Electromagnetic Scattering from Arbitrary Bodies Using Rao-Wilton-Glisson (RWG) Functions

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Arbitrarily-Shaped Body

• Arbitrary shape.
• Arbitrary material composition.
• Cavities, cables, and apertures.
• Single or multiple bodies.
• Intersecting surfaces.
• Periodic structures.
• Finite structures.
Numerical Solution Procedure

1. Describe Geometry to the computer- Planar triangular patch modeling.
2. Transform the Mathematical Equations into Matrix equation via Method of Moments.
3. Solve the Matrix equation.
4. Post-processing.
Triangulated Models
Method of Moments Solution

\[ AX = Y \] (4)

\[ X = \sum_{i=1}^{N} \alpha_i p_i \] (5)

\[ \sum_{i=1}^{N} \alpha_i A p_i = Y \] (6)

\[ \sum_{i=1}^{N} \alpha_i < q_j, A p_i > = < q_j, Y > \quad j = 1, 2, \cdots, N \] (7)

\[ \Rightarrow Z X = Y \] (8)
Applications: Electrostatic Discharge (ESD)

Method: Calculate charge distribution

\[
\int_S \frac{q_x(r')}{|r-r'|} dS' = 4\pi\epsilon_0 V_o \quad r_c S
\]

\[
\Rightarrow \sum_{i=1}^N Q_i \int_{q_i} \frac{dS'}{|r_i^e - r'|} = 4\pi\epsilon_0 V_{o_i}
\]

\[
j = 1, 2, \cdots, N
\]

\[
\Rightarrow [Z] [Q] = [V]
\]

• Electrostatic Discharge on Multi-Conductor Transmission Lines – IEEE Trans. MTT 1984

• Power-line hazard analysis - IEEE Trans. MTT 1985

• Characterization of Cross talk problem in VLSI design - IEEE Trans. MTT 1998

• Nuclear EMP Studies
Electrodynamics Problems

Arbitrary PEC Body excited by a Plane wave

**EFIE**

\[ E_{\text{tan}}^{\text{total}} = 0 \text{ for } r \in S \]

\[ \Rightarrow E_{\text{tan}}^{\text{inc}} + E_{\text{tan}}^{\text{sc}} = 0 \]

\[ \Rightarrow [j \omega A + \nabla \Phi]_{\text{tan}} = E_{\text{tan}}^{\text{inc}} \text{ for } r \in S \]

**MFIE**

\[ J = a_n \times H_{\text{tan}}^{\text{total}} \text{ for } r \in S \]

\[ \Rightarrow J = a_n \times (H^{\text{inc}} + H^{\text{sc}}) \]

\[ \Rightarrow J - a_n \times H^{\text{sc}} = a_n \times H^{\text{inc}} \text{ for } r \in S \]
Approximate the unknown current density using RWG (Rao-Wilton-Glisson) functions.

\[ J = \sum I_n f_n \]

\[ f_n = \begin{cases} 
\frac{l_n}{2A_n^+} \rho_n^+ & r \in T_n^+ \\
\frac{l_n}{2A_n^-} \rho_n^- & r \in T_n^- \\
0 & \text{otherwise}
\end{cases} \]
Method of Moments Solution Procedure

• Transform the operator equation into matrix equation using testing functions – Also RWG functions.

• Solve the matrix equation.

Testing Functions
Example of the currents on an Aircraft at 300 MHz with 2900 unknowns
Base Station Antenna Design

- 1 element placed at the center of the cylindrical ground plane.
- 16 elements distributed uniformly (45° apart) on the circumference of 2 concentric rings.
- Two driver elements shown with red ports.
- 17 elements in all.

Figure: Geometry of the array antenna.
Geometry of antenna

- 9 elements
- Driver elements are present in the inner circle and they are 135° apart.
- The elements in inner circle act as directors.
- Elements in the outer circle act as reflectors.
- Hollow cylinder acts as ground plane.

Figure: Geometry of the antenna with only essential elements.
Elements 1 and 4 are excited
8, 16, 17, 12, 9, 5, 13 are grounded
and the remaining elements are removed from the system.
(open circuited)
Far Field Gain in Horizontal Plane
Far Field Gain in Horizontal Plane
Fabrication of Antenna

- The antenna was fabricated to operate at 2GHz frequency.

- Copper sheets and rods of optimized dimensions were used for construction.

- A power splitter was designed to split the power equally between two driver elements.

Figure: Top view of the fabricated antenna
Fabricated antenna

- $hc = 1.1811$ inches (Height of the cylindrical ground plane)
- $rc = 3.3070$ inches (Radius of the cylindrical ground plane)
- $hi = 1.5029$ inches (Height of elements on inner circle)
- $ho = 4.7238$ inches (Height of elements on outer circle)
- $horg = 4.7096$ inches (Height of element at origin)
- $ri = 1.5750$ inches (Radius of inner circle)
- $ro = 3.1500$ inches (Radius of outer circle)
- $rr = 0.0625$ inches (Radius of antenna elements)

Figure: Side view of fabricated antenna
Comparison of the Radiation Patterns

Figure: The simulated and measured radiation patterns of antenna
• FERM – Lincoln Labs – This code is classified
• PATCH – Sandia National Labs – This code is classified
• IE3D – Commercial Code
• FEKO – Commercial Code
• CARLOS3D – McDonnell Douglas
Surface Formulation of Integral Equations For a Dielectric Body

\[ E_1, H_1, \mu_1, \varepsilon_1 \]

\[ E_2, H_2, \mu_2, \varepsilon_2 \]

\[ E^{inc}, H^{inc} \]
External Equivalent Problem

\[ E^S, H^S \]

\[ E^{inc}, H^{inc} \]

\[ \varepsilon_1, \mu_1 \quad S \]

\[ (0,0) \]

\[ M_s \quad J_s \]

equivalent surface
Internal Equivalent Problem
Four Governing Equations

\[
\begin{align*}
\left[ E_1^s(J_s) + E_1^s(M_s) + E^{inc} \right]_{tan} &= 0 \quad \text{for } r \in S \\
\left[ H_1^s(J_s) + H_1^s(M_s) + H^{inc} \right]_{tan} &= 0 \quad \text{for } r \in S \\
\left[ E_2^s(J_s) + E_2^s(M_s) \right]_{tan} &= 0 \quad \text{for } r \in S \\
\left[ H_2^s(J_s) + H_2^s(M_s) \right]_{tan} &= 0 \quad \text{for } r \in S
\end{align*}
\]

Where

\[
\begin{align*}
E_{1,2}^s(J_s) &= j\omega A_{1,2} + \nabla \Phi_{1,2} \\
E_{1,2}^s(M_s) &= \pm a_n \times \frac{M_s}{2} + \nabla \times F_{1,2} \\
H_{1,2}^s(M_s) &= j\omega F_{1,2} + \nabla \Psi_{1,2} \\
H_{1,2}^s(J_s) &= \mp a_n \times \frac{J_s}{2} + \nabla \times A_{1,2}
\end{align*}
\]
SIE Formulations

- PMCHWT Formulation
- EFIE Formulation
- MFIE Formulation
- CFIE Formulation
- Muller Formulation
PMCHWT Formulation

\[
\begin{align*}
[E_1^s + E_2^s]_{tan} &= -E_{tan}^{inc} \quad \text{for } r \in S \\
[H_1^s + H_2^s]_{tan} &= -H_{tan}^{inc} \quad \text{for } r \in S
\end{align*}
\]

Resulting in

\[
\begin{align*}
[j\omega (A_1 + A_2) + \nabla (\Phi_1 + \Phi_2) + \nabla \times (F_1 + F_2)]_{tan} &= E_{tan}^{inc} \\
[-\nabla \times (A_1 + A_2) + j\omega (F_1 + F_2) + \nabla (\Psi_1 + \Psi_2)]_{tan} &= H_{tan}^{inc}
\end{align*}
\]

◆ Expansion Procedure – Use RWG Functions for both \( J_s \) and \( M_s \).
◆ Testing Procedure – Use RWG Functions - similar to PEC Case.
◆ Most Efficient Solution – Acceptable accuracy.
◆ However, the same procedure fails for other formulations – Why?
EFIE Formulation

\[
\begin{align*}
[E_1^s(J_s) + E_1^s(M_s)]_{\tan} &= -E_{\tan}^{inc} \quad \text{for} \quad r \in S \\
[E_2^s(J_s) + E_2^s(M_s)]_{\tan} &= 0 \quad \text{for} \quad r \in S
\end{align*}
\]

Resulting in

\[
\begin{align*}
\left[j\omega A_1 + \nabla \Phi_1 + a_n \times \frac{M_s}{2} + \nabla \times F_1\right]_{\tan} &= E_{\tan}^{inc} \\
\left[j\omega A_2 + \nabla \Phi_2 - a_n \times \frac{M_s}{2} + \nabla \times F_2\right]_{\tan} &= 0
\end{align*}
\]

- Expansion Procedure – Let us assume RWG Functions for both \( J_s \) and \( M_s \).
- Testing Procedure – Use RWG Functions

- What happens?
RWG Testing functions

\[
\begin{pmatrix}
Z^1_E(J_s) & Z^1_E(M_s) \\
Z^2_E(J_s) & Z^2_E(M_s)
\end{pmatrix}
\]

- $Z^1_E(J_s)$ and $Z^2_E(J_s)$ are well-conditioned Submatrices.
- $Z^1_E(M_s)$ and $Z^2_E(M_s)$ are poorly-conditioned Submatrices.

Overall Result – An inaccurate solution
\[ \begin{pmatrix} Z^1_E(J_s) & Z^1_E(M_s) \\ Z^2_E(J_s) & Z^2_E(M_s) \end{pmatrix} \]

- \(Z^1_E(J_s)\) and \(Z^2_E(J_s)\) are poorly conditioned Submatrices.
- \(Z^1_E(M_s)\) and \(Z^2_E(M_s)\) are well-conditioned Submatrices.

Again, inaccurate Solution
RWG + $a_n$xRWG Testing

- $Z^1_E(J_s)$ and $Z^2_E(J_s)$ are well conditioned Submatrices.
- $Z^1_E(M_s)$ and $Z^2_E(M_s)$ are well-conditioned Submatrices.

Or

- $Z^1_E(J_s)$ and $Z^2_E(J_s)$ are poorly-conditioned Submatrices.
- $Z^1_E(M_s)$ and $Z^2_E(M_s)$ are poorly-conditioned Submatrices.
The Problem would be true for all other formulations – MFIE, CFIE, Muller formulations.

The Problem can happen for other situations – Apertures in a body and composite surfaces.

Far-fields may be acceptable with dense grids.

However, Near-fields are questionable.
Experimentation with Expansion Schemes

- Use Two separate functions to expand $J_s$ and $M_s$.
- Preferably, these two functions should be spatially orthogonal to each other.
- Use same functions (or some approximations) for testing.
Method # 1

- Use RWG functions for $J_s$.
- Use $a_n \times$ RWG for $M_s$.
- Use RWG functions for Testing.
- The EFIE Solution for this case is straightforward.

\[
\begin{align*}
\left[ j \omega A_1 + \nabla \Phi_1 + a_n \times \frac{M_s}{2} + \nabla \times F_1 \right]_{tan} &= E_{tan}^{inc} \\
\left[ j \omega A_2 + \nabla \Phi_2 - a_n \times \frac{M_s}{2} + \nabla \times F_2 \right]_{tan} &= 0
\end{align*}
\]

\[
\begin{pmatrix}
Z^1_E(J_s) & Z^1_E(M_s) \\
Z^2_E(J_s) & Z^2_E(M_s)
\end{pmatrix}
\]
• For MFIE Solution, one requires to compute the divergence of $M_s$.

• Use $a_n$ x RWG functions for Testing.

\[
\begin{align*}
\left[ -a_n \times \frac{J}{2} - \nabla \times A_1 + j\omega F_1 + \nabla \psi_1 \right]_{tan} &= H_{tan}^{inc} \\
\left[ a_n \times \frac{J}{2} - \nabla \times A_2 + j\omega F_2 + \nabla \psi_2 \right]_{tan} &= 0
\end{align*}
\]
Note that $a_n xRWG$ functions have discontinuous derivatives.

But can be handled in the following way.
Another Scheme (Method #2)

- Use the conventional triangulation scheme RWG functions to expand the electric current $J_S$.
- Develop a dual grid and polygonal basis functions to expand $M_S$. 
A sphere surface modeled by triangular and corresponding polygonal patches.
Polynomial Basis Functions
\[ \Lambda_n(r) = \begin{cases} \sum_{\ell=0}^{L_n^\pm} C_{n\ell}^\pm \Lambda_{n\ell}^\pm(r) & \text{for } r \in P_n^\pm \\ 0 & \text{otherwise} \end{cases} \]
Current flowing in the Polygon Pair
Yet Another Scheme (Method # 3)

Pulse-Like Functions on Triangle Pair

\( \hat{n} \times g_n = f_n \)

\( \hat{n} = \) outward unit normal

\[ |f_n^+| = |f_n^-| = |g_n| = 1 \]

\[ J_n = I_n \left( f_n^+ + f_n^- \right) \]

\[ M_n = I_{N+n} g_n \]
Charge Patches for $\mathbf{J}$

Let $N_p$ be the total number of triangles, and let

$$ q_s = \sum_{i=1}^{N_p} \alpha_i P_i $$

Where $P_i(\mathbf{r}) = \begin{cases} 
1, & \mathbf{r} \in T_i \\
0, & \text{otherwise}
\end{cases}$

$$
\int_{T_i} q_s ds = \int_{T_i} \frac{\nabla_s \cdot \mathbf{J}}{-j \omega} ds \\
= \frac{j}{\omega} \int_{C_i} \mathbf{J} \cdot d\mathbf{l} \\
= \frac{j}{\omega} \left[ I_{i_1} \ell_{i_1} + I_{i_2} \ell_{i_2} + I_{i_3} \ell_{i_3} \right] = \alpha_i A_i
$$

$$
\alpha_i = \frac{j}{\omega} \left[ \frac{I_{i_1} \ell_{i_1} + I_{i_2} \ell_{i_2} + I_{i_3} \ell_{i_3}}{A_i} \right]
$$
Charge Patches for $M$

Let $q_S = \sum_{i=1}^{N_n} \alpha_i P_i$, $P_i(r) = \begin{cases} 1, & r \in S_i \\ 0, & \text{otherwise} \end{cases}$

\[
\int_{S_i} q_S \, dS = \int_{S_i} \frac{\nabla_S \cdot J}{-j \omega} \, dS
= \frac{j}{\omega} \oint_{C_i} J \cdot n_\ell \, d\ell
= \frac{j}{\omega} \sum_{j=1}^{E_K} I_{ij} [\ell_{ij} \cdot (r_{ij}^{c+} \times n_{ij}^+) \times (r_{ij}^{c-} \times n_{ij}^-)]
\]

\[
\alpha_i = \frac{j}{\omega A_{S_i}} \sum_{j=1}^{E_K} I_{ij} [\ell_{ij} \cdot (r_{ij}^{c+} \times n_{ij}^+) \times (r_{ij}^{c-} \times n_{ij}^-)]
\]
Testing Vectors

\( t_m^+, t_m^- \): EFIE solutions

\( l_m \): HFIE solutions

Edge node

Centroid
An Alternate Formulation

- Consider the following Composite Body Problem

  - On surface $S_{ce}$: Only $J_{ce}$.
  - On surface $S_{de}$: $J_{de}$ and $M_{de}$.
  - On surface $S_{dc}$: Only $J_{dc}$.
  - Use PMCHWT Formulation.

- OK for far-field calculations.
- For near-field quantities, extra work is needed to obtain physical currents from equivalent currents.
Some Observations

• Dielectric Body Problem is more complicated than PEC problem.
• If RWG functions are used for expansion and testing, one must be very careful while applying the numerical procedures.
• Ideally, it is recommended to use two spatially orthogonal functions in the numerical scheme.
• It is possible to get acceptable far-field quantities using only RWG functions.
• While using software packages, one needs to know how the dielectric materials are treated.
Water meter antenna

Radiating Element

Feed

Insulator

Radiation Pattern
Time Domain Problems

• Here we solve the scattering problem directly in time-domain – Useful for Impulse radar, Wideband solutions, and signature studies.

• No matrix inversion – solution is obtained iteratively.

Time Domain Electromagnetics
Academic Press, 2001
Consider the following problem:

Figure 1: Transient pulse incident upon an arbitrarily shaped body.
Direct Time Domain Solutions are better suited to:

- Short-pulse radar systems.
- EMP studies.
- Provide better visualization.
- Provides opportunity to observe and interpret scattering behavior.
- Provides broadband information.
Incident Field

\[ E^i(r, t) = E_o \frac{4}{T \sqrt{\pi}} e^{-\gamma^2} \]

where

\[ \gamma = \frac{4}{T} (ct - c_{t_0} - r \cdot a_k), \]
Time Domain EFIE

\[
\left[ \frac{\partial A(r, t)}{\partial t} + \nabla \Phi(r, t) \right]_{tan} = E_{tan}^i(r, t)
\]

where

\[
A(r, t) = \mu \int_S \frac{J(r', t - \frac{R}{c})}{4\pi R} \, dS',
\]
\[
\Phi(r, t) = \frac{1}{\varepsilon} \int_S \frac{q_s(r', t - \frac{R}{c})}{4\pi R} \, dS'.
\]
Final Equation

\[
\left[ \kappa_{mm} \cdot \frac{l_m}{2} \left( \rho_m^{c+} + \rho_m^{c-} \right) \right] I_m(t_j) = \\
\frac{l_m}{2} (\rho_m^{c+} + \rho_m^{c-}) \cdot (\Delta t) E^i(r_m, t_{j-1}) \\
- \frac{l_m}{2} (\rho_m^{c+} + \rho_m^{c-}) \cdot \left[ A(f(r_m, t_j)) - A(r_m, t_{j-1}) \right] \\
+ l_m (\Delta t) [\Phi(r_m^{c+}, t_{j-1}) - \Phi(r_m^{c-}, t_{j-1})].
\]

Lastly, we can write Eq. (44) in matrix form as

\[
[\alpha][I(t_j)] = [F(t_j)] + [\beta][I(t_R)].
\]
Conclusions

- RWG functions have been used for a variety of problems in numerical electromagnetic problems.
- Also used in other areas – Acoustic scattering.
- New improvements include: developing faster solutions (FMM), adaptive basis functions to generate sparse moment matrix (Killian and Rao, IEEE Transactions on A&P, 2011), Domain Decomposition to handle large problems, and Adaptive Cross Approximation (Mercury MoM).
New domain Decomposition Method

- Domain Decomposition – Disjoint groups of sub-domain functions
- Functions in a group are geometrically close to one another
- Each function belongs to one and only one group.
• Decouple a given group from other groups - Can be accomplished by generating new set of basis functions.

• It is possible to solve each group separately and obtain the total solution.
Example – Two 2D strips
New basis function:

\[ g_1 = f_1 + \alpha_1 f_4 + \beta_1 f_5 + \gamma_1 f_6 \]
\[ g_2 = f_2 + \alpha_2 f_4 + \beta_2 f_5 + \gamma_2 f_6 \]
\[ g_3 = f_3 + \alpha_3 f_4 + \beta_3 f_5 + \gamma_3 f_6 \]

Criteria (g1):

\[ Z_{4,1} + \alpha_1 Z_{4,4} + \beta_1 Z_{4,5} + \gamma_1 Z_{4,6} = 0 \]
\[ Z_{5,1} + \alpha_1 Z_{5,4} + \beta_1 Z_{5,5} + \gamma_1 Z_{5,6} = 0 \]
\[ Z_{6,1} + \alpha_1 Z_{6,4} + \beta_1 Z_{6,5} + \gamma_1 Z_{6,6} = 0 \]
Solve for weights

Create new matrix
3D Results – Finite Planar Array

• Finite Periodic Array
• EFIE
• 50 x 50 grid of
  0.5 lambda x 0.5 lambda plates
• 64 unknowns per plate
• 0.75 lambda spacing
• 160,000 total unknowns
• Null fields produced
  on adjacent plates
• Redundant coefficients
• Eth = 120pi; Ephi = 0
• Theta = 45 deg Phi = 0 deg

0.75 wavelengths
3D Results – Planar Array

- 1 iteration = 0.288 average error per term
- 2 iterations = 0.122 average error per term
- ~19.5 hours wall clock time with 8 CPUs (includes time for RCS calculation on single CPU)
- Matrix approximations can be used for speedup
3D Results – Sphere

- 5 lambda radius
- CFIE
- 92550 unknowns
- 314 Groups – each roughly
  1 lambda^2 in surface area
- Null fields produced on groups
  within 2 lambda radius (typically
  around 3000 points)
- ~ 2.5 GB storage
- Eth = 120π; Ephi = 0
- Theta = 45 deg Phi = 0 deg
3D Results – Sphere

• 1 iteration = 0.096 average error per term
• 2 iterations = 0.014 average error per term
• ~26 hours wall clock time with 8 CPUs (includes time for RCS calculation on single CPU)
• Matrix approximations can be used for speedup
3D Results – Square Plate

- 12 \lambda \times 12 \lambda \ square \ plate
- EFIE
- 42883 \ unknowns
- 144 \ Groups – each roughly
  1 \lambda^2 \ in \ surface \ area
- Null \ fields \ produced \ on \ groups
  within \ 2 \ lambda \ radius \ (typically
  around \ 2800 \ points)
- \sim \ 185 \ MB \ storage
- Eth = 120\pi; \ Ephi = 0
- Theta = 45 \ deg \ Phi = 0 \ deg
3D Results – Square Plate

• 1 iteration = 0.612 average error per term
• 2 iterations = 0.232 average error per term
• ~ 2 hours 45 mins wall clock time with 8 CPUs (includes time for RCS calculation on single CPU)
3D Results – Aircraft

- French Mirage
- ~ 160,000 unknowns
- Patches represent groups.
3D Results – Aircraft

- 1 iteration = 0.224 average error per term
- 2 iterations = 0.218 average error per term
- ~ 5 days 7 hours wall clock time with 8 CPUs (includes time for RCS calculation on single CPU)