Noise Analysis and Low-Noise Design for Compact Multi-Antenna Receivers: A Communication Theory Perspective

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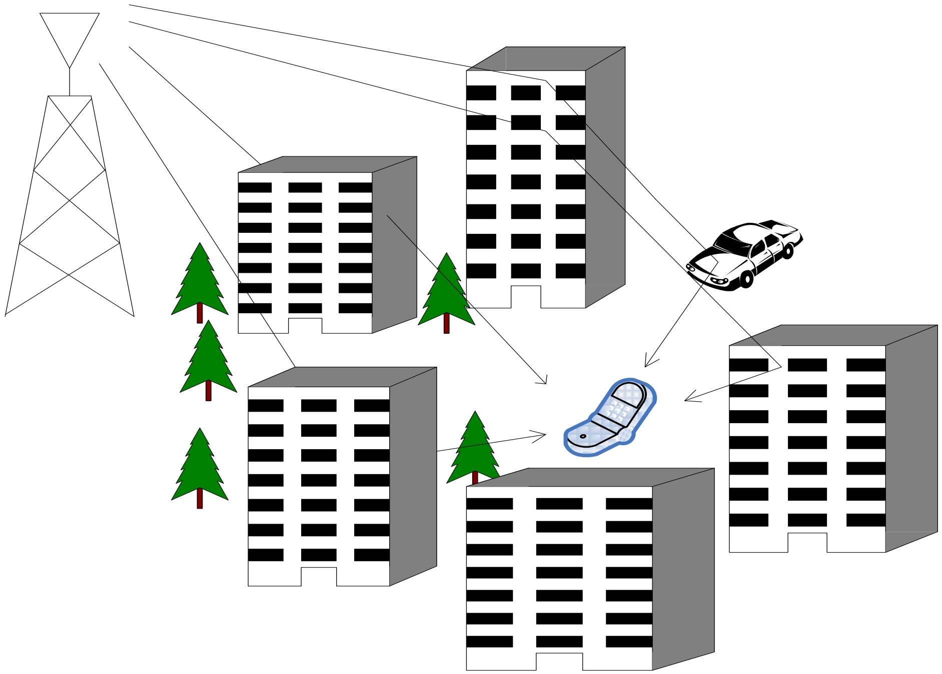
Motivation

- Multiple-input, multiple-output (MIMO) links may substantially increase spectral efficiency (e.g., IEEE 802.11n, 802.16e)
- In compact receivers, channel impairments such as antenna mutual coupling may degrade performance
- Most studies carefully model the impact of these impairments on the signal while assuming spatially white noise
- Performance depends equally on both the signal and noise, thus noise modeling warrants further consideration

Overview

- Introduction – MIMO capacity
- Prior work – mutual coupling
- Noise correlation in compact multi-antenna receivers
- Optimal front-end design for compact MIMO receivers
- Conclusions and future work

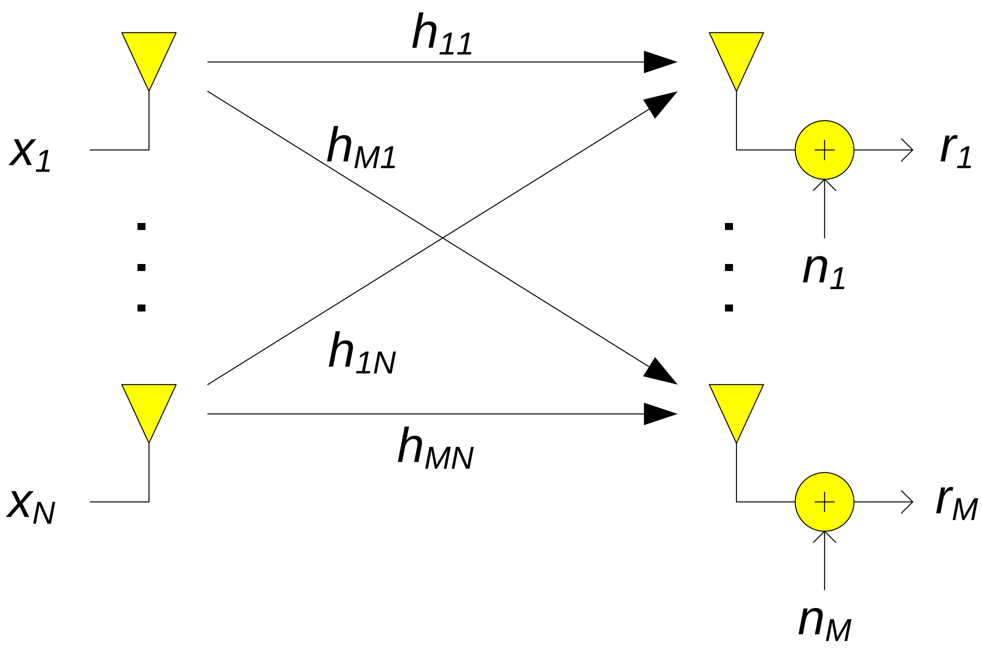
Wireless Communication and Fading



- Received signal composed of many multipath waves
- Constructive and destructive interference results in fading
  - Traditional philosophy – Multiple Rx antennas to mitigate fading
  - New philosophy (c. late '90s) – MIMO links to *exploit* fading

MIMO Channel Model

- Channel model for a frequency flat, N x M MIMO system:



$\mathbf{x} = [x_1 \cdots x_N]^T$ ~ Tx signal

$\mathbf{r} = [r_1 \cdots r_M]^T$ ~ Rx signal

$\mathbf{n} = [n_1 \cdots n_M]^T$ ~ Noise

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1N} \\ \vdots & \ddots & \vdots \\ h_{M1} & \cdots & h_{MN} \end{bmatrix}$$

h_{ij} ~ fading path gains

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

MIMO Channel Capacity

- Capacity – ultimate upper bound on spectral efficiency, introduced by Shannon ('48) for the AWGN channel:

$$r = x + n \qquad n \sim \mathcal{CN}(0, N_0 B), \quad \mathrm{E}[|x|^2] \leq P$$

$$C_{\mathrm{AWGN}} = \log_2(1+\sigma) \text{ bits/s/Hz} \qquad \sigma = \tfrac{P}{N_0 B} \sim \text{SNR}$$

- MIMO (ergodic) capacity – Telatar ('95), Foschini & Gans ('98); assumed i.i.d. Rayleigh fading and spatially white AWGN:

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \qquad \mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0 B \mathbf{I}), \quad \mathrm{tr}\{\mathrm{E}[\mathbf{x}\mathbf{x}^H]\} \leq P$$

$$h_{ij} \sim \mathcal{CN}(0,1), \quad \text{i.i.d.}$$

Rx has perfect estimate of **H**: CSIR

$$C = \mathrm{E}\left[\log_2 \det\left(\mathbf{I} + \tfrac{\sigma}{N}\mathbf{H}\mathbf{H}^H\right)\right] \xrightarrow{N \to \infty} M \cdot C_{\mathrm{AWGN}}$$

Receive Propagation Model

- Write channel matrix as $\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h}_N]$ so that

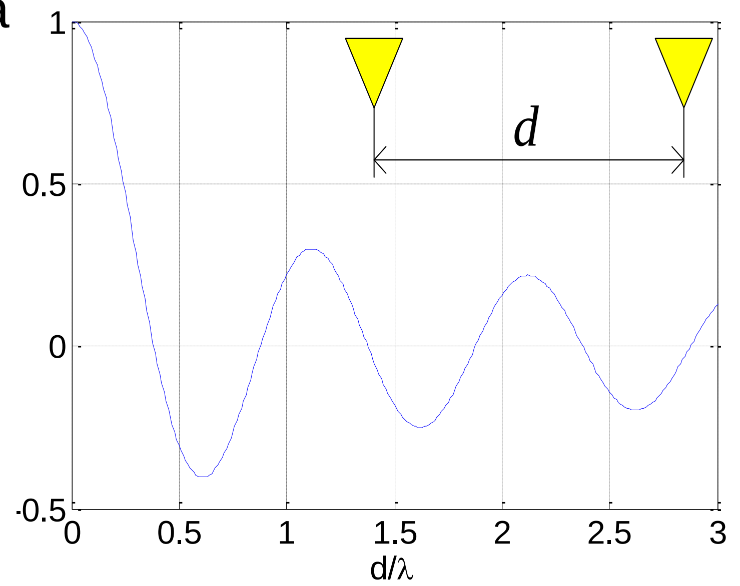
$$\mathbf{r} = \sum_{i=1}^{N} \mathbf{h}_i x_i + \mathbf{n}$$

$\mathbf{h}_i$ ~ spatial signature of i^{th} Tx antenna

- Clarke ('68): Signal from x_i received as a large number of incoherent plane waves

$$\mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma_h})$$

$$[\mathbf{\Sigma_h}]_{nm} = \mathrm{J}_0\left(2\pi \tfrac{d}{\lambda}(m-n)\right)$$

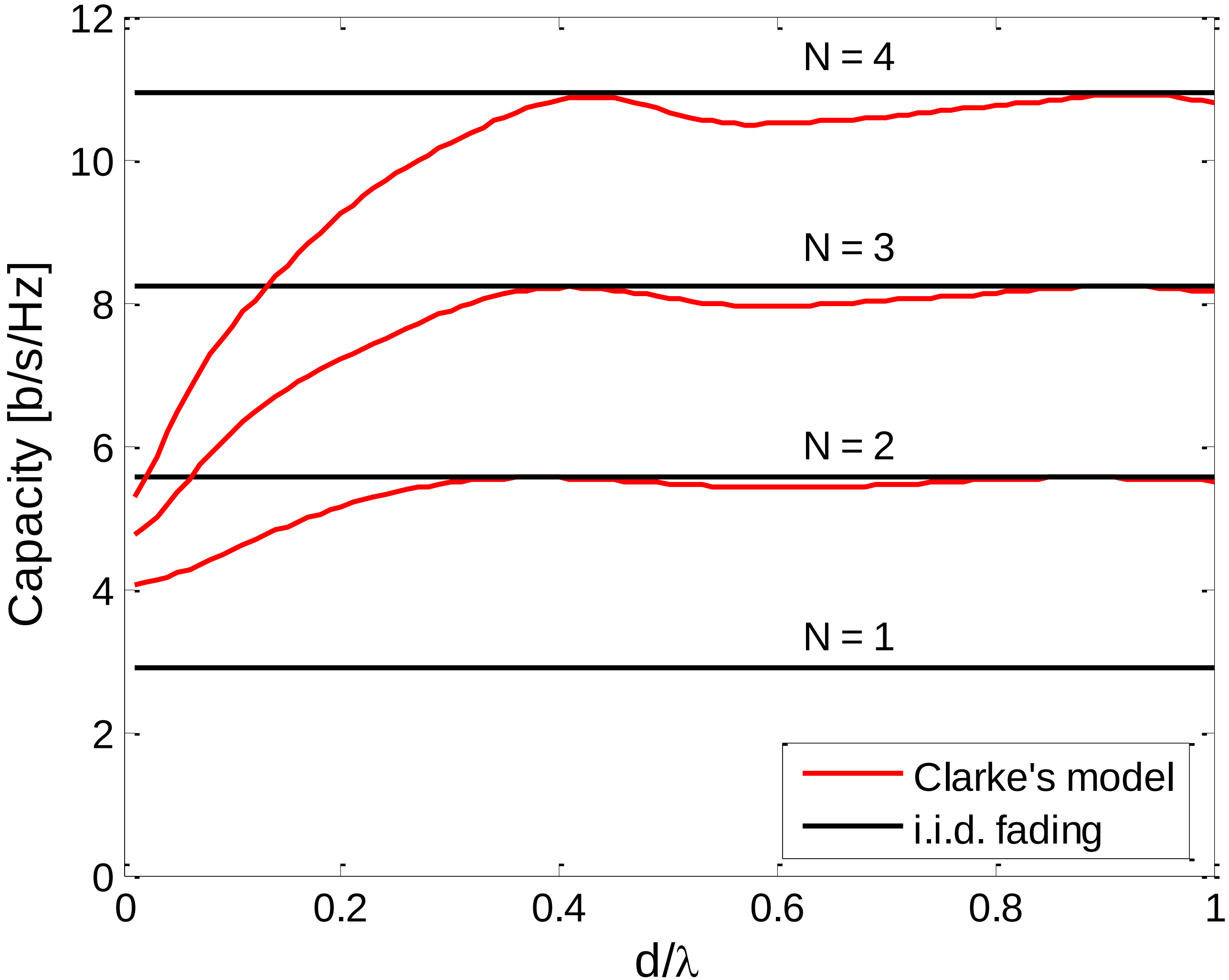


- Capacity for i.i.d. spatial signatures, cf. Shiu *et al.* ('00), Chiani *et al.* ('03):

$$C = \mathrm{E}\left[\log_2 \det\left(\mathbf{I} + \tfrac{\sigma}{N}\mathbf{\Sigma_h}\mathbf{H_w}\mathbf{H_w}^H\right)\right]$$

$\mathbf{H_w}$ ~ matrix of i.i.d. $\mathcal{CN}(0,1)$ entries

N x *N* MIMO Capacity, SNR = 10 dB



Capacity still increases with *N* at < 0.2λ!

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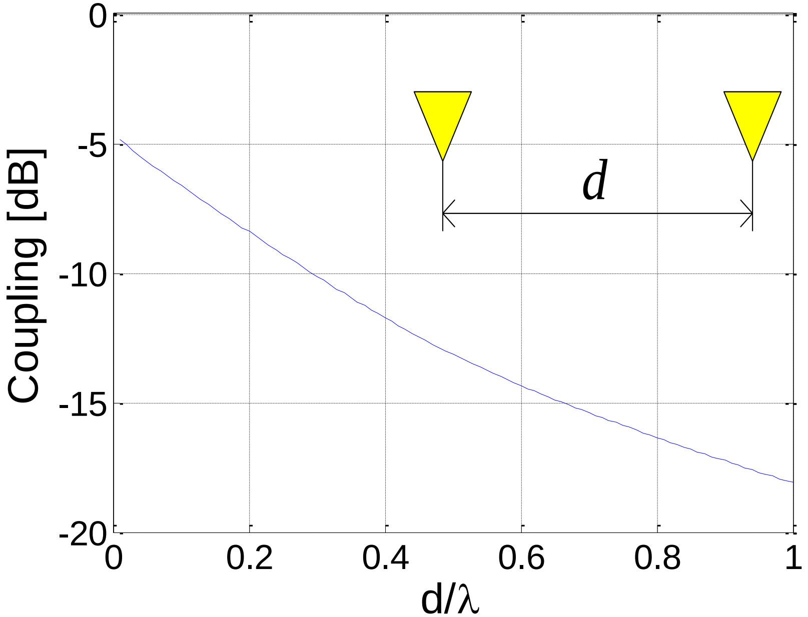
Mutual Coupling

- Previous model assumes Rx signal proportional to incident field
- At close antenna separations (< 0.5λ) interactions between array elements become non-negligible:
  - Receive pattern of each array element may be distorted
  - Mutual coupling may further correlate signals and promote power loss due to impedance mismatch



- Capacity with MC studied by Svantesson ('01), Janaswamy ('02), and others; matching for max power by Wallace & Jensen ('04)

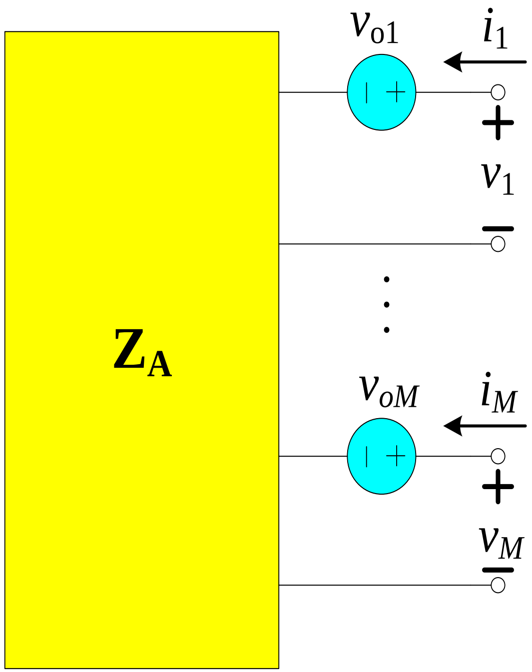
Antenna Array Circuit Model

- Model with a Thevenin equivalent network:

$$\boxed{\mathbf{v} = \mathbf{Z_A}\mathbf{i} + \mathbf{v_o}}$$

$\mathbf{Z_A}$ ~ antenna impedance matrix

$\mathbf{v_o}$ ~ open-circuit (induced) voltage

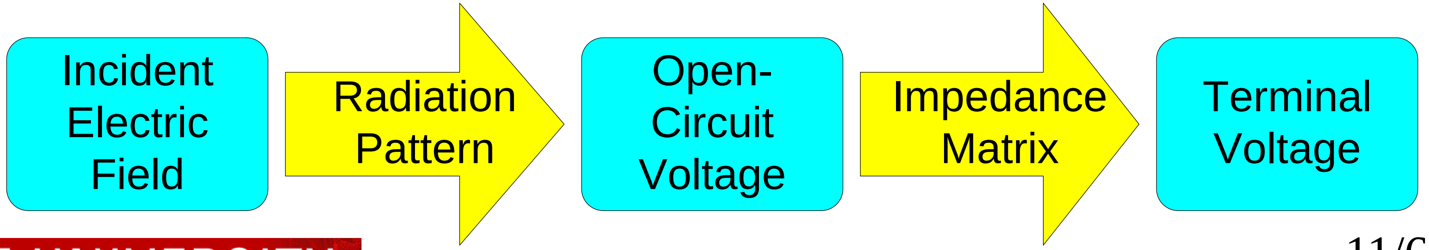


- Off-diagonal elements of $\mathbf{Z_A}$ represent mutual coupling between antennas

$$\mathbf{v_o} = \sum_{\mathbf{i}} \mathbf{h}_i x_i, \quad \mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma_h})$$

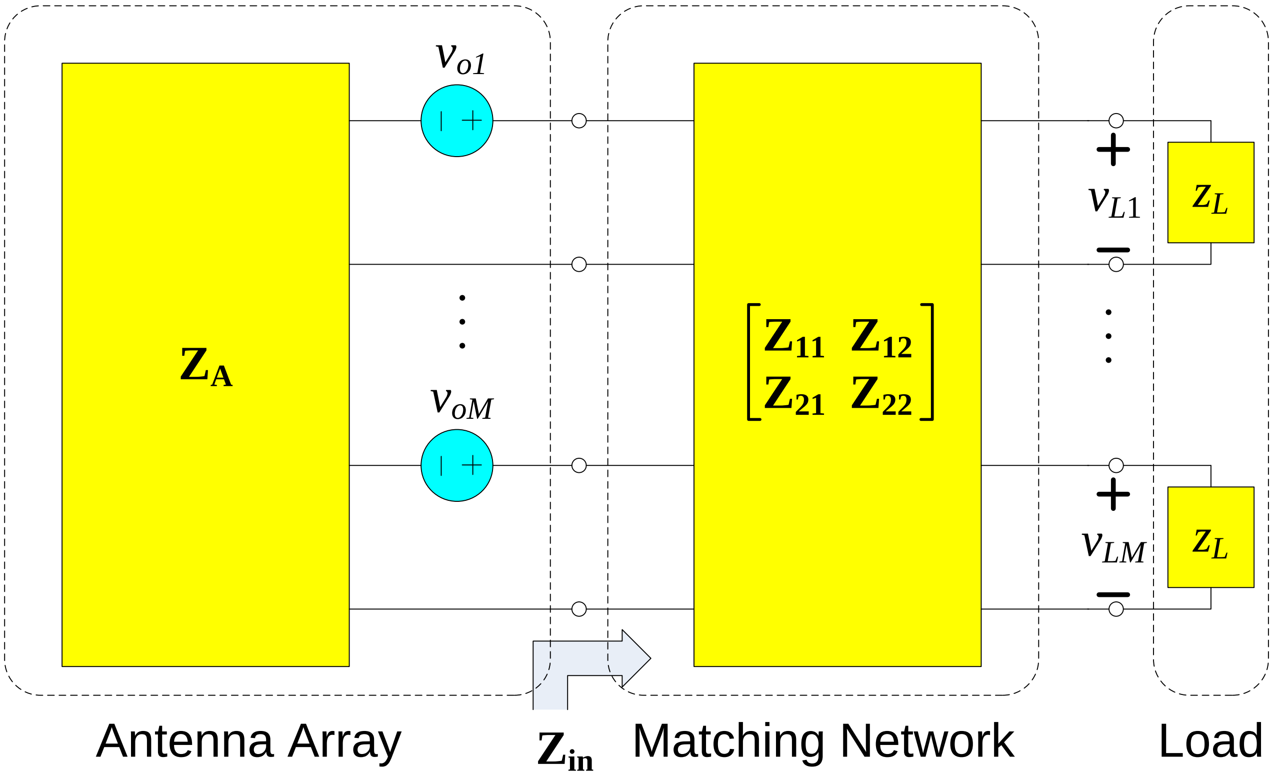
$$[\mathbf{\Sigma_h}]_{nm} = \frac{1}{2\pi}\int_0^{2\pi} g_n(\phi) g_m^*(\phi) e^{j2\pi\frac{d}{\lambda}(m-n)\cos\phi} d\phi$$

g_n ~ pattern of n^{th} element



Matching for Maximum Power Transfer

- Matching networks interface the array with rest of the receiver:

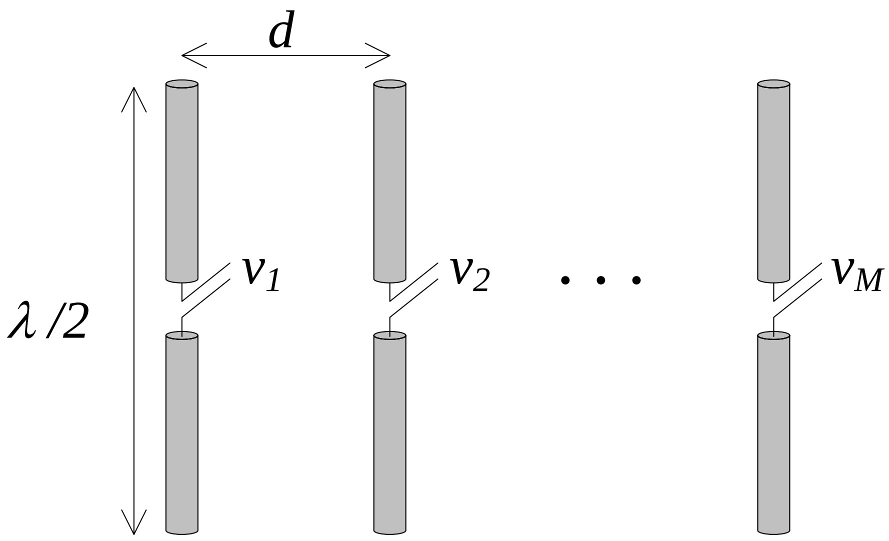


Matching network is lossless

- Maximum power delivered to load iff $\mathbf{Z_{in}} = \mathbf{Z_A}^H$ (Hermitian match)
- Practical, suboptimal solution: $\mathbf{Z_{in}} = [\mathbf{Z_A}]_{nn}^* \mathbf{I}$ (self match)

Numerical Example

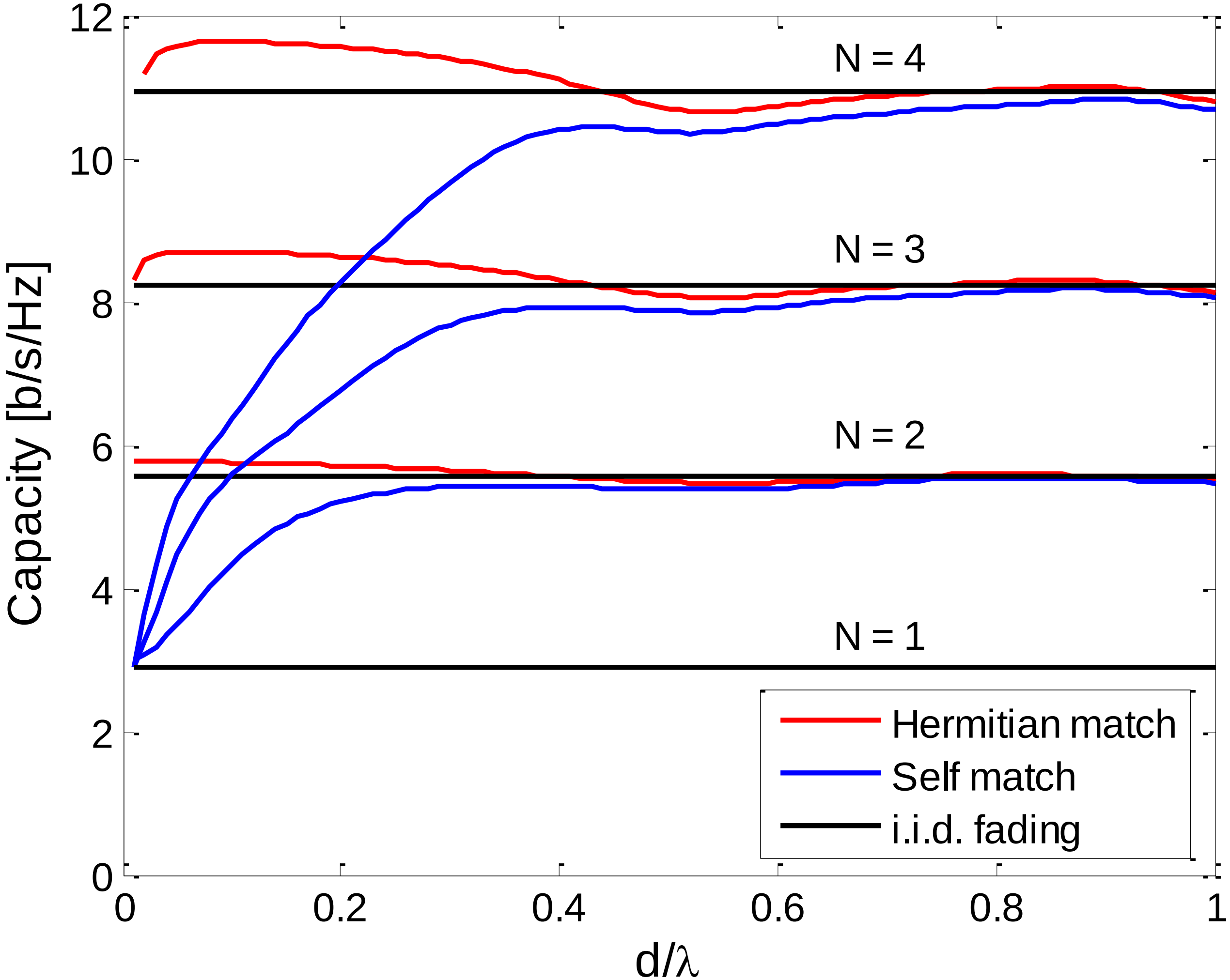
- Incident electric field – Clarke's model, 10 dB SNR
- Antenna array – ULA of half-wavelength dipoles with radius of $10^{-3}\lambda$; array pattern and impedance matrix computed with NEC



- Find load voltage for Hermitian and self match; add i.i.d. noise:

$$\mathbf{r} = \mathbf{v}_{\mathbf{L}} + \mathbf{n}, \quad \mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0 B\mathbf{I})$$

Matching Network Performance



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Noise Correlation

- Previous studies:
  - Concerned with fading correlation and how it relates to mutual coupling
  - Assumed i.i.d. AWGN by convention
  - No mention of physical noise sources
- Under certain conditions, noise could be spatially correlated:
  - External noise may correlate in the same manner as the fading signal
  - Internal noise may correlate through mutual coupling
- Performance metrics depend on both the signal and noise, so noise should warrant further consideration
- Goal: Extend the previous model to include correlated noise.

Recent Work

- Morris & Jensen ('05): Realistic model for front-end amplifiers, compared matching networks optimized for power and noise
- Gans ('06): Antenna and (spatially white) amplifier noise limited scenarios; showed matching irrelevant for the former

- Main contributions of this research:
  - Realistic noise model for a multi-antenna receiver, characterize various noise sources – noise analysis (Ch. 3)
  - Extend well-known concepts from two-port noise theory to multiport networks, develop MIMO low-noise design principles (Ch. 4)

Receive Diversity with Correlated Noise

- Consider a 1 x *M* (SIMO) receive diversity system in which both the fading and noise are spatially correlated:

$$\boxed{\mathbf{r} = \mathbf{h}x + \mathbf{n}} \qquad \mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{h}}), \quad \mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{n}})$$

Diversity combiner output: $y = \mathbf{w}^H \mathbf{r}$

Output SNR: $\gamma = P \dfrac{\mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w}}{\mathbf{w}^H \boldsymbol{\Sigma}_{\mathbf{n}} \mathbf{w}} \le P \cdot \mathbf{h}^H \boldsymbol{\Sigma}_{\mathbf{n}}^{-1} \mathbf{h} = \gamma^o, \quad \text{w. eq. iff} \;\; \mathbf{w} \propto \boldsymbol{\Sigma}_{\mathbf{n}}^{-1} \mathbf{h}$

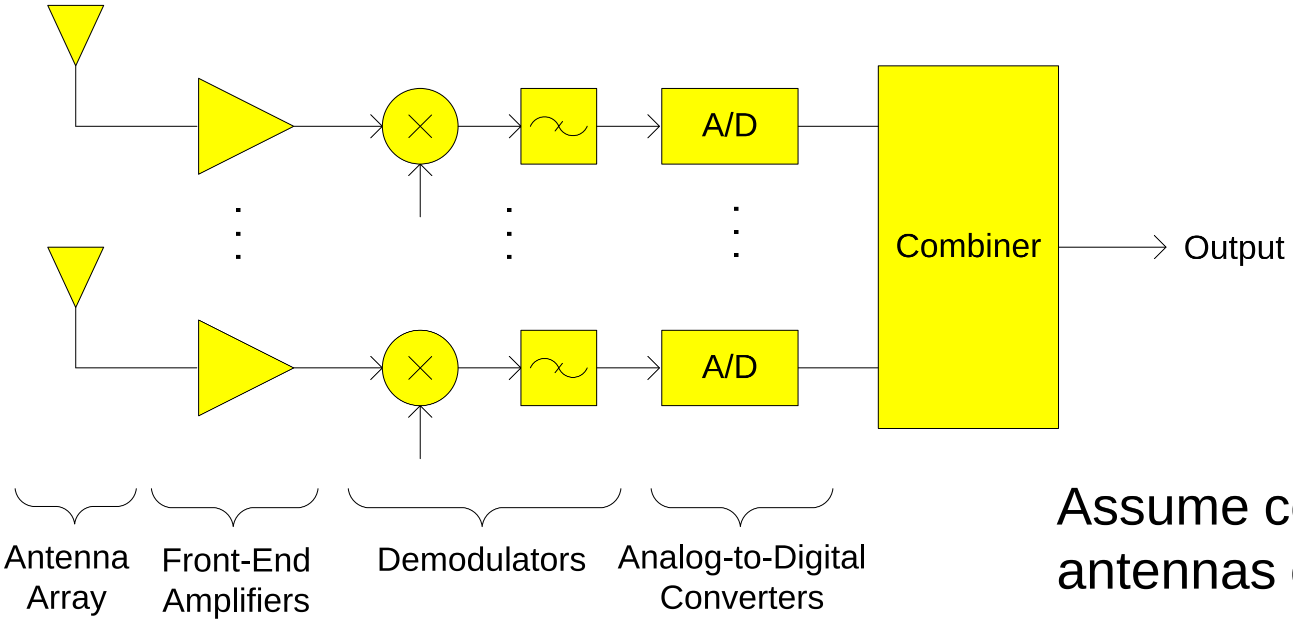
Maximum ratio combining (MRC)

Outage probability: $P_{\text{out}}(\tau) = \Pr\{\gamma^o \le \tau\}$

- P_{out} depends on eigenvalues of the SNR matrix $\boldsymbol{\Sigma} = P \cdot \boldsymbol{\Sigma}_{\mathbf{h}}^{1/2} \boldsymbol{\Sigma}_{\mathbf{n}}^{-1} \boldsymbol{\Sigma}_{\mathbf{h}}^{1/2}$
- Need a receiver noise model to determine specific form of $\boldsymbol{\Sigma}_{\mathbf{n}}$

Receiver Noise Model

- Consider a post-detection diversity receiver:

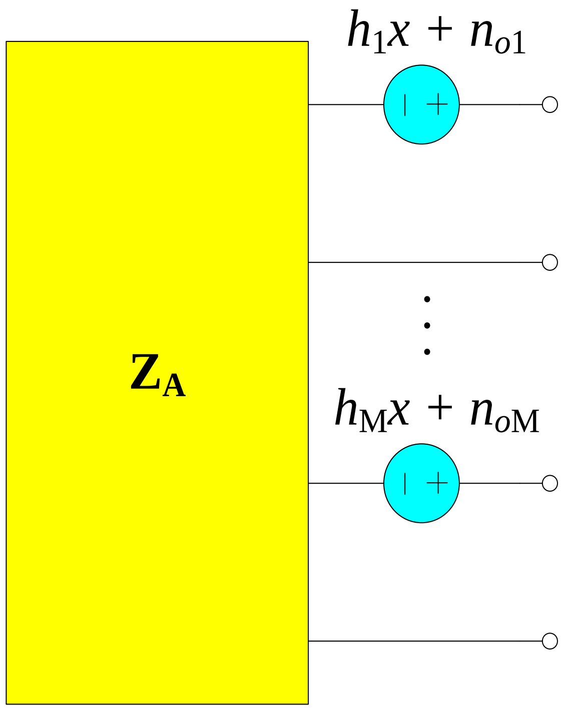


Assume coupling in antennas only

- Each stage contributes noise to the total output noise **n**
- Use noise theory to establish a noise model for each component, then calculate output noise correlation $\boldsymbol{\Sigma}_{\mathbf{n}}$

Antenna Noise

- Open-circuit voltage now contains noise, $\mathbf{v_o} = \mathbf{h}x + \mathbf{n_o}$
- Noise sources include thermal radiation, cosmic background and interference from other electronic devices



- Thermal noise from a spherically isotropic distribution of black-body radiators at temperature $T_0 = 290$ K (Twiss '55):

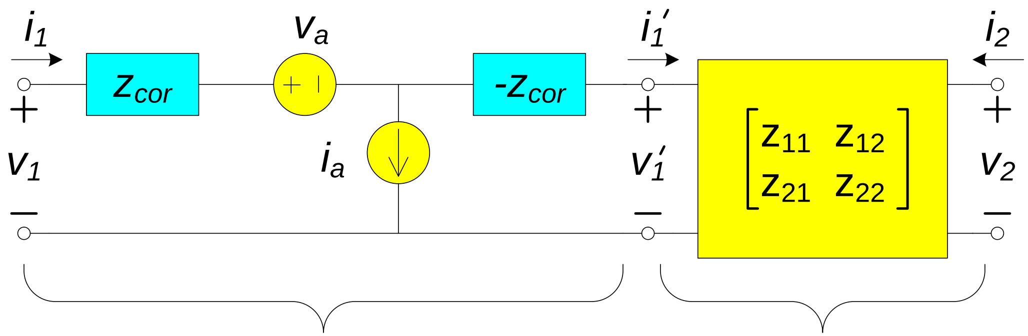
$$\mathbf{n_o} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma_{n_o}}), \quad \mathbf{\Sigma_{n_o}} = 4kT_0 B\mathbf{R_A}$$

$$\mathbf{R_A} = \tfrac{1}{2}(\mathbf{Z_A} + \mathbf{Z_A^H}) \qquad kT_0 \approx 4 \times 10^{-21} \text{ W/Hz}$$

- For antenna separations less than a few wavelengths $\mathbf{Z_A}$ is non-diagonal – noise is correlated!

Amplifier Noise

- Amplifiers typically represented by the Rothe-Dahlke ('56) model:



$$v_a \sim \mathcal{CN}(0, 4kT_0Br_a) \qquad r_a \sim \text{equivalent noise resistance}$$

$$i_a \sim \mathcal{CN}(0, 4kT_0Bg_a) \qquad g_a \sim \text{equivalent noise conductance}$$

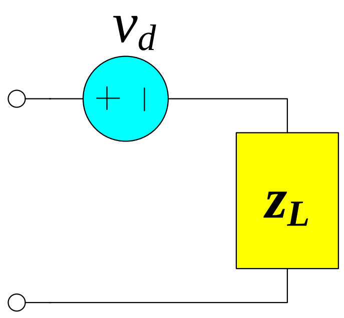
- Noise sources model thermal and shot noise
- Important amplifier metric is the noise figure NF:

$$\text{SNR}_{\text{out}} = \text{SNR}_{\text{in}} - \text{NF} \quad (\text{in dB})$$

- NF function of noise parameters $\{r_a, g_a, z_{cor}\}$ and source impedance

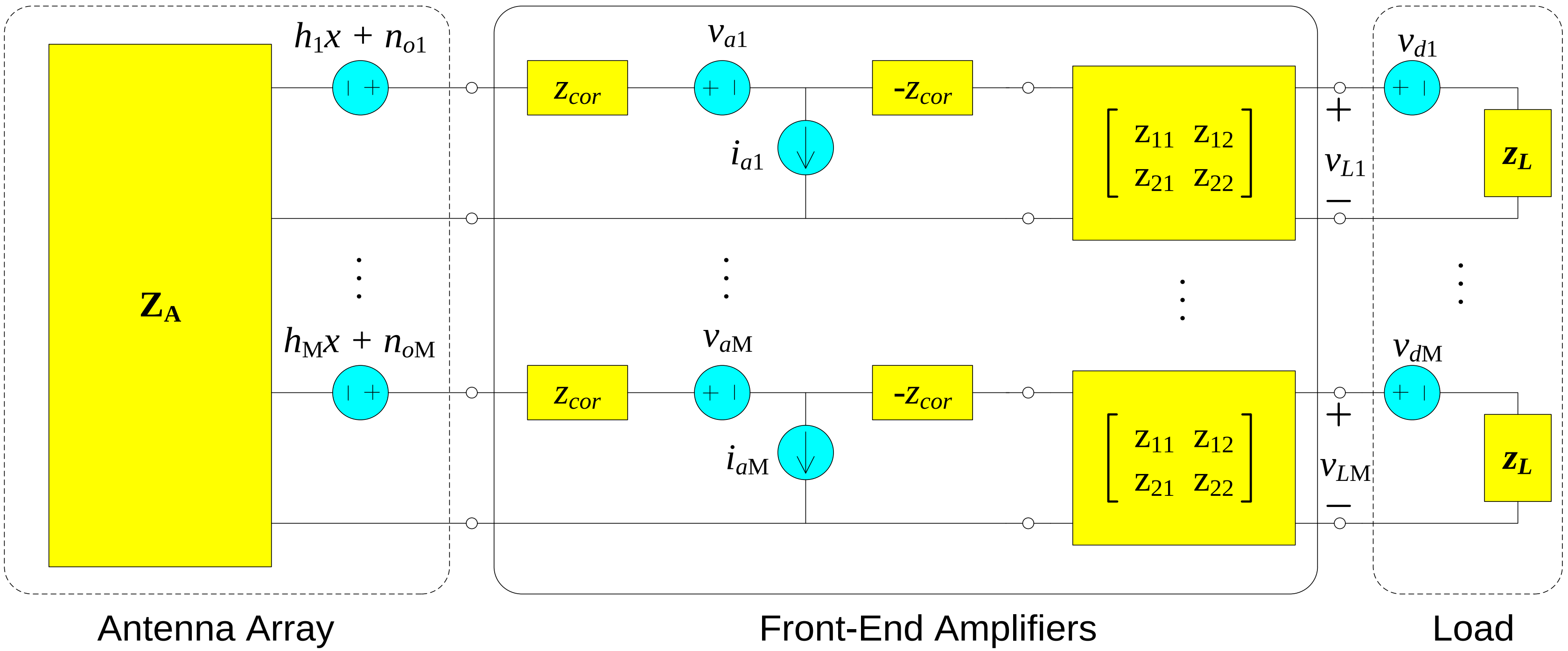
Downstream Noise

- Downstream components consist of filters, mixers, amplifiers and other noisy circuits – a detailed model would be complicated
- Alternative – assume each component performs a linear operation on the complex baseband signals and generates AWGN
- Can reference total downstream noise to the amplifier output, model with a Thevenin equivalent load:



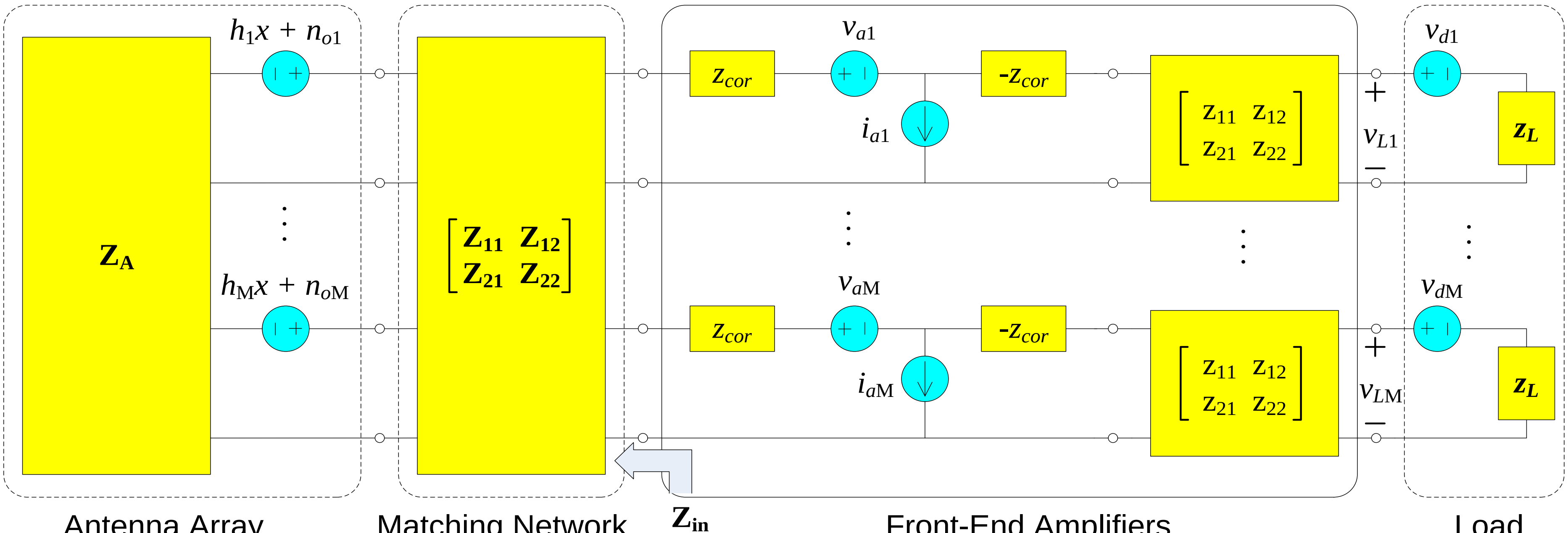
$$v_d \sim \mathcal{CN}(0, 4kT_0 B r_d)$$

Receiver Noise Model



$$\mathbf{r} = \mathbf{v}_L = \mathbf{h}x + \mathbf{n}$$

Matching for Minimum Noise Figure



- Noise figure of each amplifier minimized: $\mathbf{Z}_{\mathbf{in}} = z_{\text{opt}}\mathbf{I}$ (multiport match)
- Practical, suboptimal solution: Match for isolated dipoles (self-match)

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Numerical Results

- Front-end amplifier: Maxim 2642 LNA,

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 35.7\angle -82.0^\circ & 2.74\angle 91.8^\circ \\ 325\angle 119^\circ & 46.1\angle -23.3^\circ \end{bmatrix}$$

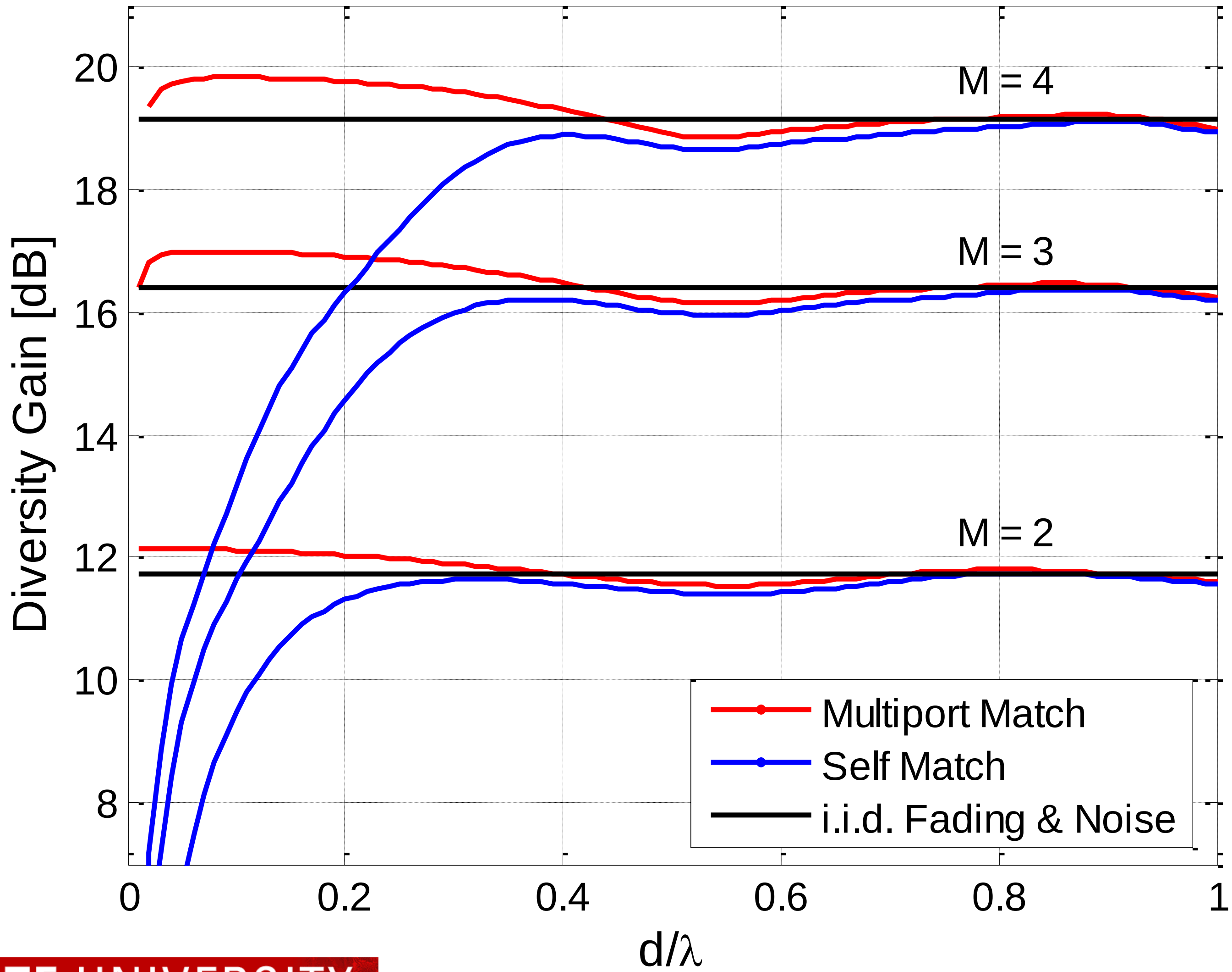
$$r_a = 9.45\,\Omega, \quad g_a = 3.24\,\text{mS}, \quad z_{\text{cor}} = 35.3\angle -114^\circ$$

- Downstream components: Mixer and IF amplifier with composite noise figure of 7.6 dB, at input impedance of 50 Ω (Pozar '05):

$$r_d = 50(F_{\text{dow}} - 1) = 240\,\Omega$$

- Calculate diversity gain at 1% outage ($P_{\text{out}} = 0.01$) for multiport matching and self matching

Matching Network Performance



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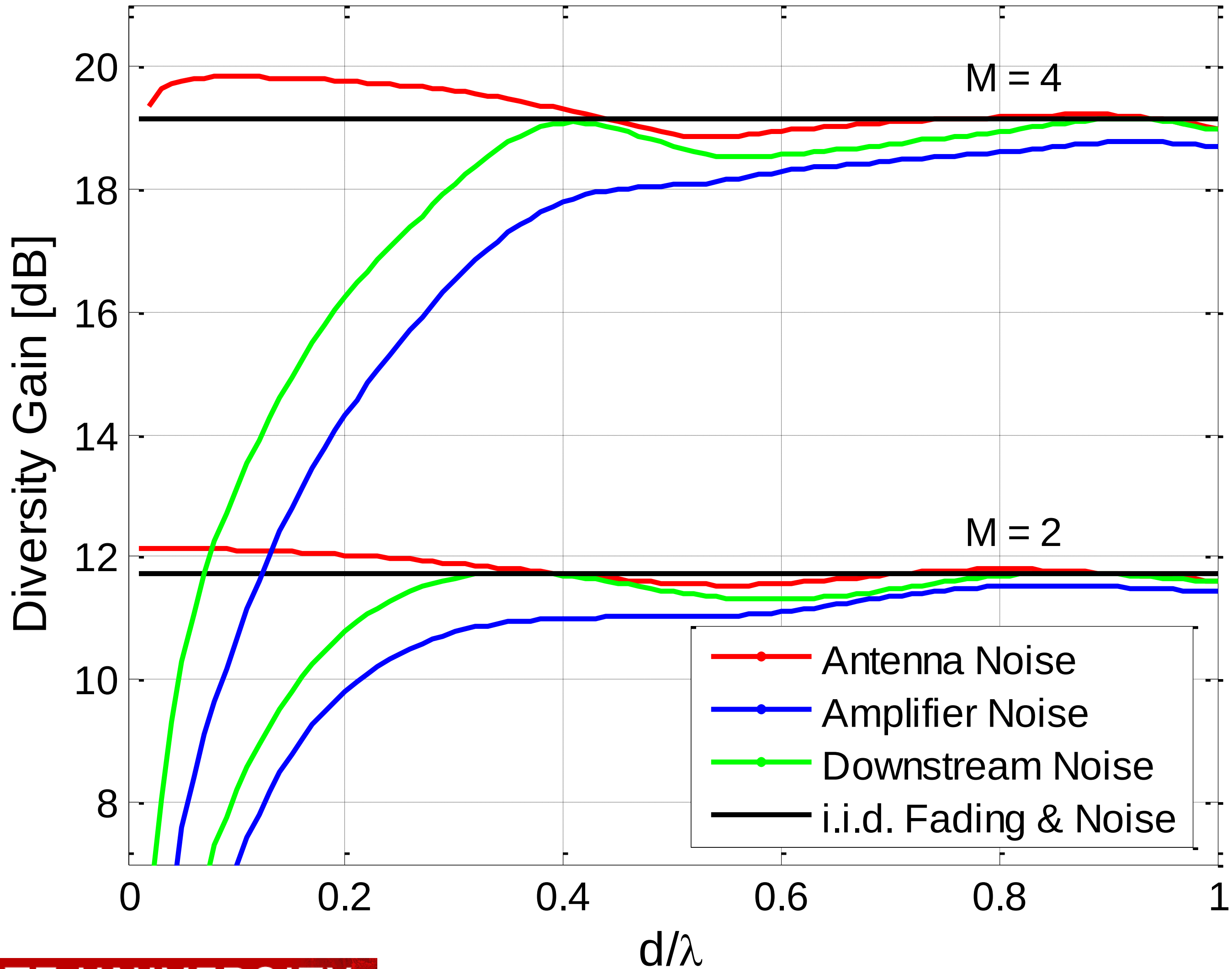
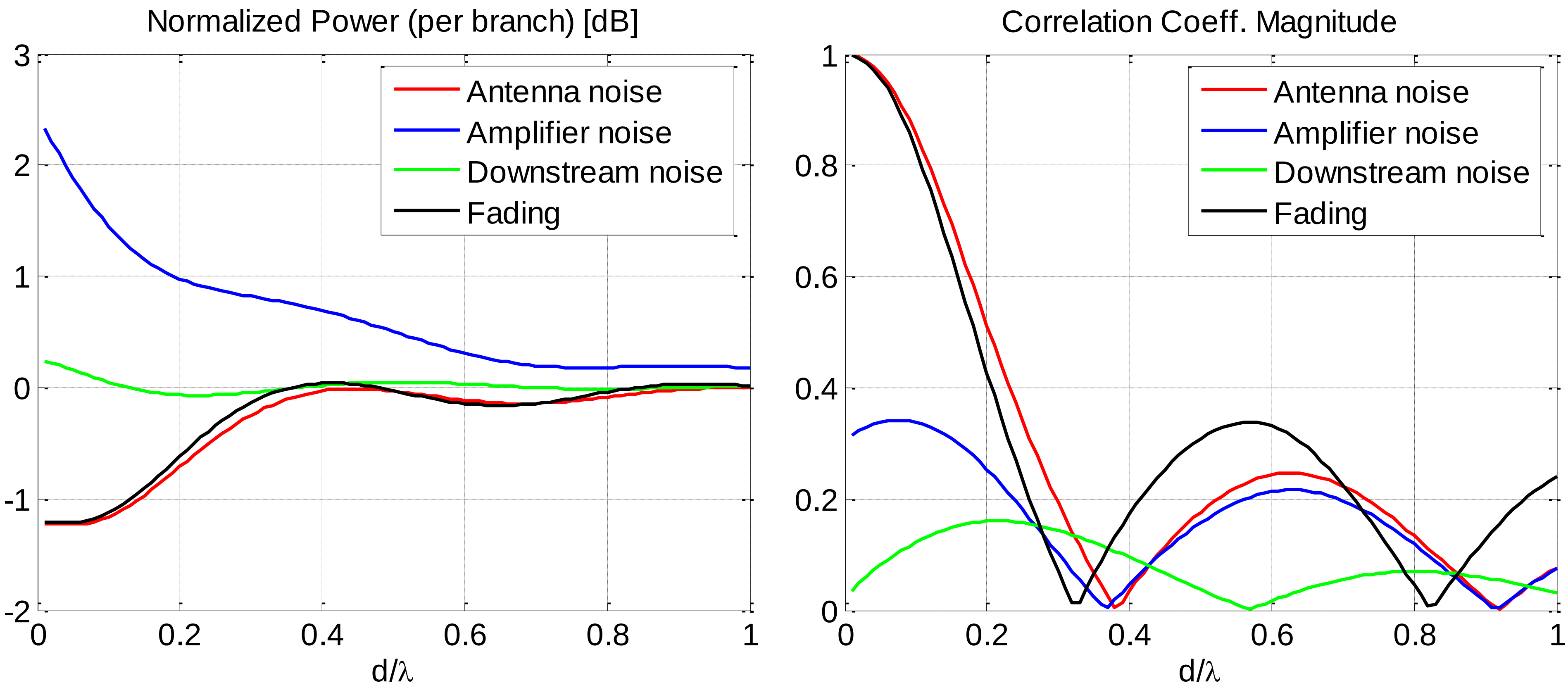
Individual Noise Sources (Self-Match)



Fading and Noise Power and Correlation



$$\boldsymbol{\Sigma}_i = P_i \begin{bmatrix} 1 & \rho_i \\ \rho_i & 1 \end{bmatrix}$$

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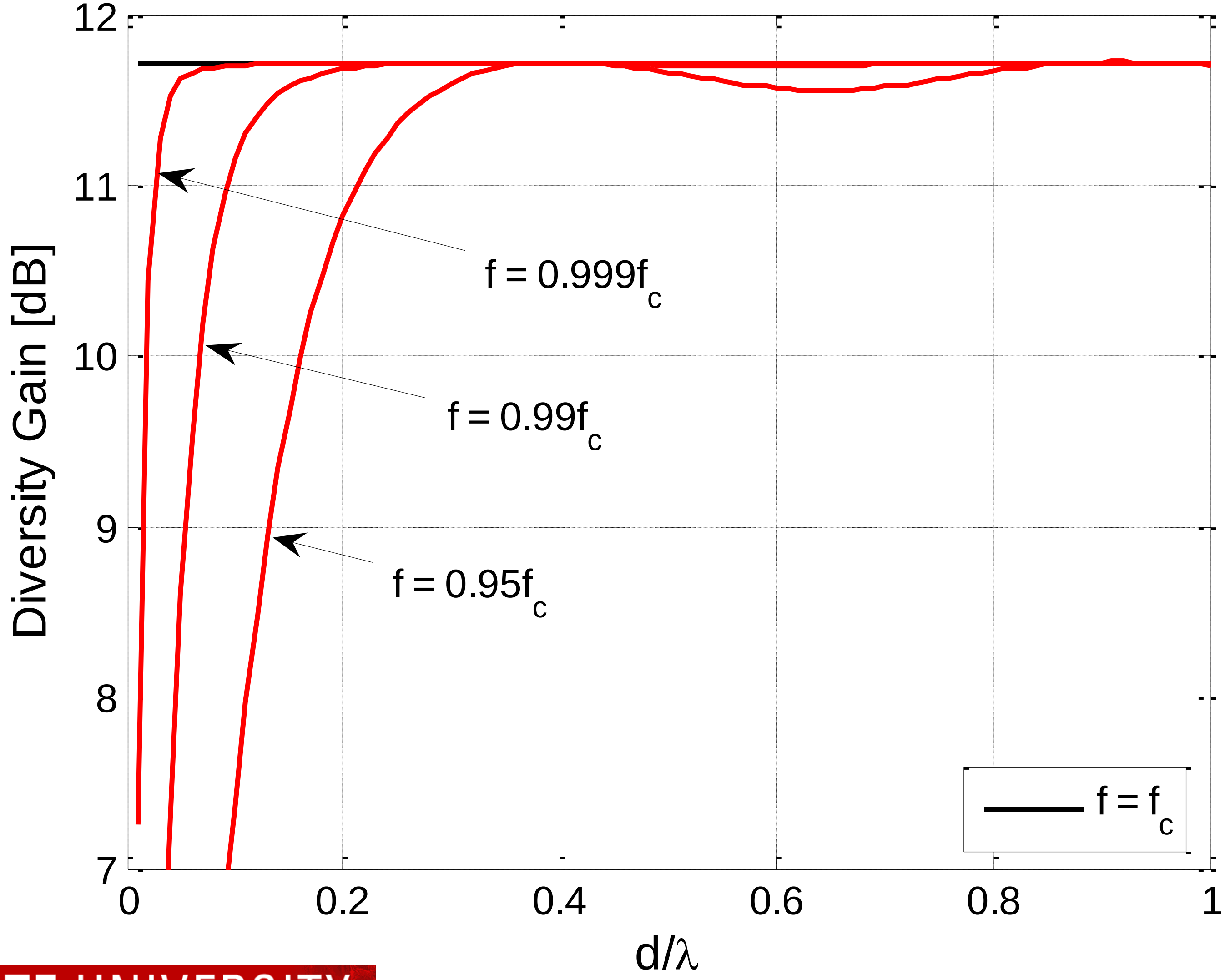
3D Scattering: Multiport Match B/W



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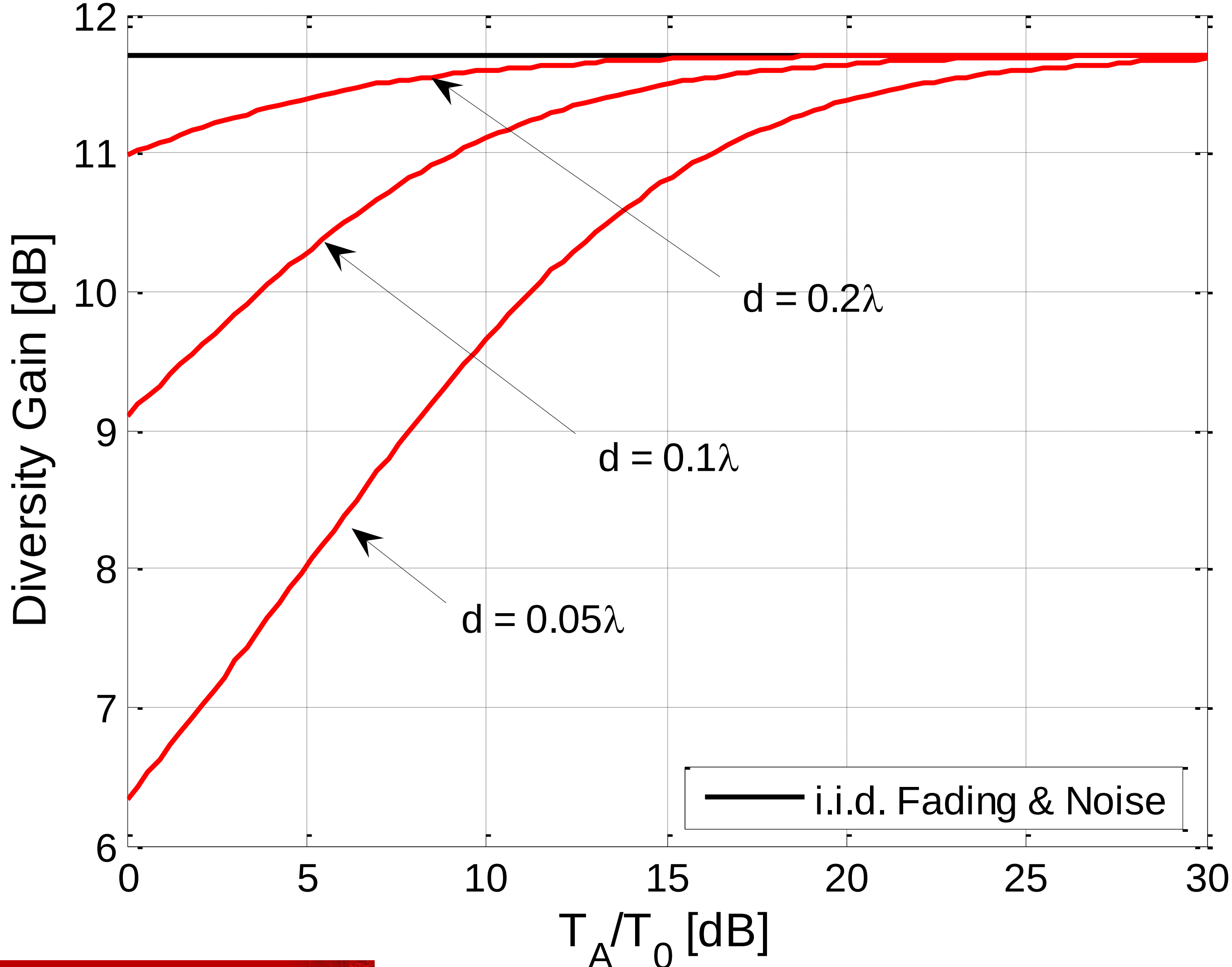
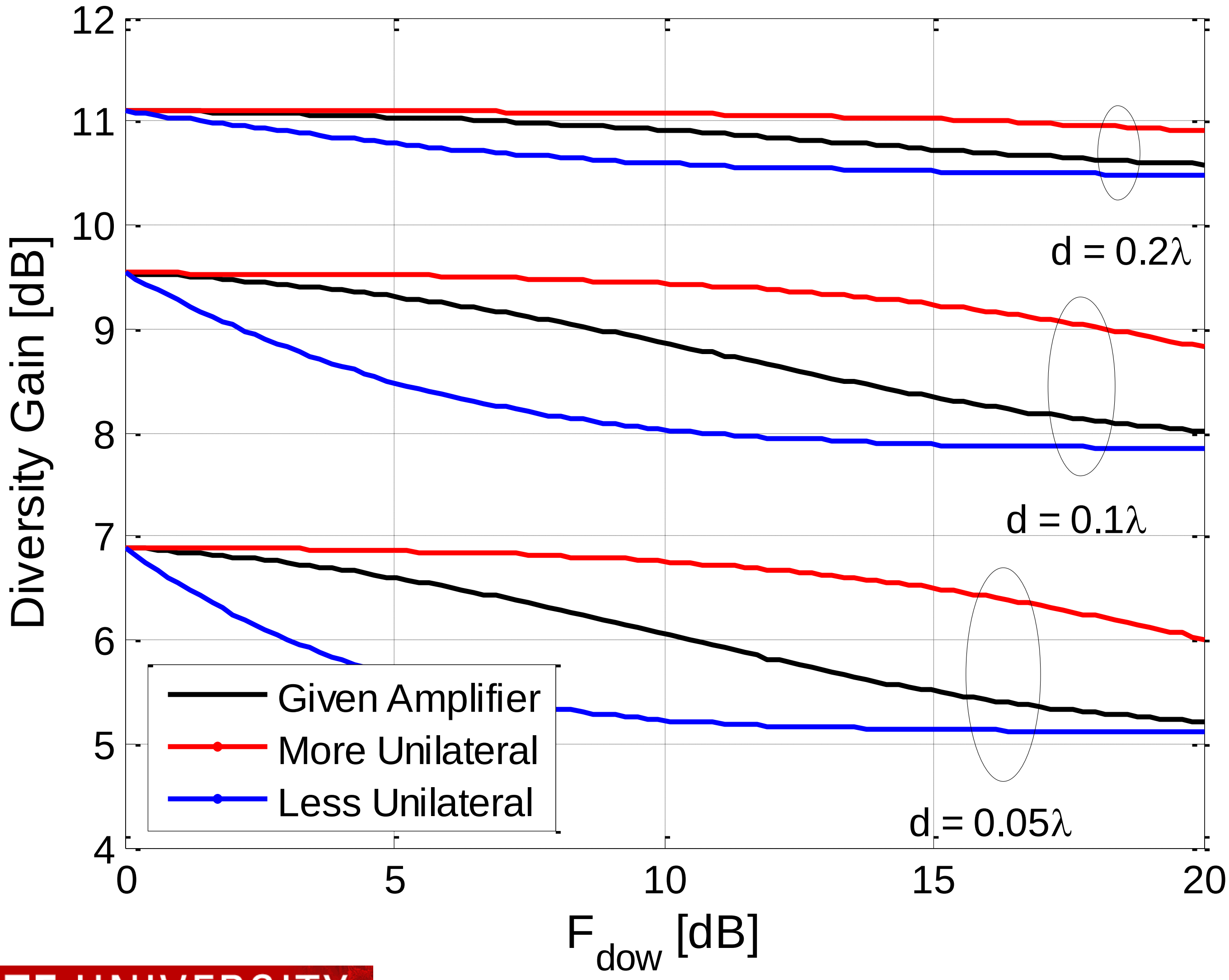
3D Scattering: Antenna Noise Strength



Downstream Noise and Amp. Unilaterality



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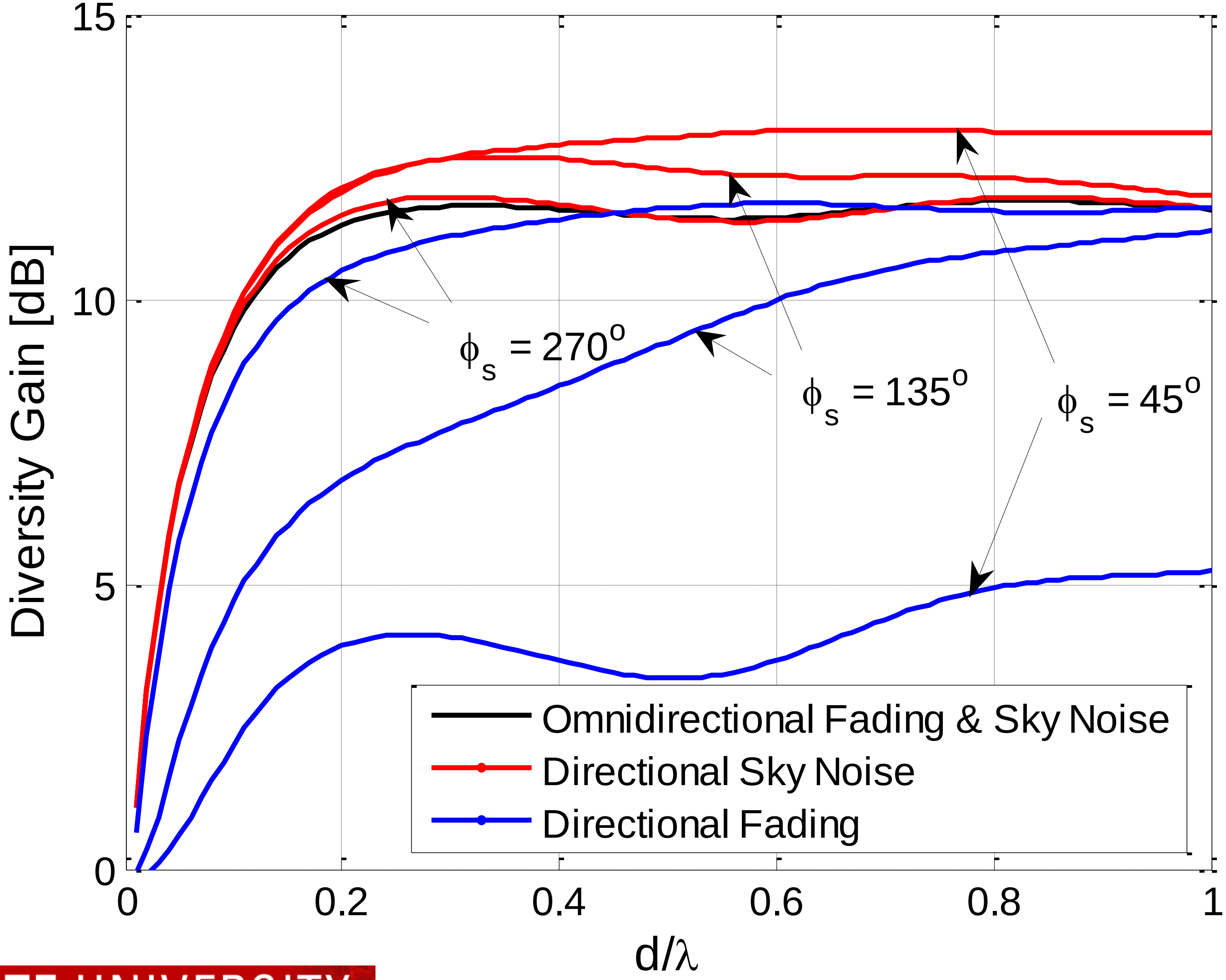
Directional Fading and Sky Noise



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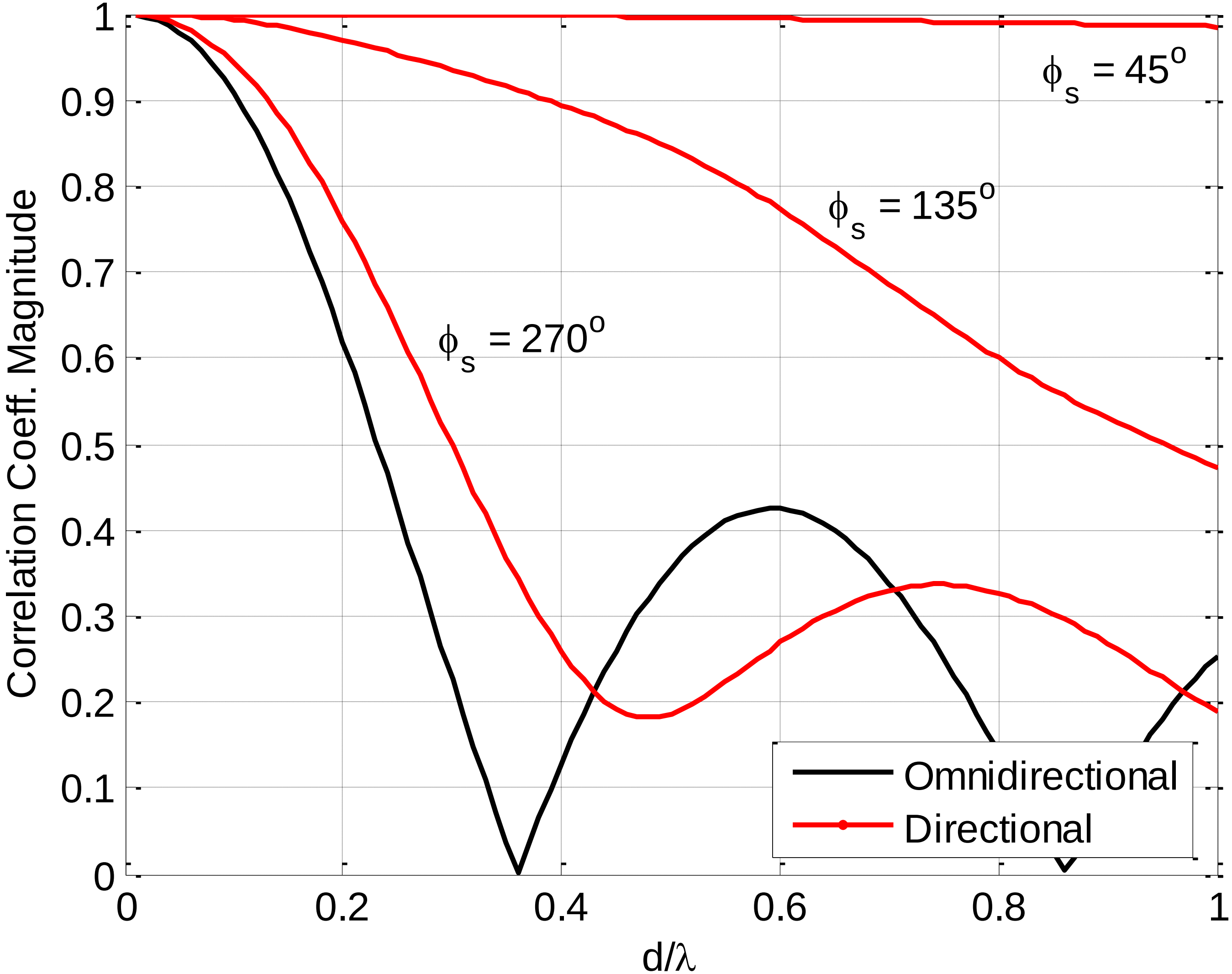
Directional Fading and Sky Noise (cont.)



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Conclusion

- In a compact receive diversity array, both the signal and noise components of the diversity branches may be correlated
- Traditional MRC is suboptimal for correlated noise
- Different noise sources can impact performance in profoundly different ways:
  - Antenna thermal noise becomes correlated as the antennas are brought closer together – the least detrimental noise
  - Amplifier noise power increases as the antennas are brought closer together – the most detrimental noise
  - Downstream noise behaves similar to i.i.d. AWGN – impact is between that of antenna and amplifier noise
- Accurate modeling of the dominant noise sources is critical to predicting performance in any multiple-antenna receiver

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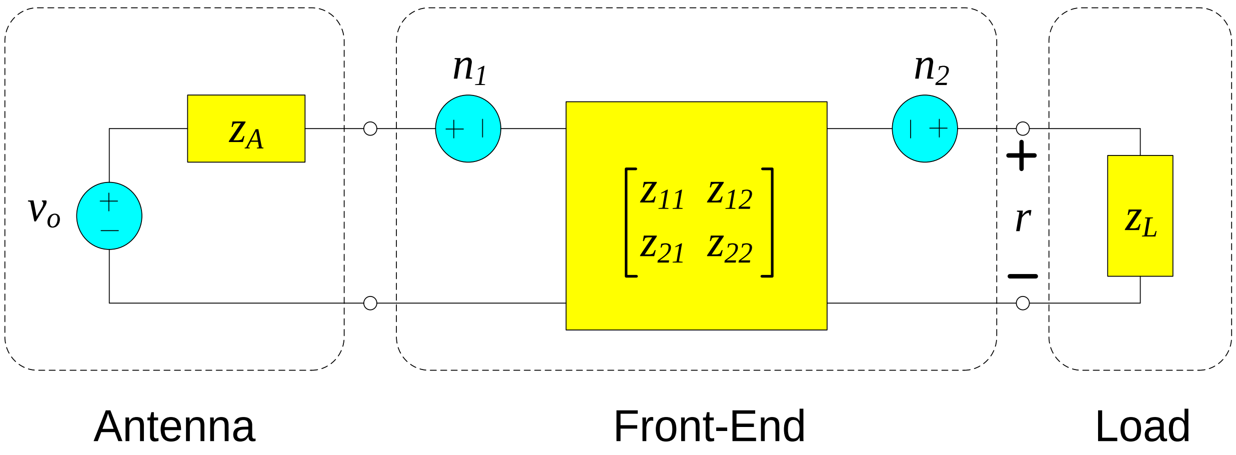
SISO Low-Noise Design



Front-end consists of amps, mixers, filters, etc.

- Antenna o/c voltage contains a signal and noise component:

$$v_o = hx + n_o$$

- At a minimum, thermal noise is present: $n_o \sim \mathcal{CN}(0, 4kT_0 B r_A)$

$kT_0 \approx 4 \times 10^{-21}$ W/Hz (at std. temp.); B ~ bandwidth; $r_A = \mathrm{Re}[z_A]$

Output signal: $r \propto hx + \underbrace{n_o + z}_{n}$ z ~ noise from front-end

SISO Low-Noise Design (cont.)

- Noise factor – measure of noise added by front-end (Friis ’44):

$$F = \frac{\mathrm{E}[|n|^2]}{\mathrm{E}[|n|^2]|_{z=0}} \qquad \left(= \frac{\text{Total noise}}{\text{Noise from antenna}} \geq 1 \right)$$

- Expression for SNR takes a convenient form:

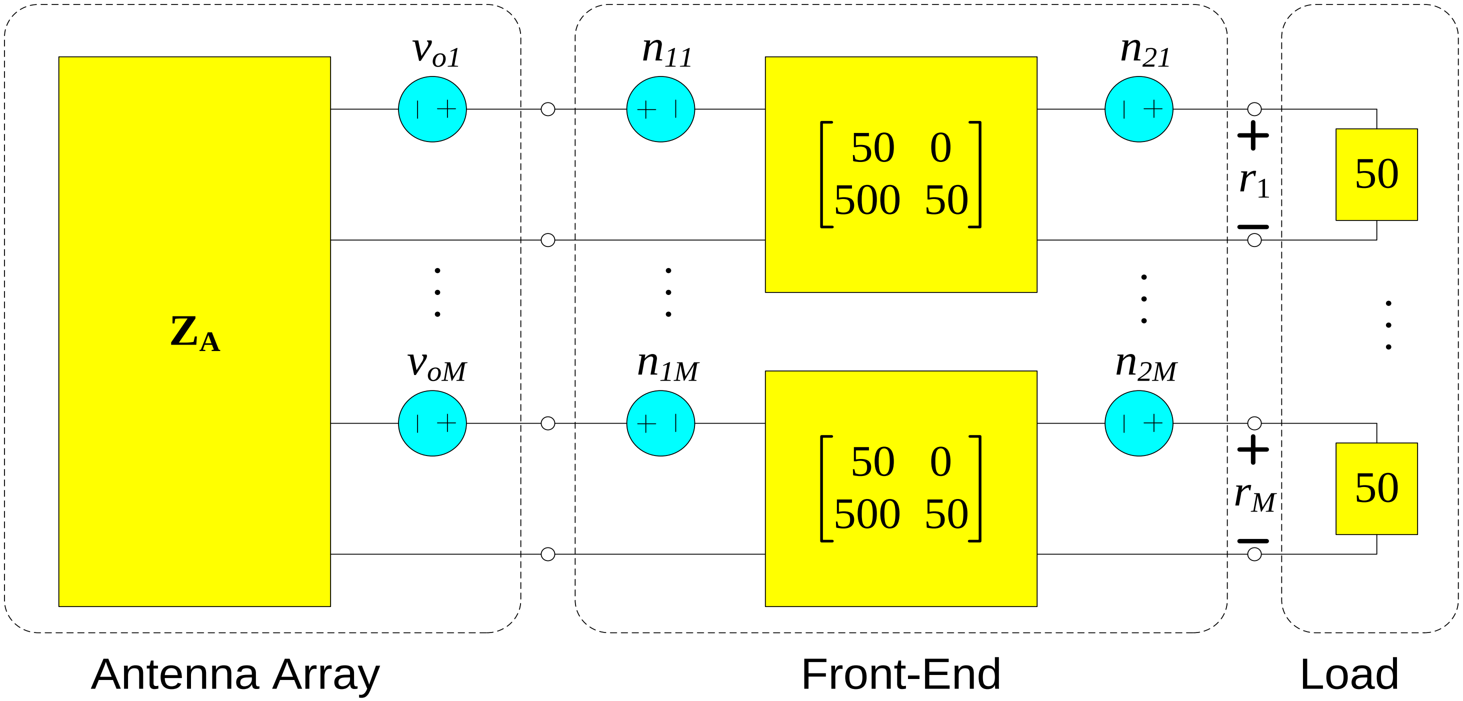
$$\sigma = \frac{P}{4kT_0 B r_A F}$$

In dB, $\mathrm{SNR}_{\mathrm{out}} = \mathrm{SNR}_{\mathrm{in}} - \mathrm{NF}$, $\quad \mathrm{NF} = 10\log_{10} F$ ~ noise figure

- Most SISO performance metrics are monotonic in the SNR, so designing the front-end for minimum noise figure is optimal
- Question: What is optimal for MIMO receivers?

Example: Uncoupled Front Ends

- Consider *M* uncoupled front-ends:



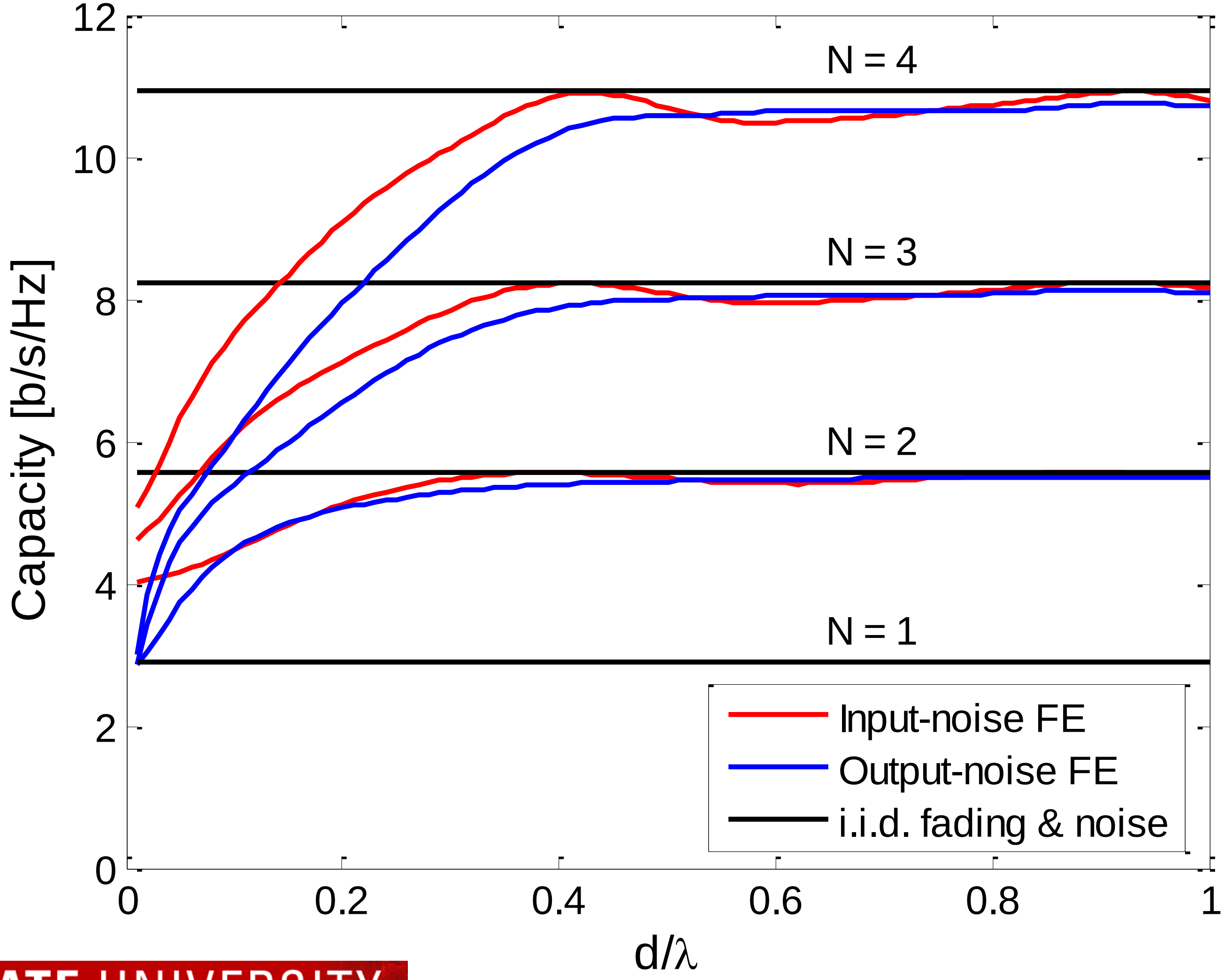
"Input-noise" FE: $\mathbf{n}_1 \sim \mathcal{CN}(\mathbf{0}, N_0 B\mathbf{I}), \quad \mathbf{n}_2 = \mathbf{0}$

"Output-noise" FE: $\mathbf{n}_1 = \mathbf{0}, \quad \mathbf{n}_2 \sim \mathcal{CN}(\mathbf{0}, GN_0 B\mathbf{I})$

} Both have 10 dB NF

$$N_0 = 36kT_0 r_A, \quad G \approx 11\text{dB}$$

Example: Uncoupled Front Ends



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Low-Noise Design Philosophy

- Goal: Develop low-noise design principles for MIMO receivers
- SISO low-noise design follows by observing that:
  1. Most SISO performance metrics are monotonic in the SNR
  2. Minimizing the front-end noise factor maximizes the SNR

  Therefore, designing for minimum noise factor is optimal
- We will develop MIMO low-noise design principles by:
  1. Demonstrating that several MIMO performance metrics are "monotonic" in the SNR matrix
  2. Show that "minimizing" a quantity referred to as the noise factor matrix "maximizes" the SNR matrix

Capacity (CSIR & Full CSI)

- CSIR capacity formula may be extended to correlated noise:

$$C_R = \mathrm{E}\left[\log_2 \det\left(\mathbf{I} + \tfrac{1}{N}\mathbf{H}_{\mathbf{w}}^H \mathbf{\Sigma} \mathbf{H}_{\mathbf{w}}\right)\right], \qquad \mathbf{\Sigma} = P \cdot \mathbf{\Sigma}_{\mathbf{h}}^{1/2} \mathbf{\Sigma}_{\mathbf{n}}^{-1} \mathbf{\Sigma}_{\mathbf{h}}^{1/2}$$

SNR matrix

- In some instances we may obtain full CSI, and the capacity is

$$C_F = \sum_{i=1}^{\min(N,M)} \mathrm{E}\left[\log\left(1 + P_i^* \lambda_i (\mathbf{H}_{\mathbf{w}}^H \mathbf{\Sigma} \mathbf{H}_{\mathbf{w}})\right)\right]$$

$\lambda_i(\mathbf{H}_{\mathbf{w}}^H \mathbf{\Sigma} \mathbf{H}_{\mathbf{w}})$ ~ i^{th} largest eigenvalue of $\mathbf{H}_{\mathbf{w}}^H \mathbf{\Sigma} \mathbf{H}_{\mathbf{w}}$

P_i^* ~ Space-time waterfilling power allocation

- Of note is <u>not</u> the *details* of these and the following metrics, but the observation that they are functions of the SNR matrix

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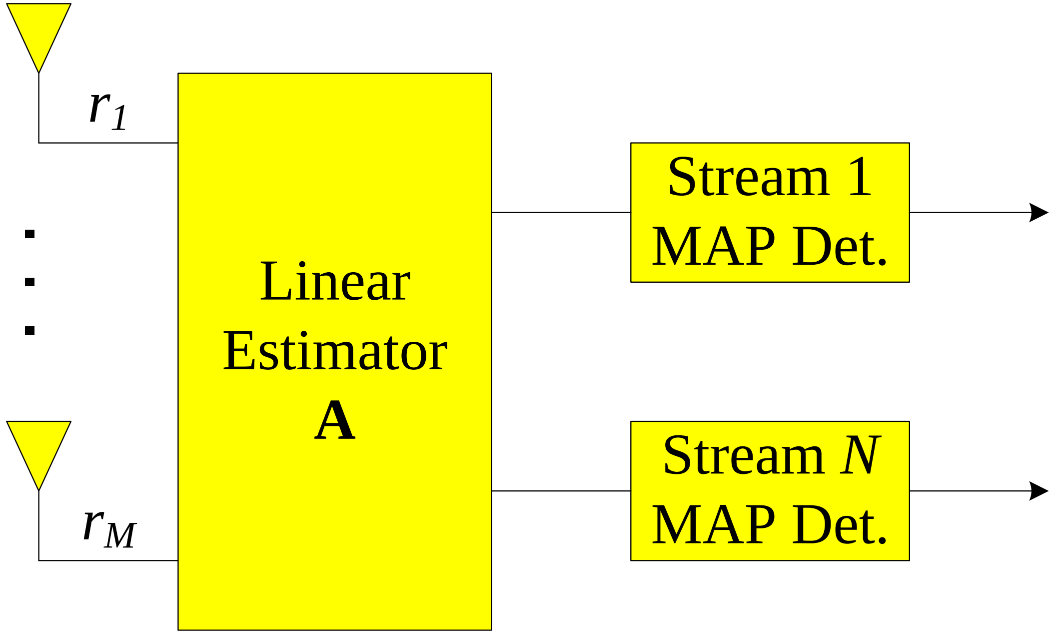
Spatial Multiplexing

- CSIR capacity derivation suggests multiplexing *N* independent data streams in space; optimal detector is joint-MAP
- Reduced-complexity V-BLAST receiver (Wolniansky *et al.* '98) forms a linear estimate of $\mathbf{x}$, then performs individual-MAP



$$\tilde{\mathbf{x}} = \mathbf{A}\mathbf{r}, \quad \mathbf{A} \in C^{N \times M}$$

V-BLAST (ZF-SC & MMSE-SC)

- Two popular criteria for choosing $\mathbf{A}$ are zero-forcing (ZF) and linear minimum mean-square error (MMSE)
- Both schemes may be used in conjunction with successive cancellation (SC) to improve performance: decode streams *sequentially*, subtracting decoded streams from Rx signal
- Capacity of V-BLAST with ZF-SC and MMSE-SC receivers:

$$C_{\text{ZF-SC}} = \sum_{i=1}^{N} \mathrm{E}\left[\log\left(1 + \frac{1}{N[(\mathbf{H}_{\mathbf{w}}^{iH}\boldsymbol{\Sigma}\mathbf{H}_{\mathbf{w}}^{i})^{-1}]_{ii}}\right)\right]$$

$\mathbf{A}^i$ ~ Delete first i - 1 columns of $\mathbf{A}$

$$C_{\text{MMSE-SC}} = C_R = \mathrm{E}\left[\log_2 \det\left(\mathbf{I} + \tfrac{1}{N}\mathbf{H}_{\mathbf{w}}^{H}\boldsymbol{\Sigma}\mathbf{H}_{\mathbf{w}}\right)\right],$$

MMSE-SC receiver is optimal!

Space-Time Coding

- Space-time block codes (STBCs), (Tarokh *et al.* ’98): encode over both space and time; may view as a matrix-valued channel:

$$\mathbf{R} = \mathbf{H}\mathbf{X} + \mathbf{N}$$

i^{th} column of $\mathbf{R}$, $\mathbf{X}$, $\mathbf{N}$ denote $\mathbf{r}$, $\mathbf{x}$, $\mathbf{n}$ during i^{th} channel use

- Maximum-likelihood (ML) detector minimizes probability of error:

$$\hat{\mathbf{X}}_{\text{ML}} = \arg\min_{\mathbf{X}_i \in \mathcal{X}} \left\| \mathbf{\Sigma}_{\mathbf{n}}^{-1/2} (\mathbf{R} - \mathbf{H}\mathbf{X}_i) \right\|_{\text{F}}^2$$

$\|\cdot\|_{\text{F}}$ ~ Frobenius norm

$\mathcal{X}$ ~ codespace

- Pairwise error probability (PEP) – error prob. of hypothetical binary decision $\mathcal{X}$ = $\{\mathbf{X}_i, \mathbf{X}_j\}$. For orthogonal STBCs the PEP is:

$$\text{P}_{ij} = \text{E}\left[\text{Q}\left(\sqrt{\frac{2}{NR} \text{tr}(\mathbf{H}_{\mathbf{w}}^{H} \mathbf{\Sigma} \mathbf{H}_{\mathbf{w}})} \right) \right]$$

BPSK modulation, rate-R OSTBC, max PEP

Beamforming (MIMO-MRC)

- Previously considered SIMO MRC; with MIMO may also perform maximum-ratio *transmission* (MRT), (Lo '99)
- Combination of MRT and MRC often referred to as MIMO-MRC:

$\mathbf{w}_T$ ~ transmit weighting vector, $\mathbf{w}_R$ ~ receive weighting vector

Output SNR: $$\gamma = P\frac{\left|\mathbf{w}_R^H\mathbf{H}\mathbf{w}_T\right|^2}{\mathbf{w}_R^H\boldsymbol{\Sigma}_{\mathbf{n}}\mathbf{w}_R} \le P\lambda_1(\mathbf{H}^H\boldsymbol{\Sigma}_{\mathbf{n}}^{-1}\mathbf{H}) = \gamma^o$$

Outage probability: $$P_{\mathrm{out}}(\tau) = \Pr\{\gamma^{\mathrm{o}} \le \tau\} = \Pr\left\{\lambda_1(\mathbf{H}_{\mathbf{w}}^H\boldsymbol{\Sigma}\mathbf{H}_{\mathbf{w}}) \le \tau\right\}$$

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Form of Performance Metrics

- We have presented several MIMO performance metrics:

CSIR Capacity	V-BLAST ZF-SC Capacity	Orthogonal STBC PEP
Full CSI Capacity	V-BLAST MMSE-SC Capacity	MIMO-MRC Outage Probability

- Each metric is the mean or cdf of a random variable of the form

$$z = g(\mathbf{H}_{\mathbf{w}}^{H} \mathbf{\Sigma} \mathbf{H}_{\mathbf{w}})$$

where for any $\mathbf{A} \geq \mathbf{B} > \mathbf{0}$ the functional g satisfies

1. $g(\mathbf{A}) \geq g(\mathbf{B})$
2. $\mathbf{A} = \mathbf{B} \Leftrightarrow g(\mathbf{C}^{H}\mathbf{A}\mathbf{C}) = g(\mathbf{C}^{H}\mathbf{B}\mathbf{C})$ for all full-rank $\mathbf{C} \in \mathcal{C}^{M \times N}$

$\mathbf{A} \geq \mathbf{B}$ means **A** - **B** is nonnegative definite

Form of Performance Metrics (cont.)

- Theorem 1: Consider two otherwise identical MIMO systems with SNR matrices $\Sigma_1 \geq \Sigma_2$ and let

$$z_1 = g(\mathbf{H}_\mathbf{w}^H \boldsymbol{\Sigma}_1 \mathbf{H}_\mathbf{w}) \quad \text{and} \quad z_2 = g(\mathbf{H}_\mathbf{w}^H \boldsymbol{\Sigma}_2 \mathbf{H}_\mathbf{w}).$$

Denoting the means and cdfs by m_i and $F_i(x)$, we have

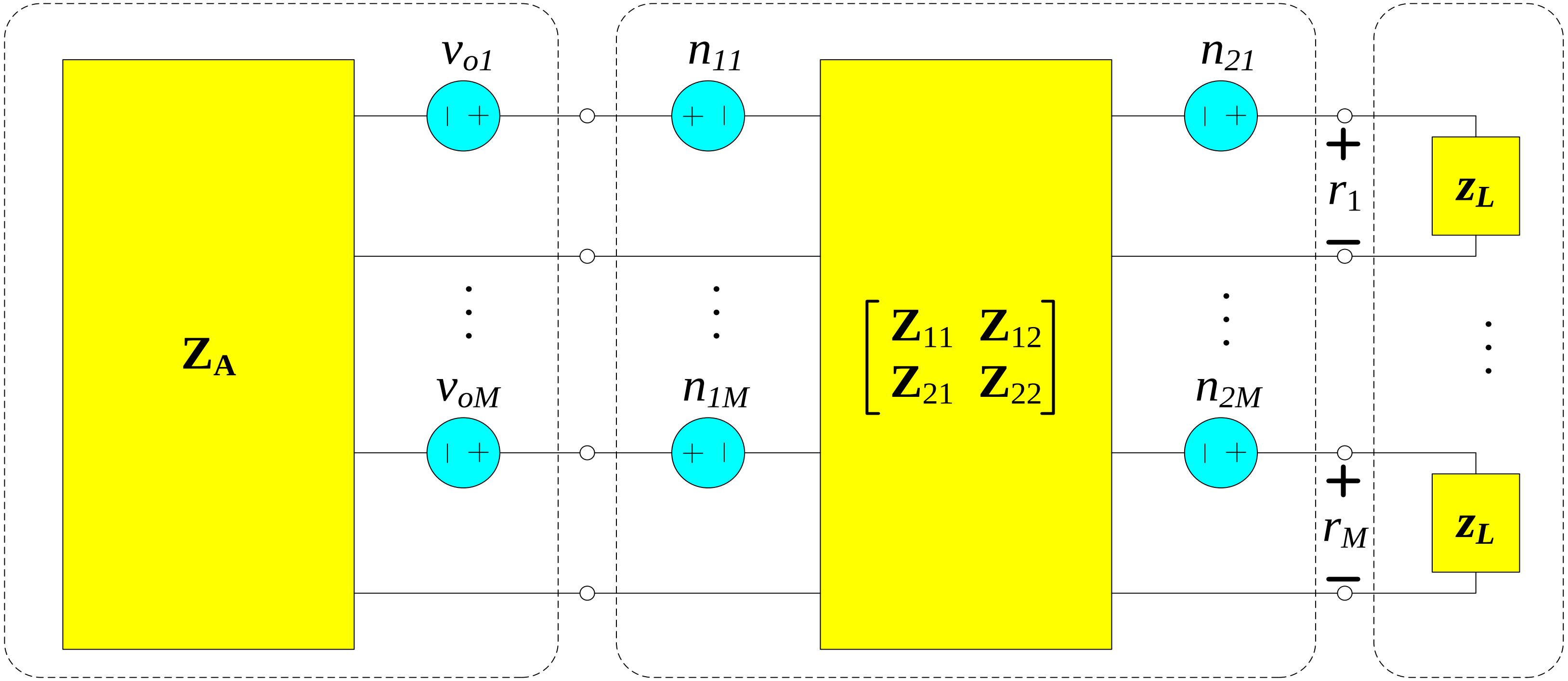
$$m_1 \geq m_2 \quad \text{and} \quad F_1(x) \leq F_2(x) \;\; \forall x \in \mathcal{R},$$

with equality if and only if $\Sigma_1 = \Sigma_2$

- This result may be thought of as a generalization of the monotonicity of SISO metrics in the SNR

Receiver Noise Model

- Model front-end as a linear, noisy 2M-port network:



Output signal: $\mathbf{r} \propto \mathbf{H}\mathbf{x} + \underbrace{\mathbf{n}_o + \mathbf{z}}_{\mathbf{n}}$

$\mathbf{z}$ ~ noise from front-end

$\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}_\mathbf{n})$

MIMO Low-Noise Design

- In a similar manner to the noise factor of a two-port, define the noise factor matrix of the front-end as:

$$\mathbf{F} = \left(\boldsymbol{\Sigma}_{\mathbf{n}}\big|_{\mathbf{z}=\mathbf{0}}\right)^{-1/2} \boldsymbol{\Sigma}_{\mathbf{n}} \left(\boldsymbol{\Sigma}_{\mathbf{n}}\big|_{\mathbf{z}=\mathbf{0}}\right)^{-1/2} \qquad \left(= F \text{ for } M = 1 \right)$$

- The SNR matrix may be expressed in terms of $\mathbf{F}$:

$$\boldsymbol{\Sigma} = \tfrac{\sigma}{N} \boldsymbol{\Sigma}_{\mathbf{h}}^{1/2} \mathbf{R}_{\mathbf{A}}^{-1/2} \mathbf{F}^{-1} \mathbf{R}_{\mathbf{A}}^{-1/2} \boldsymbol{\Sigma}_{\mathbf{h}}^{1/2} \qquad \left(\mathbf{R}_{\mathbf{A}} = \tfrac{1}{2}(\mathbf{Z}_{\mathbf{A}} + \mathbf{Z}_{\mathbf{A}}^{H}) \right)$$

- Theorem 2: For two otherwise identical systems with $\mathbf{F}_1 \le \mathbf{F}_2$,

$$C_{\mathrm{R}}^{(1)} \ge C_{\mathrm{R}}^{(2)} \qquad C_{\text{ZF-SC}}^{(1)} \ge C_{\text{ZF-SC}}^{(2)} \qquad \mathrm{P}_{ij}^{(1)} \le \mathrm{P}_{ij}^{(2)}$$

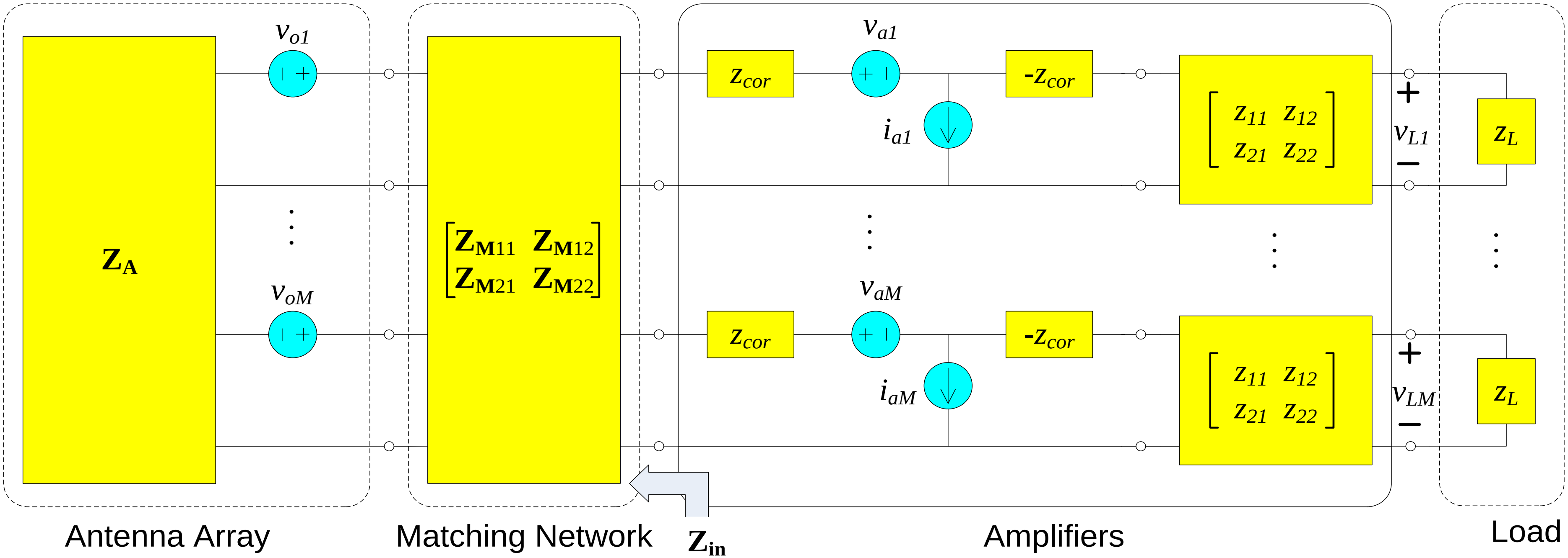
$$C_{\mathrm{F}}^{(1)} \ge C_{\mathrm{F}}^{(2)} \qquad C_{\text{MMSE-SC}}^{(1)} \ge C_{\text{MMSE-SC}}^{(2)} \qquad \mathrm{P}_{\mathrm{out}}^{(1)} \le \mathrm{P}_{\mathrm{out}}^{(2)}$$

with equality if and only if $\mathbf{F}_1 = \mathbf{F}_2$.

Design for "minimum" $\mathbf{F}$!

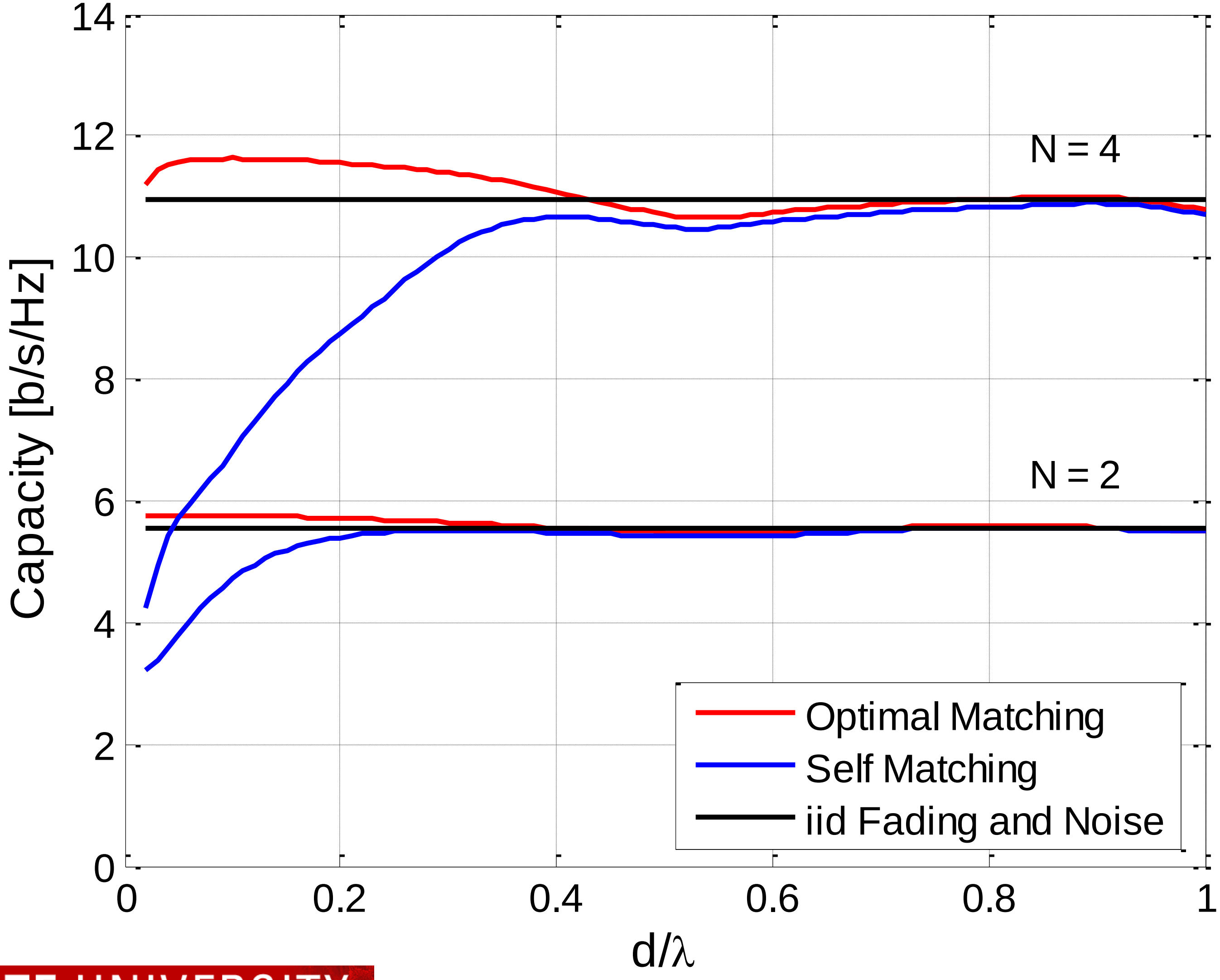
Optimal Matching for Front-End Amps

- Consider matching *M* uncoupled amplifiers to an antenna array:

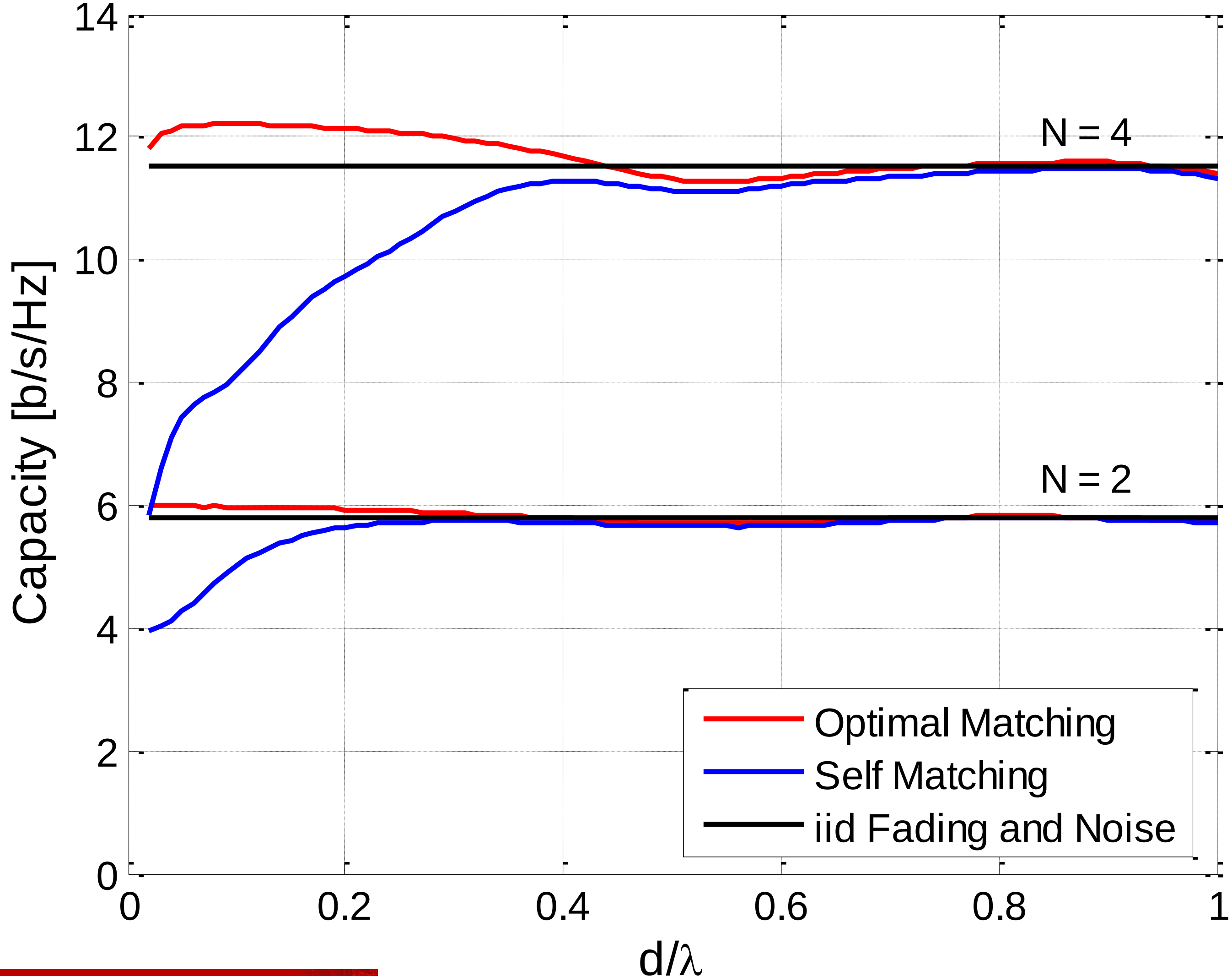


- Can show that $\mathbf{F} \geq F_{\min}\mathbf{I}$, with equality iff $\mathbf{Z_{in}} = z_{\text{opt}}\mathbf{I}$
- Prior studies have *conjectured* this matching may be optimal w.r.t. capacity; we can *prove* it is optimal for a large class of metrics!

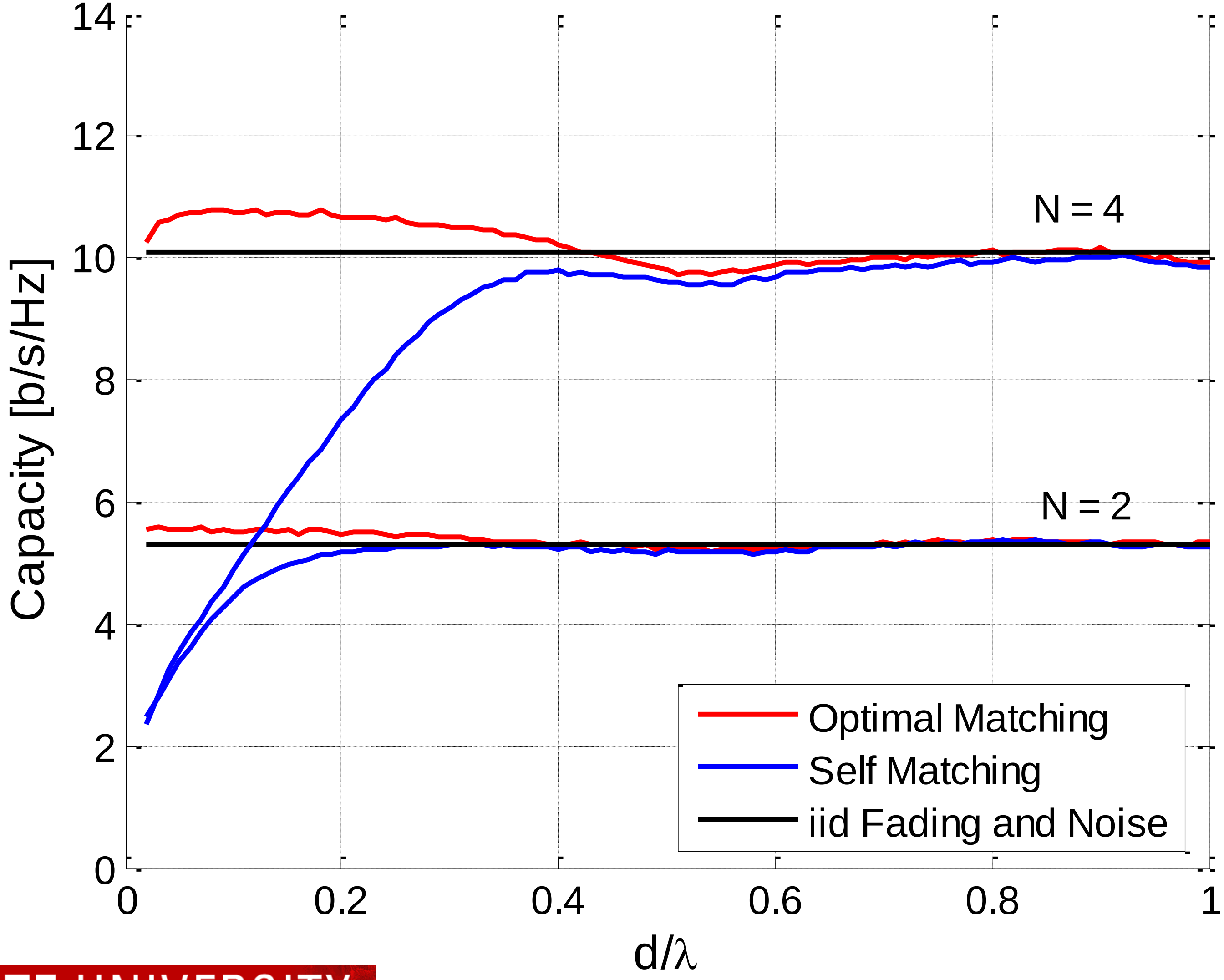
CSIR Capacity



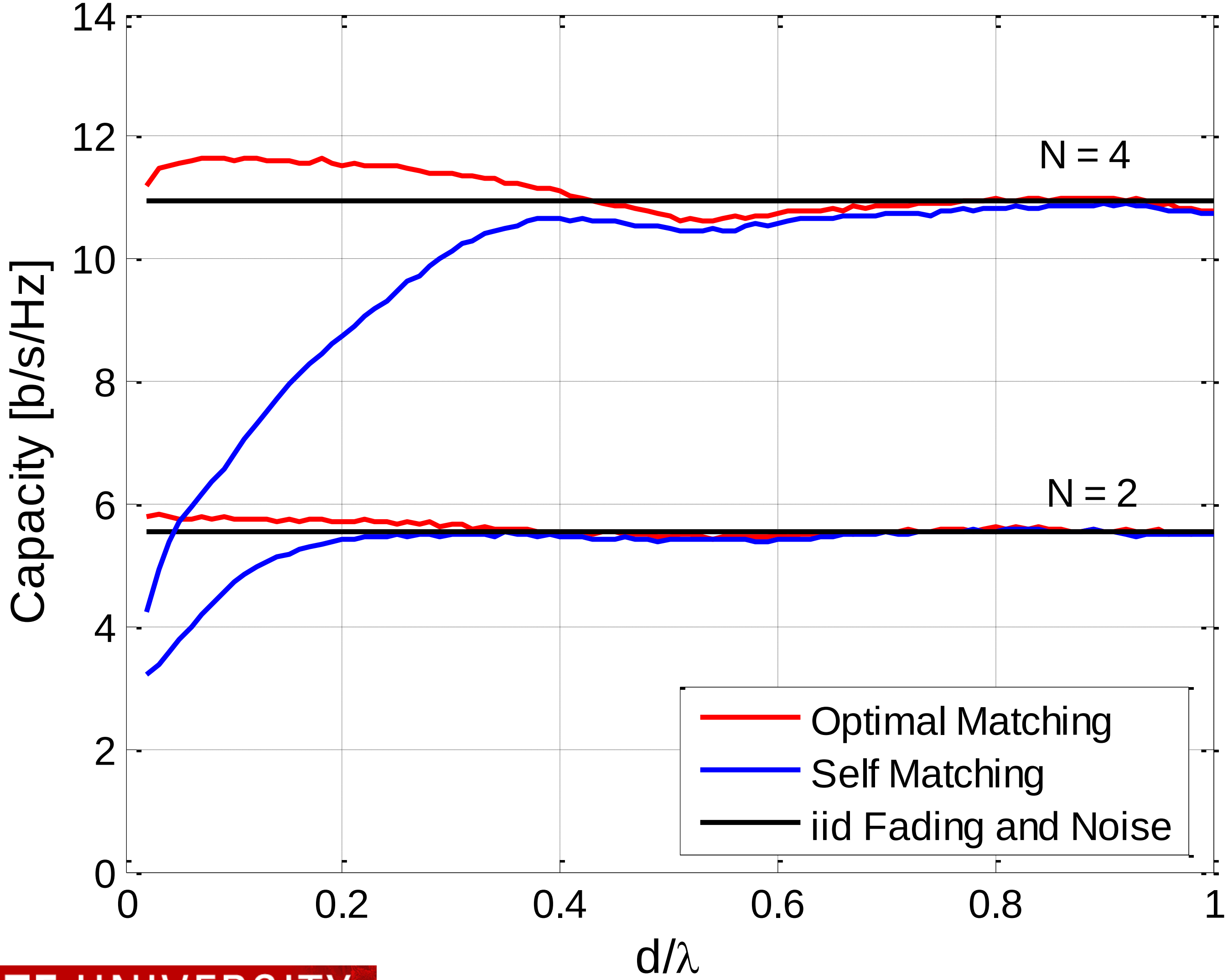
Full CSI Capacity



V-BLAST ZF-SC Capacity



V-BLAST MMSE-SC Capacity



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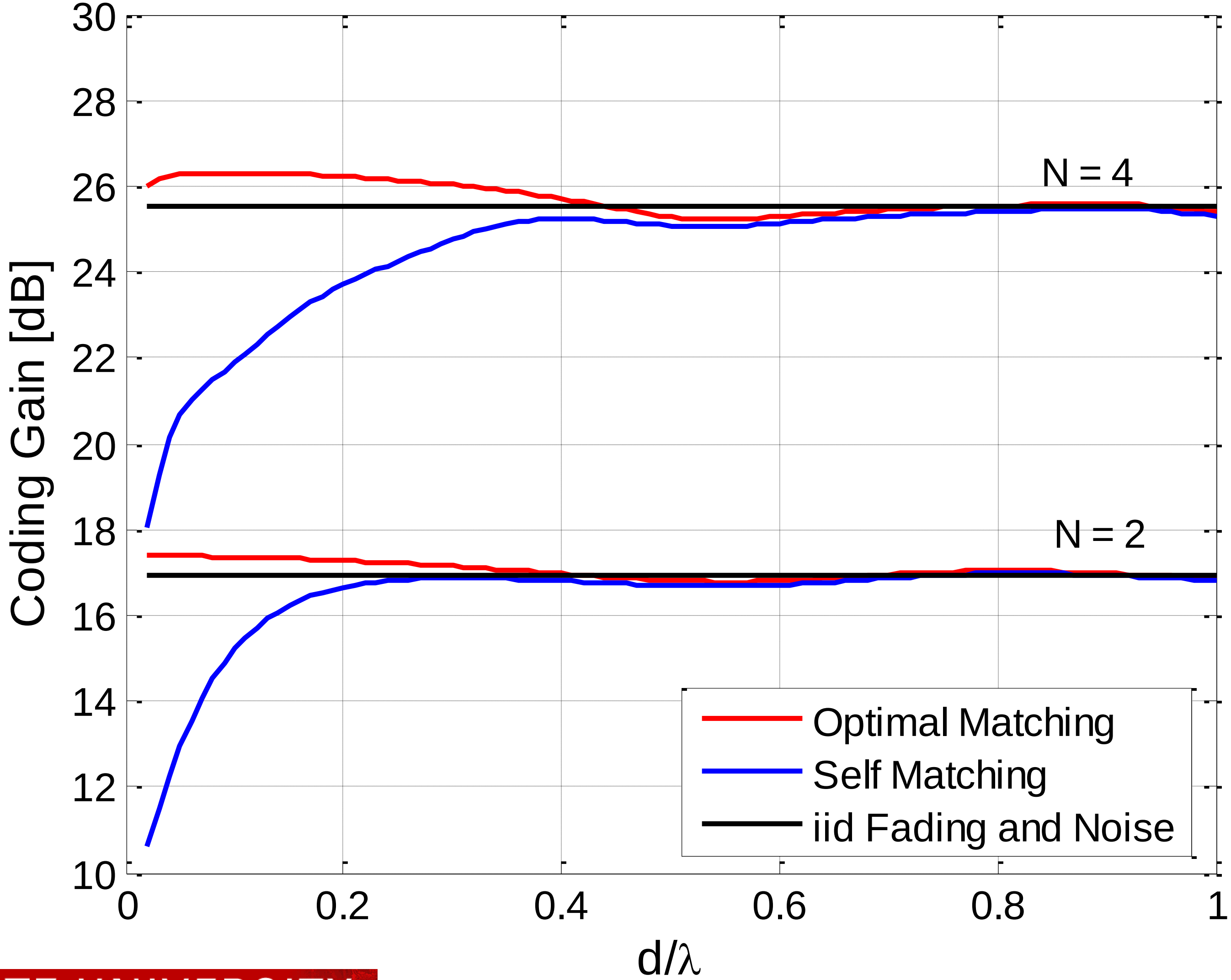
Orthogonal STBC Coding Gain at 10^{-3} PEP



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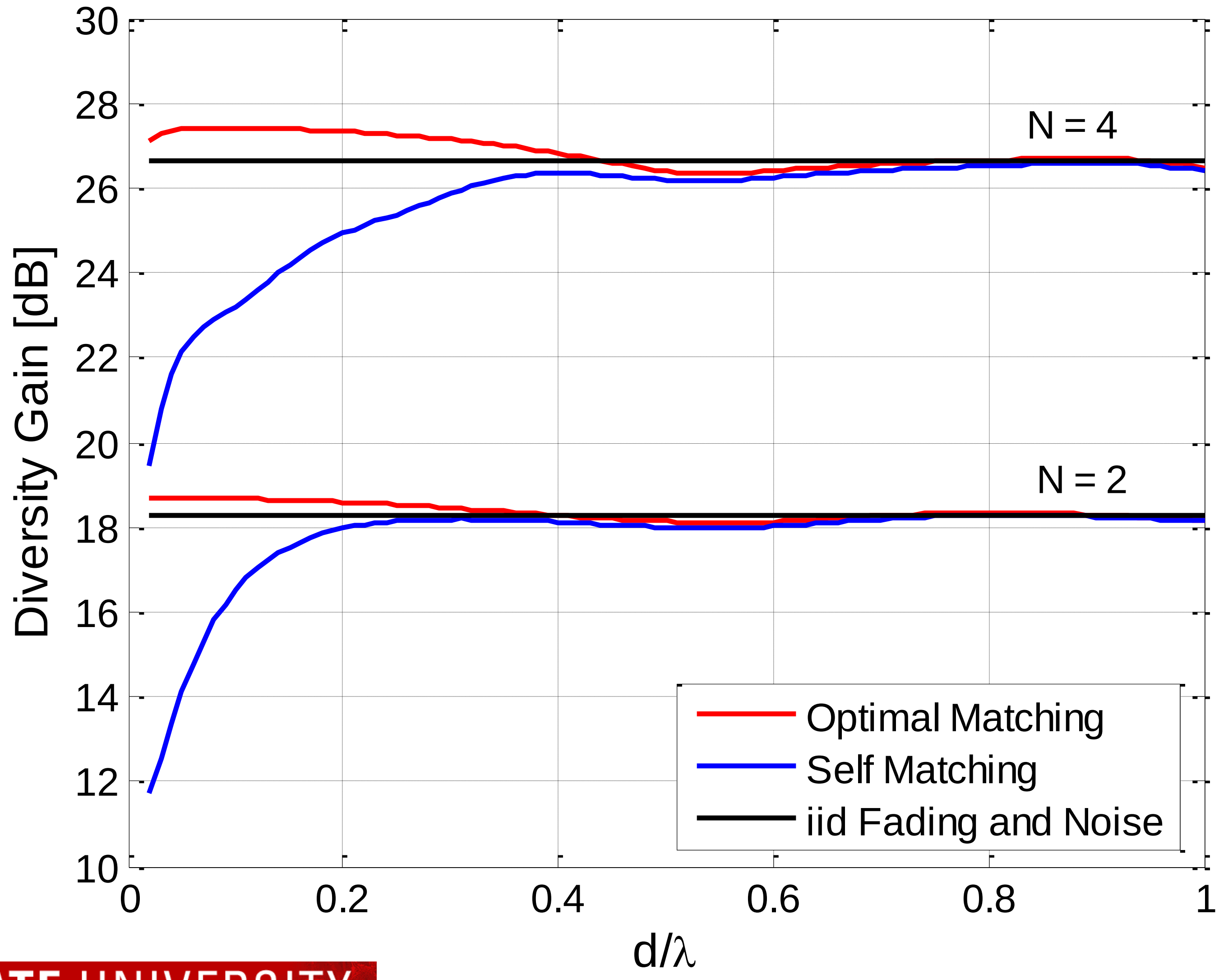
MIMO-MRC Diversity Gain at 1% Outage



Conclusion

- Scalar measures such as noise figure cannot reliably predict receiver performance; both the power and correlation of the noise is important
- Designing the front-end for “minimum” noise factor matrix is optimal for a large class of performance metrics and receiver front-ends
- Developed low-noise design principles may be readily applied to specific problems of interest, e.g., optimal amplifier matching

Overview

- Introduction – MIMO capacity
- Prior work – mutual coupling
- Noise correlation in compact multi-antenna receivers
- Optimal front-end design for compact MIMO receivers
- Conclusions and future work

Summary of Research

- Prior studies of compact MIMO receivers provided detailed models of fading; relatively little attention paid to noise
- Since fading and noise correlation play equal roles in determining most performance metrics, need to model both
- Main contributions of this research:
  - Noise analysis: Noise from the antennas, front-end amplifiers, and downstream components may affect performance in profoundly different ways
  - Low-noise design: Designing the front-end for minimum noise factor matrix is optimal for a large class of metrics and receivers, e.g., optimal matching for front-end amplifiers

Recommendations for Future Work

- Optimal broadband matching: Matching result derived earlier is optimal at the center frequency; may be suboptimal over some finite bandwidth. For a ULA of dipoles, 1^{st} order approximation

$$\mathbf{Z}_{\mathbf{A}}(\omega) = \mathbf{Z}_0 + \omega \mathbf{Z}_1$$

  seems reasonable. However, problem is still quite difficult…
- Optimal matching for coupled front-ends: We considered a bank of uncoupled, identical amplifiers; it would be useful to explore other possibilities, e.g., correlated LO noise, SoC, etc.
- Additional simulations and experimental work: Apply developed theory to other antennas (e.g., microstrip) and amplifiers (e.g., CMOS). Compare our theoretical predictions on noise correlation and its impact with experimental results.

Publications

1. C. P. Domizioli, B. L. Hughes, K. G. Gard, and G. Lazzi, “Receive diversity revisited: correlation, coupling, and noise,” in *Proc. IEEE Globecom 2007*, Washington, D.C.
2. C. P. Domizioli, B. L. Hughes, K. G. Gard, and G. Lazzi, “Optimal front-end design for MIMO receivers,” in *Proc. IEEE Globecom 2008*, New Orleans, LA
3. C. P. Domizioli, B. L. Hughes, K. G. Gard, and G. Lazzi, “Noise correlation in compact diversity receivers,” *IEEE Trans. Commun*., *IEEE Trans. Commun.*, May 2010.
4. C. P. Domizioli and B. L. Hughes, “Front-end design for compact MIMO receivers: a communication theory perspective,” submitted to *IEEE Trans. Commun.*

Publications (cont.)

5. Y. Dong, C. P. Domizioli, and B. L. Hughes, “Effects of mutual coupling and noise correlation on downlink coordinated beamforming with limited feedback,” *EURASIP J. Advances Sig. Process*., Jul. 2009
6. W. C. Cox, J. A. Simpson, C. P. Domizioli, J. F. Muth, and B. L. Hughes, “An underwater optical communication system implementing Reed-Solomon channel coding,” in *Proc. 2008 MTS/IEEE Oceans Conf.*, Quebec City, Canada

QUESTIONS?