SHAPED BEAM SYNTHESIS IN PHASED ARRAYS AND REFLECTORS

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IEEE/APS Distinguished Lecturer Program

Outline

- Introduction
- Overview of Shaped Beam Technology
- Existing Beam Shaping Algorithms
- Projection Matrix Method
- Examples
- Concluding Remarks

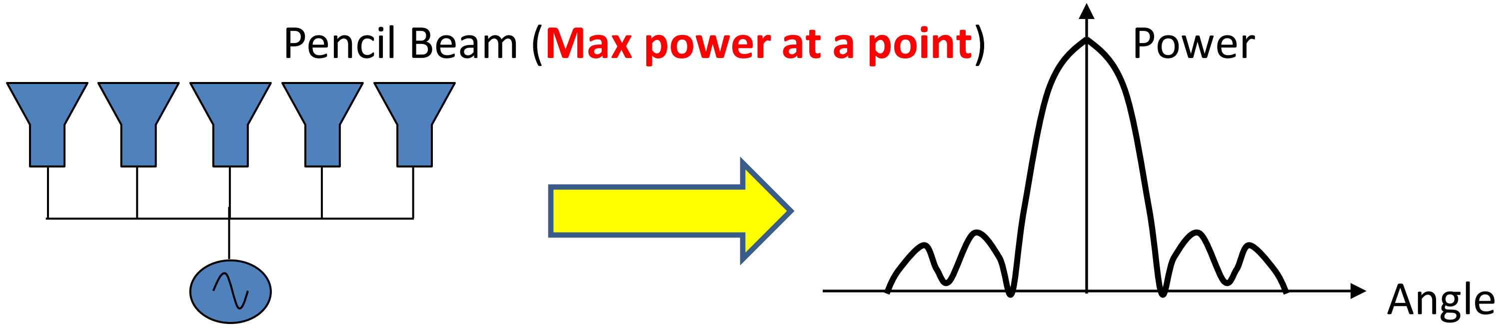
WHY SHAPED BEAMS?

- For broadcasting and communication satellites the antenna illumination should be limited to the region of interest in order to maximize the EIRP. This necessitates beam shaping capabilities for the on-board antennas
- International Telecommunication Satellite Organization (ITSO) has a strict regulation over spillover radiation

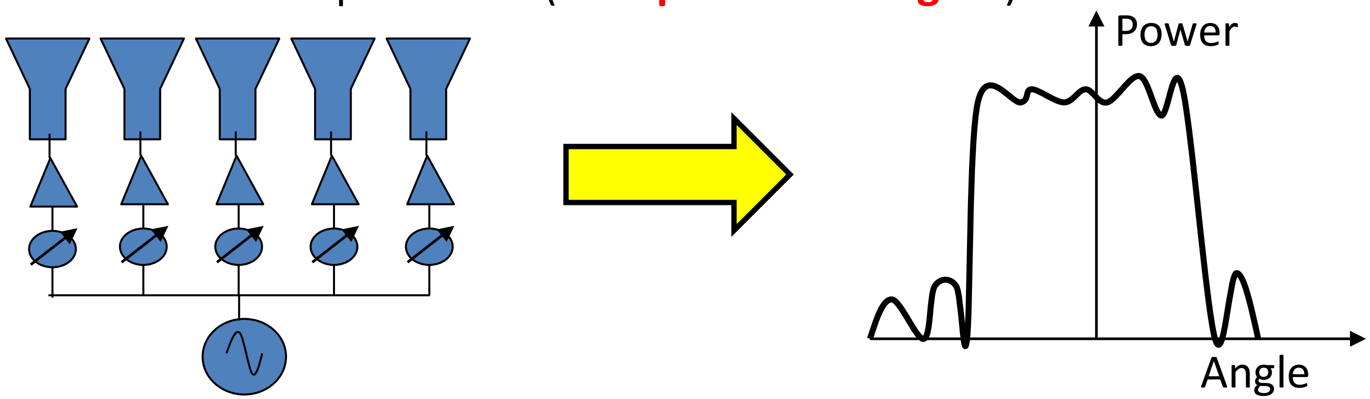
Cartoon showing a shaped beam radiation

Types of Array Beams

Pencil Beam (**Max power at a point**)



Shaped Beam (**Max power in a region**)



Analytical Solution for the **phase distribution** for a given shaped beam is impossible because the equations are non-linear

Measure of beam quality
(The Gain Area Product or GAP)

- Quality of pencil beam is measured by **Peak gain (or aperture efficiency)** and **Side lobe ratio**
- Quality of shaped beam is typically measured by **GAP**
- **GAP = Lowest power gain x Coverage area** (in square degree)
- Maximum possible GAP is 4 x π = 41253 sq-degree because

$$\int_0^{\pi}\int_0^{2\pi} G(\theta,\phi)\underbrace{\sin\theta\, d\theta\, d\phi}_{d\Omega} = 4\pi$$

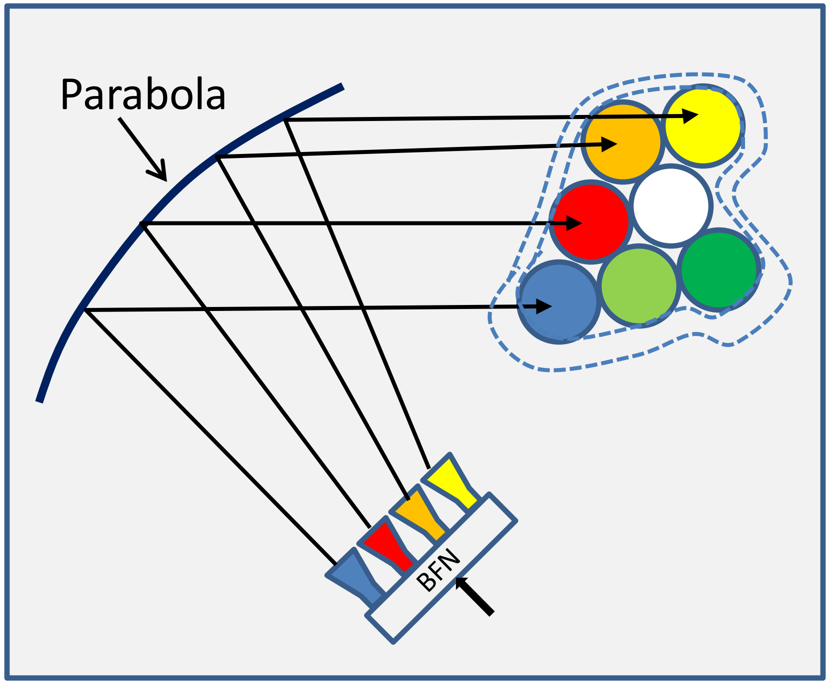
- GAP of an **ideal** flat-top beam is 4π (but that requires an infinitely large array to realize such a beam!)
- For a complex beam-shape (like too many corners), GAP is smaller
- GAP increases with the size of the array

SHAPED BEAM TECHNOLOGY

Multi-feed Parabolic Reflector

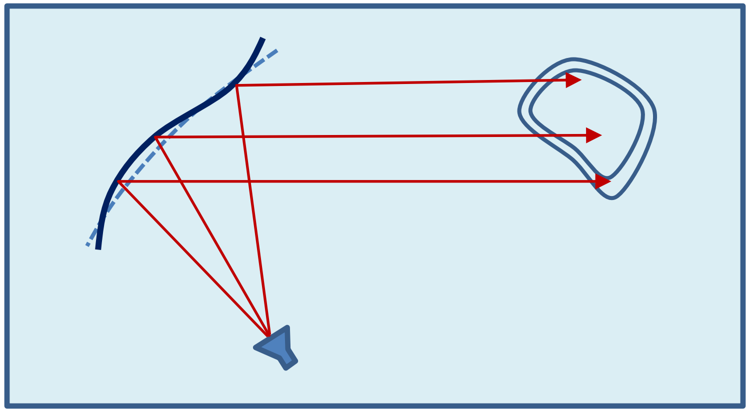
- Oldest technology (~1960), yet very useful even now
- Single parabolic surface and multiple feeds
- Each feed creates a circular (elliptical) spot beam
- The composite pattern makes a contour beam
- Requires BFN for amplitude/phase distribution
- Moderate beam reconfigurability

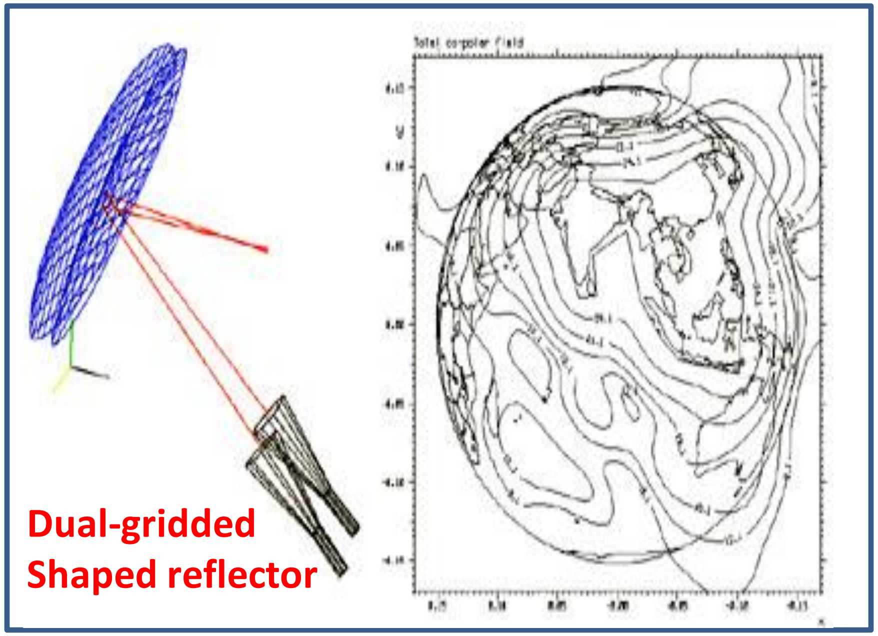
Conceptual Sketch



Shaped Surface Reflector

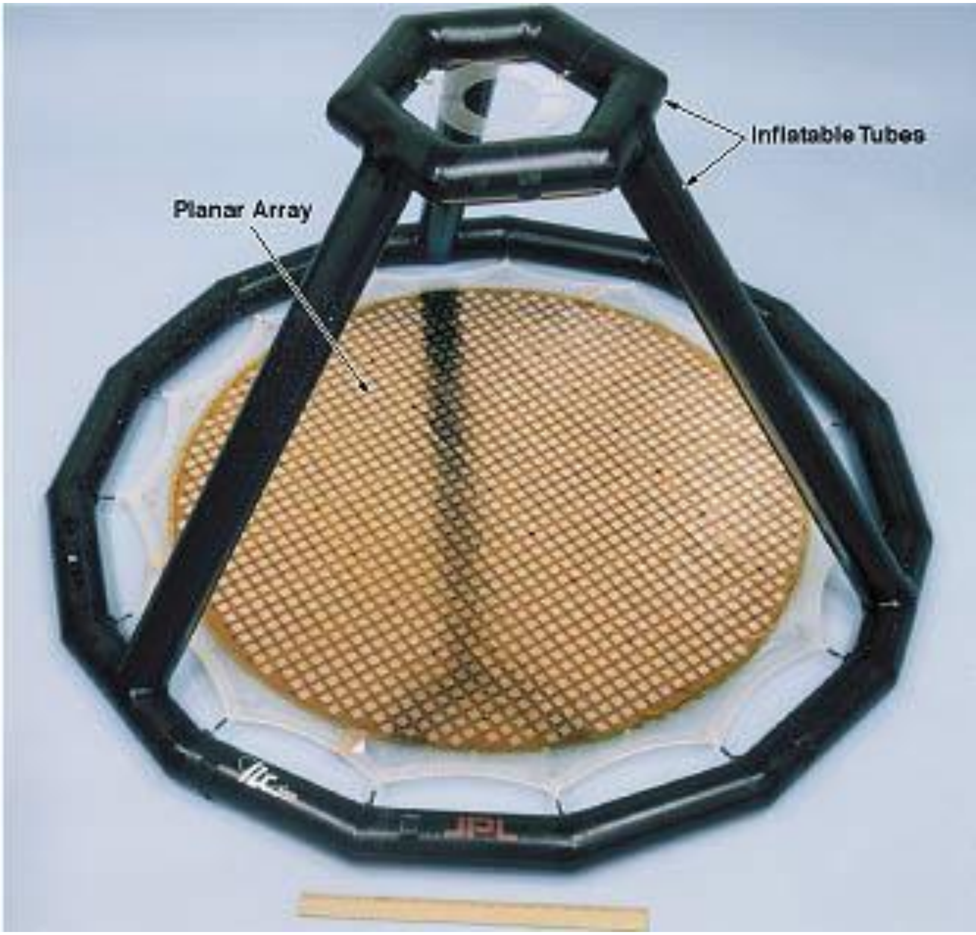
- A major technological advancement (~1980) for satellite antennas in terms of implementation cost
- Single surface, single feed per surface
- No BFN required
- Required phase distribution is realized by perturbing the surface of a parabolic reflector
- No reconfigurability
- Single point failure





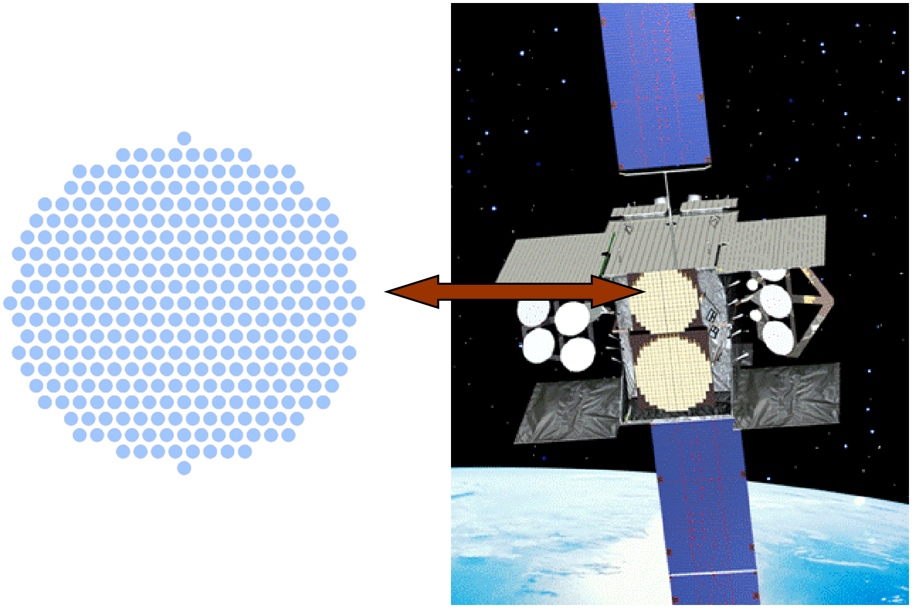
Reflectarray

- Microstrip version of a surface reflector (~1990)
- Typically flat panel with printed patch elements
- Excited with a single feed
- Phase distribution is realized by varying the sizes of patch elements *(instead of deforming the surface)*
- Light weight
- Small bandwidth (can be enhanced with slot-coupling or multiple layes)
- No beam reconfigurability



Active Phased Array

- Typically used for pencil beam scanning, but recently being considered for multiple shaped beams
- Direct radiating Phased array with electronically controlled BFN
- Each element has its own RF source (SSPA or LNA)

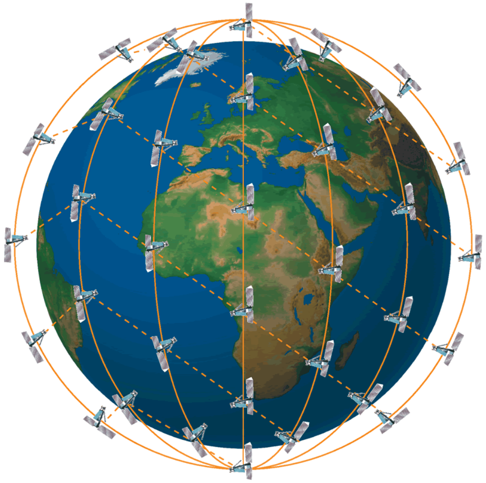


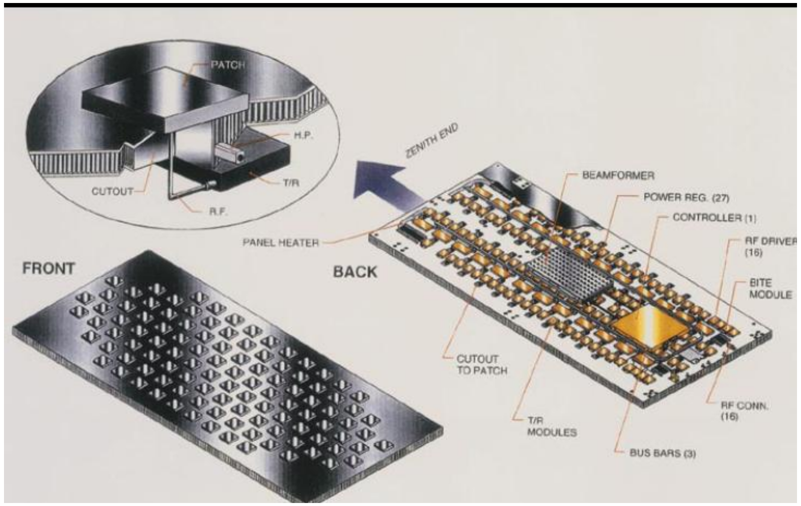
Advantages:
- Fully reconfigurable
- Multiple Shaped beams
- No single point failure. Graceful degradation of EIRP

Disadvantages:
- Low PAE; thermal dissipation; requires cooling system
- Complexity in BFN design, High implementation cost

Iridium L-Band Patch Array

Constellation consists of 66 LEO satellites at 780 KM in 6 orbital planes

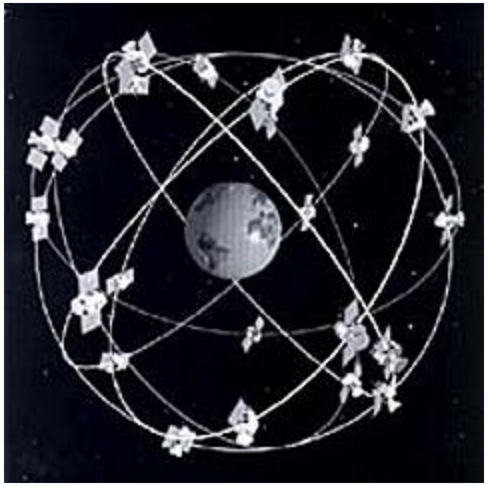


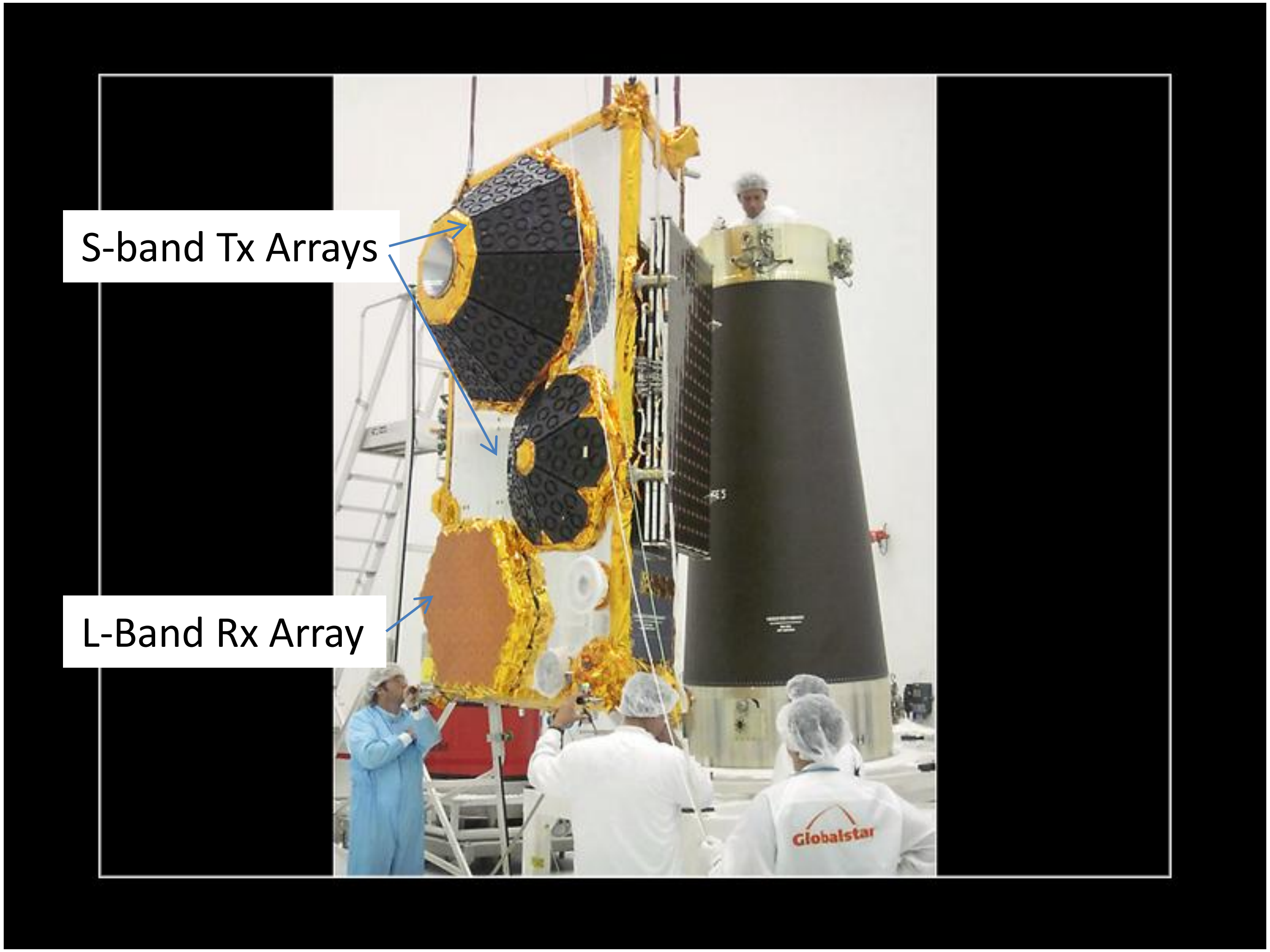


- Suspended patch elements, 106 elements per array
- 3 arrays, 16 shaped-beams per array

Source: A.B. Rohwer et al, 2010 IEEE Phased Array Symposium, Boston, pp. 504-511.

Globalstar Array

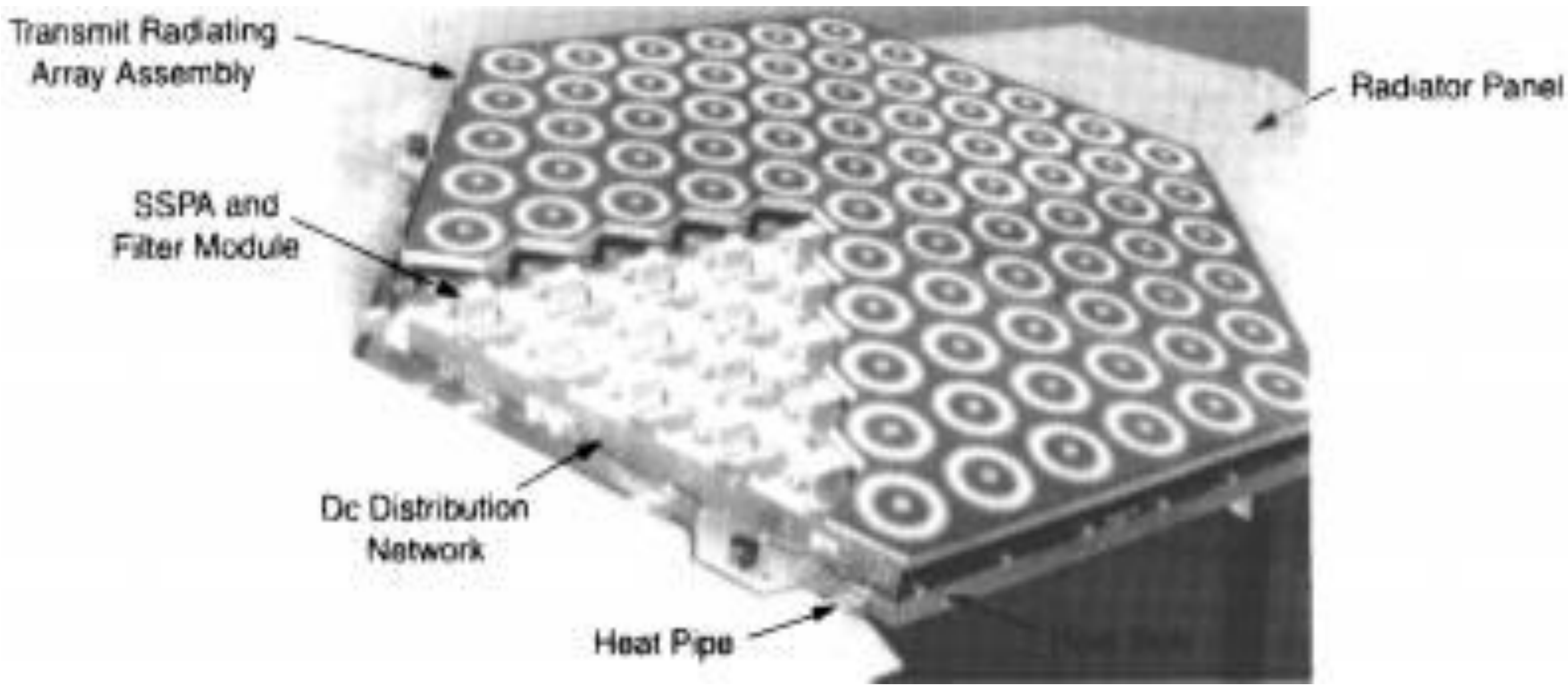




- 48 Satellites 1,045 KM LEO orbit
- Multiple Spot Beam array for satellite Telephone
- 16 beams per array

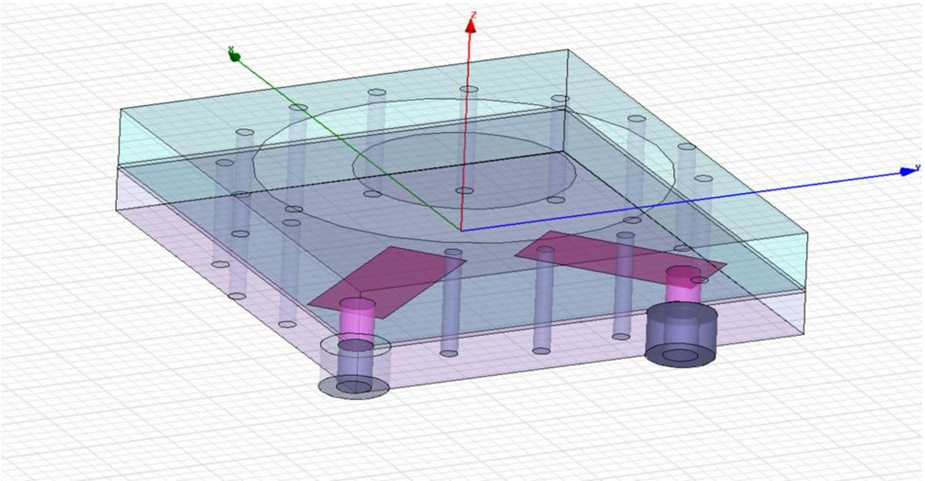
Source: www.globalstar.com/en/photoGallery/

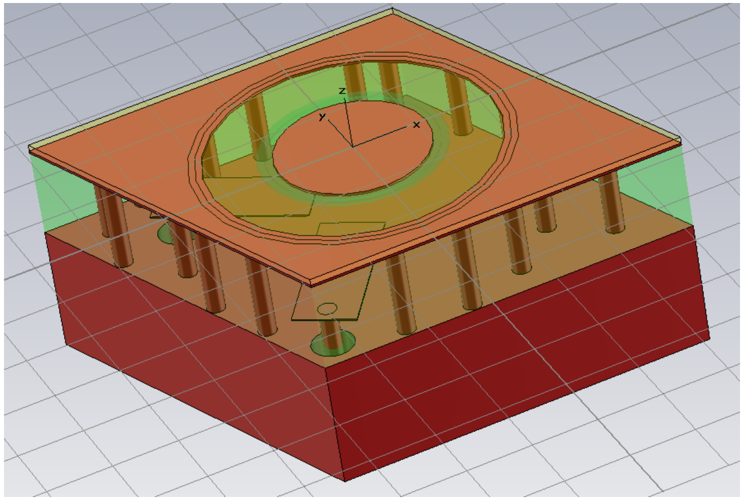
L-band Ring Slot Rx Array in Globalstar



Source: The Globalstar Cellular Satellite System, IEEE, TAP-46, June 1998.

Ring Slot Element (Globalstar)





- 2-Layered structure
- Radiation through Slot-Ring
- Slots are EM coupled by probe-fed strips
- Multiple vias between ground plane suppress surface wave/parallel plate modes
- No scan blindness; Good for wide angle scanning
- Bandwidth: 5 to 10%

Beam Shaping Algorithms

Common Beam Shaping Algorithms

ITERATIVE

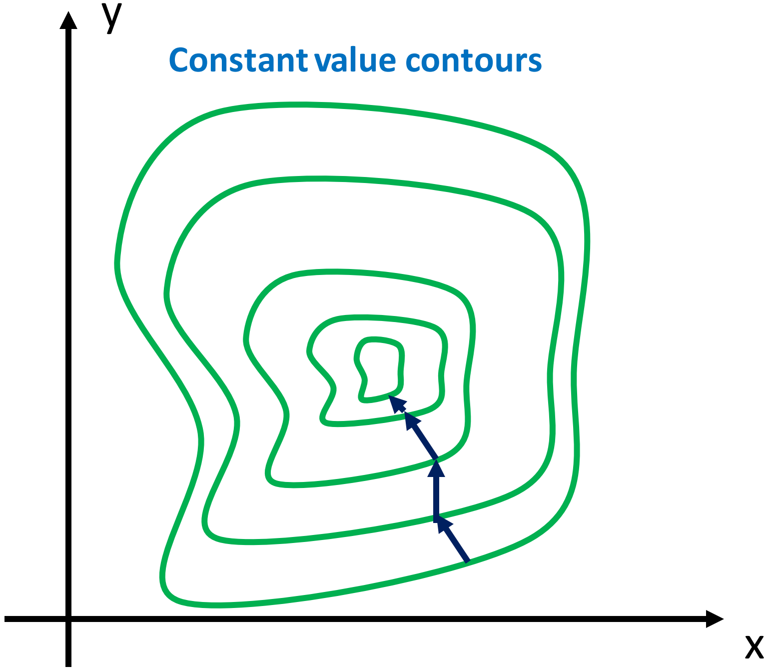
- Gradient Search
  - Most versatile
  - Minimizes a cost function
- Conjugate Match
  - Good for array opt
  - No explicit cost function
  - Improves the lowest gain point

EVOLUTIONARY

- Genetic Algorithm (GA)
  - Survival of the fittest
  - No local minimum
  - Slow
- Particle Swarm
  - Faster than conventional GA

Gradient Search Algorithm

- Finds the location of a minimum point of an objective function iteratively
- The gradient of a function represents the largest slope
- Search point moves opposite to the gradient direction



Contour of a function of two variables

Gradient Search Algorithm for Array

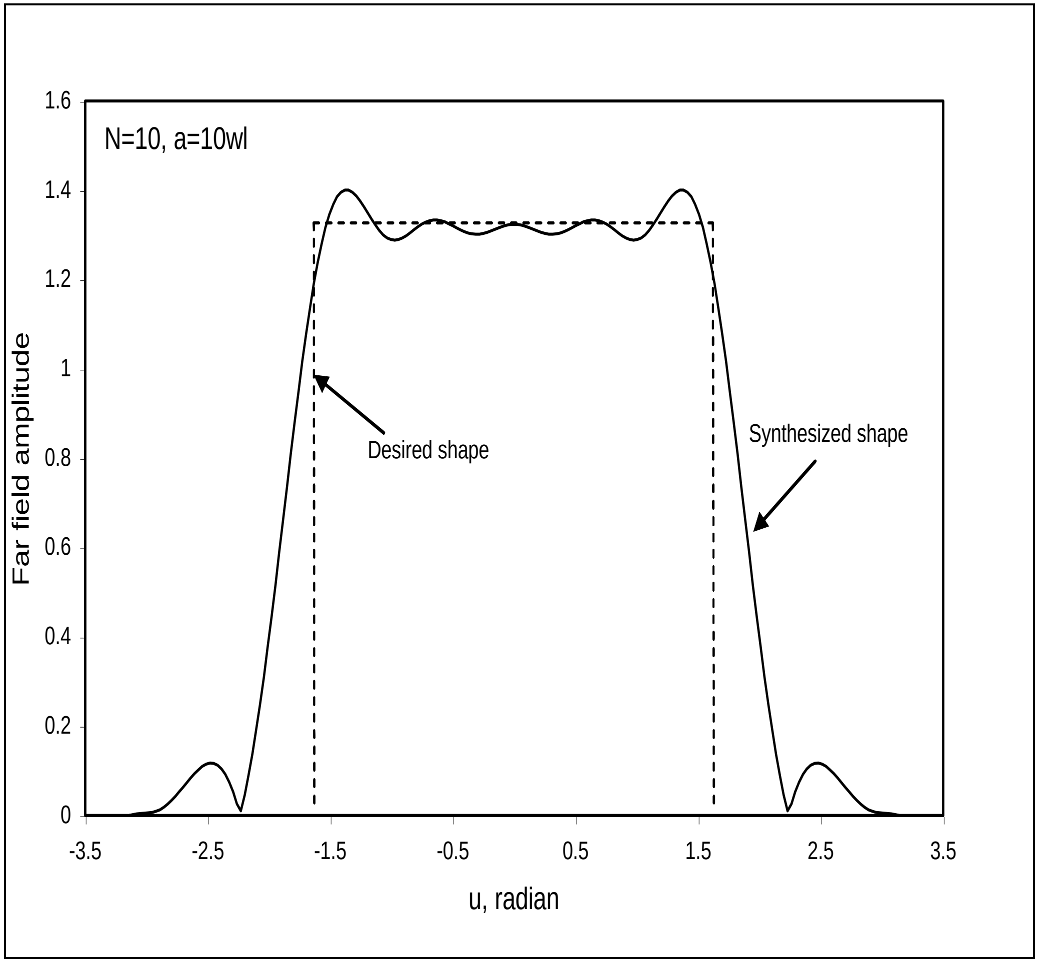
- Step-1
  - Obtain the gain as a function of amplitude and phase of the elements
  - Select finite number far field sample points covering the region
  - Compute the array gains at the far field points w.r.t. trial amplitude and phase distributions
- Step-2
  - Set the objective function as

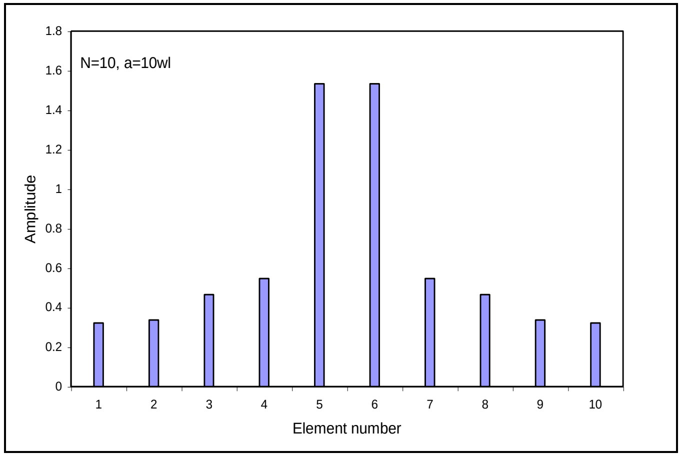
$$F = \sum_{n} |G_{desire} - G_{achieve}|^{K} ; K \geq 2$$

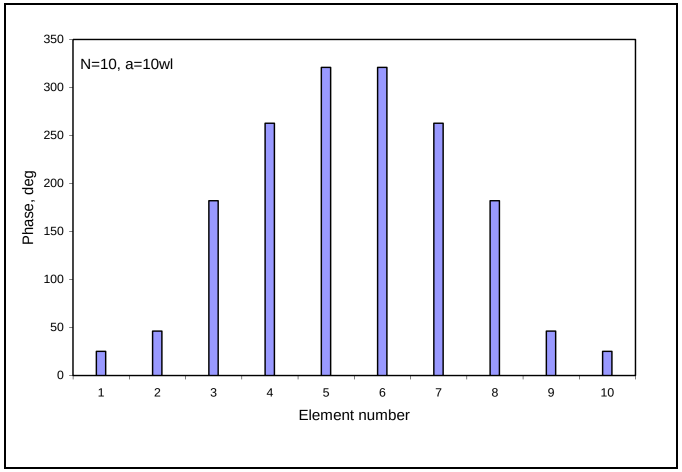
  - Compute the partial derivatives of *F* w.r.t. amplitude and phase of the elements
  - Use the relation [x(new)]=[x(old)]-R Grad(F)
  - Repeat the process until F is sufficiently small

Example: Flat top beam

(10 element array)





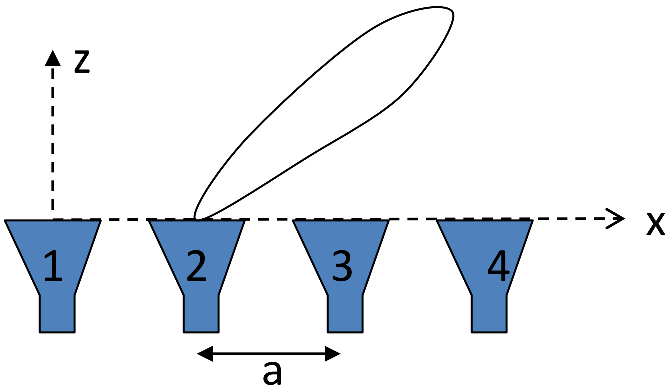


Remarks on GSA

- Very versatile algorithm, applicable to any optimization problem
- Involves numerical computations of partial derivative; computation time increases exponentially with the number of variables
- For a larger value of K (recall: K= exponent in the objective function), the GSA algorithm becomes "Mini-Max" algorithm
- "Local minimum" problem occurs if the trial solution is very far from global minimum

Conjugate Match Algorithm

- Improves the largest deviation point
- **Principle:** Peak gain at a far field point can be achieved if the excitation coefficients are complex conjugate of the far field intensities of the individual elements
- Can be understood through a scanned beam antenna

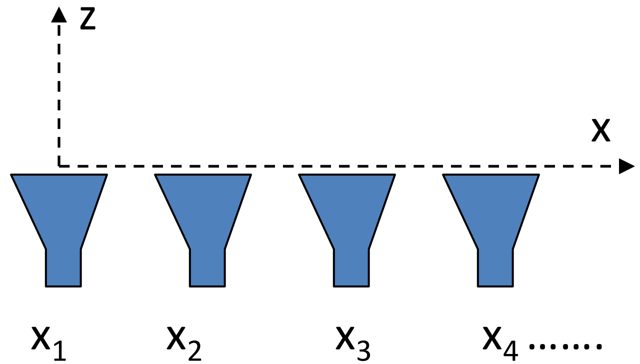


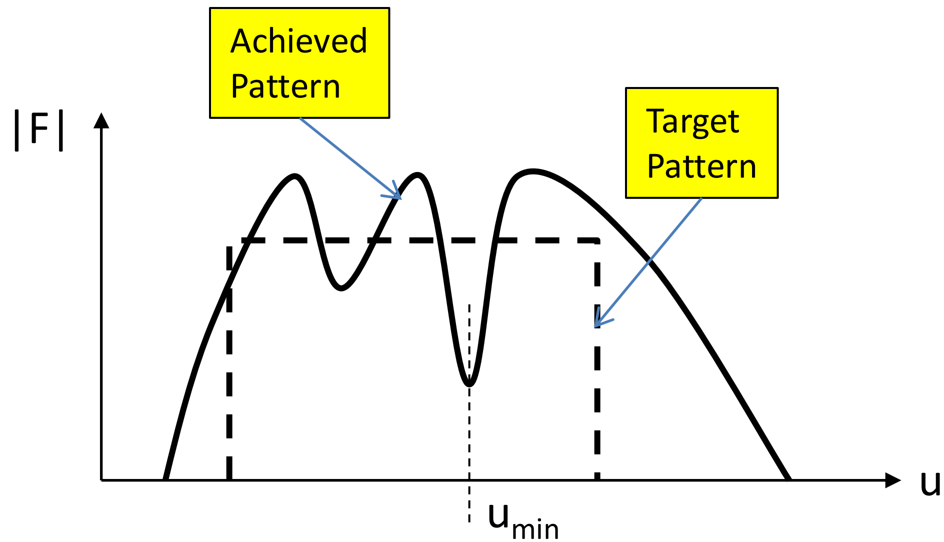
$$e_2 = f(\theta)\exp(jka\sin\theta)$$

$$A_2 = Kf^*(\theta)\exp(-jka\sin\theta)$$

Element fields add Coherently because $A_2 \times e_2$ is a real number

Conjugate Match: Phase Only Optimization





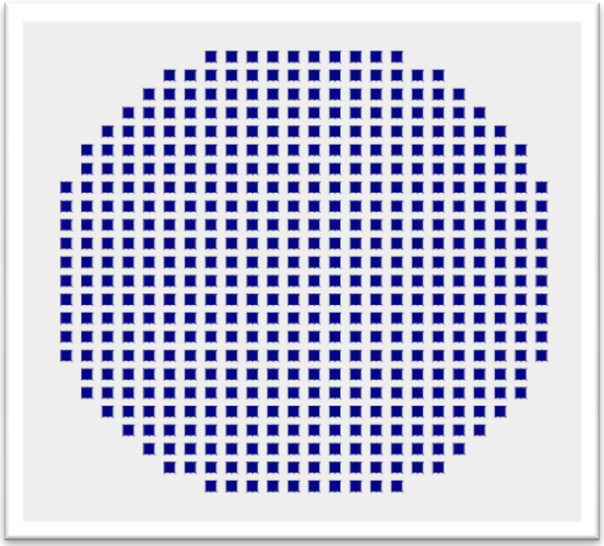
$$\text{Complex Amplitude Change for n-th element:}$$

$$\Delta A_n = F \exp(-jkx_n u_{\min})/N$$

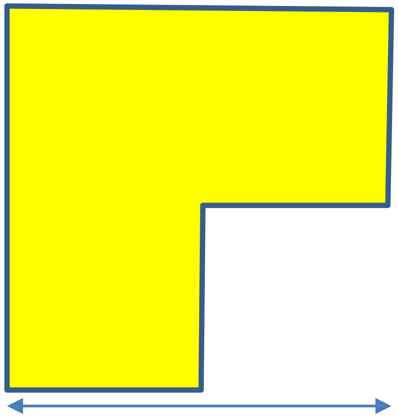
$$\text{Updated phase} = \text{Phase of } (A_n + \Delta A_n)$$

Typically, N is proportional to the number of far field points and F is a small fraction.

Shaped Beam Array Example



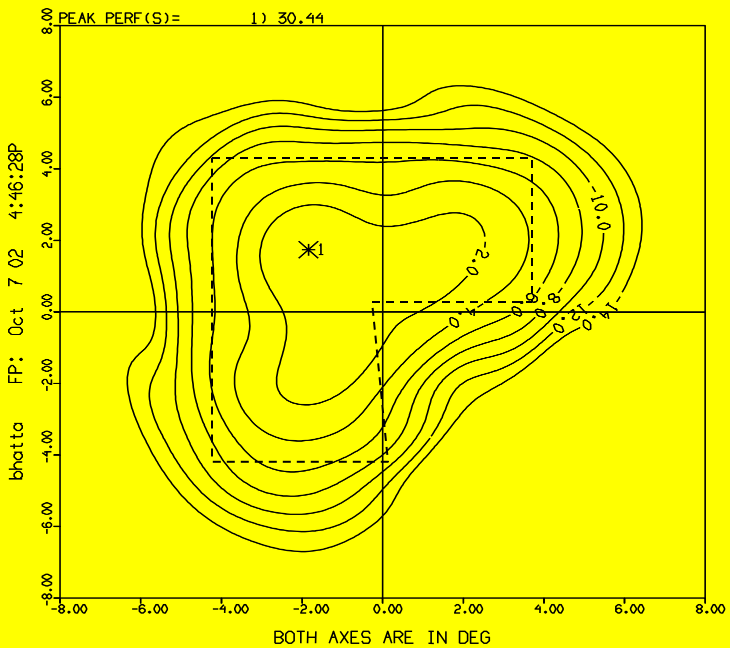
Circular Shaped Array



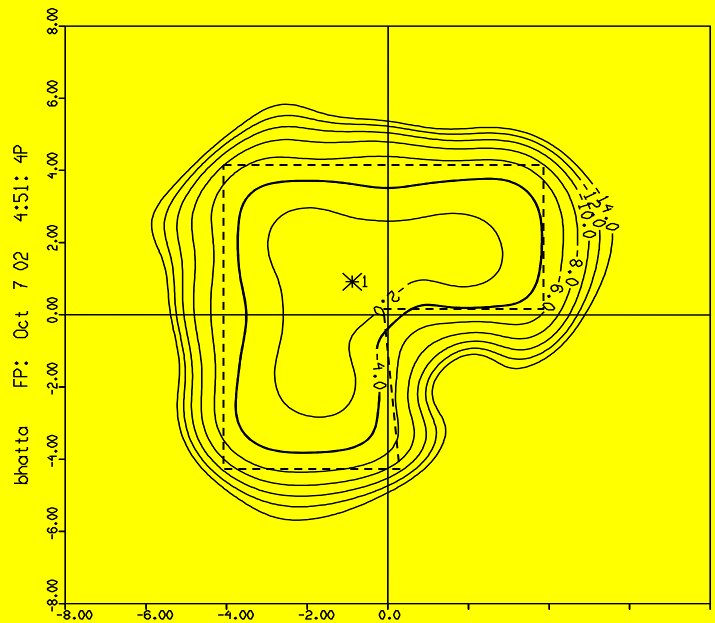
Desired inverted L-shaped beam
(beam area=48 sq-deg)

- Circular Array with 848 Elements
- Element size = 1.5 wl x 1.5 wl

GSA and Conjugate Match



GSA: GAP=16,066



Conjugate Match: GAP=17,206

- Circular Array with 848 Element
- For GSA, parameters are reduced by using polynomial functions
- Conjugate match is about **10 times** faster than GSA
- Conjugate match gives more GAP (Gain difference is about 0.3 dB)

References for GSA and CM

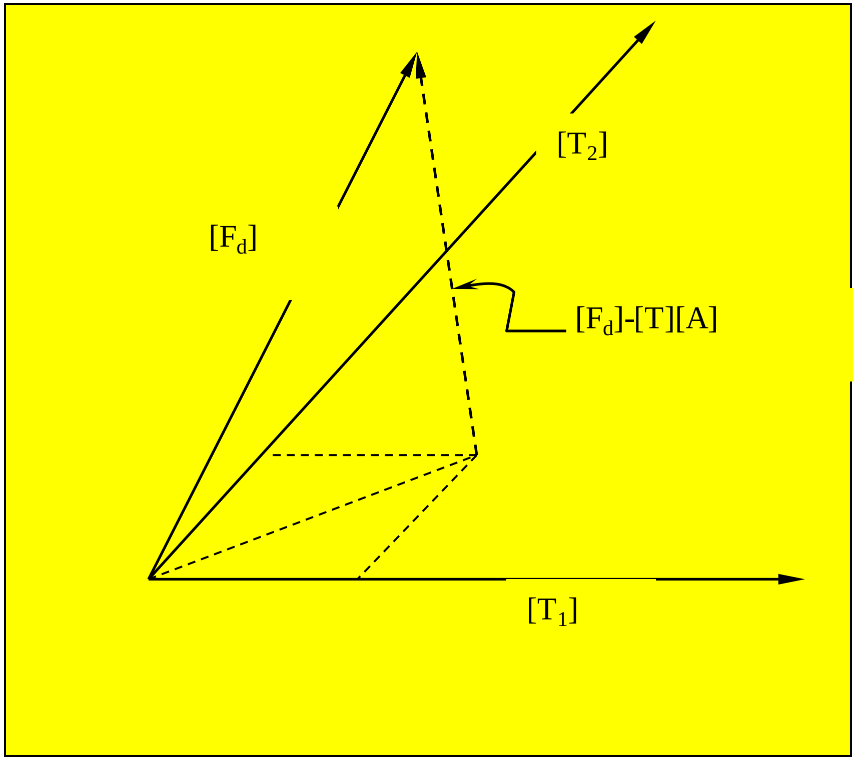
- A. R. Cherrette, S.W. Lee and R. J. Acosta, IEEE Trans., AP, pp. 698-706, June 1989. (GSA for reflector synthesis)
- L. I. Vaskelainen, IEEE Trans, AP, pp. 987-991, June 2000. (GSA for array/Phase only opt)
- P. T. Lam, S.W. Lee, D.C.D. Chang and K.C. Lang, IEEE Trans, AP, pp. 1163-1174, Nov. 1985. (CM for multi-feed reflector)
- A. K. Bhattacharyya, **Phased Array Antenna-Floquet Analysis, Synthesis, BFNs and Active Array Systems**, Wiley, 2006, Chapter 11 (detailed analysis of GSA, CM and other methods)

PROJECTION MATRIX ALGORITHM
(Principle and Applications)

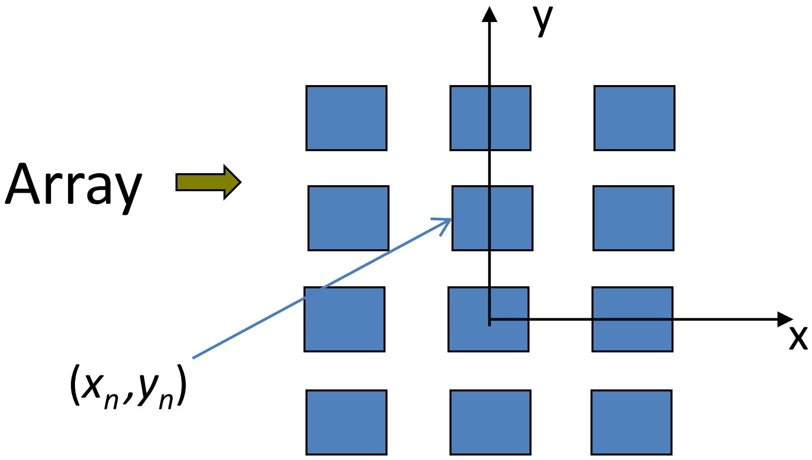
Projection Matrix Algorithm

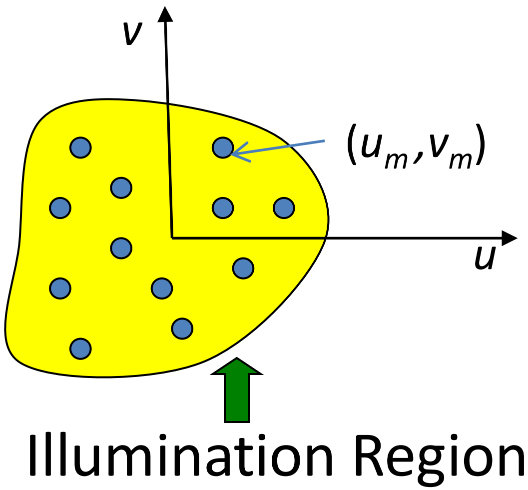
- Exists in Linear Algebra/Signal Processing/Control and other areas
- Concept of vector space is invoked
- Error vector is orthogonal to the best-fit solution vector
- The projection matrix operator eliminates orthogonal error

[F_d]=[T][A]
[A]=Unknown
[T]=Known
[F_d]="Semi" known



Array Far Field Equation





$$F_d(u_m, v_m) = \sum_{n=1}^{N} A_n E_n(u_m, v_m) \exp(jx_n u_m + jy_n v_m)$$

$$= \sum_{n=1}^{N} A_n T_{mn}$$

$$[F_d] = [T][A]$$

Far Field Intensity

$[F_d]$ is not completely known because the "phases" of Far field points are unknown

The Projection Matrix

$$\begin{bmatrix} F_{d1} \\ F_{d2} \\ . \\ . \\ F_{dM} \end{bmatrix} = \begin{bmatrix} T_{11} \; T_{12} \ldots\ldots\ldots\ldots T_{1N} \\ T_{21} \; T_{22} \ldots\ldots\ldots\ldots T_{2N} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ T_{M1} \; T_{M2} \ldots\ldots\ldots\ldots T_{MN} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ . \\ . \\ A_N \end{bmatrix}$$

$$[P][F_d] = [T][A]$$

$$[P][\Delta F_d] = [T][\Delta A]$$

$$[P] = [T][[T^*]^T [T]]^{-1} [T^*]^T$$

[P]=PROJECTION MATRIX

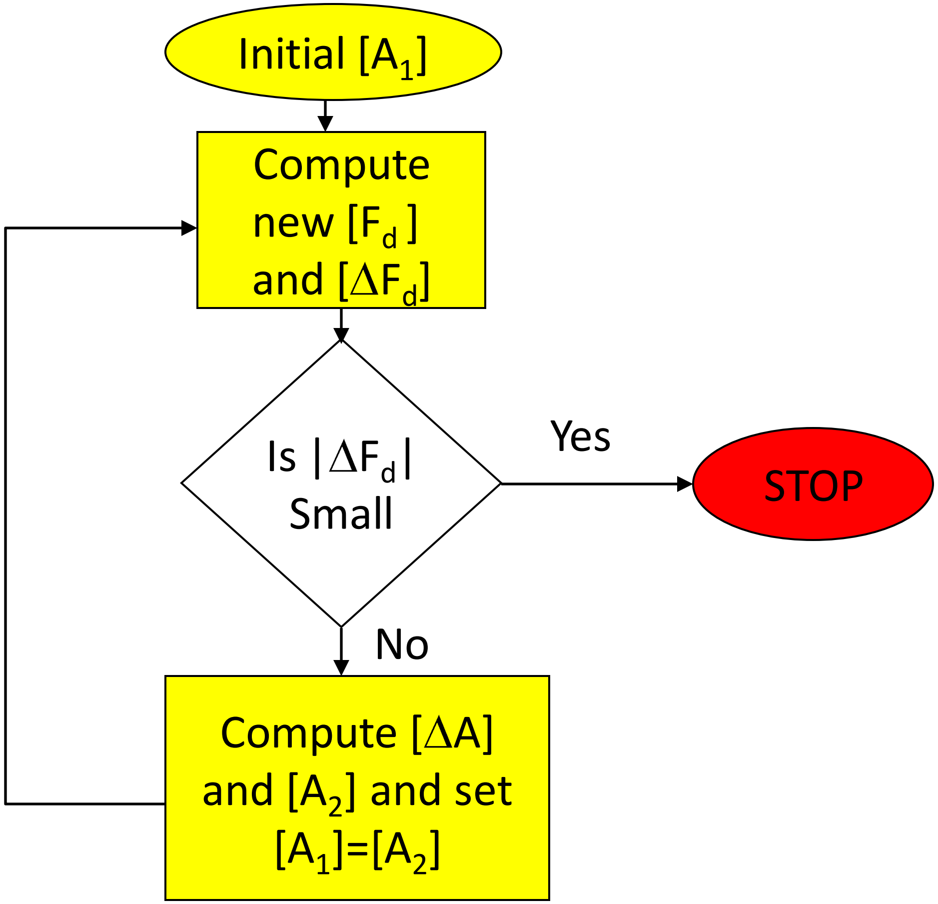
- $[F_d]$=[T][A] may not be exactly solvable because $[F_d]$ may not lie on the column space of [T]
- The projection matrix operator [P] eliminates the orthogonal component , making the equation solvable

Flow Chart of PMA

Key Equations:

$$[P][F_d] = [T][A]$$
$$[P][\Delta F_d] = [T][\Delta A]$$

- Initial phase of $[F_d]$ is estimated from initial [A]
- Phase of $[\Delta F_d]$ is taken same as that of $[F_d]$
- [A] vector is normalized to the input power at each iteration



Stability of Solution

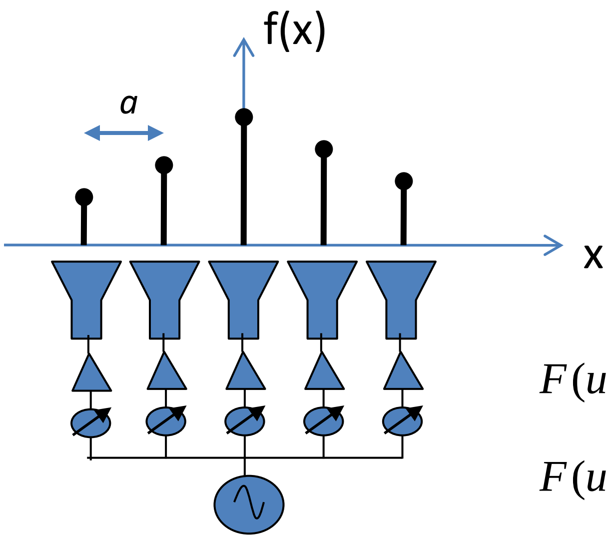
$$[P][F_d] = [T][A]$$

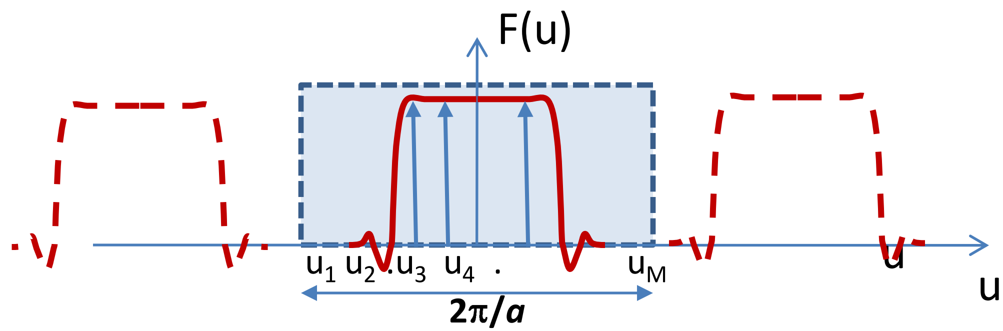
$$[P] = [T]\underbrace{[[T^*]^T[T]]}_{[\tau]}^{-1}[T^*]^T$$

- Stability of solution depends of condition number of the [τ] matrix in [P]
- Smaller condition number is better for stability
- Condition number= Norm of [τ] × Norn of $[\tau]^{-1}$
- Norm=Magnitude of the largest row vector
- Diagonal matrices have the lowest condition number

Improving condition number of [τ]

(Linear Array, Delta sources, discrete far field sampling)





$$F(u) = \int f(x)\exp(jux)dx \approx a\sum_n f(na)\exp(juna)$$

$$F(u \pm 2\pi/a) = F(u) \quad \Rightarrow \text{Grating Lobes every } 2\pi/a$$

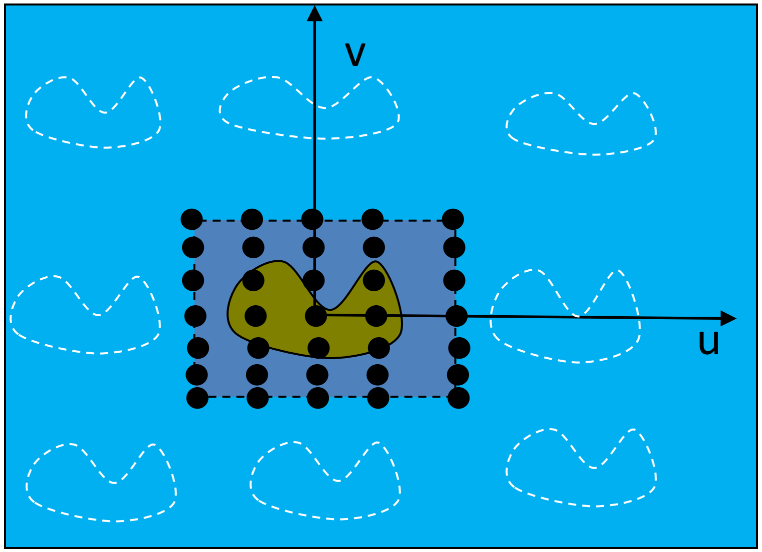
$$[T] = \begin{bmatrix} 1 & \exp(ju_1a) & \exp(2ju_1a) & \exp(3ju_1a)......\exp((N-1)ju_1a) \\ 1 & \exp(ju_2a) & \exp(2ju_2a) & \exp(3ju_2a)......\exp((N-1)ju_2a) \\ .... \\ 1 & \exp(ju_Ma) & \exp(2ju_Ma) & \exp(3ju_Ma)......\exp((N-1)ju_Ma) \end{bmatrix}$$

Diagonal matrix of identical elements has condition number unity

[τ] satisfies that condition if the columns of [T] are orthogonal. This implies $(u_M - u_1) = 2\pi/a$.

That is, the sampling interval must be extended beyond the illumination region

Far Field Sampling for Planar Array



Far-Field Sample Points

- **For a planar array** [τ] is well conditioned if

  $(u_{max}-u_{min}) = 2\pi/a$, $(v_{max}-v_{min}) = 2\pi/b$,

  $a \times b$= Cell size

- The sampling region ideally should be extended to the midway between main lobe and nearest grating lobes

- The sample interval is governed by the equation $\Delta u \times$ Dia (of array) $< 2\pi$

Convergence Test
(Condition Number)

	Array of 12 ×12 equal size elements		Array of 132 elements of 3 sizes	
$(u_{max}-u_{min})a$	Condition Number	Minimum Gain (dBi)	Condition Number	Minimum Gain (dBi)
3	1.30×10^{8}	-15**	9.25×10^{7}	10**
4	1.55×10^{8}	-21**	5.53×10^{5}	22**
5	6.00×10^{3}	4.64**	92.00	24.03
6	1.44	23.04	2.18	24.04
7	1.13	23.03	1.45	24.10
8	1.04	23.03	1.29	24.13

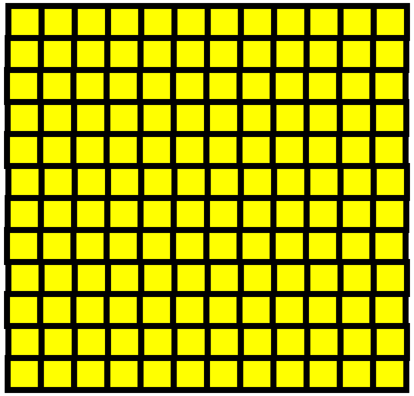
** Unstable solution

a = size of element (smallest size for the array of 3 sizes)

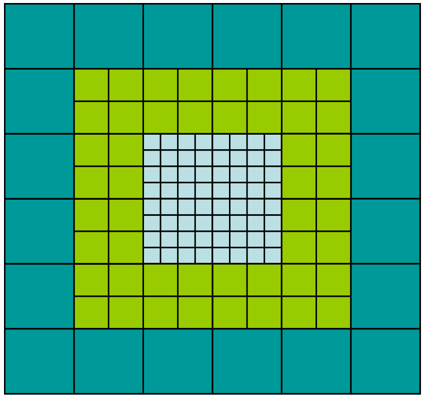
SHAPED BEAM ARRAY EXAMPLES

Numerical Computations
(Array Configurations)

- Considered two array examples:
  - Array-1: 12 x 12 identical elements
  - Array-2: 132 elements of 3 different sizes
- Uniform and tapered excitations are used for Array-1
- Only uniform excitation is considered for Array-2



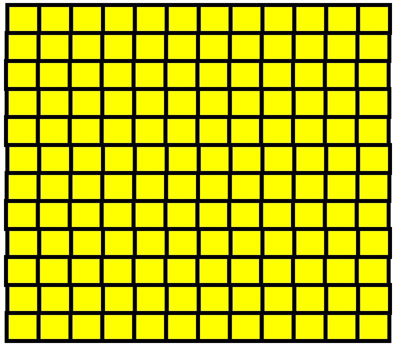
Array-1
144 elements



Array-2
132 elements

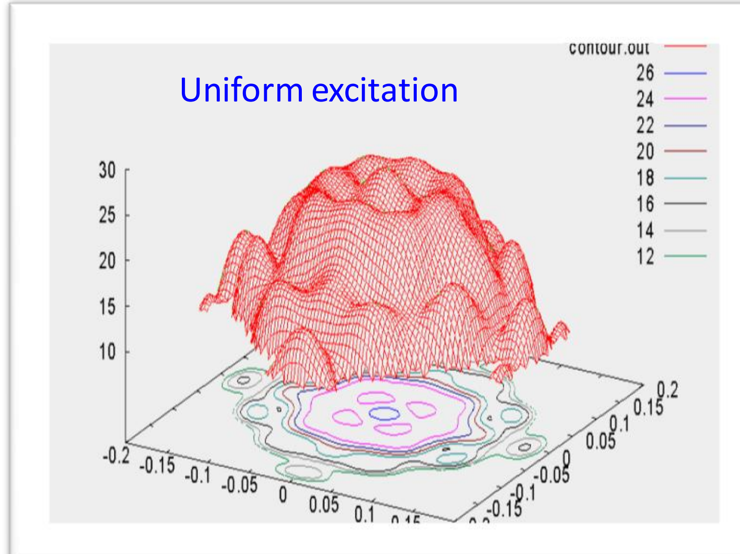
Flat Top Beams (12 x 12 element array)

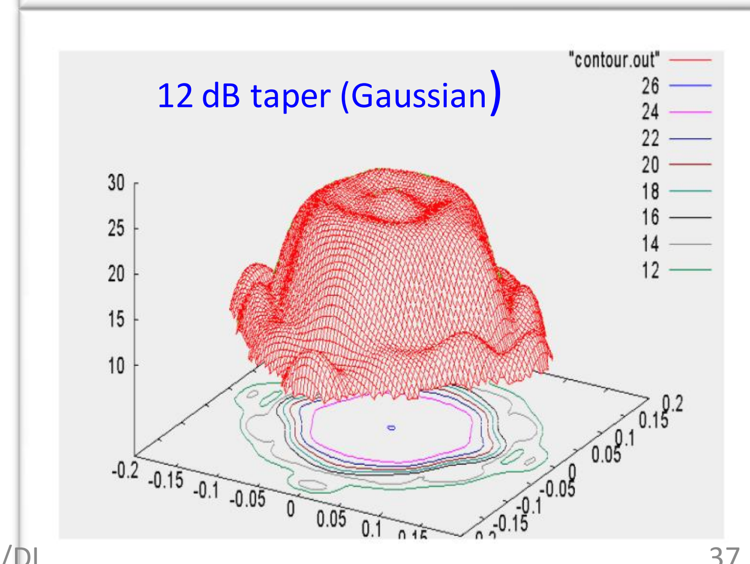
Array-1



- Element size = 1.6 x 1.6 sq-wl
- Beam dia =0.18 radian
- Sampling space =0.64 radian
- Uniform initial phase
- Computation time 38 seconds (2.4 GHz Pentium-4, 256 MB RAM)
- Lowest Gain
  - 23.03 dBi (uniform excit.)
  - 24.04 dBi (tapered excit.)

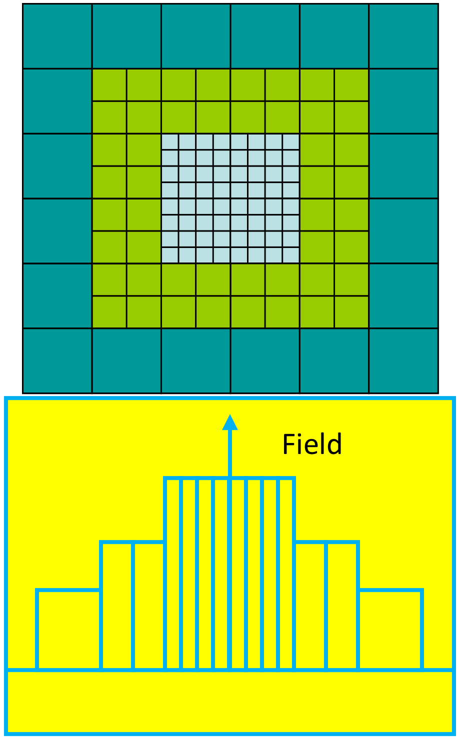
To reduce ripples amplitude taper is necessary

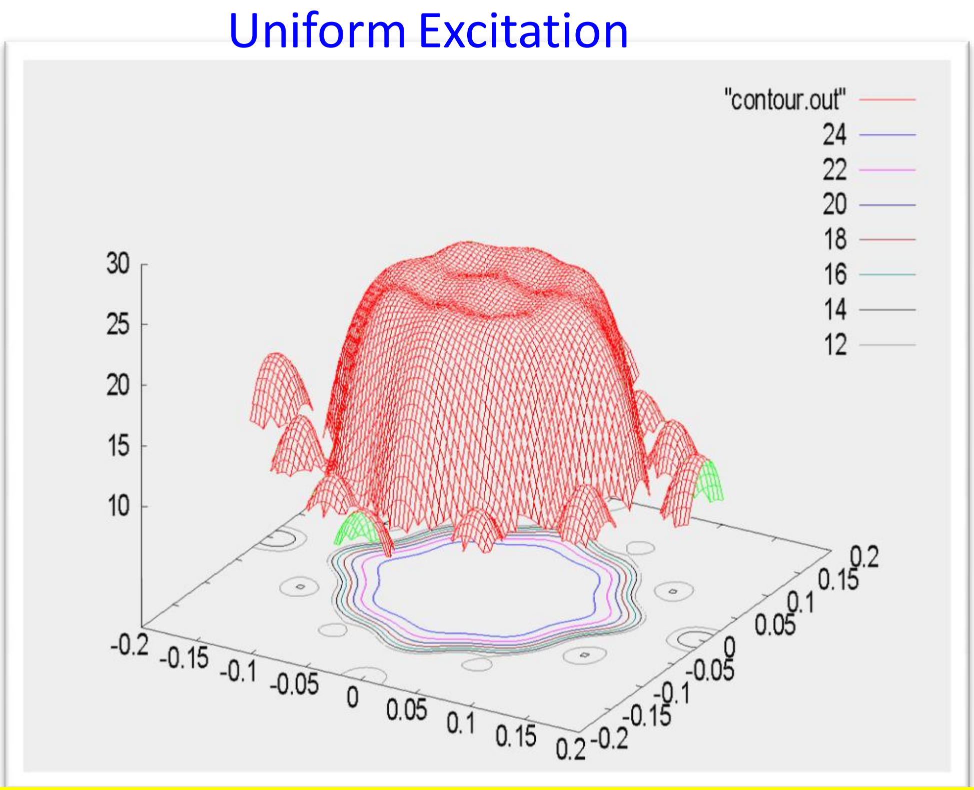




Array of 132 Elements (3 sizes)

(uniform excitation gives natural taper)

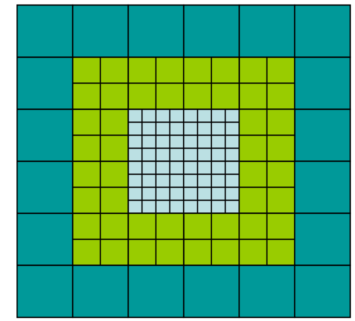




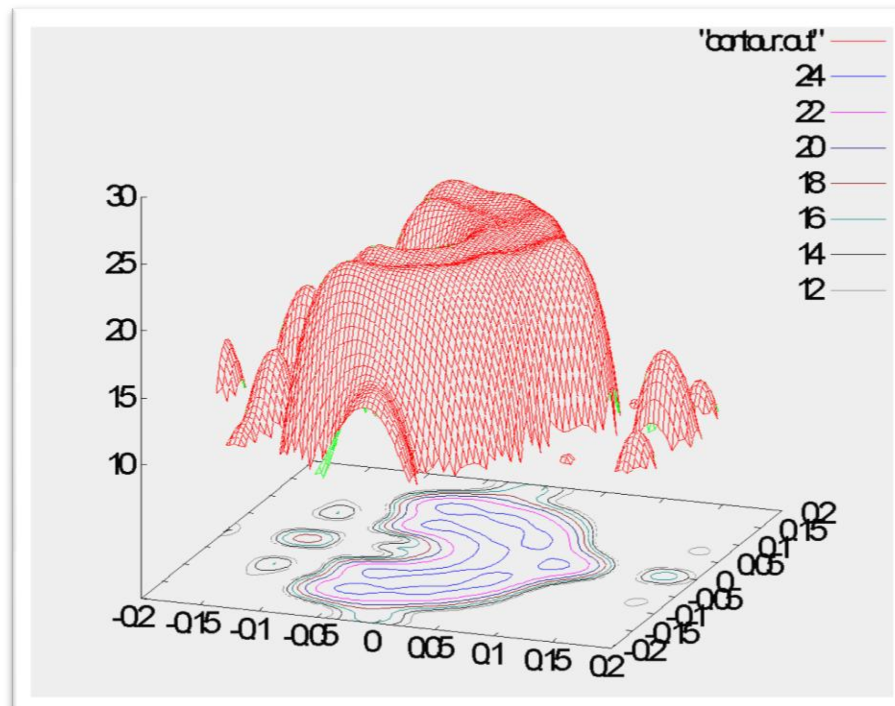
- Identical power sources yield 12 dB natural edge taper for this configuration
- The EOC gain is 24.13 dB (GAP=21596 sq-deg)
- Identical SSPAs help simplify implementation problems in active array

More Complex Contour

- **Desired Contour Shape:** 180deg annular sector
- 132-element array
- Identical power sources
- Achieved GAP=17,159 sq-deg
- Complex beams have lower GAP



Array-2



PMA Versus GSA

- Considered 400 element array, Flat top beam, 3-deg beam radius
- 27 unknowns for GSA (6-order polynomial for phase distribution)
- For larger number of iterations PMA is significantly faster than GSA
- PMA converges to a higher gain number than GSA because GSA is constrained by 6-order polynomial in this case.

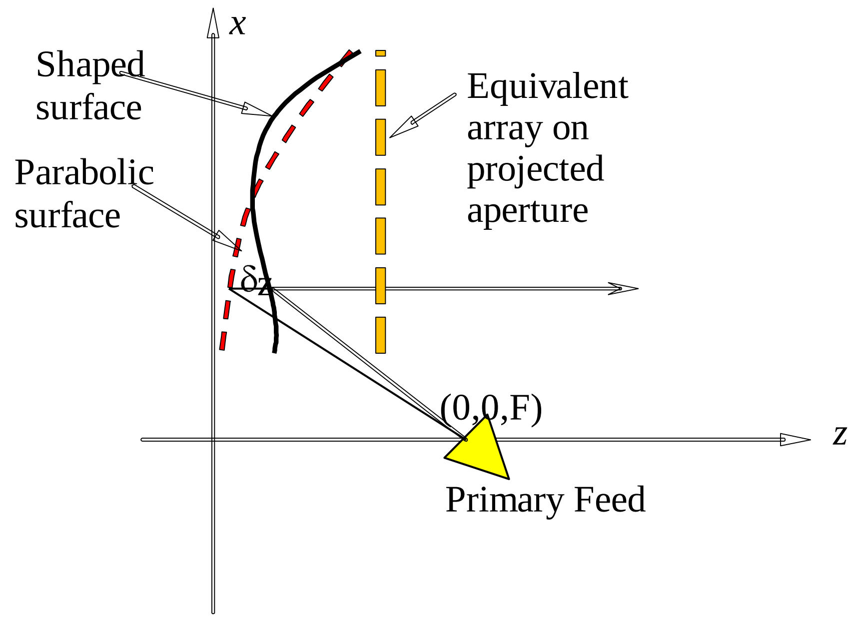
	GSA		PMA	
Number of Iterations	Time in Seconds	Minimum Gain (dBi)	Time in Seconds	Minimum Gain (dBi)
100	34	22.29	26	27.17
200	68	27.33	26	28.54
300	102	28.02	27	29.06
400	136	28.12	28	29.26
500	170	28.20	28	29.31
1000	340	28.20	29	29.34
5000	----	----	40	29.43

SHAPED REFLECTOR SYNTHESIS

Shaped Reflector Synthesis

Steps:

- Fictitious planar array assumed on projected aperture
- Phase distribution synthesized for the fictitious array
- Ray optics approximation employed to perturb the parabolic surface



Ray optics equation:

$$k_0(\delta z - \delta r) = \psi(x, y)$$

Reflector Phase Synthesis

- Typically, phase is computed using inverse tangent operation of complex amplitude. This makes discontinuous phase distribution because of multiple branches of inverse tangent
- For a realizable reflector surface the phase distribution must be continuous
- To satisfy this condition, a smooth trial phase distribution function is assumed; then it is updated by directly adding the incremental phase distribution after each iteration

Incremental Phase Computation

Complex amplitude of n-th element at i-th iteration :

$$A_n^{(i)} = |A_n^{(i)}| \exp(j\psi_n^{(i)})$$

At $(i+1)$ iteration :

$$A_n^{(i+1)} = |A_n^{(i+1)}| \exp(j\psi_n^{(i+1)})$$

$$\psi_n^{(i+1)} = \psi_n^{(i)} + \delta\psi_n^{(i)}$$

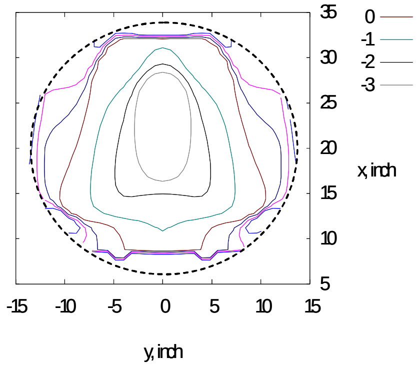
For a small $\delta\psi_n^{(i)}$, one gets

$$\delta\psi_n^{(i)} \approx \frac{1}{j}\left[\frac{A_n^{(i+1)}}{A_n^{(i)}} - 1\right]$$

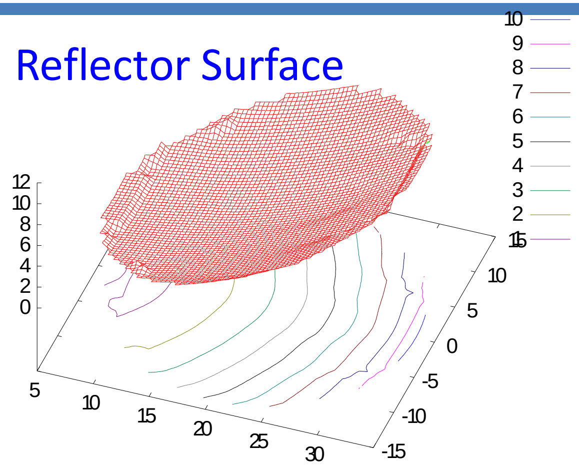
$$\psi^{(0)} = \text{Smooth Function of } x, y$$

Reflector Surface and Beam

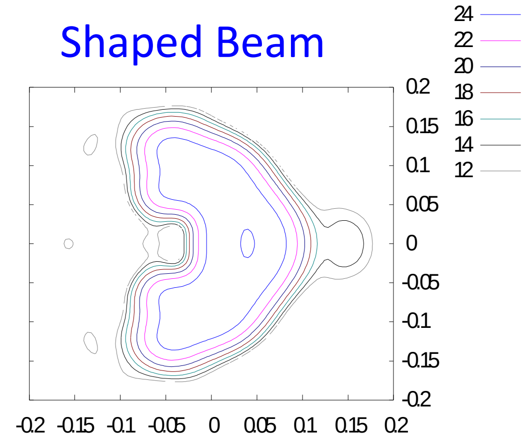
Phase Contour



Reflector Surface



Shaped Beam

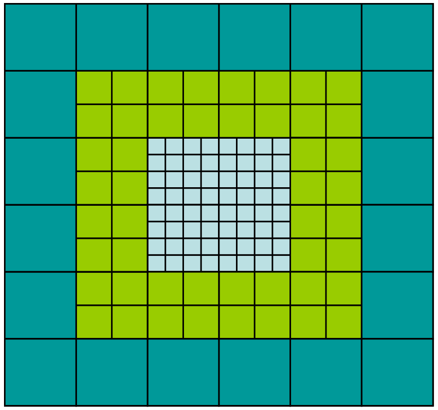


- Reflector Diameter = 22 WL, F=24 WL
- Cell size on aperture plane = 0.64 WL
- Phase distribution is a continuous function
- Smooth synthesized surface
- GAP of the beam is 20724 sq-deg
- Computation time 152 seconds

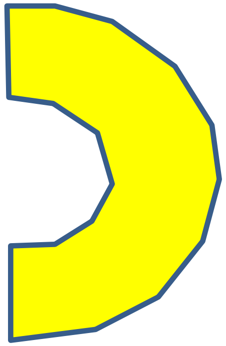
CONVERGENCE SHOW

Convergence show of a **Sector Beam**

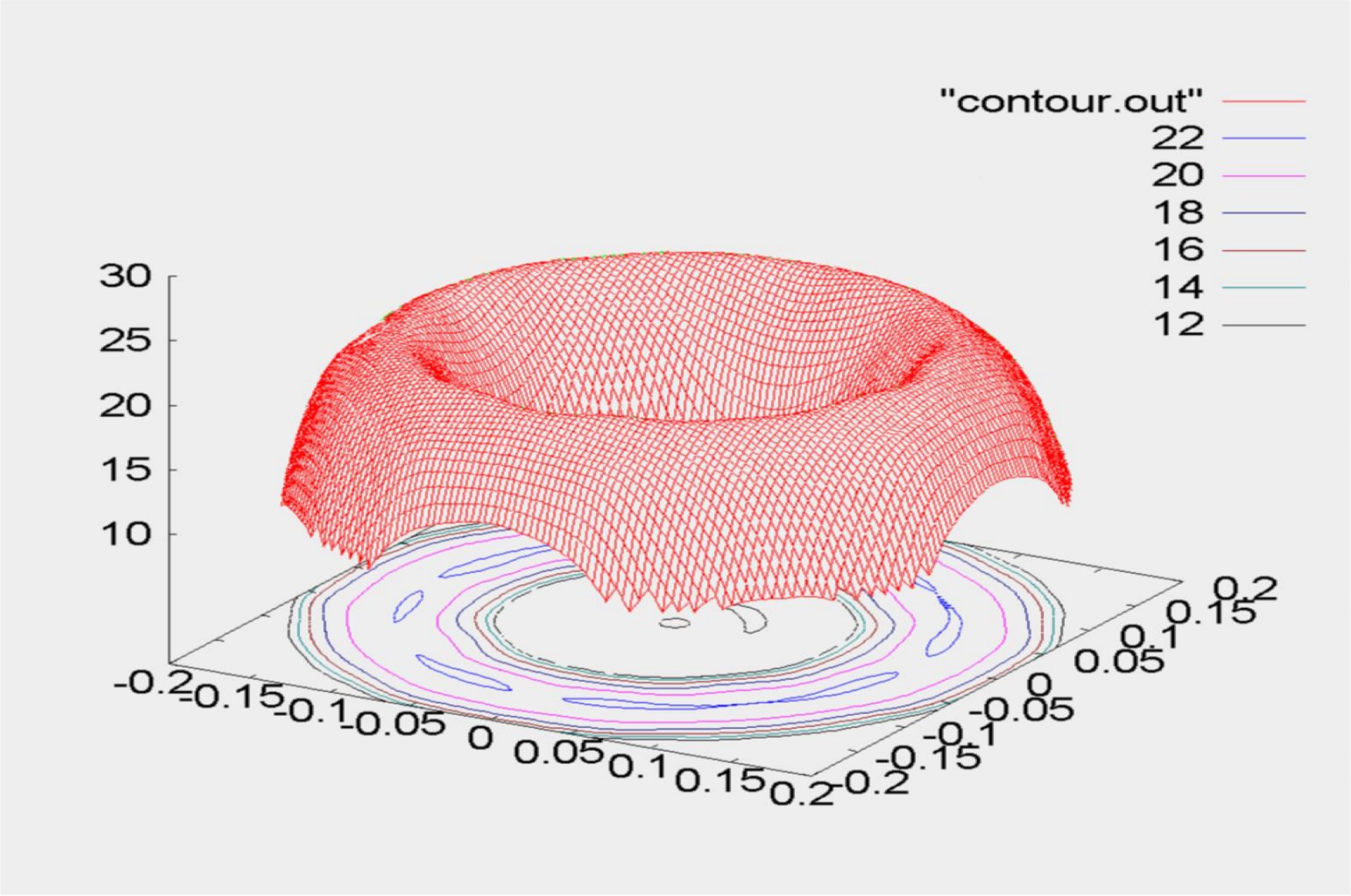
- Desired beam is a **sector beam** produced by 132 array elements
- Evolution to the desired shape from an initial shape
- Initial shape is a circular beam with a dip at the center
- Initial beam is realized by a large quadratic phase distribution in radial direction



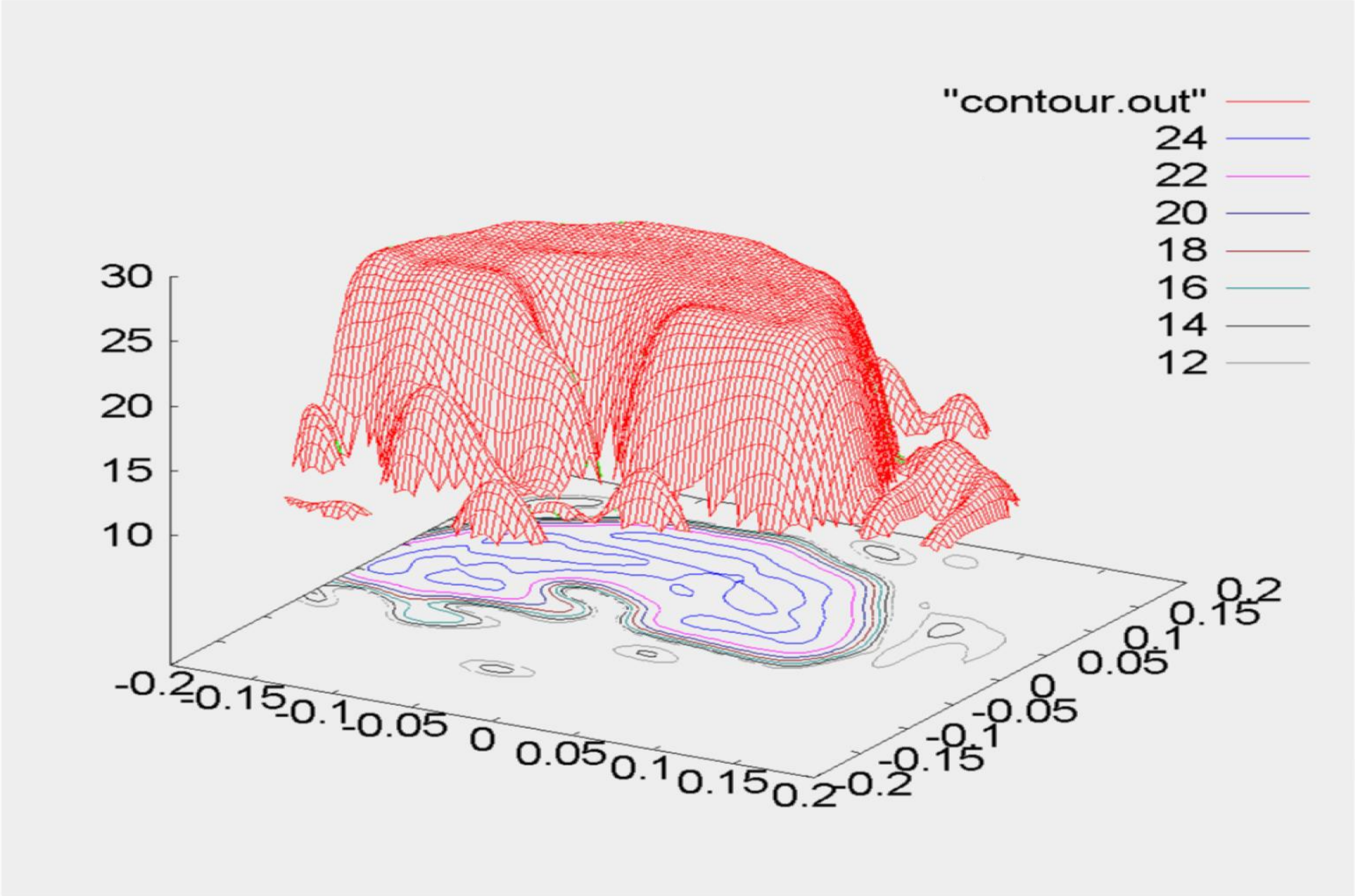
Array of 132 unequal elements



Desired sector Beam



"contour.out"
22
20
18
16
14
12
30
25
20
15
10
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2



"contour.out"
24
22
20
18
16
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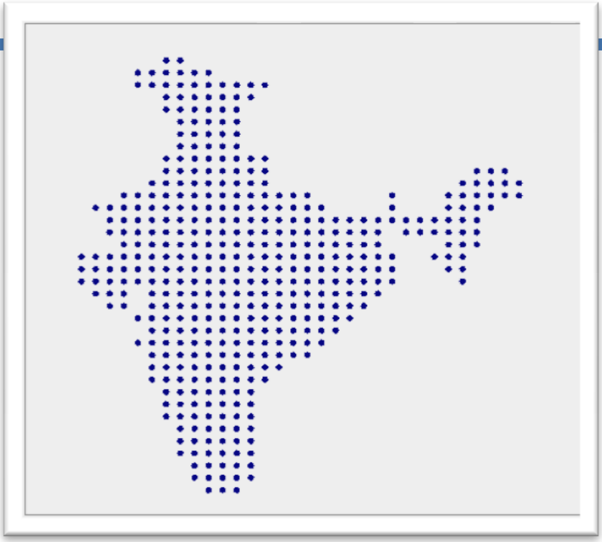
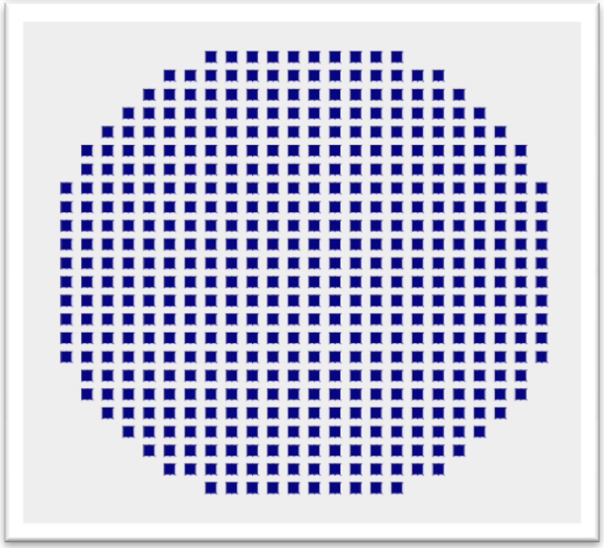
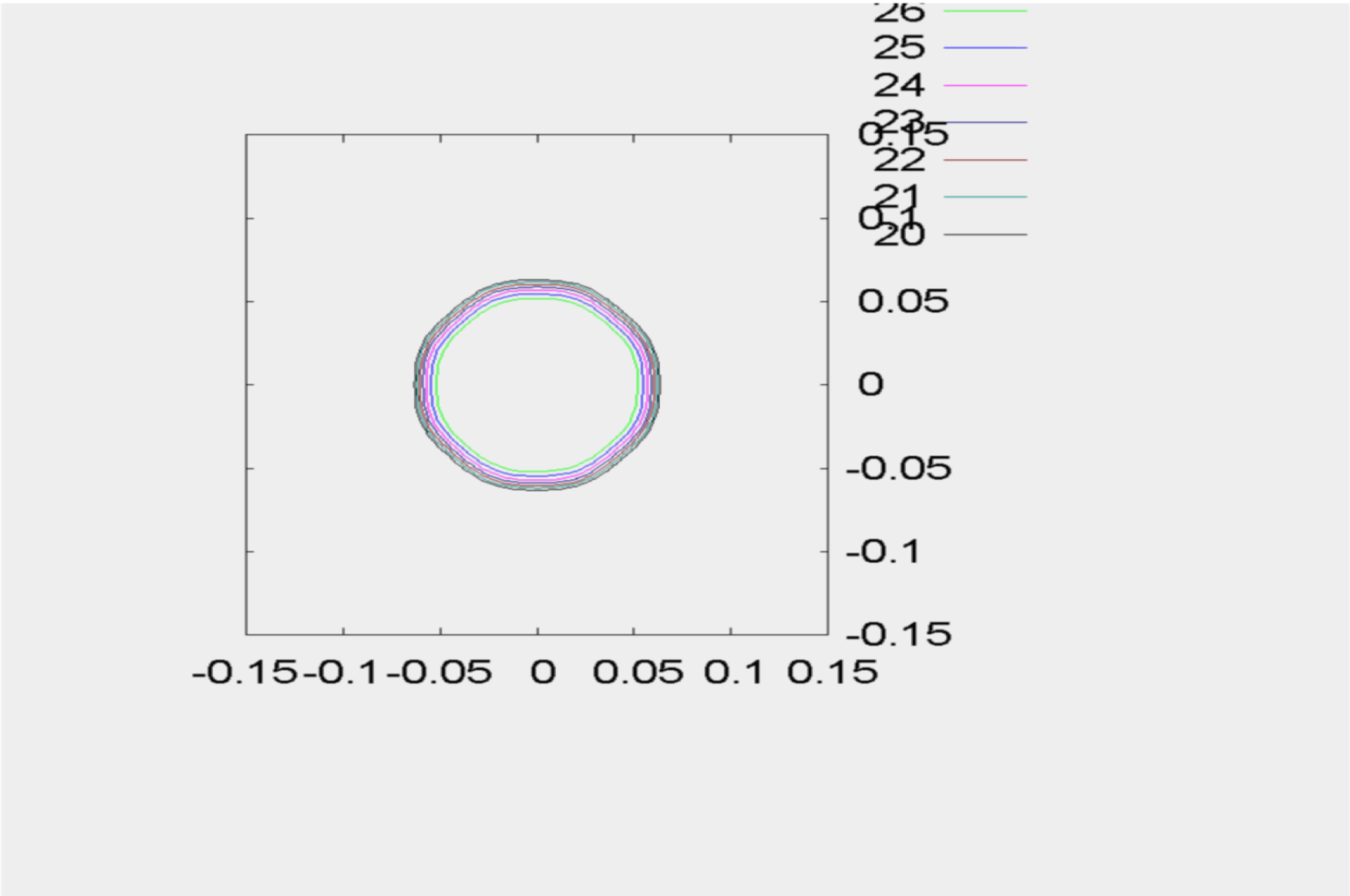
30
25
20
15
10
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2

India Beam

- Conformal beam to the shape of a country
- 484 Elements array
- Element Cell size = 2 WL x 2 WL
- 437 Samples on u-v plane
- 5000 iterations for convergence
- EOC gain=24 dBi
- GAP=17,739






26
25
24
23
22
21
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0.15
0.1
0.05
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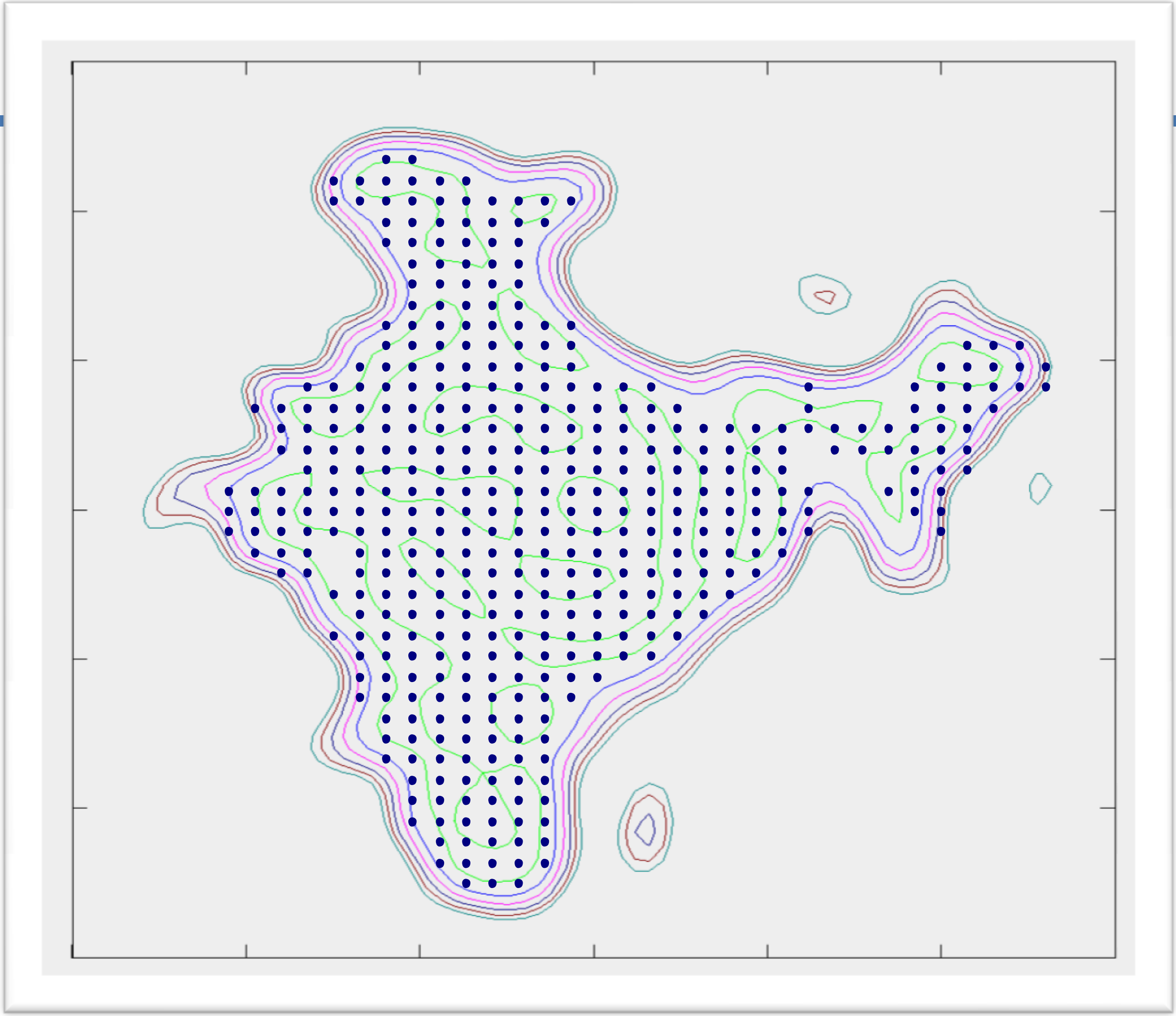
-0.05
-0.1
-0.15
-0.15 -0.1 -0.05 0 0.05 0.1 0.15

Comparison Between Desired and Achieved Shapes



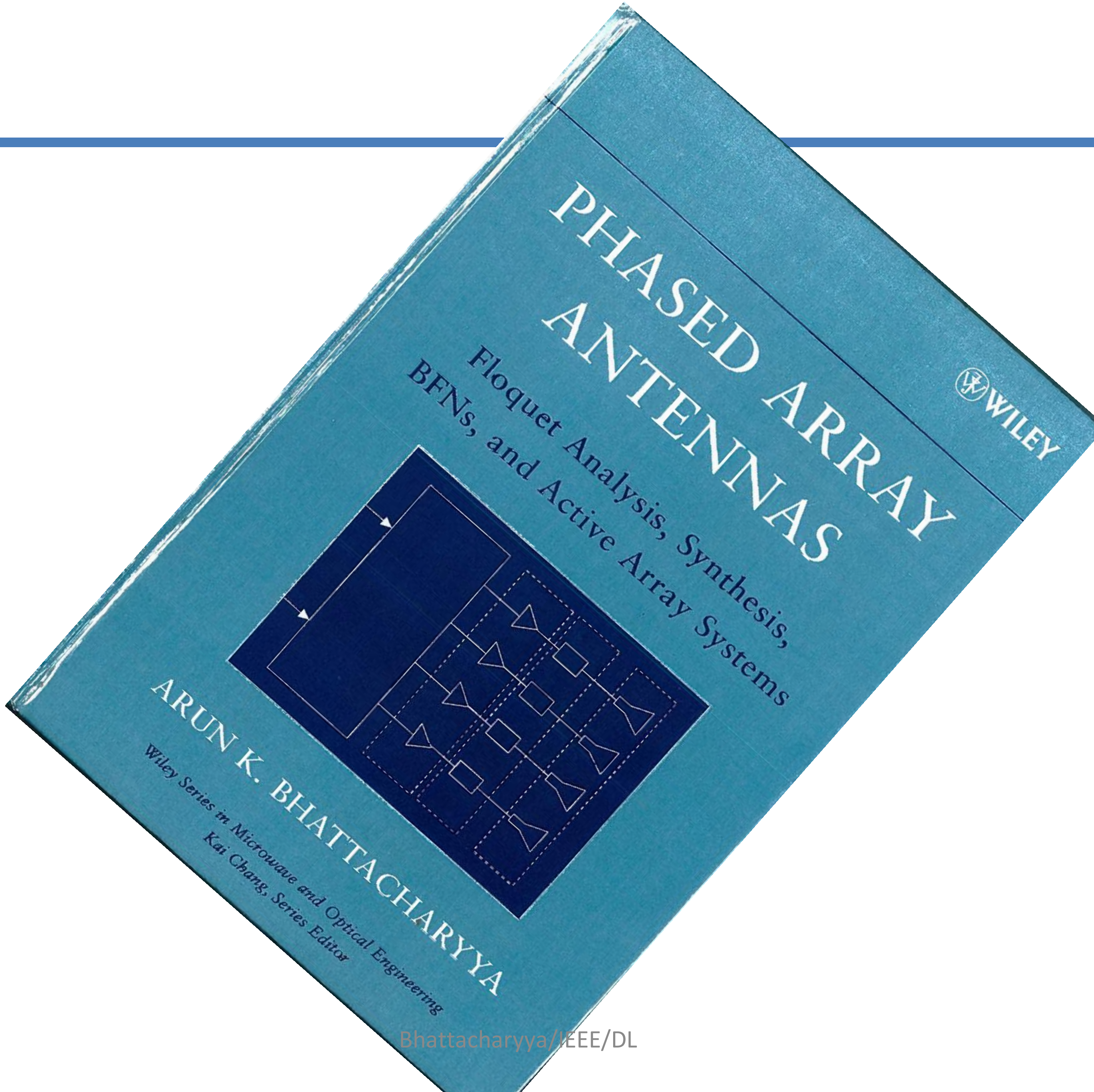
Concluding Remarks

- Successfully demonstrated the effectiveness of the PMA for shaped beam synthesis in arrays. For larger number of iterations PMA is significantly faster than GSA. Unlike a conjugate match approach, the PMA updates the complex amplitudes considering all the far field grid points **simultaneously**, resulting in a faster convergence of the final solution.

Further Reading

- A.K. Bhattacharyya, "Projection Matrix Method for Shaped Beam Synthesis in Phased Arrays and Reflectors", IEEE Trans., Antennas and Propagation, Vol. 55, pp. 675-683, March 2007.

Thank You for your attention!!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