Time-Domain Computation of Loop inductance

Edmund K. Miller, Los Alamos National Laboratory (Retired), e.miller@ieee.org

Abstract – The use of a time-domain code for modeling the electromagnetic behavior of thin wires is shown to provide an alternate way to obtain the DC inductance of wire loops. It involves computing the constant current that flows around a loop when an excitation whose time integral is non-zero is used to excite the loop as an antenna and the high-frequency energy has radiated away. Results from the time-domain model are found to agree within 0.1% of analytical values.

Index Terms – Loop inductance, time-domain electromagnetics, inductance, thin-wire loops, moment-method modeling

1. Introduction

An interesting recent article [1] described the concept of partial inductance and demonstrated how it can be used to obtain the inductance of loops of arbitrary geometry. Specific examples were given for the self inductance of square and triangular loops and shown to agree with previously derived results [2].

The purpose of this brief discussion is to show how the DC inductance of arbitrary wire loops can be obtained numerically using a time-domain computer model. The model, TWTD (Thin-Wire Time Domain) [3], is based on the time-dependent Maxwell Equations. TWTD is usually employed for modeling wire radiators and scatterers excited by impulsive voltages or plane waves to obtain their transient behavior or wide-band frequency response. But TWTD also provides an alternate approach for obtaining the inductance of wire loops. This can be done by exciting a candidate loop by a voltage whose time integral is nonzero and computing the current as a function of time until it reaches a constant value, Io, around the loop.

The TWTD model is briefly described in Section 2 below. The computational approach for obtaining loop inductance follows in Section 3, with numerical results for a variety of loop geometries included in Section 4.

2. The Thin-Wire Time-Domain (TWTD) Computer Model

One version of a time-domain integral equation for a wire object in free space has the form [3]

$$\hat{s}\cdot\mathbf{E}^{ex}(s,t) = \frac{\mu_0}{4\pi}\int_{C(r)}\left(\frac{\hat{s}\cdot\hat{s}'}{R}\frac{\partial I(s',t')}{\partial t'} + \left[\frac{c\mathbf{s}\cdot\mathbf{R}}{R^2}\right]\left[\frac{\partial I(s',t')}{\partial s'} - \frac{c}{R}Q(s',t')\right]\right)ds' \quad (1a)$$

with

$$Q(s',t') = -\int_{-\infty}^{t'}\frac{\partial I(s',t')}{\partial s'}dt' \quad (1b)$$

and where $I(s',t')$ and $Q(s,t)$ are the unknown current and the charge density at space location s' and the "retarded time" $t' = R/c$, c is the speed of light in the medium and $\mathbf{R}$ is the vector distance between the source at s' and observation point at s. Also c and μ_0 are the speed of light in, and magnetic permeability of, free space and the exciting electric field is $\mathbf{E}^{ex}(s,t)$. The unknown current and charge density on the wire can be found as the solution of an initial-value problem via a time-stepping procedure using the method of moments [3].

The TWTD model employs a nine-term polynomial basis, up to and including quadratic space-time variation and delta-weight functions to satisfy the integral equation (1). It has been well-validated by many users, one of which [4] used the inductance of a circular loop for this purpose. Since the results presented in Section 4 serve to further validate TWTD, no additional validation examples are included here.

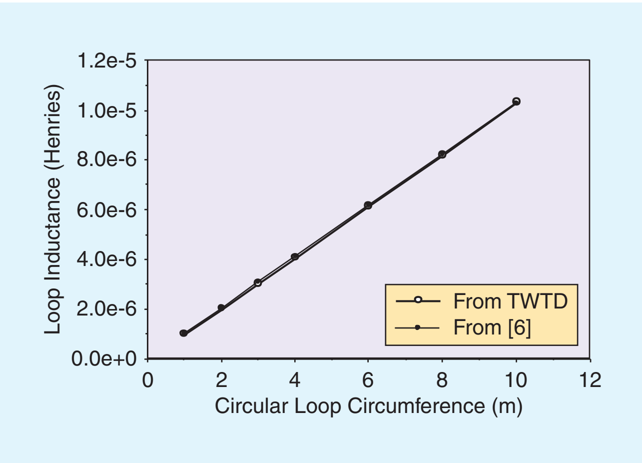

Fig. 1. The inductance of a circular loop as a function of its circumference as obtained from TWTD and Eq (6).

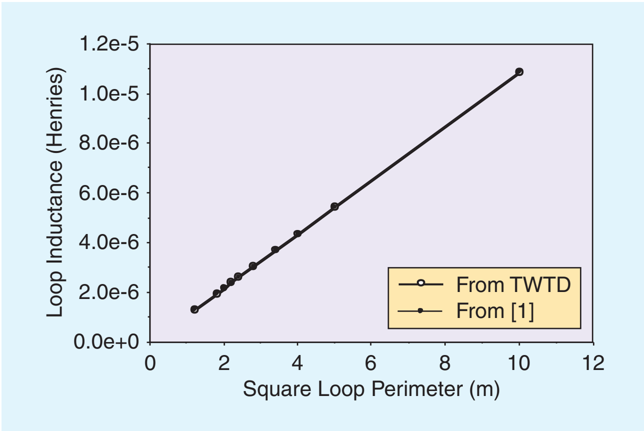

Fig. 2. The inductance of a square loop as a function of its perimeter length as obtained from TWTD and Eq. (14) of reference {1}.

3. Obtaining Loop Inductance Using TWTD

A Gaussian-pulse voltage is a convenient and effective excitation for use in time-domain modeling [3]. It is given by

$$V(t) = V_0 e^{[-A(t-T)]^2} \tag{2}$$

where V_0 is the voltage maximum, A is a width parameter, t is the time and T is the time at which the maximum voltage occurs. For the computations that follow, these parameters had the values:

$$Vo = 1-V,$$

$$A = 4.2 \times 10^8 \text{ sec}^{-1},$$

and

$$T = 1.5 \times 10^{-8} \text{ sec}.$$

For a loop whose low-frequency inductance is L, he constant current I_O is then given by [4]

$$I_O = \frac{1}{L}\int_{-\infty}^{\infty} V(t)dt \tag{3}$$

which results from integrating the defining equation for a loop. Using (2) in (3) we then find

$$I_O = \frac{\pi^{1/2}}{AL}. \tag{4}$$

For the examples that follow a constant current around the loop was typically reached in 1,000 or fewer time steps of the time-domain solution.

Thus, the inductance from the late-time TWTD current is simply obtained as

$$L = \frac{\pi^{1/2}}{AI_o}. \tag{5}$$

which was the test used previously in [4] as a validation check on the TWTD model itself.

4. Numerical Results for Loop Inductance from TWTD

Three of the loop geometries that are modeled here, circular, square and triangular loops, assumed to be perfect electric conductors, have analytical expressions for their inductance. For the circle it is [6]

$$L_{circle} = r_{loop}\,\mu_o\left[\ln\left(\frac{8r_{loop}}{r_{wire}}\right) - 2\right] \tag{6}$$

with the loop and wire radii denoted by r_{loop} and r_{wire}. For the square and triangle the inductances are given respectively by [1]

$$L_{square} = \frac{2\mu_o h_{side}}{\pi}\left[\ln\left(\frac{h_{side}}{r_{wire}}\right) - 0.774\right] \tag{7}$$

and

$$L_{triangle} = 3\frac{\mu_o}{2\pi}h_{side}\left[\ln\left(\frac{2h_{side}}{r_{wire}}\right) - 1.405\right] \tag{8}$$

with the side length of the square and triangle given by h_{side}. The results that follow were obtained from TWTD models that used 20 spatial samples, or segments, per meter. For simplicity, the wire radius was scaled to maintain a fixed ratio between the loop radius and side length in the above expressions. For the circle $r_{wire} = 2\pi r_{loop}/10^3$. For the square and triangle $r_{wire} = l_{side}/50$.

This results in the inductance for all three loops varying linearly with the loop size, as is shown in Figs. 1–3. The agreement between the TWTD values and the analytical results is within 0.1% or so. This outcome serves primarily to validate the use of TWTD for obtaining the inductance of a loop, as the analytical expressions have been independently validated.

Three other loop configurations were also modeled. The first is a square loop 0.3-m on a side having a wire radius of 6×10^{-4} m and bent into a V shape. The inductance of this loop is shown as a function of the angle in degrees between the two

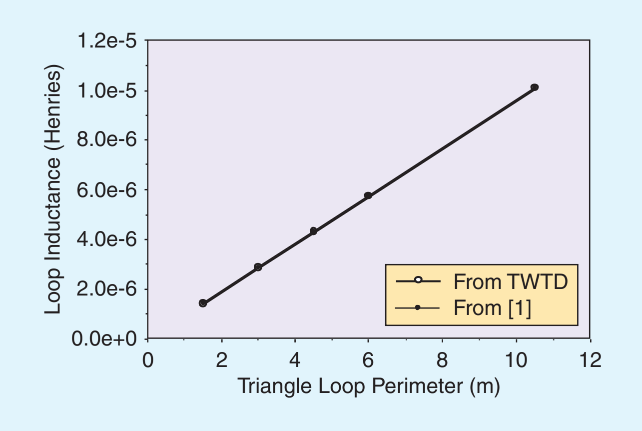

Fig. 3. The inductance of a triangle loop as a function of its perimeter length as obtained from TWTD and Eq. (19) of reference [1].

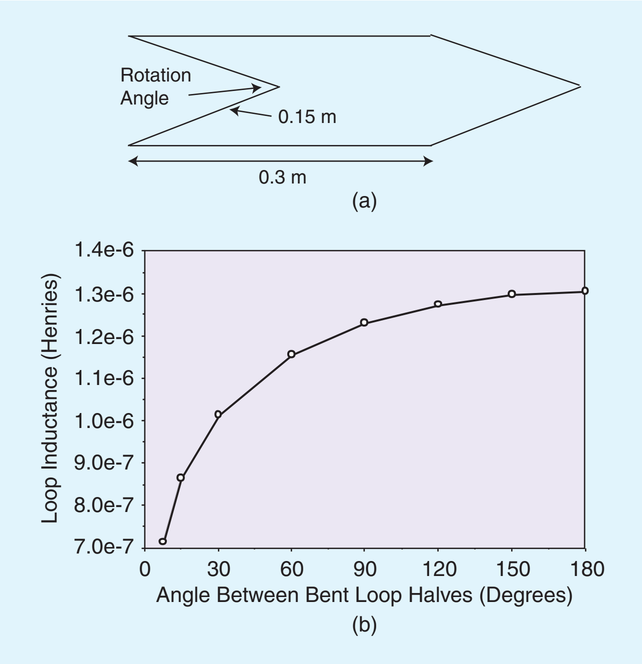

Fig. 4. (a) A square loop bent into a V-shape. (b) The inductance of the bent loop as a function of the rotation angle obtained using TWTD.

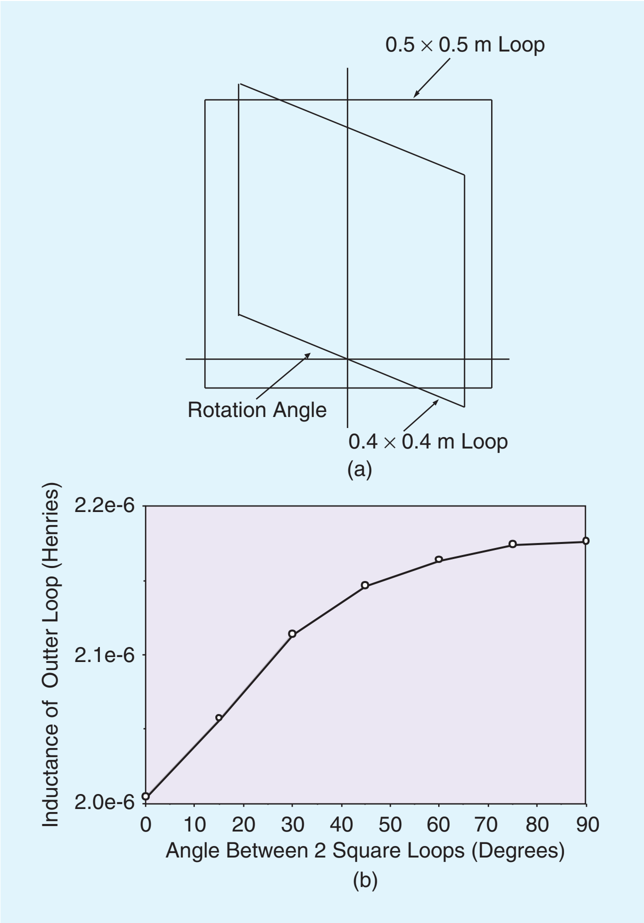

Fig. 5. (a) An inner loop rotated from the plane of a larger outer loop.(b) The inductance of the outer loop as a function of the rotation angle of the inner loop from their common plane at 0 degrees obtained using TWTD.

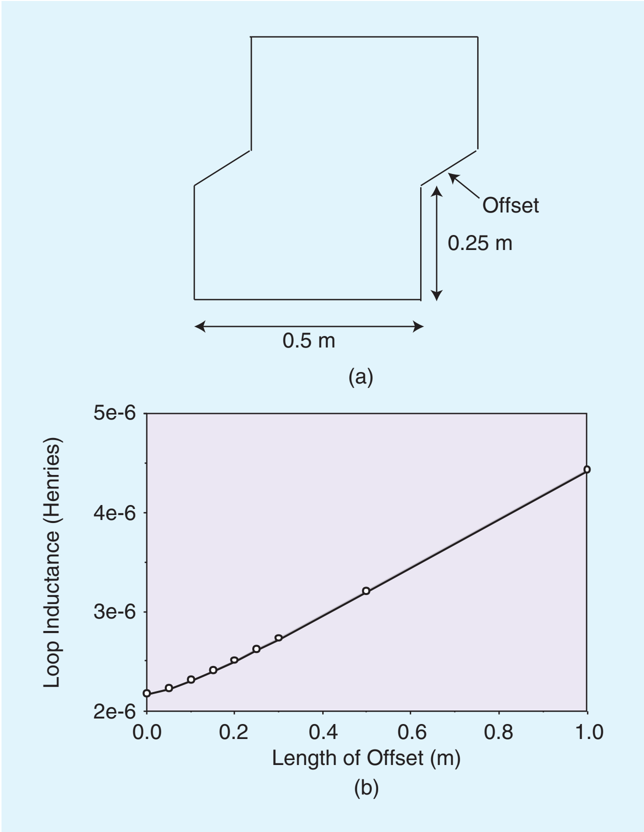

Fig. 6. The inductance of the offset loop as a function of the offset length obtained using TWTD. (a) An offset loop.

halves of the loop in Fig. 4. The inductance of the bent loop is seen to vary over a range of nearly 2:1 over the 172.5-degree variation of the included angle.

Another loop arrangement consisted of two square loops 0.5 and 0.4-m on a side with wire radii of 10^{-3}-m and having a common center. The loops are concentric when co-planar with the inductance of the larger, outer loop determined as a function of the rotation angle of the inner loop relative to it. The result of this variation is shown in Fig. 5, varying by a little less than 10% as the inner loop varies from co-planar to orthogonal relative to the outer loop.

The last example shown here is for a square loop that is initially 0.5-m on a side and wire radii of 10^{-3}-m. Its inductance is determined from TWTD as the upper half of the loop is systematically offset from the lower half while remaining joined to it by two orthogonal wires. The result for this experiment is shown in Fig. 6 where the inductance varies by more than 2:1 up to the maximum offset of 1-m, essentially increasing in proportion to the offset, essentially as might be expected from Eq. (7).

5. Concluding Comments

A possibly unanticipated use of an electromagnetics, time-domain computer model, TWTD, has been demonstrated here for determining the DC inductance of wire loops of fairly arbitrary geometry. Application of a Gaussian voltage pulse to excite the loop as an antenna results in a constant current around the loop after the higher frequency energy has radiated away. This constant current provides a straightforward way to then determine the inductance, the current being inversely proportional to it. While an analytical formula is preferable when available, the TWTD approach represents an independent way to obtain the inductance of a given loop as well as to confirm the validity of an analytical result. Furthermore, a more general time-domain model, e.g. FDTD, that permits a more complex electromagnetic environment to be handled, could be employed for printed circuit boards and other configurations that involve dielectric media.

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Biography

Edmund K. Miller *earned his Ph.D. in Electrical Engineering at the University of Michigan. Dr. E. K. Miller has held a variety of government, academic and industrial positions. These include 15 years at Lawrence Livermore National Laboratory and over four years at Los Alamos National Laboratory from which he retired as a Group Leader in 1993. His academic experience includes holding a position as Regents-Distinguished Professor at Kansas University and as Stocker Visiting Professor at Ohio University. Dr. Miller has served as an Antennas and Propagation (AP) Distinguished Lecturer, and wrote the column "PCs for AP and Other EM Reflections" from 1984 to 2000. He received (with others) a Certificate of Achievement from the IEEE Electromagnetic Compatibility Society for Contributions to Development of NEC (Numerical Electromagnetics Code) and was a recipient (with others) in 1989 of the best paper award given by the Education Society for "Computer Movies for Education."*

He served as Editor or Associate Editor of IEEE Potentials Magazine from 1985 to 2005 for which he wrote a regular column "On the Job," and in connection with which he was a member of the IEEE Technical Activities Advisory Committee of the Education Activities Board and a member of the IEEE Student Activities Committee. Dr. Miller has lectured at numerous short courses in various venues, such as Applied Computational Electromagnetics Society (ACES), AP-S, MTT-S and local IEEE chapter/section meetings, and at NATO Lecture Series and Advanced Study Institutes.

Dr. Miller edited the book "Time-Domain Measurements in Electromagnetics", Van Nostrand Reinhold, New York, NY, 1986 and was co-editor of the IEEE Press book Computational Electromagnetics: Frequency-Domain Moment Methods, 1991. He was organizer and first President of the Applied Computational Electromagnetics Society (ACES) for which he also served two terms on the Board of Directors. He served a term as Chairman of Commission A of US URSI and is or has been a member of Commissions B, C, and F, has been on the TPC for the URSI Electromagnetic Theory Symposia in 1992 and 2001, and was elected as a member of the US delegation to several URSI General Assemblies. He is a Life Fellow of IEEE from which he received the IEEE Third Millennium Medal in 2000 and is a Fellow of ACES. His research interests include scientific visualization, model-based parameter estimation, the physics of electromagnetic radiation, validation of computational software, and numerical modeling about which he has published more than 150 articles and book chapters. **EMC**