Cooperation and Competition in Multi-user Wireless Networks

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multiuser wireless networks: Motivations

Licence-free frequency band

- Deal with the inter-user interference.
- Low transmit power.

How can we increase the transmission rate?

- Cognitive radio: dynamically exploit the available radio frequency spectrum in order to efficiently avoiding interference.
- Cooperation between transmitter/receiver nodes or with additional relay nodes.

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Cognitive radio

Goal

Dynamically exploit the available radio frequency spectrum in order to efficiently avoiding interference.

Remarks

- Resource allocation game between users.
- Making use of tools from game theory.

Cooperative channels

Major information-theoretic works: [Cover and El Gamal, 1979]

- Useful to assess the benefits of cooperation in terms of communication rate (standard relay channel).
- They introduced two major relaying strategies: decode-and-forward and estimate-and-forward.
- **The relaying strategy is a key point for the cooperation between users.**

The Return of Cooperative Channels

MIMO channels

[Telatar, 1995 and 1999], [Foschini, 1996 and 1998]: Information-theoretic analysis of MIMO systems (diversity gain, multiplexing gain) → Increase communication rate.

Virtual MIMO

[Sendonaris et al., 1998 and 2003]: The benefits (rate, diversity) of MIMO systems can be obtained in a distributive manner in wireless networks.

Objectives of user cooperation

- Increase the range of wireless communications or the the communication rate.
- Increase the reliability of communications in fading environnements.

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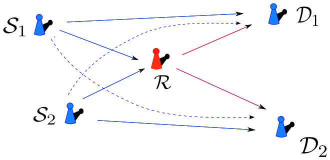
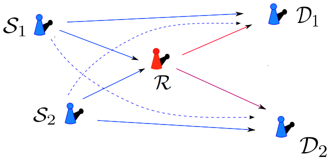
Objectives: Cooperative multi-user system + MultiMultiband radio

- Generalization of Cover and El Gamal's results.
- Introduction of a theoretic approach in a decentralized context.



- Multiband radio with Q non-overlapping frequency bands.
- Selfish users maximizing their individual transmission rate.
- Power Allocation Game.



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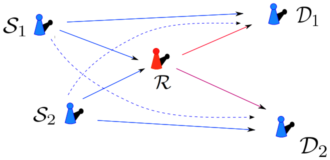

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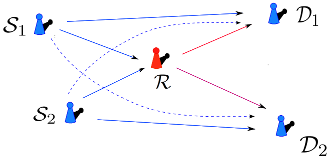

Outline of the talk

1. Shannon theory for the interference relay channel
   - Background and goal
   - The discrete case
   - The Gaussian case with only private messages
2. Power allocation Games in multiband IRCs
   - Background
   - System model
   - Equilibrium analysis for some relaying protocols
3. Conclusion and perspectives

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- Multiple access relay channel (MARC): [Sankaranarayanan et al., 2003]
- Broadcast relay channel (BRC): [Liang et al., 2007], [Kramer, 2005]

Interference channel

- Capacity region known for the special case of strong interference: [Carleial, 1975], [Sato, 1978]
- Best inner bound by [Han and Kobayashi, 1981]: rate-splitting + time-sharing.

Interference relay channel (IRC)

- Introduced by [Sahin and Erkip, 2007]: rate region for the Gaussian case with a DF-based strategy.
- Results of [Maric & al, 2008]: DF and interference forwarding.

Our contributions

- Treat both discrete and Gaussian cases.
- Coding theorems based on several strategies (DF and EF).
- Two approaches for the EF-based strategy: Bi-level and single-level compressions.
- A simple AF-based strategy for the Gaussian case.

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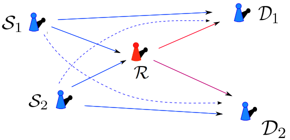

Rate-splitting ([Carleial, 1978]) at each source node

- (W_{10}, W_{11}) at $\mathcal{S}_1$ and (W_{20}, W_{22}) at $\mathcal{S}_2$.
- $\mathcal{D}_1$ decodes the triplet (W_{10}, W_{11}, W_{20}) and $R_1 = R_{10} + R_{11}$.
- $\mathcal{D}_2$ decodes the triplet (W_{10}, W_{20}, W_{22}) and $R_2 = R_{20} + R_{22}$.

Theorem (DF-based strategy)

For the DMIRC $(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_r, p(y_1, y_2, y_r|x_1, x_2, x_r), \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_r)$ with both private and common messages, any rate quadruplet $(R_{10}, R_{11}, R_{20}, R_{22})$ satisfying

$$\sum_{i \in \mathcal{I}} R_i \leq I\left(V_{\mathcal{I}}; Y_r \mid U_{\mathcal{S}}, X_r, V_{\mathcal{I}^C}\right) \quad \text{for all } \mathcal{I} \subseteq \mathcal{S} = \{10, 11, 20, 22\},$$

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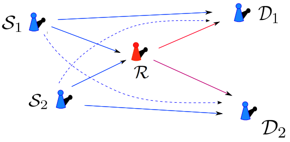
for some joint distribution $p(u_{10})p(v_{10}|u_{10})p(u_{11})p(v_{11}|u_{11})p(x_1|v_{10}, v_{11})p(u_{20})p(v_{20}|u_{20})p(u_{22})p(v_{22}|u_{22}) \times p(x_2|v_{20}, v_{22})p(x_r|u_{10}, u_{11}, u_{20}, u_{22})$, is achievable, where $\mathcal{I}^C$, $\mathcal{I}_1^C$ and $\mathcal{I}_2^C$ and the complements of $\mathcal{I}$, $\mathcal{I}_1$ and $\mathcal{I}_2$ respectively in $\mathcal{S}$, $\mathcal{S}_1$ and $\mathcal{S}_2$. We have $V_{\mathcal{I}} = \{V_j,\ j \in \mathcal{I}\}$.



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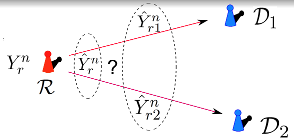

Bi-level compression feature

The relay increases interference at each receiver node.

Theorem (EF-based strategy: Bi-level resolution compression)

For the DMIRC $(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_r, p(y_1, y_2, y_r | x_1, x_2, x_r), \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_r)$ with both private and common messages, the rate quadruplet $(R_{10}, R_{11}, R_{20}, R_{22})$ is achievable, where

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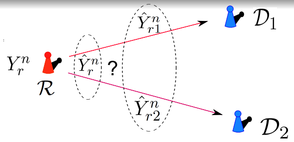

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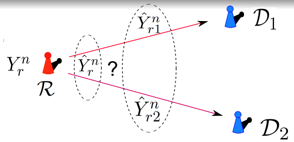

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Single-level compression feature

The estimation noise level is lower bounded by the worse receiver node.

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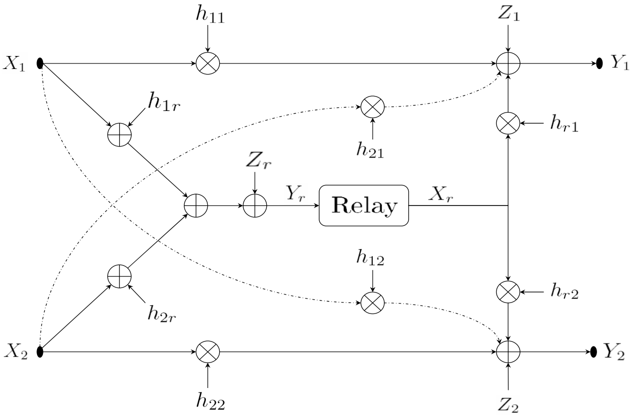
System model



power constraints: $\mathbb{E}|X_1|^2 \leq P_1$, $\mathbb{E}|X_2|^2 \leq P_2$ and $\mathbb{E}|X_r|^2 \leq P_r$.

Corollary (DF-based strategy – Sahin & Erkip, 2008)

When DF is assumed, the following region is achievable:

$$
\begin{aligned}
R_1 &\leq \min\left\{ C\left(\frac{|h_{1r}|^2(1-\tau_1)P_1}{N_r}\right), C\left(\frac{|h_{11}|^2P_1 + |h_{r1}|^2\nu_1P_r + 2\mathcal{R}e(h_{11}h_{r1}^*)\sqrt{\tau_1P_1\nu_1P_r}}{|h_{21}|^2P_2 + |h_{r1}|^2\nu_2P_r + 2\mathcal{R}e(h_{21}h_{r1}^*)\sqrt{\tau_2P_2\nu_2P_r} + N_1}\right)\right\} \\
R_2 &\leq \min\left\{ C\left(\frac{|h_{2r}|^2(1-\tau_2)P_2}{N_r}\right), C\left(\frac{|h_{22}|^2P_2 + |h_{r2}|^2\nu_2P_r + 2\mathcal{R}e(h_{22}h_{r2}^*)\sqrt{\tau_2P_2\nu_2P_r}}{|h_{12}|^2P_1 + |h_{r2}|^2\nu_1P_r + 2\mathcal{R}e(h_{12}h_{r2}^*)\sqrt{\tau_1P_1\nu_1P_r} + N_2}\right)\right\} \\
R_1 + R_2 &\leq C\left(\frac{|h_{1r}|^2(1-\tau_1)P_1 + |h_{2r}|^2(1-\tau_2)P_2}{N_r}\right),
\end{aligned}
$$

where $(\nu_1, \nu_2) \in [0,1]^2$ *s.t.* $\nu_1 + \nu_2 \leq 1$ *and* $(\tau_1, \tau_2) \in [0,1]^2$.

Corollary (EF strategy: Bi-level resolution compression with only private messages)

For the Gaussian IRC with only private messages and with the bi-level resolution estimate-and-forward strategy, the rate pair (R_{11}, R_{22}) *is achievable, where*

1. *if* $C\left(\frac{|h_{r1}|^2\nu_2 P_r}{|h_{11}|^2P_1+|h_{21}|^2P_2+|h_{r1}|^2\nu_1P_r+N_1}\right) \geq C\left(\frac{|h_{r2}|^2\nu_2 P_r}{|h_{22}|^2P_2+|h_{12}|^2P_1+|h_{r2}|^2\nu_1P_r+N_2}\right)$,

   we have

$$R_{11} \leq C\left(\frac{|h_{11}|^2P_1}{N_1+\frac{|h_{21}|^2P_2\left(N_r+N_{wz}^{(1)}\right)}{|h_{2r}|^2P_2+N_r+N_{wz}^{(1)}}}+\frac{|h_{1r}|^2P_1}{N_r+N_{wz}^{(1)}+\frac{|h_{2r}|^2P_2N_1}{|h_{21}|^2P_2+N_1}}\right),$$

$$R_{22} \leq C\left(\frac{|h_{22}|^2P_2}{N_2+|h_{r2}|^2\nu_1P_r+\frac{|h_{12}|^2P_1\left(N_r+N_{wz}^{(2)}\right)}{|h_{1r}|^2P_1+N_r+N_{wz}^{(2)}}}+\frac{|h_{2r}|^2P_2}{N_r+N_{wz}^{(2)}+\frac{|h_{1r}|^2P_1\left(|h_{r2}|^2\nu_1P_r+N_2\right)}{|h_{12}|^2P_1+|h_{r2}|^2\nu_1P_r+N_2}}\right),$$

subject to the constraints

$$N_{wz}^{(1)} \geq \frac{\left(|h_{11}|^2P_1+|h_{21}|^2P_2+N_1\right)A-A_1^2}{|h_{r1}|^2\nu_1P_r},\quad N_{wz}^{(2)} \geq \frac{\left(|h_{22}|^2P_2+|h_{12}|^2P_1+|h_{r2}|^2\nu_1P_r+N_2\right)A-A_2^2}{|h_{r2}|^2\nu_2P_r},$$

with $(\nu_1,\nu_2)\in[0,1]^2$, $\nu_1+\nu_2\leq 1$, $A=|h_{1r}|^2P_1+|h_{2r}|^2P_2+N_r$, $A_1=2\mathcal{R}e(h_{11}h_{1r}^*)P_1+2\mathcal{R}e(h_{21}h_{2r}^*)P_2$ *and* $A_2=2\mathcal{R}e(h_{12}h_{1r}^*)P_1+2\mathcal{R}e(h_{22}h_{2r}^*)P_2$. ...

The channel $\mathcal{R}$-$(\mathcal{D}_1,\mathcal{D}_2)$ is a Gaussian BC for which the capacity region is known.

Corollary (EF strategy: single-level resolution compression with only private messages)

For the Gaussian IRC with only private messages and with the bi-level resolution estimate-and-forward strategy, the rate pair (R_{11}, R_{22}) *is achievable, where*

$$R_{11} \leq C\left(\frac{|h_{11}|^2 P_1}{N_1 + \frac{|h_{21}|^2 P_2 (N_r + N_{wz})}{|h_{2r}|^2 P_2 + N_r + N_{wz}}} + \frac{|h_{1r}|^2 P_1}{N_r + N_{wz} + \frac{|h_{2r}|^2 P_2 N_2}{|h_{21}|^2 P_2 + N_1}}\right),$$

$$R_{22} \leq C\left(\frac{|h_{22}|^2 P_2}{N_2 + \frac{|h_{12}|^2 P_1 (N_r + N_{wz})}{|h_{1r}|^2 P_1 + N_r + N_{wz}}} + \frac{|h_{2r}|^2 P_2}{N_r + N_{wz} + \frac{|h_{1r}|^2 P_1 N_2}{|h_{12}|^2 P_1 + N_2}}\right),$$

subject to the constraints $N_{wz} \geq \frac{\max\left\{\sigma_1^2, \sigma_2^2\right\}}{2^{2R_0} - 1}$ *with*

$$R_0 = \min\left\{C\left(\frac{|h_{r1}|^2 P_r}{|h_{11}|^2 P_1 + |h_{21}|^2 P_2 + N_1}\right), C\left(\frac{|h_{r2}|^2 P_r}{|h_{22}|^2 P_2 + |h_{12}|^2 P_1 + N_2}\right)\right\},$$

$$\sigma_1^2 = |h_{1r}|^2 P_1 + |h_{2r}|^2 P_2 + N_r - \frac{\left(2\mathcal{R}e(h_{11} h_{1r}^*) P_1 + 2\mathcal{R}e(h_{21} h_{2r}^*) P_2\right)^2}{|h_{11}|^2 P_1 + |h_{21}|^2 P_1 + N_1}$$

$$\sigma_2^2 = |h_{1r}|^2 P_1 + |h_{2r}|^2 P_2 + N_r - \frac{\left(2\mathcal{R}e(h_{22} h_{2r}^*) P_2 + 2\mathcal{R}e(h_{12} h_{1r}^*) P_1\right)^2}{|h_{22}|^2 P_2 + |h_{12}|^2 P_1 + N_2}$$

Single-level compression vs Bi-level compression

	Single-level compression	Bi-level compression
Maximize the system sum-rate with low receiver SNRs asymmetry	×	
Maximize the rate at the "best" receiver with high received SNRs asymmetry		×
Maximize the rate at each receiver node with low receiver SNRs asymmetry	×	
Maximize the system sum-rate with high receiver SNRs asymmetry		×

Zero-delay scalar amplify-and-forward

Theorem (Transmission rate region for the IRC with ZDSAF)

Let R_i, $i \in \{1,2\}$, be the transmission rate for the source node $\mathcal{S}_i$. When ZDSAF is assumed the following region is achievable:

$$\forall i \in \{1,2\},\ R_i \leq C\left(\frac{|a_r h_{ir} h_{ri} + h_{ii}|^2 \rho_i}{|a_r h_{jr} h_{ri} + h_{ji}|^2 \rho_j \frac{N_j}{N_i} + a_r^2 |h_{ri}|^2 \frac{N_r}{N_i} + 1}\right)$$

where $\rho_i = \frac{P_i}{N_i}$ and $j = -i$.

Observation

The achievable individual rates are not always concave.

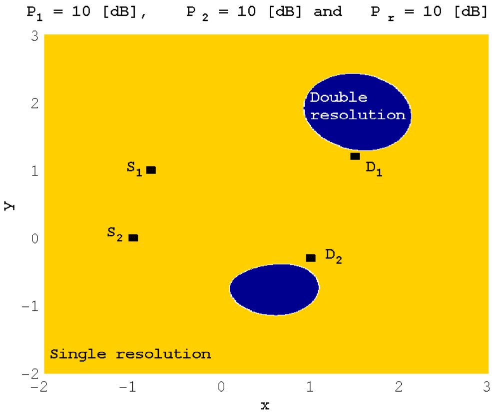
Time-Sharing Techniques [El Gamal, Mohseni and Zahedi, 2006]

$$R_i^{\mathrm{TS}} \leq \alpha_i(1-\alpha_j) C\left(\frac{|a_{r,i}^{\mathrm{TS}} h_{ir} h_{ri} + h_{ii}|^2 \rho_i}{\alpha_i[(a_{r,i}^{\mathrm{TS}})^2 |h_{ri}|^2 \frac{N_r}{N_i} + 1]}\right) + \alpha_i \alpha_j C\left(\frac{\left|a_r^{\mathrm{TS}} h_{ir} h_{ri} + h_{ii}\right|^2 \alpha_j \rho_i}{\alpha_i\left[\left|a_r^{\mathrm{TS}} h_{jr} h_{ri} + h_{ji}\right|^2 \rho_j \frac{N_j}{N_i} + \alpha_j[(a_r^{\mathrm{TS}})^2 |h_{ri}|^2 \frac{N_r}{N_i} + 1]\right]}\right)$$

$\forall i \in \{1,2\}$, where $(\alpha_1, \alpha_2) \in (0,1)^2$, $a_{r,i}^{\mathrm{TS}} = \sqrt{\frac{P_r/\mu}{|h_{ir}|^2 P_i/\alpha_i + N_r}}$, $a_r^{\mathrm{TS}} = \sqrt{\frac{P_r/\mu}{|h_{1r}|^2 P_1/\alpha_1 + |h_{2r}|^2 P_2/\alpha_2 + N_r}}$

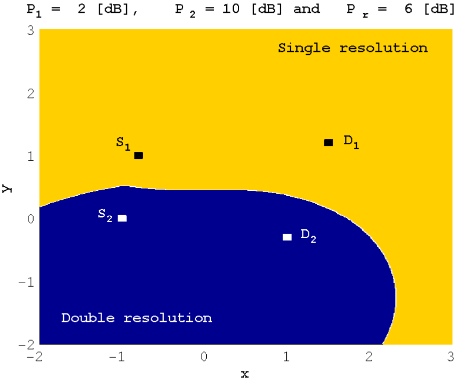
and $\mu = \max\{\alpha_1, \alpha_2\}$ are the relay amplification gains.

Bi-level resolution vs single-level resolution



- With the double resolution strategy, the cost of the additional interference by the relay is significant.

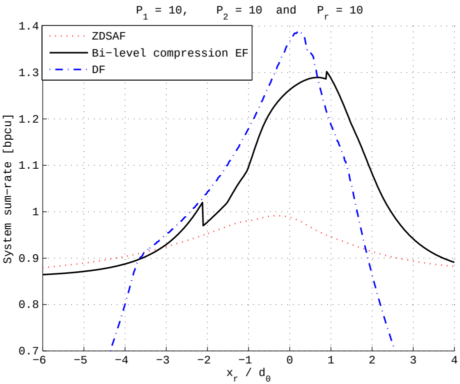
Bi-level resolution vs single-level resolution



Message

- The bi-level resolution compression is better for the "best" receiver if there is a high asymmetry in received SNRs between both receiver nodes.
- With low asymmetry in received SNRs, the single-level resolution compression is preferable to maximize the system sum-rate.

Achievable system sum-rate versus x_r (abscissa for the relay position) with AF, DF and bi-level EF.



Message

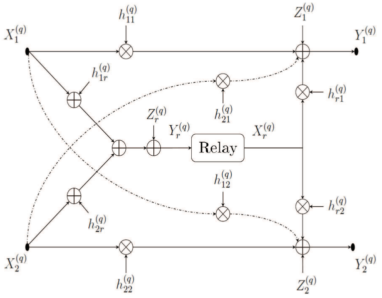
- Similar behavior as for the basic relay channel.

Outline of the talk

Related works

- [Xi & Yeh, 2008]: Traffic game in parallel relay networks with power policy to minimize a certain cost function.
- [Xi & Yeh, 2008]: Quite similar analysis for multi-hop networks.
- [Shi & al., 2008]: Special case of IRCs with the DF protocol without direct links between the sources and destinations.

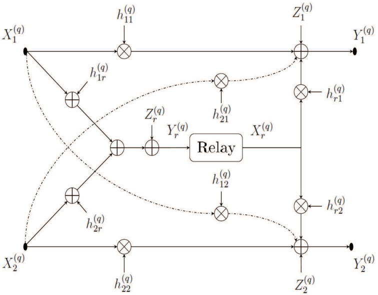
Outline of the talk



- Q non-overlapping frequency bands,
- Signal transmitted by $\mathcal{S}_i$ in band (q): $X_i^{(q)}$ with $\sum_{q=1}^{Q} \mathbb{E}|X_i^{(q)}|^2 \leq P_i., \forall i \in \{1,2\}$,
- $\theta_i^{(q)}$: fraction of power for $\mathcal{S}_i$ in band (q) ($\mathbb{E}|X_i^{(q)}|^2 = \theta_i^{(q)} P_i$),
- the channel gains are considered to be static (large scale propagation effects),
- coherent communications assumption for each transmitter-receiver pair,
- single user decoding.

Features and goals

- Each transmitter optimize its transmission rate in a selfish manner,
- a suitable model for this interaction: non-cooperative game,
- Question: do some **predictable** outcomes exist to this conflict situation?
- $\rightarrow$ a solution concept to non-cooperative game: **Nash Equilibrium** [Nash, 1950].



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Outline of the talk

The decode-and-forward case

Features

- Signal transmitted by $\mathcal{S}_i$ on band (q): $X_i^{(q)} = X_{i,0}^{(q)} + \sqrt{\frac{\tau_i^{(q)}}{\nu_i^{(q)}} \frac{\theta_i^{(q)} P_i}{P_r^{(q)}}} X_{r,i}^{(q)}$;
- Signal transmitted by $\mathcal{R}_i$ on band (q): $X_r^{(q)} = X_{r,1}^{(q)} + X_{r,2}^{(q)}$;
- Power allocation policies: $\forall i \in \{1, 2\}$, $\underline{\theta}_i = \left(\theta_i^{(1)}, \dots, \theta_i^{(Q)}\right)$.

transmission rates

the source-destination pair $(\mathcal{S}_i, \mathcal{D}_i)$ achieves the transmission rate $\sum_{q=1}^{Q} R_i^{(q),\mathrm{DF}}$ where

$$\begin{cases} R_1^{(q),\mathrm{DF}} = \min\left\{R_{1,1}^{(q),\mathrm{DF}}, R_{1,2}^{(q),\mathrm{DF}}\right\} \\ R_2^{(q),\mathrm{DF}} = \min\left\{R_{2,1}^{(q),\mathrm{DF}}, R_{2,2}^{(q),\mathrm{DF}}\right\} \end{cases}$$

with

$$\begin{cases} R_{1,1}^{(q),\mathrm{DF}} = C\left(\frac{\left|h_{1r}^{(q)}\right|^2\left(1-\tau_1^{(q)}\right)\theta_1^{(q)}P_1}{\left|h_{2r}^{(q)}\right|^2\left(1-\tau_2^{(q)}\right)\theta_2^{(q)}P_2+N_r^{(q)}}\right) \\ R_{1,2}^{(q),\mathrm{DF}} = C\left(\frac{\left|h_{11}^{(q)}\right|^2\theta_1^{(q)}P_1+\left|h_{r1}^{(q)}\right|^2\nu^{(q)}P_r^{(q)}+2\mathrm{Re}\left(h_{11}^{(q)}h_{r1}^{(q),*}\right)\sqrt{\tau_1^{(q)}\theta_1^{(q)}P_1\nu^{(q)}P_r^{(q)}}}{\left|h_{21}^{(q)}\right|^2\theta_2^{(q)}P_2+\left|h_{r1}^{(q)}\right|^2\overline{\nu^{(q)}}P_r^{(q)}+2\mathrm{Re}\left(h_{21}^{(q)}h_{r1}^{(q),*}\right)\sqrt{\tau_2^{(q)}\theta_2^{(q)}P_2\overline{\nu^{(q)}}P_r^{(q)}}+N_1^{(q)}}\right) \end{cases}$$

and $(\nu^{(q)}, \tau_1^{(q)}, \tau_2^{(q)})$ is a given triple of parameters in $[0,1]^3$, $\tau_1^{(q)} + \tau_2^{(q)} \leq 1$.

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and $(\nu^{(q)}, \tau_1^{(q)}, \tau_2^{(q)})$ is a given triple of parameters in $[0,1]^3$, $\tau_1^{(q)} + \tau_2^{(q)} \leq 1$.

The decode-and-forward case

Definition of the game: Non-cooperative strategic form game (SFG)

1. **Players:** $\mathcal{S}_1$ and $\mathcal{S}_2$;
2. **Strategy** of $\mathcal{S}_i$: $\underline{\theta}_i = (\theta_i^{(1)}, \ldots, \theta_i^{(Q)})$ in its *strategy set* $\mathcal{A}_i = \left\{\underline{\theta}_i \in [0,1]^Q \mid \sum_{q=1}^{Q} \theta_i^{(q)} \leq 1\right\}$;
3. **Utility function (or payoff)** of $\mathcal{S}_i$: $u_i^{\mathrm{DF}}(\underline{\theta}_i, \underline{\theta}_{-i}) = \sum_{q=1}^{Q} R_i^{(q),\mathrm{DF}}(\theta_i^{(q)}, \theta_{-i}^{(q)})$.

Assumption for the game

The game is played once (static game) and is with complete information *i.e.* every player knows the triplet $\mathcal{G}^{\mathrm{DF}} = (\mathcal{K}, (\mathcal{A}_i)_{i\in\mathcal{K}}, (u_i^{\mathrm{DF}})_{i\in\mathcal{K}})$, where $\mathcal{K} = \{1,2\}$

Definition [Nash Equilibrium]

The state $(\underline{\theta}_i^*, \underline{\theta}_{-i}^*)$ *is a pure NE of the SFG* $\mathcal{G}$ *if* $\forall i \in \mathcal{K}, \forall \underline{\theta}_i' \in \mathcal{A}_i,\ u_i(\underline{\theta}_i^*, \underline{\theta}_{-i}^*) \geq u_i(\underline{\theta}_i', \underline{\theta}_{-i}^*)$.

Theorem [Existence of an NE for the DF protocol]

If the channel gains satisfy the condition $\mathcal{R}e(h_{ii}^{(q)} h_{ri}^{(q)*}) \geq 0$, *for all* $i \in \{1,2\}$ *and* $q \in \{1,\ldots,Q\}$ *the game defined by* $\mathcal{G}^{\mathrm{DF}} = (\mathcal{K}, (\mathcal{A}_i)_{i\in\mathcal{K}}, (u_i^{\mathrm{DF}}(\underline{\theta}_i, \underline{\theta}_{-i}))_{i\in\mathcal{K}})$ *with* $\mathcal{K} = \{1,2\}$ *and*

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has always at least one pure NE.

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The decode-and-forward case

Proof

The proof is based on Theorem 1 of [rosen, 1965]. It states that in game with a finite number of players, if for every player

1. the strategy set is convex and compact,
2. its utility is continuous in the vector of strategies and
3. concave in its own strategy,

then the existence of at least one NE is guaranteed.

Comments

Whatever the values of the channel gains, there exists an NE. Therefore

- The transmitters are able to adapt their PA policies if the number of relay is modified,
- The transmitters are able to adapt their PA policies if the relay location is modified.

The decode-and-forward case

Proof

The proof is based on Theorem 1 of [rosen, 1965]. It states that in game with a finite number of players, if for every player

1. the strategy set is convex and compact,
2. its utility is continuous in the vector of strategies and
3. concave in its own strategy,

then the existence of at least one NE is guaranteed.

Comments

Whatever the values of the channel gains, there exists an NE. Therefore

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- The transmitters are able to adapt their PA policies if the relay location is modified.

The bi-level estimate-and-forward case

Decoding assumption and utility functions

Each receiver implements **single-user decoding** (SUD).

The utility function for $\mathcal{S}_i$ is given by: $u_i^{\mathrm{EF}}(\underline{\theta}_i, \underline{\theta}_{-i}) = \sum_{q=1}^{Q} R_i^{(q),\mathrm{EF}}$ where, for example,

$$R_1^{(q),\mathrm{EF}} = C\left(\frac{\left(\left|h_{2r}^{(q)}\right|^2\theta_2^{(q)}P_2+N_r^{(q)}+N_{wz,1}^{(q)}\right)\left|h_{11}^{(q)}\right|^2\theta_1^{(q)}P_1+\left(\left|h_{21}^{(q)}\right|^2\theta_2^{(q)}P_2+\left|h_{r1}^{(q)}\right|^2\overline{\nu^{(q)}}P_r^{(q)}+N_1^{(q)}\right)\left|h_{1r}^{(q)}\right|^2\theta_1^{(q)}P_1}{\left(N_r^{(q)}+N_{wz,1}^{(q)}\right)\left(|h_{21}^{(q)}|^2\theta_2^{(q)}P_2+|h_{r1}^{(q)}|^2\overline{\nu^{(q)}}P_r^{(q)}+N_1^{(q)}\right)+|h_{2r}^{(q)}|^2\theta_2^{(q)}P_2\left(|h_{r1}^{(q)}|^2\overline{\nu^{(q)}}P_r^{(q)}+N_1^{(q)}\right)}\right)$$

$$N_{wz,1}^{(q)} = \frac{\left(\left|h_{11}^{(q)}\right|^2\theta_1^{(q)}P_1+\left|h_{21}^{(q)}\right|^2\theta_2^{(q)}P_2+\left|h_{r1}^{(q)}\right|^2\overline{\nu^{(q)}}P_r^{(q)}+N_1^{(q)}\right)A^{(q)}-\left|A_1^{(q)}\right|^2}{\left|h_{r1}^{(q)}\right|^2\nu^{(q)}P_r^{(q)}},$$

$\nu^{(q)} \in [0,1]$, $A^{(q)} = |h_{1r}^{(q)}|^2\theta_1^{(q)}P_1 + |h_{2r}^{(q)}|^2\theta_2^{(q)}P_2 + N_r^{(q)}$ and $A_1^{(q)} = h_{11}^{(q)}h_{1r}^{(q),*}\theta_1^{(q)}P_1 + h_{21}^{(q)}h_{2r}^{(q),*}\theta_2^{(q)}P_2$.

Theorem [Existence of an NE for the bi-level EF protocol]

The game defined by $\mathcal{G}^{\mathrm{EF}} = (\mathcal{K}, (\mathcal{A}_i)_{i\in\mathcal{K}}, (u_i^{\mathrm{EF}}(\underline{\theta}_i, \underline{\theta}_{-i}))_{i\in\mathcal{K}})$ *with* $\mathcal{K}=\{1,2\}$ *and* $\mathcal{A}_i = \left\{\underline{\theta}_i \in [0,1]^Q \,\middle|\, \sum_{q=1}^{Q}\theta_i^{(q)} \leq 1\right\}$, *has always at least one pure NE.*

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The zero-delay scalar amplify-and-forward case

transmission assumption and utility functions

Each transmitter use **Time-Sharing** techniques.

The utility function for $\mathcal{S}_i$ is given by: $u_i^{\mathrm{AF}}(\underline{\theta}_i, \underline{\theta}_{-i}) = \sum_{q=1}^{Q} R_i^{(q),\mathrm{AF}}(\theta_i^{(q)}, \theta_{-i}^{(q)})$ where

$$\forall i \in \{1,2\}, \quad R_i^{(q),\mathrm{AF}} = \alpha_i^{(q)}(1-\alpha_j^{(q)})C\left(\frac{|a_{r,i}^{(q)}\ h_{ir}^{(q)}h_{ri}^{(q)}+h_{ii}^{(q)}|^2\rho_i\theta_i^{(q)}}{\alpha_i^{(q)}\left[\left(a_{r,i}^{(q)}\right)^2\left|h_{ri}^{(q)}\right|^2\frac{N_r^{(q)}}{N_i^{(q)}}+1\right]}\right)$$

$$+\alpha_i^{(q)}\alpha_j^{(q)}C\left(\frac{\left|a_r^{(q)}\ h_{ir}^{(q)}h_{ri}^{(q)}+h_{ii}^{(q)}\right|^2\alpha_j^{(q)}\rho_i}{\alpha_i^{(q)}\left[\left|a_r^{(q)}\ h_{jr}h_{ri}+h_{ji}\right|^2\rho_j\theta_j^{(q)}\frac{N_j^{(q)}}{N_i^{(q)}}+\alpha_j^{(q)}\left[\left(a_r^{(q)}\right)^2\left|h_{ri}^{(q)}\right|^2\frac{N_r^{(q)}}{N_i^{(q)}}+1\right]\right]}\right)$$

with $\forall i \in \{1,2\}$, $j=-i$ and $\rho_i^{(q)} = \frac{P_i}{N_i^{(q)}}$, $(\alpha_i^{(q)}, \alpha_j^{(q)}) \in (0,1)^2$, $a_{r,i}^{(q)} = \tilde{a}_{r,i}^{(q)}(\theta_i^{(q)}) \triangleq \sqrt{\frac{P_r/\mu^{(q)}}{\left|h_{ir}^{(q)}\right|^2 P_i/\alpha_i + N_r}}$,
$a_r^{(q)} = \tilde{a}_r^{(q)}(\theta_1^{(q)}, \theta_2^{(q)}) \triangleq \sqrt{\frac{P_r/\mu^{(q)}}{\left|h_{1r}^{(q)}\right|^2 P_1/\alpha_1^{(q)}(+|h_{2r}|^2 P_2/\alpha_2^{(q)}+N_r}}$ and $\mu^{(q)} = \max\{\alpha_1^{(q)}, \alpha_2^{(q)}\}$.

Theorem [Existence of an NE for ZDSAF when $a_r^{(q)} = \tilde{a}_r^{(q)}(\theta_1^{(q)}, \theta_2^{(q)})$]

There exists at least one pure NE in the PA game $\mathcal{G}^{\mathrm{AF}}$.

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$$+\alpha_i^{(q)}\alpha_j^{(q)} C\left(\frac{\left|a_r^{(q)} \; h_{ir}^{(q)} h_{ri}^{(q)} + h_{ii}^{(q)}\right|^2 \alpha_j^{(q)} \rho_i}{\alpha_i^{(q)}\left[\left|a_r^{(q)} \; h_{jr} h_{ri} + h_{ji}\right|^2 \rho_j \theta_j^{(q)} \frac{N_j^{(q)}}{N_i^{(q)}} + \alpha_j^{(q)}\left[\left(a_r^{(q)}\right)^2 \left|h_{ri}^{(q)}\right|^2 \frac{N_r^{(q)}}{N_i^{(q)}} + 1\right]\right]}\right)$$

with $\forall i \in \{1,2\}$, $j = -i$ and $\rho_i^{(q)} = \frac{P_i}{N_i^{(q)}}$, $(\alpha_i^{(q)}, \alpha_j^{(q)}) \in (0,1)^2$, $a_{r,i}^{(q)} = \tilde{a}_{r,i}^{(q)}(\theta_i^{(q)}) \triangleq \sqrt{\frac{P_r/\mu^{(q)}}{\left|h_{ir}^{(q)}\right|^2 P_i/\alpha_i + N_r}}$,

$a_r^{(q)} = \tilde{a}_r^{(q)}(\theta_1^{(q)}, \theta_2^{(q)}) \triangleq \sqrt{\frac{P_r/\mu^{(q)}}{\left|h_{1r}^{(q)}\right|^2 P_1/\alpha_1^{(q)} (+|h_{2r}|^2 P_2/\alpha_2^{(q)} + N_r}}$ and $\mu^{(q)} = \max\{\alpha_1^{(q)}, \alpha_2^{(q)}\}$.

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There exists at least one pure NE in the PA game $\mathcal{G}^{\mathrm{AF}}$.

The zero-delay scalar amplify-and-forward case: a special case

The special case: parameters

$\alpha_i^{(q)} = 1$, $Q = 2$ and $\forall q \in \{1,2\}$, $a_r^{(q)} = a_{r,1}^{(q)} = a_{r,2}^{(q)} = A_r^{(q)} \in [0, \tilde{a}_r(1,1)]$ are constant.

Best Response (BR) functions

$$\mathrm{BR}_i(\theta_j) = \arg\max_{\theta_i} u_i(\theta_i, \theta_j) = \begin{vmatrix} F_i(\theta_j) & \text{if } 0 < F_i(\theta_j) < 1 \\ 1 & \text{if } F_i(\theta_j) \geq 1 \\ 0 & \text{otherwise} \end{vmatrix}$$

where $j = -i$, $h_{ij} = h_{ij}^{(1)}$, $g_{ij} = h_{ij}^{(2)}$, $F_i(\theta_j) \triangleq -\frac{c_{ij}}{c_{ii}}\theta_j + \frac{d_i}{c_{ii}}$ is an affine function of θ_j; for $(i,j) \in \{(1,2),(2,1)\}$, $c_{ii} = 2|A_r^{(1)} h_{ri} h_{ir} + h_{ii}|^2 |A_r^{(2)} g_{ri} g_{ir} + g_{ii}|^2 \rho_i$;

$c_{ij} = |A_r^{(1)} h_{ri} h_{ir} + h_{ii}|^2 |A_r^{(2)} g_{ri} g_{jr} + g_{ji}|^2 \rho_j + |A_r^{(1)} h_{ri} h_{jr} + h_{ji}|^2 |A_r^{(2)} g_{ri} g_{ir} + g_{ii}|^2 \rho_j$;

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Theorem [Number of Nash equilibria for ZDSAF]

For the game $\mathcal{G}^{\mathrm{AF}}$ with fixed amplification gains at the relays, (i.e., $\frac{\partial a_r}{\partial \theta_i^{(q)}} = 0$), there can be a unique NE, two NE, three NE or an infinite number of NE, depending on the channel parameters (i.e., h_{ij}, g_{ij}, ρ_i, $A_r^{(q)}$, $(i,j) \in \{1,2,r\}^2$, $q \in \{1,2\}$).

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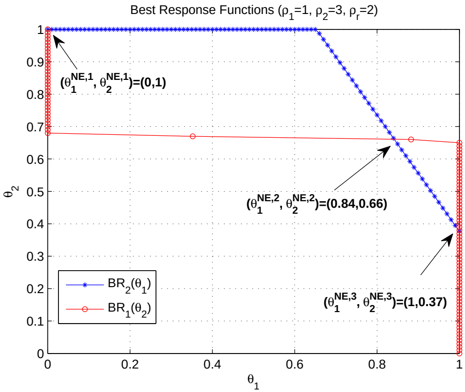
Simulation: Number of Nash equilibria for the ZDSAF protocol



Message

- Three Nash equilibria, in general.
- The NE point can be predicted from the sole knowledge of the starting point of the game when making use of the Cournot tatnnement.

Example of application: Optimal relay location

Stackelberg formulation

- Introduction of a leader in the game (the network provider for example).
- A bi-level game
- At a first stage: The leader chooses its strategy.
- At a second stage: The remaining players react according to the decision of the leader.

Strategy of the leader

- 2D propagation scenario.
- Strategy: The pair of coordinates $(x_{\mathcal{R}}; y_{\mathcal{R}})$ corresponding to the relay location.
- Utility function:
  - The social welfare $u(x_{\mathcal{R}}, y_{\mathcal{R}}) = u_1\left[\underline{\theta}^*(x_{\mathcal{R}}, y_{\mathcal{R}})\right] + u_2\left[\underline{\theta}^*(x_{\mathcal{R}}, y_{\mathcal{R}})\right]$;
  - The utility function of one of the users.

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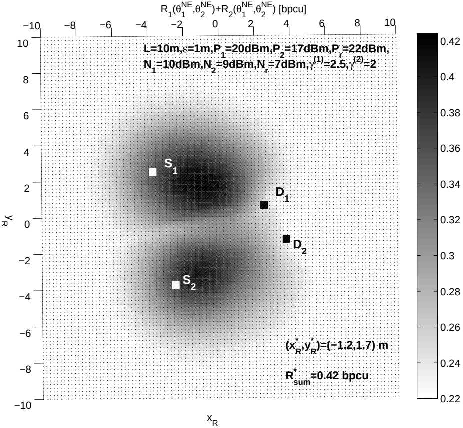
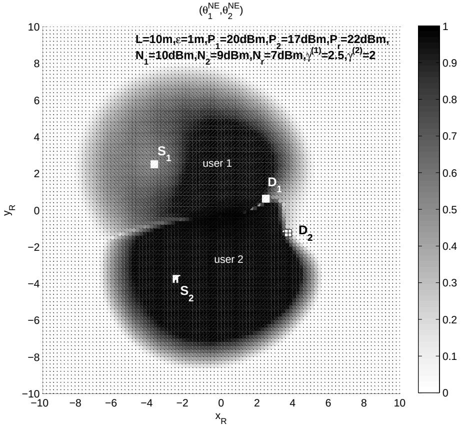
Optimal relay location for the ZDSAF protocol with full power regime



Message

- The optimal relay location for the individual rates is one of the segments between $\mathcal{S}_i$ and $\mathcal{D}_i$.

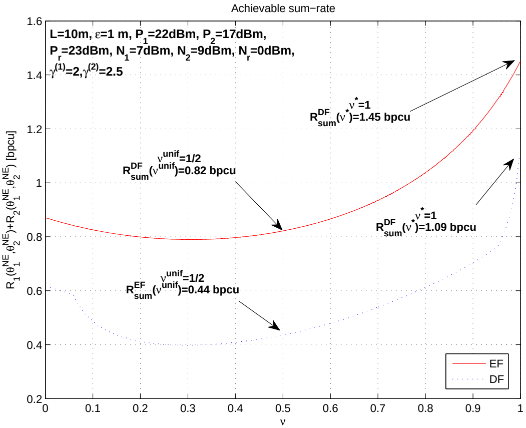
Optimal relay location for the ZDSAF protocol with full power regime



Message

- The selfish behavior of the transmitters leads to self-regulating the interference in the network.

Optimal power allocation at the relay for DF and EF



Message

- The relay allocates all its available power to the better receiver.

Conclusion

- Multiband interference relay channels.
- Shannon theory for the IRC.
- Power allocation game for the decentralized multiband IRCs.

Perspectives

- Improve the characterization of NE: analyze the uniqueness issue, for example.
- Consider a more general game.
- Distributed iterative algorithms that converge to NE.

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Questions?