

Short Circuit Analysis of IEEE Test Feeders

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Abstract – In 1991 a paper giving the data for four distribution system test feeders was published [1]. The purpose of the test feeders was to give software developers a common set of data that could be used to verify the correctness of their power flow analysis programs. Initially there were four test feeders and they were used primarily to check the accuracies of new power flow analyses. There have not been any comparisons of the results of short circuit studies. This paper will address a method of short circuit analysis and apply it to the initial four test feeders. Only the results for the IEEE 13 Node Test Feeder will be presented in this paper. The total feeder data and short circuit results for all four feeders will be found on the IEEE website [2].

Index Terms–Test feeders, distribution lines, transformers, component models, short circuit analysis

I. INTRODUCTION

Each of the original test feeders had special characteristics that provided a test for the accuracy of the distribution component models and the convergence characteristics of the power flow analysis program being tested. The original four test feeders are:

- 13 Node Test Feeder – provided a good test of the convergence of a program for a very unbalanced system
- 34 Node Test Feeder – a very long feeder requiring the application of voltage regulators to satisfy ANSI voltage standards
- 37 Node Test Feeder – a three wire delta underground system
- 123 Node Test Feeder – a large system consisting of overhead and underground single phase, two phase and three phase laterals along with step voltage regulators and shunt capacitors

Additional test feeders have been added for the special purpose of testing transformer connection models and induction machine models.

The test feeders have been used by many program developers as example feeders to demonstrate their method of modeling and power flow analysis. Because of the successful application of the test feeders for power flow analysis it is time to develop and demonstrate short circuit analyses on the

four Test feeders.

II. GENERAL SYSTEM

A diagram of the major components of a test feeder is shown in Figure 1.

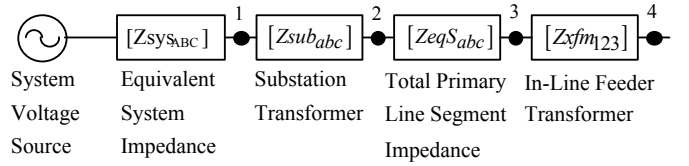


Figure 1 – Components of a Test Feeder

A model is required for each of the components in Figure 1. The short circuit analysis program must be capable of analyzing all possible types of short circuits at all nodes. With reference to Figure 1 short circuits at node 1 will come from the short circuit analysis of a balanced transmission system. Short circuits at all the other nodes will be computed in the distribution short circuit analysis program.

III. COMPONENT MODELS

Each of the components in Figure 1 can be modeled as shown in Figure 2.

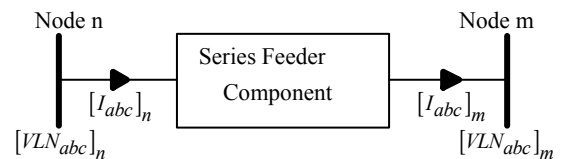


Figure 2 – General Component Model

For the general model in Figure 2 generalized matrices can be developed to model the input/output characteristics of the component [3]. The generalized equations are in the form of:

$$\begin{aligned} [VLN_{abc}]_n &= [a] \cdot [VLN_{abc}]_m + [b] \cdot [I_{abc}]_m \\ [I_{abc}]_n &= [c] \cdot [VLN_{abc}]_m + [d] \cdot [I_{abc}]_m \end{aligned} \quad (1)$$

Equations 1 are applied when the voltages and currents at the input are computed knowing the output voltages and currents.

$$[V_{abc}]_m = [A] \cdot [V_{abc}]_n + [B] \cdot [I_{abc}]_m \quad (2)$$

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Equation 2 is slightly different in that the output voltages are computed based upon the input voltages and the output currents. Equation 2 is recognized as the “forward” sweep equation used in power flow analysis. Typically only the current equation of Equation 1 is used in the “backward” sweep. Although these equations were primarily developed for the modified ladder power flow iterative technique, they become the foundation for the short circuit analysis.

Equivalent System Model

The equivalent system model is somewhat different from the rest of the component models. The basic model is shown in Figure 3.

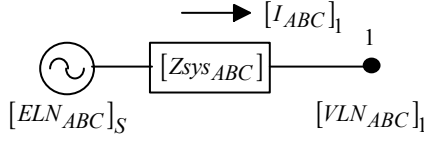


Figure 3 – Equivalent System Model

In order to model the system, the equivalent phase impedance matrix is required. Typically it will be necessary to obtain the three-phase and line-to-ground short circuit currents from a transmission network analysis. From such a study the positive and zero sequence impedances will be obtained. The sequence impedance matrix is converted to the phase impedance matrix by:

$$[Z_{012}] = \begin{bmatrix} Z_{zero} & 0 & 0 \\ 0 & Z_{pos} & 0 \\ 0 & 0 & Z_{pos} \end{bmatrix} \quad (3)$$

$$[Z_{sys_{ABC}}] = [A_{sym}] \cdot [Z_{012}] \cdot [A_{sym}]^{-1}$$

In Equation 3 the matrix $[A_{sym}]$ is the symmetrical component transformation matrix. The equation for the system model is:

$$\begin{aligned} [VLN_{abc}]_1 &= [A_s] \cdot [ELN_{ABC}]_S = [B_s] \cdot [I_{abc}]_1 \\ \text{where: } [A_s] &= \text{unity matrix} \\ [B_s] &= [Z_{sys_{ABC}}] \end{aligned} \quad (4)$$

Substation Transformer Model

All transformer connections (three-phase, two phase and single-phase) use the circuit shown in Figure 4. Figure 4 shows the connections for the step-down connection.

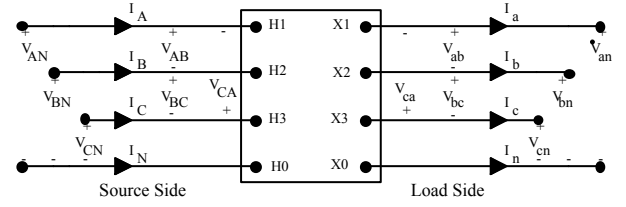


Figure 4 – Transformer Bank Step-Down Model

Note in Figure 4 that the high voltage terminals are labeled with H while the low voltage terminals are labeled with X. On the source side capital letters are used to denote the high voltage phases while lower case letters denote the load side. For the system of Figure 1 the transformer model equations are:

$$\begin{aligned} [VLN_{abc}]_2 &= [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}]_2 \\ [I_{ABC}] &= [c_t] \cdot [VLN_{abc}]_2 + [d_t] \cdot [I_{abc}]_2 \end{aligned} \quad (5)$$

The matrices in Equation 5 are defined for all possible transformer connections in Reference [3].

In Figure 1 for the short circuit studies it will be necessary to compute the Thevenin equivalent of the system referenced to Node 2. The equations [3] are:

$$\begin{aligned} [E_{th}] &= [A_t] \cdot [VLN_{ABC}]_1 \\ [Z_{th}] &= [A_t] \cdot [Z_{sys_{ABC}}] \cdot [d_t] + [B_t] \end{aligned} \quad (6)$$

Distribution Line Model

The model for all distribution lines is shown in Figure 5:

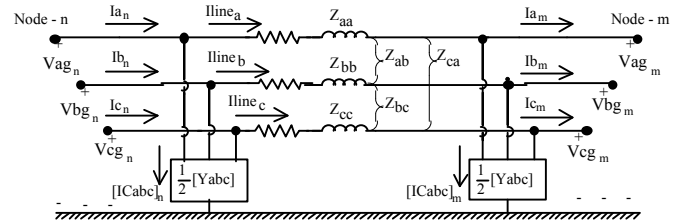


Figure 5 – Distribution Line Model

Figure 5 shows the model for a three-phase distribution line. For two-phase and single-phase lines the same model can be used by eliminating the phases that are not present. Note in Figure 5 that the shunt admittance of the line is included. For most cases the shunt admittance is so small that it can be ignored. However, for long and in particular underground cables, the shunt admittance should be included. With reference to Figure 1 the distribution line equations: [3]

$$\begin{aligned} [VLN_{abc}]_3 &= [A_{line}] \cdot [VLN_{abc}]_2 - [B_{line}] \cdot [I_{abc}]_3 \\ [I_{abc}]_2 &= [d_{line}] \cdot [I_{abc}]_3 \end{aligned} \quad (7)$$

The distribution line impedances are computed using Carson's equations resulting in the 4x4 primitive matrix. The Kron reduction is used to create the 3x3 phase impedance matrix needed for the computation of the model matrices. With this application the correct mutual coupling between phases is computed. For two-phase lines the elements in the row and column of the missing phase will be zero. The same is true for a single-phase line. In all cases only 3x3 matrices are computed. In Figure 1 the matrix $[Z_{sys_{abc}}]$ will be the sum of all impedances from the source down to the primary terminals of the distribution transformer.

Distribution Transformer Model

The model for a step-down distribution transformer bank is the same as for the substation transformer. This transformer bank may include a center tapped transformer for the usual service to single-phase customers. The model for center tapped transformers is given in Reference [3].

IV. EQUIVALENT MODEL AT THE FAULTED NODE

To start the short circuit analysis the equivalent phase impedance matrix from the source to the faulted node must be determined. For example, if Node 3 is the faulted node, the total equivalent phase impedance matrix will be:

$$[Z_{eq_{abc}}] = [Z_{th}] + [Z_{sys_{abc}}]$$

where:

$[Z_{th}]$ = Thevenin equivalent of source and substation transformer

$[Z_{sys_{abc}}]$ = Sum of line sections from the substation to Node 3

(8)

If the faulted node is Node 4, then the Thevenin equivalent circuit at node 4 must be computed using the matrix $[Z_{eq_{abc}}]$ and the voltage $[E_{th}]$ computed at the substation transformer.

Figure 6 is the three-phase Thevenin circuit at the faulted node.

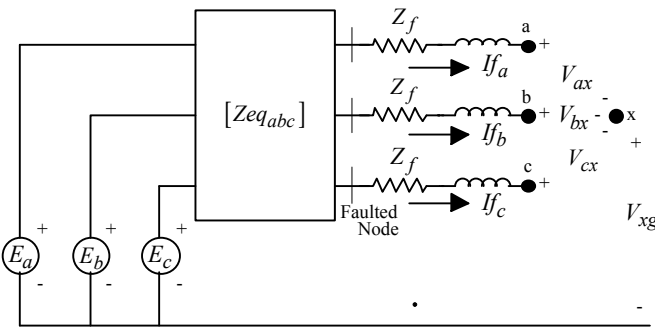


Figure 6 – Thevenin Three-Phase Circuit

In Figure 6 KVL can be written around the three loops:

$$[E_{abc}] = [Z_{eq_{abc}}] \cdot [I_{f_{abc}}] + [Z_{f_{abc}}] \cdot [I_{f_{abc}}] + [V_{abc}]_x + [V_{xg}] \quad (9)$$

Reference [3] reduces Equation 9 to a general form of:

$$[IP_{abc}] = [I_{f_{abc}}] + [Y] \cdot [V_{abc}]_x + [Y] \cdot [V_{xg}]$$

$$\text{where: } [Y] = [ZTOT]^{-1}$$

$$[ZTOT] = [Z_{eq_{abc}}] + [ZF]$$

$$[IP_{abc}] = [Y] \cdot [E_{abc}]$$

$$[I_{f_{abc}}] = \begin{bmatrix} I_{f_a} \\ I_{f_b} \\ I_{f_c} \end{bmatrix}$$

$$[V_{abc}]_x = \begin{bmatrix} V_{ax} \\ V_{bx} \\ V_{cx} \end{bmatrix}$$

$$[V_{xg}] = \begin{bmatrix} V_{xg} \\ V_{xg} \\ V_{xg} \end{bmatrix}$$

(10)

In Figure 6 there are 7 unknowns. Equations 10 will give 3 independent equations. The type of fault modeled will yield the remaining 4 independent equations resulting in the matrix equation:

$$\begin{bmatrix} IP_a \\ IP_b \\ IP_c \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Y_{1,1} & Y_{1,2} & Y_{1,3} & Y_{s1} \\ 0 & 1 & 0 & Y_{2,1} & Y_{2,2} & Y_{2,3} & Y_{s2} \\ 0 & 0 & 1 & Y_{3,1} & Y_{3,2} & Y_{3,3} & Y_{s3} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \end{bmatrix} \cdot \begin{bmatrix} I_{f_a} \\ I_{f_b} \\ I_{f_c} \\ V_{ax} \\ V_{bx} \\ V_{cx} \\ V_{xg} \end{bmatrix} \quad (11)$$

Reference [2] gives the equations for the last four rows of the matrix in Equation 11. As example, if a b-c fault is to be simulated the four needed equations are:

$$\begin{aligned} V_{bx} &= 0 \\ V_{cx} &= 0 \\ I_{f_b} + I_{f_c} &= 0 \\ I_{f_a} &= 0 \end{aligned} \quad (12)$$

For this faulted condition the Matrix of 11 becomes:

$$\begin{bmatrix} IP_a \\ IP_b \\ IP_c \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Y_{1,1} & Y_{1,2} & Y_{1,3} & Y_{S1} \\ 0 & 1 & 0 & Y_{2,1} & Y_{2,2} & Y_{2,3} & Y_{S2} \\ 0 & 0 & 1 & Y_{3,1} & Y_{3,2} & Y_{3,3} & Y_{S3} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} If_a \\ If_b \\ If_c \\ V_{ax} \\ V_{bx} \\ V_{cx} \\ V_{xg} \end{bmatrix} \quad (13)$$

In reduced notation Equation 13 is:

$$[IP] = [C] \cdot [X] \quad (14)$$

and: $[X] = [C]^{-1} \cdot [IP]$

The models developed in this section are used in the Radial Distribution Analysis Program (RDAP) [4] as well as Milsoft Utility Solutions' Windmil program in an effort to compare and validate the results. Both programs were used to determine the short circuit currents for the four IEEE test feeders. Only the 13 node feeder will be listed in this paper. The complete short circuit currents for this feeder and the other three will appear on the IEEE website [2].

IV. IEEE 13 NODE TEST FEEDER

The one-line diagram of the IEEE 13 Node Test Feeder is shown in Figure 7.

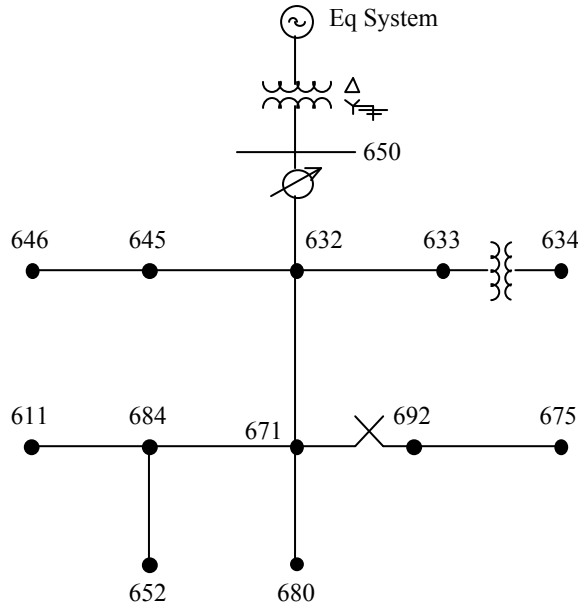


Figure 7 – IEEE 13 Node Test Feeder

The basic data for the test feeder can be found on the website [2]. Not included in that data are the equivalent sequence impedances of the equivalent system. The positive and zero sequence impedances of the equivalent system are:

$$\begin{aligned} Z_{zero} &= 0.3474 + j8.5180 \, \Omega \\ Z_{pos} &= 0.7673 + j4.7852 \, \Omega \end{aligned} \quad (15)$$

For these values of sequence impedances the sequence impedance matrix is:

$$Z_{012} = \begin{pmatrix} 0.3474 + 8.518j & 0 & 0 \\ 0 & 0.7673 + 4.7852j & 0 \\ 0 & 0 & 0.7673 + 4.7852j \end{pmatrix}$$

Converting the sequence impedance matrix to the phase impedance matrix results in:

$$Z_{sys \, ABC} = \begin{pmatrix} 0.6273 + 6.0295j & -0.14 + 1.2443j & -0.14 + 1.2443j \\ -0.14 + 1.2443j & 0.6273 + 6.0295j & -0.14 + 1.2443j \\ -0.14 + 1.2443j & -0.14 + 1.2443j & 0.6273 + 6.0295j \end{pmatrix}$$

Short Circuit Study Assumptions

- Source voltage is balanced 115 kV
- Voltage regulators are set on unity taps
- Voltage regulator impedance is zero
- Shunt capacitor banks neglected
- Load current is neglected
- Fault impedance is zero

Sample results from RDAP and Windmil for the short circuit study are given in Appendix A.

The third table in Appendix A lists the percent differences between RDAP and Windmil for each fault type. Note the values are extremely small, most likely due to rounding in the programs.

The IEEE website will also list the results from other distribution analysis programs used in the industry.

V. CONCLUSIONS

A method for analyzing short circuit currents for radial distribution feeders was outlined. The short circuit currents for the IEEE 13 Node Test Feeder were computed using RDAP and Windmil and the sample results are posted in the Appendix.

1. IEEE Distribution Planning Working Group Report, "Radial distribution test feeders", *IEEE Transactions on Power Systems*, August 1991, Volume 6, Number 3, pp 975-985.
2. <http://ewh.ieee.org/soc/pes/dsacom/testfeeders/index.html>
3. W. H. Kersting, *Distribution System Modeling and Analysis*, 2nd Edition, CRC Press, Boca Raton, Florida, 2007.
4. <http://www.zianet.com/whpower/>

Appendix A:

RDAP

Short Circuit Currents - IEEE 13 Node (Amps)										
Node	Phase	Three Phase Fault (LLL)			Single Line-To-Ground Fault			Three-Phase To Ground Fault		
		A	B	C	A	B	C	A	B	C
RG60	ABC	8416.8	8416.8	8416.8	8479.3	8479.3	8479.3	8416.8	8416.8	8416.8
692	ABC	3350.4	3271.6	2964.7	2196.4	2156.9	2173.9	3317.5	3268.1	3009.6
684	AC	0.0	0.0	0.0	2019.5	0.0	2001.8	0.0	0.0	0.0
680	ABC	2909.9	2839.5	2549.7	1851.9	1817.0	1832.1	2880.6	2836.9	2589.6
675	ABC	3121.1	3088.8	2778.2	2077.0	2049.9	2057.6	3091.6	3087.1	2816.4
671	ABC	3350.4	3271.6	2964.7	2196.4	2156.9	2173.9	3317.5	3268.1	3009.6
652	A	0.0	0.0	0.0	1795.7	0.0	0.0	0.0	0.0	0.0
646	BC	0.0	0.0	0.0	0.0	2516.5	2524.3	0.0	0.0	0.0
645	BC	0.0	0.0	0.0	0.0	2806.5	2817.8	0.0	0.0	0.0
634	ABC	15276	15135	14720	13046	12962	12986	15191	15150	14796
633	ABC	4150.2	4023.2	3802.5	2950.6	2910.3	2921.8	4115.6	4028.3	3837.1
632	ABC	4801.3	4705.2	4392.2	3495.5	3444.4	3466.5	4759.2	4698.3	4449.3
611	C	0.0	0.0	0.0	0.0	0.0	1852.0	0.0	0.0	0.0

Windmil

Short Circuit Currents - IEEE 13 Node (Amps)										
Node	Phase	Three Phase Fault (LLL)			Single Line-To-Ground Fault			Three-Phase To Ground Fault		
		A	B	C	A	B	C	A	B	C
REG2	ABC	8416.3	8416.3	8416.3	8479.0	8479.0	8479.0	8416.3	8416.3	8416.3
692	ABC	3350.1	3271.4	2964.7	2196.3	2156.9	2173.9	3317.1	3268.0	3009.6
684	AC	0.0	0.0	0.0	2019.4	0.0	2001.7	0.0	0.0	0.0
680	ABC	2909.6	2839.3	2549.7	1851.8	1817.0	1832.0	2880.3	2836.7	2589.6
675	ABC	3120.8	3088.6	2778.3	2076.8	2049.8	2057.5	3091.3	3086.9	2816.4
671	ABC	3350.1	3271.4	2964.7	2196.3	2156.9	2173.9	3317.1	3268.0	3009.6
652	A	0.0	0.0	0.0	1795.6	0.0	0.0	0.0	0.0	0.0
646	BC	0.0	0.0	0.0	0.0	2516.5	2524.2	0.0	0.0	0.0
645	BC	0.0	0.0	0.0	0.0	2806.5	2817.8	0.0	0.0	0.0
634	ABC	15276	15135	14720	13046	12962	12986	15190	15150	14796
633	ABC	4149.9	4023.1	3802.4	2950.5	2910.3	2921.7	4115.4	4028.2	3837.0
632	ABC	4800.9	4704.9	4392.1	3495.4	3444.4	3466.4	4758.8	4698.0	4449.2
611	C	0.0	0.0	0.0	0.0	0.0	1852.0	0.0	0.0	0.0

[illegible]

Windmil

[illegible]

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