RF IV Waveform Measurement and Engineering
- Role in Supporting Non-Linear CAD Design -

Centre for High Frequency Engineering
School of Engineering
Cardiff University

Contact information
Prof. Paul J Tasker – tasker@cf.ac.uk
website: www.engin.cf.ac.uk/chfe
Non-Linear CAD Models:  
- *state function based formulation*

- Time domain formulations
  - Physics Based State function formulation: I & Q
    - Four quasi-static I and Q surface functions
      - Advanced formulations include time delays

\[
i_{gs}(t) = I_{g}\left(v_{gs}(t), v_{ds}(t)\right) + \frac{\partial Q_{g}\left(v_{gs}(t), v_{ds}(t)\right)}{\partial t} \\
i_{ds}(t) = I_{d}\left(v_{gs}(t), v_{ds}(t)\right) + \frac{\partial Q_{d}\left(v_{gs}(t), v_{ds}(t)\right)}{\partial t}
\]

This fundamental formulation is followed by all analytical models and the Root lookup table model.
Non-Linear CAD Models:
- behavioral “black box: based formulation

- Frequency or Time domain formulations
  - Behavioral based formulation
    - Many different formulations
      - Analytical or experimental data based

\[ i_d(t) = a_0 + a_1 v_g(t - \tau_1) + a_2 v_g(t - \tau_2)^2 + a_3 v_g(t - \tau_3)^3 + \ldots \]

\[ b_2(\omega) = f(a_1(\omega), a_1(2\omega), \ldots, a_1(n\omega), a_2(\omega), a_2(2\omega), \ldots, a_2(n\omega)) \]

Generally focus on describing a specific behavior
RF I-V Waveform Measurement & Engineering
- role in CAD modelling

- State Function I(V) - Q(V) Non-Linear Models
  - Directly Measures Model related parameters I & V
    - I-Q function Extraction
      - Data Lookup Model Generation
    - Analytical Model validation and Optimization

- Behavioural “Black Box” Non-Linear Models
  - Directly Measures Non-Linear Behaviour
    - Directly Import into CAD Tool
      - Data Lookup behavioural model
    - Indirectly Import into CAD Tool
      - Formulated behavioural models (Volterra)
      - Emerging non-linear parameter equivalent to linear s-parameters (X-parameters)
Non-Linear CAD Models:  
- *state function based formulation*

- Requires measurement of the state functions: I & Q
  - DC I-V provides Current State Function
    - Static measurements: trapping and thermal issues
  - S-parameters measure differential of state functions
    - Trapping and thermal issues

Ideally require direct dynamic measurement of state functions
Non-Linear State Function CAD Models:
- indirect extraction from bias dependent s-parameters

\[ i_{gs}(t) = I_{g} \left( v_{gs}(t), v_{ds}(t) \right) + \frac{\partial Q_{g}(v_{gs}(t), v_{ds}(t))}{\partial t} \]

\[ i_{ds}(t) = I_{d} \left( v_{gs}(t), v_{ds}(t) \right) + \frac{\partial Q_{d}(v_{gs}(t), v_{ds}(t))}{\partial t} \]

**Small Signal Equivalent Circuit Model**

\[ y_{gs} = y_{11} \cdot v_{gs} \]
\[ y_{gd} = y_{12} \cdot v_{ds} \]
\[ y_{gm} = y_{21} \cdot v_{gs} \]
\[ y_{ds} = y_{22} \cdot v_{ds} \]

Linearize to get s-parameters

\[ Q_{g} = \int \Re(y_{11}) \cdot v_{gs} + \int \Re(y_{12}) \cdot v_{ds} \]
\[ I_{d} = \int \Im(y_{21}) \cdot v_{gs} + \int \Im(y_{22}) \cdot v_{ds} \]

Integrate bias dependent linear s-parameters to get non-linear parameters

Integrates bias dependent linear s-parameters to get non-linear parameters

Provides data for Root model or for analytical curve-fitting
Non-Linear State Function CAD Models: 
- *direct extraction from RF I-V Waveforms*

Model uses state functions to describe the arbitrary time dependent terminal current flow resulting from the applied arbitrary time dependent terminal voltages.

If we have measured the terminal current flow resulting from an applied and measured terminal voltage and we reverse process and determine state functions?

**YES:** Solutions in both the time and frequency domain.
Non-Linear State Function CAD Models:
- direct extraction from RF I-V Waveforms

- One-Port Problem
  - Two State functions
  - Depend on one variable

\[
i_{gs}(t) = I_g(v_{gs}(t)) + Q_{g}(v_{gs}(t)) \frac{\partial v_{gs}(t)}{\partial t} \\
i_{gs}(t) = I_g(v_{gs}(t_1)) + C_g \frac{\partial v_{gs}(t_1)}{\partial t}
\]

Two equations with two unknowns, so solve for \( I_g \) and \( C_g \)

Schreurs et al, EuMC 1997
Non-Linear State Function CAD Models:  
- direct extraction from RF I-V Waveforms

- Extraction of state functions for all measured values of $V_{gs}(t)$
- Only one large signal measurement needed
- Model extraction or model validation
Non-Linear State Function CAD Models:
- direct extraction from RF I-V Waveforms

- Waveform Measurement
  Power Sweep @ 1.8 GHz

- Performance Understanding
  Reactive Non-Linearity’s

- Extraction State Functions I & Q
  Reactive strongly second order

Transistor Behavior

State Functions
Stimulate with appropriate engineered waveforms $V_1(t)$ and $V_2(t)$ voltages and perform measurements

Input

Reverse Modelling Process

Extract $I$ and $Q$ state functions: i.e., their dependence on voltages $V_1$ and $V_2$

- Extracted Fully Dynamic Intrinsic I and Q Surfaces of a pHEMT transistor

Morgan et al, IMS 2001
Waveform Measurement and Engineering
- are we looking at the device or the system?

- **time domain**
  - $a_2/b_2$ or $b_2/a_2$

- **frequency domain**
  - $b_1/a_1$ or $a_1/b_1$

**DUT**

- measure & engineer
- measure
- measure & engineer
Waveform Measurement and Engineering
- are we looking at the device or the system?

- “Forward and Reverse Looking” Measurements
  - separation in the frequency domain
    - fundamental
      - Input Impedance $S_{11}(b_1/a_1)$
    - harmonics
      - source Impedance $(a_1/b_1)$
Waveform Measurement and Engineering
- *are we looking at the device or the system?*

- “Forward and Reverse Looking” Measurements
  - separation in the time domain
    - Load Impedance \(a_2/b_2\) when current generator is active
    - Device Impedance \(b_2/a_2\) when current generator is inactive

Extract Output Capacitance \(C_{ds} = 0.4\) pF/mm
RF I-V Waveform Measurement & Engineering
- role in CAD modelling

- State Function I(V) - Q(V) Non-Linear Models
  - Directly Measures Model related parameters I & V
    - I-Q function Extraction
      - Data Lookup Model Generation
    - Analytical Model validation and Optimization
Transistor RF I-V Waveforms
– Verification of non-linear CAD models

- Control mode of excitation
  - Similar to circuit operation
  - Maximize coverage of output I-V space, state variable space.

Measure: $V_1(t)$, $V_2(t)$ and $I_1(t)$, $I_2(t)$

Import Measured $V_1(t)$ and $V_2(t)$ into the simulator

Compare Simulated $I_1(t)$, $I_2(t)$ with Measured $I_1(t)$, $I_2(t)$
Transistor RF I-V Waveforms
– Verification/Optimization of non-linear CAD models

Provides insight to why and where the model is failing to accurately predict non-linear behavior.

Some divergence near knee

More robust and useful than what is typically done: simply comparing simulated and measured Power performance.

Some divergence at pinch-off
Transistor RF I-V Waveforms
– Verification/Optimization of non-linear CAD models

- Separate the measured currents into their individual components
  - Displacement and real contributions
- Results can be presented as a function of input or output voltage
  - Check model formulations
  - Validity as a function of bias

- Result, in this case, show that the LDMOS model used is not accurately modelling the variation of output capacitance as a function of gate bias.
RF I-V Waveform Measurement & Engineering
- role in CAD modelling

- **State Function I(V) - Q(V) Non-Linear Models**
  - Directly Measures Model related parameters I & V
    - I-Q function Extraction
      - *Data Lookup Model Generation*
    - Analytical Model validation and Optimization

- **Behavioural “Black Box” Non-Linear Models**
  - Directly Measures Non-Linear Behaviour
    - Directly Import into CAD Tool
      - *Data Lookup behavioural model*
    - Indirectly Import into CAD Tool
      - Formulated behavioural models (Volterra)
      - Emerging non-linear parameter equivalent to linear s-parameters (X-parameters)
Review Linear Design Situation
- back to basics: s-parameters behavioral models

- utilize measured s-parameter data tables in RF CAD Tools

Can simply transform s-parameter to any arbitrary impedance environment

Can also measure say \( S_{21} \) and \( S_{11} \) as function on input drive to get a very basic non-linear behavioral model

DUT

Vector Network Analyzer

“datasets” modeling

Amplifier

Gain, Bandwidth, Stability, Matching

Characterization and CAD Design Enabling Tool
Consider Non-Linear Design Situation
- large signals: waveform based behavioral models

Waveform Measurement and Engineering

- utilize measured waveform data tables in RF CAD Tools

"datasets" modeling

Cannot simply transform waveforms to any arbitrary impedance environment

Can measure as a function of input and/or output fundamental and harmonic load impedances: CAD interpolation and extrapolation
CAD Enabled Waveform Engineering
- Direct Waveform Look-up (DWLU) Data Model

Measure Waveforms

250W 20A

Klopfenstein Impedance Transformers
High Power Bias-tee

Populate multi-dimensional datasets

Formulate in frequency domain

P_in, Γ_LOAD, Bias

Sweep:

\[ i_1(n, \omega) = M\left( |V_1(\omega)|, Z_L, V_1, V_2 \right) V_1(\omega) \]

\[ i_2(n, \omega) = N\left( |V_1(\omega)|, Z_L, V_1, V_2 \right) V_1(\omega) \]
Data Import Unit constructed in Agilent ADS using FDD & DAC

“Black-Box” Non-Linear Model

Requires $Z_L$ determination

DAC Component links to the Generic MDIF data file: $V_{in}$, Bias look-up

CAD Enabled Waveform Engineering
- DWLU Data Model Implementation

AWR Microwave Office
version presently being verified
CAD Enabled Waveform Engineering
- DWLU Data Model Utilization

100W LDMOS

Simulate on Data Look-up Grid

--- Measured
--- Simulated

**DWLU Accurately regenerates RF waveforms**
CAD Enabled Waveform Engineering
- DWLU Data Model Utilization

100W LDMOS

I/P Voltage Waveform

Output Power vs Pin

Gain vs Pin

O/P Voltage Waveform

Output Phase vs Pin

\[ Z_{\text{REF}} = \gamma \Omega \]

Waveforms can be used directly within CAD Tools

Simulate off Data Look-up Grid

--- Measured
--- Simulated

**DWLU Accurately interpolates RF waveforms**
CAD Based Waveform Engineering
- Parameter Based Data Models: Formulation Concepts

- Non-linear Data look-up
  - Direct looks up measured waveform data
    - Stored in the frequency domain

- Non-linear Data Formulation: Parameter look-up
  - Transform waveform data into “circuit parameters”
    - Equivalent functionality to linear data formulation: s-parameters
      - Circuit analysis and design formulation
      - Travelling wave a-b rather than I-V formulations

- Agilent Solution: X-parameters
  - Natural extension of linear s-parameters data-set to non-linear data-set

- Cardiff Formulations
  - Natural extension of X-parameters. Cardiff “Mixing” Formulation for load-pull contours: contains higher order mixing terms
CAD Based Waveform Engineering
- Formulated Based Data Lookup Models FDLU

Good simulation accuracy can be kept for quite a large area on Smith Chart

Fast and robust simulation implementation

\[ b_k = \sum_{m=0}^{n-1} C_{k,m} \left( \frac{Q}{P} \right)^m a_1 + \sum_{m=0}^{n-1} U_{k,m} \left( \frac{P}{Q} \right)^m a_2 \]

\[ Q = \text{phase}(a_1), \quad P = \text{phase}(a_2) \]

\[ C_{k,m} = f(|a_1(f_o)|, |a_2(f_o)|) \quad \text{and} \quad U_{k,m} = f(|a_1(f_o)|, |a_2(f_o)|) \]

Good accuracy for different drive power levels
CAD based Waveform Engineering
- Parameter Based Data Models: Formulation Concept

- “Circuit” Formulation Requirement: *remove direct reference to load*
  - Component dependency: $f(|a_1|,|a_2|,(Q/P))$

\[
\begin{align*}
|a_1|^2 - |b_1|^2 &= a_1 \text{ stimulus} \\
|a_2|^2 - |b_2|^2 &= a_2 \text{ stimulus} \\
|b_1| &= P \cdot f(|a_1|,|a_2|,(Q/P)) \\
|b_2| &= P \cdot f(|a_1|,|a_2|,(Q/P))
\end{align*}
\]

Linear System uses $s$-parameters: 1st order system

\[
\begin{align*}
b_1 &= \{S_{11} \cdot |a_1| \cdot P + S_{12} \cdot |a_2| \cdot Q\}
\end{align*}
\]
CAD based Waveform Engineering
- Parameter Based Data Models: Formulation Concept

- Use of s-parameters in non-linear design: “Hot” S-parameters
  - Wrong functionality:
    - model circular function
    - measurement elliptical functionality

\[
b_1 = P \left\{ S_{11} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + S_{12} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 \right\}
\]

\[
b_2 = P \left\{ S_{21} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + S_{22} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 \right\}
\]

Parameter dependency:
\[ S_{m,n}(|a_1|,|a_2|) \]
CAD based Waveform Engineering
- Parameter Based Data Models: Formulation Concept

- Non-Linear System: include mixing components
  - Weakly Non-Linear System: 3rd order: relates to S&T Parameters (X-parameters)

\[
\begin{align*}
b_1 &= \left\{ X_{12}^{T} \cdot |a_2| \cdot \frac{P^2}{Q} + X_{11}^{S} \cdot |a_1| \cdot P + X_{12}^{S} \cdot |a_2| \cdot Q + X_{11}^{T} \cdot |a_1| \cdot \frac{Q^2}{P} \right\} \\
b_2 &= \left\{ X_{22}^{T} \cdot |a_2| \cdot \frac{P^2}{Q} + X_{21}^{S} \cdot |a_1| \cdot P + X_{22}^{S} \cdot |a_2| \cdot Q + X_{21}^{T} \cdot |a_1| \cdot \frac{Q^2}{P} \right\}
\end{align*}
\]

For small perturbation reduces to three parameters: X-parameters

\[
\begin{align*}
b_1 &= P \cdot \left\{ X_{12}^{T} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{-1} + X_{11}^{S} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + X_{12}^{S} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 + X_{11}^{T} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^2 \right\} \\
b_2 &= P \cdot \left\{ X_{22}^{T} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{-1} + X_{21}^{S} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + X_{22}^{S} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 + X_{21}^{T} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^2 \right\}
\end{align*}
\]

Parameter dependency: \( X_{m,n}(|a_1|) \)
CAD based Waveform Engineering
- Parameter Based Data Models: Formulation Concept

- 3rd Order Mixing Model: S&T-parameters (X-parameters)
  - Significantly improved functionality:
    - model is now an elliptical function
    - measurement elliptical functionality
  - Next Step
    - Compute local X-parameters
      - function of load
    - Allow for full amplitudes dependence
    - Increase order of mixing

\[
\begin{align*}
b_1 &= P \cdot \left\{ X_{12}^{T} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{-1} + X_{11}^{S} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^{0} + X_{12}^{S} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{1} + X_{11}^{T} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^{2} \right\} \\
b_2 &= P \cdot \left\{ X_{22}^{T} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{-1} + X_{21}^{S} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^{0} + X_{22}^{S} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{1} + X_{21}^{T} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^{2} \right\}
\end{align*}
\]

Parameter dependency: \( X_{m,n}(|a_1|, |a_2|) \)
CAD based Waveform Engineering
- Parameter Based Data Models: Formulation Concept

- Non-Linear System: *include mixing components*
  - Strongly Non-Linear System: n\textsuperscript{th} order: relates to C&U Parameters (R-parameters)

\[
b_1 = P \left\{ S_{11} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + S_{12} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 \right\}
\]
\[
b_2 = P \left\{ S_{21} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + S_{22} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 \right\}
\]

\[
b_1 = P \left\{ X_{12}^T \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{-1} + X_{11}^* \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + X_{12}^* \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 + X_{11}^T \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^2 \right\}
\]
\[
b_2 = P \left\{ X_{22}^T \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{-1} + X_{21}^* \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + X_{22}^* \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 + X_{21}^T \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^2 \right\}
\]

\[
b_1 = P \left\{ R_{1,2} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^2 + R_{1,1} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{-1} + R_{1,0} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + R_{1,2} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 + R_{1,3} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^2 + R_{1,3} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^3 \right\}
\]
\[
b_2 = P \left\{ R_{2,2} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^2 + R_{2,1} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^{-1} + R_{2,0} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^0 + R_{2,2} \cdot |a_2| \cdot \left( \frac{Q}{P} \right)^1 + R_{2,3} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^2 + R_{2,3} \cdot |a_1| \cdot \left( \frac{Q}{P} \right)^3 \right\}
\]
CAD based Waveform Engineering
- Parameter Based Data Models: Cardiff Formulation

- Modelling with Extracted Fundamental $R_{m,n}$ components: $g(|a_1|, |a_2|)$

- Accurate reproduction of measured $b_2$ contours (load-pull contours) with 7th order $(Q/P)$ phase model

- Avoids any implicit load based lookup

$$b_m = P \cdot f\left(|a_1|, |a_2|, \left(\frac{Q}{P}\right)^n\right) = P \cdot \sum_{n=-\left\lfloor \frac{N+1}{2} \right\rfloor}^{\left\lfloor \frac{N+1}{2} \right\rfloor} R_{m,n} \left(\frac{Q}{P}\right)^n$$

Parameter dependency: $R_{m,n}(|a_1|, |a_2|)$

7th order model: R-parameters

Collapse large data lookup to small (6*12 or 8x12) $R_{m,n}$ parameter lookup
CAD based Waveform Engineering
- Parameter Based Data Models: Cardiff Formulation

- Cardiff “Circuit” Parameter Formulation
  - Generalized to n\textsuperscript{th} order in terms of the relative phase component (Q/P)
    \[
    b_m = P \cdot f\left(|a_1|, |a_2|, \left(\frac{Q}{P}\right)\right) = P \cdot \sum_{n=-\left(\frac{\pi}{2}\right)^{-1}}^{n=\left(\frac{\pi}{2}\right)} R_{m,n} \left(\frac{Q}{P}\right)^n
    \]
    where \(R_{m,n} = g(|a_1|, |a_2|)
  - Determination of parameters \(R_{m,n}\) requires measurements at constant \(|a_1|\) and \(|a_2|\) while sweeping relative phase component (Q/P), normalized to optimum load: easy to achieve with active load-pull
CAD based Waveform Engineering
- Parameter Based Data Models: Cardiff Formulation

- “Circuit” Formulation that is an extension of linear s-parameters
  - *remove direct reference to load*
  - Formulation dependency: \( f(|a_1|,|a_2|, (Q/P)) \)
- Function dependency: \( f(|a_1|,|a_2|) \)

\[
\begin{align*}
|a_1|^2 - |b_1|^2 &= a_1 \text{ Stimulus} \\
|a_2|^2 - |b_2|^2 &= a_2 \text{ Response}
\end{align*}
\]

\[
\begin{align*}
b_1 &= P \cdot f \left( |a_1|, |a_2|, \left( \frac{Q}{P} \right) \right) \\
a_1 &= |a_1| e^{j\theta_1} = |a_1| P
\end{align*}
\]

\[
\begin{align*}
b_2 &= P \cdot f \left( |a_1|, |a_2|, \left( \frac{Q}{P} \right) \right) \\
a_2 &= |a_2| e^{j\theta_2} = |a_2| Q
\end{align*}
\]

Parameter dependency:
\[
R_{m,n}(|a_1|,|a_2|)
\]
**CAD based Waveform Engineering**  
*Parameter Based Data Models: Cardiff Formulation*

- Magnitude Function Fitting to Extracted Fundamental $R_{m,n}$ components: $g(|a_1|, |a_2|)$
  - $R_{m,n} = \alpha_0 + \alpha_1|a_2| + \alpha_2|a_2|^2 + \alpha_3|a_2|^3 + \alpha_4|a_2|^4 + \alpha_5|a_2|^5 + \alpha_6|a_2|^6 + \alpha_7|a_2|^7$
  - Only 20 relevant coefficients
  - Accurate reproduction of measured $b_2$ contours (load-pull contours) with 7th order model

![Graph showing 7th order model: R-parameters](image)
CAD based Waveform Engineering
- Parameter Based Data Models: CAD Implementation

Schematic of ADS Simulation

The model can be directly imported into CAD software, after processing the measurement data.

Simulated load-pull Contours (Left)

Load-pull Power Contours

Simulated load-pull Contours

Measured load-pull Contours (Right)

S-parameter equivalent non-linear formulations are emerging
RF I-V Waveform Measurement & Engineering
- role in CAD modelling

- **State Function I(V) - Q(V) Non-Linear Models**
  - Directly Measures Model related parameters I & V
    - Analytical Model validation and optimization
    - I-Q function Extraction
      - Data Lookup Model Generation

- **Behavioral “Black Box” Non-Linear Models**
  - Directly Measures Non-Linear Behaviour
    - Directly Import into CAD Tool
      - Data Lookup behavioural model
    - Indirectly Import into CAD Tool
      - Formulated behavioural models (Volterra)
      - Emerging non-linear parameter equivalent to linear s-parameters (X-parameters)