Space-Time Adaptive Processing (STAP) Some Performance Limiting Factors

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Agenda

- The Airborne Clutter Environment
- Space-Time Adaptive Processing
  – What is it?
    - Natural Evolution of Radar Signal Process
  – Why Put the Adaptive in Space-Time Processing?
    - Short Answer - Realities, Imperfect Knowledge, Uncertainties and Imperfections
    - Simple case in point is open loop DPCA
  – How is it Implemented?
- Limiting Factors List & Examples
- Current Investigations to Mitigate Limiting Factors

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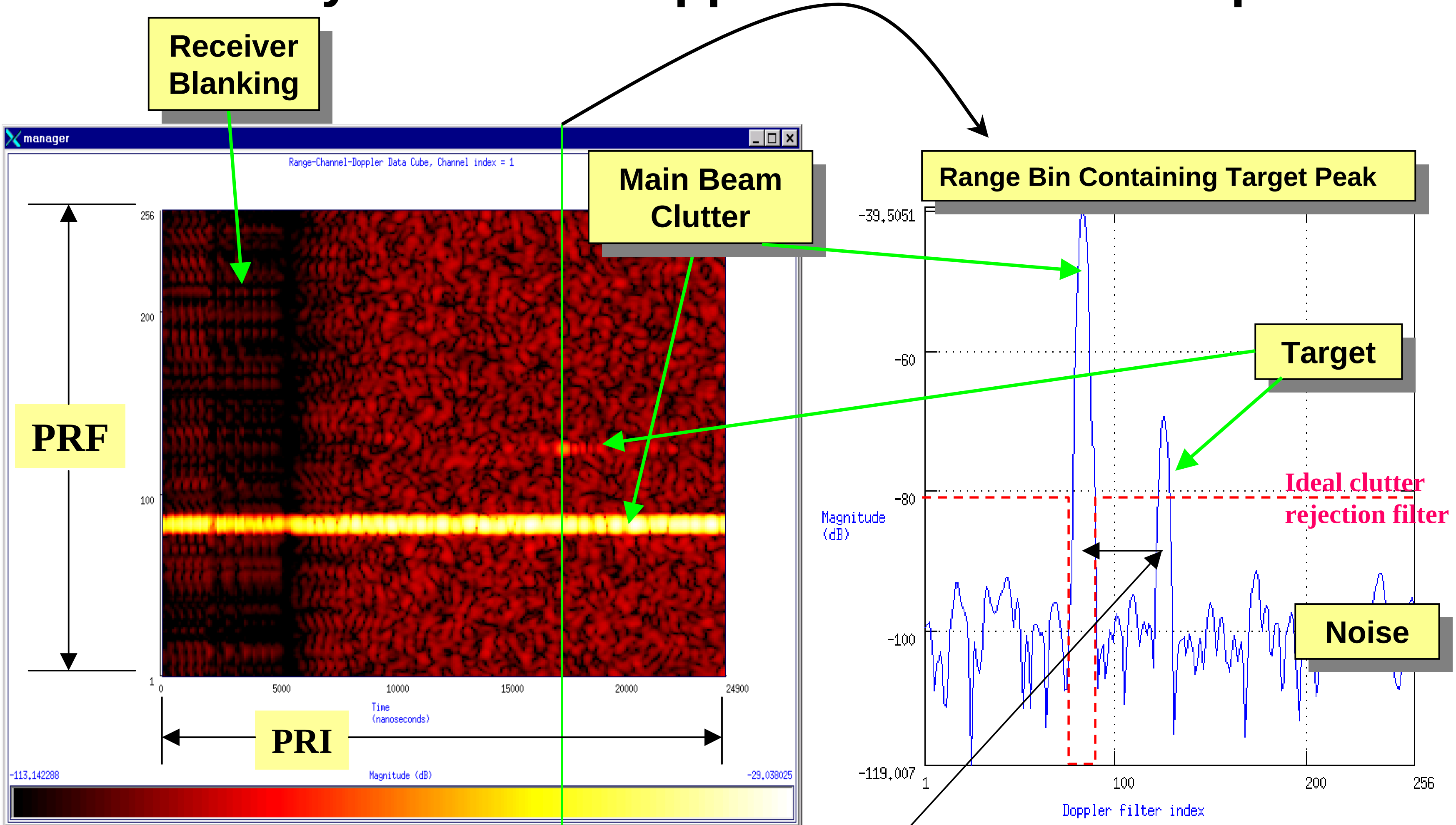
Simple Example of target and interference environment as observed by a Pulsed-Doppler Airborne Intercept Radar



Slow Moving Targets require Fast Transition from Stopband to Passband

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Space-Time Adaptive Processing is a Natural Evolution of Radar Signal Process

- Time Only Processing - Single Channel
  - MTI Processing
  - Pulse Doppler Processing
- Space Only Processing - Multiple Channel
  - Jammer Cancellation
- Space-Time Processing (Non-Adaptive)
  - Displaced Phase Center Array (DPCA) Processing
  - Simultaneous DPCA
- Space-Time Adaptive Processing
  - Segmented Antenna Co-Aligned with Velocity Vector
  - Arbitrary Antenna Manifolding and Alignment

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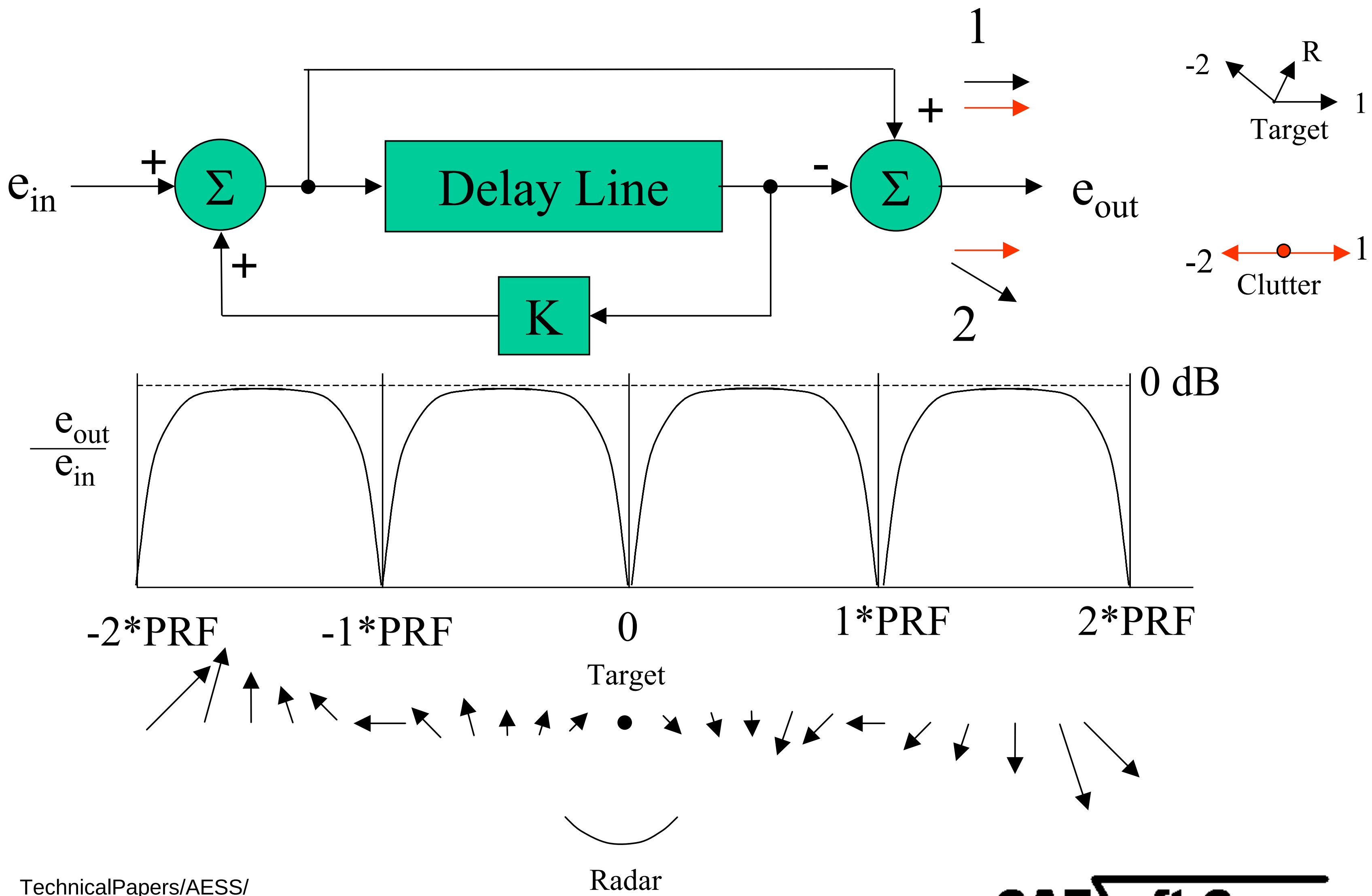
Simple MTI Radar (Time) Processing



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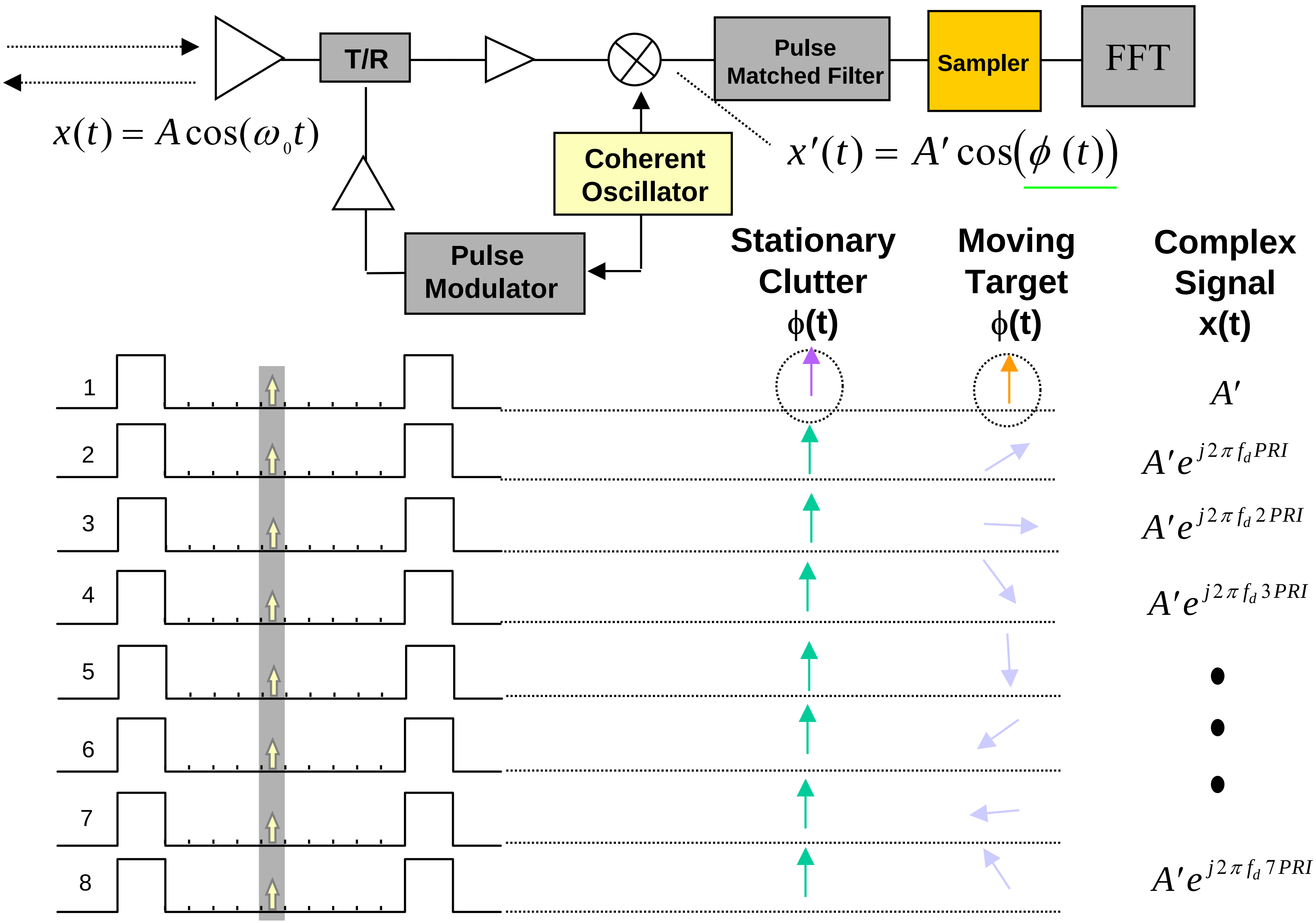
Pulse Doppler Processing Of Moving Targets (Time)



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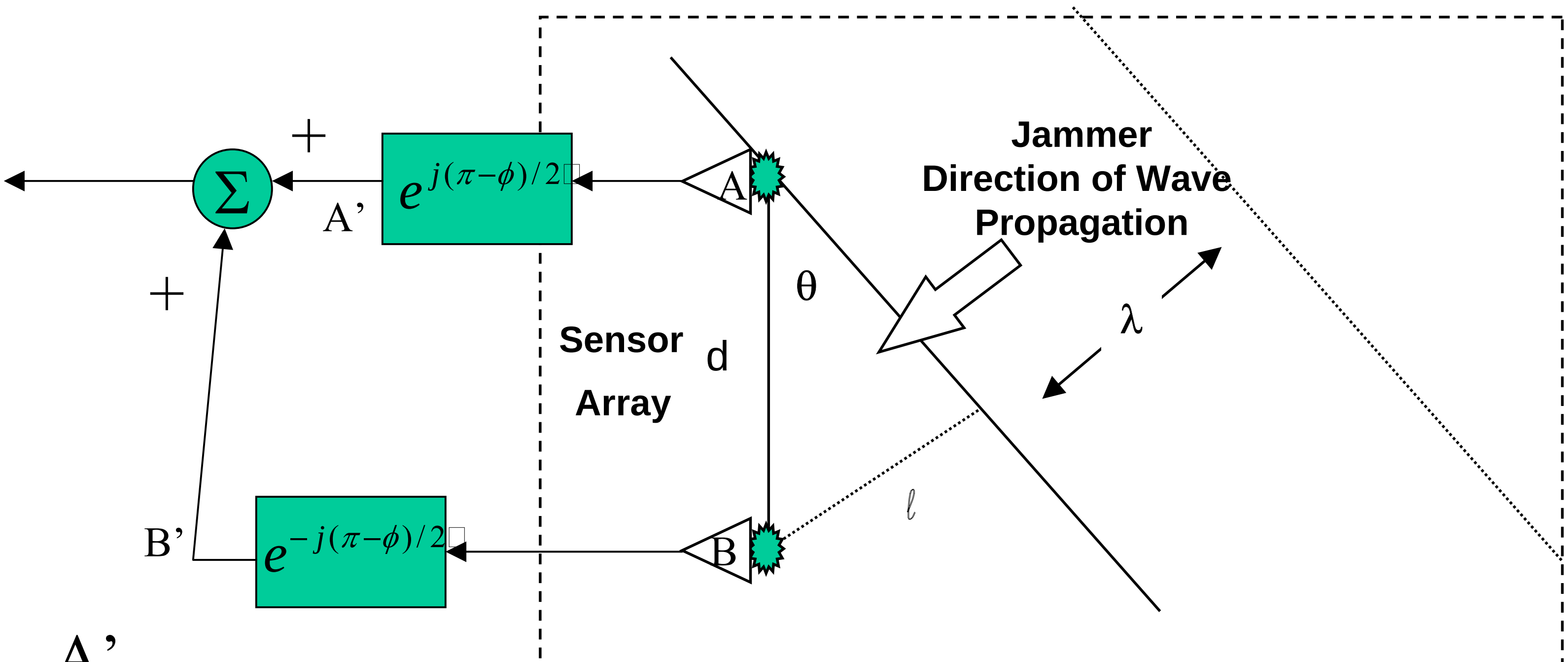
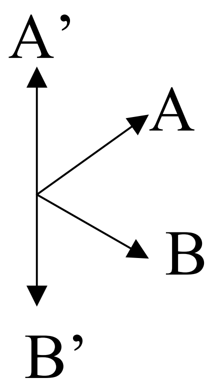
Jammer Nulling (Space)





The Path Difference Between Elements Is

$$l = d\sin(\theta)$$

Electrical Phase Difference Between of Jammer's Channels Signal in Channels A & B As a Result of The Angle of Arrival

$$\phi = -\frac{2\pi}{\lambda}d\sin(\theta)$$

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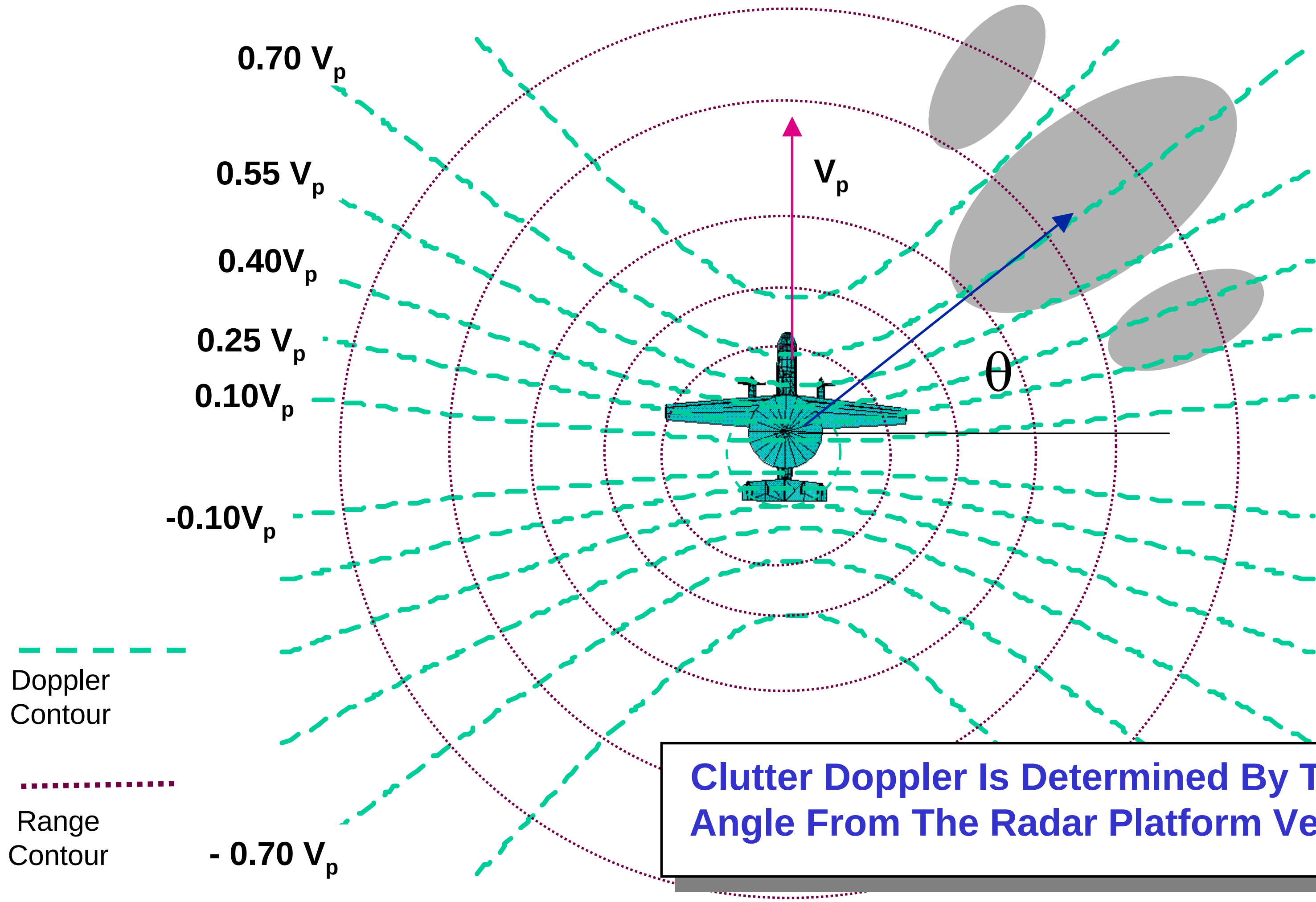
Range, Doppler & Angle of Ground Clutter

$$f_{clutter} = -\frac{2}{\lambda}\left(V_p \bullet U_{p\_to\_clutter}\right) \approx -\frac{2}{\lambda}\left(V_p \times \sin(\theta_{clutter})\right)$$



Clutter Doppler Is Determined By The Look Angle From The Radar Platform Velocity

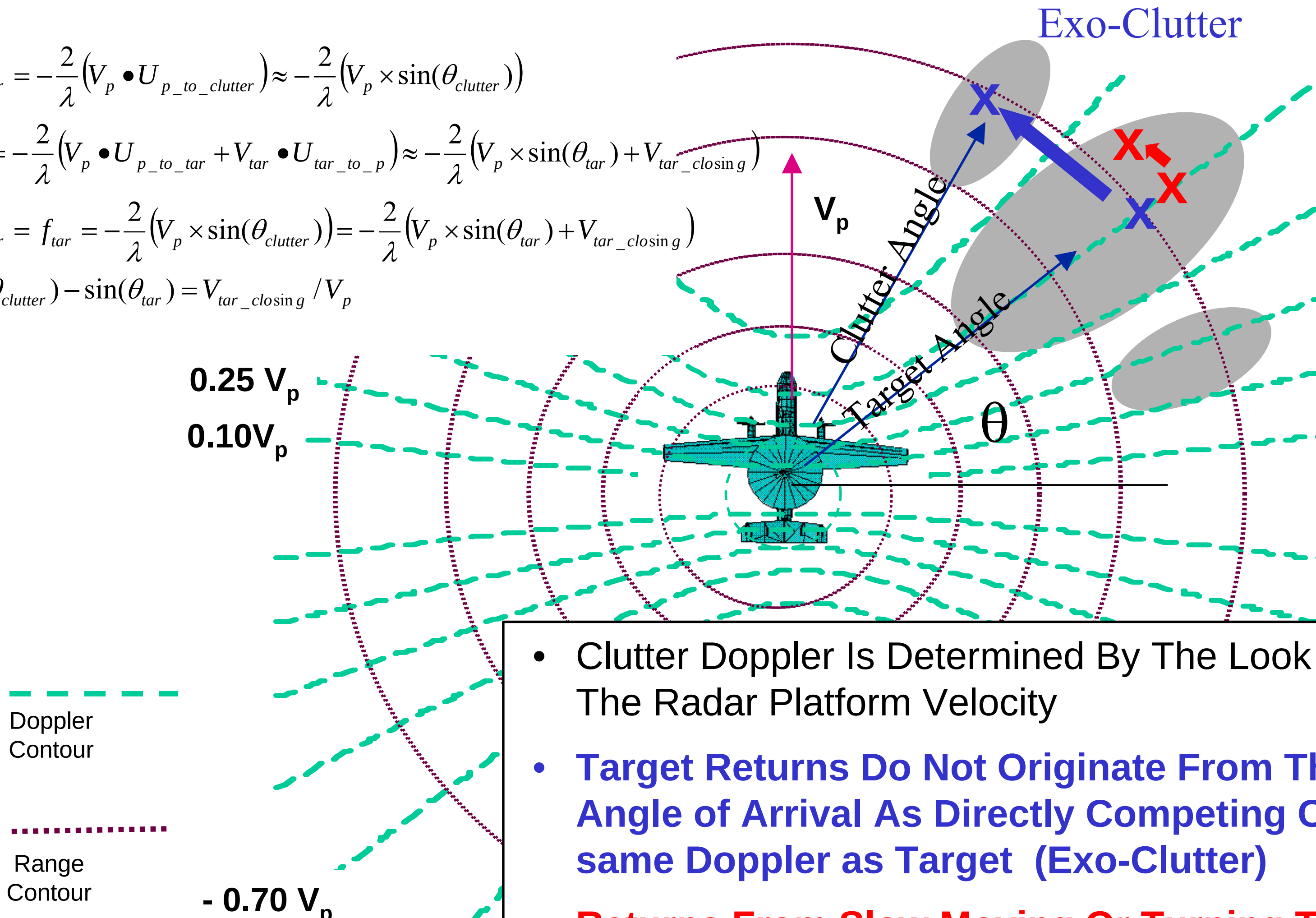
Range, Doppler & Angle of Moving Targets

$$f_{clutter} = -\frac{2}{\lambda}\left(V_p \bullet U_{p\_to\_clutter}\right) \approx -\frac{2}{\lambda}\left(V_p \times \sin(\theta_{clutter})\right)$$

$$f_{tar} = -\frac{2}{\lambda}\left(V_p \bullet U_{p\_to\_tar} + V_{tar} \bullet U_{tar\_to\_p}\right) \approx -\frac{2}{\lambda}\left(V_p \times \sin(\theta_{tar}) + V_{tar\_closing}\right)$$

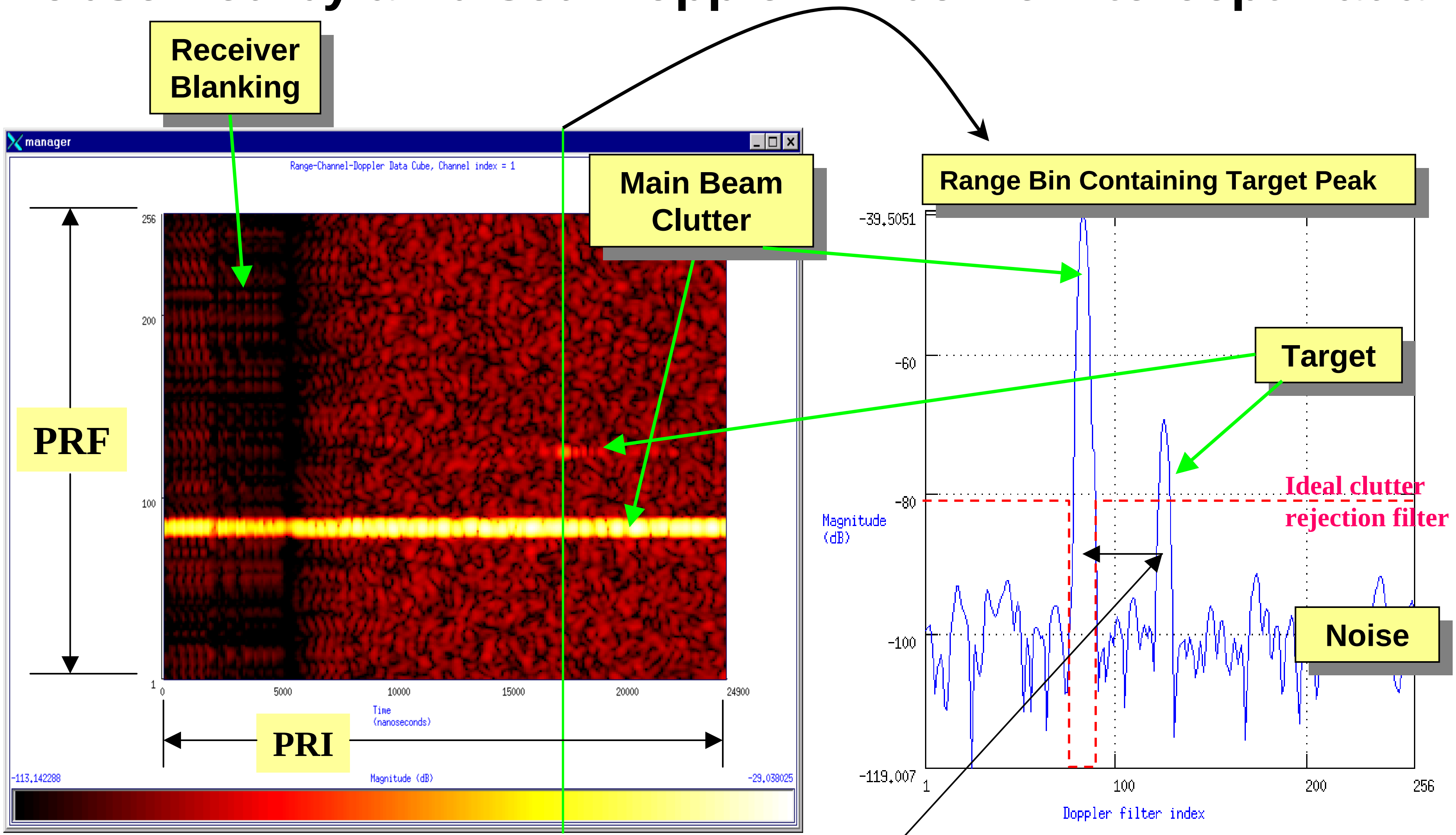
$$f_{clutter} = f_{tar} = -\frac{2}{\lambda}\left(V_p \times \sin(\theta_{clutter})\right) = -\frac{2}{\lambda}\left(V_p \times \sin(\theta_{tar}) + V_{tar\_closing}\right)$$

$$\sin(\theta_{clutter}) - \sin(\theta_{tar}) = V_{tar\_closing} / V_p$$



- Clutter Doppler Is Determined By The Look Angle From The Radar Platform Velocity
- **Target Returns Do Not Originate From The Same Angle of Arrival As Directly Competing Clutter at same Doppler as Target (Exo-Clutter)**
- **Returns From Slow Moving Or Turning Targets Must Compete With Mainlobe Clutter (Endo-Clutter)**

Simple Example of target and interference environment as observed by a Pulsed-Doppler Airborne Intercept Radar



More Clutter Rejection Required With Slower Target

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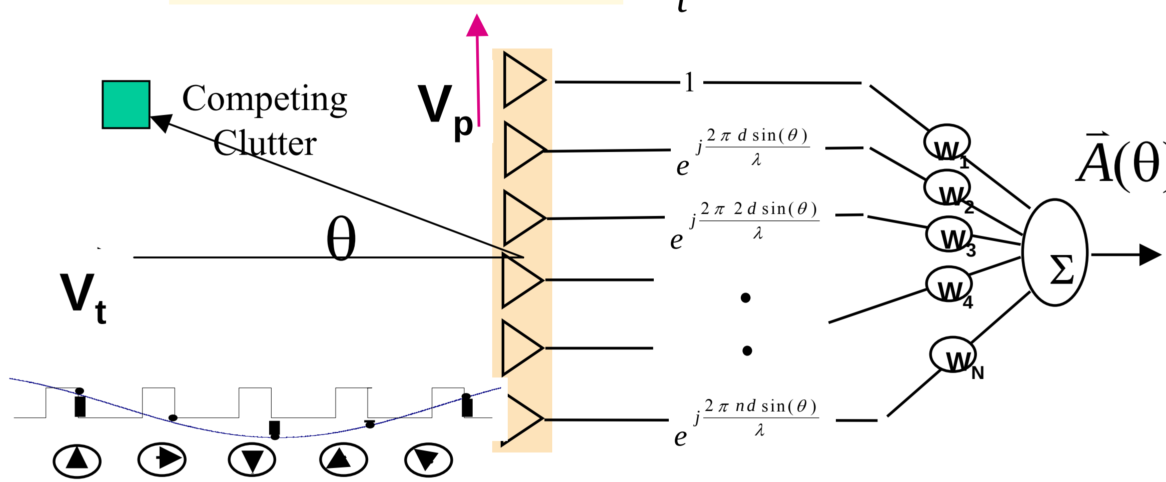
How Space relates to Time (Doppler) in Space-Time Processing (Co-Aligned Array)

Target Signal Vector $S_t = x \otimes A^T$

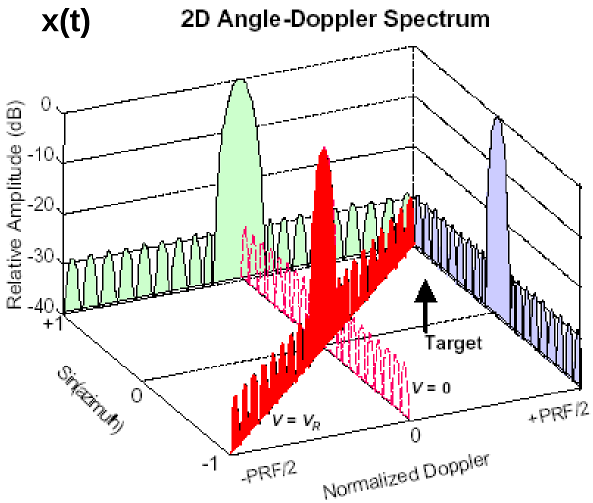
$$\begin{bmatrix} A' \\ A' e^{j 2\pi f_d PRI} \\ A' e^{j 2\pi f_d 2 PRI} \\ A' e^{j 2\pi f_d 3 PRI} \\ \vdots \\ A' e^{j 2\pi f_d (m-1) PRI} \end{bmatrix}$$

Complex Pulsed Signal x(t)



$$\vec{A}(\theta) = \begin{bmatrix} 1 \\ e^{j\frac{2\pi}{\lambda} d \sin(\theta)} \\ e^{j\frac{2\pi}{\lambda} 2d \sin(\theta)} \\ \vdots \\ e^{j\frac{2\pi}{\lambda}(N-1) d \sin(\theta)} \end{bmatrix}$$

Spatial Steering Vector



$$\text{Normalized Doppler} = \bar{f}_d = f_d / PRF = f_d PRI = \frac{2V_p \times PRI}{\lambda} \sin\theta_{clutter}$$

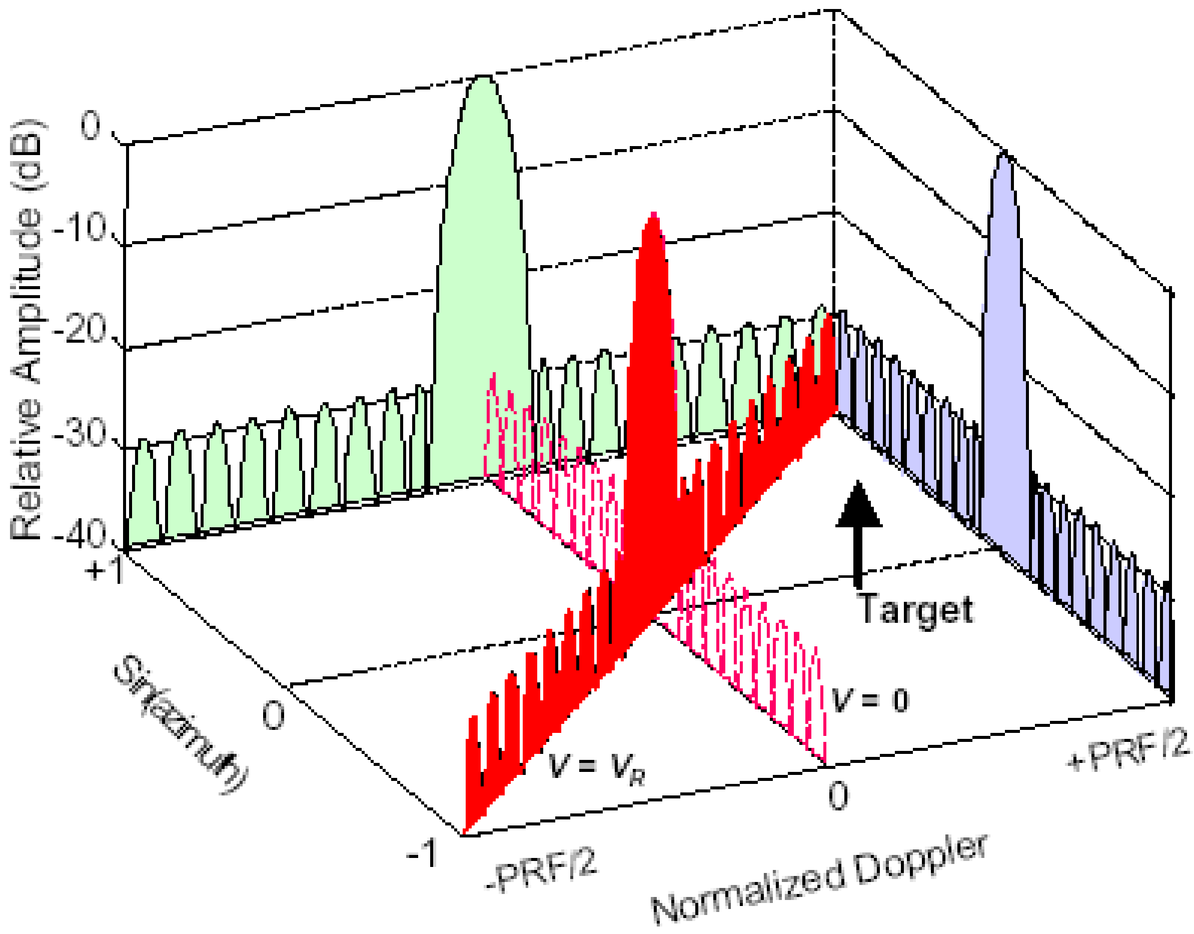
$$\bar{f}_d = \frac{2V_p \times PRI}{\lambda}\left(\frac{\lambda}{d}\right)\left(\frac{d}{\lambda}\right)\sin\theta_{clutter} = \frac{2V_p \times PRI}{d}\left(\frac{d}{\lambda}\sin\theta_{clutter}\right)$$

$$\bar{\theta} = \frac{d}{\lambda}\sin(\theta_{clutter}) \quad \text{(Spatial Frequency Normalized Angle)}$$

$$\beta = \frac{2V \times PRI}{d} \quad \text{(Slope of the Clutter Ridge = Spatial/Doppler Frequency}$$

$$\bar{f}_d = \beta\bar{\theta}$$

2 Dimensional Space-Time Filtering



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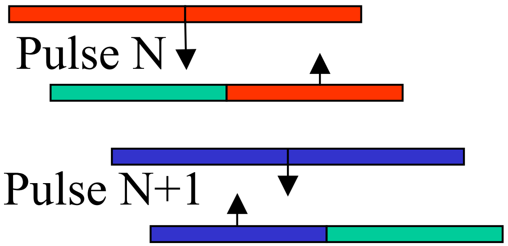
Full Aperture TX & Segmented RX Aperture DPCA



Pulse	Phase Center Displacement		
	TX	RX	Total
N	0	V*Tr+d/2	V*Tr+d/2
N+1	d/2	V*Tr	V*Tr+d/2

1. Delay Line Cancellation Processing
2. Cancels clutter from all angles and Dopplers
3. PRF tied to Aircraft Velocity
4. Matched Sub-Arrays & Channels
5. Constraints on Antenna mounting & Aircraft Motion
6. Not satisfying 3 & 4 Degrades Performance

$$\text{Normalized Doppler} = \bar{f}_d = f_d / PRF = f_d PRI = \frac{2V_p \times PRI}{\lambda} \sin\theta_{clutter}$$

$$\bar{f}_d = \frac{2V_p \times PRI}{\lambda}\left(\frac{\lambda}{d}\right)\left(\frac{d}{\lambda}\right)\sin\theta_{clutter} = \frac{2V_p \times PRI}{d}\left(\frac{d}{\lambda}\sin\theta_{clutter}\right)$$

$$\bar{\theta} = \frac{d}{\lambda}\sin(\theta_{clutter}) \quad \text{(Spatial Frequency/Normalized Angle)}$$

$$\beta = \frac{2V \times PRI}{d} \quad \text{(Slope of the Clutter Ridge = Spatial/Doppler Frequency)}$$

$$\bar{f}_d = \beta\bar{\theta}$$

$$(\beta = 1 \text{ for DPCA}); \frac{2V \times PRI}{d} = 1; PRF = \frac{2V}{d}$$

Phase Center Displacement during PRI :

$$\text{Displacement} = V * Time = V * 1/PRF = V * \frac{d}{2V} = \frac{d}{2}$$

Why Adaptive Processing

- DPCA & Non-Adaptive Space-Time Processing Implementations are limited by:
  - Channel Matching (both spatial and temporal)
  - Errors in the knowledge of Hardware Characteristics
  - Trajectories/Antenna Mounting Limitations
    - Nose Mounted
    - CRAB Angles
- Adaptive Processing
  - Dynamic Compensation Technique
  - Flexible in that it automatically adjusts to the interference environment

Adaptive Processing Math

Target Signal Vector

$$s_t = x(t) \otimes A(n)^T$$

Target Signal Vector

Received Signal Vector

$$\mathbf{x} = \alpha \mathbf{s}_t + \mathbf{n}$$

Thermal Noise + Interference

STAP Filter Output

$$y = \mathbf{w}^H \mathbf{x} = \alpha \mathbf{w}^H \mathbf{s}_t + \mathbf{w}^H \mathbf{n}$$

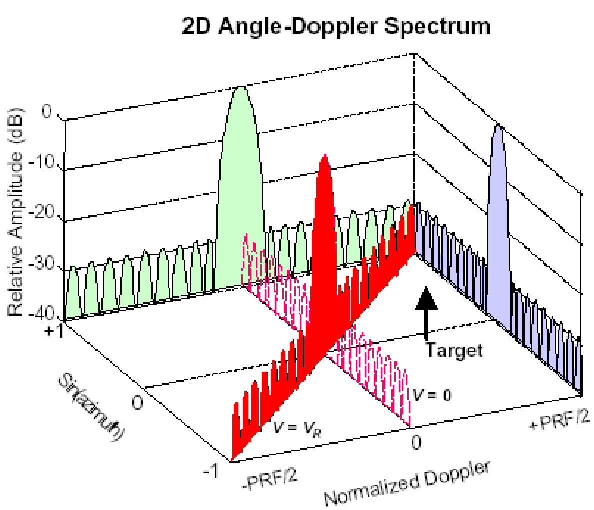
Constrained STAP Weight Vector*

$$\mathbf{w} = \kappa \tilde{\mathbf{R}}^{-1} \mathbf{s}_d = \frac{\tilde{\mathbf{R}}^{-1} \mathbf{s}_d}{\mathbf{s}_d^H \tilde{\mathbf{R}}^{-1} \mathbf{s}_d}$$

Desired Signal Vector

Taper

Interference Covaraince Matrix



SINR

$$SINRout = \frac{Ps}{Pn} = \frac{\alpha^2 \left| \mathbf{w}^H \mathbf{s}_t \right|^2}{\mathbf{w}^H \mathbf{R} \mathbf{w}}$$

Desired Signal Vector

* Brennan & Reed (1973)

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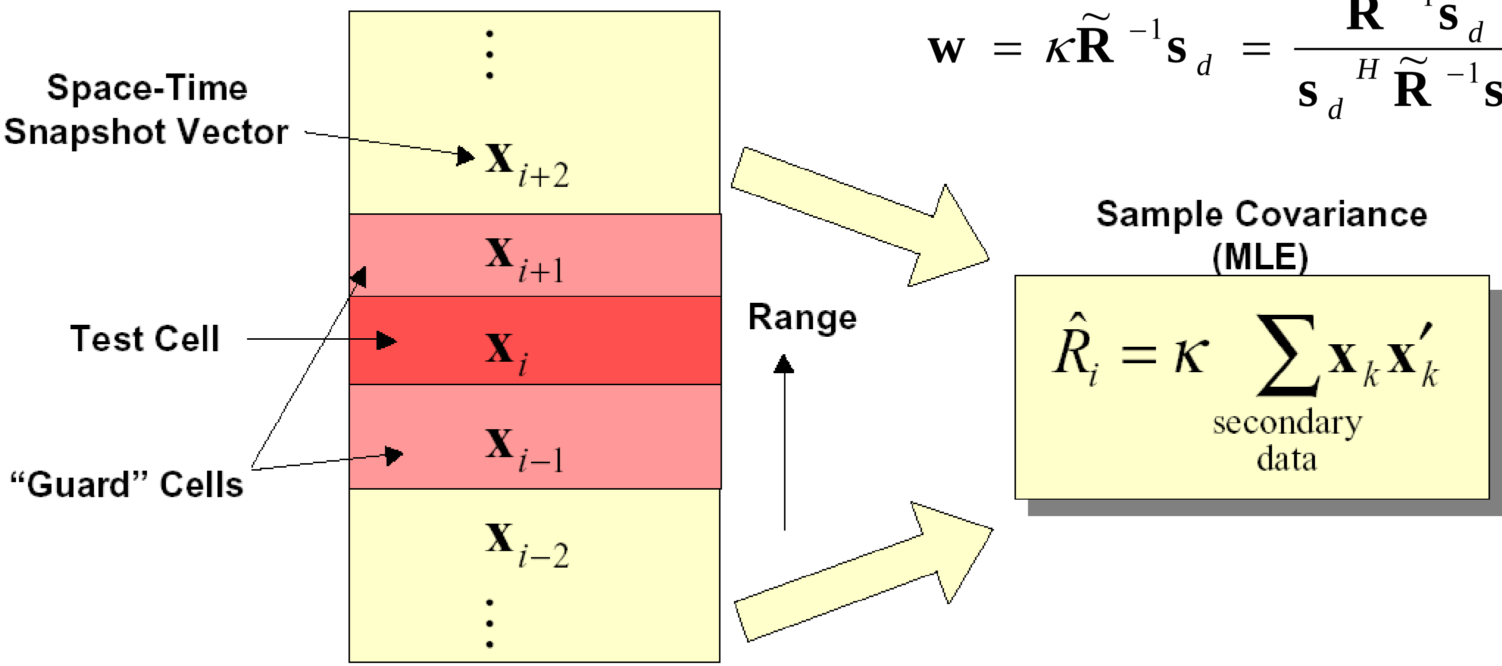
Clutter Covariance Estimation

- In practice, the interference covariance matrix is estimated from auxiliary data obtained from surrounding range bins
  - Stationarity/homogeneity assumption required w.r.t. range

$$\mathbf{w} = \kappa \widetilde{\mathbf{R}}^{-1}\mathbf{s}_d = \frac{\widetilde{\mathbf{R}}^{-1}\mathbf{s}_d}{\mathbf{s}_d^H \widetilde{\mathbf{R}}^{-1}\mathbf{s}_d}$$



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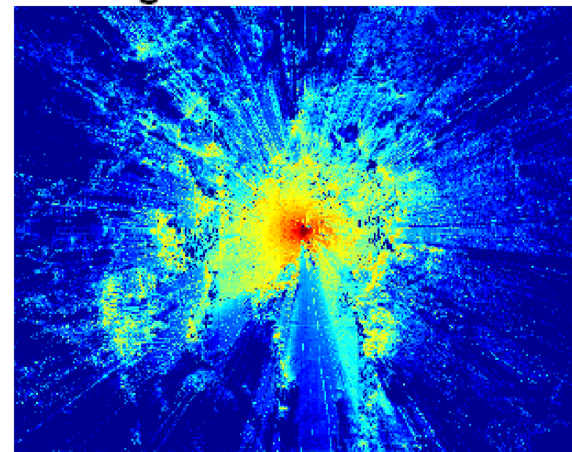
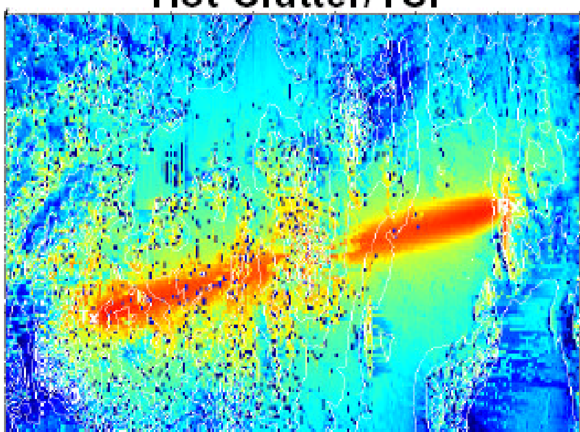
Fundamental Issues in Realizing STAP Potential (R_i)

- **Heterogeneous Clutter**
  - Inhomogeneous land clutter often violates "stationarity" assumption required for covariance estimation
    - Clutter undernulling, poor MDV
- **Large Clutter Discretes**
  - Land clutter is often "spiky"
    - Increased false alarms
- **Air/Ground Traffic Induced Distortions**
  - Strong presence of background traffic in training region can cause target cancellation
    - Reduced MDV (e.g., MCARM data set)
- **Joint Hot & Cold Clutter Cancellation**
  - 3D nature of problem exacerbates adaptive weights

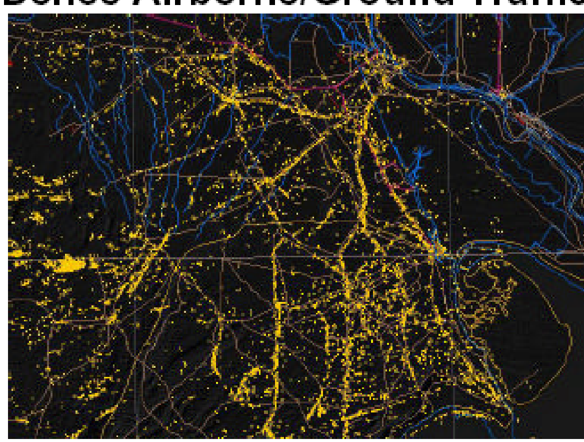
Inhomogeneous Terrain/Clutter



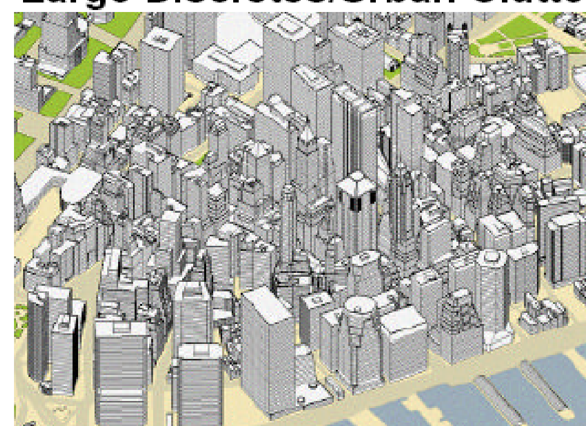
Hot Clutter/TSI



Dense Airborne/Ground Traffic



Large Discretes/Urban Clutter



Fundamental Issues in Realizing STAP Potential

- Interference "Spreading" Mechanisms
  - Internal clutter motion (ICM)
  - Clutter scintillation
  - Diffuse multipath
  - Transmitter instabilities
  - Antenna "jitter"
  - Bandwidth (dispersion)
  - Antenna "crab"
  - etc.
- Less useable angle-Doppler
  - Clutter is spread in angle and Doppler

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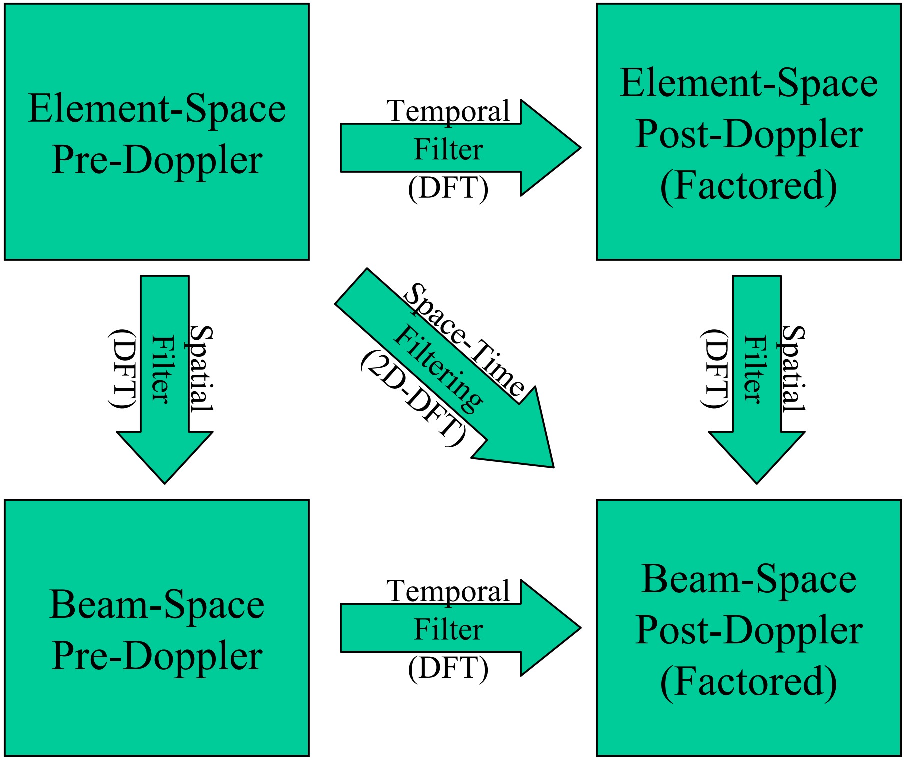
STAP Integration Into Radar Signal Processing



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Limiting Factors we will demonstrate

- Spatial Mismatch between antenna phase centers
- Non-Homogeneity
  - Terrain Shadowing
  - Discretes
  - Backscatter Variations
- Estimation
  - Moving Window vs Global
  - Movers in training set
- Under Nulling / Over Nulling
- Internal clutter motion
- Mis-alignment between Antenna Center Line and Velocity Vector, i.e. Crabbing
- Range Ambiguities
- Non Planar Antenna Arrays
  - Conformal
  - Deformation

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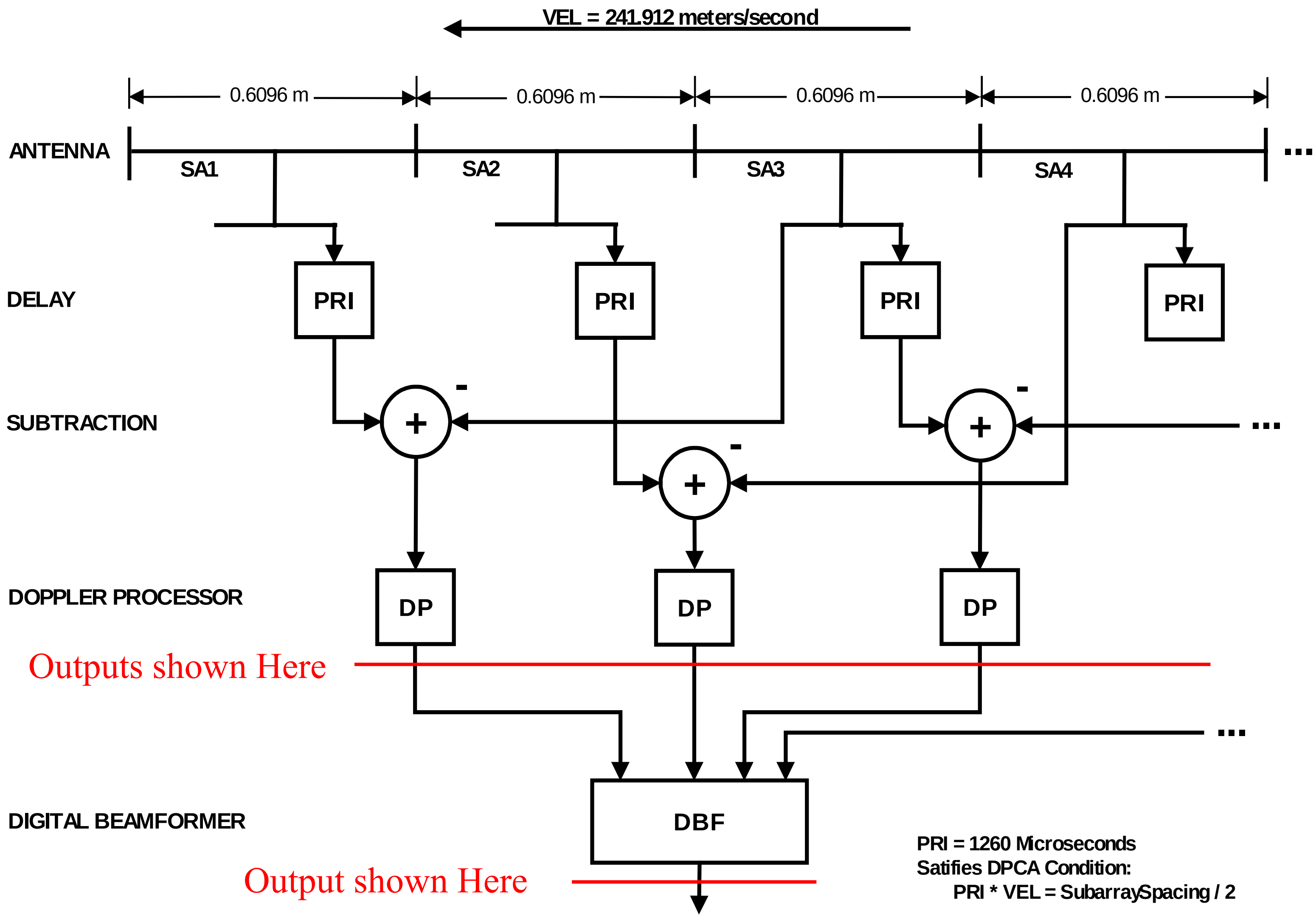
Space-Time (DPCA) Example with Airframe Near Field Scattering

DPCA + Doppler Processing



PRI = 1260 Microseconds
Satifies DPCA Condition:
PRI * VEL = SubarraySpacing / 2

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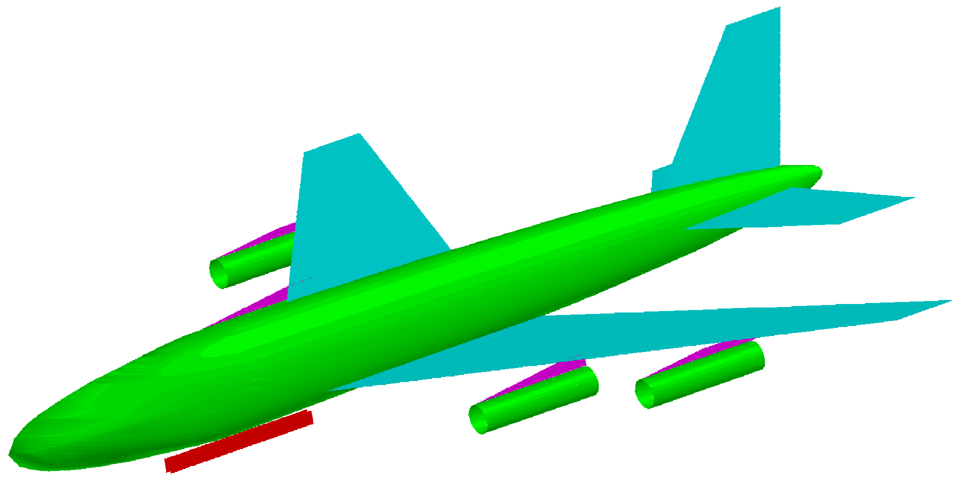
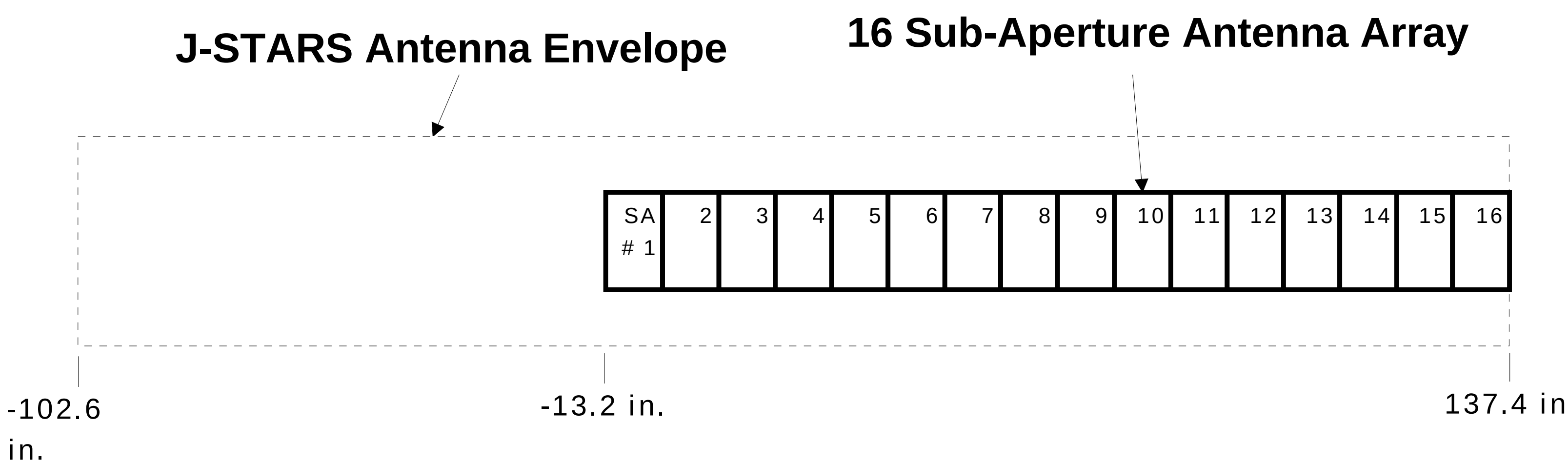
Diagram illustrating the relationship between the current J-Stars Array and the 16 Subaperture Array used in the Example





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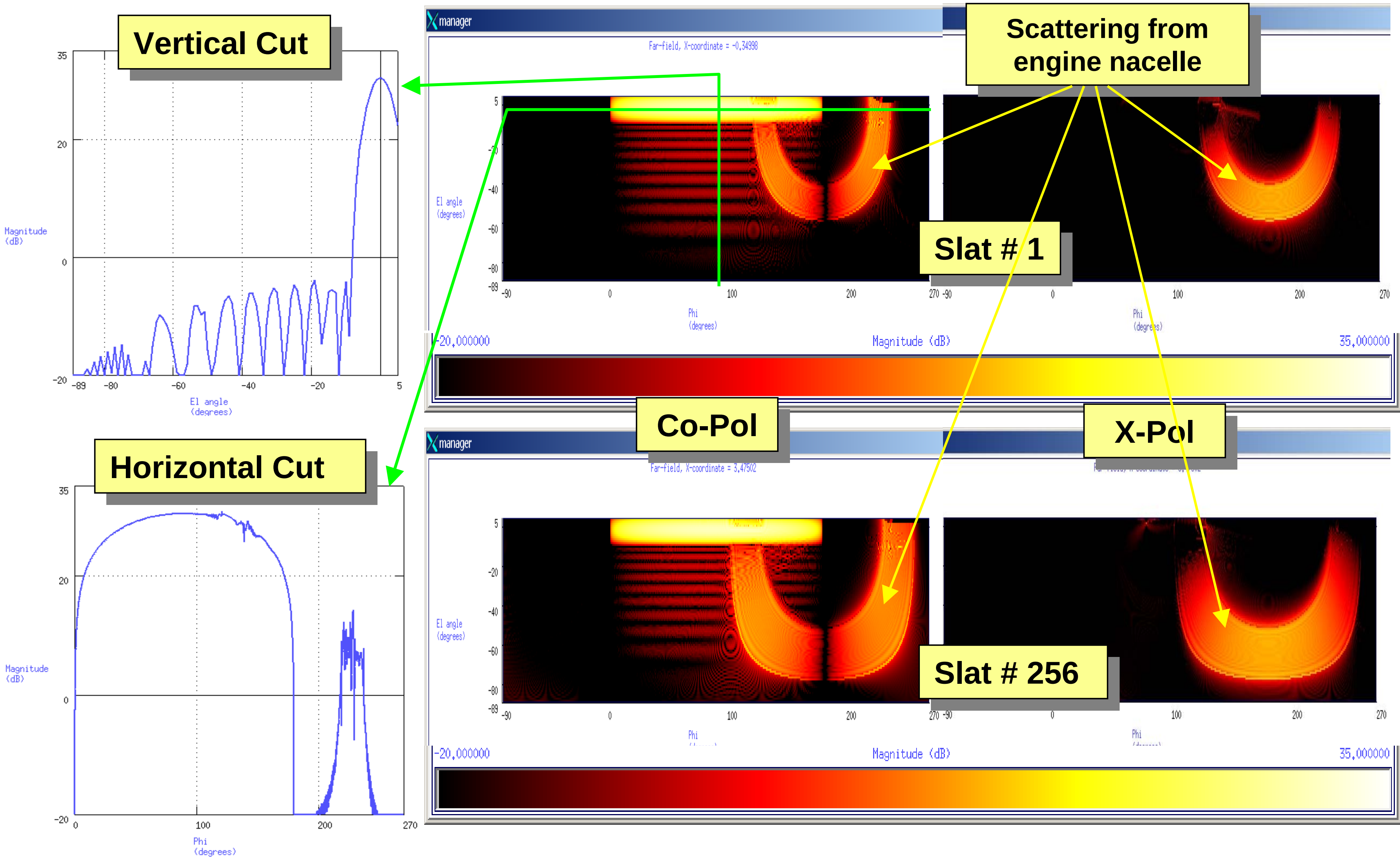
Far-Field from Column Subarrays (Slats) at Front and Rear of Array



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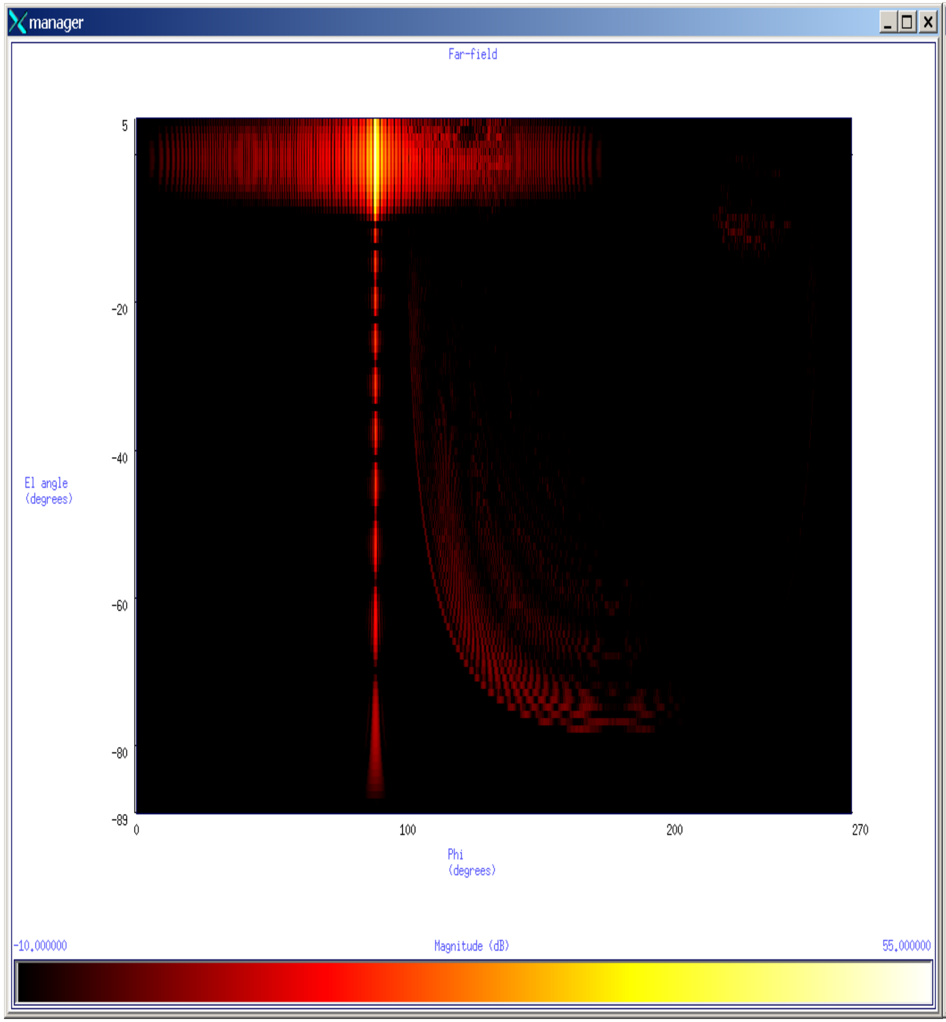
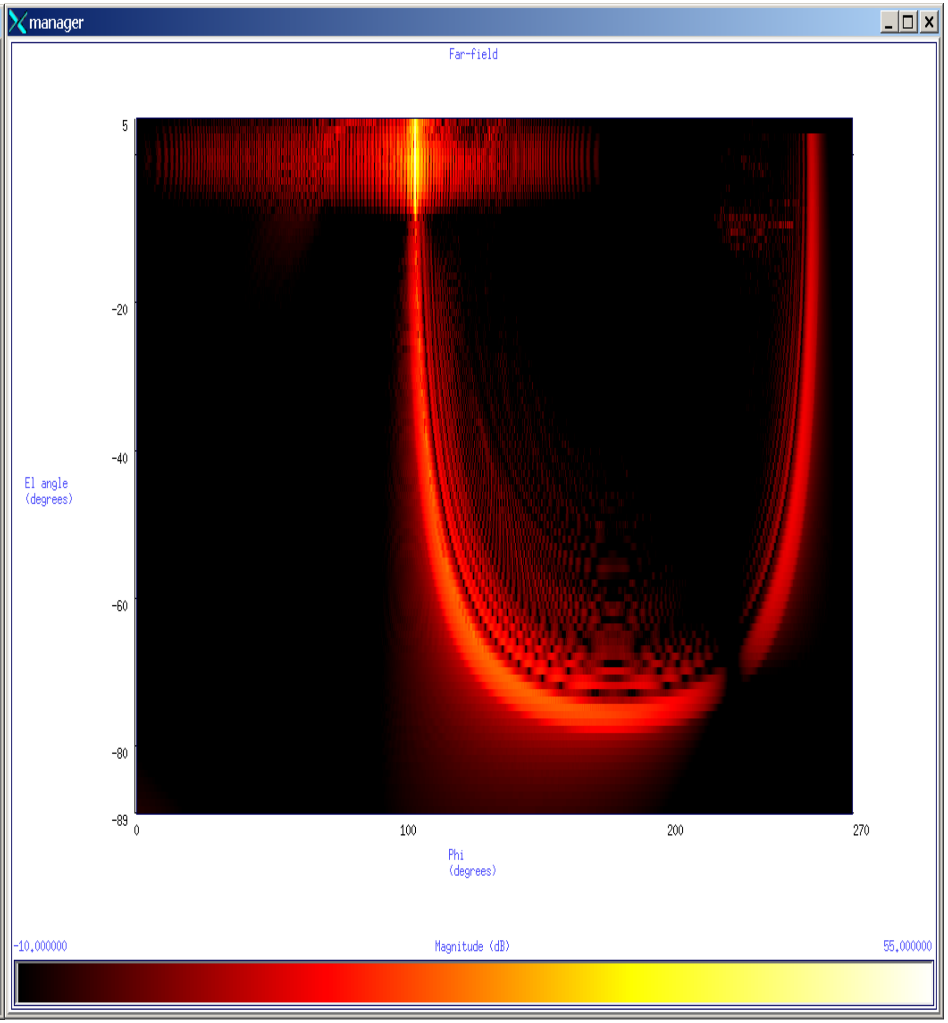
Tx Antenna Patterns at Az = 0 and 15 Deg.





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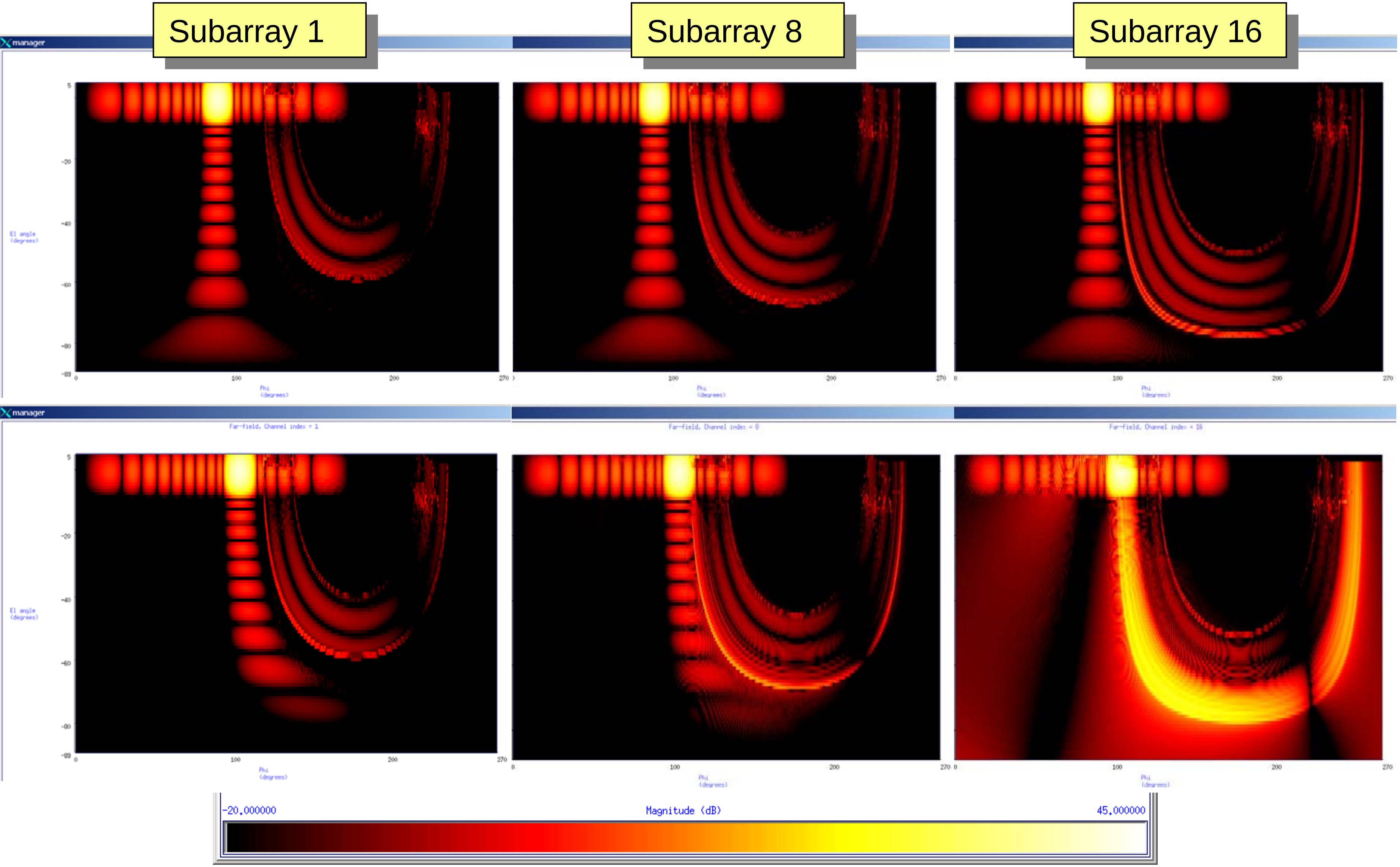
Rx Patterns for Subarrays 1, 8, 16 at Az = 0 and 15 Deg.



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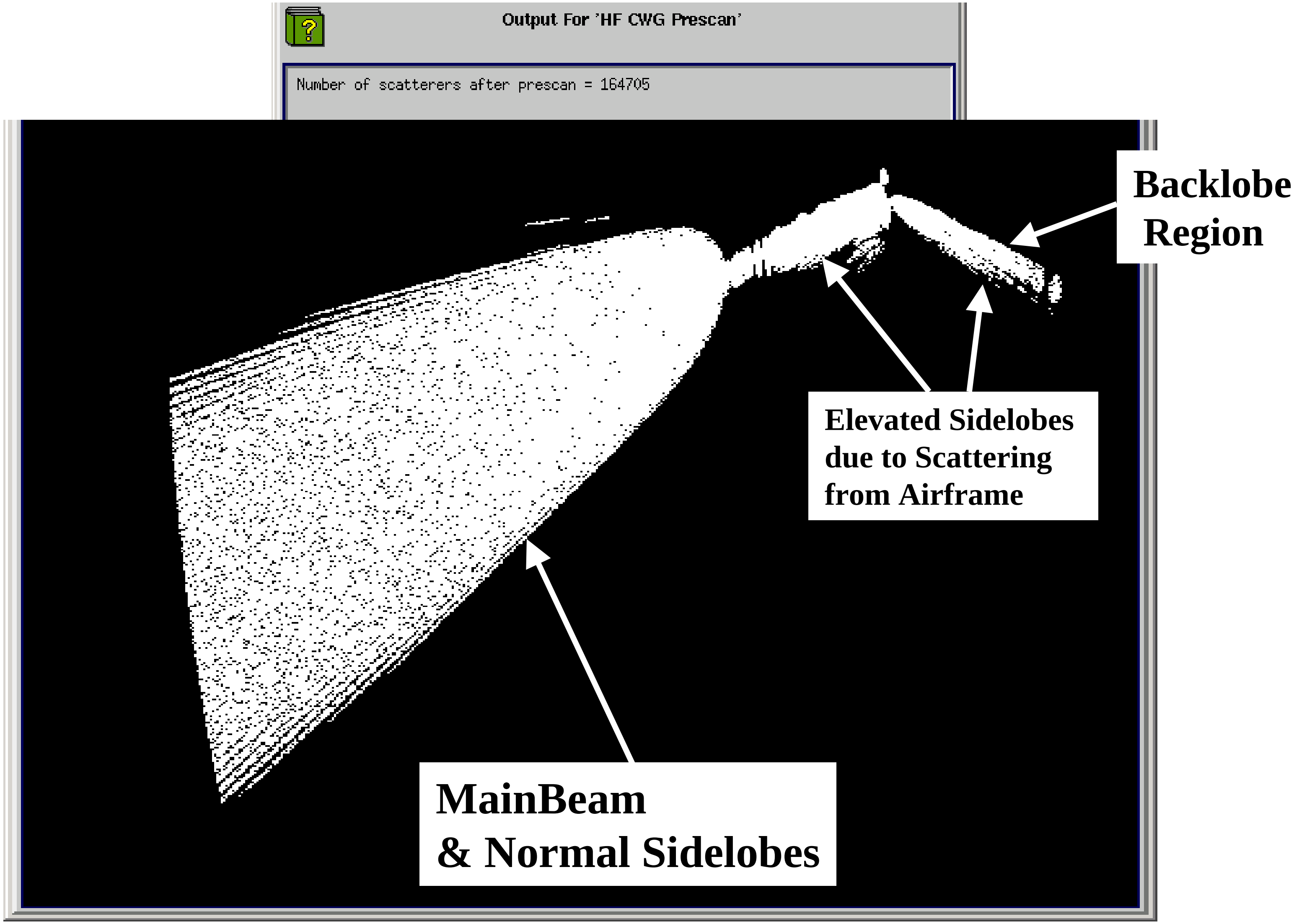
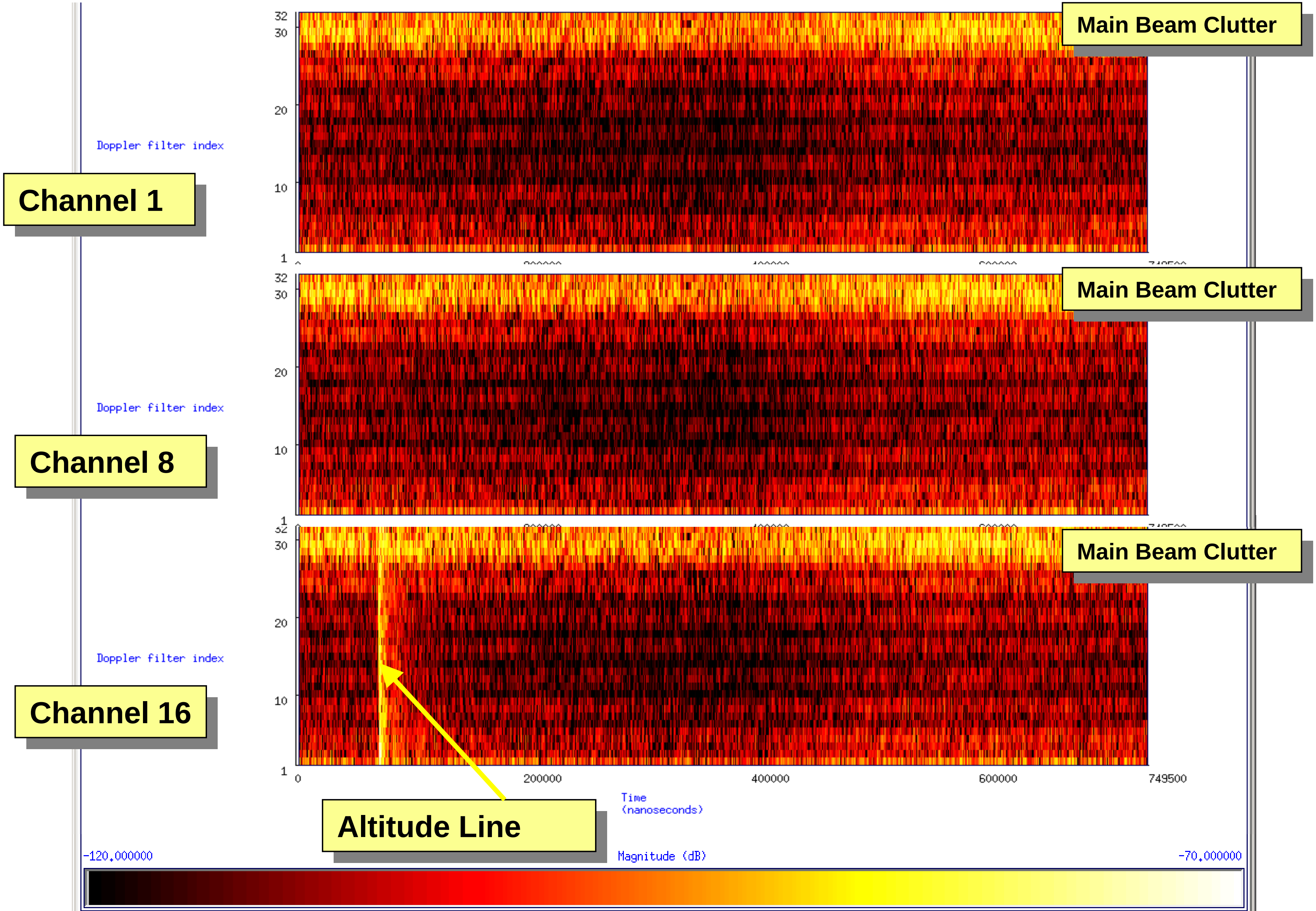
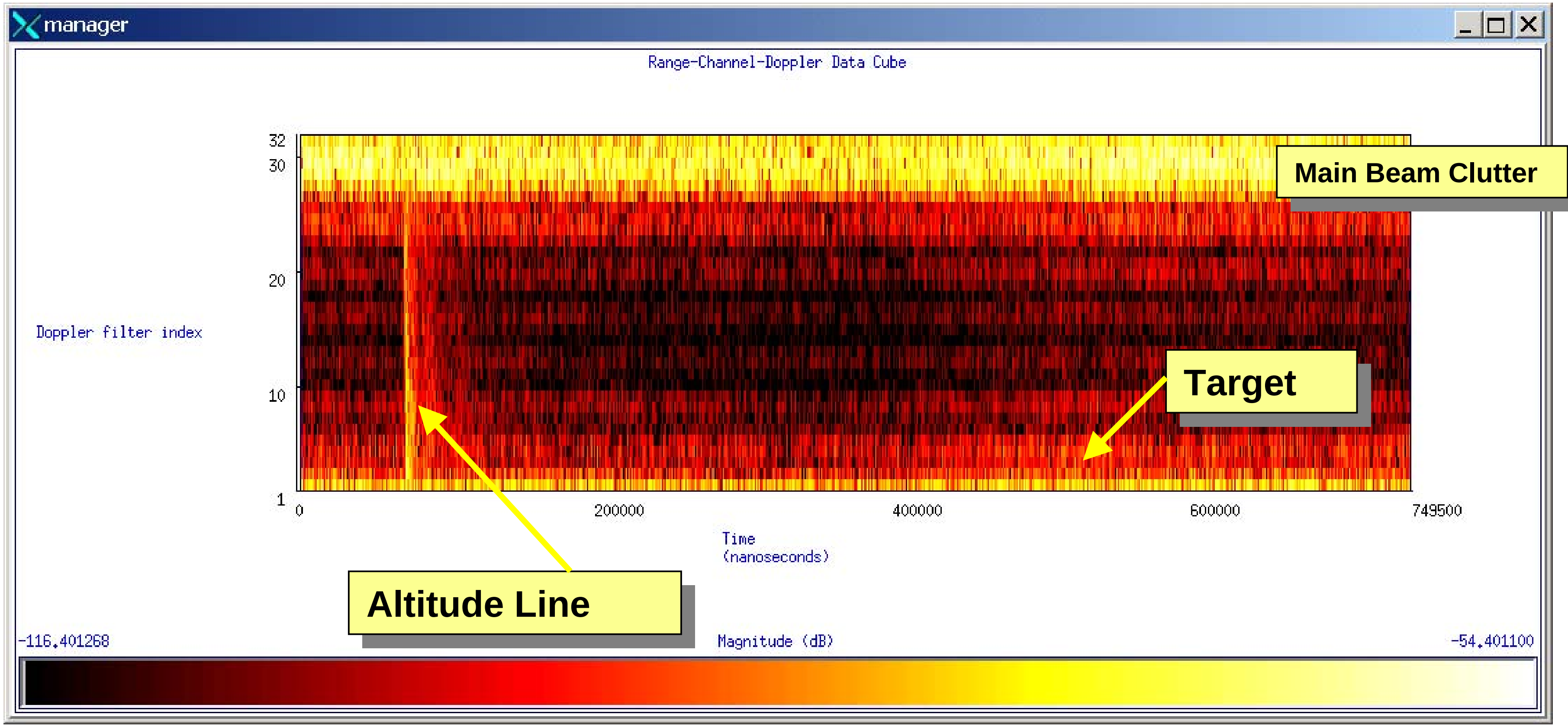
Scatterer Field after Prescan, Azimuth = 15 Deg.



Conventional Doppler Processor Output, Azimuth = 15 Deg.



Doppler Processing + Digital Beamforming, Azimuth = 15 Deg.



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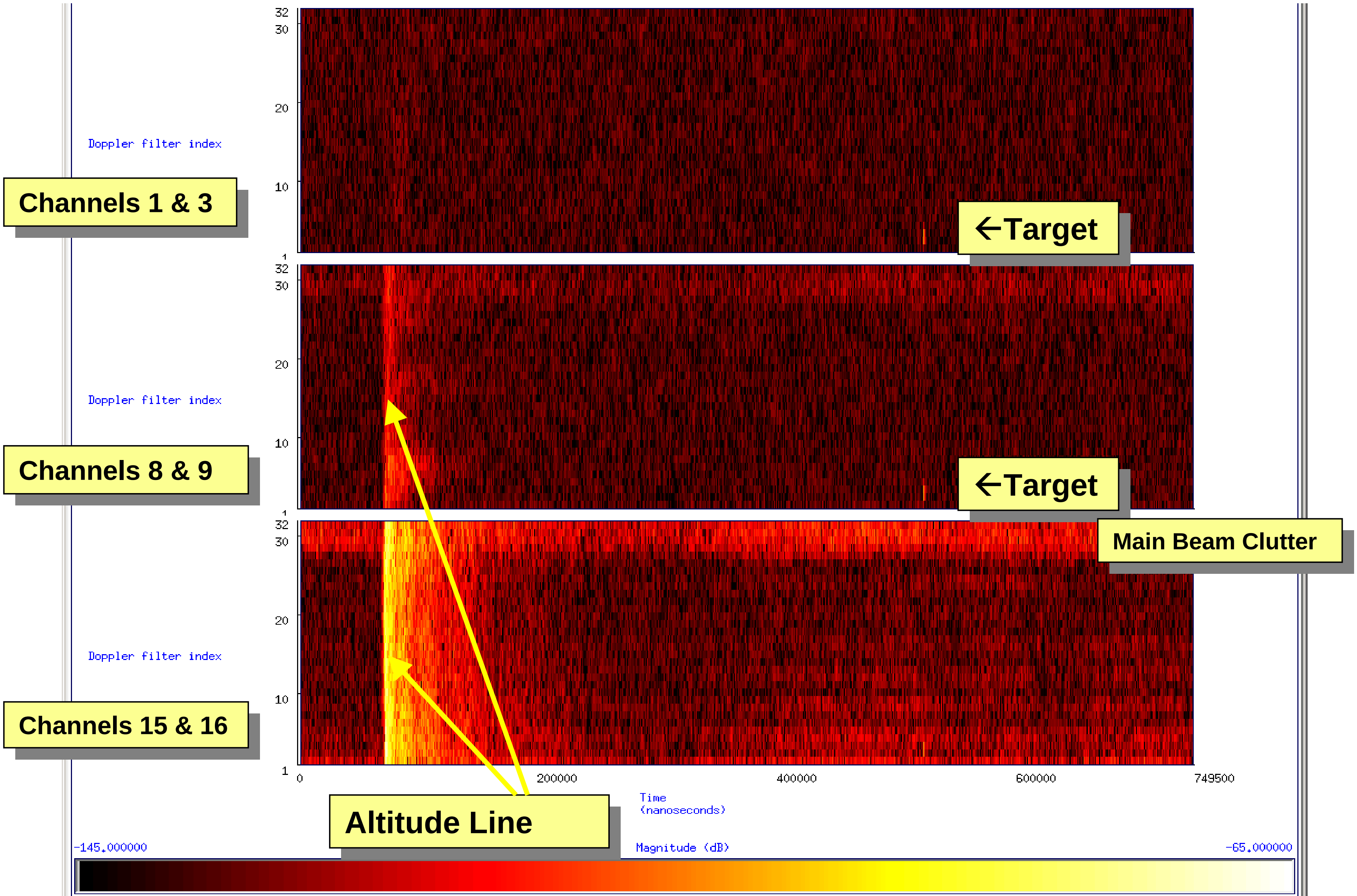
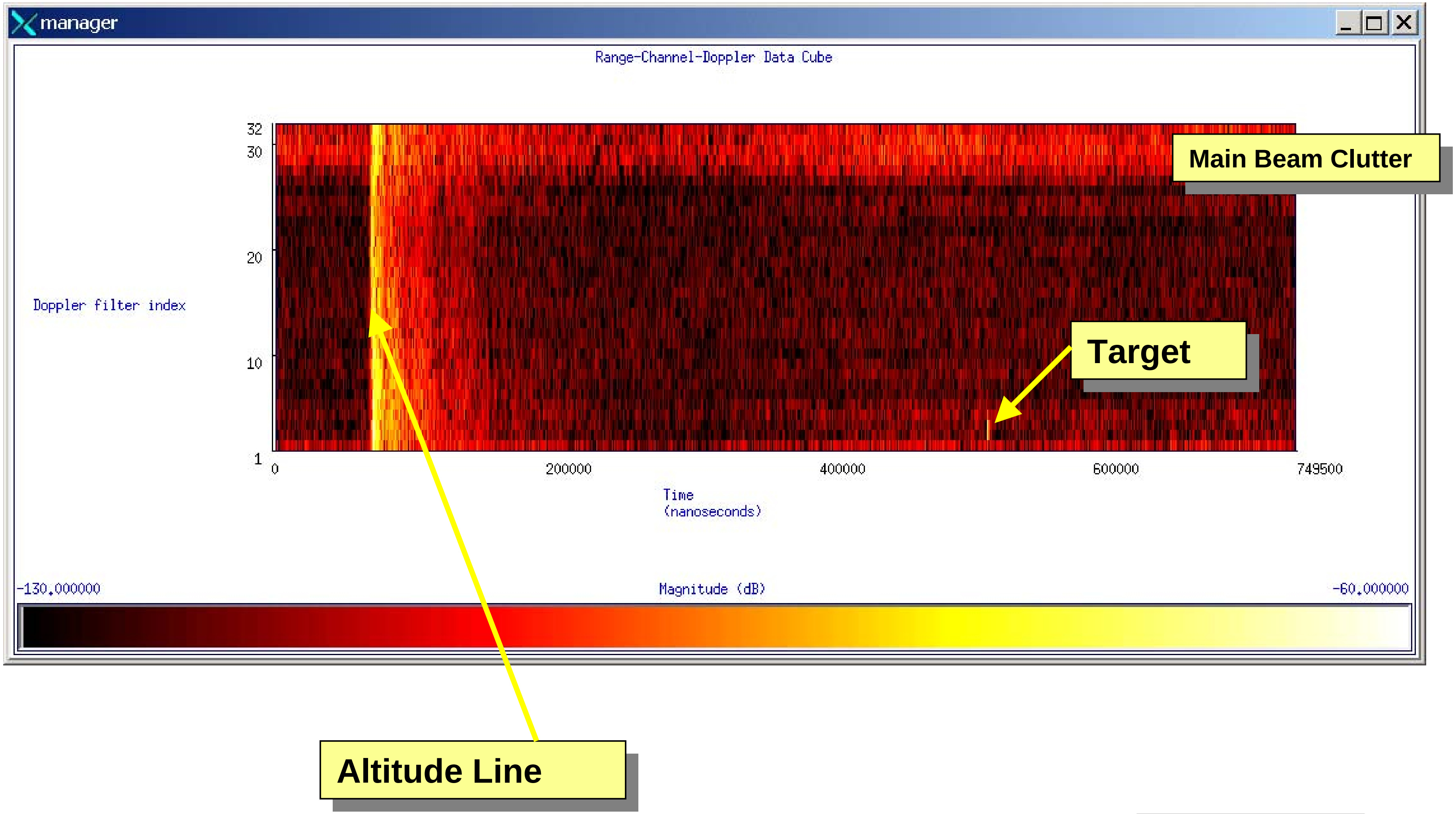
DPCA + Doppler Processing, Azimuth = 15 Deg.



DPCA + Doppler Processing + Digital Beamforming, Azimuth = 15 Deg.



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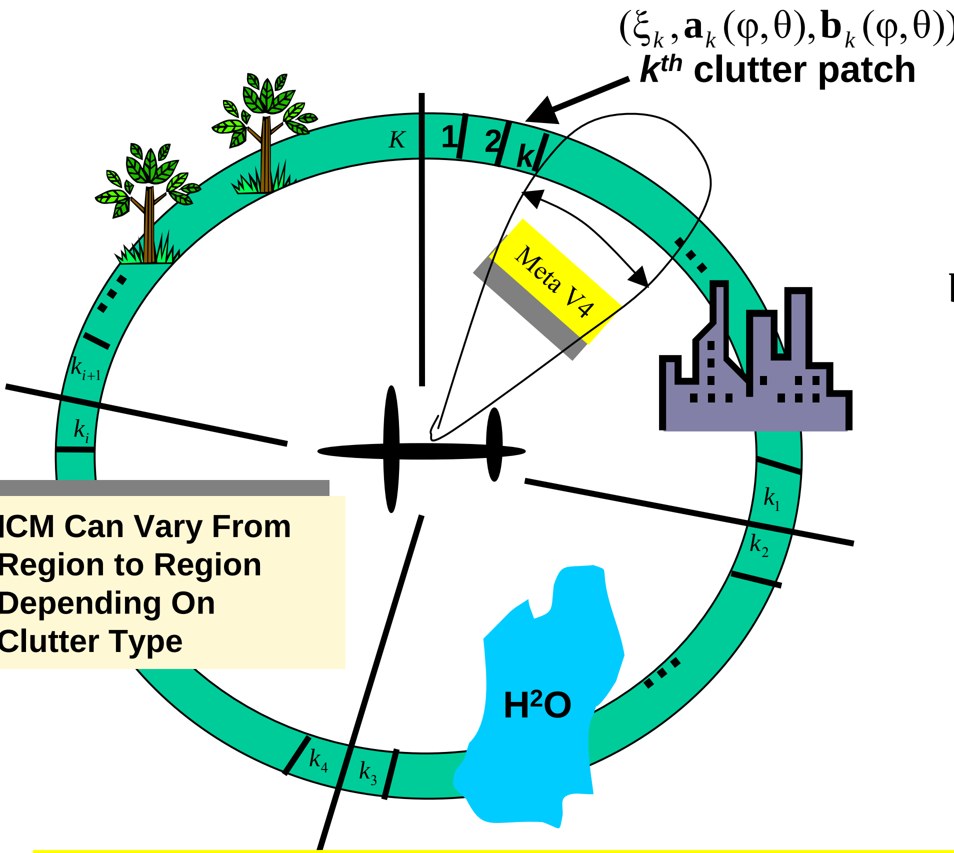
Clutter Analytic (Clairvoyant) Covariance Matrix Calculation Technique



ICM Can Vary From Region to Region Depending On Clutter Type

Spatial Vector

$$\mathbf{a}_k = \exp(j2\pi(0:N-1)\frac{d}{\lambda}\sin(\phi_k)\cos(\theta_k))$$

Temporal Vector

$$\mathbf{b}_k = \exp(j2\pi(0:M-1)\frac{2v}{\lambda \bullet prf}\sin(\phi_k + \phi_{vma})\cos(\theta))$$

Space Time Vector

$$\mathbf{v}_k = (\mathbf{b}_k \otimes \mathbf{a}_k) \in C^{NM\times 1}$$

Analytic Covariance Matrix Calculation

$$\mathbf{R} = \sum_{k=1}^{K} \xi_k^2 \mathbf{v}_k \mathbf{v}_k^H$$

Homogeneous Clutter in Low Fidelity Meta-Model
Heterogeneous Clutter in Higher Fidelity

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Adaptive Processing Metrics

- GMTI Requires Robust MDV Performance
- Moving Platform – Clutter Coupled in Angle and Doppler – Use *STAP*
- Adaptive Matched Filter With Known Covariance Matrix R
- Metric Used: SINR Loss

Received Signal Vector

$$\mathbf{x} = \alpha \mathbf{s}_t + \mathbf{n}$$

Target Signal Vector ($\mathbf{s}_t$); Thermal Noise + Interference ($\mathbf{n}$)

STAP Filter Output

$$y = \mathbf{w}^H \mathbf{x} = \alpha \mathbf{w}^H \mathbf{s}_t + \mathbf{w}^H \mathbf{n}$$

Constrained STAP Weight Vector

$$\mathbf{w} = \kappa \tilde{\mathbf{R}}^{-1} \mathbf{s}_d = \frac{\tilde{\mathbf{R}}^{-1} \mathbf{s}_d}{\mathbf{s}_d^H \tilde{\mathbf{R}}^{-1} \mathbf{s}_d}$$

Desired Signal Vector ($\mathbf{s}_d$); Estimated Interference Covariance Matrix ($\tilde{\mathbf{R}}$)

The Known Interference Covariance Matrix, R, Was Used

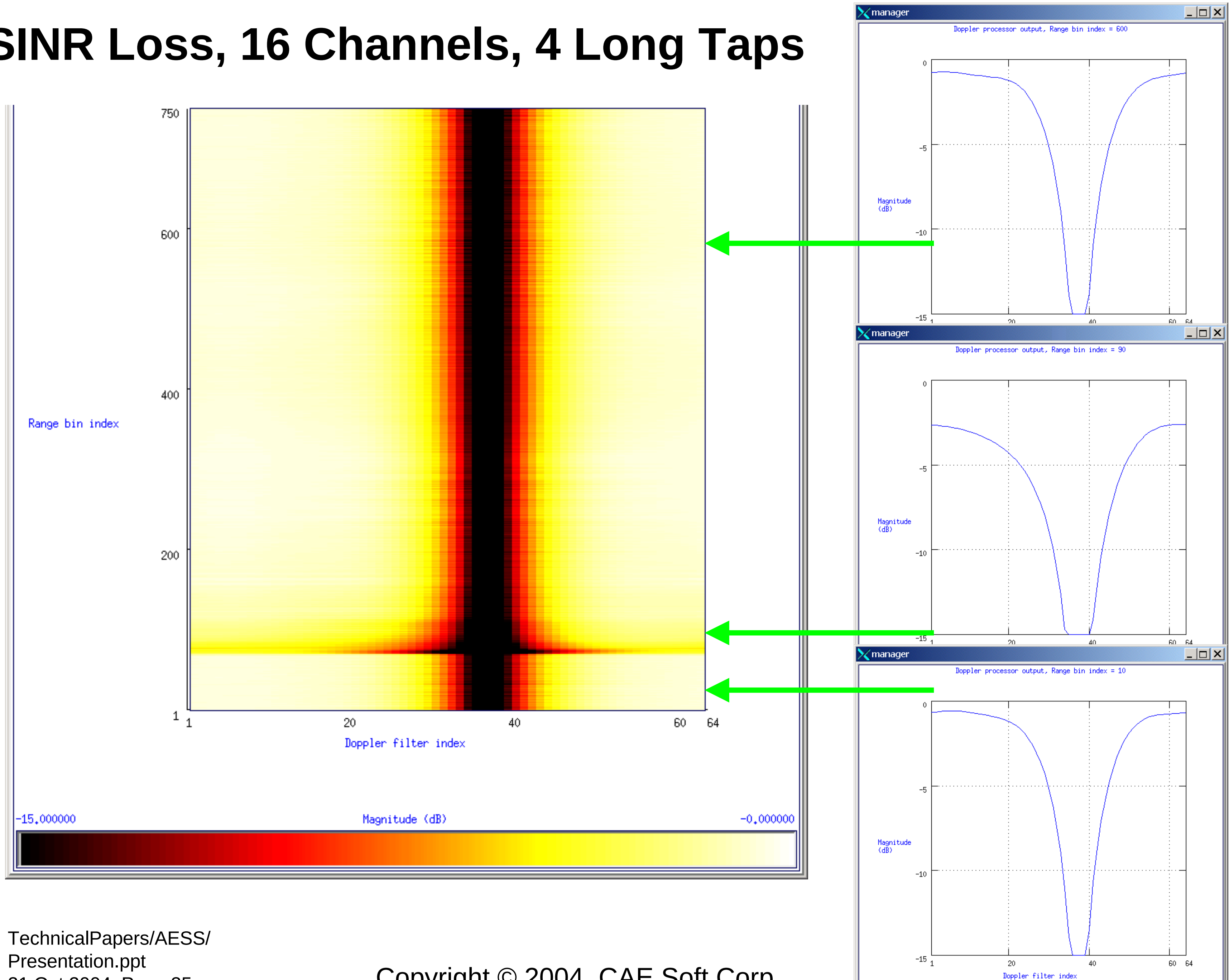
SNR $\mathbf{w} = \mathbf{s}$, $SNR = \dfrac{\alpha^2 \left|\mathbf{s}^H \mathbf{s}\right|^2}{\mathbf{s}^H \left(\sigma^2 \mathbf{I}\right) \mathbf{s}} = \dfrac{NM\alpha^2}{\sigma^2}$

SINR $SINRout = \dfrac{Ps}{Pn} = \dfrac{\alpha^2 \left|\mathbf{w}^H \mathbf{s}_t\right|^2}{\mathbf{w}^H \mathbf{R} \mathbf{w}}$

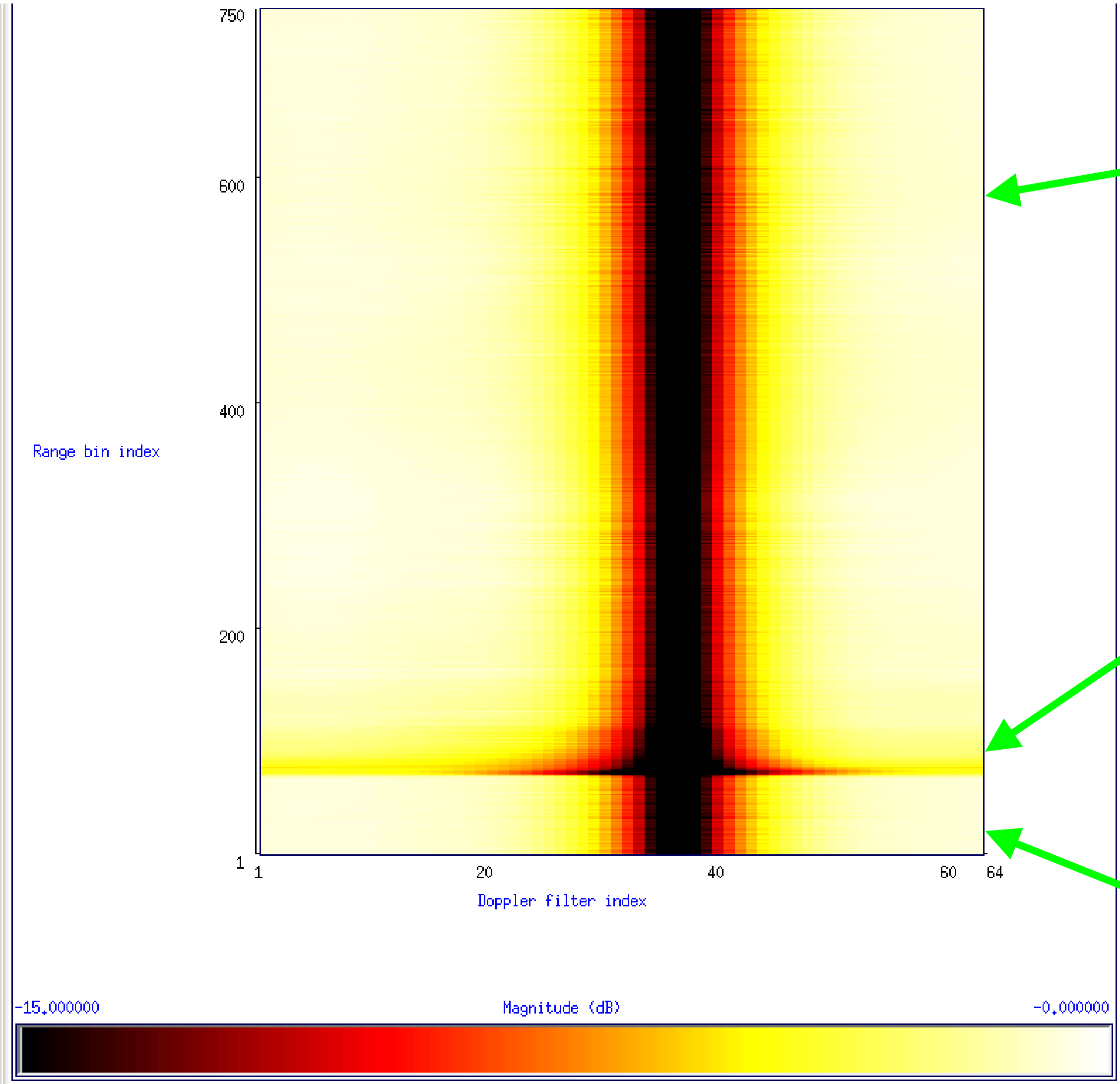
SINR Loss $SINR \;\; Loss = \dfrac{SINR}{SNR}$

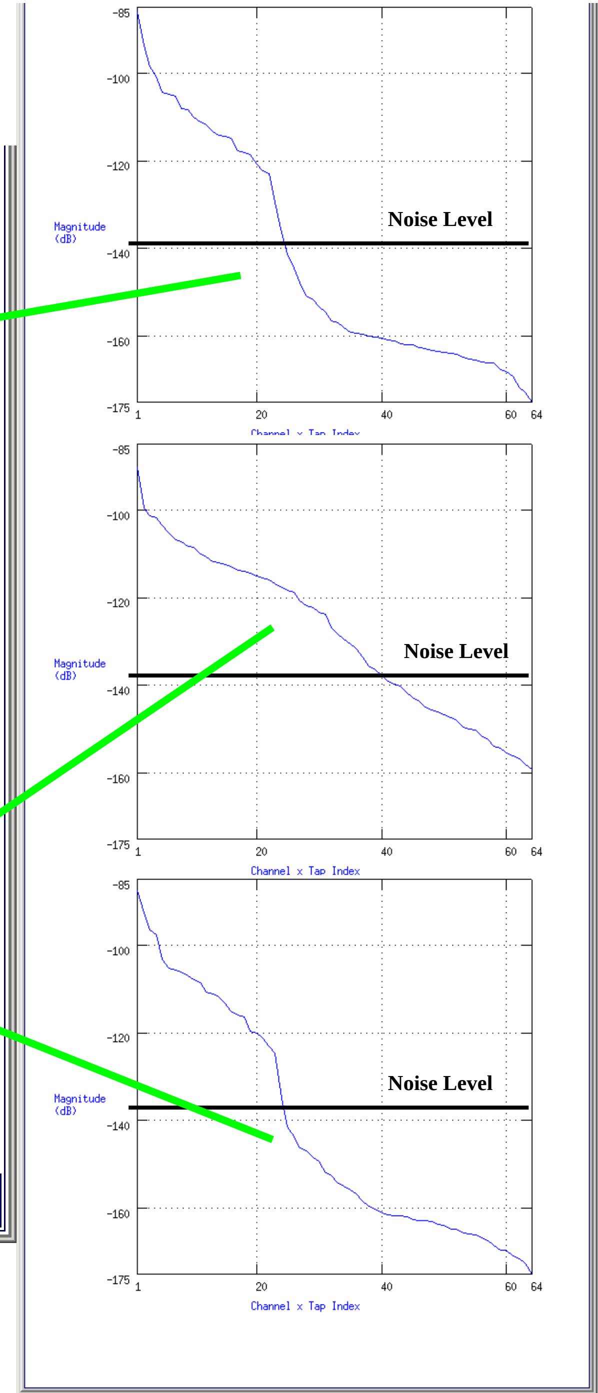
SINR Loss – Performance of STAP Filter Relative to Interference Free Case

SINR Loss, 16 Channels, 4 Long Taps



Eigen-Decomposition

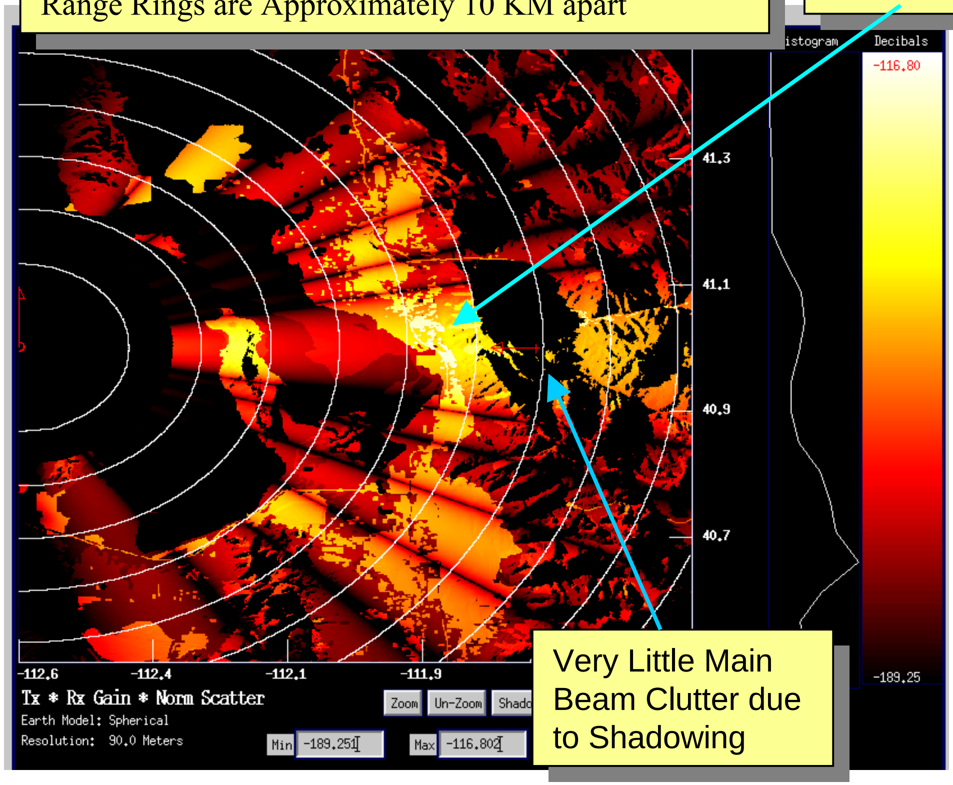




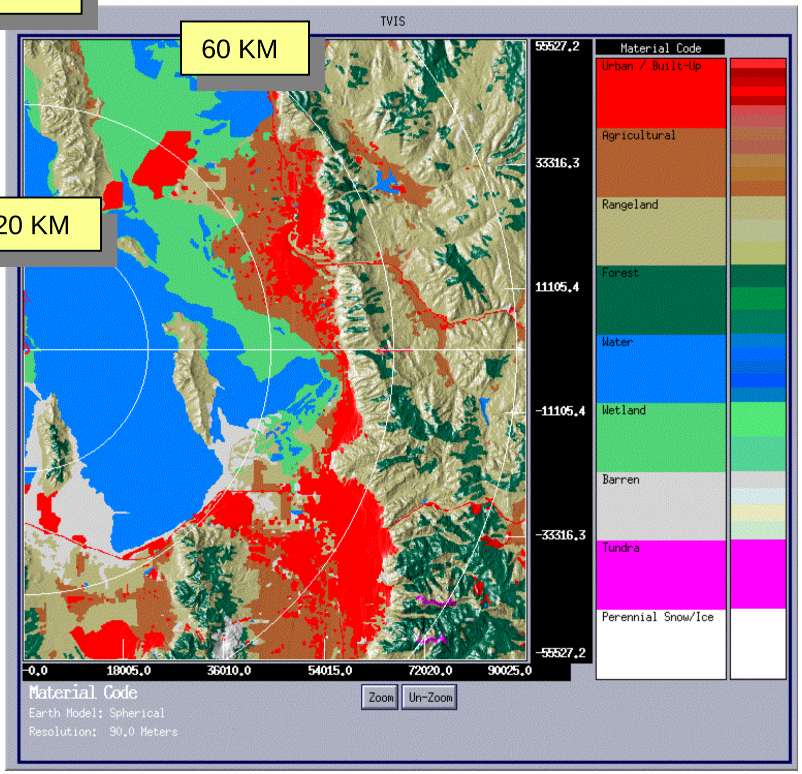
Example of Heterogeneous Clutter Background

- Radar positioned just on left border of plot, ½ way down plot
- Looking out over Great Salt Lake towards SLC and Wasatch Mt. Range

Clutter Power



Land Cover



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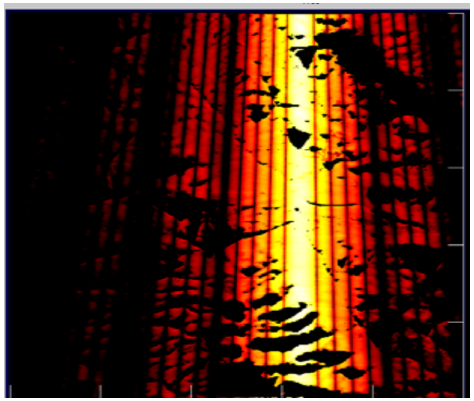
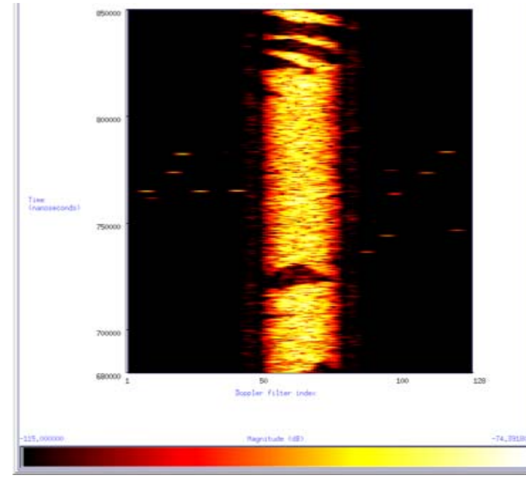
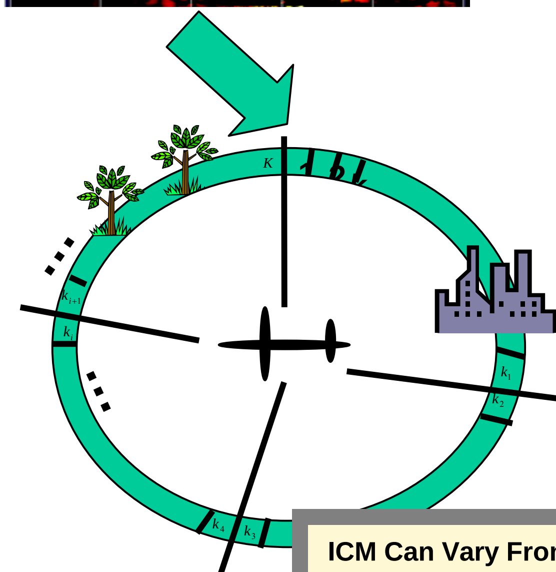
Appearance of Heterogeneous Backgrounds in High and Medium Fidelities



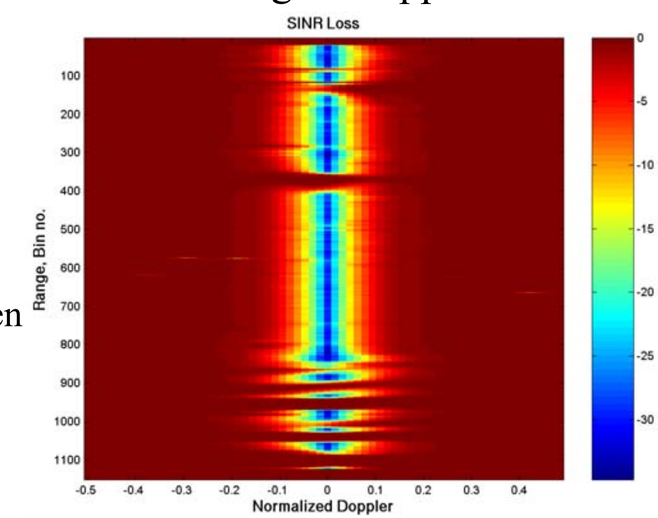
High Fidelity Waveform Gen.
& Doppler Processing



Range - Doppler



Analytic Covariance Gen
& Adaption

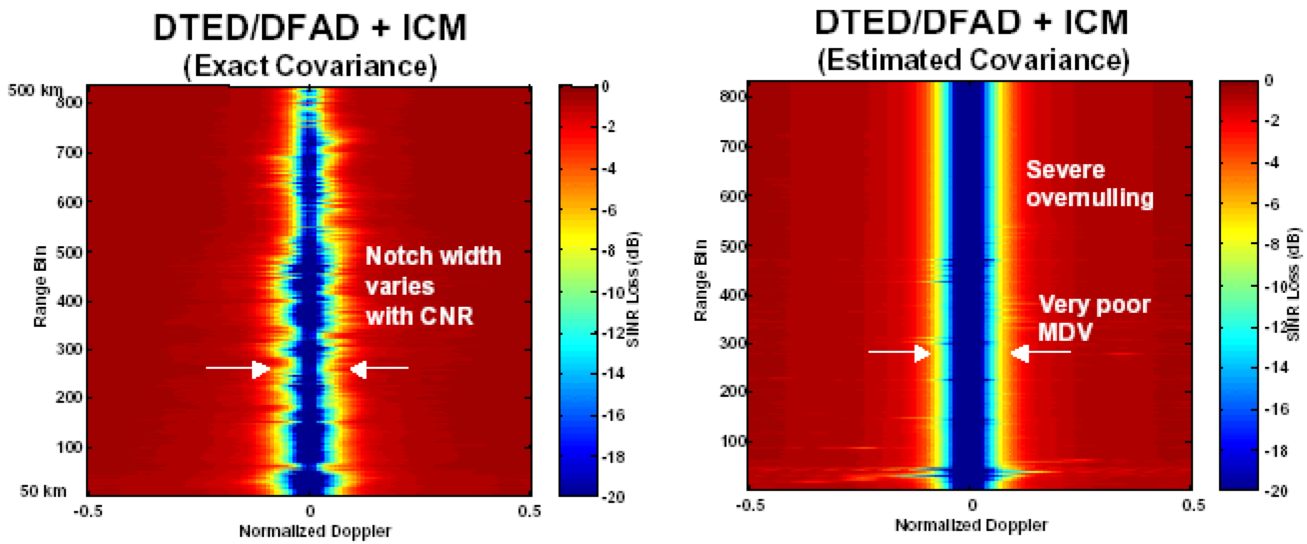


SINR Loss

ICM Can Vary From Region to Region Depending On Clutter Type

CAE Soft Corp.

Impact of Real-World Effects



- When estimating covariance over large sample support, range cells with strong CNR dominant
  - Strong CNR cells also have high rank (e.g., Doppler spread)
  - Results in severe overnulling and poor MDV
- Localized training required to track “true” clutter rank

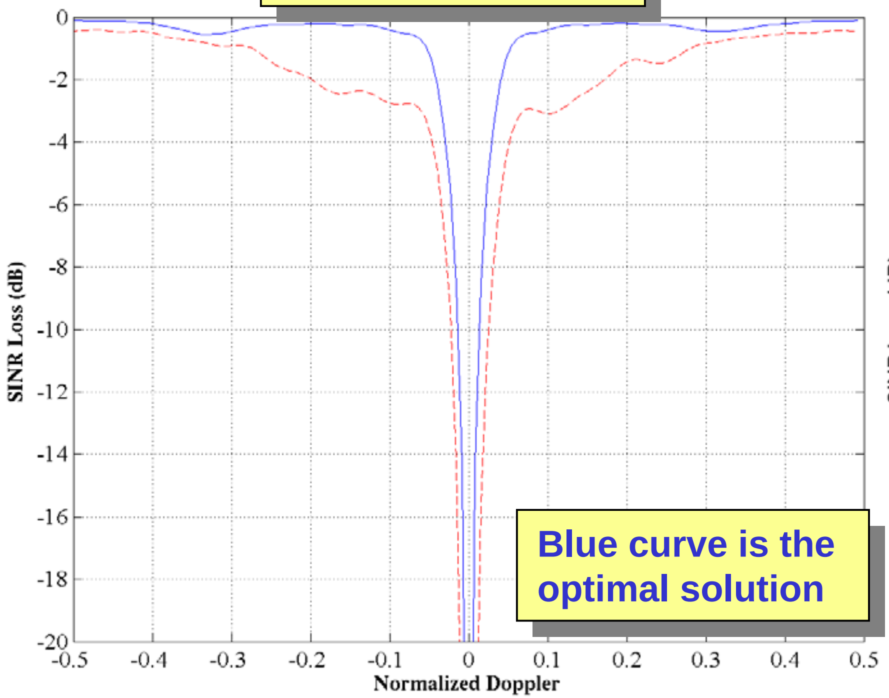
Requires Minimal Sample Support Solution

* Data courtesy of Information Systems Laboratories, Vienna, VA

CAESoft Corp.

Additional SINR Loss due to errors in estimation process

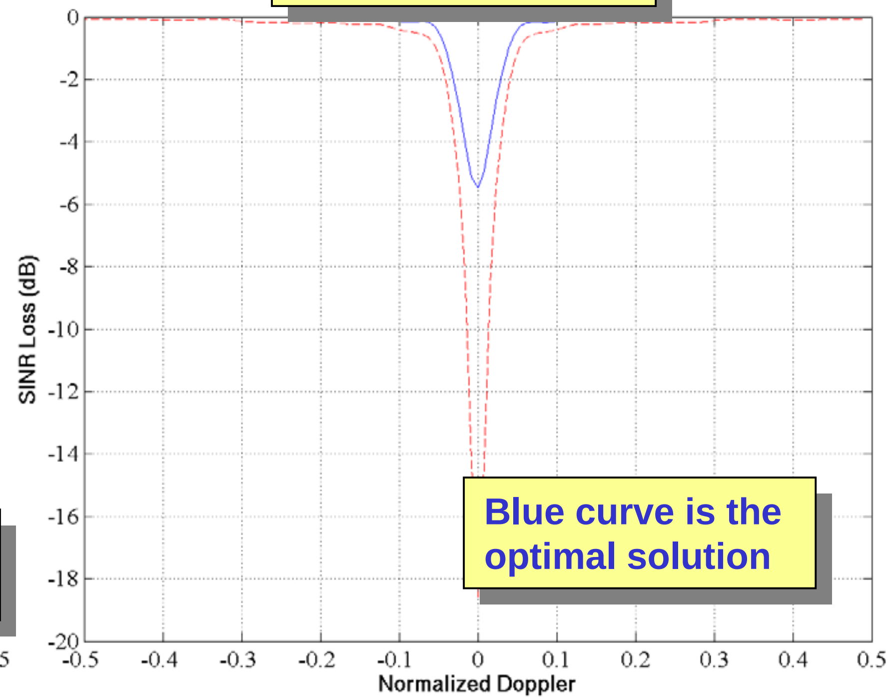
Under-Nulling



Blue curve is the optimal solution

Medium CNR = 20 dB region was used to estimate STAP weights that were applied to High CNR = 40 dB region

Over-Nulling

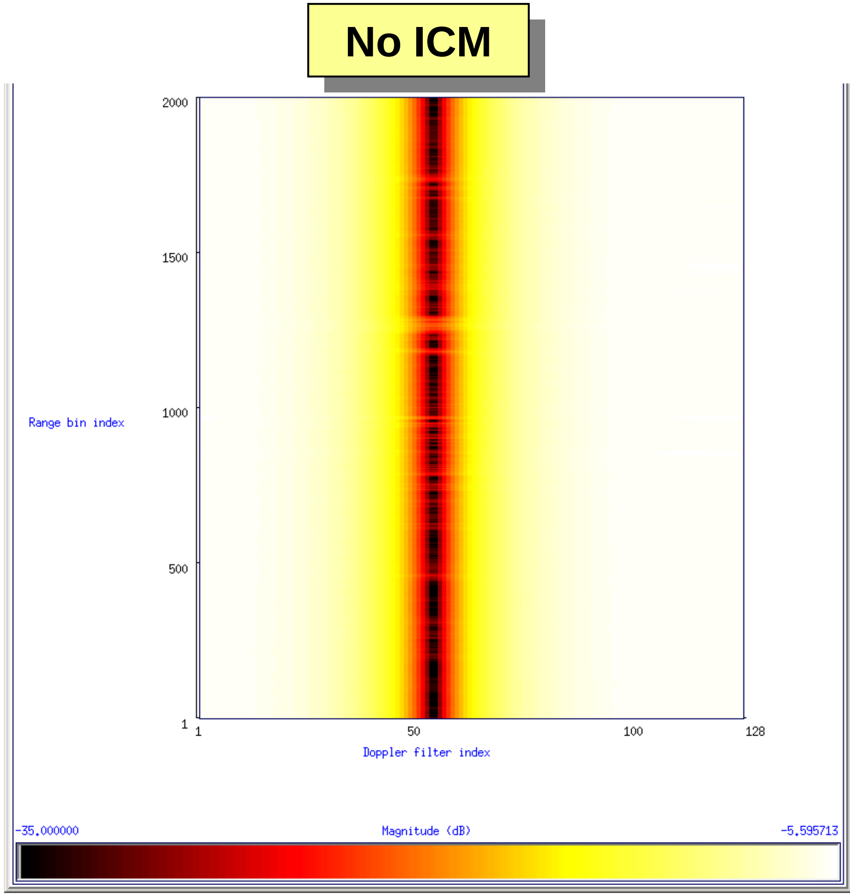


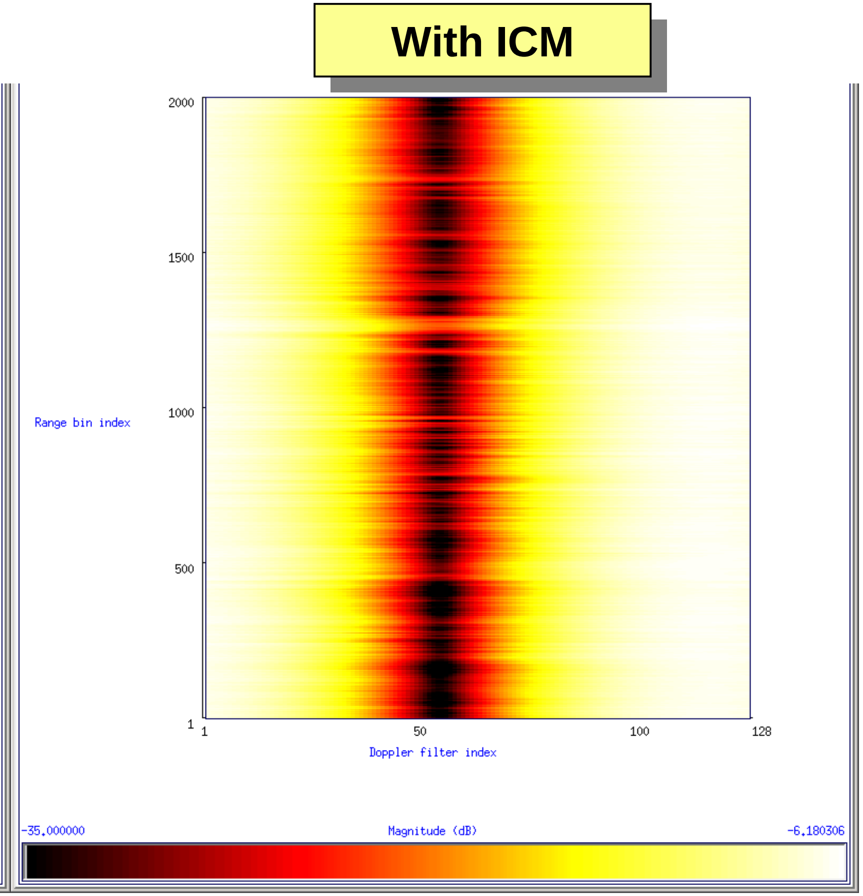
Blue curve is the optimal solution

High CNR = 40 dB region was used to estimate STAP weights that were applied to Low CNR = 10 dB region

CAE Soft Corp.

SINR Loss without and with Internal Clutter Motion





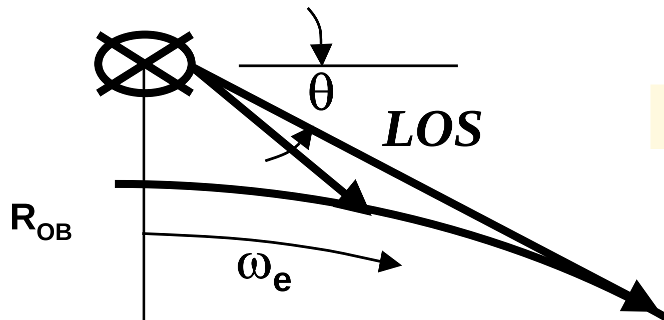
CAE Soft Corp.

Doppler Warping due to earth's rotation

H_{OB} = 500 KM

V_I = 7613 m/s

$R_{OB}\,\omega_e$ = 458 m/s



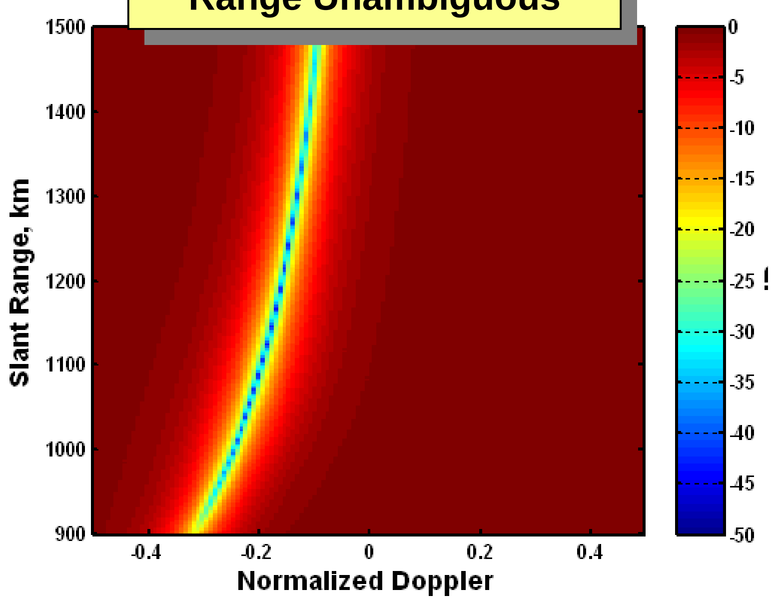
Doppler Shift Varies With Range

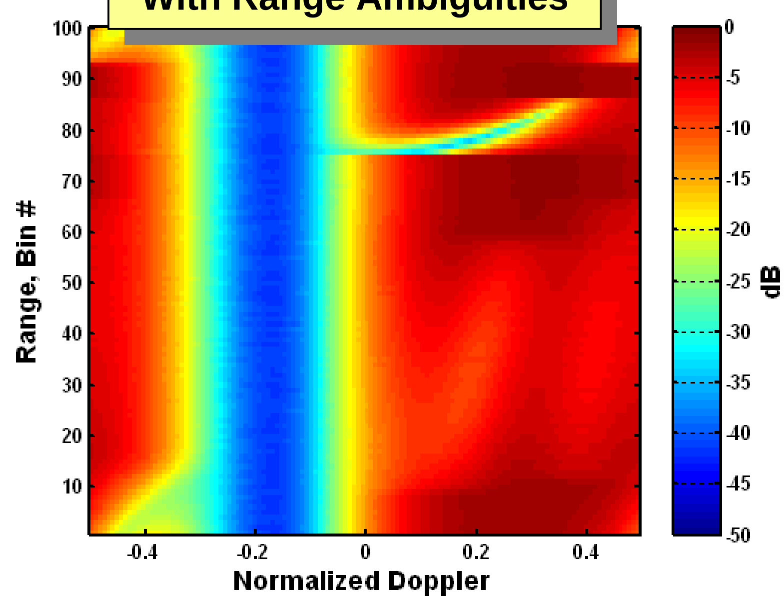
$$f_{dop_c} = \frac{-2\dot{R}}{\lambda} = \frac{-2\langle LOS \mid V_{sat}\rangle_{ECEF}}{\lambda} = -2\,R_{OB}\,\omega_e \cos\theta$$

$$|V_{sat}\rangle_{ECEF} = \begin{bmatrix} V_I \\ -R_{OB}\,\omega_e \\ 0 \end{bmatrix}; \quad |LOS\rangle_{ECEF} = \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix} \text{ for a radial along equator}$$

$\omega_e = 7.2722 \times 10^{-5}$ rad / sec = earth rotation angle rate

R_{OB} = distance from earth center to satellite

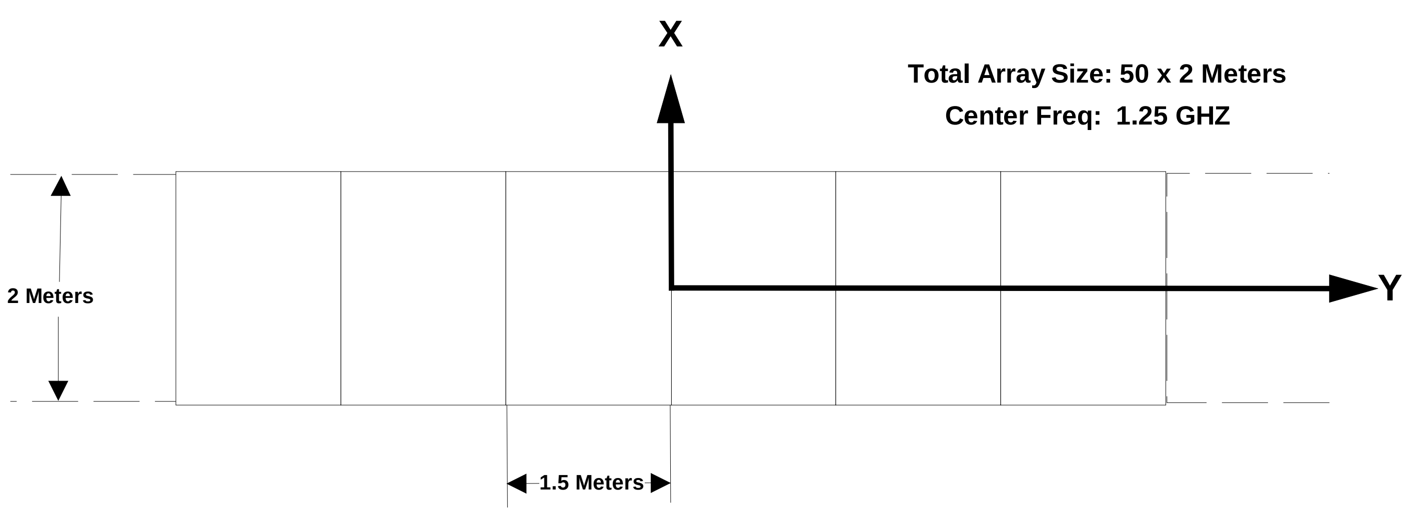




Te
Pr
21

Space Radar Systems

- Present a new set of problems due to weight and packaging constraints:
    - The array must be folded up to fit into the launch vehicle shroud
    - On orbit the array is unfolded and attached to a truss structure
- The resulting array will not be flat to within $\lambda/20$ because distortions will occur which cannot be controlled by mechanical means alone. These may be due to:
    - Static distortions due to deployment errors
    - Dynamic distortions due to station keeping, heating, etc. The dynamic distortions must be sensed at a temporal rate higher than the natural frequency of the structure and spatially at a rate greater than the highest order mode of significance.
- A robust realtime metrology scheme coupled with dynamic electronic compensation is required to live with expected mechanical deformations.



Volumetric patterns with no errors, out of plane, and orientation errors

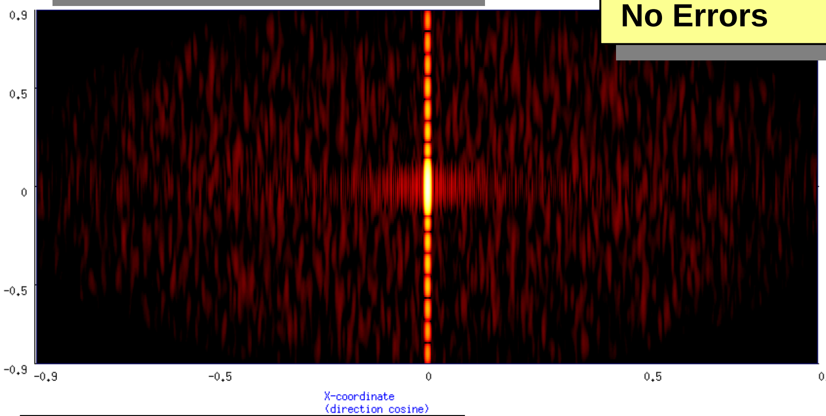
L-Band

50m x 2.5m array

32 panels

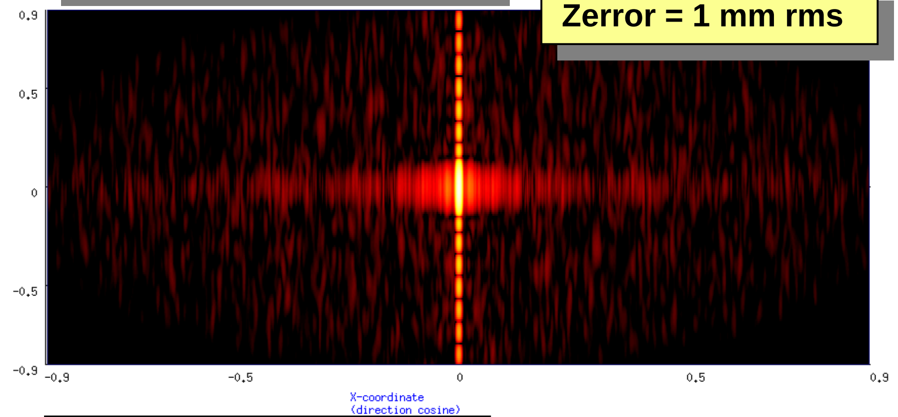
Pk Directive Gain = 41.7 dBI
RMS Sidelobes = - 55.7 dBML

No Errors



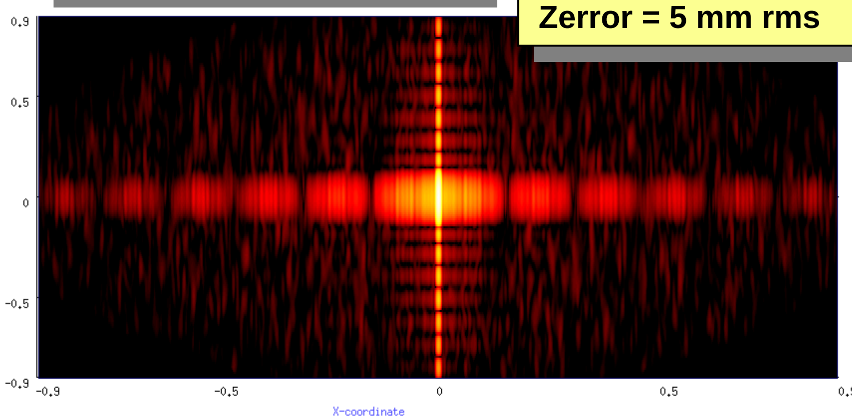
Pk Directive Gain = 41.7 dBI
RMS Sidelobes = - 55.5 dBML

Zerror = 1 mm rms



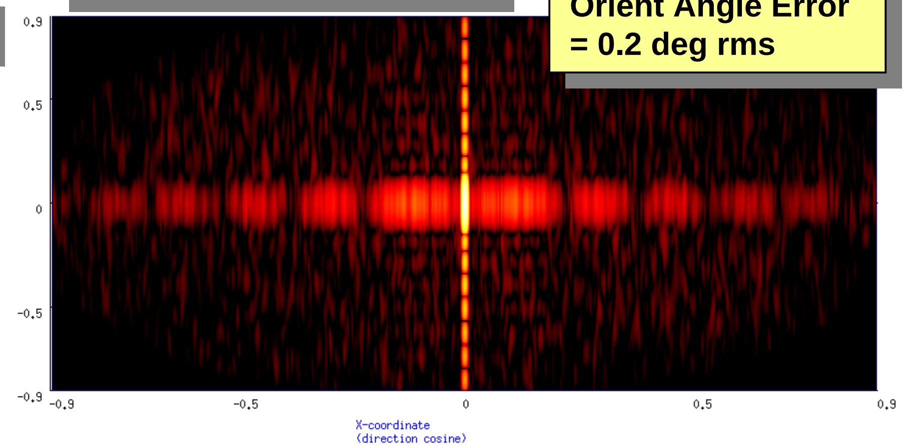
Pk Directive Gain = 41.6 dBI
RMS Sidelobes = - 51.8 dBML

Zerror = 5 mm rms

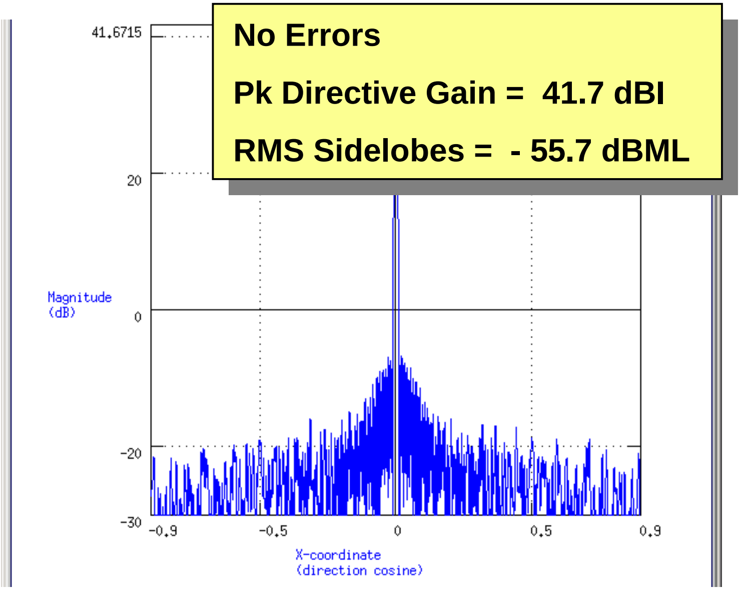


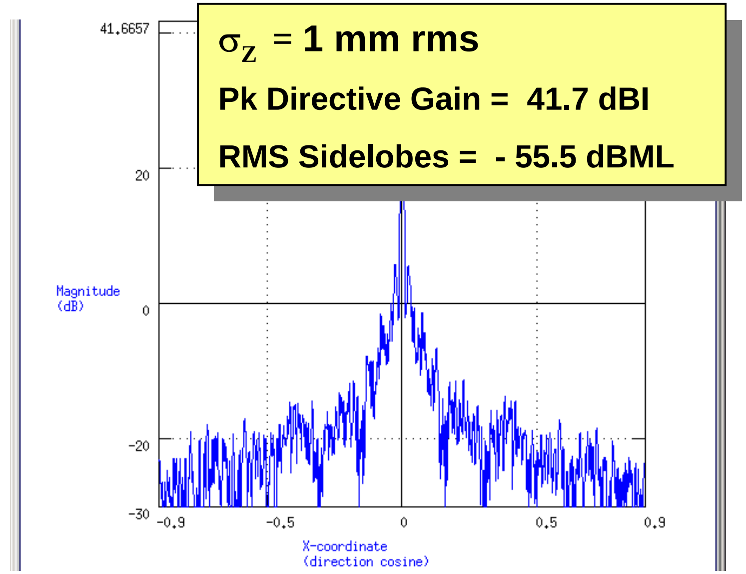
Pk Directive Gain = 41.6 dBI
RMS Sidelobes = - 54.8 dBML

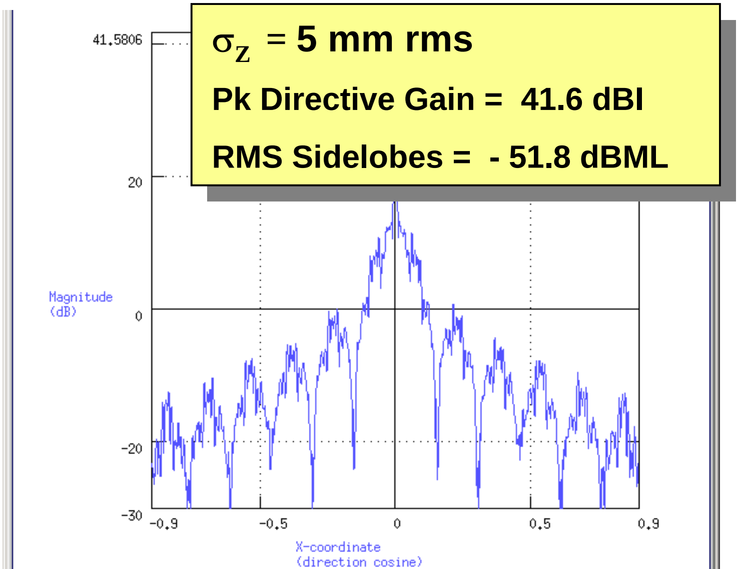
Orient Angle Error = 0.2 deg rms

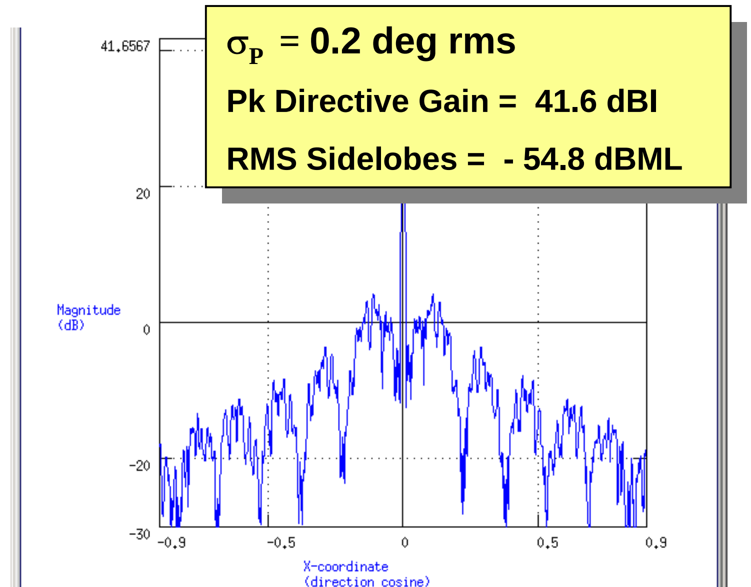


Azimuthal Cuts to show the Effect of some Errors

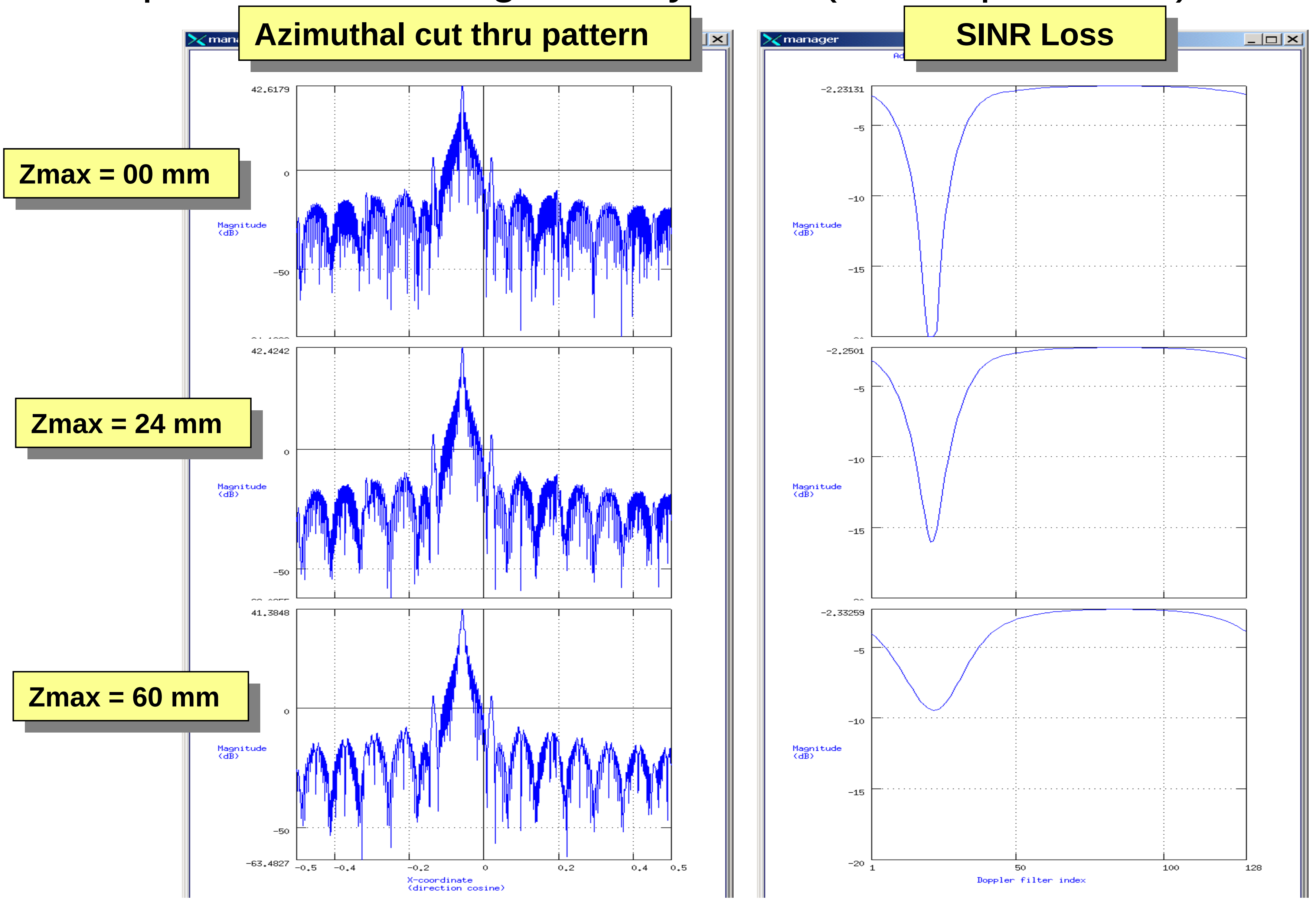








Antenna Performance & SINR Loss changes with parabolic bending of array truss (uncompensated)



Ending

Current Investigations to Mitigate Limiting Factors

- Knowledge-Based Approaches
  - Select Training Set
  - Mix Adaptive and Non-Adaptive Techniques
- Techniques that require less Sample Support in Training Set
- Orthogonal Waveforms
- Many Others