhung, 2006];
  - 2-group-decodable square SMC for 2^m (integer m > 1) transmit antennas [K. Pavan Srinath and B. Sundar Rajan, 2009];

  → **Not available for any tx antenna number, any code rate.**

Group Decodable STBC with Rate > 1

Definition 2 (Group-Decodable STBC, T. P. Ren et al). An STBC with dispersion matrices $\mathbf{C}_1, \cdots, \mathbf{C}_L$ is said to be Γ-group-decodable if

(i) $\mathbf{C}_p^H \mathbf{C}_q = -\mathbf{C}_q^H \mathbf{C}_p, \ \forall p \in \Theta_i, \forall q \in \Theta_{i'}, i \neq i'$; ← **block diagonal property of $H_c^H H_c$**

(ii) $\begin{bmatrix}\mathbf{C}_{i_1}^R \\ \mathbf{C}_{i_1}^I\end{bmatrix}, \cdots, \begin{bmatrix}\mathbf{C}_{i_k}^R \\ \mathbf{C}_{i_k}^I\end{bmatrix}, \cdots, \begin{bmatrix}\mathbf{C}_{i_{L_i}}^R \\ \mathbf{C}_{i_{L_i}}^I\end{bmatrix}$ are linearly independent with $i_k \in \Theta_i$, $k = 1, 2, \cdots, L_i$, where $i = 1, 2, \cdots, \Gamma$, Θ_i denotes the set of indexes of symbols in the ith group with $|\Theta_i| = L_i$ and $\sum_{i=1}^{\Gamma} L_i = L$.

Signal Model

$$\tilde{\mathbf{R}} = \sqrt{\rho}\mathbf{X}\tilde{\mathbf{H}} + \tilde{\mathbf{Z}} \quad \Longrightarrow \quad \mathbf{r} = \sqrt{\rho}\mathbf{H}\mathbf{s} + \mathbf{z}$$

where $l = 1, 2, \cdots, L$ and

$$\mathbf{r} = \begin{bmatrix} \tilde{\mathbf{r}}_1^R \\ \tilde{\mathbf{r}}_1^I \\ \vdots \\ \tilde{\mathbf{r}}_{N_r}^R \\ \tilde{\mathbf{r}}_{N_r}^I \end{bmatrix}, \ \bar{\mathbf{h}} = \begin{bmatrix} \tilde{\mathbf{h}}_1^R \\ \tilde{\mathbf{h}}_1^I \\ \vdots \\ \tilde{\mathbf{h}}_{N_r}^R \\ \tilde{\mathbf{h}}_{N_r}^I \end{bmatrix}, \ \mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_L \end{bmatrix}, \ \mathbf{z} = \begin{bmatrix} \tilde{\mathbf{z}}_1^R \\ \tilde{\mathbf{z}}_1^I \\ \vdots \\ \tilde{\mathbf{z}}_{N_r}^R \\ \tilde{\mathbf{z}}_{N_r}^I \end{bmatrix},$$

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_L] = \begin{bmatrix} \mathscr{C}_1\bar{\mathbf{h}} & \mathscr{C}_2\bar{\mathbf{h}} & \cdots & \mathscr{C}_L\bar{\mathbf{h}} \end{bmatrix},$$

$$\mathscr{C}_l = \begin{bmatrix} \mathcal{C}_l & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathcal{C}_l & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathcal{C}_l \end{bmatrix}_{N_r \times N_r}, \ \mathcal{C}_l = \begin{bmatrix} \mathbf{C}_l^R & -\mathbf{C}_l^I \\ \mathbf{C}_l^I & \mathbf{C}_l^R \end{bmatrix}_{2\times 2}.$$

Code Construction

- Step 1: Pick a full-rank matrix of size 2x2

$$\mathbf{C}_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- Step 2: Write (i) as $C\bar{y} = 0$ ← **over-determined**

where

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\bar{y} = \begin{bmatrix} y_{11}^R & y_{11}^I & y_{21}^R & y_{21}^I & y_{12}^R & y_{12}^I & y_{22}^R & y_{22}^I \end{bmatrix}^T$$

T. P. Ren, Y. L. Guan, C. Yuen, E. Gunawan and E. Y. Zhang, "Unbalanced and Balanced 2-Group Decodable Spatial Multiplexing Code," IEEE VTC-Fall, Sep 2009.

- Step 3: Obtain the solution space $\overline{y_i}$ of (i):

$$\overline{y} = k_1\overline{y}_1 + k_2\overline{y}_2 + k_3\overline{y}_3 + k_4\overline{y}_4$$

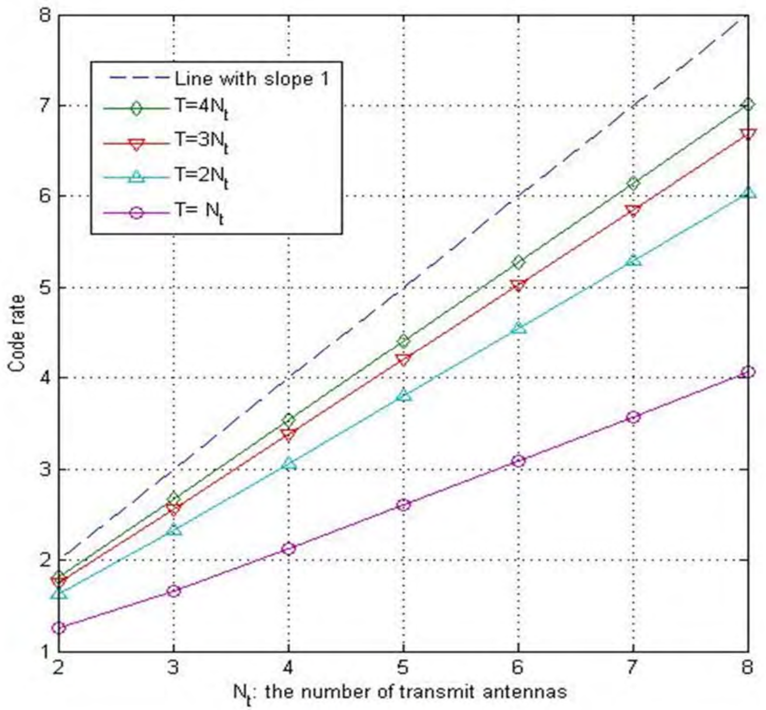
$$= k_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} + k_3 \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + k_4 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

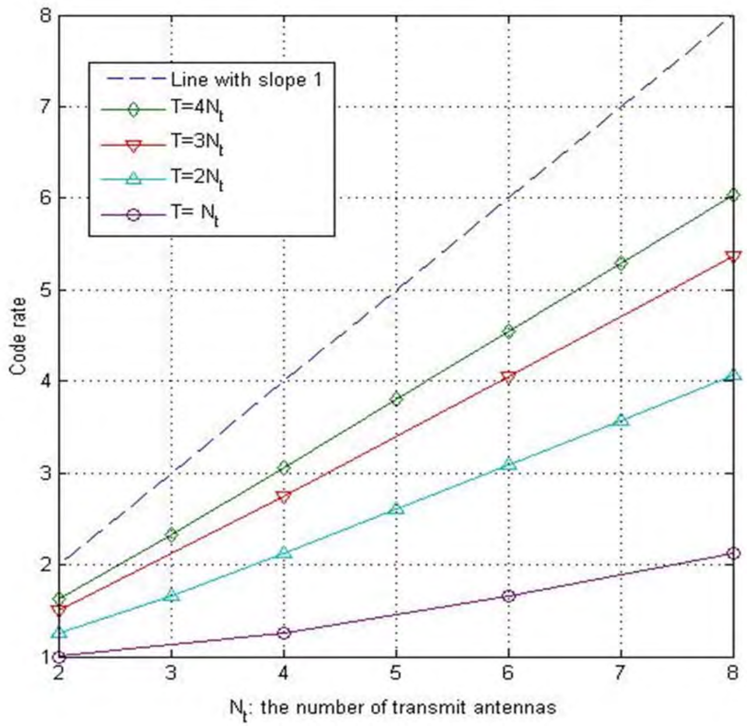
- Step 4: Transform $\overline{y_i}$ to form dispersion matrices of 2nd group:

$$Y_1 = \begin{bmatrix} j & j \\ j & -j \end{bmatrix}, Y_2 = \begin{bmatrix} j & -j \\ j & j \end{bmatrix}, Y_3 = \begin{bmatrix} j & j \\ -j & j \end{bmatrix}, Y_4 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

Achievable Code Rates

	Unbalanced	Balanced
$T < N_t$	$\frac{2TN_t - N_t^2 + 1}{2T}$	$\frac{TN_t - N_t^2 + 1}{T}$
$T \geq N_t$	$\frac{T^2 + 1}{2T}$	$\frac{T^2 + 4}{4T}$





Group-Decodable STBC with Rate > 1

Fast-Group-Decodable STBC with Rate > 1

T. P. Ren

National University of Defense Technology, China

<u>Y. L. Guan</u>

Nanyang Technological University, Singapore

C. Yuen

Instititute for Infocomm Research, Singapore

Fast Decoding of STBC

$$\mathbf{r}=\mathbf{Hs}+\mathbf{z} \Longrightarrow \mathbf{r}'=\mathbf{Rs}+\mathbf{z}'$$

where $\mathbf{s}=[s_1\ s_2\ \ldots\ s_L]^T$, $\mathbf{H}=\mathbf{QR}$ is QR decomposition,

$$\mathbf{r}'=[r'_1,\ r'_2,\ \ldots\ ]^T=\mathbf{Q}^T\mathbf{r},\ \mathbf{z}'=\mathbf{Q}^T\mathbf{z}.$$

The code is fast-decodable if $\mathbf{R}=\begin{bmatrix} f_{11} & 0 & \cdots & f_{1L} \\ 0 & f_{22} & \cdots & f_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & f_{LL} \end{bmatrix}$

Fast decoding:

1) $r'_1, r'_3, \cdots, r'_{TN_r} \longrightarrow$ ML joint decoding $\longrightarrow s_1^{ML}$

2) $r'_2, r'_3, \cdots, r'_{TN_r} \longrightarrow$ ML joint decoding $\longrightarrow s_2^{ML}$

3) $r'_3, \cdots, r'_{TN_r} \longrightarrow$ ML joint decoding $\xrightarrow{s_1^{ML},\, s_2^{ML}} s_3^{ML}, \cdots, s_L^{ML}$

Motivation

- Fast-decodable (FD) STBC was proposed by Biglieri et al in 2009 to reduce the QRM decoding complexity of high-rate STBC. But complexity reduction is limited.
- Group-Decodable (GD) STBC has more complexity reduction, but code rate is low.

➔ Nice to have STBC with both FD and GD advantages

Relationship between FD, GD and FGD

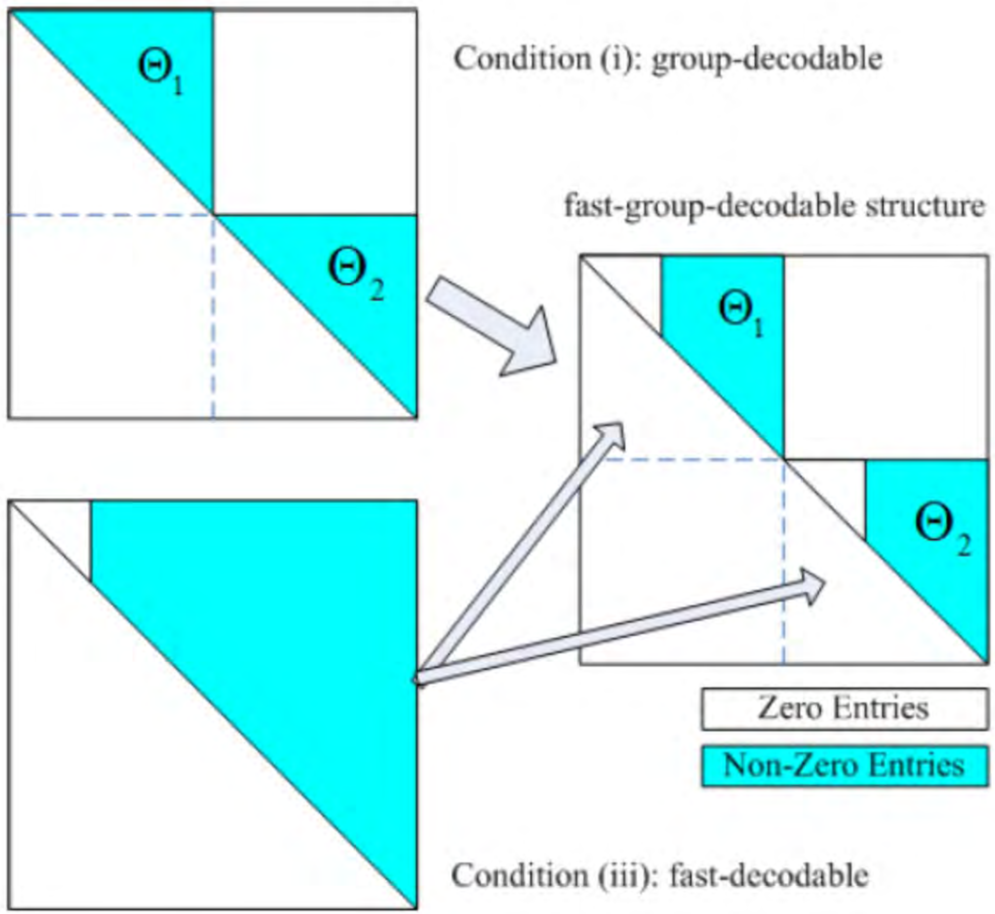


Fig. 1. Upper-triangular matrices **R**s for the 2-group-decodable (GD), fast-decodable (FD) and fast-group-decodable (FGD) code structures.

Outline

- Motivation
- Code Construction
- Performance
- Conclusions

Group-Decodable STBC

Definition 2 (Group-Decodable STBC, T. P. Ren et al). An STBC with dispersion matrices $\mathbf{C}_1, \cdots, \mathbf{C}_L$ is said to be Γ-group-decodable if

(i) $\mathbf{C}_p^H \mathbf{C}_q = -\mathbf{C}_q^H \mathbf{C}_p, \ \forall p \in \Theta_i, \forall q \in \Theta_{i'}, i \neq i'$;

(ii) $\begin{bmatrix} \mathbf{C}_{i_1}^R \\ \mathbf{C}_{i_1}^I \end{bmatrix}, \cdots, \begin{bmatrix} \mathbf{C}_{i_k}^R \\ \mathbf{C}_{i_k}^I \end{bmatrix}, \cdots, \begin{bmatrix} \mathbf{C}_{i_{L_i}}^R \\ \mathbf{C}_{i_{L_i}}^I \end{bmatrix}$ are linearly independent

with $i_k \in \Theta_i$ $, k = 1, 2, \cdots, L_i$, where $i = 1, 2, \cdots, \Gamma$, Θ_i denotes the set of indexes of symbols in the ith group with $|\Theta_i| = L_i$ and $\sum_{i=1}^{\Gamma} L_i = L$.

Fast-Group-Decodable STBC

Definition 3 (Fast-Group-Decodable STBC). An STBC with dispersion matrices $\mathbf{C}_1, \cdots, \mathbf{C}_L$ is said to be fast-group-decodable if

(i) $\mathbf{C}_p^H \mathbf{C}_q = -\mathbf{C}_q^H \mathbf{C}_p, \ \forall p \in \Theta_i, \forall q \in \Theta_{i'}, i \neq i'$;

(ii) $\begin{bmatrix} \mathbf{C}_{i_1}^R \\ \mathbf{C}_{i_1}^I \end{bmatrix}, \cdots, \begin{bmatrix} \mathbf{C}_{i_l}^R \\ \mathbf{C}_{i_l}^I \end{bmatrix}, \cdots, \begin{bmatrix} \mathbf{C}_{i_{L_i}}^R \\ \mathbf{C}_{i_{L_i}}^I \end{bmatrix}$ are linearly independent with $i_l \in \Theta_i, \ l = 1, 2, \cdots, L_i$;

(iii) $\mathbf{C}_{i_{l_1}}^H \mathbf{C}_{i_{l_2}} = -\mathbf{C}_{i_{l_2}}^H \mathbf{C}_{i_{l_1}}$ with $i_{l_1}, i_{l_2} \in \Theta_i, \ l_1 = 1, 2, \cdots, K_i - 1, \ l_2 = l_1 + 1, \cdots, K_i$ and $K_i \leq L_i$, where $i = 1, 2, \cdots, \Gamma$, Θ_i denotes the set of indexes of symbols in the ith group with $|\Theta_i| = L_i$ and $\sum_{i=1}^{\Gamma} L_i = L$. Then K_i levels are removed from the real SD tree for the ith group.

Ref: T. P. Ren, Y. L. Guan, C. Yuen and R. J. Shen, "Fast-Group-Decodable Space-Time Block Code,“ IEEE ITW'10, 6-8 Jan. 2010.

Code Construction (max complexity reduction)

Suppose that $\mathbf{C}_1, \cdots, \mathbf{C}_K$ are the dispersion matrices of an orthogonal STBC with the maximum code rate.

Step 1: Choose $\mathbf{C}_1$ as the seed matrix, then obtain the QOC equation and its J-dimension solution space $\{\bar{\mathbf{y}}\}$, denoted as $\{\bar{\mathbf{y}}_1, \bar{\mathbf{y}}_2, \cdots, \bar{\mathbf{y}}_J\}$.

Step 2: Under the vector-to-matrix mapping function g^{-1}, obtain $\{\begin{bmatrix}\mathbf{Y}_1^R\\ \mathbf{Y}_1^I\end{bmatrix}, \begin{bmatrix}\mathbf{Y}_2^R\\ \mathbf{Y}_2^I\end{bmatrix}, \cdots, \begin{bmatrix}\mathbf{Y}_J^R\\ \mathbf{Y}_J^I\end{bmatrix}\}$ where $\begin{bmatrix}\mathbf{Y}_i^R\\ \mathbf{Y}_i^I\end{bmatrix} = g^{-1}(\bar{\mathbf{y}}_i)$ with $i = 1, \cdots, J$.

Step 3: Let the basis of $\{\begin{bmatrix}\mathbf{C}_2^R\\ \mathbf{C}_2^I\end{bmatrix}, \cdots, \begin{bmatrix}\mathbf{C}_K^R\\ \mathbf{C}_K^I\end{bmatrix}\}' = \{\begin{bmatrix}\mathbf{Y}_1^R\\ \mathbf{Y}_1^I\end{bmatrix}, \begin{bmatrix}\mathbf{Y}_2^R\\ \mathbf{Y}_2^I\end{bmatrix}, \cdots, \begin{bmatrix}\mathbf{Y}_J^R\\ \mathbf{Y}_J^I\end{bmatrix}\} - \{\begin{bmatrix}\mathbf{C}_2^R\\ \mathbf{C}_2^I\end{bmatrix}, \cdots, \begin{bmatrix}\mathbf{C}_K^R\\ \mathbf{C}_K^I\end{bmatrix}\}$ be denoted as $\{\begin{bmatrix}\mathbf{C}_{K+1}^R\\ \mathbf{C}_{K+1}^I\end{bmatrix}, \cdots, \begin{bmatrix}\mathbf{C}_{J+1}^R\\ \mathbf{C}_{J+1}^I\end{bmatrix}\}$, then the resultant STBC with dispersion matrices $\mathbf{C}_1$ to $\mathbf{C}_{J+1}$ are fast-group-decodable that achieves the lowest complexity.

Code Example

FGD-STBC of size 4x4 can be constructed based on O-STBC of size 4x4.

Such an O-STBC [Ganesan and Stoica, 2001] has dispersion matrices:

$$\mathbf{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{C}_2 = \begin{bmatrix} j & 0 & 0 & 0 \\ 0 & j & 0 & 0 \\ 0 & 0 & -j & 0 \\ 0 & 0 & 0 & -j \end{bmatrix},$$

$$\mathbf{C}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \mathbf{C}_4 = \begin{bmatrix} 0 & 0 & j & 0 \\ 0 & 0 & 0 & -j \\ j & 0 & 0 & 0 \\ 0 & -j & 0 & 0 \end{bmatrix},$$

$$\mathbf{C}_5 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{C}_6 = \begin{bmatrix} 0 & 0 & 0 & j \\ 0 & 0 & j & 0 \\ 0 & j & 0 & 0 \\ j & 0 & 0 & 0 \end{bmatrix}.$$

Code Example

In GD-STBC construction, let C_1 be the seed matrix in the first group. There will be $2N_tT - N_tN_t$ = 2x4x4 - 4x4 = 16 dispersion matrices in the second group, hence the code rate is 17/8. Use C_2-C_6 to obtain the other 11 dispersion matrices in the second group:

$$\mathbf{C}_7 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \ \mathbf{C}_8 = \begin{bmatrix} 0 & j & 0 & 0 \\ j & 0 & 0 & 0 \\ 0 & 0 & 0 & -j \\ 0 & 0 & -j & 0 \end{bmatrix},$$

$$\mathbf{C}_9 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{C}_{10} = \begin{bmatrix} 0 & 0 & 0 & j \\ 0 & 0 & -j & 0 \\ 0 & -j & 0 & 0 \\ j & 0 & 0 & 0 \end{bmatrix},$$

Code Example

$$\mathbf{C}_{11} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathbf{C}_{12} = \begin{bmatrix} 0 & 0 & j & 0 \\ 0 & 0 & 0 & j \\ j & 0 & 0 & 0 \\ 0 & j & 0 & 0 \end{bmatrix},$$

$$\mathbf{C}_{13} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{C}_{14} = \begin{bmatrix} 0 & j & 0 & 0 \\ j & 0 & 0 & 0 \\ 0 & 0 & 0 & j \\ 0 & 0 & j & 0 \end{bmatrix},$$

$$\mathbf{C}_{15} = \begin{bmatrix} j & 0 & 0 & 0 \\ 0 & -j & 0 & 0 \\ 0 & 0 & -j & 0 \\ 0 & 0 & 0 & j \end{bmatrix}, \mathbf{C}_{16} = \begin{bmatrix} j & 0 & 0 & 0 \\ 0 & j & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & j \end{bmatrix},$$

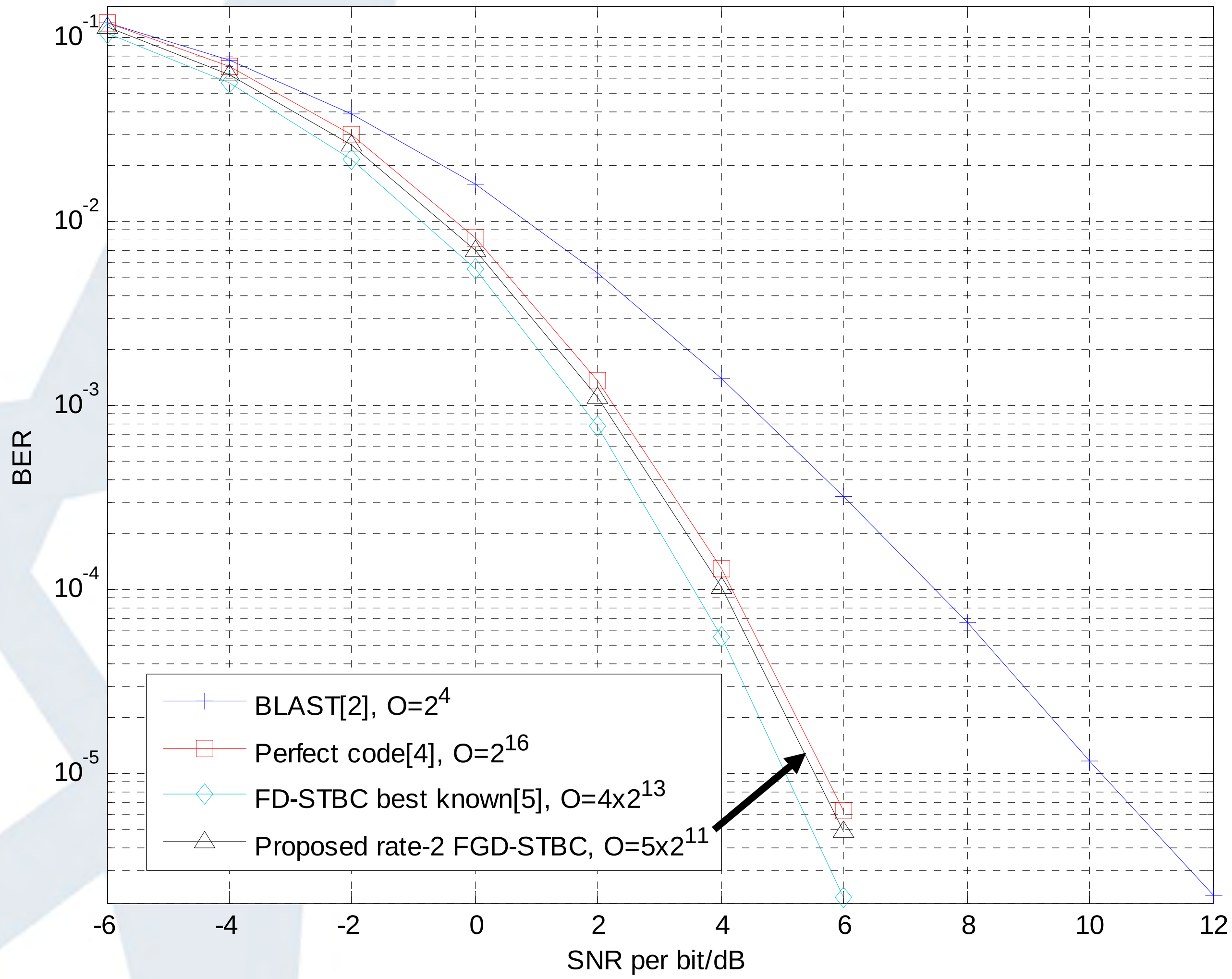
$$\mathbf{C}_{17} = \begin{bmatrix} j & 0 & 0 & 0 \\ 0 & -j & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & -j \end{bmatrix}.$$

Complexity Comparison

	Code Rate (R)	Group size (G)	Level removal of real SD (K)	Complexity order (O)
BLAST [2]	4	8	1	2^b
Perfect Code [4]	4	32	1	2^{4b}
FD-STBC [5]	2	16	4	$4 \cdot 2^{\frac{13b}{4}}$
FGD-STBC proposed	2	15	5	$5 \cdot 2^{\frac{11b}{4}}$

Note: For FGD-STBC proposed, only C_1-C_{16} are applied for a rate-2 code.

BER Performance



Remarks

- FD and GD code structure can be integrated to generate FGD-STBC
- A scalable code construction for FGD-STBC with max complexity reduction is presented
- The proposed FGD-STBC achieves
  - Full transmit diversity gain
  - Best known rate-complexity trade off

References

- Tian Peng Ren; Yong Liang Guan; Chau Yuen, Gunawan, E.; Er Yang Zhang;, "Group-Decodable Space-Time Block Codes with Code Rate > 1", IEEE Transactions on Communications, accepted, 2010.
- C. Yuen; Y. L. Guan; T. T. Tjhung, "On the Search for High-Rate Quasi-Orthogonal Space-Time Block Code", *International Journal of Wireless Information Network (IJWIN)*, May 2006, Pages 1 – 12. http://dx.doi.org/10.1007/s10776-006-0033-2.
- T. P. Ren, Y. L. Guan, C. Yuen, E. Gunawan and E. Y. Zhang, "Unbalanced and Balanced 2-Group Decodable Spatial Multiplexing Code,“ IEEE VTC-Fall, Sep 2009.
- T. P. Ren, Y. L. Guan, C. Yuen and R. J. Shen, "Fast-Group-Decodable Space-Time Block Code,“ IEEE ITW'10, Cairo, Egypt, 6-8 Jan. 2010.

Outline

- Group-Decodable STBC with Rate ≤ 1
- Fast-Group-Decodable STBC with Rate > 1
- Shift-Orthogonal STBC
- 2-Step 2-Way MIMO Relaying

MIMO RELAYING (1)

Async Cooperative Diversity Relaying

元永亮

Ren Tian Peng [1], Guan Yong Liang[2], Yuen Chau[3], Erry Gunawan[2], and Rong Jun Shen[1]

[1] National University of Defense Technology, China
[2] Nanyang Technological University, Singapore
[3] Institute of Infocomm Research, Singapore

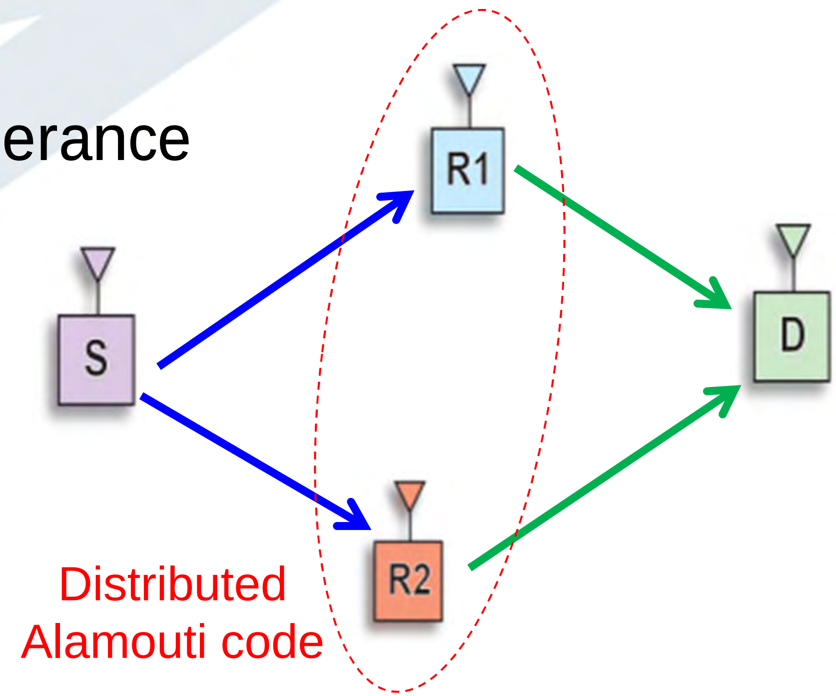
Dec 2010

Outline

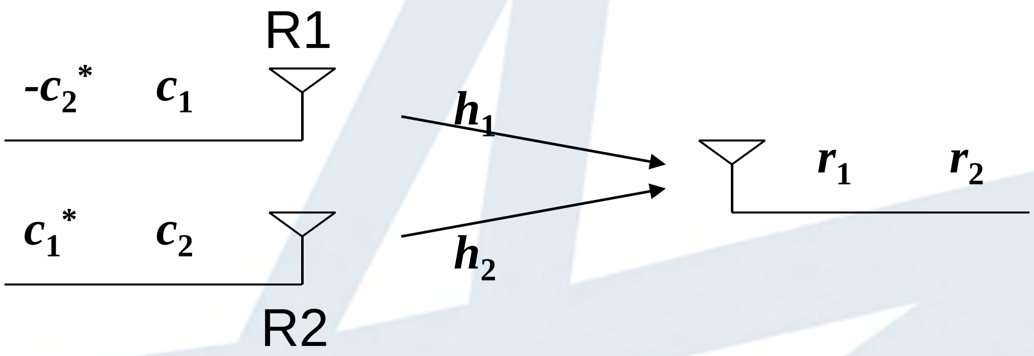
- Introduction
- Shift-Orthogonal STBC (SO-STBC)
- Delay-Tolerant Cooperative Diversity Routing
- Performance
- Conclusion

MIMO Relaying

- Benefits
  - Diversity via single-antenna base stations/relays (cooperative diversity)
  - Range extension
  - Relay (router) failure tolerance



Distributed Alamouti Coding



$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2^* \end{bmatrix}$$

$\mathbf{H_c}$ is unitary

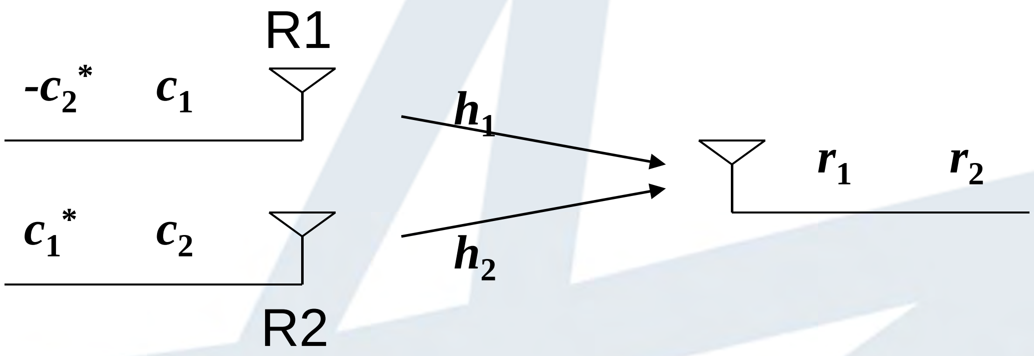
$$\begin{bmatrix} \hat{c}_1 \\ \hat{c}_2 \end{bmatrix} = \mathbf{H}_{\mathbf{C}}^{\mathrm{H}} \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix}$$

$$= \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} h_1^* \eta_1 + h_2 \eta_2^* \\ h_2^* \eta_1 - h_1 \eta_2^* \end{bmatrix}$$

$$\mathbf{H_c}^{\mathbf{H}} \mathbf{H_c} = \alpha \mathbf{I}$$

1. **c_1 and c_2 decoupled**
2. **MRC effect achieved**

Distributed Alamouti Coding



$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2^* \end{bmatrix}$$

$\mathbf{H_c}$ is unitary

What if

- **Received STBC signals are not synchronized in time?**
- **OFDM (high PAPR) or joint detection (high complexity) cannot be afforded?**

Asynchronous MIMO Relaying

STBC matrix **X** of length L:

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1L} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2L} \end{bmatrix}$$

Asynchronous version of **X** received:

$$\mathbf{X}^a = \begin{bmatrix} \mathbf{x}_1^a \\ \mathbf{x}_2^a \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1L} & \star \\ \star & x_{21} & x_{22} & \cdots & x_{2L} \end{bmatrix}$$

<u>**Design objectives on X**</u>

A sub-matrix of X*^a* (an async version of X) should have full row-rank and be unitary.

→ Rows of X should have aperiodic correlation = 0

Shift-Orthogonal STBC

Construction : Assuming that $\mathbf{A}_{M\times M} \triangleq [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_M]^T$ is a Fourier matrix, $\mathbf{1}_M$ denotes a row vector with M entries 1, $\mathbf{S}_{M\times L_s} \triangleq [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_M]^T \triangleq [s_{ml}]$ where information symbol s_{ml} is the entry in the mth row and lth column of matrix $\mathbf{S}$, then

$$\mathbf{X}_{M\times ML_s} = (\mathbf{1}_M \otimes \mathbf{S}) \odot (\mathbf{A} \otimes \mathbf{1}_{L_s}) \quad (1)$$

is a shift-orthogonal STBC where $[\cdot]^T$ denotes the transpose of a matrix; $\odot$ means Hadamard product and $\otimes$ means Kronecker product; $m = 1, 2, \cdots, M$, and $l = 1, 2, \cdots, L_s$.

$\mathbf{A}_{M\times M}$ can also be formed using zero correlation sequence set.

[T Ren, Y L Guan, E Gunawan, E Y Zhang, IEEE Trans Comm, June 2010]

Shift-Orthogonal STBC

Construction : Assuming that $\mathbf{A}_{M\times M} \triangleq [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_M]^T$ is a Fourier matrix, $\mathbf{1}_M$ denotes a row vector with M entries 1, $\mathbf{S}_{M\times L_s} \triangleq [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_M]^T \triangleq [s_{ml}]$ where information symbol s_{ml} is the entry in the mth row and lth column of matrix $\mathbf{S}$, then

$$\mathbf{X}_{M\times ML_s} = (\mathbf{1}_M \otimes \mathbf{S}) \odot (\mathbf{A} \otimes \mathbf{1}_{L_s}) \quad (1)$$

is a shift-orthogonal STBC where $[\cdot]^T$ denotes the transpose of a matrix; $\odot$ means Hadamard product and $\otimes$ means Kronecker product; $m = 1, 2, \cdots, M$, and $l = 1, 2, \cdots, L_s$.

code rate of $\mathbf{X} = L_s / (M L_s + L_p)$
$\approx 1/M$ when $L_s >> L_p$

[T Ren, Y L Guan, E Gunawan, E Y Zhang, IEEE Trans Comm, June 2010]

Code example for 2 relays

$$\mathbf{s}_s = [s_{s,1} \; s_{s,2} \; s_{s,3} \; s_{s,4}]$$

$$\mathbf{A}_{2\times 2} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

2 relays

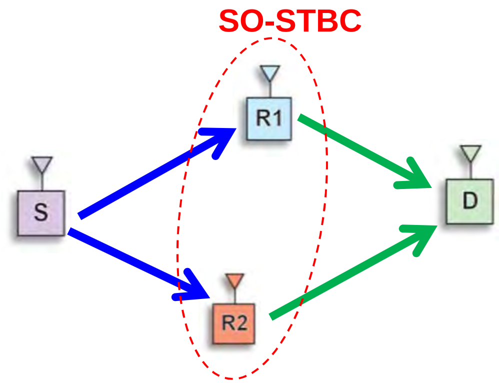
Following the construction above with Fourier matrix $\mathbf{A}_{2\times 2}$, the shift-orthogonal STBC transmitted by I_{11} and I_{12} is:

differential delay < 2

$$\mathbf{x}_{11} = [\; s_{11,3} \quad s_{11,4} \;|\; s_{11,1} \; \cdots \; s_{11,4} \quad s_{11,1} \; \cdots \quad s_{11,4} \;],$$
$$\mathbf{x}_{12} = [-s_{12,3} \; -s_{12,4} \;|\; s_{12,1} \; \cdots \; s_{12,4} \; -s_{12,1} \; \cdots \; -s_{12,4} \;].$$

where the solid box contains space-time coded signalsand the dashed box contains the cyclic prefixes of length $L_p = 2$.

Code example for 2 relays



Async received signal:

$\tilde{\mathbf{r}}_{21}$

$$\mathbf{r}_{21} = [h_{11,21}\ h_{12,21}] \cdot \begin{bmatrix} \star & s_{11,3} & s_{11,4} & s_{11,1} & s_{11,2} & s_{11,3} & s_{11,4} & s_{11,1} & s_{11,2} & s_{11,3} & s_{11,4} \\ -s_{12,3} & -s_{12,4} & s_{12,1} & s_{12,2} & s_{12,3} & s_{12,4} & -s_{12,1} & -s_{12,2} & -s_{12,3} & -s_{12,4} & \star \end{bmatrix} + \mathbf{n}_{21}$$

Equivalent channel matrix:

$$\tilde{\mathbf{H}} \triangleq [\tilde{\mathbf{h}}_1\ \tilde{\mathbf{h}}_2\ \tilde{\mathbf{h}}_3\ \tilde{\mathbf{h}}_4] = \begin{bmatrix} 0 & h_{12,12} & h_{11,12} & 0 & 0 & -h_{12,12} & h_{11,12} & 0 \\ 0 & 0 & h_{12,12} & h_{11,12} & 0 & 0 & -h_{12,12} & h_{11,12} \\ h_{11,12} & 0 & 0 & h_{12,12} & h_{11,12} & 0 & 0 & -h_{12,12} \\ -h_{12,12} & h_{11,12} & 0 & 0 & h_{12,12} & h_{11,12} & 0 & 0 \end{bmatrix}$$

Decode SO-STBC by: $\tilde{\mathbf{r}}_{21}\tilde{\mathbf{H}}^H \triangleq [v_1\ v_2\ v_3\ v_4]$

to obtain:

$$v_1 = 2|h_{11,21}|^2 \cdot s_{11,1} + 2|h_{12,21}|^2 \cdot s_{12,1},$$
$$v_2 = 2|h_{11,21}|^2 \cdot s_{11,2} + 2|h_{12,21}|^2 \cdot s_{12,2},$$
$$v_3 = 2|h_{11,21}|^2 \cdot s_{11,3} + 2|h_{12,21}|^2 \cdot s_{12,3},$$
$$v_4 = 2|h_{11,21}|^2 \cdot s_{11,4} + 2|h_{12,21}|^2 \cdot s_{12,4}.$$

Code example for 3 relays

postfix

$$
\begin{array}{l}
\text{relay1:} \\
\text{relay2:} \\
\text{relay3:}
\end{array}
\begin{bmatrix}
s_1 & s_2 & s_3 & s_1 & s_2 & s_3 & s_1 & s_2 & s_3 & s_1 & s_2 \\
s_2 & s_3 & s_1 & e^{j2\pi/3}s_2 & e^{j2\pi/3}s_3 & e^{j2\pi/3}s_1 & e^{j4\pi/3}s_2 & e^{j4\pi/3}s_3 & e^{j4\pi/3}s_1 & s_2 & s_3 \\
s_3 & s_1 & s_2 & e^{j4\pi/3}s_3 & e^{j4\pi/3}s_1 & e^{j4\pi/3}s_2 & e^{j2\pi/3}s_3 & e^{j2\pi/3}s_1 & e^{j2\pi/3}s_2 & s_3 & s_1
\end{bmatrix}
$$

Despite shifts in 2nd and 3rd rows, sub-matrix still orthogonal and rank 3

$$
\begin{bmatrix}
s_1 & s_2 & s_3 & s_1 & s_2 & s_3 & s_1 & s_2 & s_3 & s_1 & s_2 & - & - \\
- & - & s_2 & s_3 & s_1 & e^{j2\pi/3}s_2 & e^{j2\pi/3}s_3 & e^{j2\pi/3}s_1 & e^{j4\pi/3}s_2 & e^{j4\pi/3}s_3 & e^{j4\pi/3}s_1 & s_2 & s_3 \\
- & s_3 & s_1 & s_2 & e^{j4\pi/3}s_3 & e^{j4\pi/3}s_1 & e^{j4\pi/3}s_2 & e^{j2\pi/3}s_3 & e^{j2\pi/3}s_1 & e^{j2\pi/3}s_2 & s_3 & s_1 & -
\end{bmatrix}
$$

An Application -- Async MANET

In a SPR (single-path routing) MANET, a link from a source node to a destination node is typically accomplished by finding the best single path through some relay nodes.

However, **when a node fails, re-routing is necessary, leading to delays.**

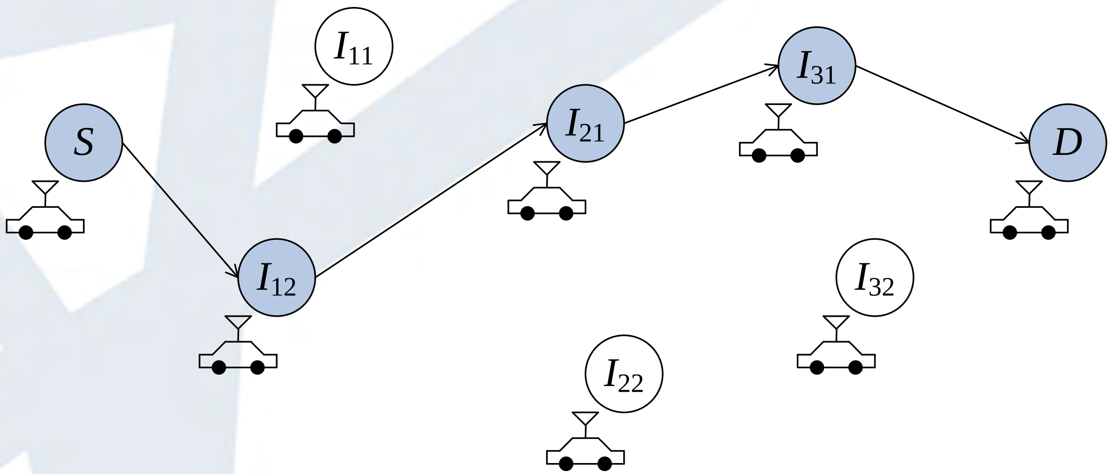


Fig. 1. Single-path routing (SPR) MANET for inter-vehicle communications

An Application -- Async MANET

In the DR (dispersity routing) MANET, several parallel routing paths are permitted in order to transmit information with redundancy.

However, DR **requires additional bandwidth to prevent self-interference among the parallel paths**.

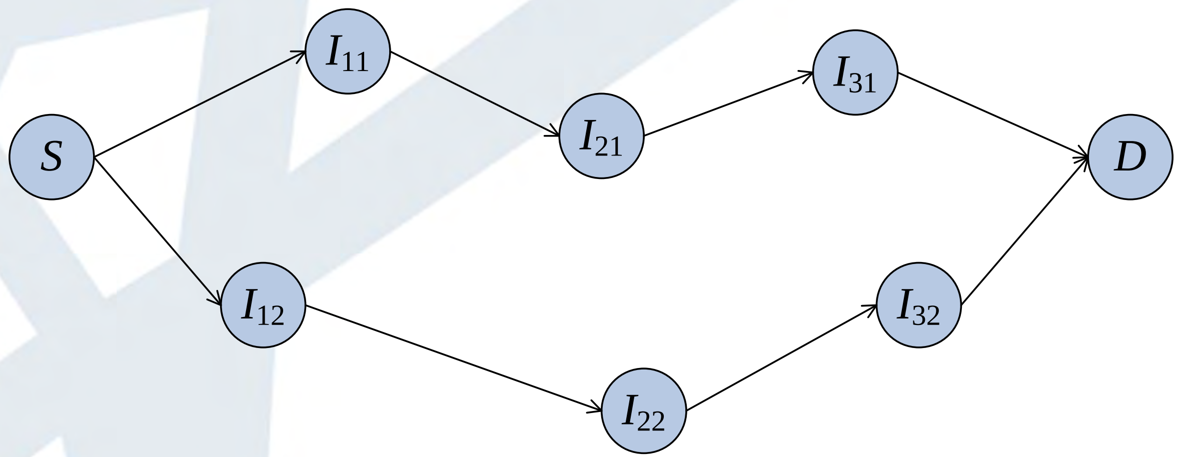


Fig. 3. Dispersity routing (DR) MANET

[N F Maxemchuk, IEEE MILCOM 2007]

An Application -- Async MANET

In a cooperative diversity routing (CDR) MANET, data is transported **from relay cluster to relay cluster**. Any two adjacent relay clusters form a virtual MIMO system and apply distributed STBC to obtain cooperative diversity.

However, cooperative diversity works only if there is **timing alignment among the routing nodes**, this requires complex MAC protocol and may be impossible if node mobility is high.

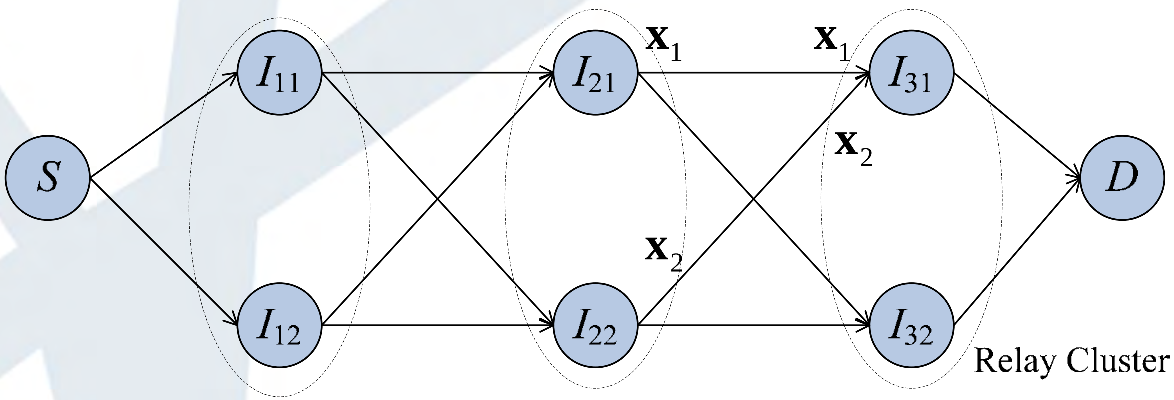


Fig. 4. Synchronous cooperative diversity routing (S-CDR) MANET

[M. A. Tope, J. C. McEachen, and A. C. Kinney, ISCC, June 2006]

Delay-Tolerant Cooperative Diversity Routing MANET

Using SO-STBC, delay-tolerant cooperative diversity routing (DT-CDR) with simple symbol-wise decoding is possible.

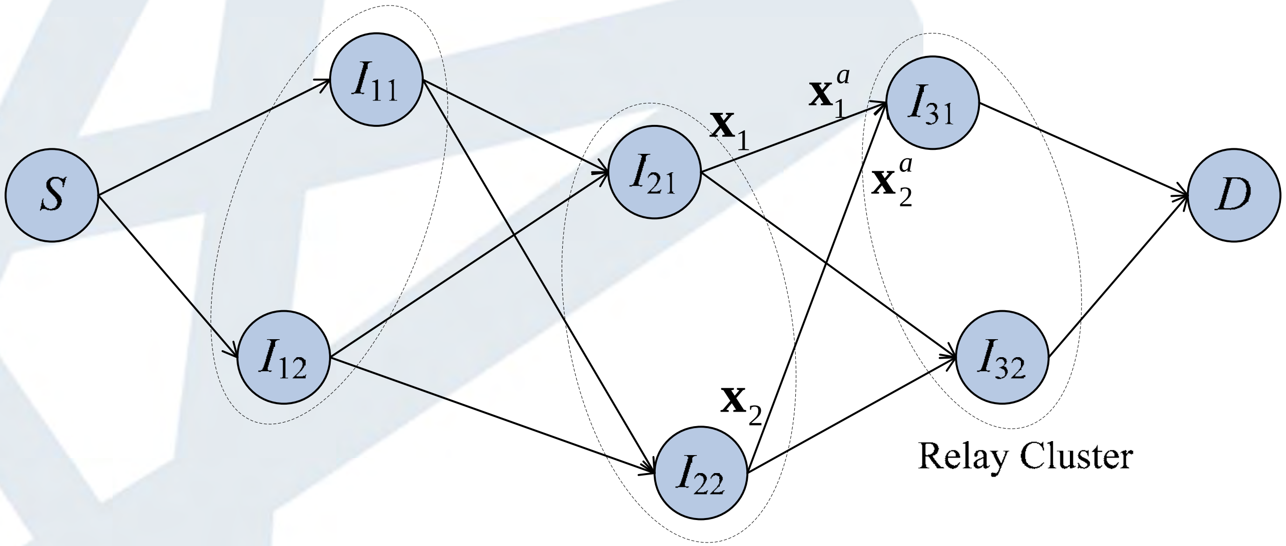


Fig. 5. Delay-tolerant cooperative diversity routing (DT-CDR) MANET

[T Ren, Y L Guan, et. al., VTC-Fall, 2010]

Simulation

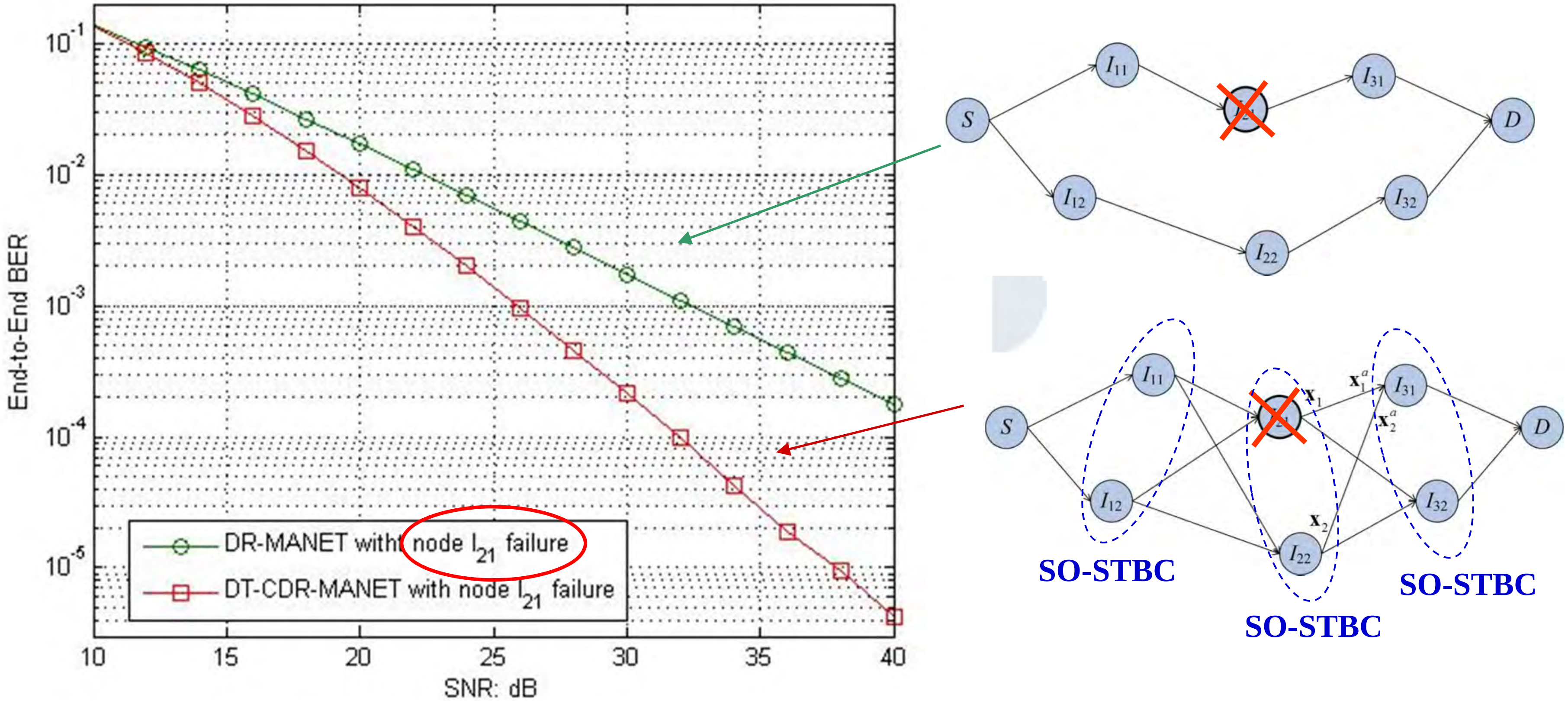
In the simulations, we assume that the channel is quasi-static Rayleigh fading, the channel state information including the timing difference can be obtained from the channel estimation at the downstream nodes.

Naturally, the transmit power used for the whole network is assumed to be shared equally among every relay cluster and the nodes in the same relay cluster have the same transmit power.

To achieve a bit rate of 1 bit/sec/Hz, BPSK, BPSK and QPSK are applied in the SPR MANET, the DR MANET and the DT-CDR MANET, respectively.

All nodes (source, destination, relays/routers) use only 1 antenna

Simulation – DF Example



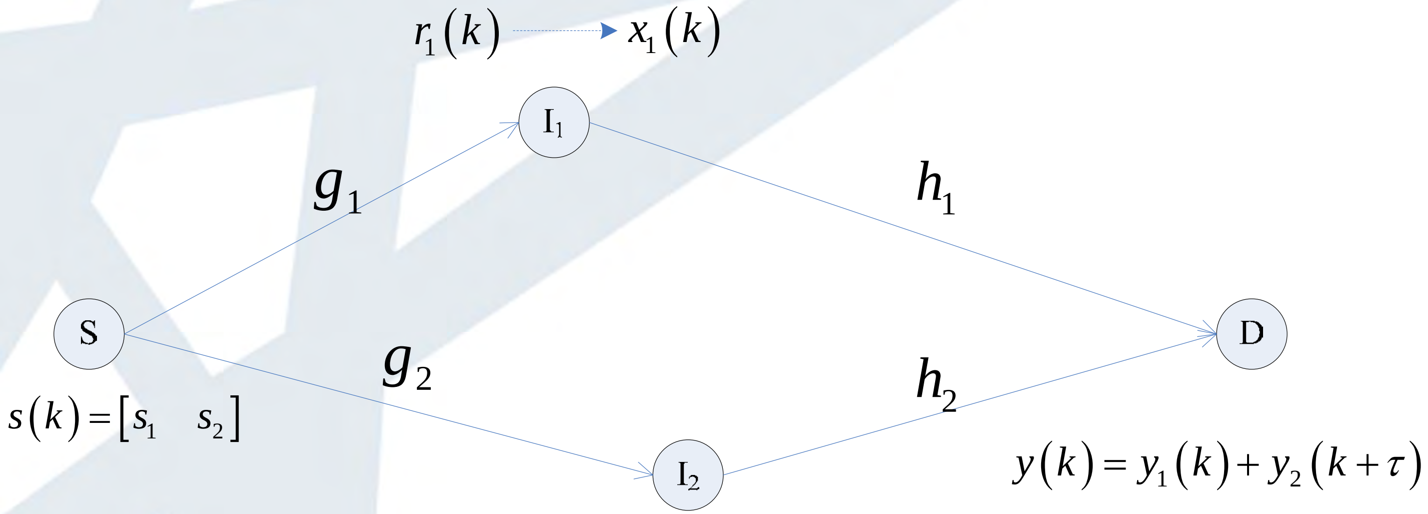
- DR-MANET (all radio links synchronous) loses diversity when routing node I_{21} fails
- CDR-MANET (all radio links asynchronous) maintains diversity

[T Ren, Y L Guan, et. al., VTC-Fall, 2010]

Asynchronous AF Relaying

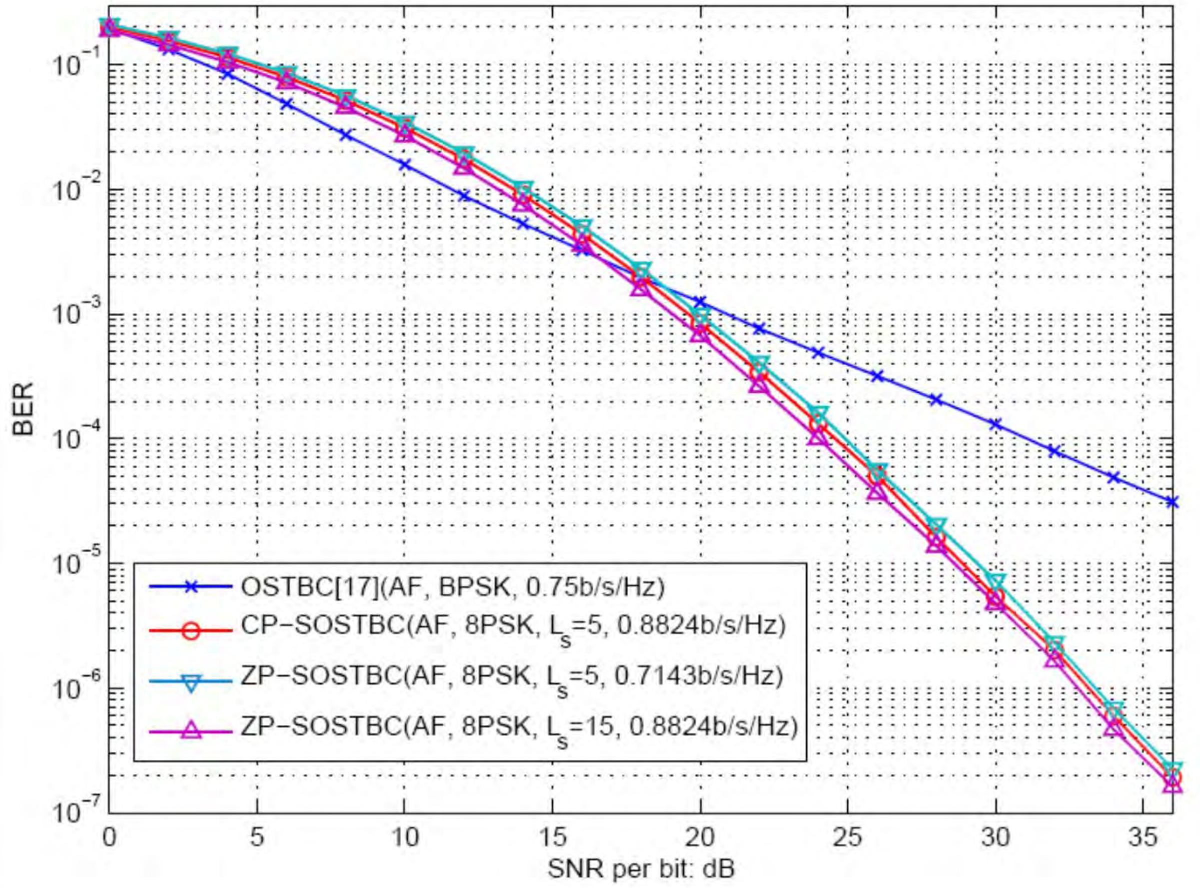
In DF (decode and forward) relaying, every relay detects (decode) the symbols and uses only the “cleaned” symbols in the next hop.

In AF (amplify and forward) relaying, the relays use the noisy symbols to form the next-hop signal.



Due to its row-wise design, SO-STBC supports async AF relaying + full diversity + linear decoding

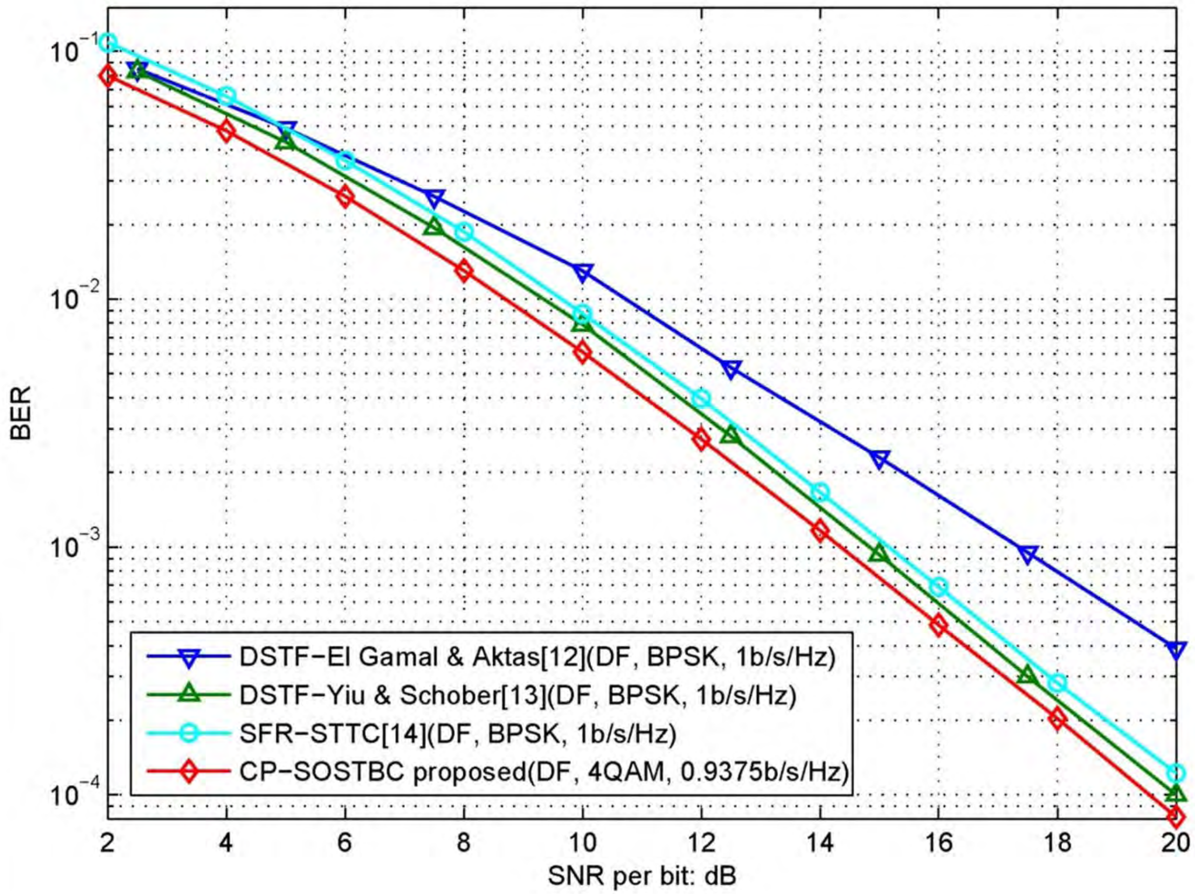
Simulation: AF Example



ML decoding BER of SO-STBC and O-STBC for an asynchronous AF wireless relay network with 3 relays and maximum time difference 2

[T Ren, Y L Guan, E Gunawan, E Y Zhang, IEEE Trans Comm, June 2010]

Comparison: Delay-Tolerant STC



The other delay-tolerant STC's require trellis decoding

Conclusion

- Shift-Orthogonal STBC (SO-STBC) provides distributed, open-loop spatial diversity for async MANET or async Network MIMO.
- SO-STBC can be systematically constructed based on Fourier Matrix or Zero Periodic Correlation sequence set.
- Symbol-wise ML decoding of SO-STBC can be done using very simple Alamouti-like linear decoding.
- SO-STBC supports both async DF relaying and async AF relaying.
- SO-STBC simplifies the routing protocol and reduces the network latency of async MANET.

Outline

- Group-Decodable STBC with Rate ≤ 1
- Fast-Group-Decodable STBC with Rate > 1
- Shift-Orthogonal STBC
- 2-Step 2-Way MIMO Relaying

MIMO RELAYING (2)

2-Step Bi-Directional Relaying

M. Eslamifar (Nanyang Technological University, Singapore)

C. Yuen (Institute for Infocomm Research, Singapore)

W. H. Chin (Toshiba Research Lab, UK)

Y. L. Guan (Nanyang Technological University, Singapore)

Dec 2010

Outline

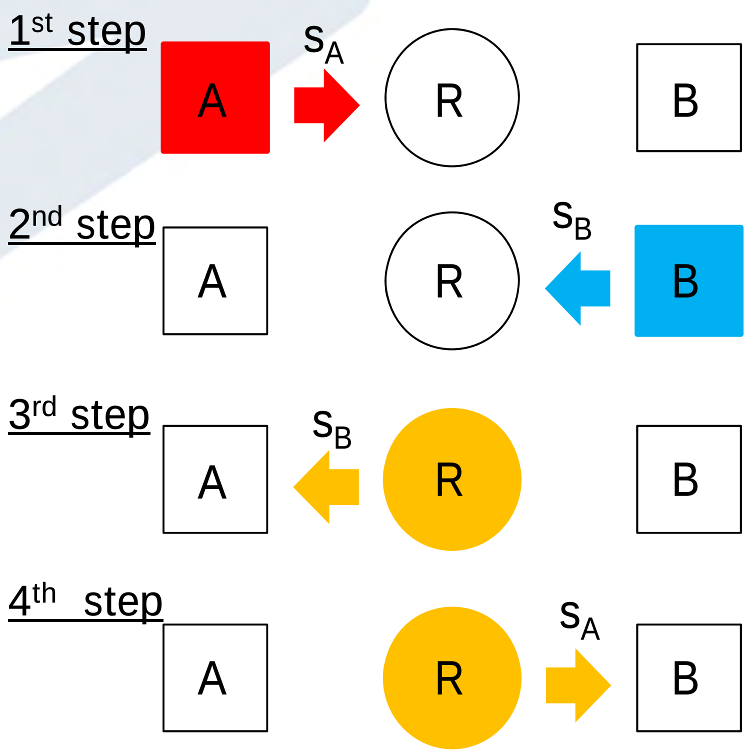
- 2-Step Bi-Directional Relaying
- Uplink
- Downlink
  - Antenna Selection
  - Antenna Selection with Binary Network Coding
  - STBC with Binary Network Coding
  - Transmit Beamforming
  - Max-Min AS-BNC
- Performance
- Conclusion

Introduction

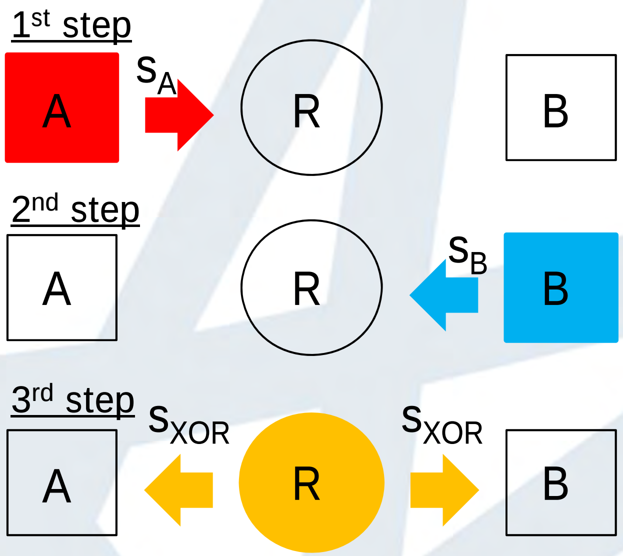
- **Bi-directional relaying:**

- **Conventional routing:**

  4 time slots

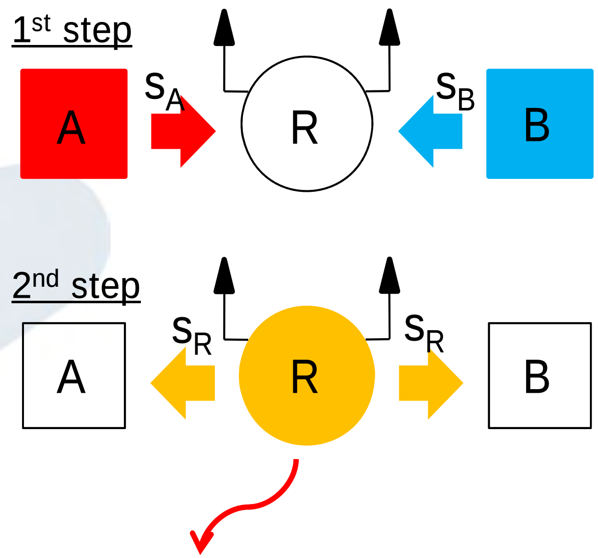


- Network coding with **single-antenna relay**: **3** time slots



$$s_{XOR} = \hat{s}_A \oplus \hat{s}_B$$

- Network coding with **multi-antenna relay**: **2** time slots

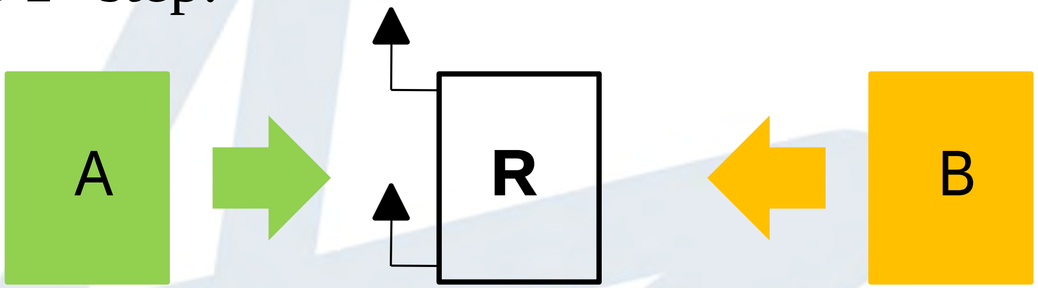


Multi-Antenna Downlink options:

- Spatial diversity gain?
- Binary/Analog network coding?
- Open/Closed loop (need CSI)?
- Single/Multiple relays?

Uplink

- In the 1st step:



$$\boldsymbol{y}_R = \boldsymbol{H}_u \boldsymbol{s} + \boldsymbol{n}_R$$

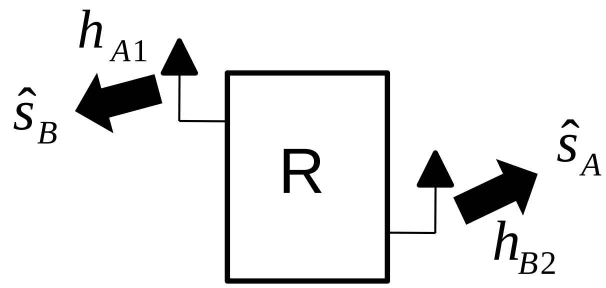
- The relay performs joint ML decoding:

$$\hat{s} = \arg\min_{\boldsymbol{s}} \|\boldsymbol{y}_R - \boldsymbol{H}_u \boldsymbol{s}\|^2$$

- Diversity order of uplink = N (number of relay antennas)

Downlink 1: Antenna Selection

- The best antenna channel to node A is selected to transmit $\hat{s}_B$
- Likewise, the best antenna channel to node B is selected to tx $\hat{s}_A$
- e.g. *if* $|h_{A1}| > |h_{A2}|$ and $|h_{B2}| > |h_{B1}|$

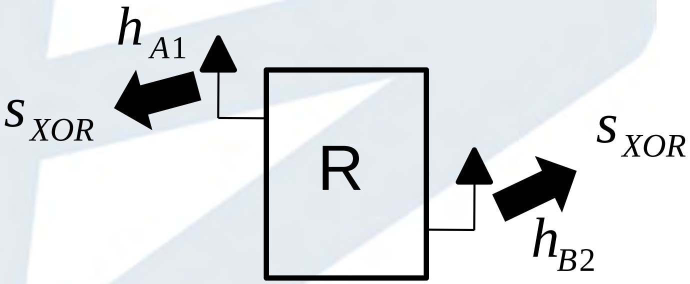


node A receives: $y_A = \dfrac{(h_{A1}\hat{s}_B + h_{B2}\hat{s}_A)}{\sqrt{2}} + n_A$

- After canceling off $h_{A1}\hat{s}_A$, $y_A \rightarrow \hat{y}_A = h_{A1}\hat{s}_B + n_A$

Downlink 2: Antenna Selection with Binary Network Coding

- Same as "Antenna Selection" scheme, except that both selected antennas transmit $s_{XOR} = \hat{s}_A \oplus \hat{s}_B$

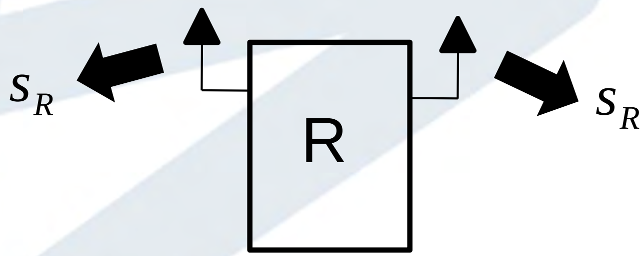


- Node A receives $y_A = \frac{(h_{A1} + h_{B2})s_{XOR}}{\sqrt{2}} + n_A$, cancels $h_{A1}\, s_{XOR}$,

  decodes $h_{B2}\, s_{XOR}$, then obtain $s_{XOR} \oplus \hat{s}_A \rightarrow \hat{s}_B$.

Downlink 3: Alamouti STBC with Binary Network Coding

- Since both nodes A and B are able to extract the other node's data from s_{XOR} , it may be transmitted using STBC such as Alamouti Code.



$$\mathbf{s_R} = \begin{bmatrix} s_{XOR}(n) & -s^*_{XOR}(n+1) \\ s_{XOR}(n+1) & -s^*_{XOR}(n) \end{bmatrix}$$

- <u>Note:</u> 2 time slots are used to transmit 2 symbols → rate = 1

Downlink 4: Tx Beamforming

- Relay transmits: $s_R = \sqrt{\mathcal{P}}(\frac{v_A \hat{s}_A + v_B \hat{s}_B}{\sqrt{2}})$

where $v_A = \frac{h_B^*}{\|h_B\|}$ (maximal ratio transmission)

$v_B = \frac{h_A^*}{\|h_A\|}.$

- Node A receives: $y_A = h_A^T s_R + n_A.$

- After canceling off $\sqrt{\mathcal{P}}\frac{h_{RA}^T v_A \hat{s}_A}{\sqrt{2}}$, $y_A \rightarrow \hat{y}_A = \sqrt{\mathcal{P}}\frac{h_A^T v_B s_B}{\sqrt{2}} + n_A.$

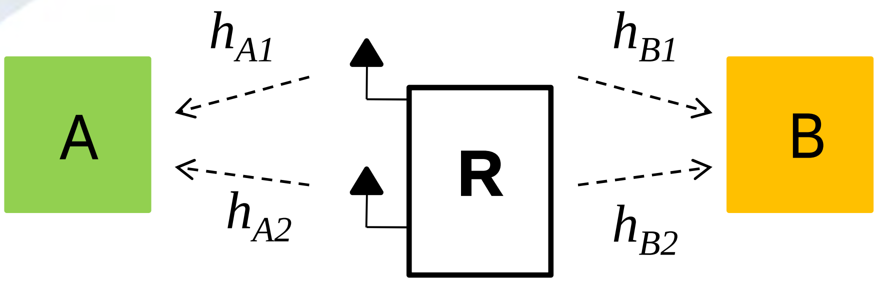
Downlink 5: Max-Min AS-BNC

- 1 out of *N* relay antennas is selected to transmit $s_{XOR} = \hat{s}_A \oplus \hat{s}_B$ Antenna selection is determined in 2 steps.

  Step 1: $g_k = \min\{|h_{Ak}|^2, |h_{Bk}|^2\}, \quad 1 \leq k \leq N.$

  Step 2: $j = \arg \max_{k=\{1,\dots,N\}} g_k$

- E.g. with *N* = 2 relay antennas:



$$s_R = \begin{cases} \sqrt{\mathcal{P}} \begin{pmatrix} s_{XOR} \\ 0 \end{pmatrix} & if \min\{|h_{A1}|, |h_{B1}|\} > \min\{|h_{A2}|, |h_{B2}|\} \\ \sqrt{\mathcal{P}} \begin{pmatrix} 0 \\ s_{XOR} \end{pmatrix} & if \min\{|h_{A2}|, |h_{B2}|\} > \min\{|h_{A1}|, |h_{B1}|\} \end{cases}$$

Max-Min AS-BNC: Analysis

- Moment Generating Function: $\varphi_\gamma(t) = \frac{N!}{\prod_{i=0}^{N-1}(i + 1 + t\varsigma\sigma^2/2)}$

- Diversity Order:

$$G_d = \lim_{\varsigma \to \infty} -\frac{\log N!}{\log \varsigma} + \lim_{\varsigma \to \infty} \frac{\sum_{i=0}^{N-1} \log(i + 1 + g_{MPSK}\sigma_0^2\varsigma/2)}{\log \varsigma}$$

$$= N. \qquad (33)$$

- Average Symbol Error Probability (SEP):

$$P_d^{Max-Min} \geq \frac{1}{\Pi} \int_0^{\Pi - \Pi/M} \varphi_\gamma\left(\frac{g_{MPSK}}{\sin \Theta^2}\right) d\Theta$$

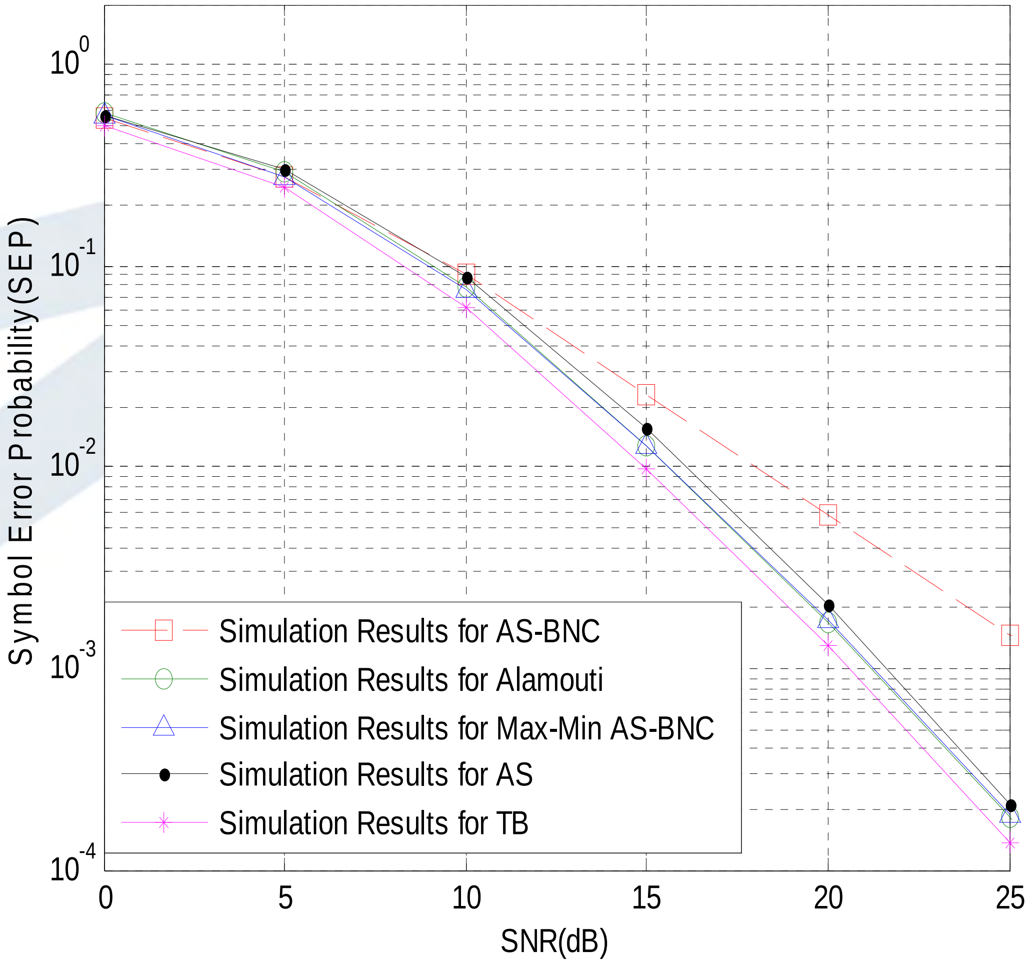
$g_{MPSK} = \sin^2(\Pi/M)$, γ = instantaneous received SNR ,
$\varphi_\gamma(t)$ = moment generating function (MGF) of γ and $\varsigma = \mathcal{P}/\sigma_s^2$.

Performance

Comparing all 5 schemes

- At high SNR,
  - TB ← best
  - Max-Min AS- BNC
  - Alamouti-BNC
  - AS
  - AS-BNC ← worst

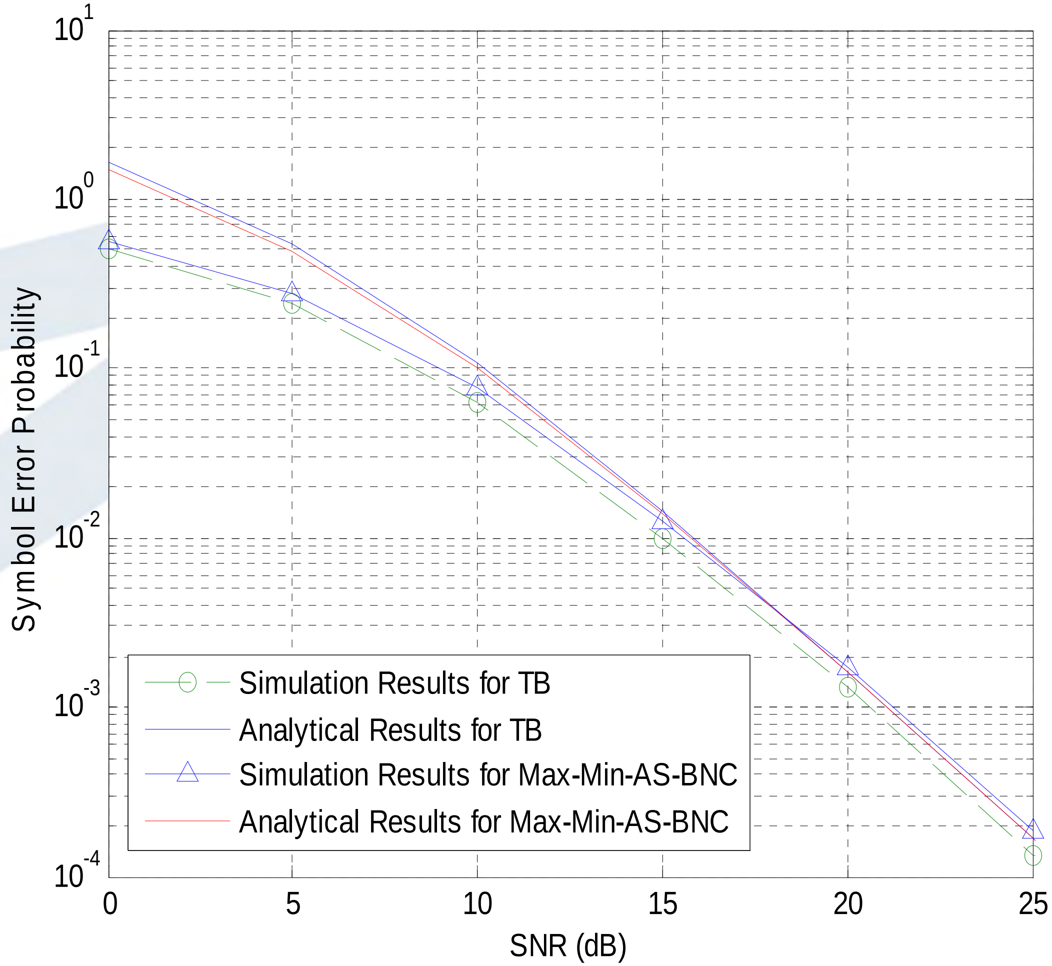
- AS-BNC has no spatial diversity gain



Performance

Comparing best 2 schemes
(TB and Max-Min AS BNC)

- SEP upper bounds verified by simulation
- Both have diversity order N
- TB outperforms Max-Min AS-BNC



Remarks

- Downlink CSI (channel state info) knowledge
  - TB needs full and **accurate** CSI at the relay.
  - Max-Min AS-BNC only needs **coarse** CSI (which channel is better) at the relay. No phase info needed.
- Max-Min AS-BNC can be **implemented across multiple relays** using a distributed protocol:
  - Each relay waits for a delay inversely proportional to its own downlink channel gain before transmitting
  - While waiting, listen to other relays. If another relay transmit first, abort own transmission.

Conclusions

- Multi-antenna relaying with binary/analog network coding achieves bi-directional communication in 2 time slots (shortest possible) + spatial diversity.
- TB has the best performance but requires full downlink CSI (channel state information) at the relay
- Max-Min AS-BNC has comparable performance as TB but only requires course CSI and can also be extended to multiple relays with distributed antennas
- STBC-BNC comes next in performance and does not require CSI, but suffers rate loss or more complex decoding if the number of relay antennas is more than 2.

References

- Mahshad Eslamifar, Woon Hau Chin, Chau Yuen, Y L Guan, “Performance Analysis of Bi-Directional Relaying with Multiple Antennas”, IEEE Trans Wireless Comms, 2nd review.
- --, “Performance Analysis of Two-Way Multiple-Antenna Relaying with Network Coding”, VTC-Fall 2009
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Thank You