First-Photon Imaging and Other Imaging with Few Photons

Modeling at the right scale

Inverse-problem mentality >> denoising mentality

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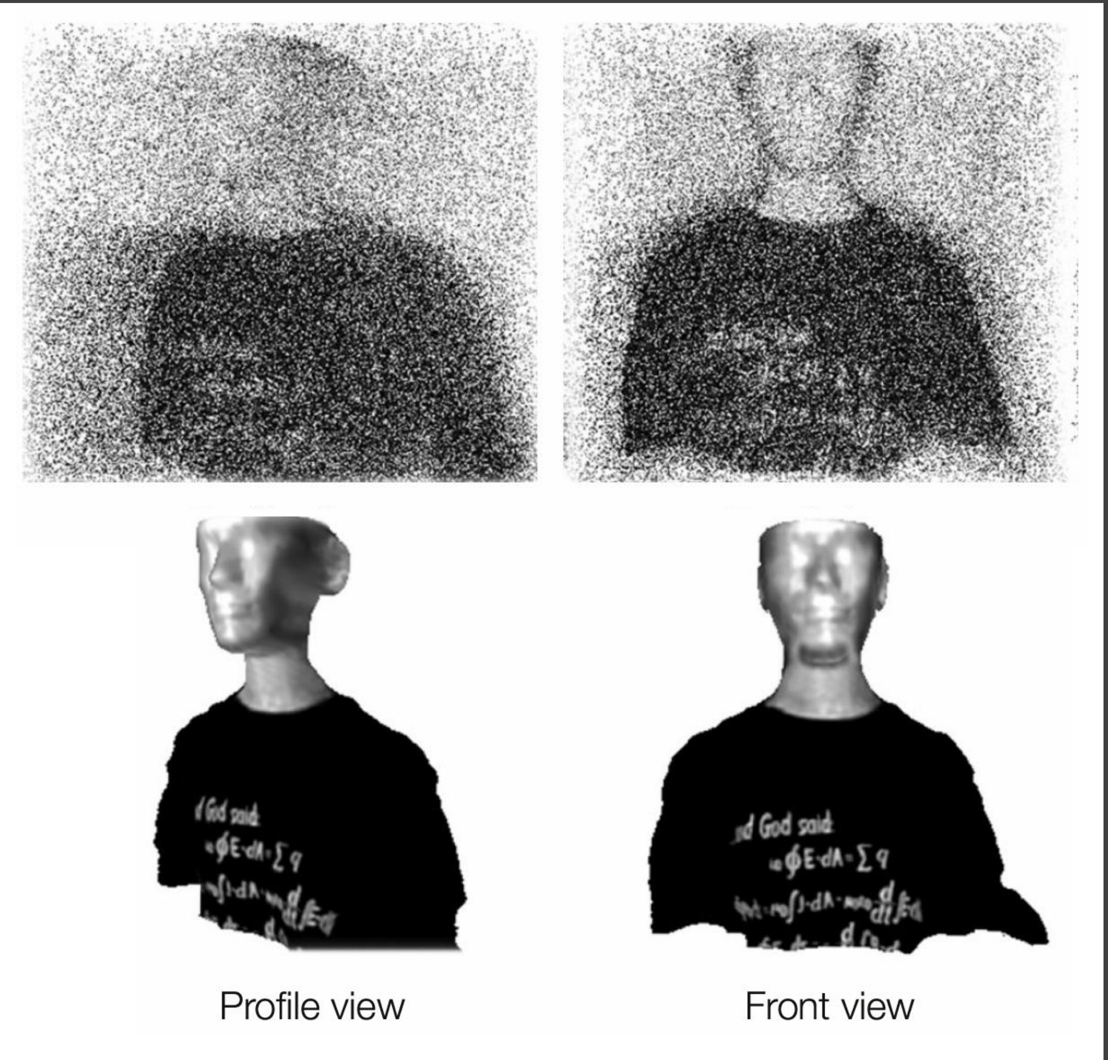
Reflectivity and depth from 1 detected photon per pixel

Half from active source, half from background light and dark counts



Key idea: Image formation that integrates physical modeling of acquisition and scene modeling can provide dramatic improvements

- A. Kirmani, D. Venkatraman, D. Shin, A. Colaço, F. N. C. Wong, J. H. Shapiro, and V. K. Goyal, "First-photon imaging," *Science*, 343(6166):58-61, 2014

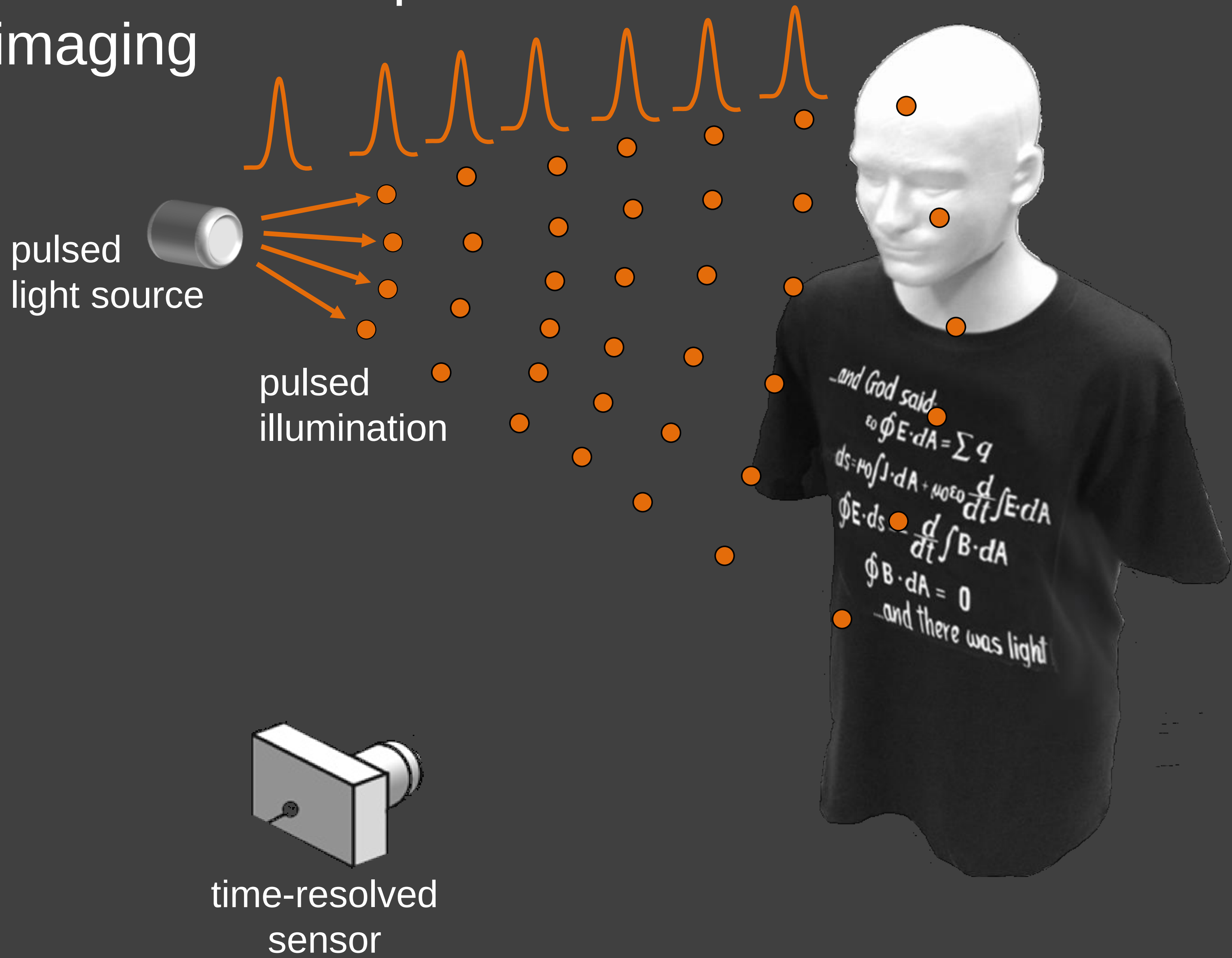
Photon-efficient depth+reflectivity imaging: Variations

	Deterministic acquisition time	Exploit pulse shape	Exploit transverse smoothness	Uncalibrated background	Estimate multiple layers	Compensate for array properties
Kirmani, Venkatraman, Shin, Colaço, Wong, Shapiro, Goyal, *Science*, 343(6166):58-61, 2014		✓	✓			
Shin, Kirmani, Shapiro, Goyal, *IEEE Trans. Computational Imaging*, 1(2):112-125, 2015	✓	✓	✓			
Shin, Shapiro, Goyal, *IEEE Signal Processing Letters*, 22(12):2254-2258, 2015	✓	✓		✓		
Shin, Xu, Wong, Shapiro, Goyal, *Optics Express*, 24(3):1873-1888, 2016	✓	✓			✓	
Shin, Xu, Venkatraman, Lussana, Villa, Zappa, Goyal, Wong, Shapiro, submitted, 2015	✓	✓	✓			✓

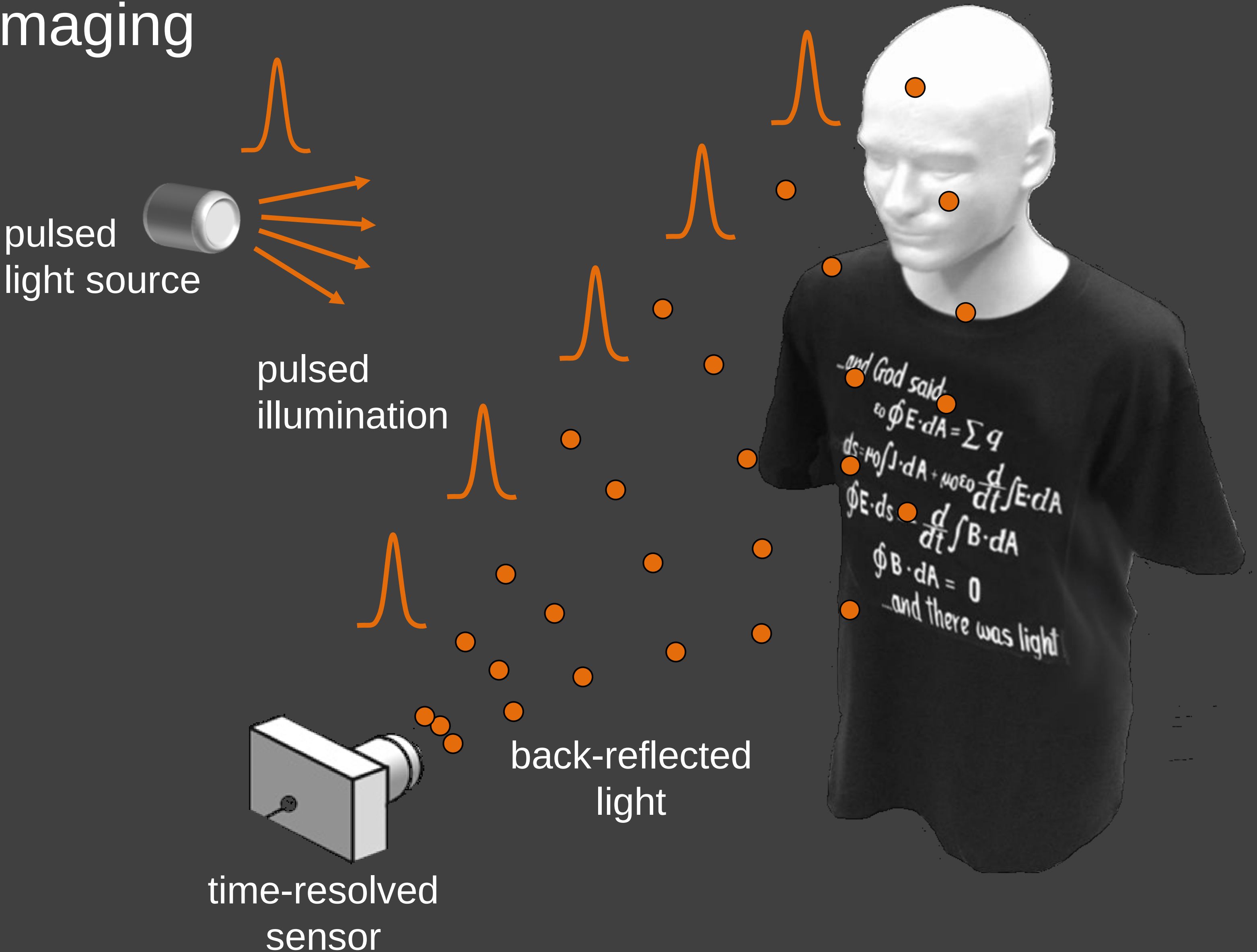
In prep: fluorescence lifetime imaging, transverse super-resolution, unambiguous range extension

Time-of-flight depth imaging

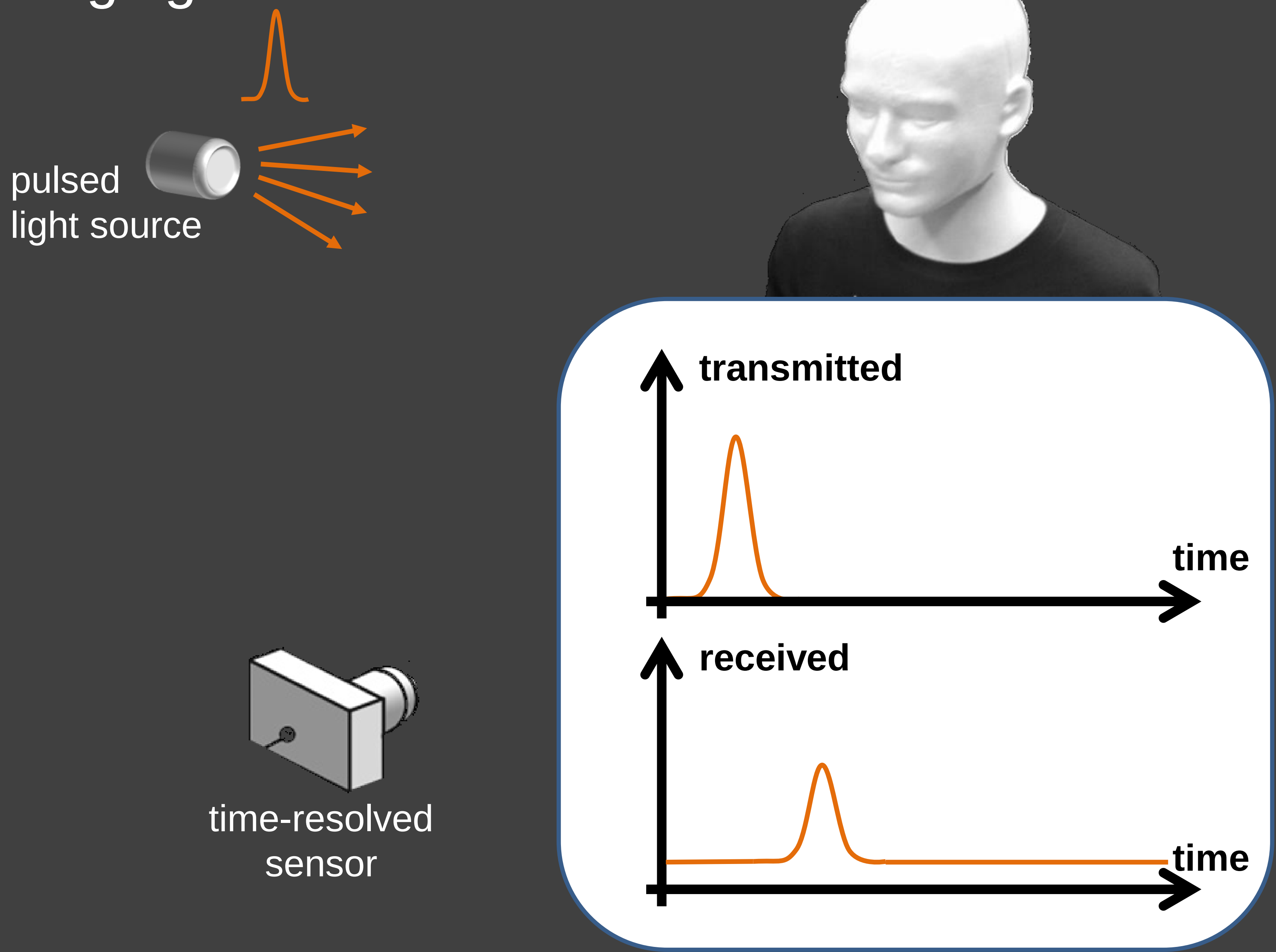
Conventional active optical depth imaging



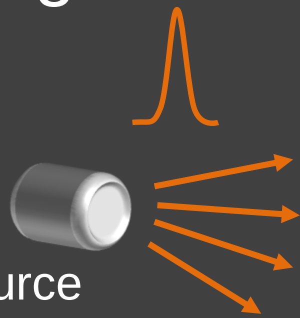
Conventional active optical depth imaging



Conventional active optical depth imaging

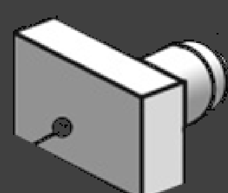


Conventional active optical depth imaging

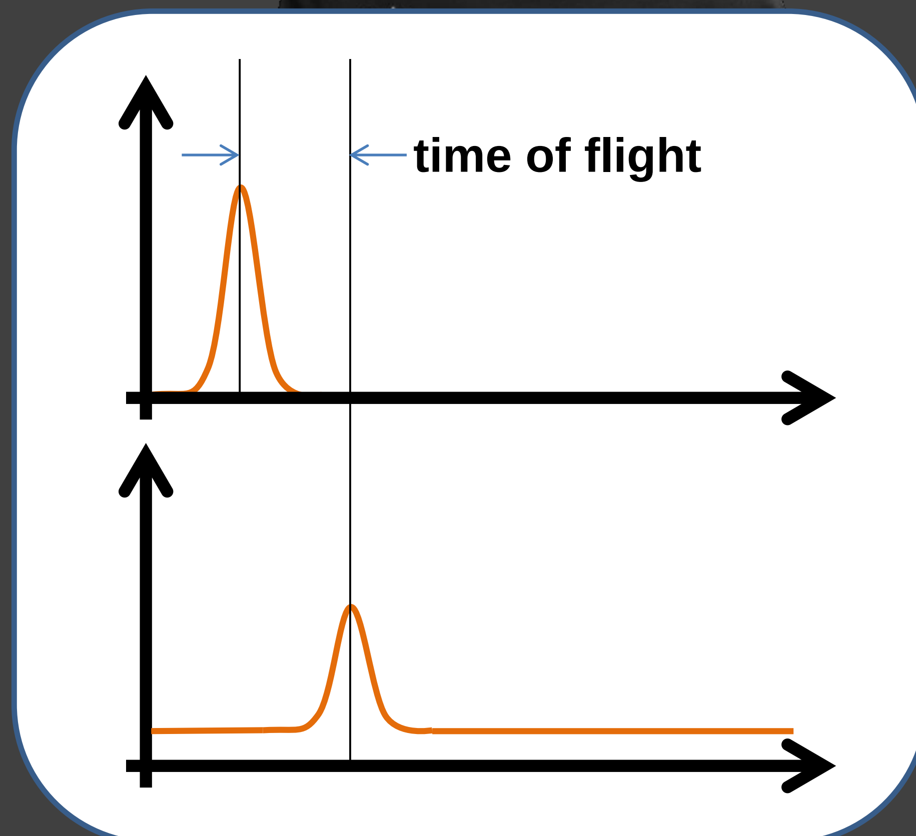


pulsed
light source

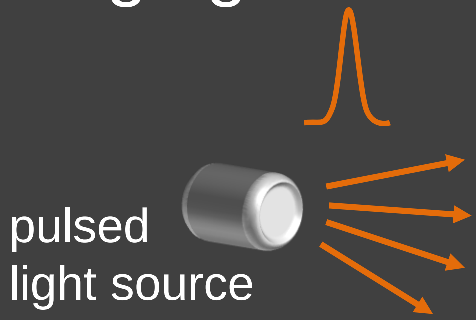
Pixelwise measurement
- distance



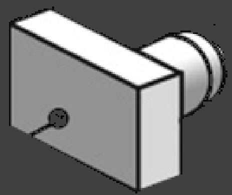
time-resolved
sensor

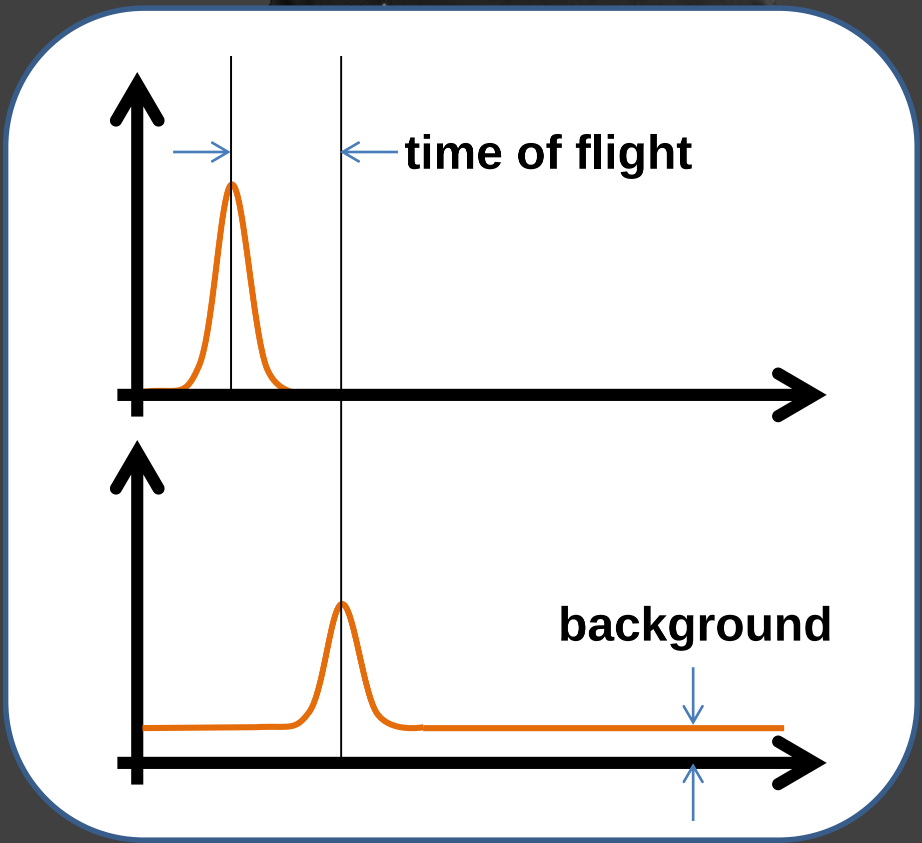


Conventional active optical depth imaging

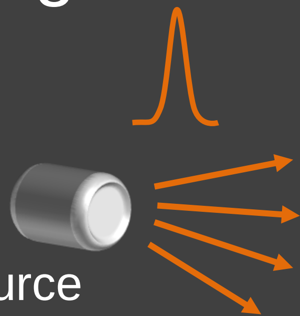


Pixelwise measurement
- distance
- ambient light



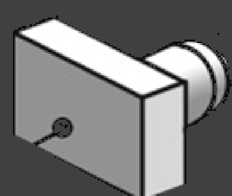


Conventional active optical depth imaging

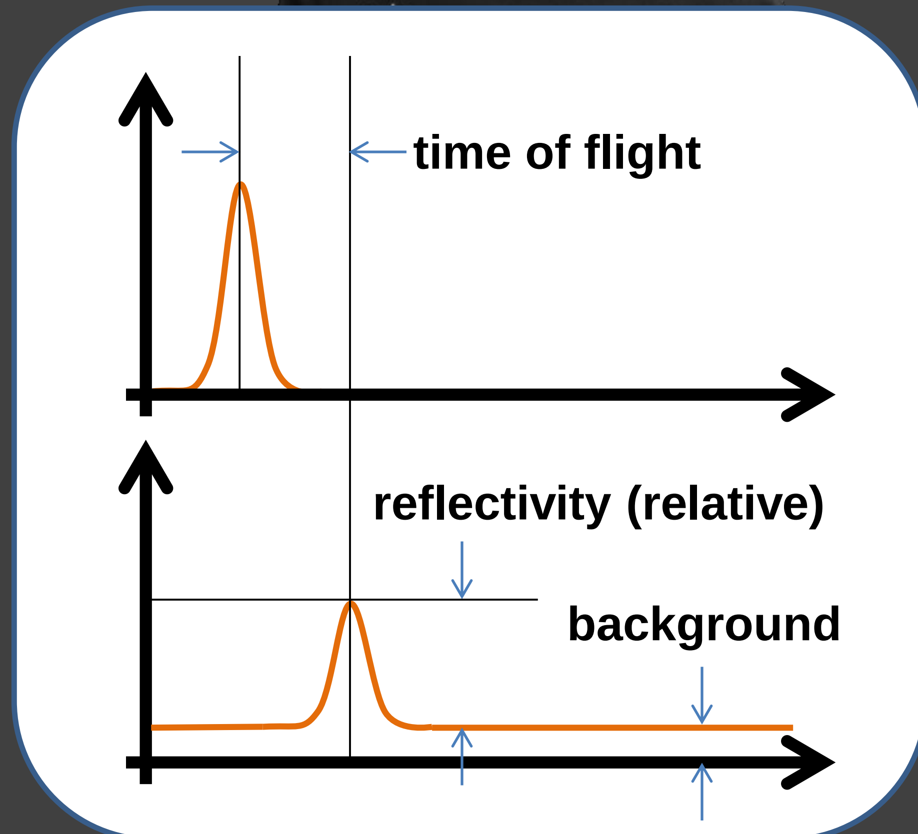


pulsed
light source

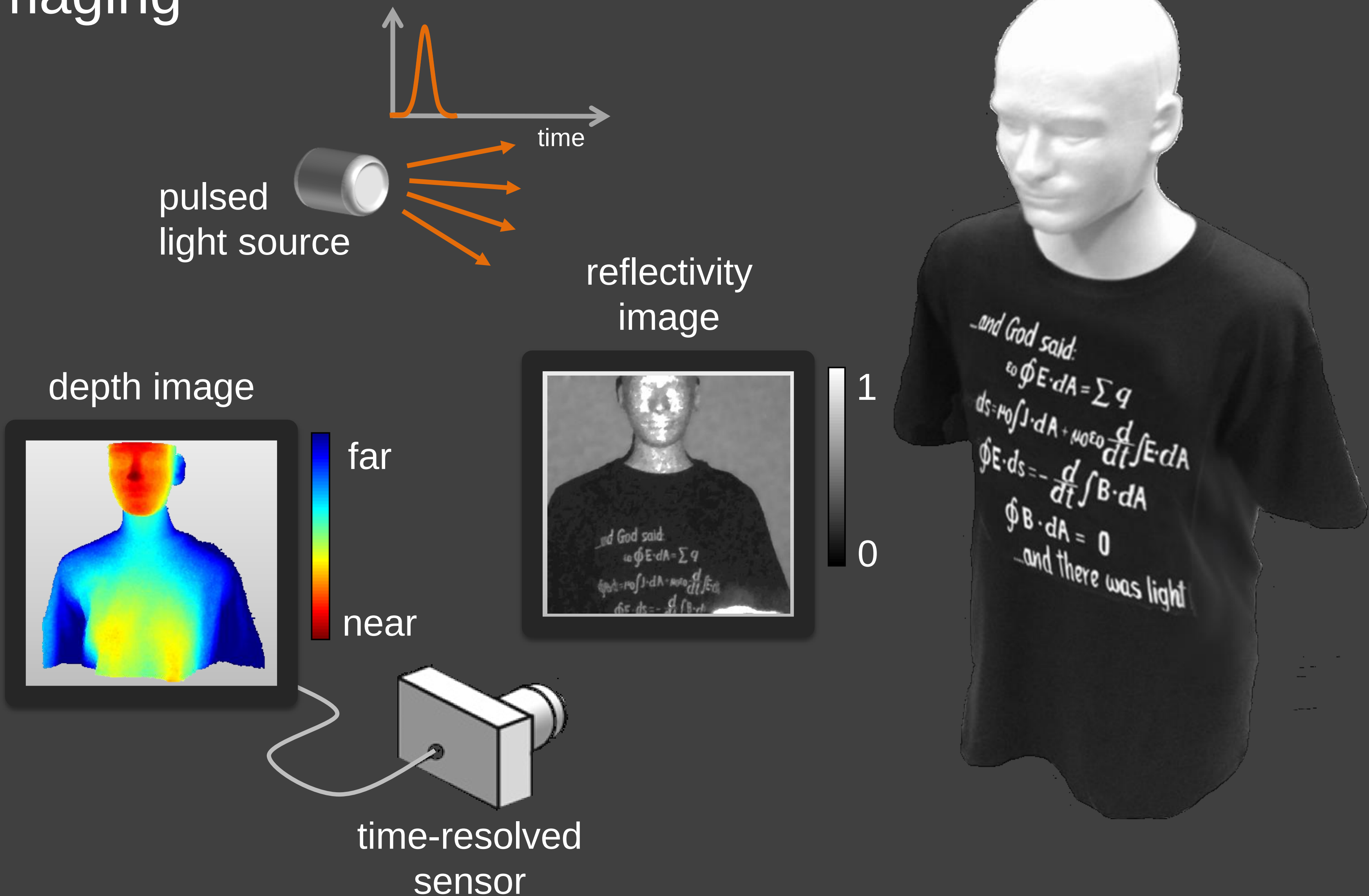
Pixelwise measurement
- distance
- ambient light
- reflectivity



time-resolved
sensor



Conventional active optical depth imaging



Photon-efficient implementation

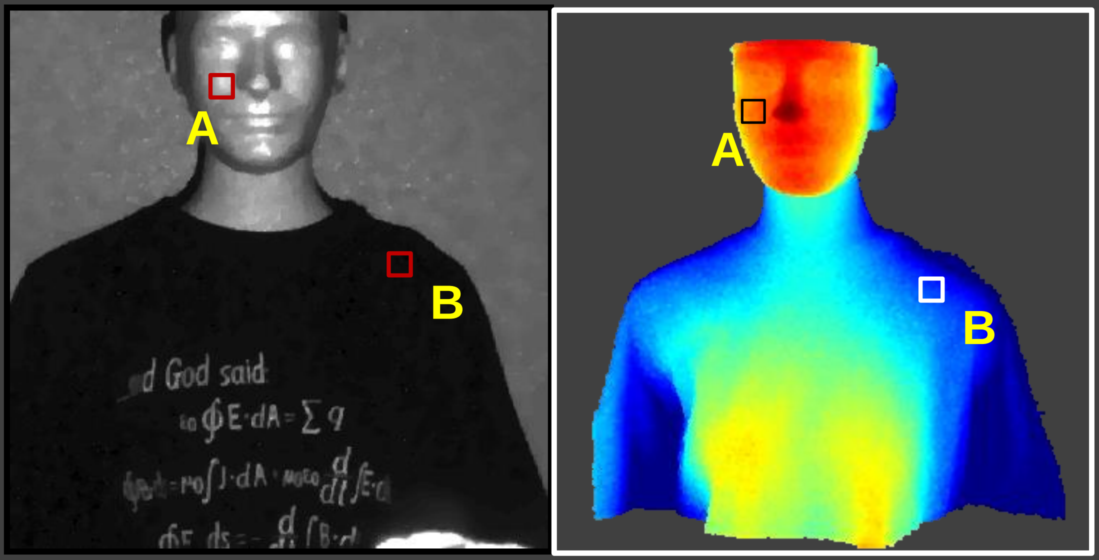
Detector sensitive to individual photons

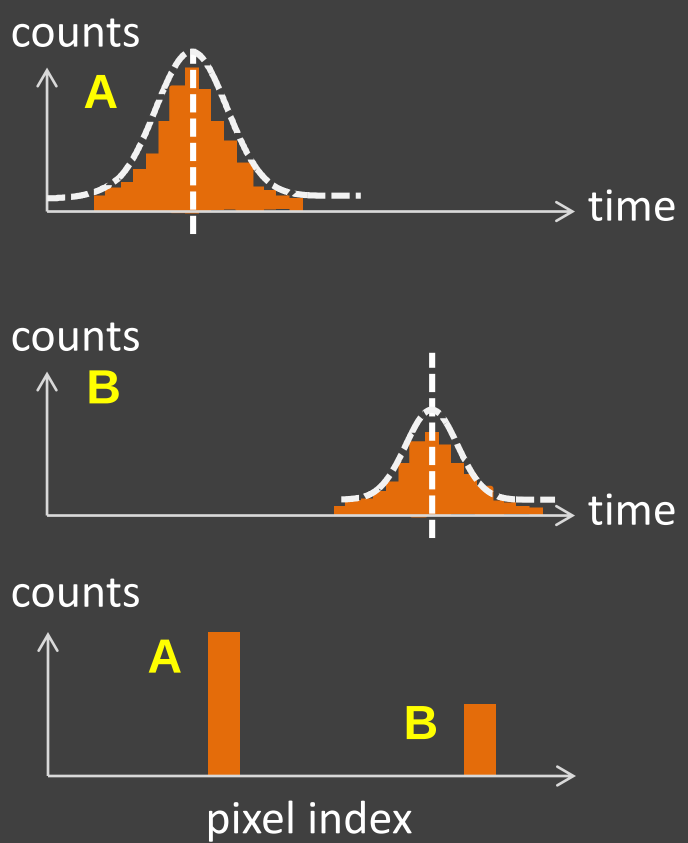
- Micro Photon Devices single-photon avalanche diode: 35% quantum efficiency, 100 μm x 100 μm

Finite-resolution time tagging

- PicoQuant HydraHarp time-correlated single-photon counting module: 8 ps

Histogram as proxy for waveform





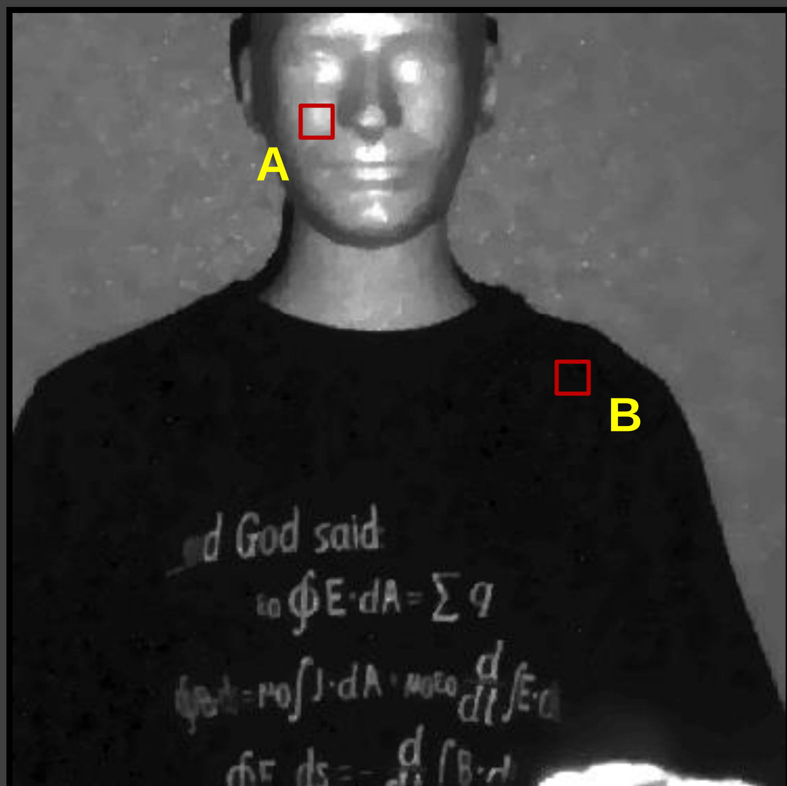
A: brighter, nearer

B: darker, farther

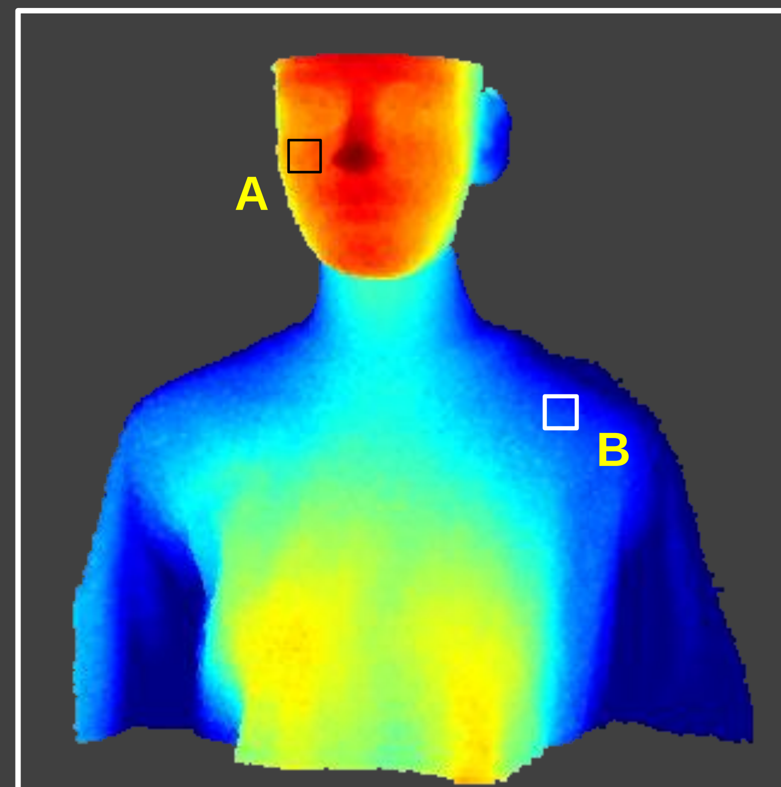
Classical noise models

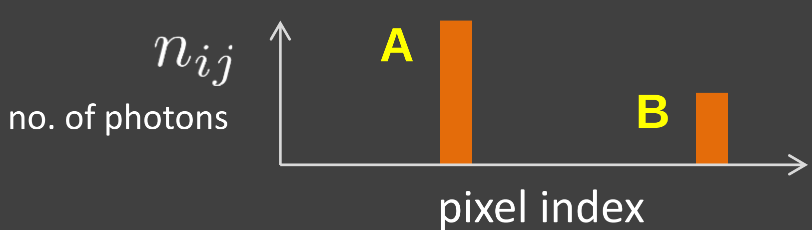
Detector noise models for optical imaging systems
(ground truth)

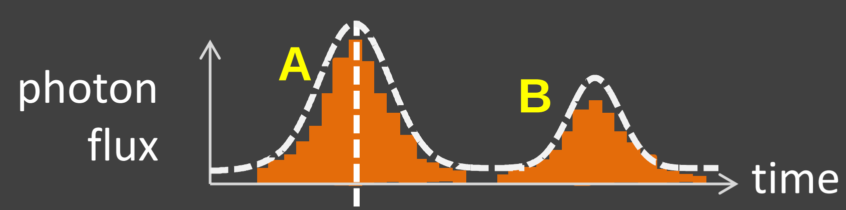
Reflectivity $\{\alpha_{ij}\}$



Range (depth) $\{Z_{ij}\}$

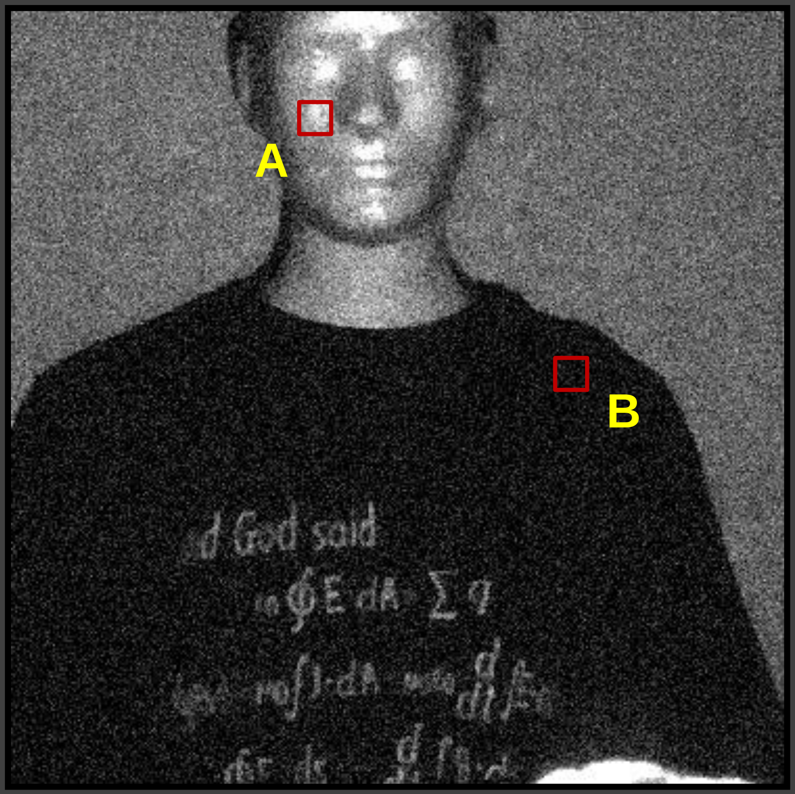




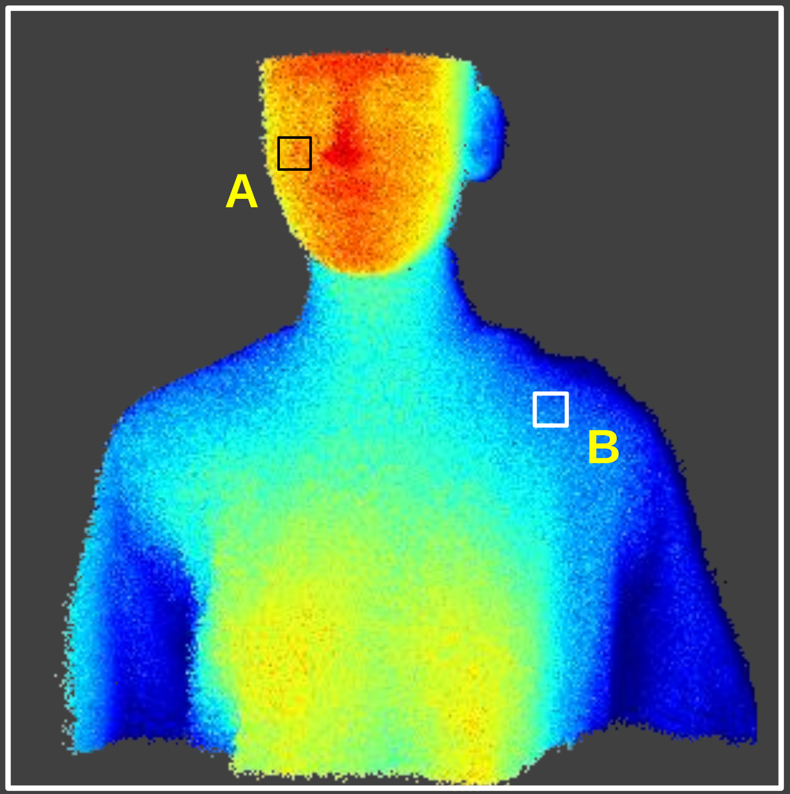


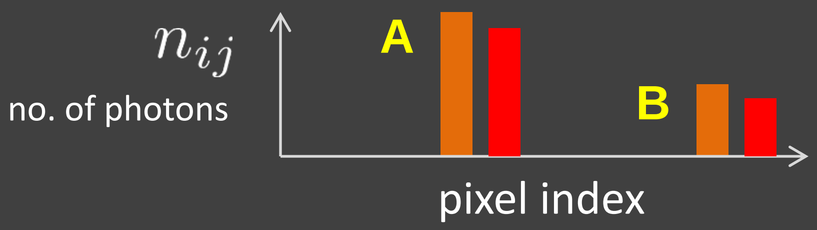
>≈10^4 detected photons/pixel [no background]

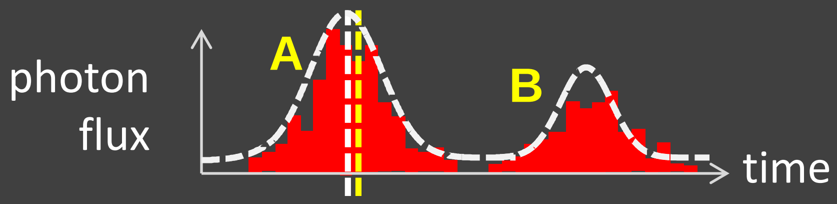
error in photon count
≈ Gaussian



error in ML depth est.
≈ Gaussian

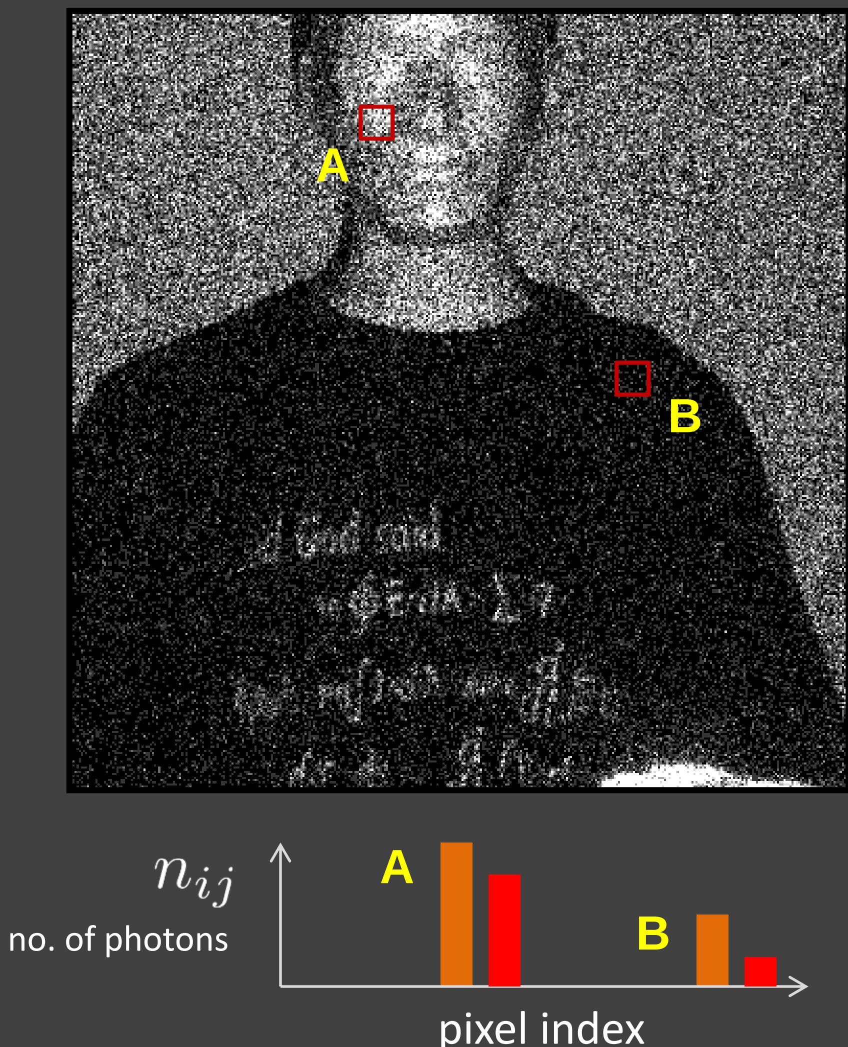




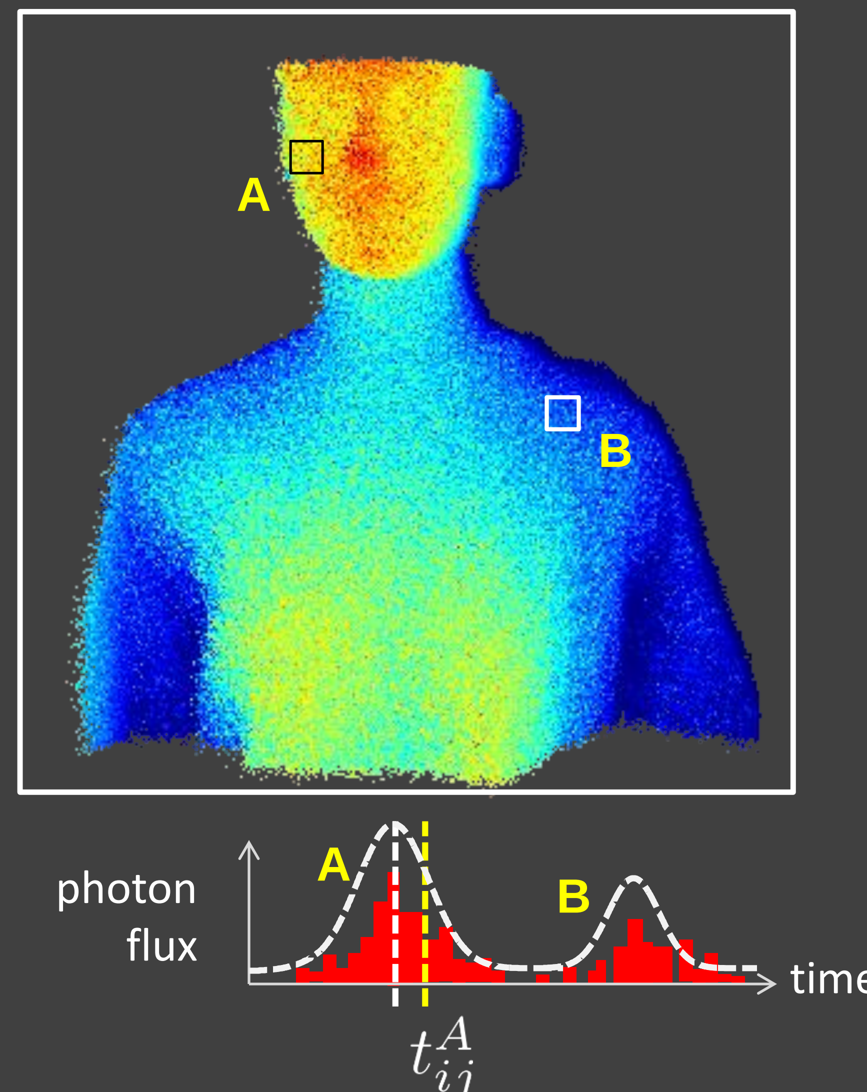


<≈10^4 detected photons/pixel [no background]

photon count
= Poisson r.v.



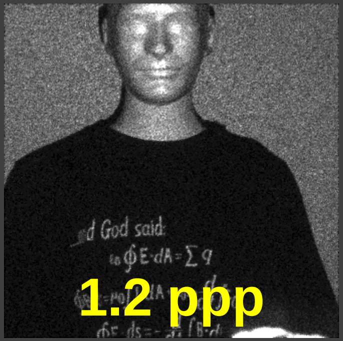
error in ML depth est.
≈ Gaussian

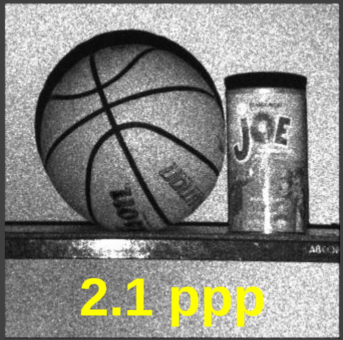


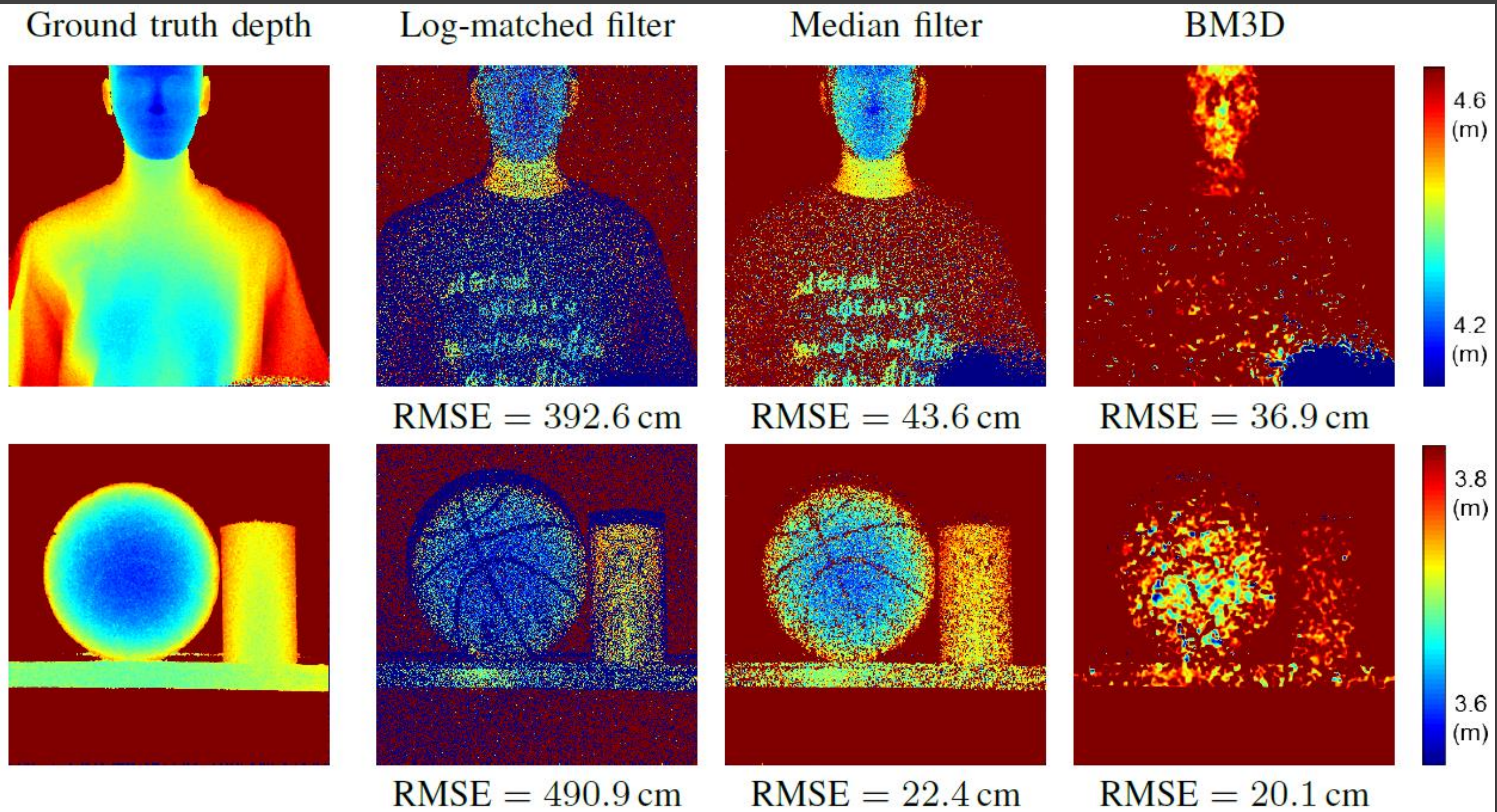
≈1 photon/pixel

(half signal, half noise …)

Depth imaging of two scenes
(few photons per pixel)





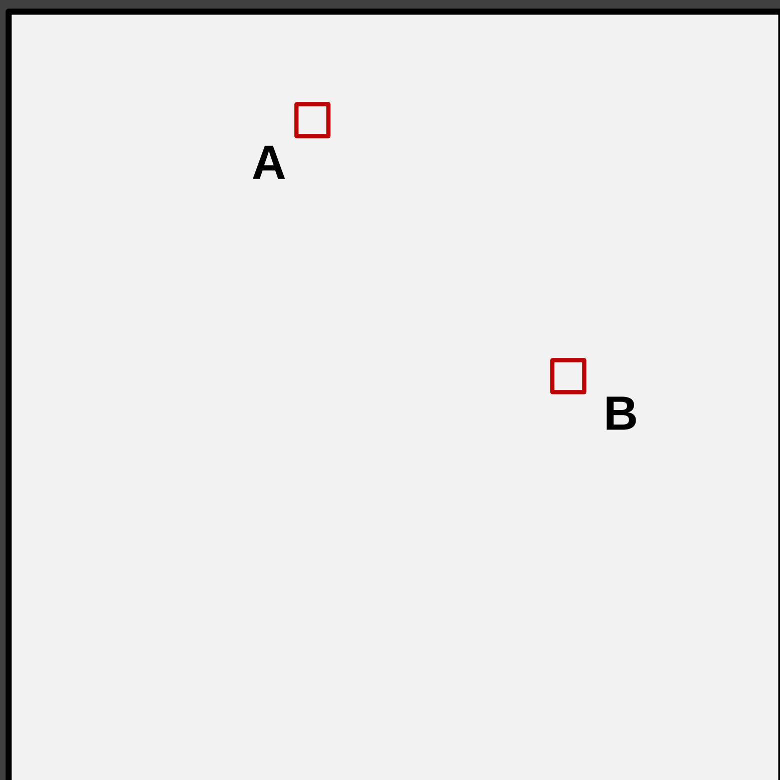


Imaging from only first photon detection

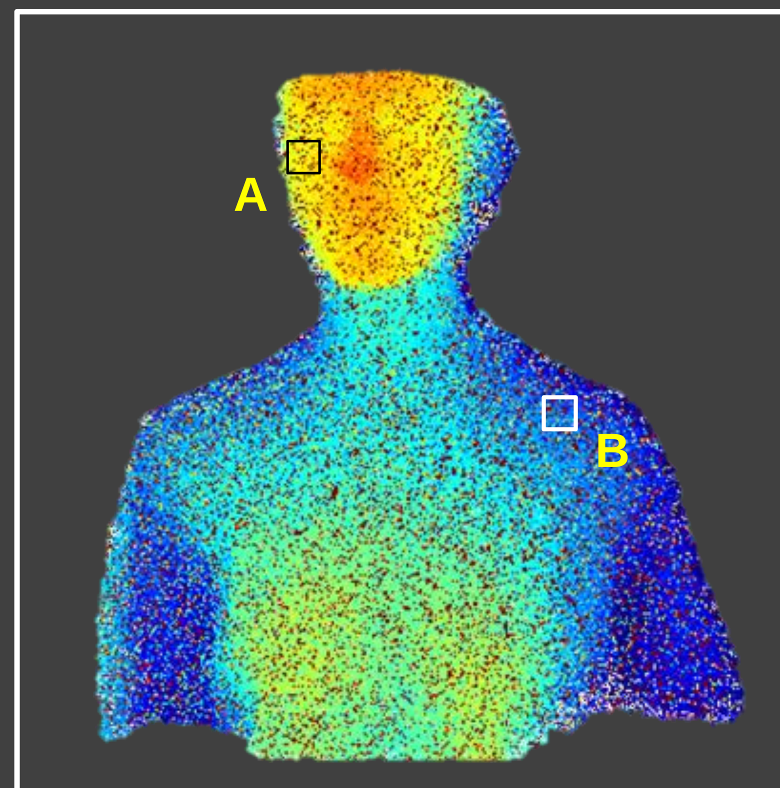
Conventional image formation

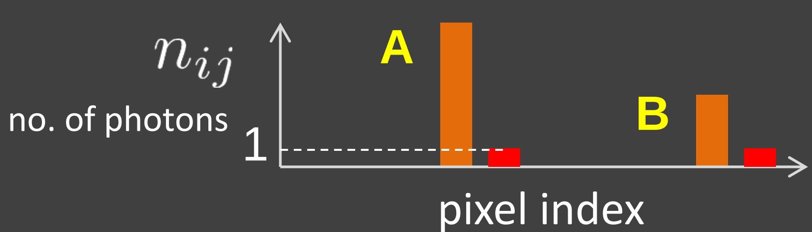
(one detected photon/pixel) [no background]

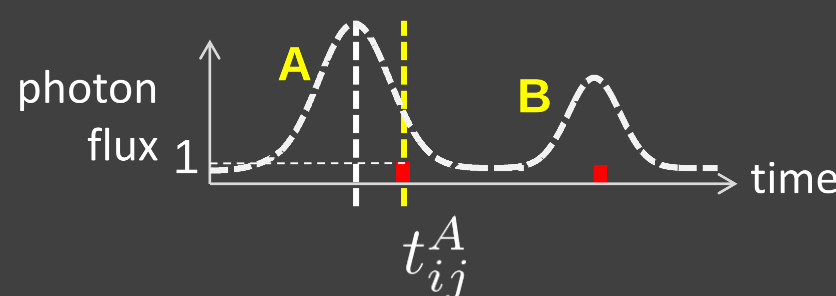
Featureless image



var(t_{ij}) ∝ mean-square pulse width

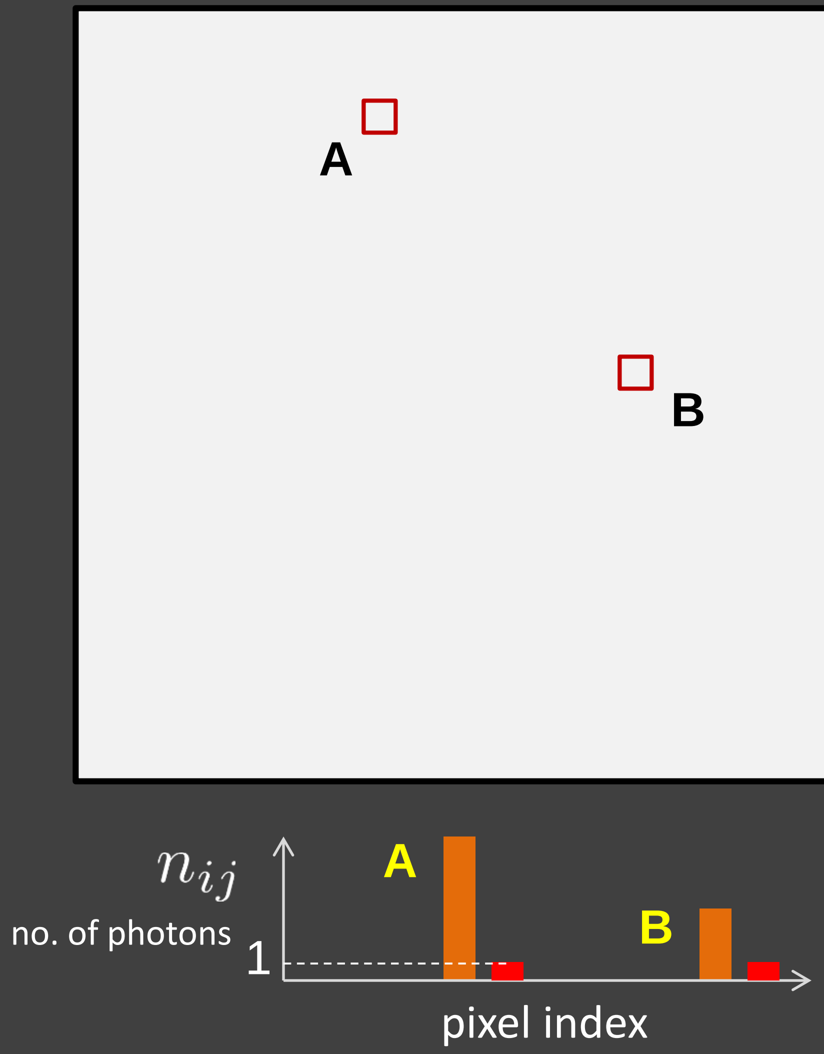




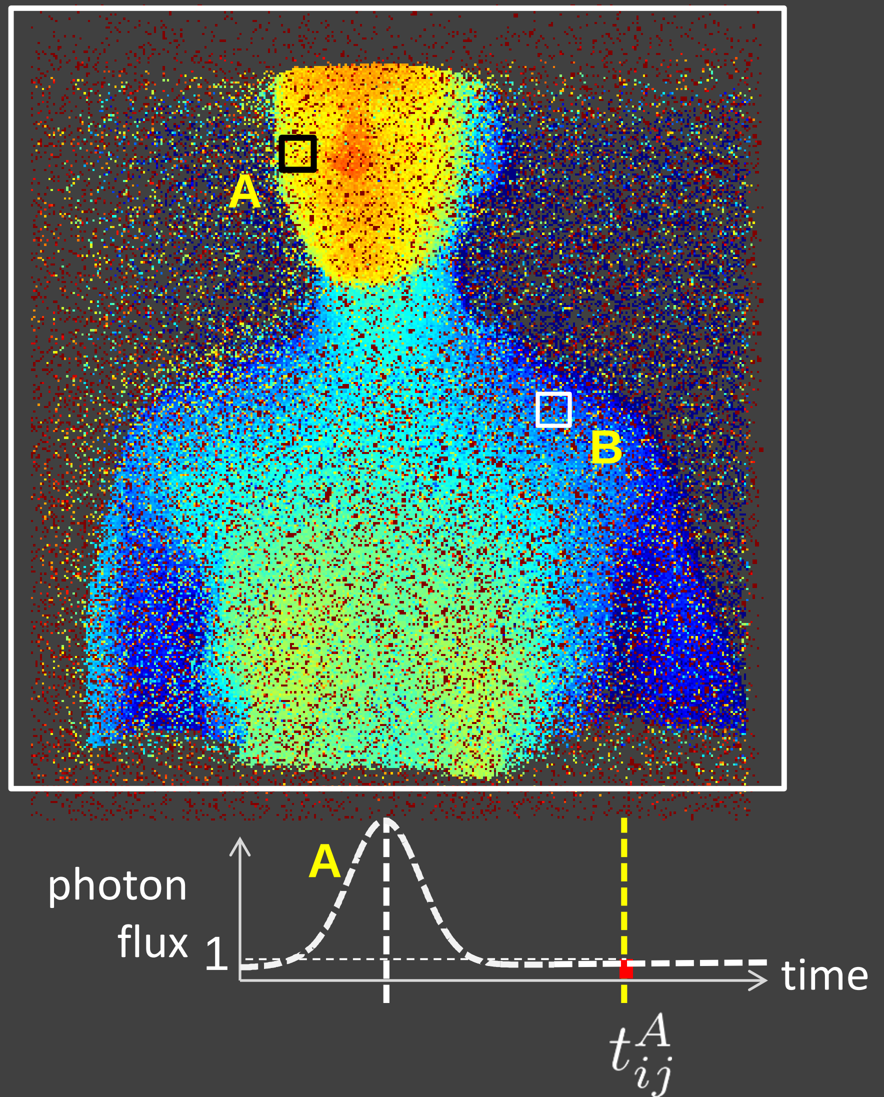


Conventional image formation with background noise (one detected photon/pixel)

Featureless image

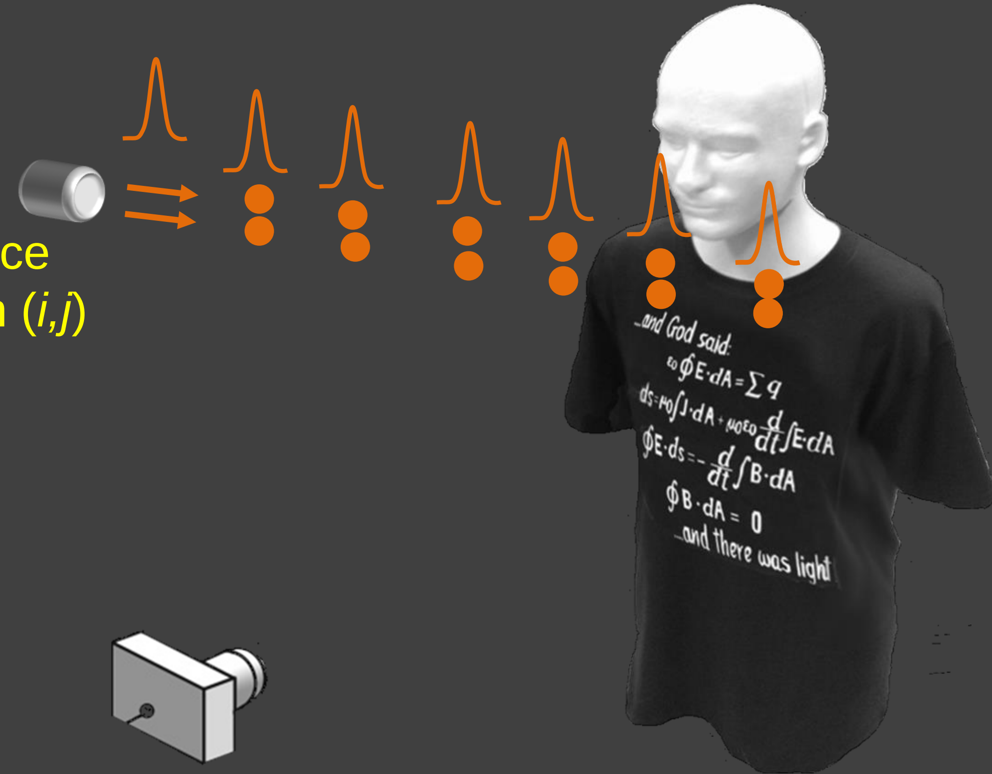


$\mathrm{var}(t_{ij}) \approx (\text{pulse-period})^2/12$

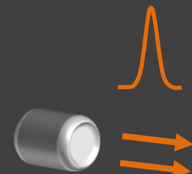


Quantum nature of photon detection

raster-scanned,
pulsed light source
illuminates patch (*i*,*j*)

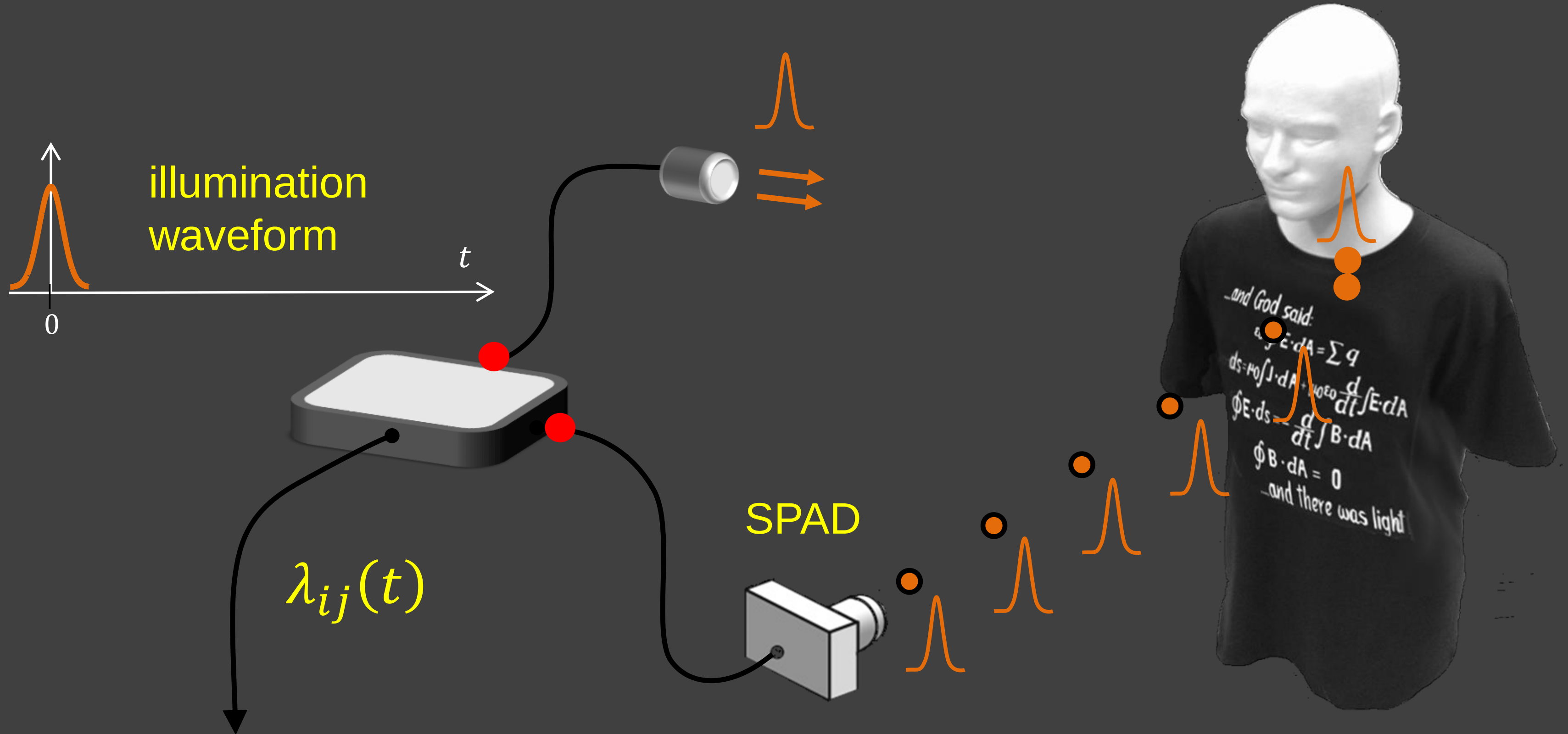


raster-scanned,
pulsed light source

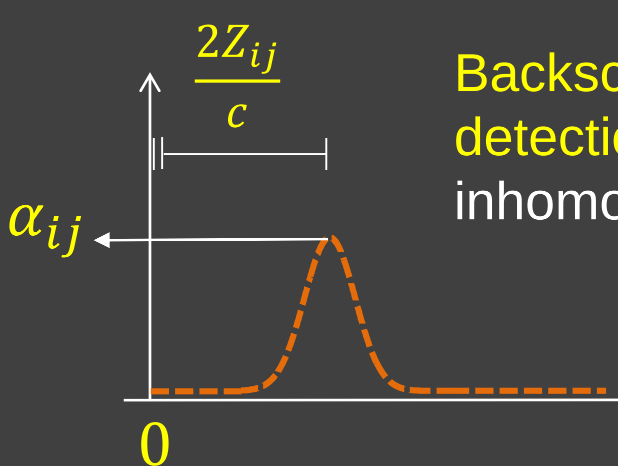


single-photon
avalanche
detector
(SPAD)

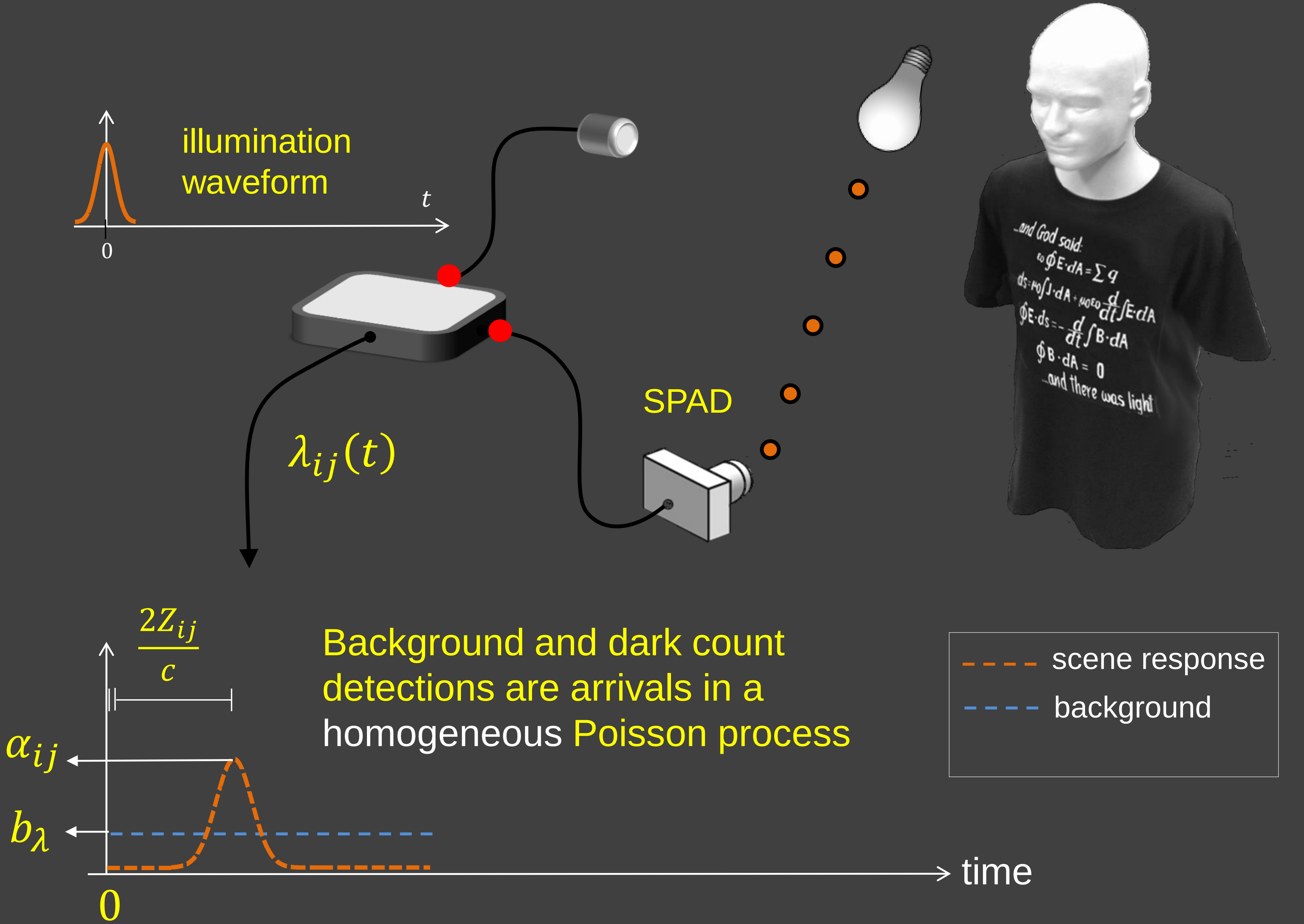
Poisson photo-detection statistics



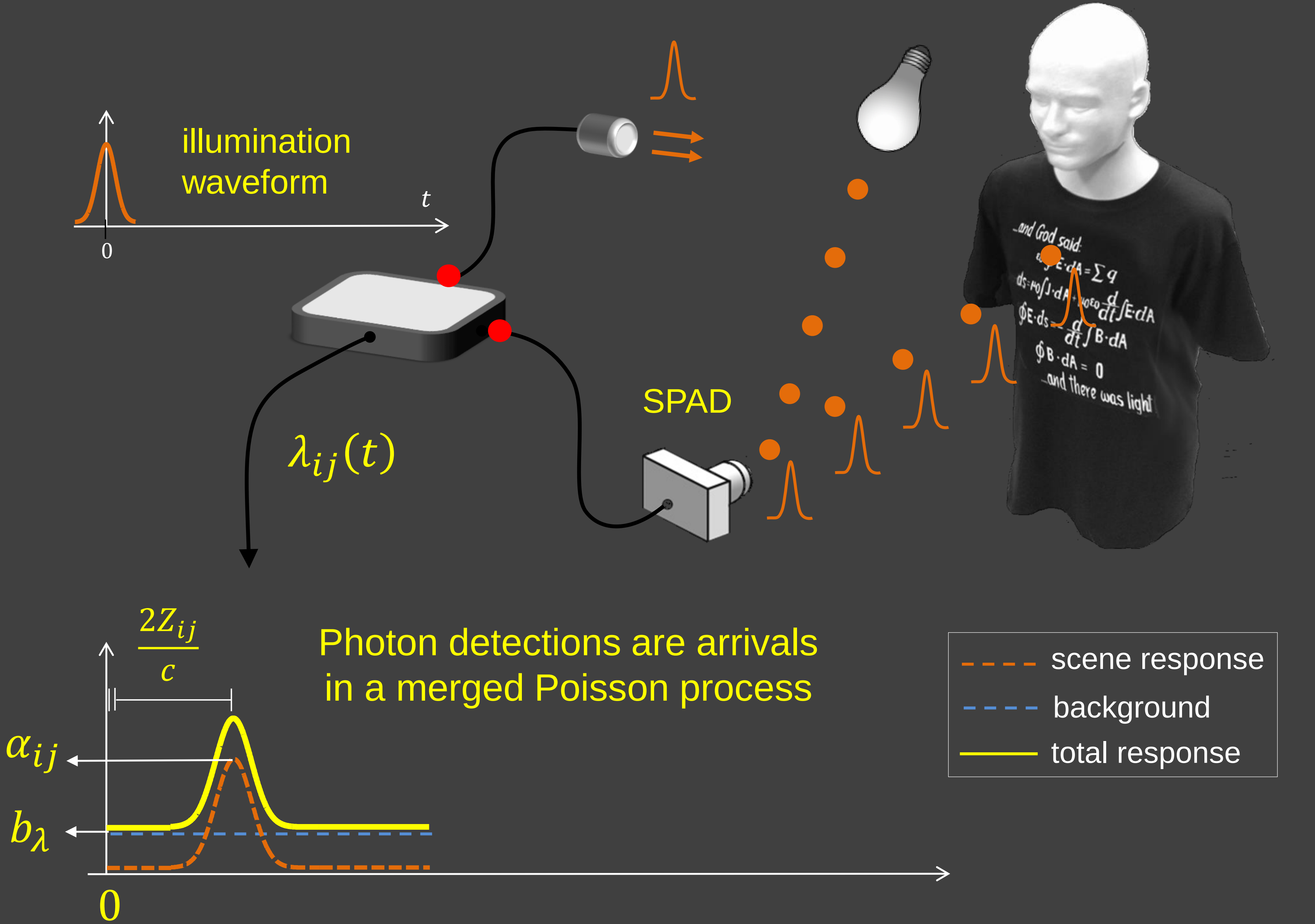
Backscattered signal photons detections are arrivals in an inhomogeneous Poisson process
[Saleh et al.]



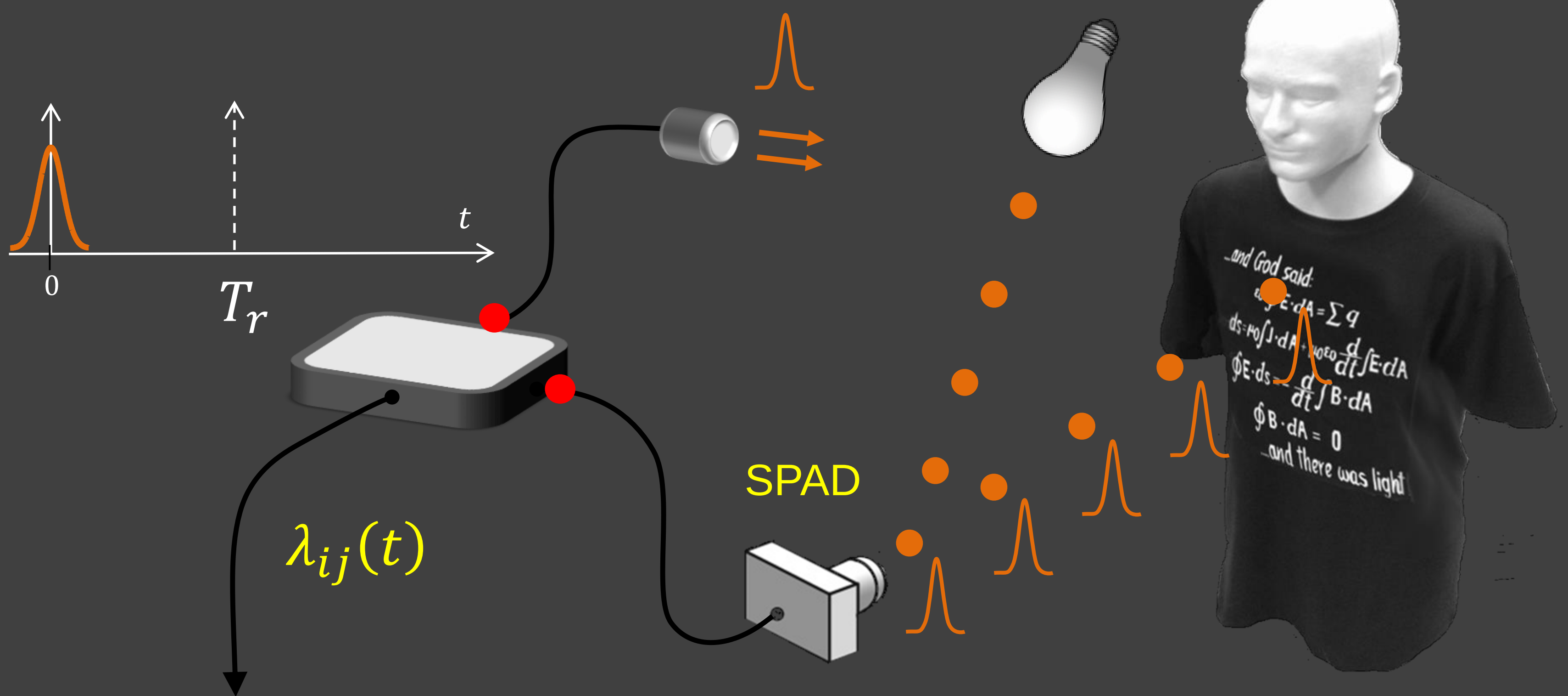
Poisson photo-detection statistics

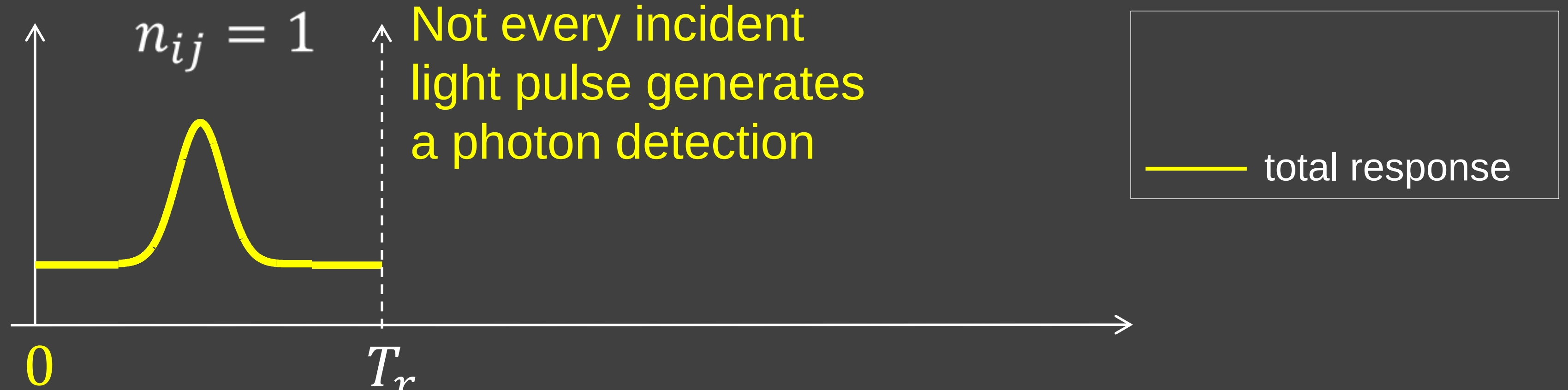


Poisson photo-detection statistics



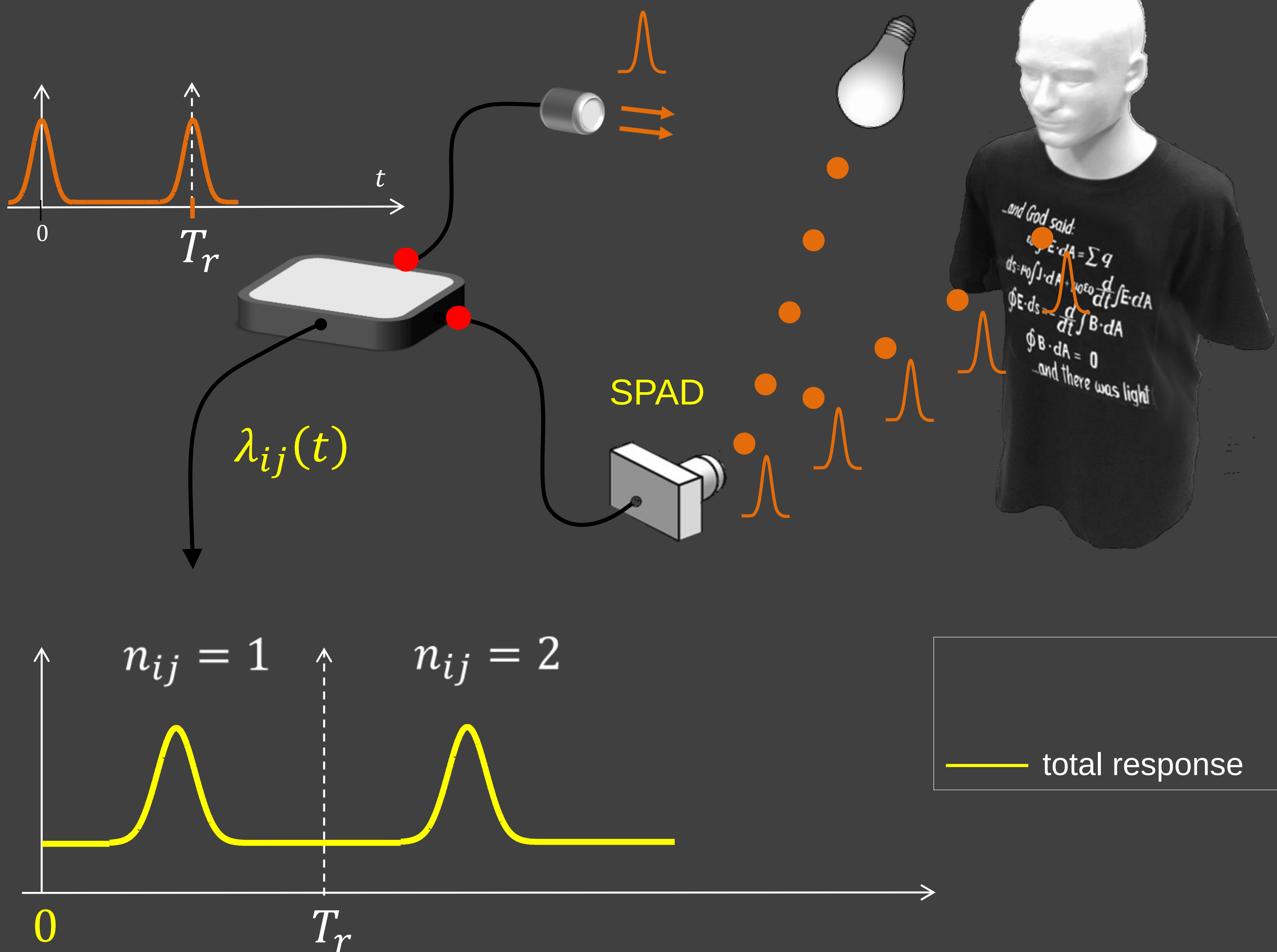
Low-light level photo-detection



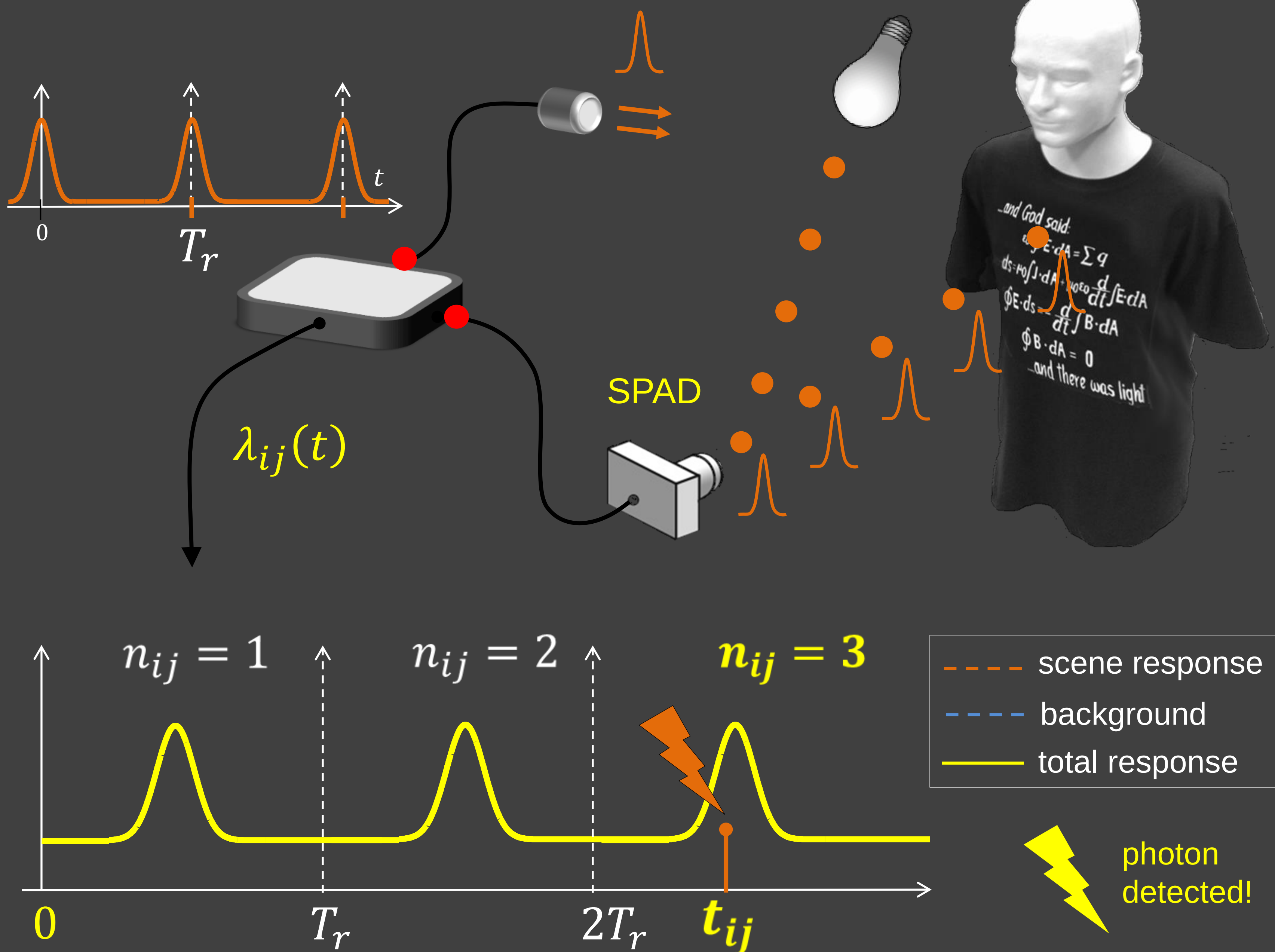


Not every incident light pulse generates a photon detection

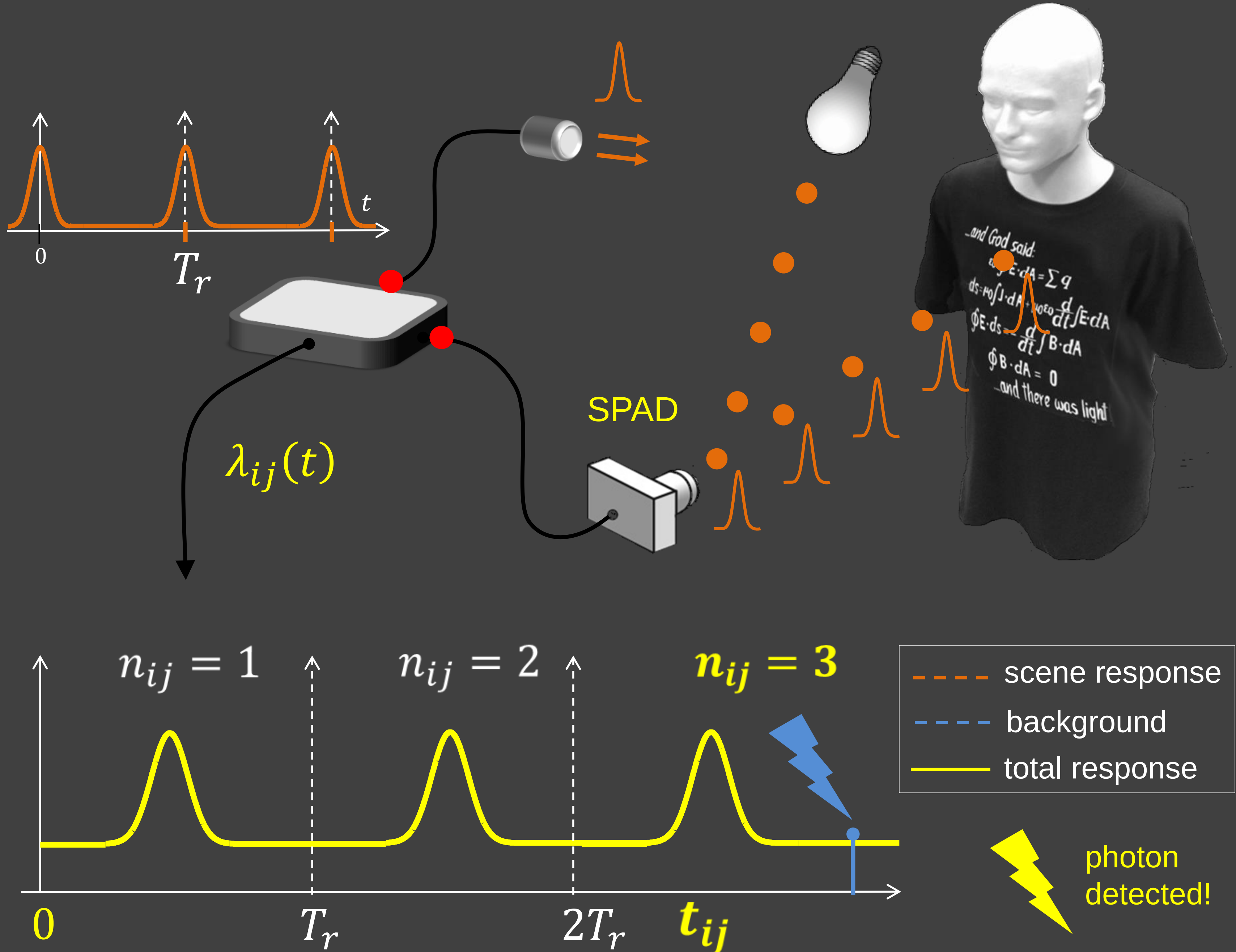
Low-light level photo-detection



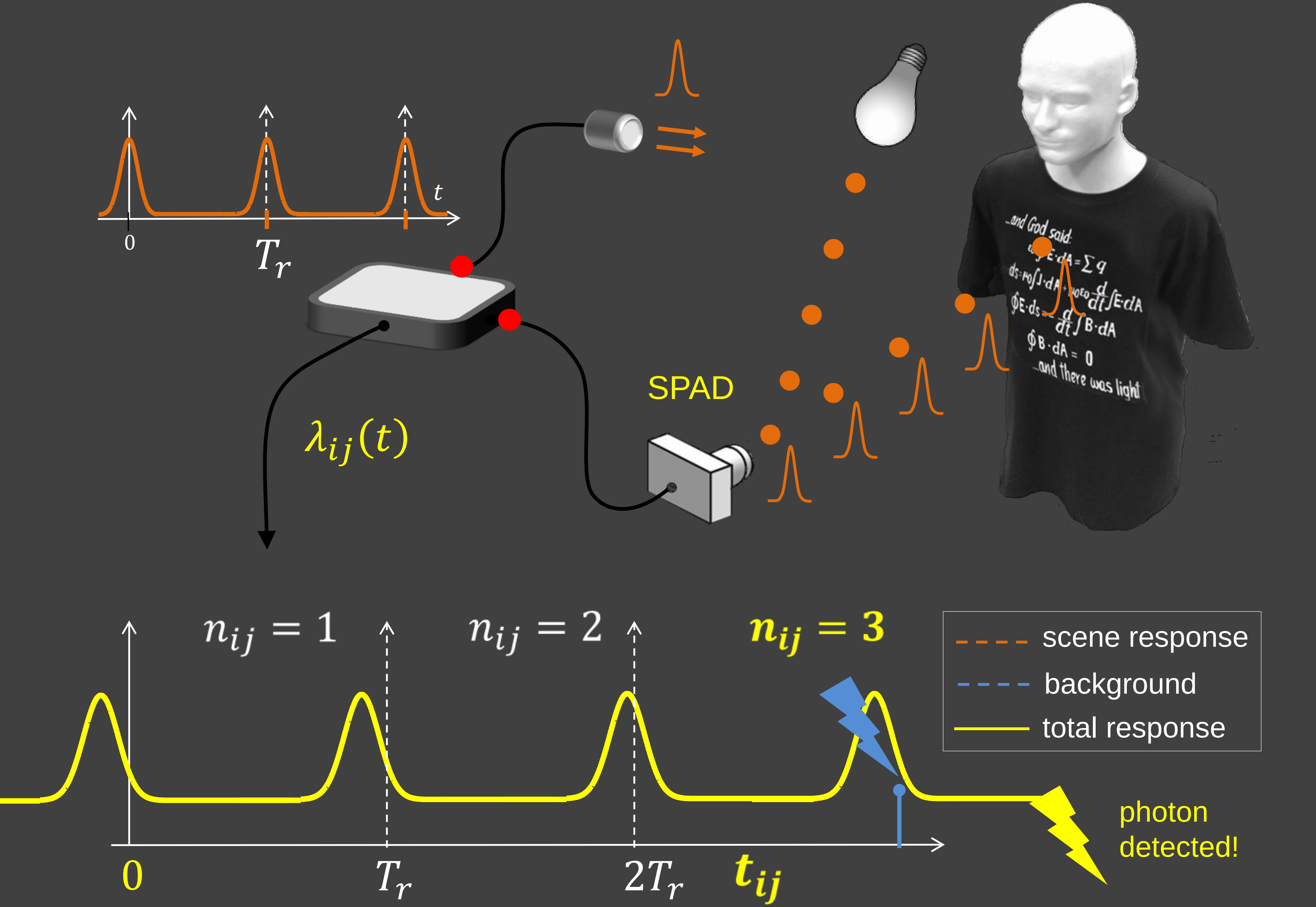
Low-light level photo-detection



Low-light level photo-detection

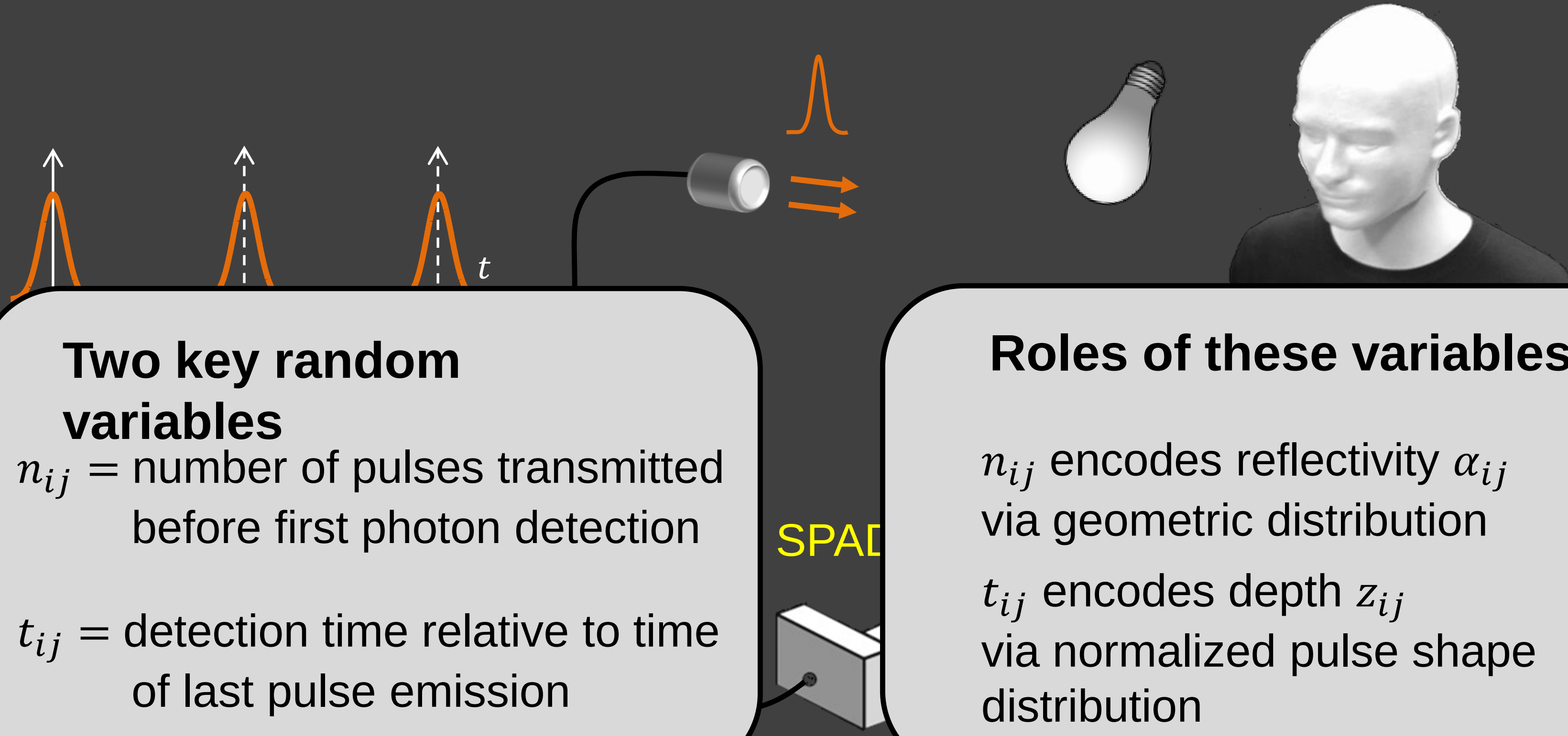


Low-light level photo-detection



Reinterpret what was observed

Pulse until one detection



Two key random variables

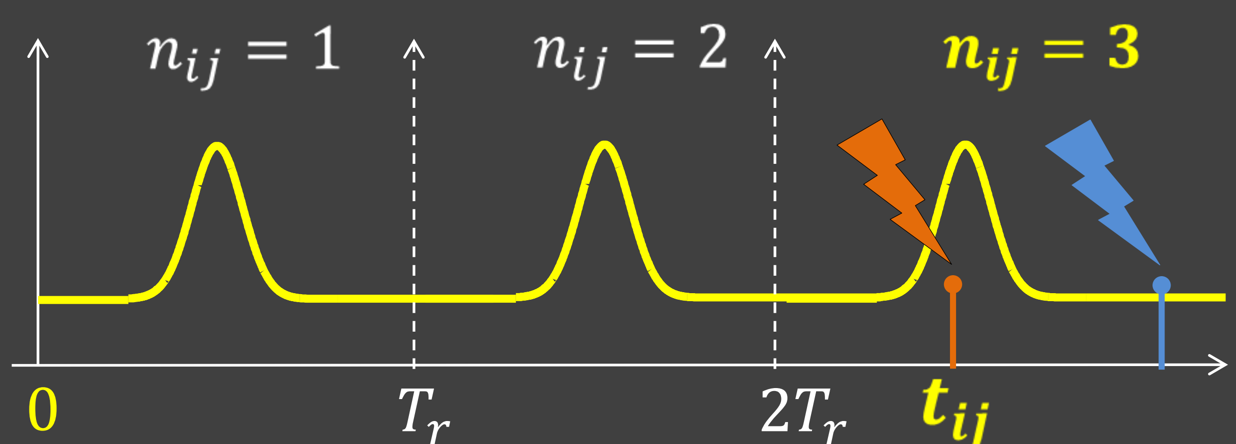
n_{ij} = number of pulses transmitted before first photon detection

t_{ij} = detection time relative to time of last pulse emission

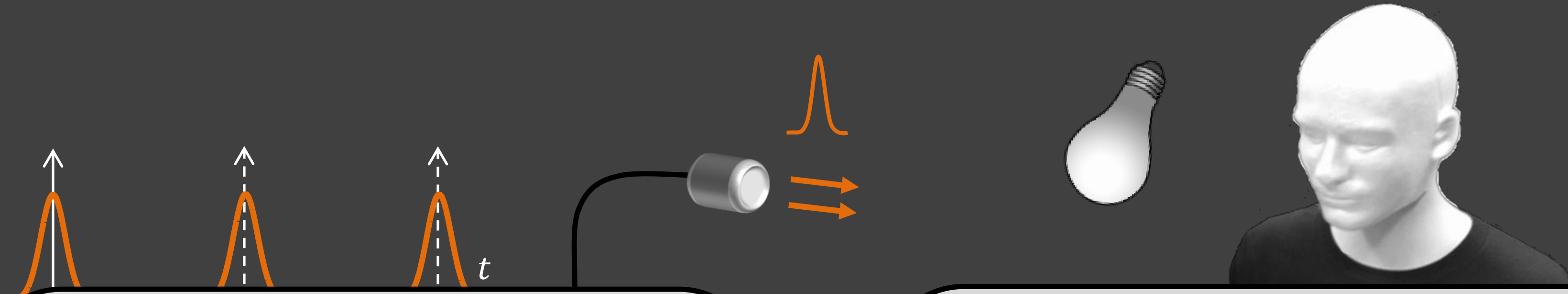
Roles of these variables

n_{ij} encodes reflectivity α_{ij} via geometric distribution

t_{ij} encodes depth z_{ij} via normalized pulse shape distribution



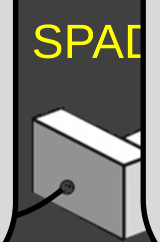
Aside: **Fixed** number of pulses



Key random variables

k_{ij} = number of photon detections

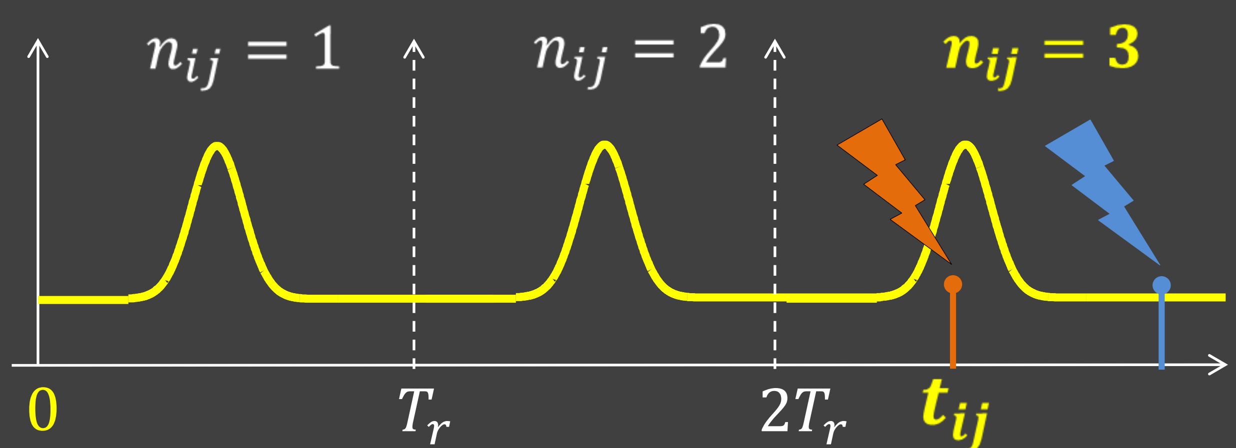
$t_{ij1}, t_{ij2}, \ldots, t_{ijk_{ij}}$ = detection times relative to times of last pulse emission



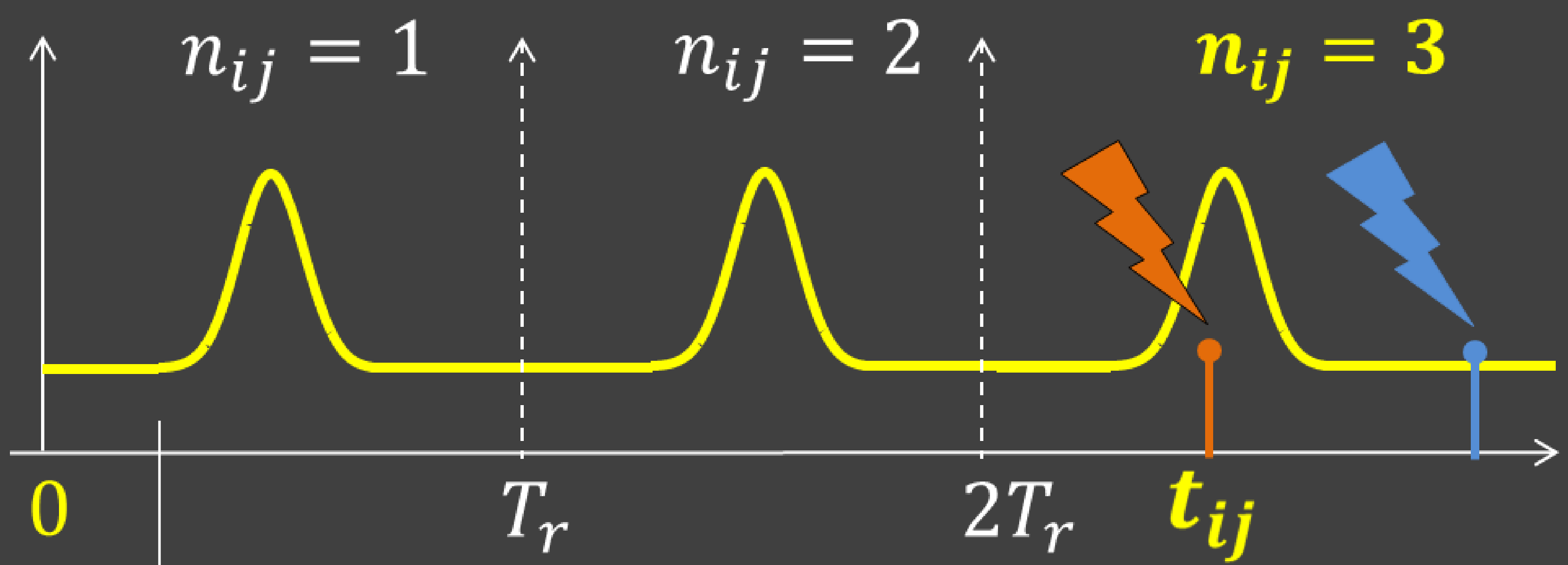
Roles of these variables

k_{ij} encodes reflectivity α_{ij} via **binomial** distribution

t_{ijk}'s encode depth z_{ij} via normalized pulse shape distribution



Quantitative acquisition modeling



$\Pr[\text{no photodetection in one period}] = e^{-(\alpha_{ij}+b_\lambda)T}$

$\Pr[n_{ij} = k; \alpha_{ij}] = e^{-(\alpha_{ij}+b_\lambda)T(k-1)}\left[1 - e^{-(\alpha_{ij}+b_\lambda)T}\right]$

$f_{t_{ij}}\left(\tau \mid \text{detected signal photon}\,; z_{ij}\right) \propto s\left(\tau - 2z_{ij}/c\right)$

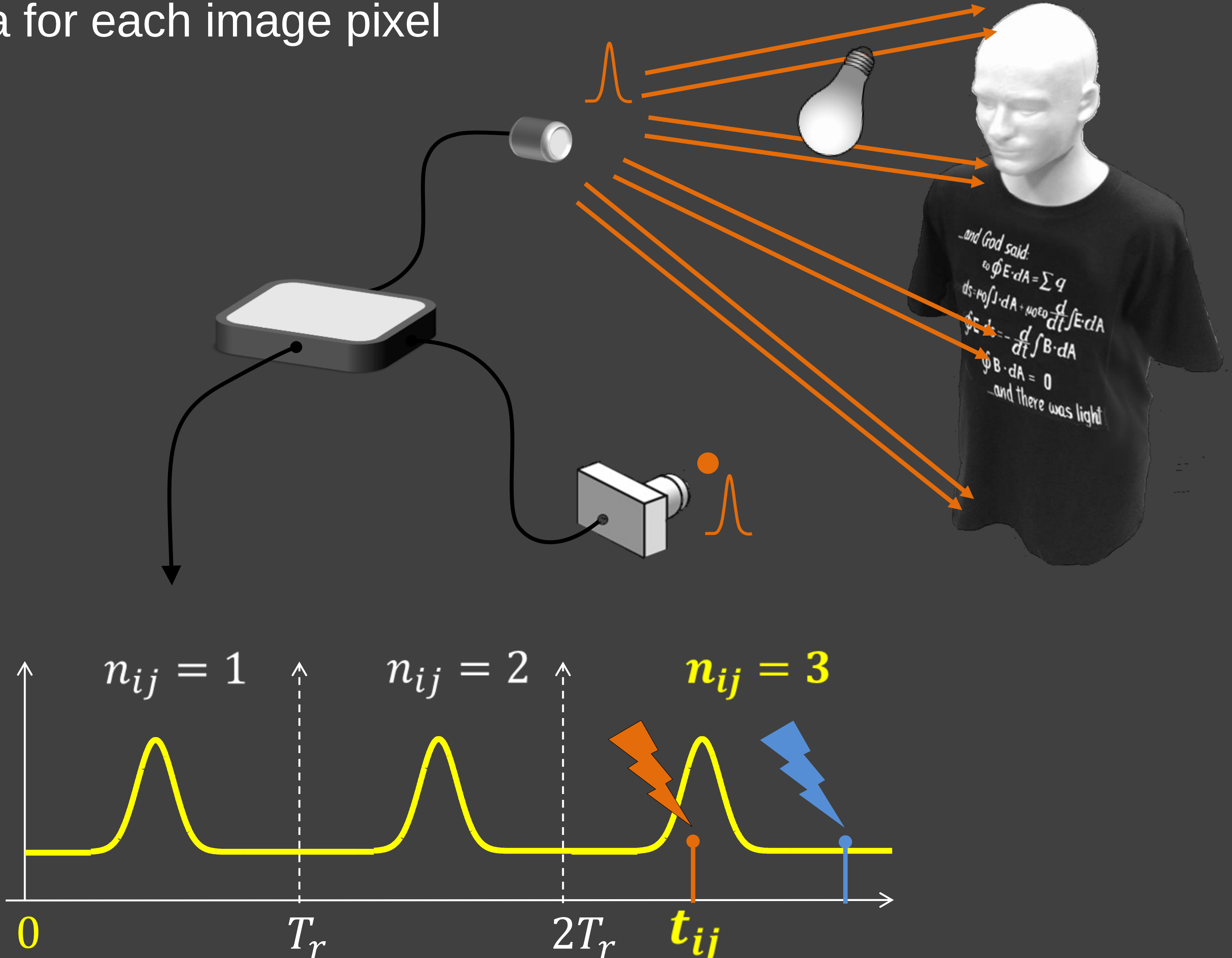
$$\Pr[\text{background photon detection}] = \frac{b_\lambda}{\alpha_{ij} + b_\lambda}$$

$$= 1 - \Pr[\text{signal photon detection}]$$

$$f_{t_{ij}}\left(\tau \mid \text{detected background photon}\,; z_{ij}\right) = \frac{1}{T_r}$$

In our experiment $\Pr[\text{background photon detection}] = 0.5$

Raster scanning is used to collect first-photon data for each image pixel



First-photon reflectivity data

Number of pulses transmitted
before first photon detection

$$n_{ij}$$

$$\Pr[n_{ij} = k; \alpha_{ij}] = e^{-(\alpha_{ij} + b_\lambda T)(k-1)}\left[1 - e^{-(\alpha_{ij} + b_\lambda T)}\right]$$

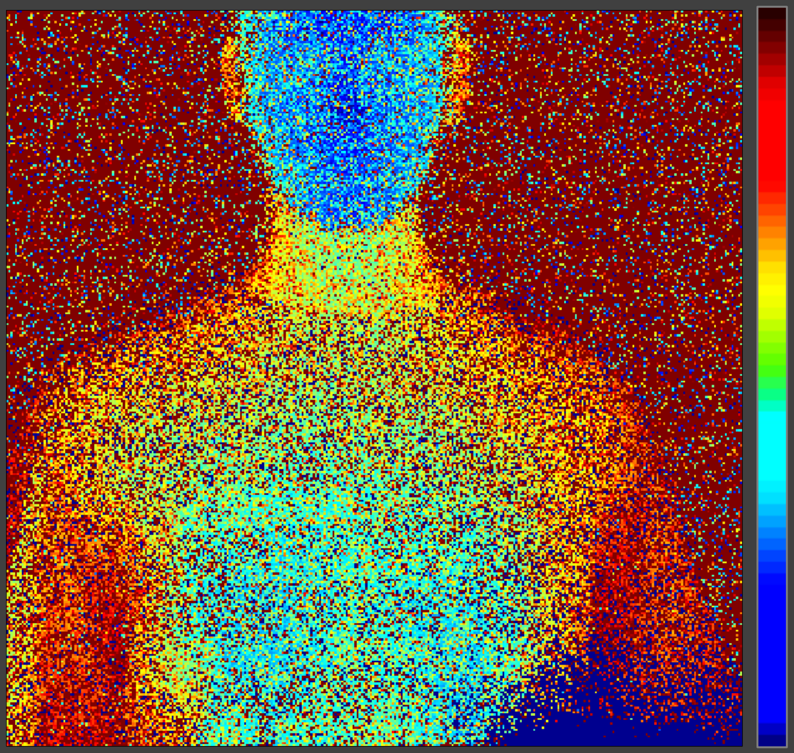
First-photon reflectivity data

Number of pulses transmitted before first photon detection

$$n_{ij}$$

$$\Pr[n_{ij} = k; \alpha_{ij}] = e^{-(\alpha_{ij}+b_\lambda T)(k-1)}\left[1 - e^{-(\alpha_{ij}+b_\lambda T)}\right]$$

First-photon time-of-flight data

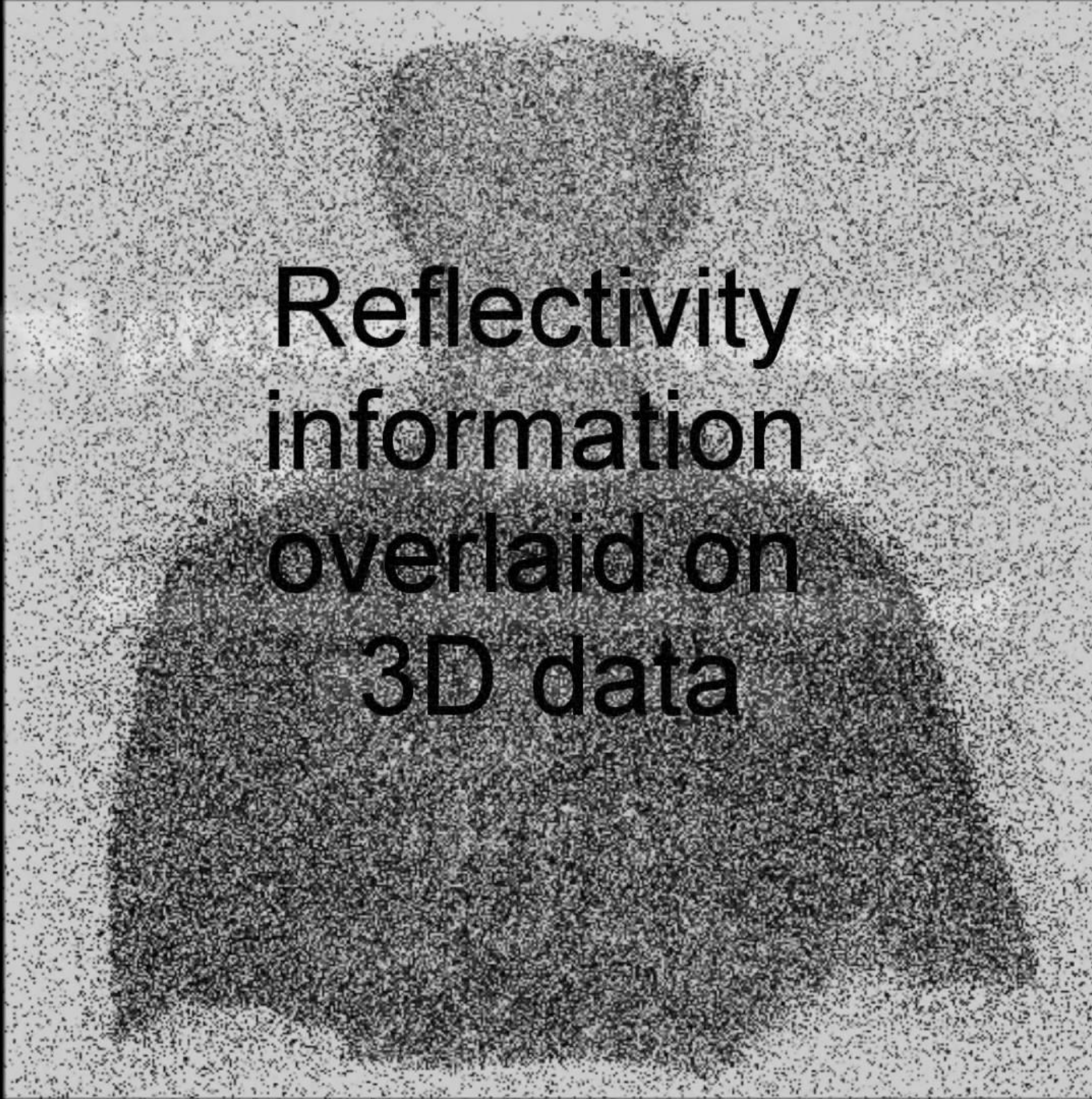


First-photon detection time relative to last transmitted pulse

$$t_{ij}$$

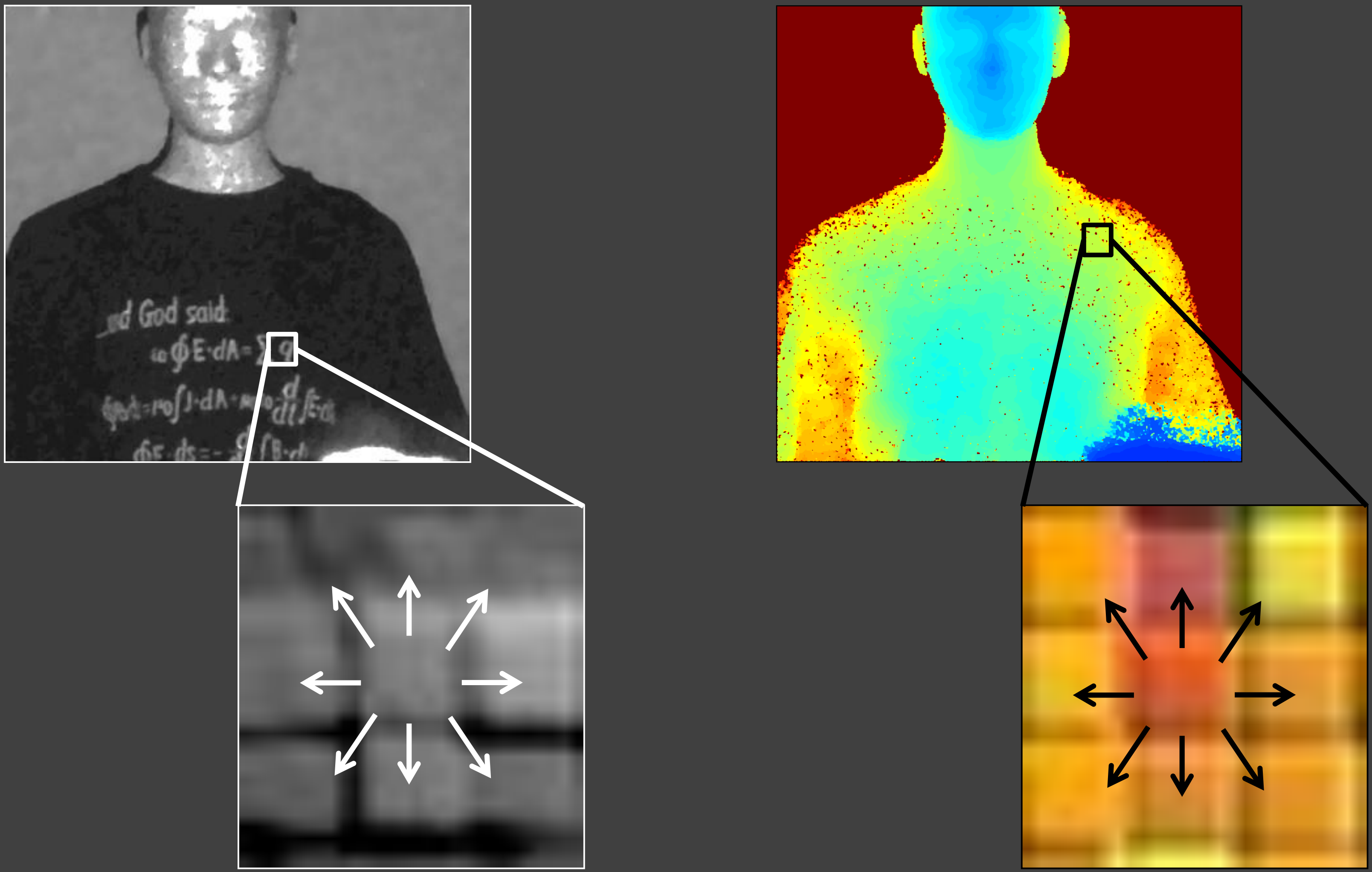
$$f_{t_{ij}}(\tau\,;z_{ij}\ |\text{detected signal photon}) \propto s(\tau - 2z_{ij}/c)$$

Pointwise estimation


Reflectivity
information
overlaid on
3D data

Novel image formation

Combining first-photon physics with spatial correlations



Combining first-photon physics with spatial correlations

Image formation method

Step 1: Estimate reflectivity from elapsed pulse data $\{n_{ij}\}$

Step 2: Censor background noise photons using ROAD filtering

Step 3: Estimate depth from uncensored time-of-arrival data $\{t_{ij}\}$

Step 1: Reflectivity estimation using regularized maximum likelihood estimation

data likelihood

$$\Pr[n_{ij} = k; \alpha_{ij}] = e^{-(\alpha_{ij}+b_{\lambda})T(k-1)}\left[1 - e^{-(\alpha_{ij}+b_{\lambda})T}\right]$$

parameter

regularized ML estimation

$$\underset{A=\{\alpha_{11}\ldots\alpha_{NN}\}}{\operatorname{argmin}} \sum_{i}\sum_{j} -\log \Pr[n_{ij}; a_{ij}] \quad + \quad \beta \parallel \Phi_{\alpha}\, A \parallel_{1}$$

data fidelity term

analysis with sparsity-promoting basis (wavelet)

Step 1

Reflectivity Reconstruction

(Mannequin Dataset)

Combining first-photon physics with spatial correlations

Image reconstruction method

Step 1: Estimate reflectivity from elapsed pulse data $\{n_{ij}\}$

Step 2: Censor background noise photons using ROAD filtering

Step 3: Estimate depth from uncensored time-of-arrival data $\{t_{ij}\}$

Step 2: Background photon censoring

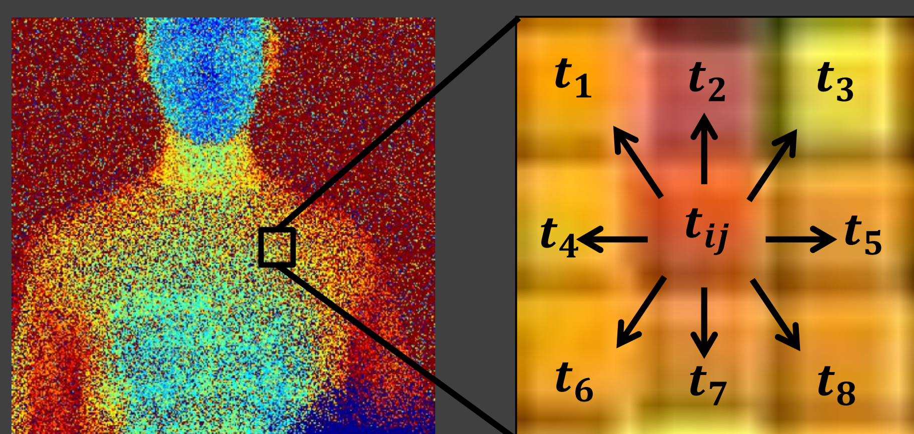
data likelihood

$$\Pr[\text{background photon detection}] = \frac{b_\lambda}{\alpha_{ij} + b_\lambda}$$

$$f_{t_{ij}}\left(\tau \mid \text{detected signal photon}\,;z_{ij}\right) \propto s\left(\tau - 2z_{ij}/c\right)$$

$$f_{t_{ij}}\left(\tau \mid \text{detected ambient photon}\,;z_{ij}\right) = \frac{1}{T_r}$$

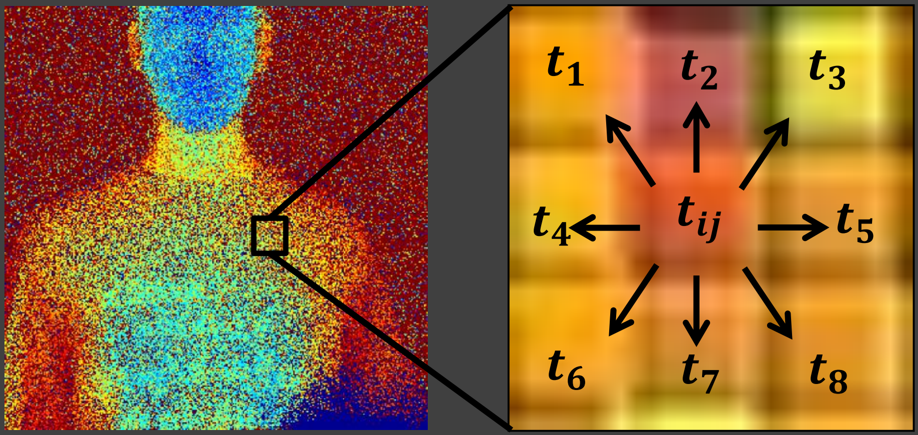
rank-ordered absolute difference (ROAD)-based test



$$\left\{\begin{array}{c} |t_{ij} - t_1| \\ |t_{ij} - t_2| \\ |t_{ij} - t_3| \\ |t_{ij} - t_4| \\ \vdots \\ \vdots \\ |t_{ij} - t_8| \end{array}\right\} = \mathrm{ROAD}_{ij}$$

Step 2: Background photon censoring

ROAD filtering



$$\left\{ \begin{array}{c} |t_{ij} - t_1| \\ |t_{ij} - t_2| \\ |t_{ij} - t_3| \\ |t_{ij} - t_4| \\ \vdots \\ \vdots \\ |t_{ij} - t_8| \end{array} \right\} = \mathrm{ROAD}_{ij}$$

RMS pulse-width

background level

if $\mathrm{ROAD}_{ij} \geq 4T_p \dfrac{b_\lambda}{\alpha_{ij} + b_\lambda}$ **then** (i, j) is censored.

estimated reflectivity

Step 2

Background Noise Censoring

(Mannequin Dataset)

Combining first-photon physics with spatial correlations

Image reconstruction method

Step 1: Estimate reflectivity from elapsed pulse data $\{n_{ij}\}$

Step 2: Censor background noise photons using ROAD filtering

Step 3: Estimate depth from uncensored time-of-arrival data $\{t_{ij}\}$

Step 3: Depth estimation using regularized maximum likelihood estimation

data likelihood

$$f_{t_{ij}}\left(\tau \mid \text{detected signal photon}\,;\ z_{ij}\right) \propto s\left(\tau - 2z_{ij}/c\right)$$

regularized ML estimation

$$\underset{D=\{d_{ij}\}}{\operatorname{argmin}} \sum\sum_{\substack{\textit{uncensored}\\ \textit{locations}}} -\log f_{t_{ij}}\left(\tau \mid \text{signal}\,;d_{ij}\right) + \beta \,\|\, \Phi_\alpha D \,\|_1$$

parameter

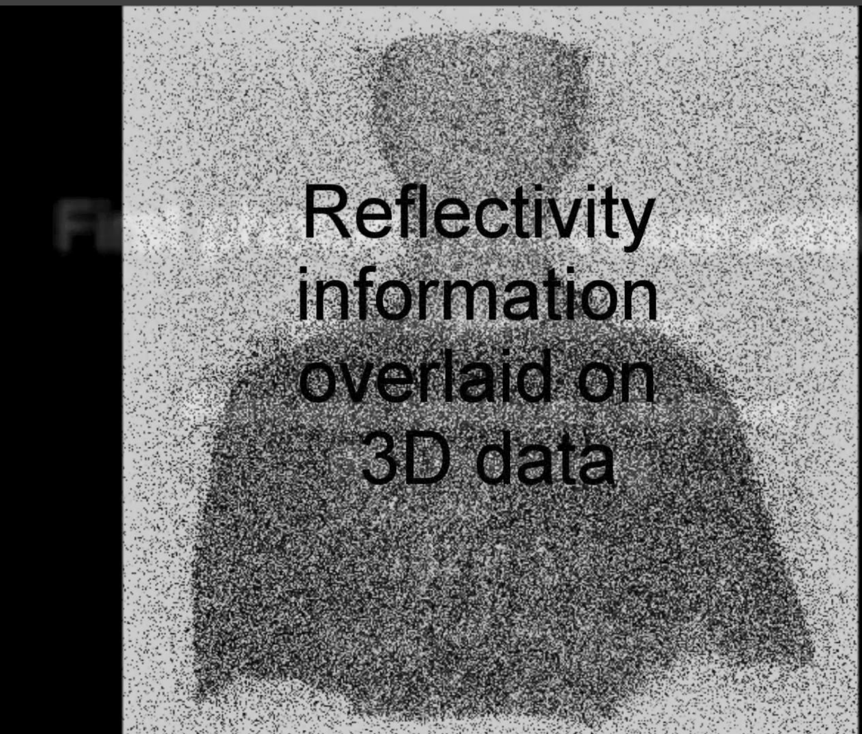
data fidelity term
at uncensored pixels

sparsity promoting
bases (wavelets)

Step 3

3D form Reconstruction

(Mannequin Dataset)

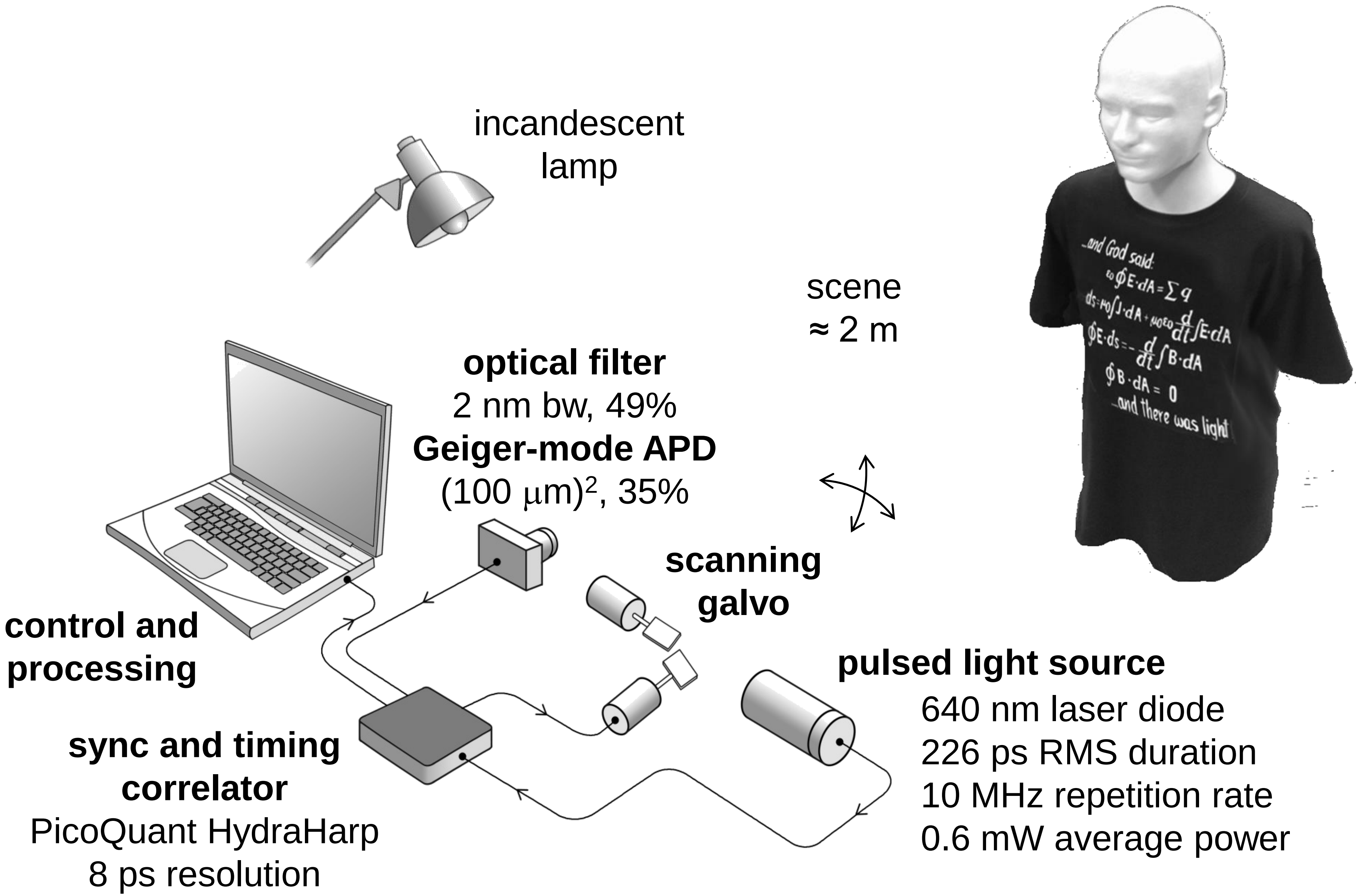


Conventional pixelwise
maximum likelihood estimates

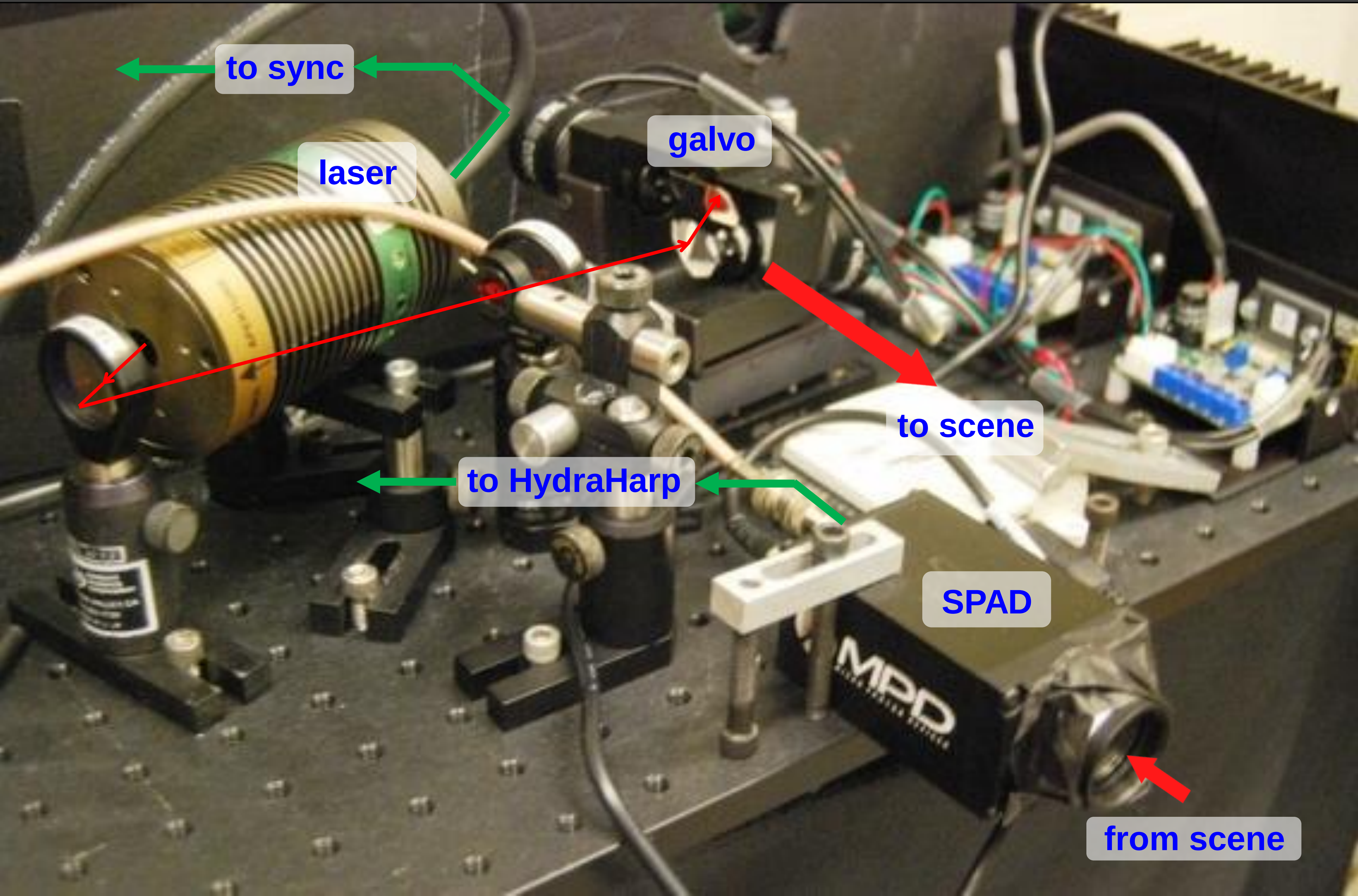
First-photon imaging

Experiments and evaluation

Experimental setup

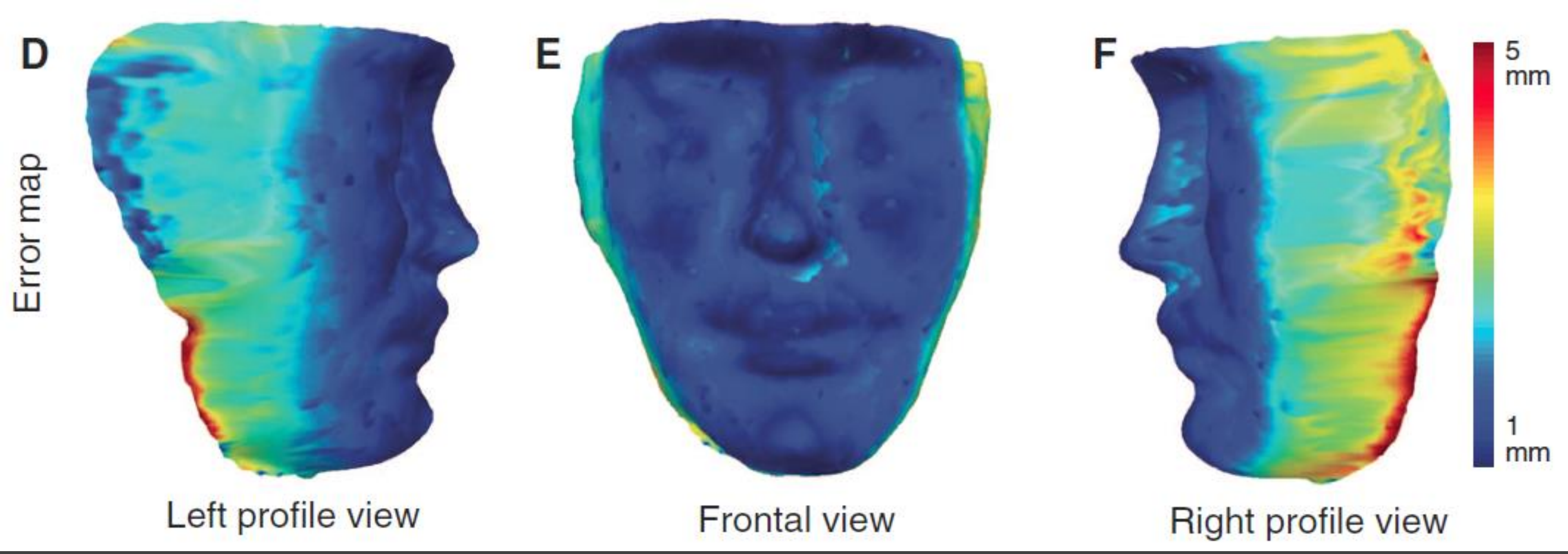


Experimental setup



Limitations

- High error at lateral surfaces, edges, and corners
- High optical flux (almost every pulse leads to detection)



	Deterministic acquisition time	Exploit pulse shape	Exploit transverse smoothness	Uncalibrated background	Estimate multiple layers	Compensate for array properties
Kirmani, Venkatraman, Shin, Colaço, Wong, Shapiro, Goyal, *Science*, 343(6166):58-61, 2014		✓	✓			
Shin, Kirmani, Shapiro, Goyal, *IEEE Trans. Computational Imaging*, 1(2):112-125, 2015	✓	✓	✓			
Shin, Shapiro, Goyal, *IEEE Signal Processing Letters*, 22(12):2254-2258, 2015	✓	✓		✓		
Shin, Xu, Wong, Shapiro, Goyal, *Optics Express*, 24(3):1873-1888, 2016	✓	✓			✓	
Shin, Xu, Venkatraman, Lussana, Villa, Zappa, Goyal, Wong, Shapiro, submitted, 2015	✓	✓	✓			✓

In prep: fluorescence lifetime imaging, transverse super-resolution, unambiguous range extension

Related work with fixed dwell time

- Parallelizable (detector array)
- For low photon count on average, many pixels have 0 detections
- Performance can be even better than FPI

		Random dwell	Fixed dwell
Mannequin	Mean T_a	244 μs	244 μs
	Mean $k_{i,j}$	1 ppp	2.7 ppp
	Pixels missing data	0%	33%
	PSNR	35 dB	37 dB
	RMSE	0.4 cm	0.3 cm
Sunflower	Mean T_a	15 μs	15 μs
	Mean $k_{i,j}$	1 ppp	8.7 ppp
	Pixels missing data	0%	18%
	PSNR	15 dB	16 dB
	RMSE	0.8 cm	0.5 cm
Basketball-and-can	Mean T_a	181 μs	181 μs
	Mean $k_{i,j}$	1 ppp	1.7 ppp
	Pixels missing data	0%	24%
	PSNR	40 dB	40 dB
	RMSE	1.1 cm	1.1 cm
Reflectivity chart	Mean T_a	120 μs	120 μs
	Mean $k_{i,j}$	1 ppp	1.7 ppp
	Pixels missing data	0%	27%
	PSNR	40 dB	42 dB
Depth chart	Mean T_a	6.2 μs	6.2 μs
	Mean $k_{i,j}$	1 ppp	1.1 ppp
	Pixels missing data	0%	35%
	RMSE	0.4 cm	0.4 cm

	Deterministic acquisition time	Exploit pulse shape	Exploit transverse smoothness	Uncalibrated background	Estimate multiple layers	Compensate for array properties
Kirmani, Venkatraman, Shin, Colaço, Wong, Shapiro, Goyal, *Science*, 343(6166):58-61, 2014		✓	✓			
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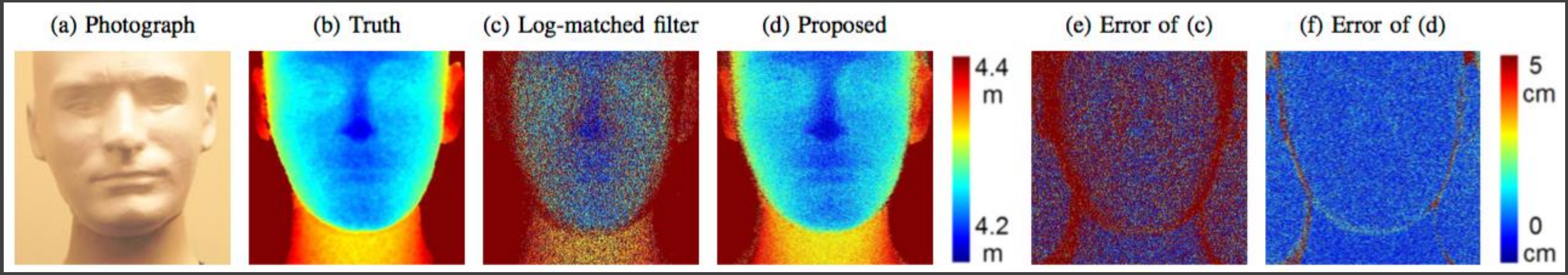
Shin, Shapiro, Goyal, *IEEE Signal Processing Letters*, 22(12):2254-2258, 2015	✓	✓		✓		
Shin, Xu, Wong, Shapiro, Goyal, *Optics Express*, 24(3):1873-1888, 2016	✓	✓			✓	
Shin, Xu, Venkatraman, Lussana, Villa, Zappa, Goyal, Wong, Shapiro, submitted, 2015	✓	✓	✓			✓

In prep: fluorescence lifetime imaging, transverse super-resolution, unambiguous range extension

Unknown background and no transverse regularization

- Exploit union-of-subspaces model for each pixel separately
- CoSaMP-inspired efficient algorithm
- Compared to log-matched filter, MAE reduction factor of 6 using 15 photons per pixel



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Shin, Kirmani, Shapiro, Goyal, *IEEE Trans. Computational Imaging*, 1(2):112-125, 2015	✓	✓	✓			
Shin, Shapiro, Goyal, *IEEE Signal Processing Letters*, 22(12):2254-2258, 2015	✓	✓		✓		
Shin, Xu, Wong, Shapiro, Goyal, *Optics Express*, 24(3):1873-1888, 2016	✓	✓			✓	
Shin, Xu, Venkatraman, Lussana, Villa, Zappa, Goyal, Wong, Shapiro, submitted, 2015	✓	✓	✓			✓

In prep: fluorescence lifetime imaging, transverse super-resolution, unambiguous range extension

Multiple depths per pixel and no transverse regularization

- Exploit longitudinal sparsity for each pixel separately
- ISTA-inspired efficient algorithm for convex relaxation of problem
- Compare to mixture of Gaussians fit with EM (shown at 19 ppp)

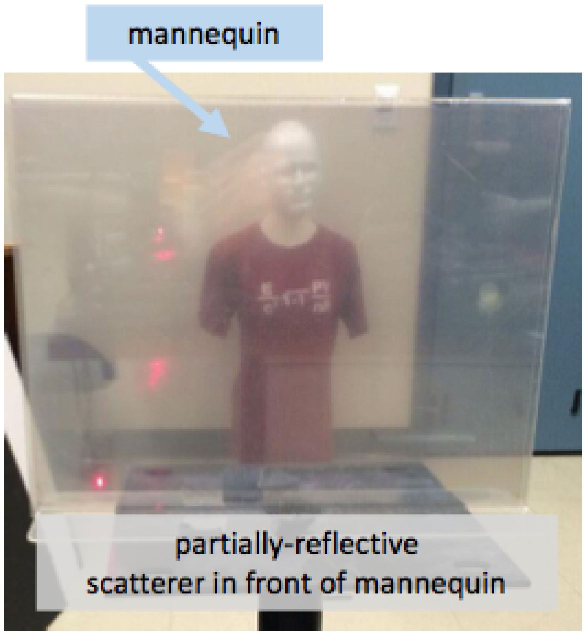
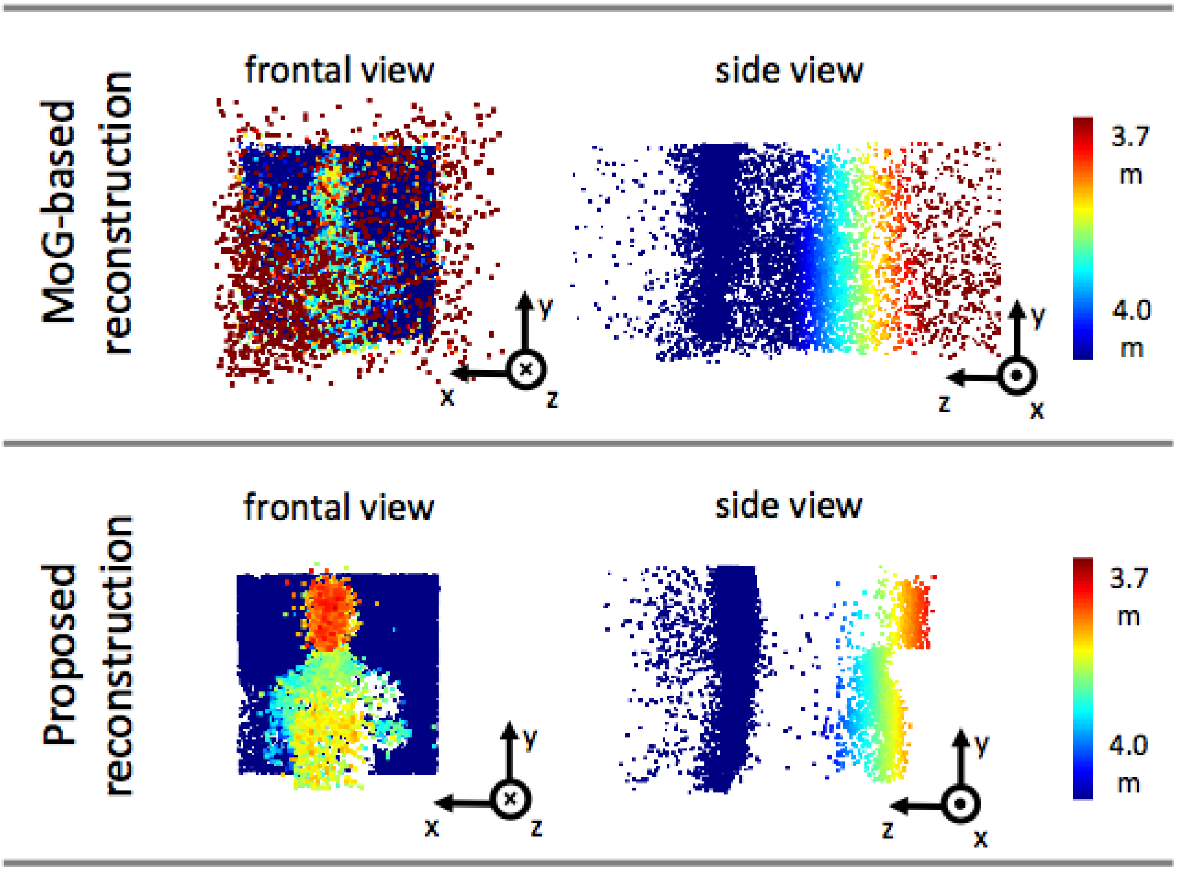


Photo of scene with object behind scatterer

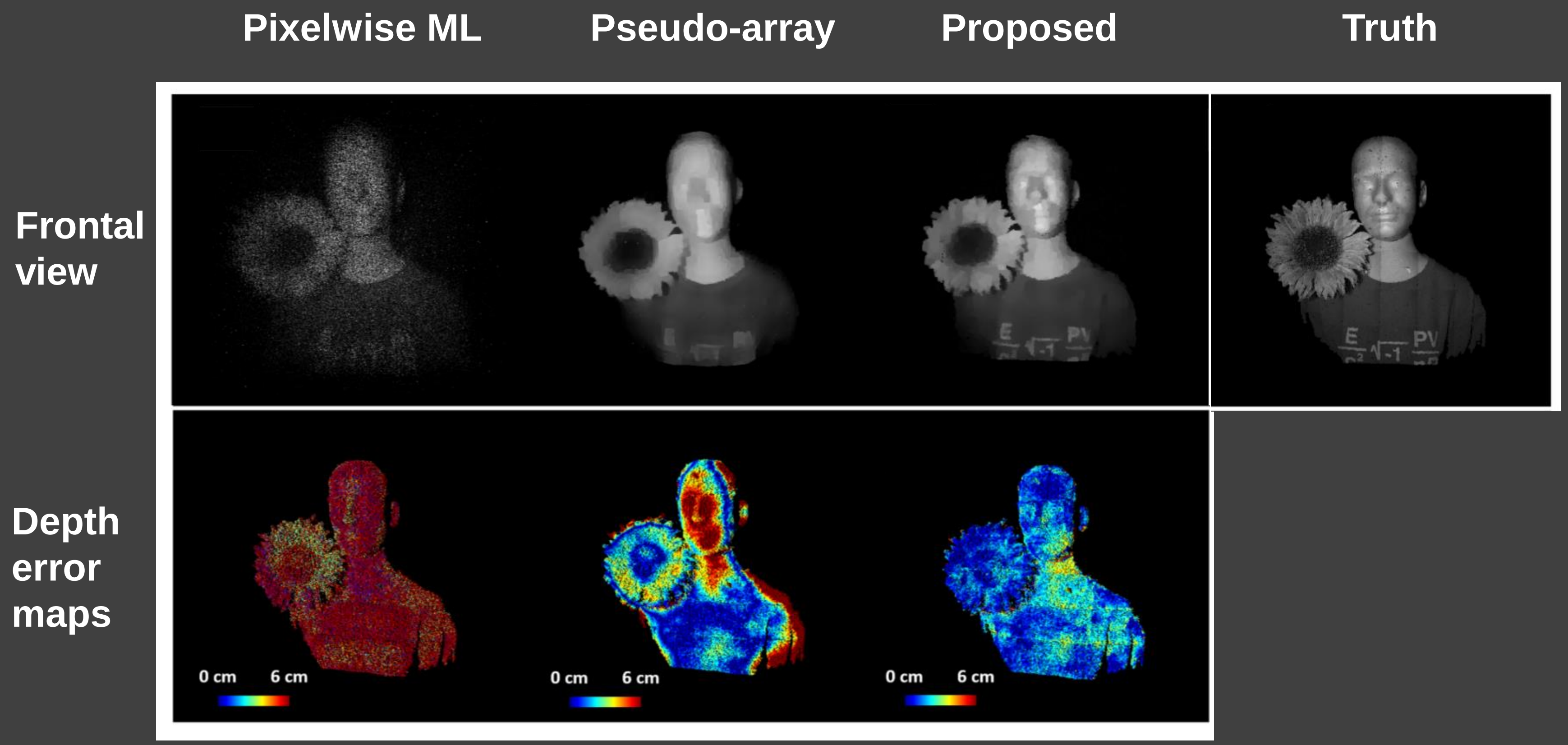


	Deterministic acquisition time	Exploit pulse shape	Exploit transverse smoothness	Uncalibrated background	Estimate multiple layers	Compensate for array properties
Kirmani, Venkatraman, Shin, Colaço, Wong, Shapiro, Goyal, *Science*, 343(6166):58-61, 2014		✓	✓			
Shin, Kirmani, Shapiro, Goyal, *IEEE Trans. Computational Imaging*, 1(2):112-125, 2015	✓	✓	✓			
Shin, Shapiro, Goyal, *IEEE Signal Processing Letters*, 22(12):2254-2258, 2015	✓	✓		✓		
Shin, Xu, Wong, Shapiro, Goyal, *Optics Express*, 24(3):1873-1888, 2016	✓	✓			✓	
Shin, Xu, Venkatraman, Lussana, Villa, Zappa, Goyal, Wong, Shapiro, submitted, 2015	✓	✓	✓			✓

In prep: fluorescence lifetime imaging, transverse super-resolution, unambiguous range extension

Mitigate challenges of SPAD array

- Much coarser time resolution, hot pixels
- Developed improved noise censoring, longitudinal super-resolution
- RMS error at one-third RMS pulse width with 1 signal photon/pixel



First-Photon Imaging and Other Imaging with Few Photons

Image formation that integrates physical modeling of acquisition and scene modeling can provide dramatic improvements

Model at the right scale

Apply an inverse problem mentality

First-Photon Imaging and Other Imaging with Few Photons