Electromagnetic Scattering from Arbitrary Bodies Using Rao-Wilton-Glisson (RWG) Functions

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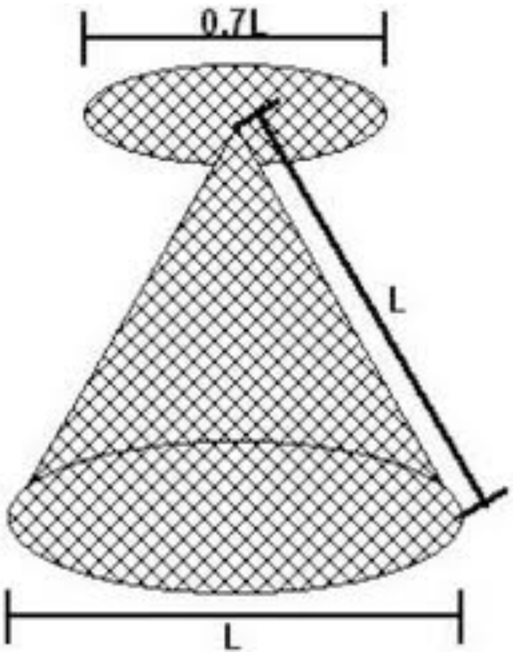
Arbitrarily-Shaped Body

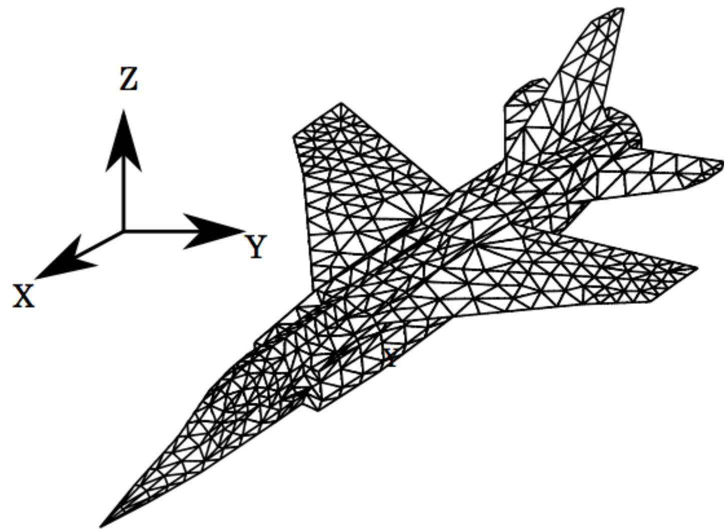
- Arbitrary shape.
- Arbitrary material composition.
- Cavities, cables, and apertures.
- Single or multiple bodies.
- Intersecting surfaces.
- Periodic structures.
- Finite structures.

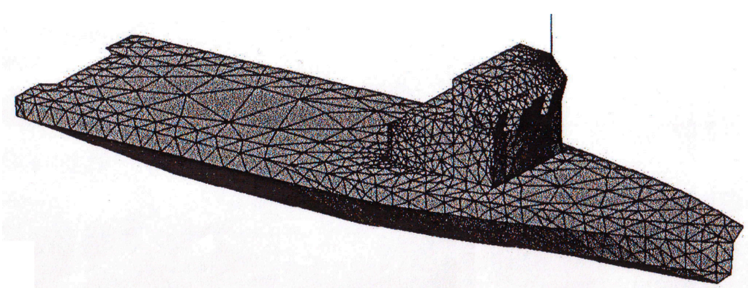
Numerical Solution Procedure

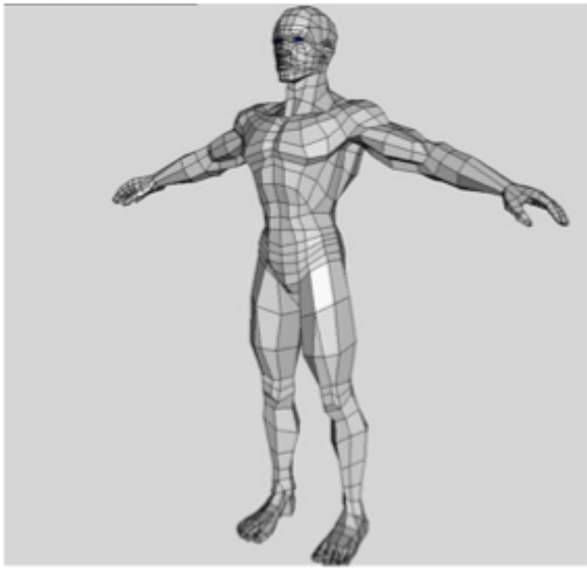
1. Describe Geometry to the computer- Planar triangular patch modeling.
2. Transform the Mathematical Equations into Matrix equation via Method of Moments.
3. Solve the Matrix equation.
4. Post-processing.

Triangulated Models









Method of Moments Solution

$$AX = Y \tag{4}$$

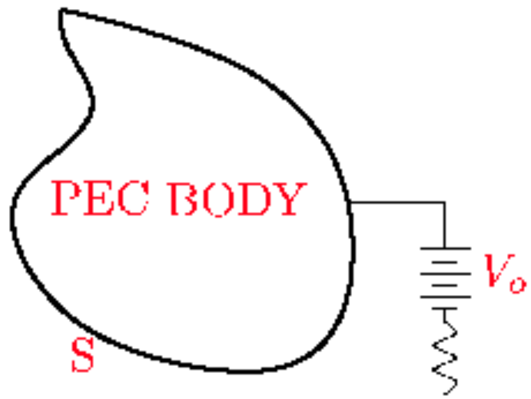
$$X = \sum_{i=1}^{N} \alpha_i p_i \tag{5}$$

$$\sum_{i=1}^{N} \alpha_i A p_i = Y \tag{6}$$

$$\sum_{i=1}^{N} \alpha_i < q_j, A p_i > \quad = \quad < q_j, Y > \qquad j = 1, 2, \cdots, N \tag{7}$$

$$\Rightarrow \boldsymbol{Z}\boldsymbol{X} = \boldsymbol{Y} \tag{8}$$

Electrostatic Problems



Applications: Electrostatic Discharge (ESD)

Method: Calculate charge distribution

$$\int_S \frac{q_s(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\, dS' = 4\pi\epsilon_o V_o \quad \mathbf{r} \in S$$

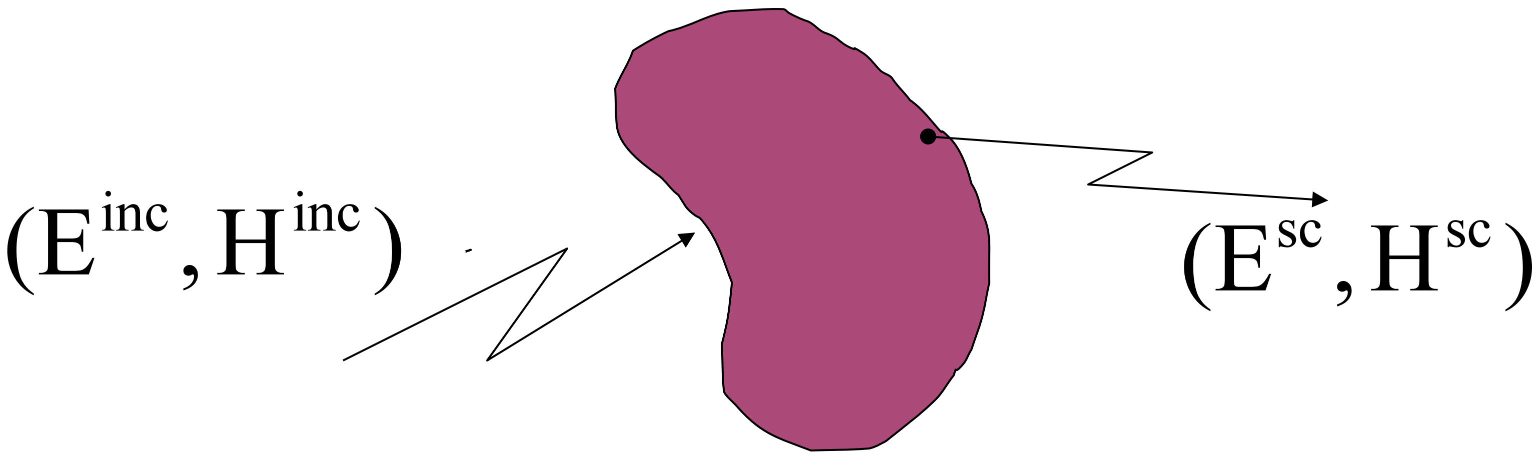
$$\Longrightarrow \quad \sum_{i=1}^{N} Q_i \int_{T_i} \frac{dS'}{\left|\mathbf{r_j^c}-\mathbf{r}'\right|} = 4\pi\epsilon_o V_{o_j}$$

$$j = 1, 2, \cdots, N$$

$$\Longrightarrow \quad [Z]\,[Q] = [V]$$

- Charge calculation – IEEE Trans. A&P 1979
- Electrostatic Discharge on Multi-Conductor Transmission Lines – IEEE Trans. MTT 1984
- Power-line hazard analysis - IEEE Trans. MTT 1985
- Characterization of Cross talk problem in VLSI design - IEEE Trans. MTT 1998
- Nuclear EMP Studies

Electrodynamics Problems



Arbitrary PEC Body excited by a Plane wave

EFIE

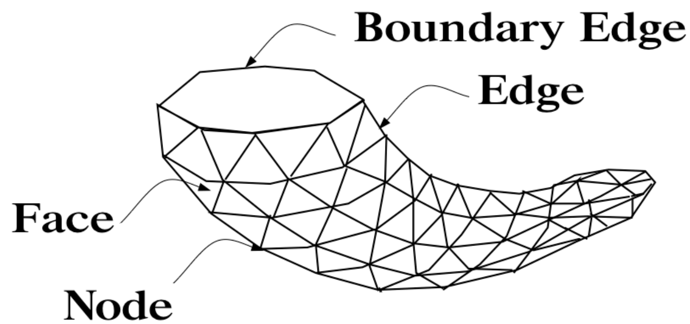
$$\boldsymbol{E}_{tan}^{total} = 0 \;\text{ for }\; \boldsymbol{r} \in S$$
$$\Rightarrow \boldsymbol{E}_{tan}^{inc} + \boldsymbol{E}_{tan}^{sc} = 0$$
$$\Rightarrow [j\omega \boldsymbol{A} + \nabla \Phi]_{tan} = \boldsymbol{E}_{tan}^{inc} \;\text{ for }\; \boldsymbol{r} \in S$$

MFIE

$$\boldsymbol{J} = \boldsymbol{a}_n \times \boldsymbol{H}^{total} \;\text{ for }\; \boldsymbol{r} \in S$$
$$\Rightarrow \boldsymbol{J} = \boldsymbol{a}_n \times (\boldsymbol{H}^{inc} + \boldsymbol{H}^{sc})$$
$$\Rightarrow \boldsymbol{J} - \boldsymbol{a}_n \times \boldsymbol{H}^{sc} = \boldsymbol{a}_n \times \boldsymbol{H}^{inc} \;\text{ for }\; \boldsymbol{r} \in S$$

Method of Moments Solution Procedure

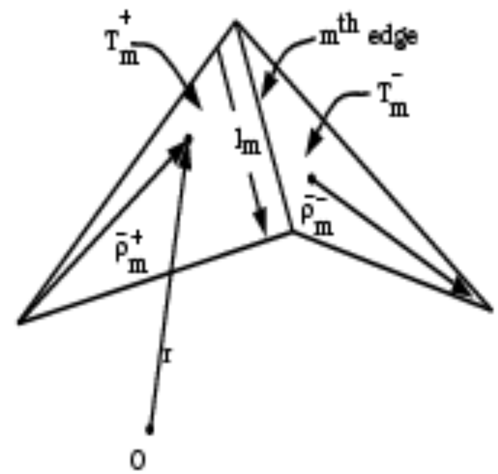
Approximate the unknown current density using RWG (Rao-Wilton-Glisson) functions.



Triangulated Body

$$\underline{\mathbf{J}} = \sum I_n \underline{\mathbf{f}}_n$$

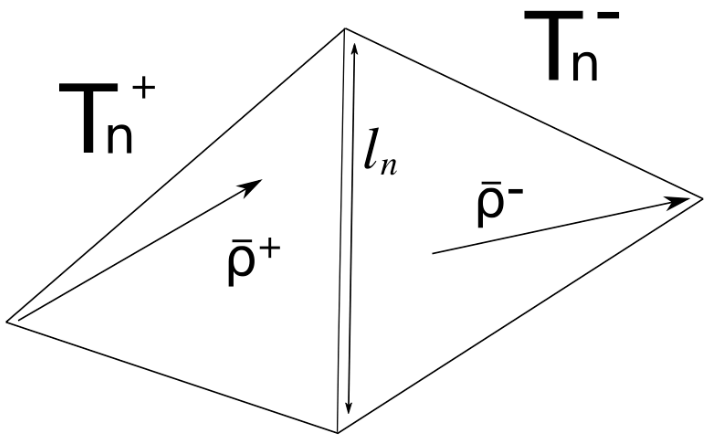
$$\mathbf{f}_n = \begin{cases} \dfrac{l_n}{2A_n^+}\,\rho_n^+ & \mathbf{r} \in \mathbf{T}_n^+ \\[2ex] \dfrac{l_n}{2A_n^-}\,\rho_n^- & \mathbf{r} \in \mathbf{T}_n^- \\[2ex] \mathbf{0} & \text{otherwise} \end{cases}$$



RWG Basis functions

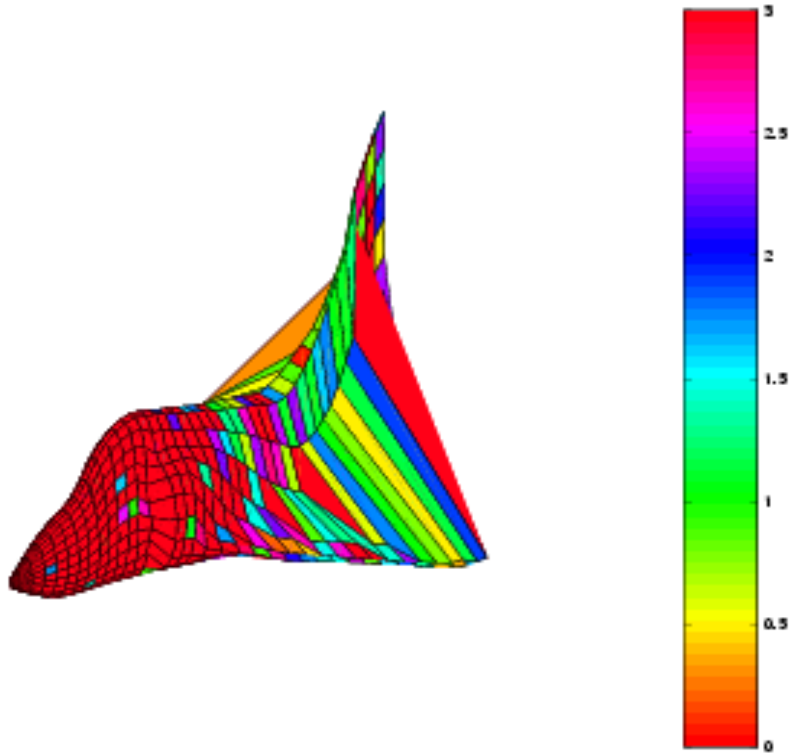
Method of Moments Solution Procedure

- Transform the operator equation into matrix equation using testing functions – Also RWG functions.
- Solve the matrix equation.



Testing Functions

Example of the currents on an Aircraft at 300 MHz with 2900 unknowns



Base Station Antenna Design

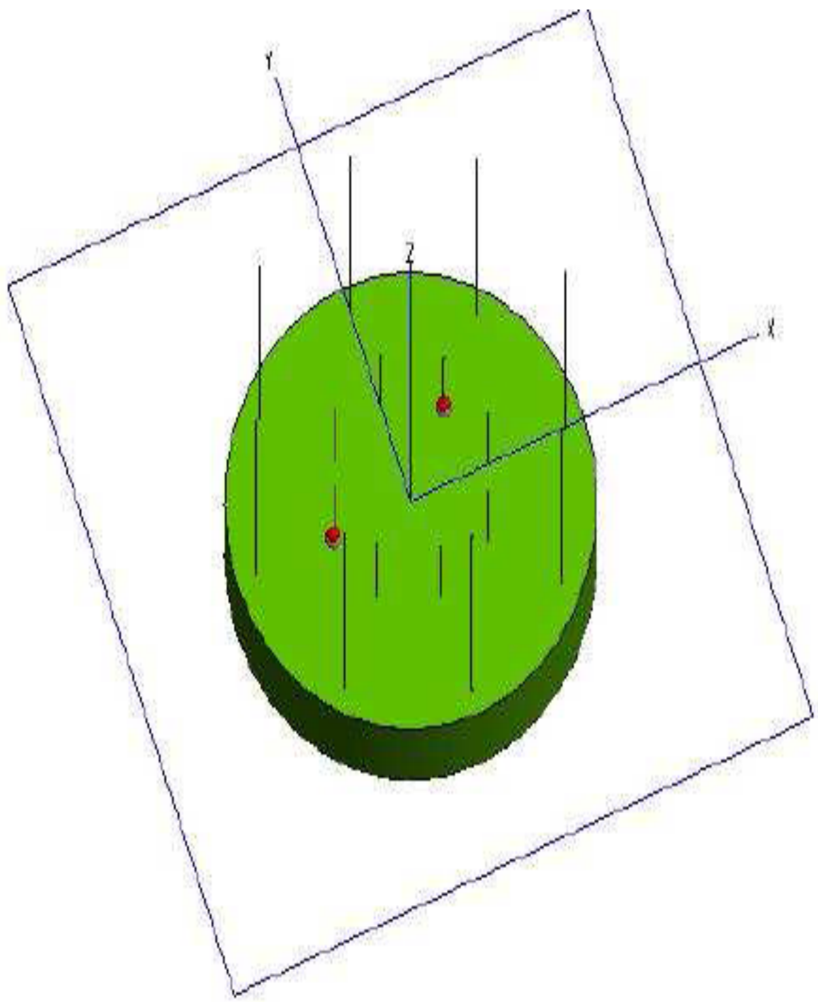


Figure: Geometry of the array antenna

- 1 element placed at the center of the cylindrical ground plane.
- 16 elements distributed uniformly (45 ° apart) on the circumference of 2 concentric rings.
- Two driver elements shown with red ports.
- 17 elements in all.

Geometry of antenna

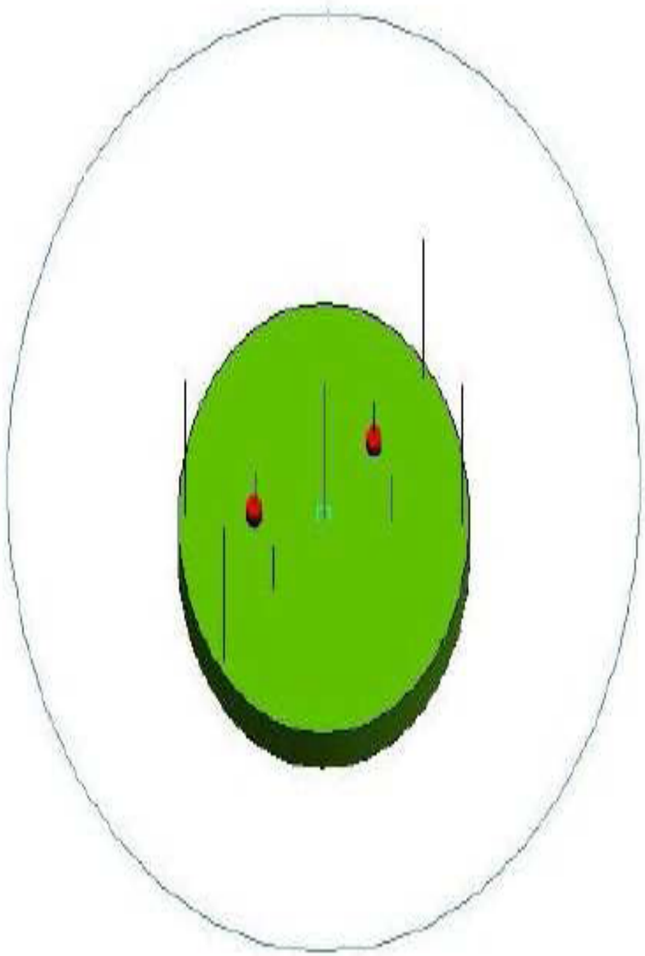
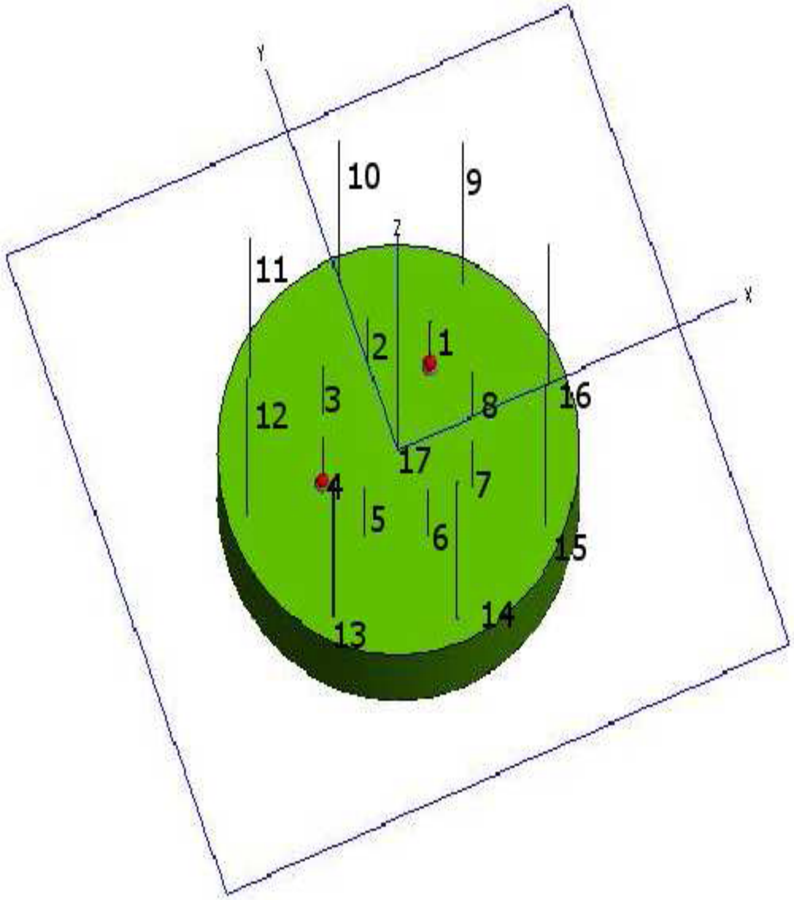
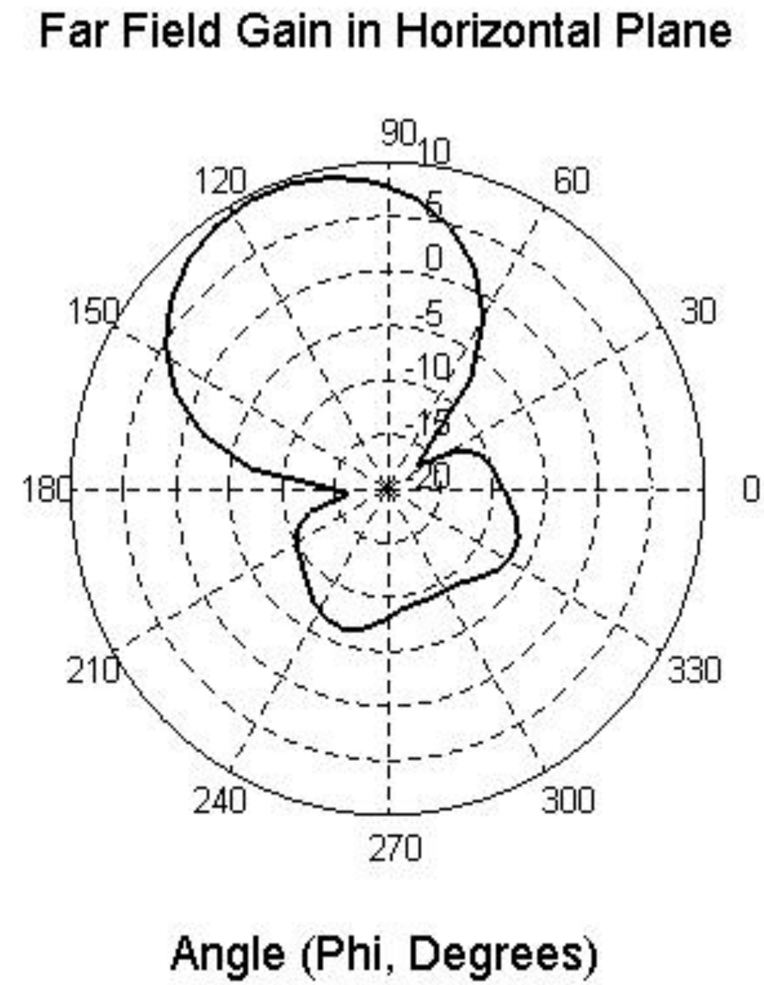


Figure : Geometry of the antenna with only essential elements.

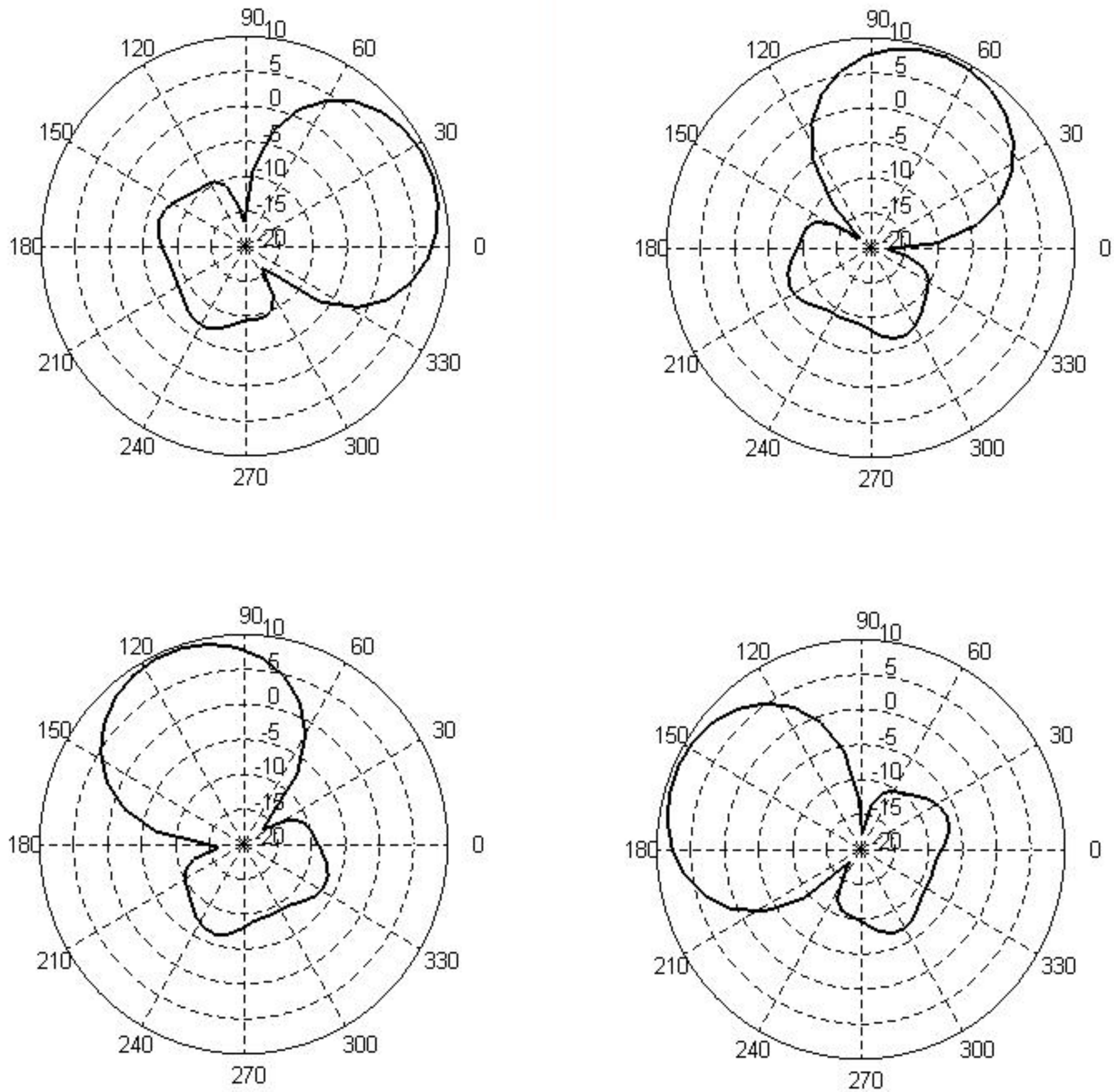
- 9 elements
- Driver elements are present in the inner circle and they are 135° apart.
- The elements in inner circle act as directors.
- Elements in the outer circle act as reflectors.
- Hollow cylinder acts as ground plane.



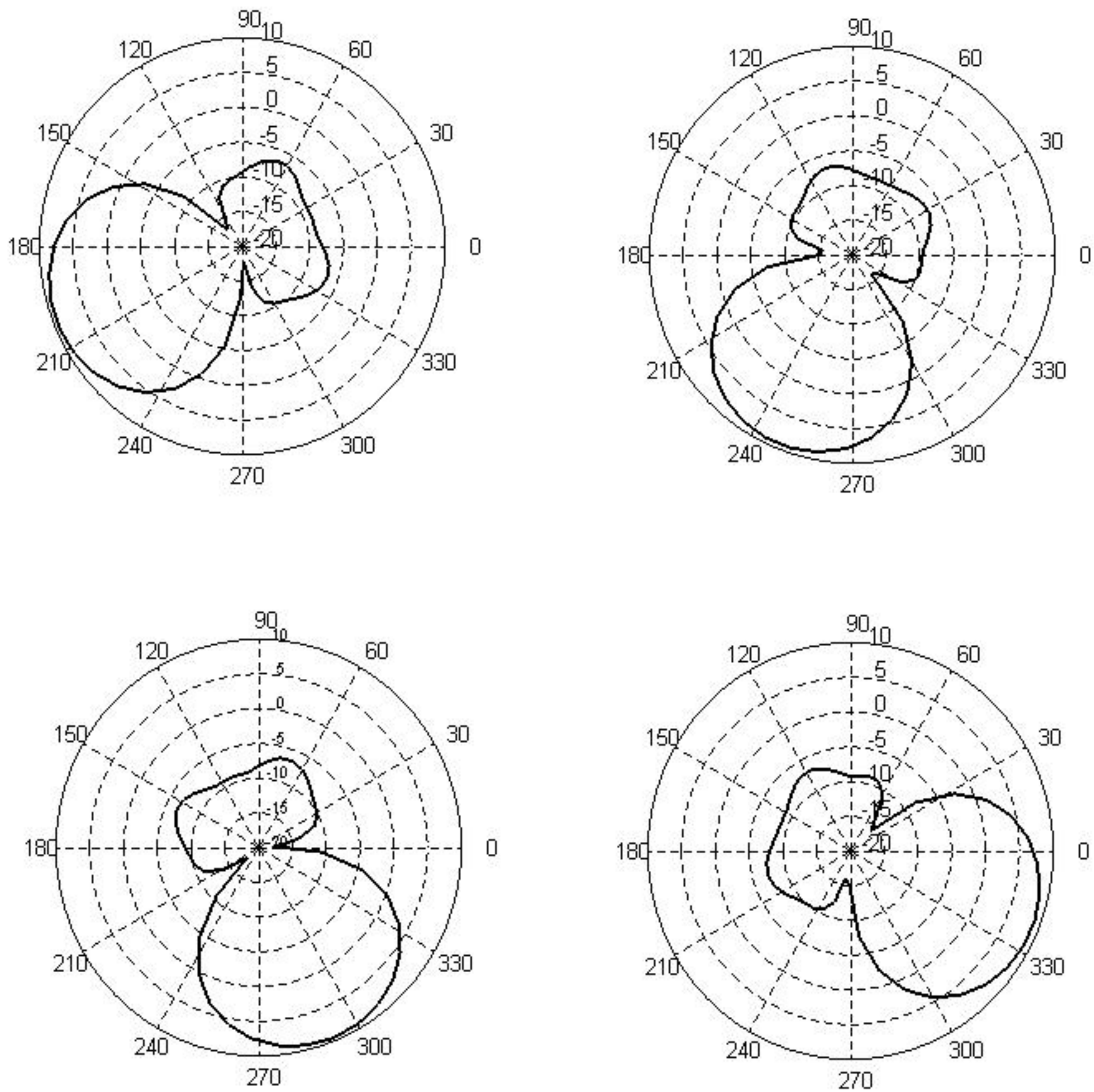


Elements 1 and 4 are excited
8, 16, 17, 12 , 9, 5, 13 are grounded
and the remaining elements are removed from the system.
(open circuited)

Far Field Gain in Horizontal Plane



Far Field Gain in Horizontal Plane



Fabrication of Antenna

- The antenna was fabricated to operate at 2GHz frequency.
- Copper sheets and rods of optimized dimensions were used for construction.
- A power splitter was designed to split the power equally between two driver elements.

Figure: Top view of the fabricated antenna

Fabricated antenna

- hc= 1.1811 inches (Height of the cylindrical ground plane)
- rc =3.3070 inches (Radius of the cylindrical ground plane)
- hi =1.5029 inches (Height of elements on inner circle)
- ho = 4.7238 inches (Height of elements on outer circle)
- horg =4.7096 inches (Height of element at origin)
- ri = 1.5750 inches (Radius of inner circle)
- ro = 3.1500 inches (Radius of outer circle)
- rr = 0.0625 inches (Radius of antenna elements)

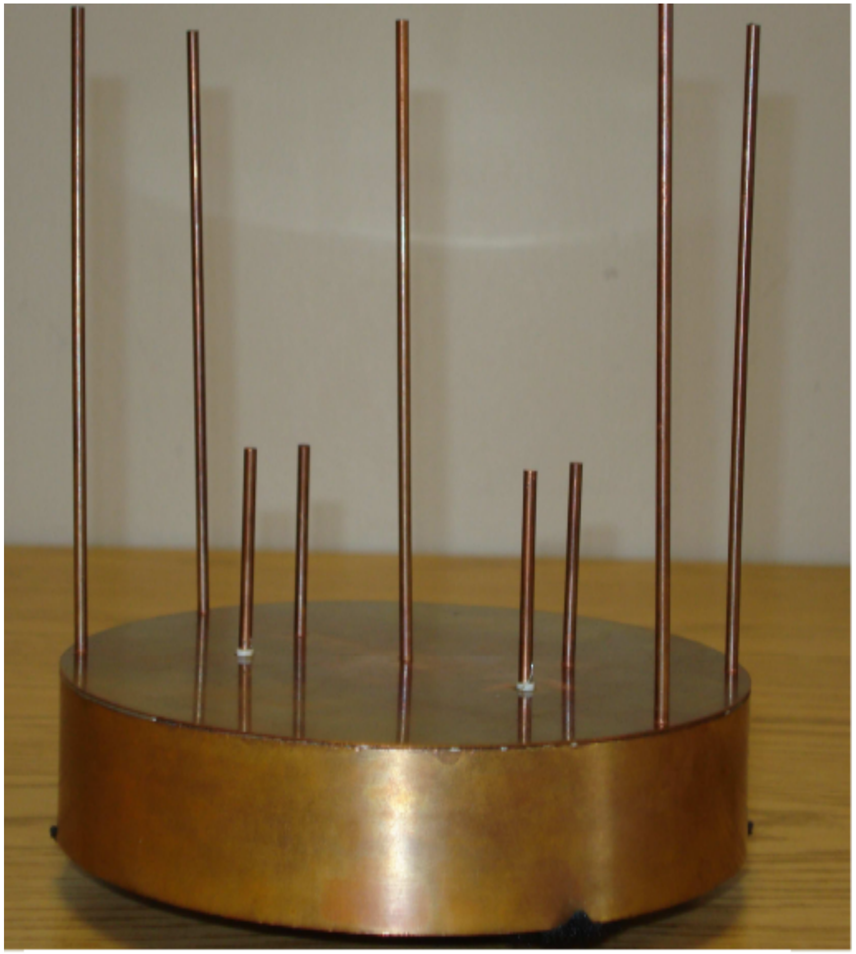


Figure: Side view of fabricated antenna

Comparison of the Radiation Patterns

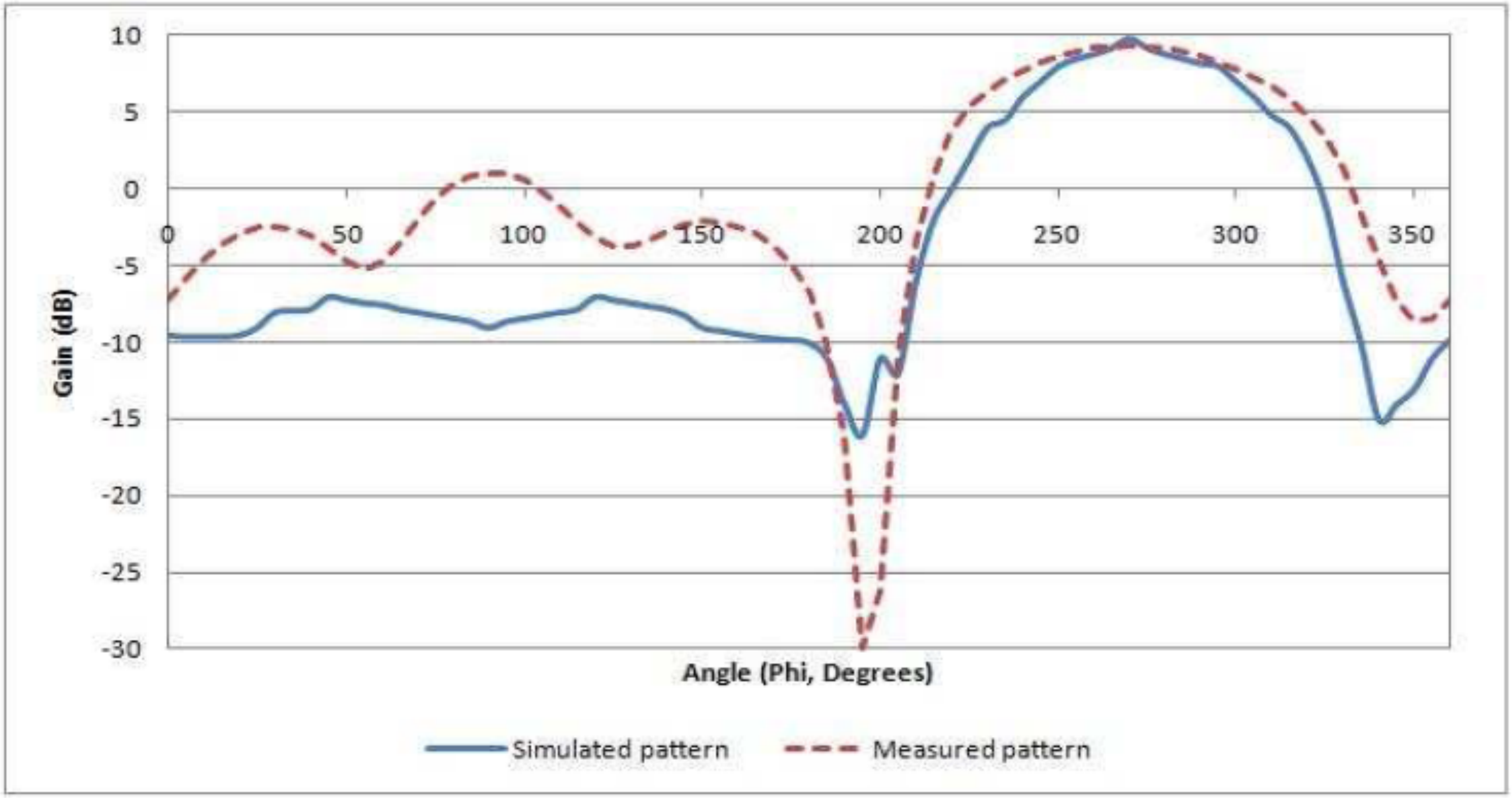
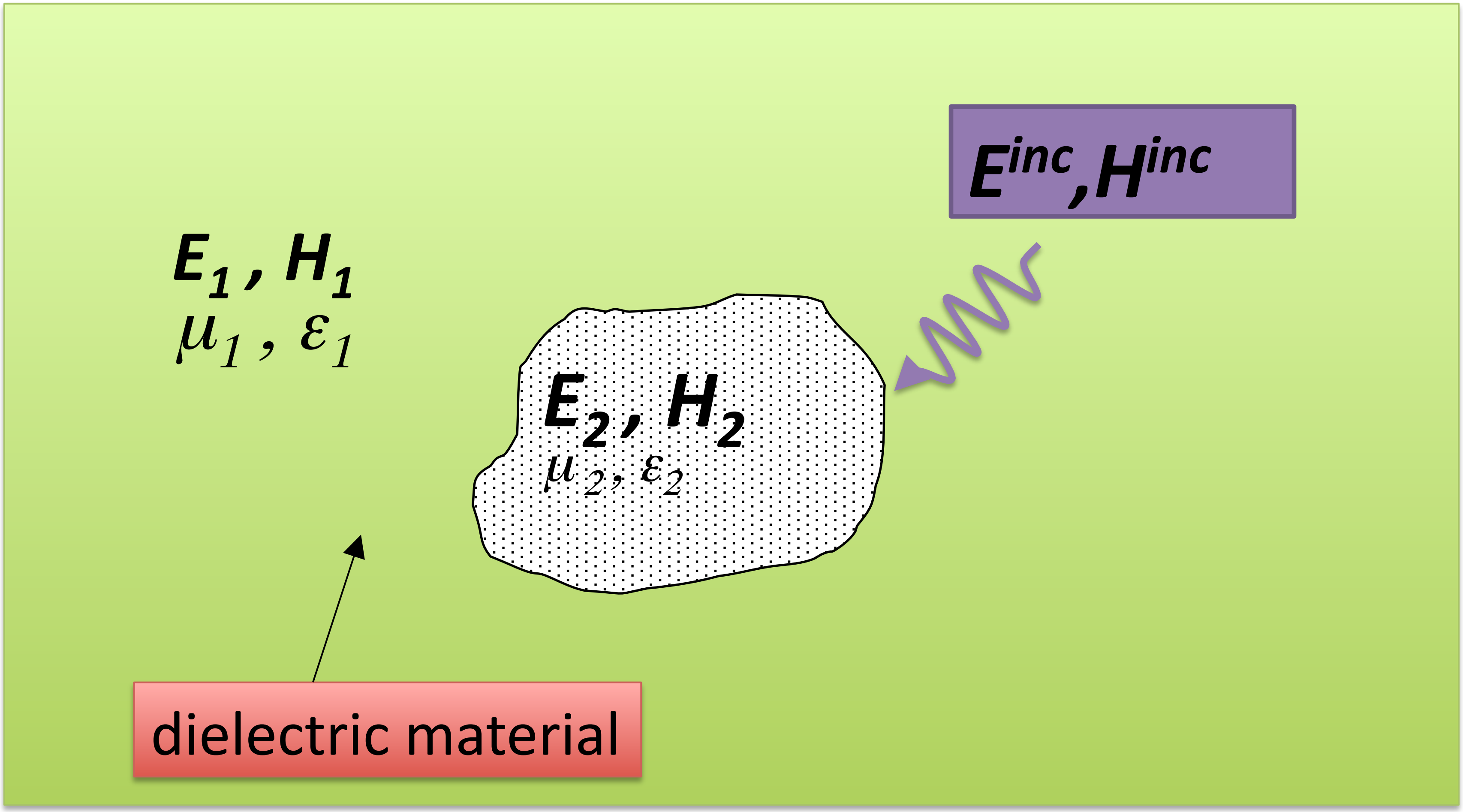


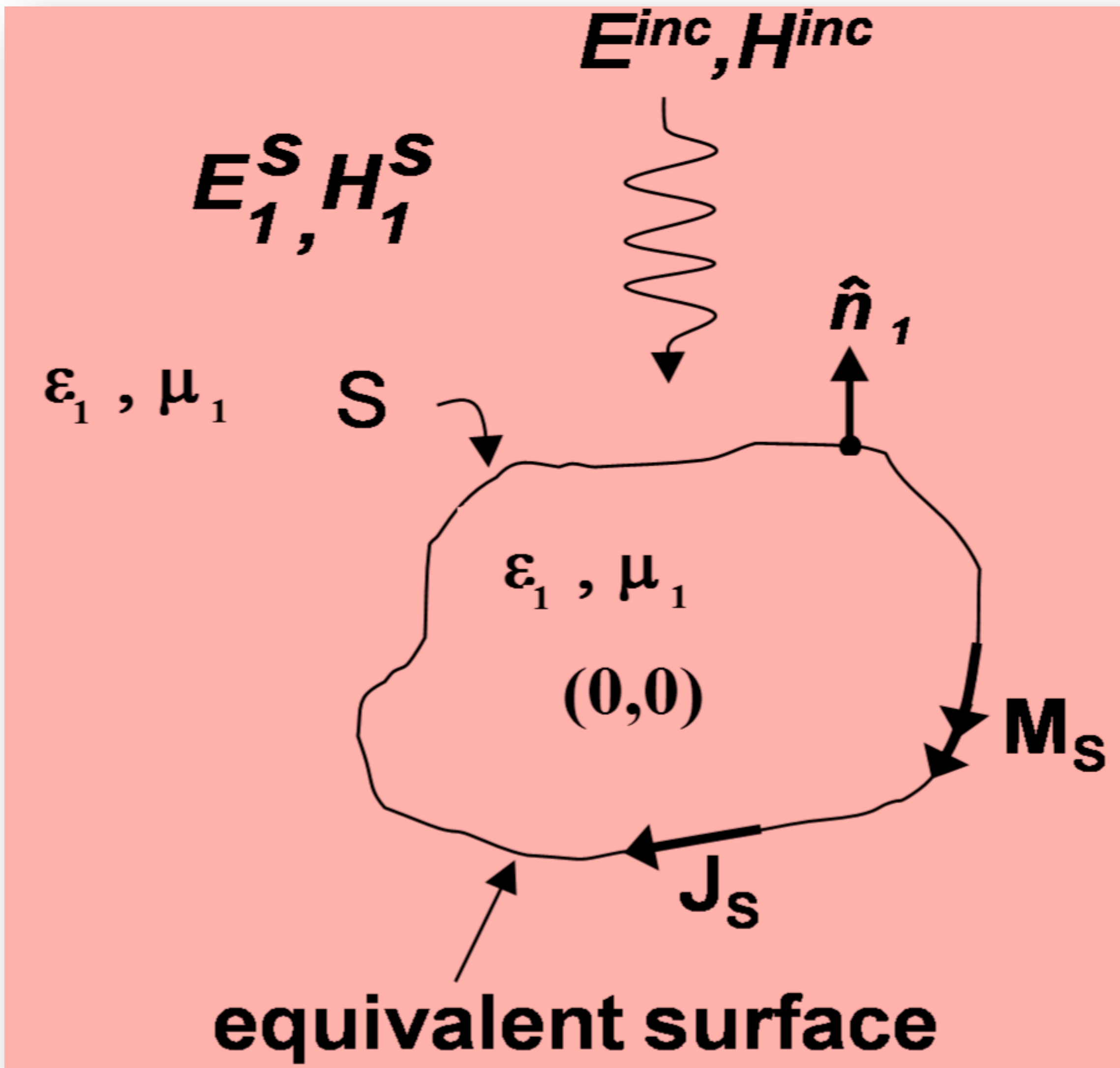
Figure: The simulated and measured radiation patterns of antenna

- FERM – Lincoln Labs – This code is classified
- PATCH – Sandia National Labs – This code is classified
- IE3D – Commercial Code
- FEKO – Commercial Code
- CARLOS3D – McDonnell Douglas

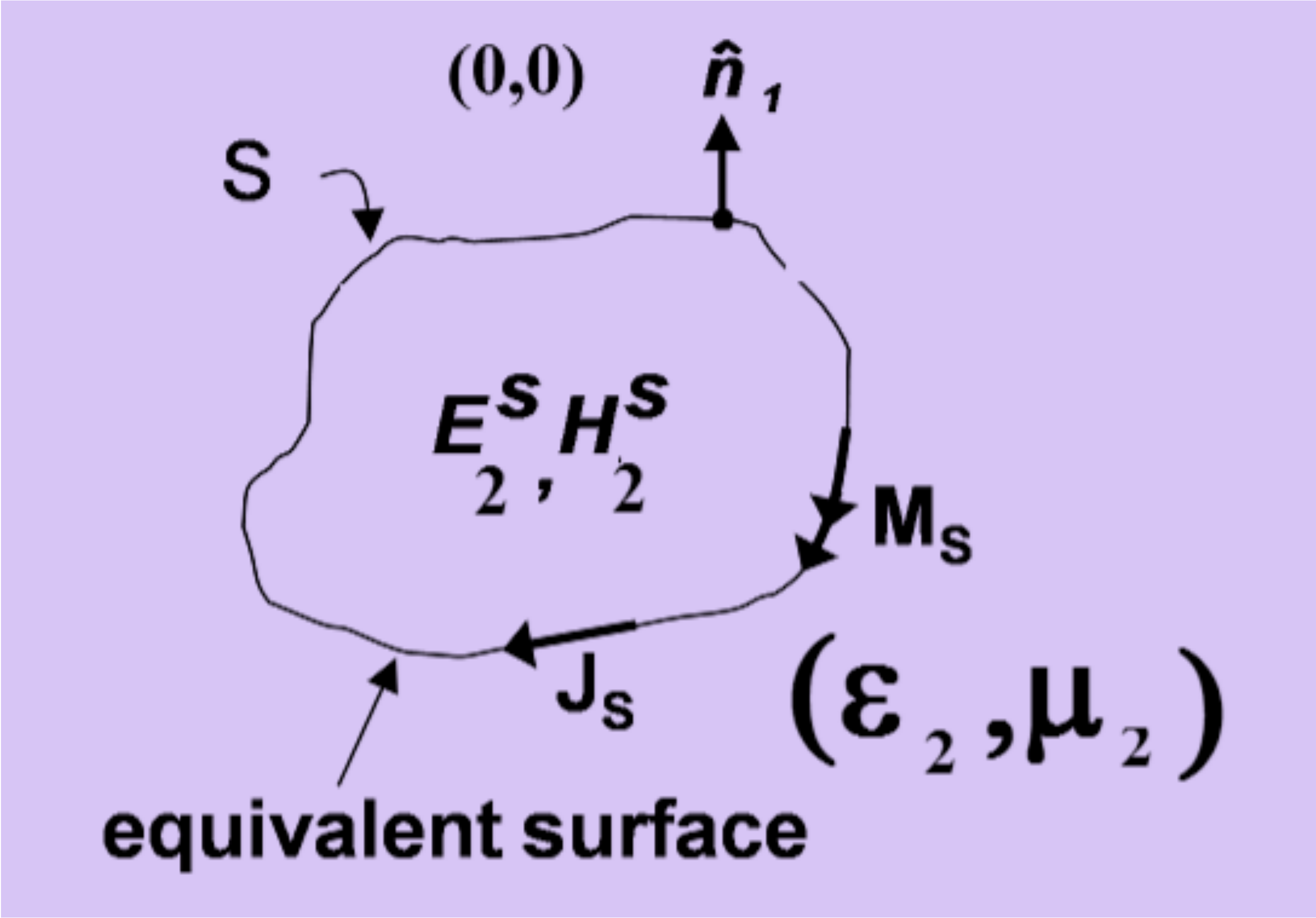
Surface Formulation of Integral Equations For a Dielectric Body



External Equivalent Problem



Internal Equivalent Problem



Four Governing Equations

$$
\begin{aligned}
\left[\boldsymbol{E}_1^s(\boldsymbol{J}_s) + \boldsymbol{E}_1^s(\boldsymbol{M}_s) + \boldsymbol{E}^{inc}\right]_{tan} &= 0 \quad \text{for} \quad \boldsymbol{r} \in S \\
\left[\boldsymbol{H}_1^s(\boldsymbol{J}_s) + \boldsymbol{H}_1^s(\boldsymbol{M}_s) + \boldsymbol{H}^{inc}\right]_{tan} &= 0 \quad \text{for} \quad \boldsymbol{r} \in S
\end{aligned}
$$

$$
\begin{aligned}
\left[\boldsymbol{E}_2^s(\boldsymbol{J}_s) + \boldsymbol{E}_2^s(\boldsymbol{M}_s)\right]_{tan} &= 0 \quad \text{for} \quad \boldsymbol{r} \in S \\
\left[\boldsymbol{H}_2^s(\boldsymbol{J}_s) + \boldsymbol{H}_2^s(\boldsymbol{M}_s)\right]_{tan} &= 0 \quad \text{for} \quad \boldsymbol{r} \in S
\end{aligned}
$$

Where

$$
\begin{aligned}
\boldsymbol{E}_{1,2}^s(\boldsymbol{J}_s) &= j\omega \boldsymbol{A}_{1,2} + \nabla \Phi_{1,2} \\
\boldsymbol{E}_{1,2}^s(\boldsymbol{M}_s) &= \pm \boldsymbol{a}_n \times \frac{\boldsymbol{M}_s}{2} + \nabla \times \boldsymbol{F}_{1,2} \\
\boldsymbol{H}_{1,2}^s(\boldsymbol{M}_s) &= j\omega \boldsymbol{F}_{1,2} + \nabla \Psi_{1,2} \\
\boldsymbol{H}_{1,2}^s(\boldsymbol{J}_s) &= \mp \boldsymbol{a}_n \times \frac{\boldsymbol{J}_s}{2} + \nabla \times \boldsymbol{A}_{1,2}
\end{aligned}
$$

SIE Formulations

- PMCHWT Formulation
- EFIE Formulation
- MFIE Formulation
- CFIE Formulation
- Muller Formulation

PMCHWT Formulation

$$
\begin{aligned}
[\boldsymbol{E}_1^s + \boldsymbol{E}_2^s]_{tan} &= -\boldsymbol{E}_{tan}^{inc} \quad \text{for} \quad \boldsymbol{r} \in S \\
[\boldsymbol{H}_1^s + \boldsymbol{H}_2^s]_{tan} &= -\boldsymbol{H}_{tan}^{inc} \quad \text{for} \quad \boldsymbol{r} \in S
\end{aligned}
$$

Resulting in

$$
\begin{aligned}
[j\omega\left(\boldsymbol{A}_1 + \boldsymbol{A}_2\right) + \nabla\left(\Phi_1 + \Phi_2\right) + \nabla \times \left(\boldsymbol{F}_1 + \boldsymbol{F}_2\right)]_{tan} &= \boldsymbol{E}_{tan}^{inc} \\
[-\nabla \times \left(\boldsymbol{A}_1 + \boldsymbol{A}_2\right) + j\omega\left(\boldsymbol{F}_1 + \boldsymbol{F}_2\right) + \nabla\left(\Psi_1 + \Psi_2\right)]_{tan} &= \boldsymbol{H}_{tan}^{inc}
\end{aligned}
$$

- ◆Expansion Procedure – Use RWG Functions for both J_s and M_s.
- ◆Testing Procedure – Use RWG Functions - similar to PEC Case.
- ◆Most Efficient Solution – Acceptable accuracy.
- ◆However, the same procedure fails for other formulations – Why?

EFIE Formulation

$$
\begin{aligned}
[\boldsymbol{E}_1^s(\boldsymbol{J}_s) + \boldsymbol{E}_1^s(\boldsymbol{M}_s)]_{tan} &= -\boldsymbol{E}_{tan}^{inc} \quad \text{for} \quad \boldsymbol{r} \in S \\
[\boldsymbol{E}_2^s(\boldsymbol{J}_s) + \boldsymbol{E}_2^s(\boldsymbol{M}_s)]_{tan} &= 0 \quad \text{for} \quad \boldsymbol{r} \in S
\end{aligned}
$$

Resulting in

$$
\begin{aligned}
\left[j\omega \boldsymbol{A}_1 + \nabla \Phi_1 + \boldsymbol{a}_n \times \frac{\boldsymbol{M}_s}{2} + \nabla \times \boldsymbol{F}_1 \right]_{tan} &= \boldsymbol{E}_{tan}^{inc} \\
\left[j\omega \boldsymbol{A}_2 + \nabla \Phi_2 - \boldsymbol{a}_n \times \frac{\boldsymbol{M}_s}{2} + \nabla \times \boldsymbol{F}_2 \right]_{tan} &= 0
\end{aligned}
$$

- Expansion Procedure – Let us assume RWG Functions for both J_s and M_s.
- Testing Procedure – Use RWG Functions

- What happens?

RWG Testing functions

$$\begin{pmatrix} \boldsymbol{Z}_E^1(\boldsymbol{J}_s) & \boldsymbol{Z}_E^1(\boldsymbol{M}_s) \\ \boldsymbol{Z}_E^2(\boldsymbol{J}_s) & \boldsymbol{Z}_E^2(\boldsymbol{M}_s) \end{pmatrix}$$

- $\boldsymbol{Z}_E^1(\boldsymbol{J}_s)$ and $\boldsymbol{Z}_E^2(\boldsymbol{J}_s)$ are well-conditioned Submatrices.
- $\boldsymbol{Z}_E^1(\boldsymbol{M}_s)$ and $\boldsymbol{Z}_E^2(\boldsymbol{M}_s)$ are poorly-conditioned Submatrices.

Overall Result – An inaccurate solution

a_nxRWG Testing

$$\begin{pmatrix} \boldsymbol{Z}_E^1(\boldsymbol{J}_s) & \boldsymbol{Z}_E^1(\boldsymbol{M}_s) \\ \boldsymbol{Z}_E^2(\boldsymbol{J}_s) & \boldsymbol{Z}_E^2(\boldsymbol{M}_s) \end{pmatrix}$$

- $\boldsymbol{Z}_E^1(\boldsymbol{J}_s)$ and $\boldsymbol{Z}_E^2(\boldsymbol{J}_s)$ are poorly conditioned Submatrices.
- $\boldsymbol{Z}_E^1(\boldsymbol{M}_s)$ and $\boldsymbol{Z}_E^2(\boldsymbol{M}_s)$ are well-conditioned Submatrices.

Again, inaccurate Solution

RWG + a_nxRWG Testing

- $\boldsymbol{Z}_E^1(\boldsymbol{J}_s)$ and $\boldsymbol{Z}_E^2(\boldsymbol{J}_s)$ are well conditioned Submatrices.
- $\boldsymbol{Z}_E^1(\boldsymbol{M}_s)$ and $\boldsymbol{Z}_E^2(\boldsymbol{M}_s)$ are well-conditioned Submatrices.

Or

- $\boldsymbol{Z}_E^1(\boldsymbol{J}_s)$ and $\boldsymbol{Z}_E^2(\boldsymbol{J}_s)$ are poorly-conditioned Submatrices.
- $\boldsymbol{Z}_E^1(\boldsymbol{M}_s)$ and $\boldsymbol{Z}_E^2(\boldsymbol{M}_s)$ are poorly-conditioned Submatrices.

- The Problem would be true for all other formulations – MFIE, CFIE, Muller formulations.

- The Problem can happen for other situations – Apertures in a body and composite surfaces.

- Far-fields may be acceptable with dense grids.

- However, Near-fields are questionable.

Experimentation with Expansion Schemes

- Use Two separate functions to expand $\mathbf{J}_s$ and $\mathbf{M}_s$.
- Preferably, these two functions should be spatially orthogonal to each other.
- Use same functions (or some approximations) for testing.

Method # 1

- Use RWG functions for $\mathbf{J}_s$.
- Use a_nxRWG for $\mathbf{M}_s$.
- Use RWG functions for Testing.
- The EFIE Solution for this case is straightforward.

$$\left[j\omega \boldsymbol{A}_1 + \nabla\Phi_1 + \boldsymbol{a}_n \times \frac{\boldsymbol{M}_s}{2} + \nabla \times \boldsymbol{F}_1 \right]_{tan} = \boldsymbol{E}^{inc}_{tan}$$

$$\left[j\omega \boldsymbol{A}_2 + \nabla\Phi_2 - \boldsymbol{a}_n \times \frac{\boldsymbol{M}_s}{2} + \nabla \times \boldsymbol{F}_2 \right]_{tan} = 0$$

$$\begin{pmatrix} \boldsymbol{Z}^1_E(\boldsymbol{J}_s) & \boldsymbol{Z}^1_E(\boldsymbol{M}_s) \\ \boldsymbol{Z}^2_E(\boldsymbol{J}_s) & \boldsymbol{Z}^2_E(\boldsymbol{M}_s) \end{pmatrix}$$

- For MFIE Solution, one requires to compute the divergence of M_s.
- Use a_nxRWG functions for Testing.

$$
\begin{aligned}
\left[-\boldsymbol{a}_n \times \frac{\boldsymbol{J}}{2} - \nabla \times \boldsymbol{A}_1 + j\omega \boldsymbol{F}_1 + \nabla \Psi_1\right]_{tan} &= \boldsymbol{H}_{tan}^{inc} \\
\left[\boldsymbol{a}_n \times \frac{\boldsymbol{J}}{2} - \nabla \times \boldsymbol{A}_2 + j\omega \boldsymbol{F}_2 + \nabla \Psi_2\right]_{tan} &= 0
\end{aligned}
$$

- Note that a_nxRWG functions have discontinuous derivatives.
- But can be handled in the following way.

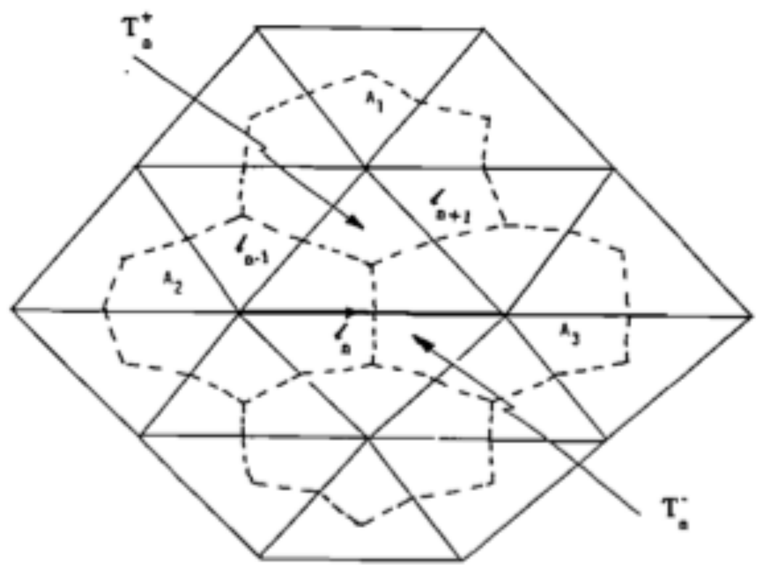
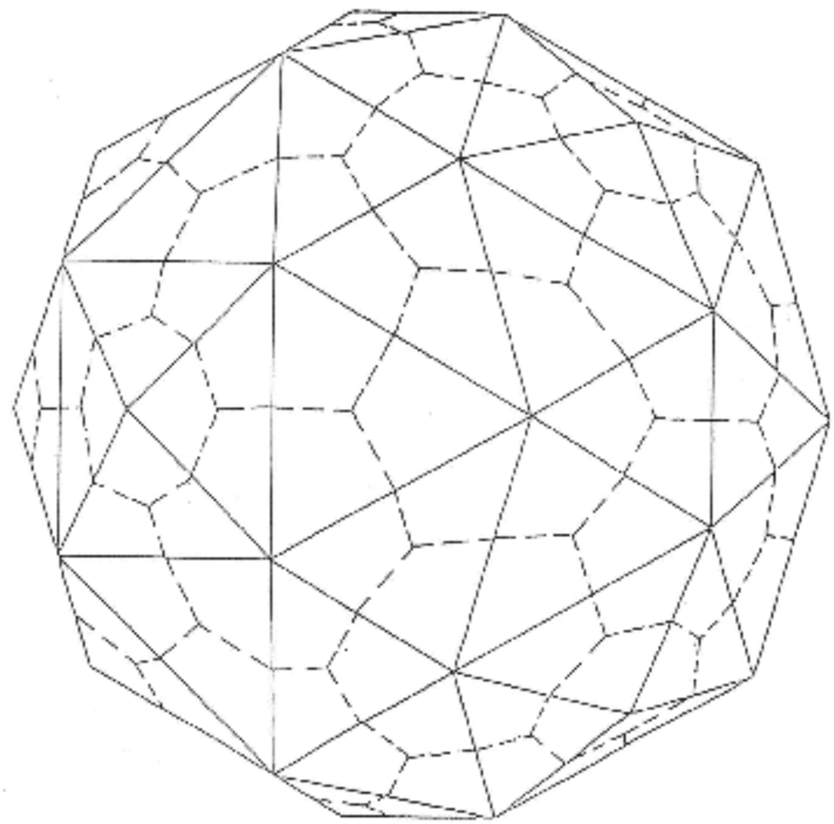


FIGURE 6. Magnetic charge patches associated with the nth edge.

Another Scheme (Method #2)

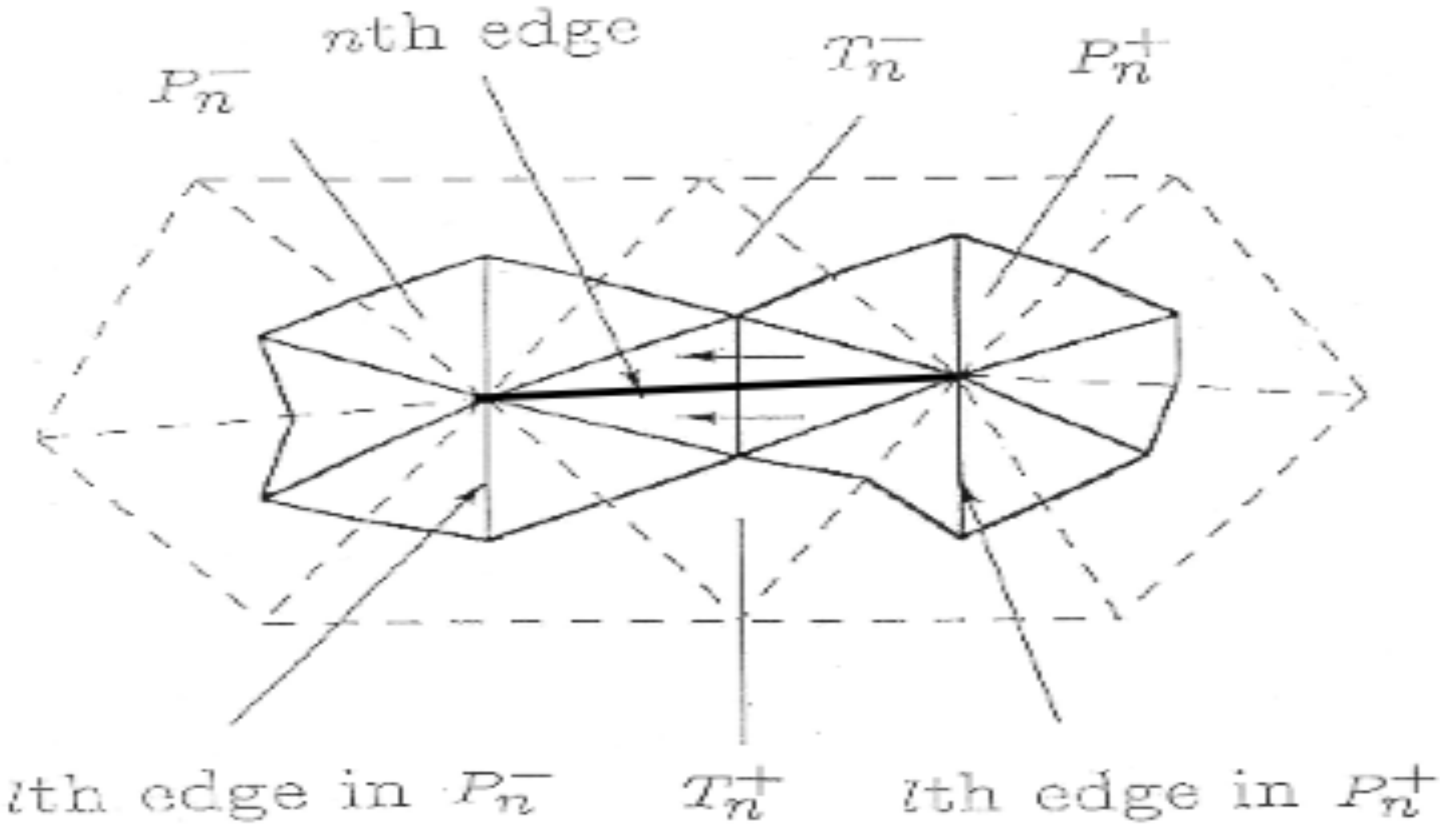
- Use the conventional triangulation scheme RWG functions to expand the electric current J_{S}.
- Develop a dual grid and polygonal basis functions to expand M_s.

Dual Grid



A sphere surface modeled by triangular and corresponding polygonal patches.

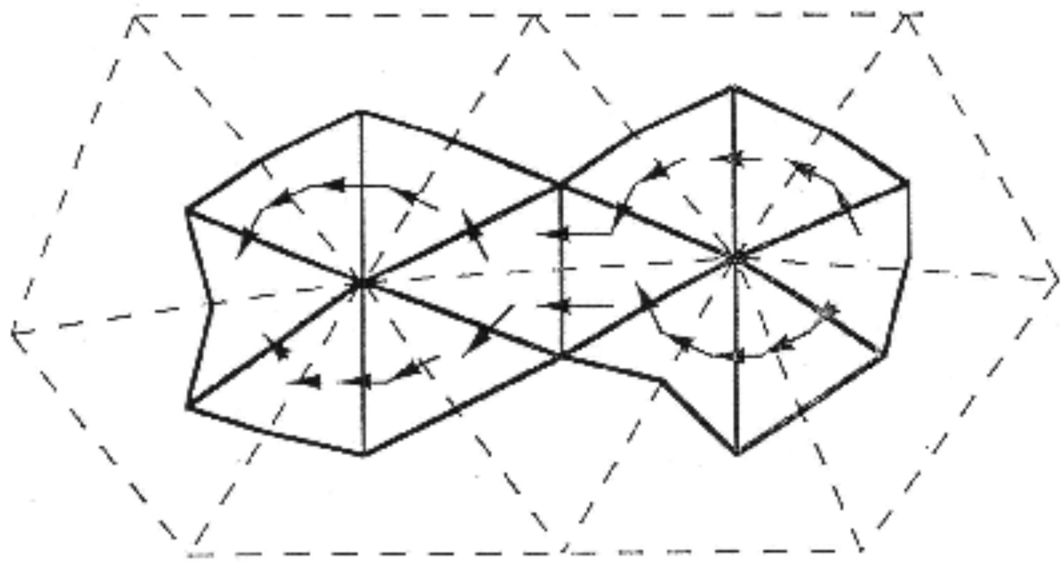
Polynomial Basis Functions



Mathematical Representation

$$\Lambda_n(\boldsymbol{r}) = \begin{cases} \displaystyle\sum_{\ell=0}^{L_n^\pm} C_{n\ell}^\pm \Lambda_{n\ell}^\pm(\boldsymbol{r}) & \text{for } \boldsymbol{r} \in P_n^\pm \\ 0 & \text{otherwise} \end{cases}$$

Current flowing in the Polygon Pair



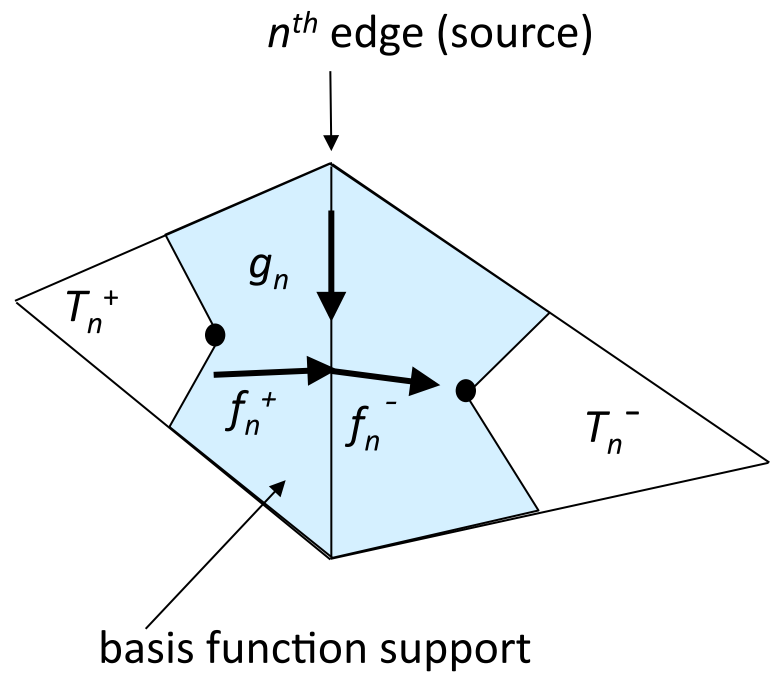
Yet Another Scheme (Method # 3)

Pulse-Like Functions on Triangle Pair

$$\hat{\boldsymbol{n}} \times \boldsymbol{g_n} = \boldsymbol{f_n}$$

$\hat{\boldsymbol{n}} =$ outward unit normal

$$\left|\boldsymbol{f_n^+}\right| = \left|\boldsymbol{f_n^-}\right| = \left|\boldsymbol{g_n}\right| = 1$$



$$\boldsymbol{J_n} = I_n\left(\boldsymbol{f_n^+} + \boldsymbol{f_n^-}\right)$$

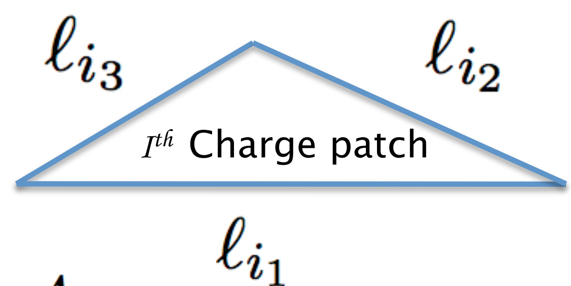
$$\boldsymbol{M_n} = I_{N+n}\boldsymbol{g_n}$$

Charge Patches for J

Let N_p be the total number of triangles, and let

$$q_s = \sum_{i=1}^{N_p} \alpha_i P_i \qquad \text{Where} \quad P_i(\boldsymbol{r}) = \begin{cases} 1, & \boldsymbol{r} \in T_i \\ 0, & \text{otherwise} \end{cases}$$

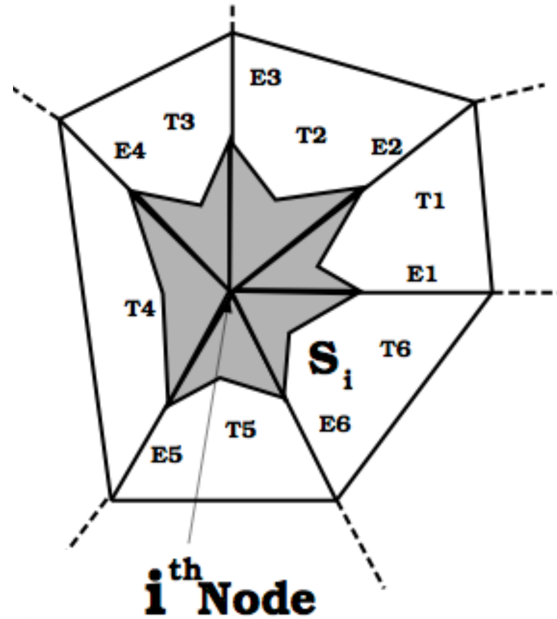
$$\begin{aligned} \int_{T_i} q_s ds &= \int_{T_i} \frac{\nabla_s \bullet \boldsymbol{J}}{-j\omega} ds \\ &= \frac{j}{\omega} \oint_{C_i} \boldsymbol{J} \bullet \boldsymbol{dl} \\ &= \frac{j}{\omega} [I_{i_1} \ell_{i_1} + I_{i_2} \ell_{i_2} + I_{i_3} \ell_{i_3}] \quad = \quad \alpha_i A_i \end{aligned}$$



$$\alpha_i = \frac{j}{\omega} \left[\frac{I_{i_1} \ell_{i_1} + I_{i_2} \ell_{i_2} + I_{i_3} \ell_{i_3}}{A_i} \right]$$

Charge Patches for M

Let $$q_S = \sum_{i=1}^{N_n} \alpha_i P_i \qquad P_i(\boldsymbol{r}) = \begin{cases} 1, & \boldsymbol{r} \in S_i \\ 0, & \text{otherwise.} \end{cases}$$



$$\int_{S_i} q_S \, dS = \int_{S_i} \frac{\nabla_s \bullet \boldsymbol{J}}{-j\omega} dS$$

$$= \frac{j}{\omega} \oint_{C_i} \boldsymbol{J} \bullet n_\ell \, d\ell$$

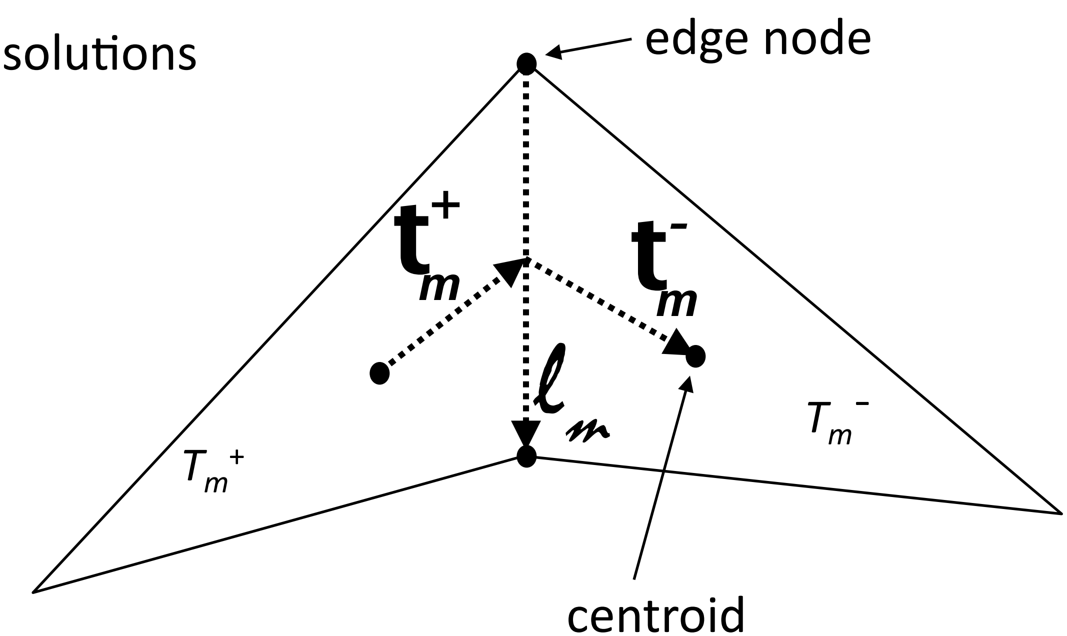
$$= \frac{j}{\omega} \sum_{j=1}^{E_K} I_{ij} [\boldsymbol{\ell}_{ij} \bullet (\boldsymbol{r}_{ij}^{c+} \times n_{ij}^{+} + (\boldsymbol{r}_{ij}^{c-} \times n_{ij}^{-})]$$

$$\alpha_i = \frac{j}{\omega A_{Si}} \sum_{j=1}^{E_K} I_{ij} [\boldsymbol{\ell}_{ij} \bullet (\boldsymbol{r}_{ij}^{c+} \times n_{ij}^{+} + (\boldsymbol{r}_{ij}^{c-} \times n_{ij}^{-})]$$

Testing Vectors

$\mathbf{t}_m^+$, $\mathbf{t}_m^-$: EFIE solutions

ℓ_m : HFIE solutions



An Alternate Formulation

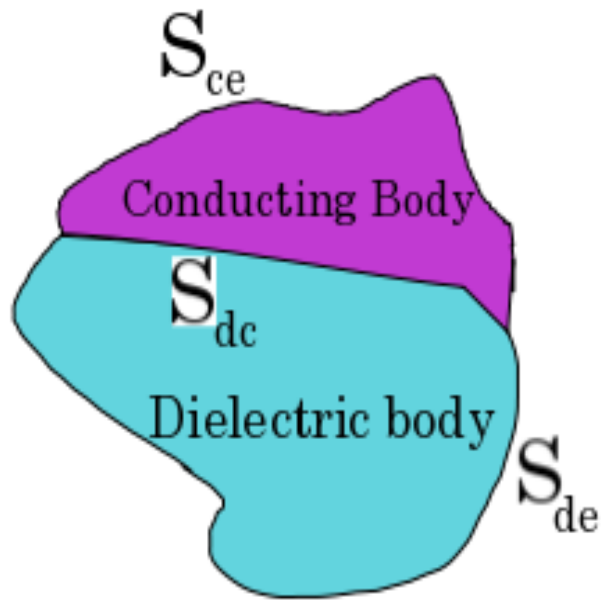
- Consider the following Composite Body Problem

•On surface S_{ce}---------- Only J_{ce}.

•On surface S_{de}-----------J_{de} and M_{de}.

•On surface S_{dc}------------Only J_{dc}.

•Use PMCHWT Formulation.



•OK for far-field calculations.

•For near-field quantities, extra work is needed to obtain physical currents from equivalent currents.

Some Observations

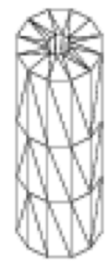
- Dielectric Body Problem is more complicated than PEC problem.
- If RWG functions are used for expansion and testing, one must be very careful while applying the numerical procedures.
- Ideally, it is recommended to use two spatially orthogonal functions in the numerical scheme.
- It is possible to get acceptable far-field quantities using only RWG functions.
- While using software packages, one needs to know how the dielectric materials are treated.

Water meter antenna

Radiating Element

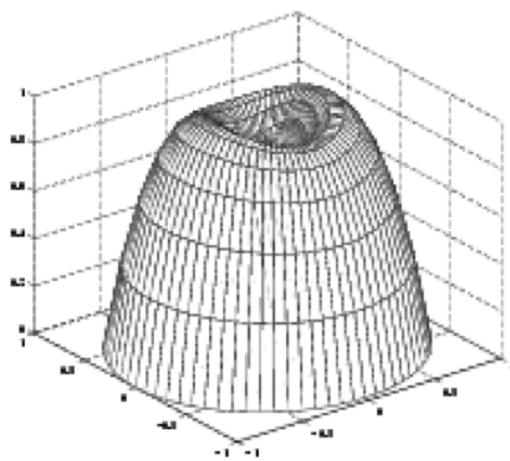
(a)

Feed



(b)

Insulator



Radiation Pattern

Time Domain Problems

- Here we solve the scattering problem directly in time-domain – Useful for Impulse radar, Wideband solutions, and signature studies.
- No matrix inversion – solution is obtained iteratively.

Time Domain Electromagnetics

Academic Press, 2001

Consider the following problem:

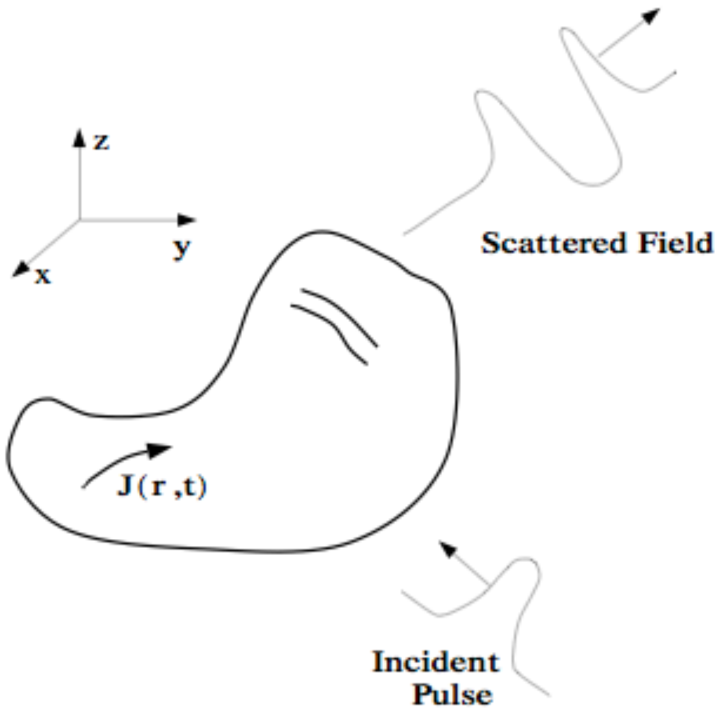
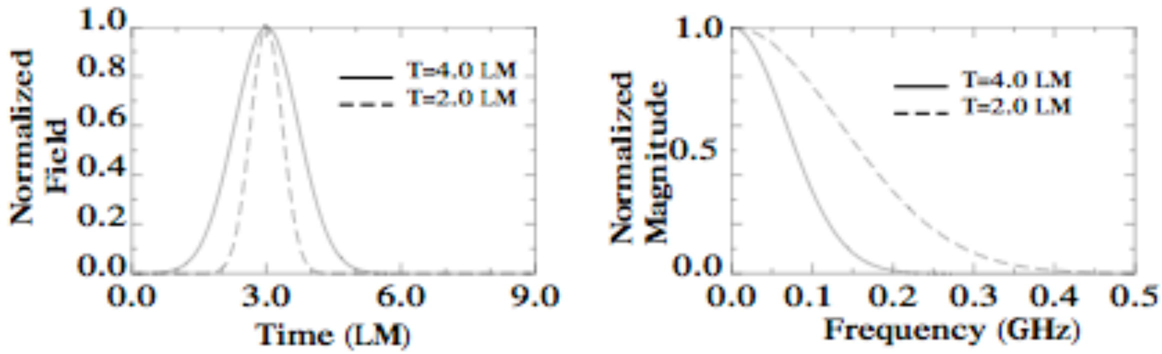


Figure 1: Transient pulse incident upon an arbitrarily shaped body.

Direct Time Domain Solutions are better suited to:

- Short-pulse radar systems.
- EMP studies.
- Provide better visualization.
- Provides opportunity to observe and interpret scattering behavior.
- Provides broadband information.

Incident Field



(a) (b)

$$\boldsymbol{E}^i(\boldsymbol{r},t) = \boldsymbol{E}_o \frac{4}{T\sqrt{\pi}} \mathrm{e}^{-\gamma^2}$$

where

$$\gamma = \frac{4}{T}(ct - ct_o - \boldsymbol{r}\cdot\boldsymbol{a}_k),$$

Time Domain EFIE

$$\left[\frac{\partial \boldsymbol{A}(\boldsymbol{r},t)}{\partial t} + \nabla\Phi(\boldsymbol{r},t)\right]_{tan} = \boldsymbol{E}^{i}_{tan}(\boldsymbol{r},t)$$

where

$$\begin{aligned} \boldsymbol{A}(\boldsymbol{r},t) &= \mu \int_S \frac{\boldsymbol{J}(\boldsymbol{r}',t-\frac{R}{c})}{4\pi R}\,dS' \ , \\ \Phi(\boldsymbol{r},t) &= \frac{1}{\epsilon}\int_S \frac{q_s(\boldsymbol{r}',t-\frac{R}{c})}{4\pi R}\,dS' \ , \end{aligned}$$

Final Equation

$$
\begin{aligned}
&[\boldsymbol{\kappa}_{mm} \cdot \frac{l_m}{2}(\boldsymbol{\rho}_m^{c+} + \boldsymbol{\rho}_m^{c-})]I_m(t_j) = \\
&\frac{l_m}{2}(\boldsymbol{\rho}_m^{c+} + \boldsymbol{\rho}_m^{c-}) \cdot (\Delta t)\boldsymbol{E}^i(\boldsymbol{r}_m, t_{j-1}) \\
&-\frac{l_m}{2}(\boldsymbol{\rho}_m^{c+} + \boldsymbol{\rho}_m^{c-}) \cdot \left[\cancel{\boldsymbol{A}}(\boldsymbol{r}_m, t_j) - \boldsymbol{A}(\boldsymbol{r}_m, t_{j-1}\right] \\
&+l_m(\Delta t)[\Phi(\boldsymbol{r}_m^{c+}, t_{j-1}) - \Phi(\boldsymbol{r}_m^{c-}, t_{j-1})].
\end{aligned}
$$

Lastly, we can write Eq. (44) in matrix form as

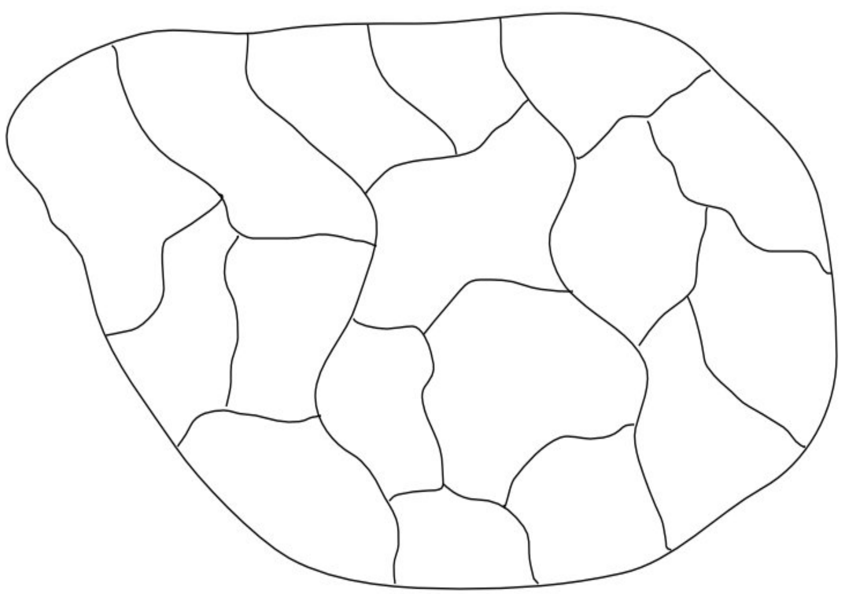
$$
[\boldsymbol{\alpha}][\boldsymbol{I}(t_j)] = [\boldsymbol{F}(t_j)] + [\boldsymbol{\beta}][\boldsymbol{I}(t_R)].
$$

Conclusions

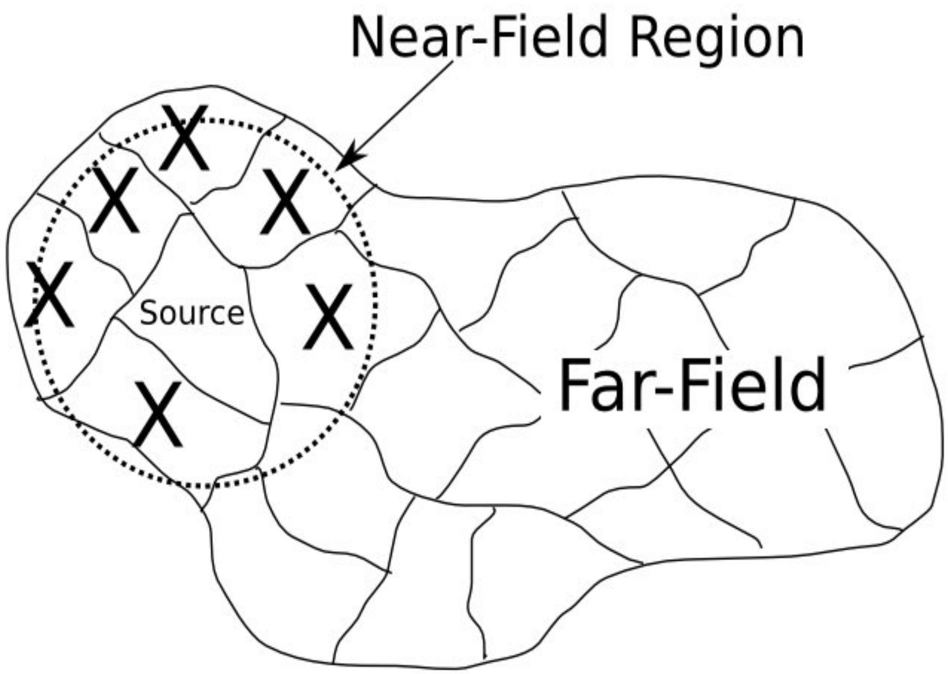
- RWG functions have been used for a variety of problems in numerical electromagnetic problems.
- Also used in other areas – Acoustic scattering.
- New improvements include: developing faster solutions (FMM), adaptive basis functions to generate sparse moment matrix (Killian and Rao, IEEE Transactions on A&P, 2011), Domain Decomposition to handle large problems, and Adaptive Cross Approximation (Mercury MoM).

New domain Decomposition Method

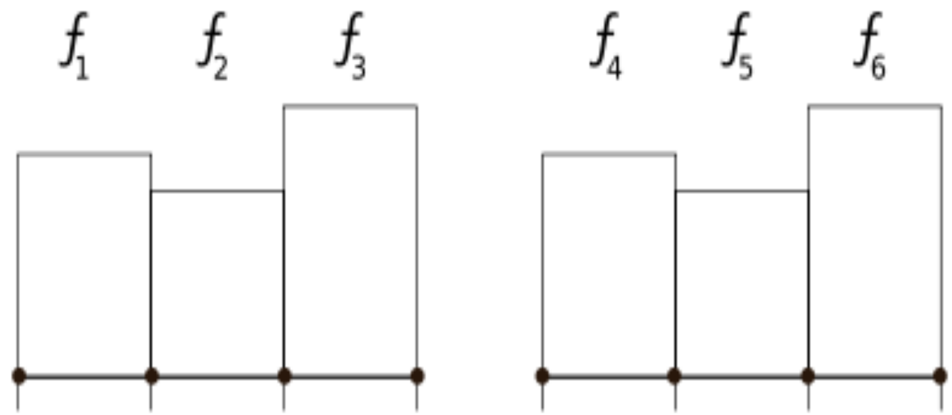
- Domain Decomposition – Disjoint groups of sub-domain functions
- Functions in a group are geometrically close to one another
- Each function belongs to one and only one group.



- Decouple a given group from other groups- Can be accomplished by generating new set of basis functions.
- It is possible to solve each group separately and obtain the total solution.



Example – Two 2D strips



$$
Z = \begin{bmatrix} \begin{bmatrix} Z_{1,1} & Z_{1,2} & Z_{1,3} \\ Z_{2,1} & Z_{2,2} & Z_{2,3} \\ Z_{3,1} & Z_{3,2} & Z_{3,3} \end{bmatrix} & \begin{bmatrix} Z_{1,4} & Z_{1,5} & Z_{1,6} \\ Z_{2,4} & Z_{2,5} & Z_{2,6} \\ Z_{3,4} & Z_{3,5} & Z_{3,6} \end{bmatrix} \\ \begin{bmatrix} Z_{4,1} & Z_{4,2} & Z_{4,3} \\ Z_{5,1} & Z_{5,2} & Z_{5,3} \\ Z_{6,1} & Z_{6,2} & Z_{6,3} \end{bmatrix} & \begin{bmatrix} Z_{4,4} & Z_{4,5} & Z_{4,6} \\ Z_{5,4} & Z_{5,5} & Z_{5,6} \\ Z_{6,4} & Z_{6,5} & Z_{6,6} \end{bmatrix} \end{bmatrix}
= \begin{bmatrix} [Z_{G_1G_1}] & [Z_{G_1G_2}] \\ [Z_{G_2G_1}] & [Z_{G_2G_2}] \end{bmatrix}
$$

New basis function:

$$
\begin{aligned}
g_1 &= f_1 + \alpha_1 f_4 + \beta_1 f_5 + \gamma_1 f_6 \\
g_2 &= f_2 + \alpha_2 f_4 + \beta_2 f_5 + \gamma_2 f_6 \\
g_3 &= f_3 + \alpha_3 f_4 + \beta_3 f_5 + \gamma_3 f_6
\end{aligned}
$$

Criteria (g1) :

$$
\begin{aligned}
Z_{4,1} + \alpha_1 Z_{4,4} + \beta_1 Z_{4,5} + \gamma_1 Z_{4,6} &= 0 \\
Z_{5,1} + \alpha_1 Z_{5,4} + \beta_1 Z_{5,5} + \gamma_1 Z_{5,6} &= 0 \\
Z_{6,1} + \alpha_1 Z_{6,4} + \beta_1 Z_{6,5} + \gamma_1 Z_{6,6} &= 0
\end{aligned}
$$

$$\begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix} = -\begin{bmatrix} Z_{4,4} & Z_{4,5} & Z_{4,6} \\ Z_{5,4} & Z_{5,5} & Z_{5,6} \\ Z_{6,4} & Z_{6,5} & Z_{6,6} \end{bmatrix}^{-1} \begin{bmatrix} Z_{4,1} \\ Z_{5,1} \\ Z_{6,1} \end{bmatrix}$$

Solve for weights

$$\tilde{Z}_{2,1} = Z_{2,1} + \alpha_1 Z_{2,4} + \beta_1 Z_{2,5} + \gamma_1 Z_{2,6}$$

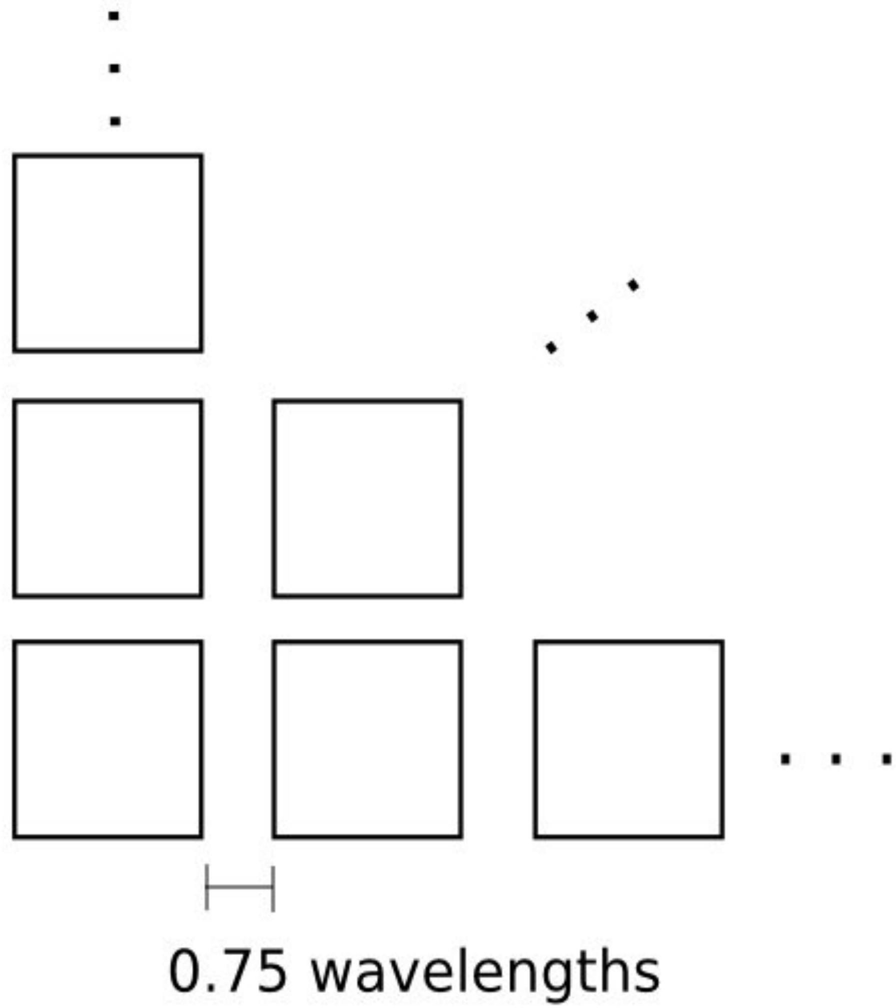
Create new matrix

$$\tilde{Z} = \begin{bmatrix} \tilde{Z}_{G_1G_1} & 0 \\ 0 & \tilde{Z}_{G_2G_2} \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 & \alpha_4 & \alpha_5 & \alpha_6 \\ 0 & 1 & 0 & \beta_4 & \beta_5 & \beta_6 \\ 0 & 0 & 1 & \gamma_4 & \gamma_5 & \gamma_6 \\ \alpha_1 & \alpha_2 & \alpha_3 & 1 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 & 0 & 1 & 0 \\ \gamma_1 & \gamma_2 & \gamma_3 & 0 & 0 & 1 \end{bmatrix}$$

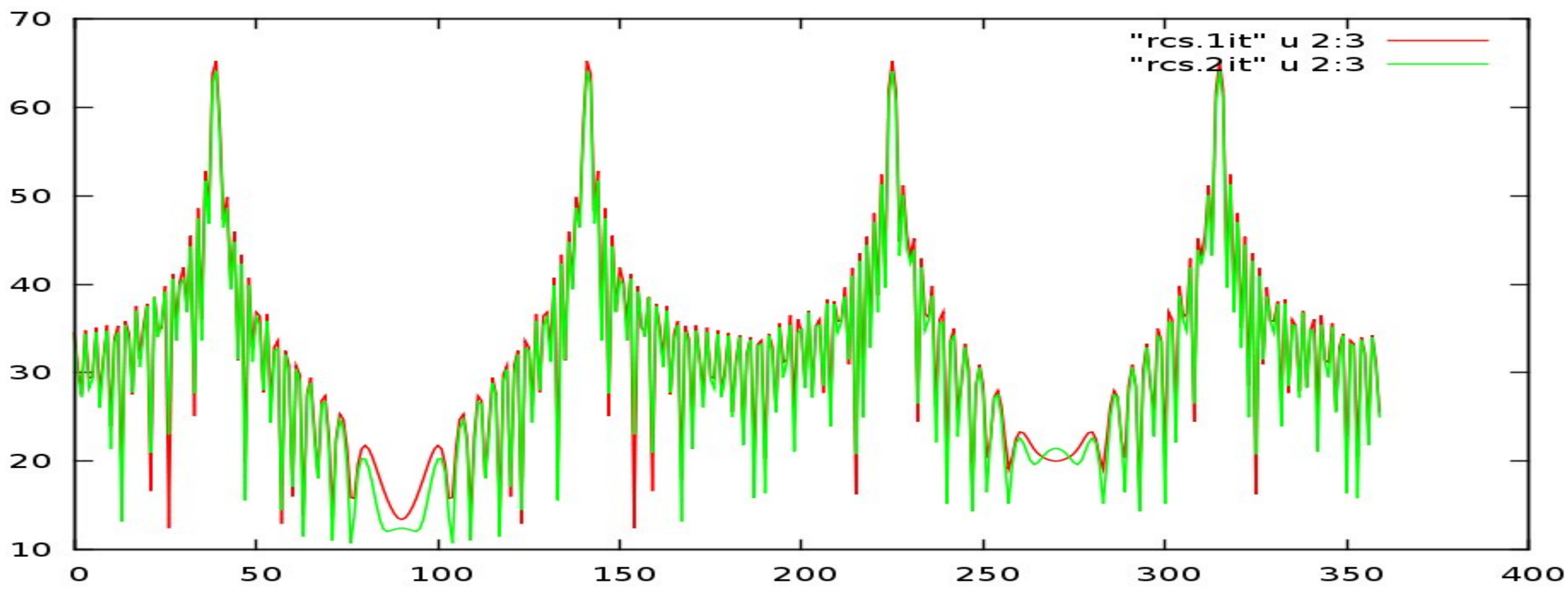
3D Results – Finite Planar Array

- Finite Periodic Array
- EFIE
- 50 x 50 grid of
  0.5 lambda x 0.5 lambda plates
- 64 unknowns per plate
- 0.75 lambda spacing
- 160,000 total unknowns
- Null fields produced
  on adjacent plates
- Redundant coefficients
- Eth = 120pi; Ephi = 0
- Theta = 45 deg Phi = 0 deg



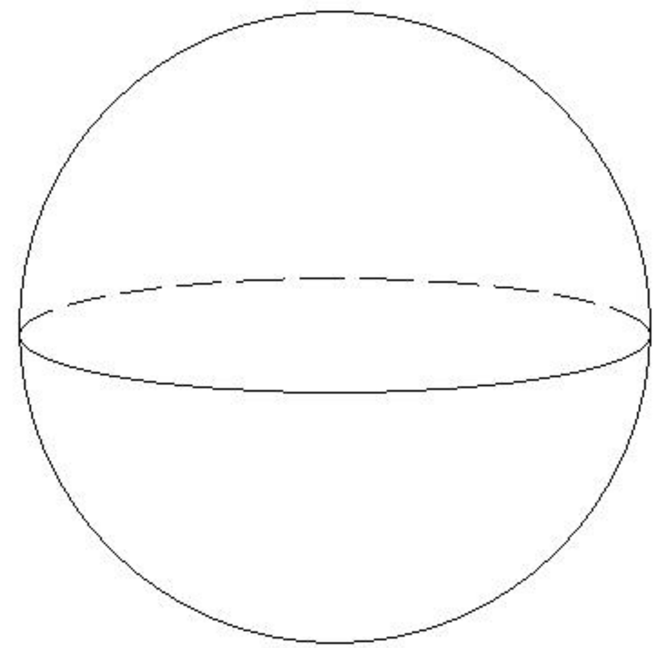
3D Results – Planar Array

- 1 iteration = 0.288 average error per term
- 2 iterations = 0.122 average error per term
- ~19.5 hours wall clock time with 8 CPUs (includes time for RCS calculation on single CPU)
- Matrix approximations can be used for speedup



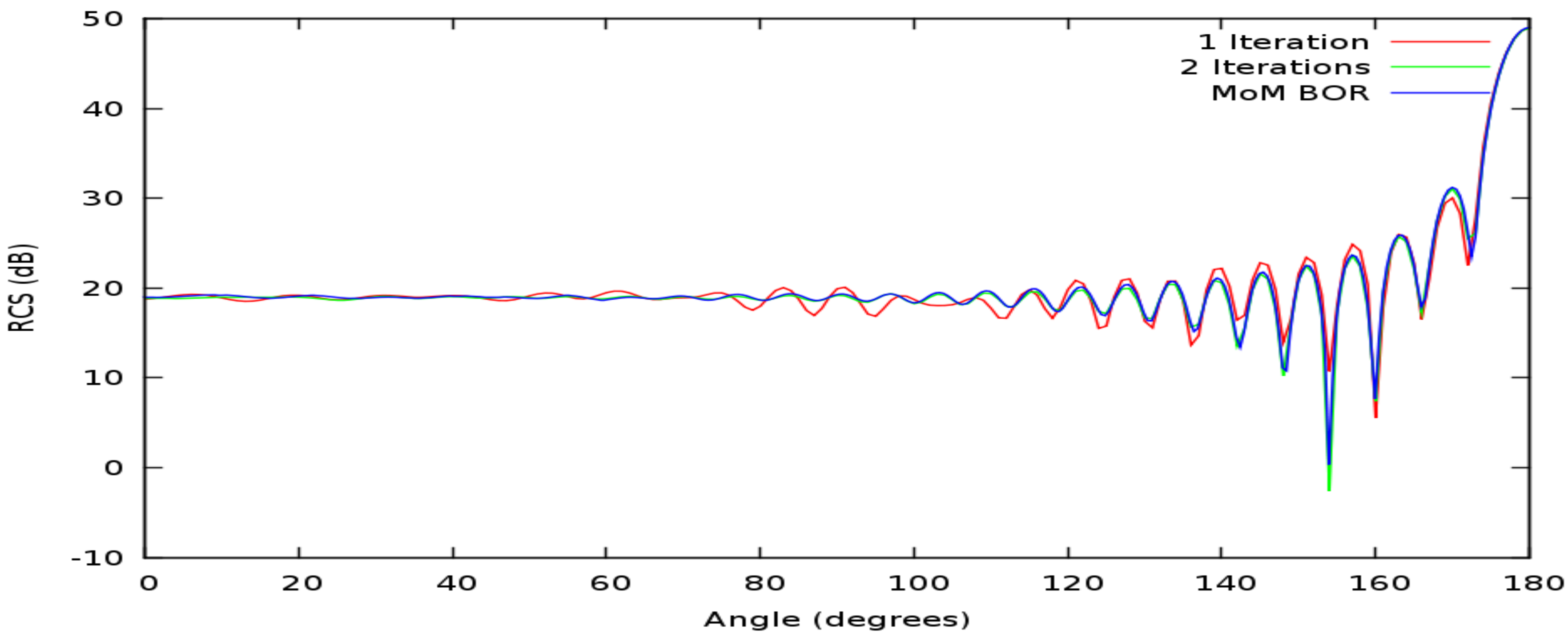
3D Results - Sphere

- 5 lambda radius
- CFIE
- 92550 unknowns
- 314 Groups – each roughly 1 lambda^2 in surface area
- Null fields produced on groups within 2 lambda radius (typically around 3000 points)
- ~ 2.5 GB storage
- Eth = 120pi; Ephi = 0
- Theta = 45 deg Phi = 0 deg



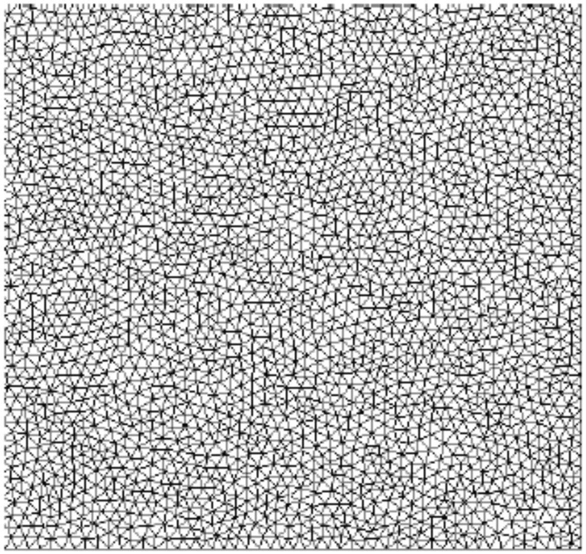
3D Results – Sphere

- 1 iteration = 0.096 average error per term
- 2 iterations = 0.014 average error per term
- ~26 hours wall clock time with 8 CPUs (includes time for RCS calculation on single CPU)
- Matrix approximations can be used for speedup



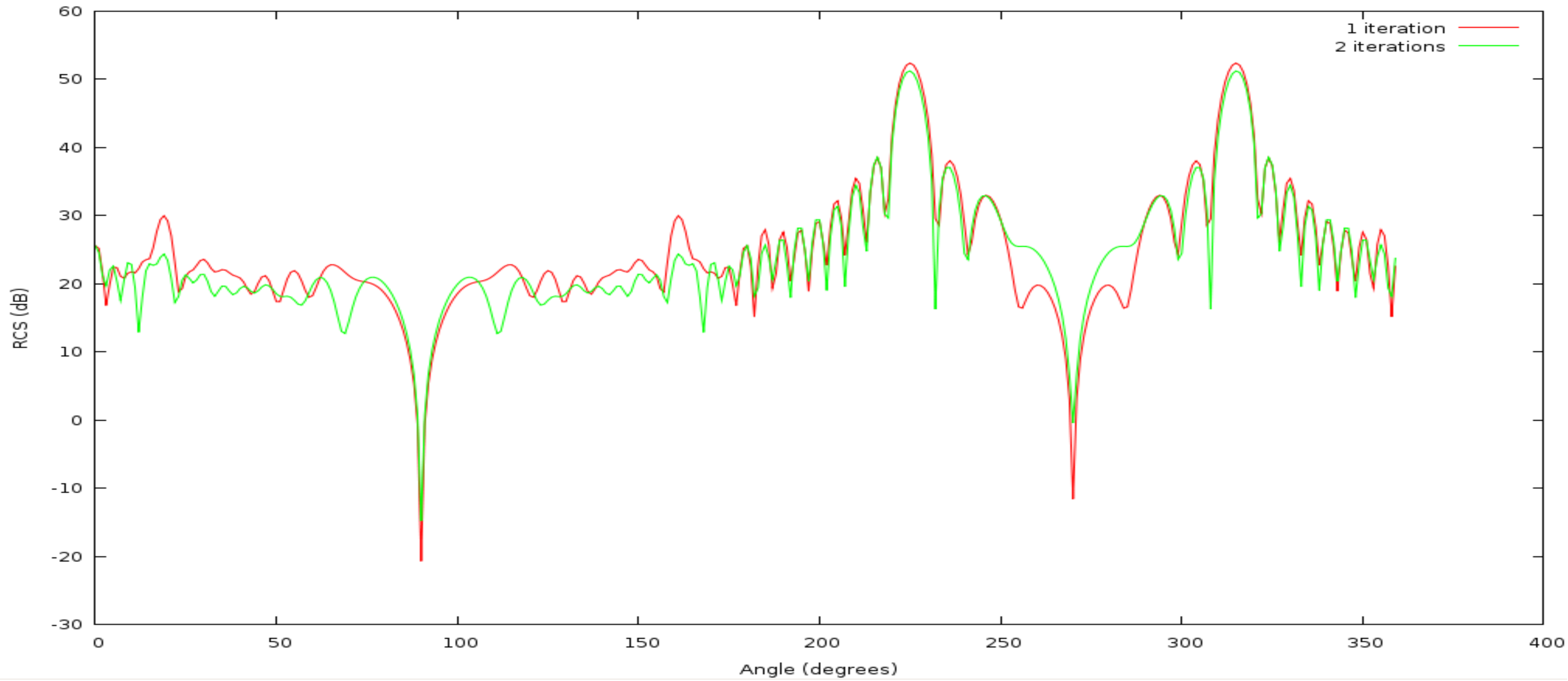
3D Results – Square Plate

- 12 λ x 12 λ square plate
- EFIE
- 42883 unknowns
- 144 Groups – each roughly 1 λ^2 in surface area
- Null fields produced on groups within 2 lambda radius (typically around 2800 points)
- ~ 185 MB storage
- Eth = 120pi; Ephi = 0
- Theta = 45 deg Phi = 0 deg



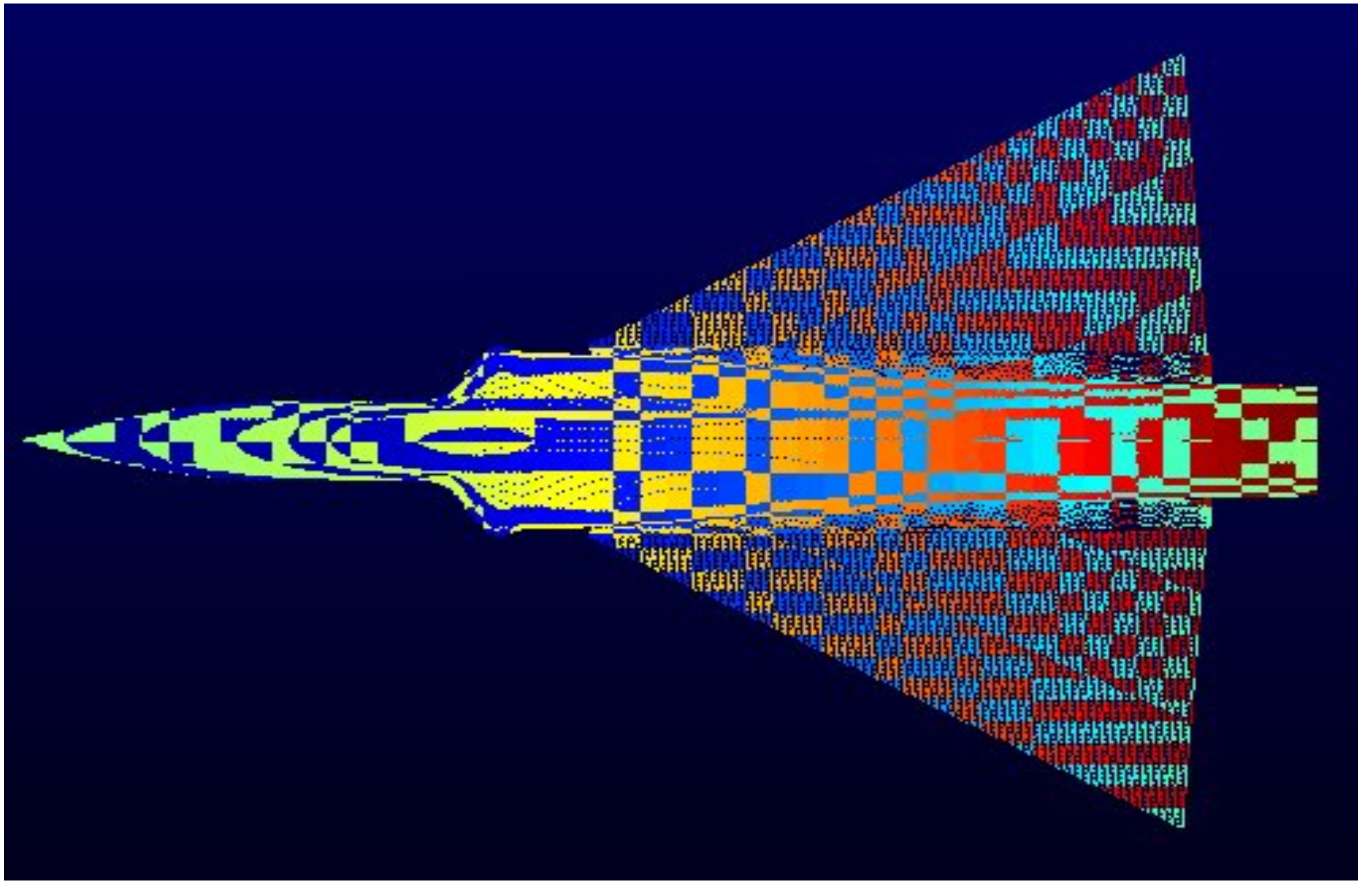
3D Results – Square Plate

- 1 iteration = 0.612 average error per term
- 2 iterations = 0.232 average error per term
- ~ 2 hours 45 mins wall clock time with 8 CPUs (includes time for RCS calculation on single CPU)



3D Results - Aircraft

- French Mirage
- ~ 160,000 unknowns
- Patches represent groups.



3D Results – Aircraft

- 1 iteration = 0.224 average error per term
- 2 iterations = 0.218 average error per term
- ~ 5 days 7 hours wall clock time with 8 CPUs (includes time for RCS calculation on single CPU)