DSP Based Corrections of Analog Components in Digital Receivers

IEEE Communications, Signal Processing, and Vehicular Technology Chapters Coastal Los Angeles Section

24-April 2008
It’s all done with Computer Chips
We each own a Billion Transistors

We have an amazing wealth of resources at our disposal! Just how big is a Billion?
A stack of a billion dollar notes would be 76.2 kilometers High.
A billion seconds ago was 32.5 years ago.
We each own a Billion Transistors

For Comparison, the Eiffel Tower Contains 18,084 Parts
It is Fastened together by 2.5 Million Rivets
We each own a Billion Transistors

The world manufactures more transistors than it grows grains of rice.
How big is a billion grains of rice?

- 8mm x 2mm x 2mm (Long Grain)
- 1-billion grains of rice
- 8 Meters x 2 Meters x 2 Meters
- Or 32 Cubic Meters
- Or a cube 3.2 Meters on a side
- It weighs 24,000 kgr (26.6 tons)
- Market price, $1,000/ton; $26,600
- CLS-350 Mercedes Benz weighs 2,200 kgr (gross wt)
From Where Did DC Come?

- Self Mixing in Analog Down Converter
- ADC Injects DC
- 2's Complement Arithmetic Injects DC
Self Mixing in Down Converter

DC Offset: Self Mixing
Desired Component: Mixing
Spectral Image: I-Q Imbalance
DC From A-to-D Converter

X(n) $\rightarrow$ ADC $\rightarrow$ $x_q(n)$

CLK

$x_q$ $\rightarrow$ $x$

Rounding Quantizer

$X(n)$ $\rightarrow$ ADC $\rightarrow$ $X_q(n)$

CLK

$x_q$ $\rightarrow$ $x$

Truncating Quantizer
Truncating Quantizer

Highest Allowable Output Value Not Greater than Measured Value

Sampled Signal Values → Quantized Sample Values

Quantization Errors

Sample Time

Amplitude

(N+1)q

(N-1)q
2's Complement
Number Representation

Quantized Number Line

Negative numbers:
Measure displacement from reference.
Reference = -Nmin

Positive numbers:
Measure displacement from reference.
Reference = 0
Errors Due to finite Arithmetic

Quantize Addition  \[ s = x_1 + x_2 + q_A \]
Quantize Coefficient  \[ h \]
Quantize Multiplication  \[ p = x \cdot h_Q + q_M \]
Finite Arithmetic in Radix-2 FFT Algorithm
Radix-2 FFT
Signal Flow Diagram
Algorithm Noise Due to Finite Arithmetic, Coefficient Noise
Algorithm Noise due to Finite Arithmetic, Scaling Noise
DC Canceller, DC Notch Filter

\[ H(Z) = \frac{Z - 1}{Z - (1 - \alpha)} \]
Spectral Response

magnitude response

phase response
DC Canceller with Embedded Sigma-Delta Converter

Suppress Bit Growth with Sigma-Delta Converter in Feedback Path
DC Canceller Time Series

Output Time Series, DC Canceller

DC Estimate, of DC Canceller

DC Canceller with ε-Δ Quantizer

DC Canceller with ε-Δ Quantizer
Spectra
Input and Output of Canceller

Spectrum, Input to DC Canceller

Spectrum, Output DC Canceller

Spectrum, Output Quantized DC Canceller

Spectrum, Output DC Canceller with Σ-Δ Quantizer
Tunable Notch, Spin the Delay Line

\[ H(Z) = \frac{Z - e^{j\phi}}{Z - (1 - \alpha)e^{j\phi}} \]
Spectral Response
Tuning With LP-to-BP Transformation

\[ H(Z) = \frac{Z^2 - 2cZ + 1}{Z^2 - 2c(1 - \alpha)Z + (1 - 2\alpha)} : c = \cos(\phi) \]
Implementing LP-to-BP Transformation

\[
\begin{align*}
1 - cZ & \quad Z - c \\
Y(z) & = y(z) + b_1 \frac{1}{Z} x(z) \\
Y(z) z^{-1} & = y(z) + b_1 \frac{1}{Z} x(z) \\
X & \quad Y \\
\end{align*}
\]
Spectral Response

Magnitude Response

Phase Response
Self Tuning: Reference Canceling

\[ y(n) = \frac{x(n)}{e^{j\theta}} \]

\[ e^{j\theta} \]

\[ Z^{-1} \]

\[ w(n) \]

\[ w(n+1) \]

\[ \mu \]
Filters have Same Transfer Function

\[ y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k) \]

\[ e^{j\theta} \]

\[ z^{-1} \]

\[ w(n) \]

\[ w(n+1) \]

\[ e^{j\theta} \]

\[ x(n) \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow y(n) \]

\[ x(z) \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow f(z) \rightarrow - \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow y(z) \]

\[ e^{j\phi} \]

\[ z^{-1} \]

\[ w(n) \]

\[ w(n+1) \]

\[ e^{j\theta} \]

\[ x(n) \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow y(n) \]

\[ x(z) \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow f(z) \rightarrow - \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow + \rightarrow y(z) \]

\[ e^{j\phi} \]

\[ z^{-1} \]
I-Q Imbalance
Ideal I-Q Mixing
Real Sinusoids, Time and Frequency

![Diagram showing real sinusoids](image)

- $A/2$ at $f_0$ and $-f_0$ on the Real axis.
- $A/2$ at $f_0$ and $-f_0$ on the Imag axis.

Graphs of $A \cos(\omega_0 t)$ and $A \sin(\omega_0 t)$ vs. time ($t$).
Complex Sinusoids-I

- $A \cos(\omega_0 t)$
- $j A \sin(\omega_0 t)$

Graphs showing real and imaginary components of sinusoidal functions.
Complex Sinusoids-II

- \( \frac{A}{2} \) Real
- \(-f_0\)
- Imag

- \( f_0 \)

- \( f \)

- \( A \) Real
- \(-f_0\)
- Imag

- \( f_0 \)

- \( A \) Real
- \(-f_0\)
- Imag

- \( f_0 \)

- \( f \)

- \( A \)

- \( \theta(t) = -\omega_0 t \)
- \( e^{-j\omega_0 t} \)
- \( jA \sin(\omega_0 t) \)
- \( A \cos(\omega_0 t) \)
Imbalance: New Spectral Terms

\[ x = \{ A \cos(2\pi f_0 t) \} \]

\[ i y = \{ i \ A \sin(2\pi f_0 t) \} \]

\[ x + iy = \{ A \exp(j 2\pi f_0 t) \} \]

\[ x = \mathcal{F}\{ A \cos(2\pi f_0 t) \} \]

\[ i y = \mathcal{F}\{ i (1 + \varepsilon) A \sin(2\pi f_0 t + \alpha) \} \]

\[ x + iy = \mathcal{F}\{ \text{sum} \} \]
Arbitrary Signals, Time and Frequency

Real Signal: Hermitian Transform

Imaginary Signal: Anti-Hermitian Transform
Complex Baseband & Complex Band-Centered

\[ [x(t) + j y(t)] \]

\[ H(\omega) \]

\[ [x(t) + j y(t)] e^{+j \omega_c t} \]

\[ H(\omega - \omega_c) \]

\[ [x(t) - j y(t)] \]

\[ H^*(\omega) \]

\[ [x(t) - j y(t)] e^{-j \omega_c t} \]

\[ H^*(\omega + \omega_c) \]
Complex Baseband & Real Band-Centered

\[ H^*(\omega + \omega_c) \]

\[ [x(t) + jy(t)] \]

\[ e^{j\omega_c t} + [x(t) - jy(t)] e^{-j\omega_c t} \]

\[ x(t) \cos(\omega_c t) - y(t) \sin(\omega_c t) \]
Complex Down Conversion

\[ H^*(\omega + \omega_c) \]

\[ H^*(\omega + 2\omega_c) \]

\[ H(\omega) \]

\[ H(\omega - \omega_c) \]

\[ [x(t) + jy(t)] e^{j\omega c} + [x(t) - jy(t)] e^{-j\omega c} \]

\[ \{ [x(t) + jy(t)] e^{j2\omega_c} + [x(t) - jy(t)] e^{-j2\omega_c} \} \]

\[ [x(t) + jy(t)] e^{+j\omega c} + [x(t) - jy(t)] e^{-j\omega c} \]

\[ [x(t) + jy(t)] e^{+j2\omega c} + [x(t) - jy(t)] e^{-j2\omega c} \]
Gain and Phase Imbalance in I-Q Mixers

\[ \cos(\omega_0 t) \]

\[ \sin(\omega_0 t) \]

Balanced

\[ \text{Match} \]

\[ \hat{i}(t) \]

\[ \hat{Q}(t) \]

\[ \cos(\hat{\omega}_0 t) \]

\[ (1+\epsilon) \sin(\hat{\omega}_0 t+\alpha) \]

Imbalanced
Balanced Mixers

Input Spectrum: Signal at $f = +0.15$

Quadrature Local Oscillator Spectrum: at $f = -0.1$

Down-converted Spectrum: Desired Signal at $f = +0.05$

Filtered Spectrum: Desired Signal at $f = +0.05$
Gain Imbalance

Input Spectrum: Signal at $f = 0.15$

Quadrature Local Oscillator Spectrum: at $f = -0.1$

Down Converted Spectrum: Desired Signal at $f = +0.05$

Filtered Spectrum: Desired Signal at $f = +0.05$

Downconverted Tone

Gain-Imbalance

Phase-Imbalance

Filter-Imbalance

Balance
Phase Imbalance

- Input Spectrum: Signal at f = 0.15
- Quadrature Local Oscillator Spectrum: at f = -0.1
- Down Converted Spectrum: Desired Signal at f = 0.05
- Filtered Spectrum: Desired Signal at f = 0.05

- Downconverted Tone

- Gain-Imbalance
- Phase-Imbalance
- Filter-Imbalance

Balance
Gain and Phase Imbalance

Input Spectrum: Signal at f = 0.15

Quadrature Local Oscillator Spectrum: at f = -0.1

DownConverted Spectrum: Desired Signal at f = 0.05

Filtered Spectrum: Desired Signal at f = 0.05
Filter Imbalance

Input Spectrum: Signal at $f = +0.15$

Quadrature Local Oscillator Spectrum: at $f = -0.1$

Down Converted Spectrum: Desired Signal at $f = +0.05$

Filtered Spectrum: Desired Signal at $f = -0.05$

Downconverted Tone

Gain-Imbalance

Phase-Imbalance

Filter-Imbalance

Balance
Effect of Gain Imbalance

\[ \frac{A}{2} \quad \text{Real} \]
\[ -f_0 \quad \text{Imag} \]
\[ \frac{A}{2} \quad f_0 \]
\[ f \]

\[ \frac{A}{2} (1 + \varepsilon) \quad \text{Real} \]
\[ -f_0 \quad \text{Imag} \]
\[ f_0 \quad f \]

\[ \frac{A}{2} (1 + \varepsilon) \quad \text{Real} \]
\[ -f_0 \quad \text{Imag} \]
\[ f_0 \quad f \]

\[ A (1 + \frac{\varepsilon}{2}) \quad \text{Real} \]
\[ -f_0 \quad \text{Imag} \]
\[ f_0 \quad f \]

\[ A (1 + \frac{\varepsilon}{2}) \quad \text{Real} \]
\[ -f_0 \quad \text{Imag} \]
\[ f_0 \quad f \]

Image Spectra
Due to gain imbalance

\[ H(\omega) \left(1 + \frac{\varepsilon}{2}\right) \quad \text{Real} \]
\[ -f_0 \quad \text{Imag} \]
\[ f_0 \quad f \]

\[ -H^*(\omega) \frac{\varepsilon}{2} \quad \text{Real} \]
\[ -f_0 \quad \text{Imag} \]
\[ f_0 \quad f \]
Effect of Phase Imbalance
Description and Model of Baseband Down Conversion

\[ H'(\omega) = H(\omega) + 0.5(\varepsilon + j\alpha)H(\omega) - 0.5(\varepsilon - j\alpha)H^*(\omega) \]

\[ (x' + jy) = (x + jy) + 0.5 \varepsilon (x + jy) + j0.5 \alpha (x + jy) \]
\[ - 0.5 \varepsilon (x - jy) + j0.5 \alpha (x - jy) \]

\[ (x' + jy) = x + j[y(1 + \varepsilon) + \alpha x] \]
Effect of I-Q Imbalance on 16-QAM Constellation

I-Q samples of 16-QAM signal with I-Q gain and phase mismatch.
Input Signal Conditioning and Correction

- Analog I/Q Down Convert
- Analog Low Pass Filters
- A-to-D Converter
- DC Cancel
- Phase Balance
- Gain Balance
- Fixed Equalizer

Analog Signals

Digital Signals
DC-Cancel and I-Q Imbalance Correction

\[ \hat{I} \]

\[ \hat{Q} \]
I-Q Imbalance
Correction Algorithms
Gain and Phase Parameter Trajectories

**Gain Parameter**
- **Amplitude** scale: 0.85 to 1.05
- **Time (in samples)** range: 0 to 8000

**Phase Parameter**
- **Amplitude** scale: 0 to 0.5
- **Time (in samples)** range: 0 to 8000
Constellation after Correcting Gain and Phase Imbalance

![Constellation of I-Q Samples After Phase and Gain Adaptation](image-url)
• That was the Easy part

• The Easy part is over

• Consider a channelizer!
Analog Block Conversion
Digital Second Conversion

\[ B(\omega) \]

\[ 0.5 (\varepsilon + j\alpha) B(\omega) \]

\[ -0.5 (\varepsilon - j\alpha) A^*(\omega) \]

\[ A(\omega) \]

\[ 0.5 (\varepsilon + j\alpha) A(\omega) \]

\[ -0.5 (\varepsilon - j\alpha) B^*(\omega) \]
Description and Model of Block Down Conversion

\[ A'(\omega) = A(\omega) + 0.5(\varepsilon + j\alpha)A(\omega) - 0.5(\varepsilon - j\alpha)B^*(\omega) \]

\[ B'(\omega) = B(\omega) + 0.5(\varepsilon + j\alpha)B(\omega) - 0.5(\varepsilon - j\alpha)A^*(\omega) \]
Constellations of channel $+k$ and $-k$
Crosstalk Between Channels $k$ and $-k$
Due to gain and Phase Imbalance
Constellation after Gradient Descent Correction of Gain and Phase Imbalance
Crosstalk Between Channels $k$ and Empty Channel-$k$
Constellation after Gradient Descent Correction of Gain and Phase Imbalance
Estimating Correcting
Gain Terms

\[ A' = A + 0.5(\varepsilon + j\alpha) A - 0.5(\varepsilon - j\alpha) B^* \]
\[ B' = B + 0.5(\varepsilon + j\alpha) B - 0.5(\varepsilon - j\alpha) A^* \]
\[ A'B' = -0.5(\varepsilon - j\alpha) AA^* - 0.5(\varepsilon - j\alpha) BB^* + AB + AB(\mathcal{O}(\varepsilon^2, \alpha^2)) \]
\[ A'B' = -0.5(\varepsilon - j\alpha)(AA' + BB') \]
\[ (\varepsilon - j\alpha) \approx -2 \quad \frac{A'B'}{(AA^* + BB^*)} \approx \frac{A'B'}{(A'A'^* + B'B'^*)} \]
The Plot Thickens

Analog Block Down Conversion
With a Frequency Offset
Removed by Digital Down Conversion
Analog Block Conversion with Frequency Offset
Digital Down Conversion and Removal of Frequency Offset

\[ B(\omega+\Delta\omega) \]

\[ 0.5 (\varepsilon+j\alpha) B(\omega+\Delta\omega) \]

\[ 0.5 (\varepsilon-j\alpha) A^*(\omega-\Delta\omega) \]

\[ A(\omega+\Delta\omega) \]

\[ 0.5 (\varepsilon+j\alpha) A(\omega+\Delta\omega) \]

\[ 0.5 (\varepsilon-j\alpha) B^*(\omega-\Delta\omega) \]

\[ B(\omega) \]

\[ 0.5 (\varepsilon+j\alpha) B(\omega) \]

\[ 0.5 (\varepsilon-j\alpha) A^*(\omega-2\Delta\omega) \]

\[ A(\omega) \]

\[ 0.5 (\varepsilon+j\alpha) A(\omega) \]

\[ 0.5 (\varepsilon-j\alpha) B^*(\omega-2\Delta\omega) \]

\[ 0 \]

\[ \Delta\omega \]
Description of Model and Block Down Conversion With frequency Offset

\[ A'(\omega) = A(\omega) + 0.5(\varepsilon + j\alpha)A(\omega) - 0.5(\varepsilon - j\alpha)B^*(\omega - 2\Delta\omega) \]

\[ B'(\omega) = B(\omega) + 0.5(\varepsilon + j\alpha)B(\omega) - 0.5(\varepsilon - j\alpha)A^*(\omega - 2\Delta\omega) \]
Estimating Correcting Gain Terms with Frequency Offset

\[
A' = A + 0.5(\varepsilon + j\alpha)A - 0.5(\varepsilon - j\alpha)e^{j2\Delta\omega}B^*
\]

\[
B' = B + 0.5(\varepsilon + j\alpha)B - 0.5(\varepsilon - j\alpha)e^{j2\Delta\omega}A^*
\]

\[
A'B'e^{-j2\Delta\omega} = -0.5(\varepsilon - j\alpha)AA^* - 0.5(\varepsilon - j\alpha)BB^* + AB + AB(O(\varepsilon^2, \alpha^2))
\]

\[
A'B'e^{-j2\Delta\omega} = -0.5(\varepsilon - j\alpha)(AA' + BB')
\]

\[
(\varepsilon - j\alpha) \equiv -2 \frac{A'B'e^{-j2\Delta\omega}}{(AA^* + BB^*)} \equiv \frac{A'B'e^{-j2\Delta\omega}}{(A'A'^* + B'B'^*)}
\]
Constellations with I-Q Imbalance with and without Frequency Offset

Constellations, With I-Q Imbalance

Constellations, With I-Q Imbalance and Frequency Offset
Constellation Trajectories During Adaptive Convergence

Constellations, With I-Q Imbalance and Frequency Offset

Constellations, Compensated for I-Q Imbalance
Processing Task

Variable BW
Variable Length
FIR Filter
Narrower Bandwidth Filters Have Narrower Transition Bandwidths Hence Are Longer Filters
Spectrum and Group Delay
IIR Filter and Equalizer
Spectrum and Group Delay
2- BW IIR Filters and Equalizers
Pole-Zero Diagrams for IIR Filter and Equalizer

Roots: 11-Pole Inverse Tchebyschev, $f_{cd} = 4 \text{ MHz}$

Roots: 11-Pole Filter, 16-Pole Phase Equalizer

Roots: 11-Pole Inverse Tchebyschev, $f_{cd} = 3 \text{ MHz}$

Roots: 11-Pole Filter, 16-Pole Phase Equalizer
Filter Coefficient Variation with Filter Bandwidth

\[ H(Z) = \frac{Z^2 + b_1 Z + 1}{Z^2 + a_1 Z + a_2} \]
Equalizer Coefficient Variation with Filter Bandwidth

\[ E(Z) = \frac{a_2 Z^4 + a_1 Z^2 + 1}{Z^4 + a_1 Z^2 + a_2} \]
Recursive Filter for Variable Bandwidth with Companion All-Pass Equalizer

\[ a_n(bw) \quad b_n(bw) \quad c_n(bw) \quad d_n(bw) \]
Compare FIR Filter and Equalized IIR Filter
Narrower Bandwidth $\rightarrow$ Longer Filter
Longer Filter $\rightarrow$ More Taps

True for Fixed Sample Rate
Narrower Bandwidth $\rightarrow$ Longer Filter,  
Longer Filter $\rightarrow$ Same Taps  
at Lower Sample Rate

31-Sample Impulse Response; $f_{BW} = 1.0, f_s = 5$

31-Sample Impulse Response; $f_{BW} = 0.5, f_s = 2.5$

Spectrum; $f_{BW} = 1.0, f_s = 5, f_{BW}/f_s = 1/5$

Spectrum; $f_{BW} = 0.5, f_s = 2.5, f_{BW}/f_s = 1/5$
Continuously Variable Bandwidth FIR Filter with Fixed Number of Taps
Spectra at Input and Output of the Three Processing blocks of the Variable Bandwidth Filter

Spectrum, Input to System and Low Pass Filter at Input Rate

Spectrum, Output of 4-to-3 Down-Sampler and Low Pass at Same Reduced Rate

Spectrum, Output of Lowpass Filter at Reduced Rate

Spectrum, Output of 3-to-4 Up-sampler at Original Input Rate
Impulse Response of Variable BW FIR Filter

Impulse Response: Prototype Lowpass Filter

Spectrum, Prototype Filter

Impulse Response: 4/3 Upsampled Lowpass Filter

Spectrum, Interpolated Filter
Power Amplifier Linearization

PA Linearization
Peak-to-Average Ratio Control
Non Linear Amplifier and Precompensating Gain

Nonlinear Transfer Function of Amplifier

Compensating Gain of Transfer Function

1-dB Compression Point
Transition Diagram Input and Output of Amplifier and input and output of Precompensator
Spectra: Input and Output of Amplifier and Output of Precompensator and Precompensated Amplifier
To Clip or Not to Clip; That is the Question!

\[ s(t) = s(t) - s(t) + L_{\text{CLIP}} - L_{\text{CLIP}} - L_{\text{CLIP}} - L_{\text{CLIP}} - L_{\text{CLIP}} - L_{\text{CLIP}} \]

\[ s_1(t) \]

\[ s_2(t) \]

\[ s_3(t) = s_2(t) - s_1(t) \]
Band Limited Subtractive Clipping

\[ S(n) \]

\[ C(k_1) \]

\[ C(k_2) \]
Band-Limited Clipping

\[ s_1(t) \xrightarrow{\text{CLIP-1}} s_2(t) \xrightarrow{-} s_3(t) \]

\[ s_1(t) \xrightarrow{\text{CLIP-2}} s_2(t) \xrightarrow{\text{Filter}} s_4(t) \]
Spectra: Input Signal, Clipping Component and Clipped Signal

Magnitude of Input Signal

Magnitude of Input Signal Exceeding Top

Magnitude of Input Signal - Level Exceeding Top
Spectra: Input Signal, Band Limited Clipping Component and Clipped Signal

Magnitude of Input Signal

Magnitude of Input Signal - Level Exceeding Top and Filtered Version

Magnitude of Input Signal - Filtered Level Exceeding Top
Spectra: Input, Clip Component, Band Limited Clip, and Band Limited Clip
Professor harris, may I be excused?
My brain is full.
SOFTWARE DEFINED RADIO MAN

Is Open For Questions