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A Stability Criterion for High–Accuracy Digital Resonators based on Δ–Σ Schemes

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Abstract – *The continuous evolution of high performance mixed–signal integrated circuits requires to use increasingly sophisticated measurement and testing procedures, whose cost may currently cover almost 50% of the overall production budget [1]. This major increment is due to several reasons such as the increasing complexity and duration of the tests, the high cost and the quick obsolescence of the external Automatic Test Equipment (ATE) and, last but not least, the implementation of quality assurance programs in compliance with ISO 9000 series of standards. In this paper a method for determining the degree of stability of Δ–Σ harmonic resonators is described.*

Keywords – *BIST, digital resonator, root locus, delta–sigma.*

I. INTRODUCTION

A valuable solution to reduce considerably both testing times and instrumentation costs is provided by Built–In Self–Test (BIST) schemes. Generally speaking, a BIST scheme consists of both stimulus generation and measurement–oriented on–chip components. Of course, in order to make the BIST effective, such a scheme must be programmable, flexible and, above all, inexpensive in terms of integration resources. As analogue–to–digital Converters (ADCs) are usually the key devices of mixed–signal integrated circuits, the ability of characterizing accurately their metrological performances is one of the most important tasks in designing a BIST scheme. To this purpose, many different solutions have been presented and various kinds of test stimuli have been used [2], [4]. Among them, high–quality, programmable sine waves are probably the most suitable test signals because they are commonly employed in many standard ADC testing procedures [5]. Moreover, sinusoidal oscillators based on a Δ–Σ topology, i.e. digital resonators exploiting 1–bit delta–sigma properties, are particularly suitable for BIST purposes because they can be implemented without hardware multipliers and with a minimum amount of analogue circuitry (i.e., a 1–bit DAC followed by a low–order filter). Also, they are able to generate an output sinusoidal signal of elevated spectral purity [6]–[9]. Unfortunately, most Δ–Σ–based resonators have proved to work correctly only under certain conditions, while exhibiting serious stability problems when some circuit parameters are changed [10]. Up to now, these phenomena have not been properly investigated during the design phase. In fact, all published results rely mostly on rules of thumb and extensive computer simulations. Hence, the aim of this paper is to provide a general insight about the stability issue of this kind of devices, thus determining a stabilization criterion. To this purpose, in the following, a method for determining the degree of stability of Δ–Σ harmonic resonators is described. Eventually, this method is applied to the specific oscillator presented in [7].

II. DESCRIPTION OF STABILITY ANALYSIS

In general, harmonic digital resonators are based on the cascade of two discrete–time integrators in a loop, with the sign of one integrator being positive and the other negative. In these structures, the oscillation frequency depends on one or two multiplicative coefficients [7].

In order to avoid the use of hardware multipliers and multi–bit D/A converters, a 1–bit Δ–Σ modulator is inserted in the loop as shown in Fig. 1. The increment in circuital complexity due to the introduction of this modulator is partially counterbalanced by the benefit of generating a high–quality, single–bit output signal that can be multiplied by a constant coefficient simply by means of a multiplexer. Obviously, the implementation cost as well as the spectral quality of the generated signal depend on the architecture of the Δ–Σ modulator employed. In particular, using the well–known additive white noise model to represent the behavior of the 1–bit quantizer, the modulator should be designed so that the Noise Transfer Function (NTF) exhibits an in–band Signal–to–Noise Ratio

(SNR) higher than a given value [9]. However, the additive noise model is too coarse to cope with stability issues. Indeed, even if this kind of circuits seems to be stable, it has been shown that, under certain initial conditions, the amplitude of the internal signals may diverge suddenly [8]. The reasons of this behavior can be understood more clearly if an analysis of the dynamics of the resonator is performed by modeling the 1–bit quantizer with a time–varying gain [10]. This parameter, which in the following will be referred to as λ, results from the ratio between the constant unit output amplitude of the modulator and the variable input amplitude of the quantizer. Using this model, it follows that any nonlinear resonator can be regarded as a sequence of parametric linear systems depending on different values of λ. Furthermore, the order N of each system is equal to the sum between the order M of the Δ–Σ modulator inserted in the loop and the order K of the digital resonator. As a typical harmonic oscillator is a second–order device, it usually results that $N = M + 2$. Thus, by assuming that $x[\cdot]$ is a Nx1 state variable vector and that $F(\lambda)$ is the NxN state transition matrix of each linear system, it results that the 1–step updating law for the state variables is:

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$$x[n+1] = F(\lambda) \cdot x[n] \tag{1}$$

Accordingly, it can be easily shown that the z–transform of the free evolution of the system is given by [11]:

$$X(z) = z[zI - F(\lambda)]^{-1} \cdot x[0] = \frac{N(z, \lambda, x[0])}{P(z, \lambda)}, \tag{2}$$

where I is a NxN unit matrix, $x[0]$ is the vector representing the initial values of the state variables, $P(\cdot, \cdot)$ is the characteristic polynomial of the matrix $F(\cdot)$, and $N(\cdot, \cdot, \cdot)$ is a vector of polynomials depending on the initial conditions $x[0]$. Observe that both system eigenvalues, i.e. the zeros of the characteristic polynomial, and the mode amplitudes depend on the parameter λ. Since the system stability is related to the position of the poles on the complex plane, the root locus technique is a valid method to investigate the degree of stability of parametric Δ–Σ structures [12]. Generally speaking, the positive root locus associated with a $N-$th order feedback system consists of N curves representing the positions of the system poles on the complex plane for different values of the loop gain $\lambda \geq 0$. Therefore, even though the sequence of λ values, which depends on the chaotic dynamics of the internal Δ–Σ modulator, cannot be predicted analytically, it is at least possible to determine which modes are liable of possible malfunctions and whether the existing limit cycles are stable. Limit cycles occur when the poles of the system exhibit a module equal to 1, i.e. when the root locus intersects the unit circle. The fundamental radian frequency of each limit cycle is given by the angle of these intersection points in polar coordinates. The stability of each limit cycles depends instead on how the module of the corresponding pole varies for different values of λ. In fact, if the pole module tends to move out of the unit circle when λ decreases (i.e. when the amplitude of the internal signal increases) it means that the considered limit cycle is always critically stable, i.e. the amplitude of the system internal state is destined to diverge. Conversely, if the pole position tends to move into the unit circle when λ decreases, the growing amplitude of the state signals tends to produce stable modes (i.e. poles whose module is lower than 1), thus leading to a stable oscillation. In the final paper a more complete explanation about the influence of multiple time–varying modes on the overall stability of Δ–Σ resonators will be provided. Actually, in order to clarify the proposed approach, the results of a stability analysis carried out on a particular kind of resonator is presented in the following.

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III. SIMULATION RESULTS

Suppose to insert in the general Δ–Σ resonator scheme shown in Fig. 1, the fourth-order loop filter $H(z)$ displayed in Fig. 2. The resulting structure is a $6-$th order system, in which every state variable x_i, $i = 1, ..., 6$ represents the content of a different register. The multiplicative coefficients of the filter $H(z)$ have been set to $A_3 = 2^{-6}$, $A_4 = A_5 = -2^{-4}$, $A_6 = 2^{-7}$, $B_3 = 2^{14}$, $B_4 = -2^{10}$, $B_5 = -2^{9}$, $B_6 = 2^{6}$ in order to optimize the spectral purity of the output signal [7]. By manipulating the algebraic definition of each state variable of the system, after some algebraic steps, it results immediately that the state transition matrix is equal to: where the eigenvalues are a function of λ and depend on the parameters k_0 and k_1. In Fig. 3(a) and 3(b) the full root locus of the system and a zoomed part of the plot around the point $(1, 0)$ are shown, by assuming that $k_0 = 9.15 \cdot 10^{-6}$ and $k_1 = 2^{-14}$. Observe that the branches of the root locus b, b^*, c, c^* intersect the unit circle in two couple of symmetric points. A spectral analysis revealed that the radian fundamental frequency of the output signal coincides with the absolute value of the angle associated with the intersection points between the branches c, c^* and the unit circle. Further numerical analyses also proved that when $\lambda \to 0$ the poles related to the branches c and c^* are slightly inside the unit circle, whereas when $\lambda \to \infty$ such poles lie outside the unit circle, thus leading to the conclusion that the corresponding limit cycle is stable. Nevertheless, repeated simulation sessions pointed out that the system is not always stable as expected. In fact, even if the resonator works correctly under certain conditions [7], the amplitude of the internal state signals may suddenly diverge when different initial values for the state variables are set. For instance, if the initial register values $x_1[0] = 0.0055$ and $x_2[0] = 3.51 \cdot 10^{-6}$ and $x_i[0] = 0$, $i = 3, ..., 6$ are chosen, the peak–to–peak amplitude of the internal state signals tends to infinity after about $8 \cdot 10^7$ iterations. Root locus of the Δ–Σ resonator after modeling the 1–bit quantizer as a time–varying gain when $k_0 = 9.15 \cdot 10^{-6}$ and $k_1 = 2^{-14}$. In (a) the full root locus is shown, while in (b) a zoomed portion of (a) is plotted around the point $(1, 0)$ of the complex plane. The arrows highlight the direction along which the gain λ grows. This is clearly recognizable in Fig. 4(a), where the curves representing the envelope of absolute values of the time–varying state variables are plotted as a function of the number of iterations on a logarithmic scale. In this way, the chaotic pattern of some state variables does not hide the information about the amplitude of the internal signals. According to the root locus analysis, the possible unstable behavior is due to the critical modes associated with the locus branches b, b^*. Indeed, the limit cycle corresponding to the intersection points between such branches and the unit circle is clearly unstable because when $\lambda \rightarrow 0$ the poles tend to move out of the unit circle, thus further stressing the incoming instability. The system will be certainly stable only if the amplitude of the quantizer input, namely the sequence of λ values, is maintained higher than the critical parameter λ_c corresponding to the intersection points mentioned above. If this condition is not met, the behavior of the resonator becomes unpredictable, thus making the whole device unreliable. Of course, λ_c depends on the chosen values for circuital parameters. In the case considered, for instance, λ_c is equal to about 0.47. In order to verify the validity of this assumption, the minimum values of λ have been recorded. The collected results as well as the maxima of the most critical state variable x_6, chosen as reference, are shown in Fig. 4(b). Observe that as soon as the sequence of λ values becomes lower than the critical threshold $\lambda_c = 0.47$, the amplitude of x_6 increases abruptly. From this analysis it follows that once λ_c is known, the stability of oscillators based on a Δ–Σ topology can be obtained simply by assuring that $\lambda > \lambda_c$. However, since a simple clipping of the state variables would deteriorate excessively the spectral purity of the generated waveform, a different approach has to be followed. Good results in terms of output accuracy have been achieved by clipping only minima and maxima of the resonator internal signals, which are recognized as the main responsible of an incoming instability. This strategy is justified by the fact that not all signals increase simultaneously at the same rate. In particular, the state variable x_6 exhibits an almost linear trend which is higher than the trend of the others state variables so that it seems to anticipate the actual breakdown. Therefore, a proper monitoring and a careful clipping of the more rapidly increasing state variables is sufficient to keep under control the values of λ, thus assuring system stability. A more complete explanation about this approach as well as a description of the stabilization results will be provided in the final paper.

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IV. CONCLUSIONS

The correct operation of Δ–Σ harmonic resonators for BIST purposes depends on the stability of the self–generated limit cycles. In this paper, by modeling the 1–bit quantizer inside the Δ–Σ modulator as a time–varying gain, the stability of this kind of oscillators is analyzed using the root locus technique. The obtained results not only provide useful guidelines to understand whether a certain structure is critically stable, but also promote the implementation of a flexible clipping strategy which increases the stability of the resonator without deteriorating the spectral purity of the output signal.

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