Advanced Computational Methods in Lightning Performance

The Numerical Electromagnetics Code (NEC-2)

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Abstract: NEC-2 code, a standard tool for numerical analysis on electromagnetic field around antennas, can be applied to analysis of lightning transient overvoltages. The code solves three-dimensional boundary problems by using the electric-field integral equation. This tutorial introduces the basic theory of NEC-2, and explains how to tune the original program to fit for time-domain analyses. Then the set up of the input data for a simple test case is described. The accuracy of the analysis is demonstrated by comparison with an experiment, and finally, results of advanced study are presented.

Keywords: lightning surge, numerical electromagnetic analysis

I. INTRODUCTION

For practical purposes, the analysis of lightning transients in large conducting structures has mostly been made through modeling the structures by transmission lines. In this approach, the parameters of a transmission line such as the surge impedance, the velocity of the traveling wave etc. need to be determined. This is physically correct if the electromagentic field around a conductor is in the TEM mode, in that the distribution of the electric field is the same as the electrostatic field. Parameters of the modeled transmission line for parallel conductors or a horizontal conductor above a ground plane can be determined on physical basis.

The electromagnetic field around non-parallel conductors or a non-horizontal conductor above a ground plane is not in the TEM mode during transient periods. In modeling such conductor systems by transmission lines, their parameters need to be determined experimentally or hypothetically. Electromagnetic field around a conductor system needs to be solved to produce induced voltages and distribution of currents, which facilitates modeling with equivalent circuits without carrying out experiments.

The idea to numerically solve transient electromagnetic field associated with lightning transients is not new. However, it has not been recognized as a practical method, partly because the verification of the accuracy of the result has been difficult, and reproduction of the result by a third party is difficult if a dedicated computer code was developed for the analysis.

Numerical Electromagnetics Code is a widely used computer code in analyzing three-dimensional electromagnetic field around antennas and scatterers in the frequency domain. The Lawrence Livermore Laboratory developed it and its second version, NEC-2, is publicly released [1]. It was applied to time-domain analysis, combined with Fourier transform, of lightning surge response of a transmission tower, and the accuracy of the computed result was verified through comparison with an experiment [2]. Thus its effectiveness in the application to time-domain analysis of lightning transients of a conductor system was demonstrated.

NEC solves integral equations at the boundary numerically by the method of moments. In this tutorial, the basic theory of NEC and methods to obtain the code is described. Then its application to time-domain analysis is demonstrated by using a simple test case. Examples of extended applications are also presented.

II. BASIC THEORY

A. Electric Field Integral Equation

NEC allows to use both the electric-field integral equation and the magnetic-field integral equation. The former is suited for analysis of thin-wire structures, while the latter is suitable for structures having large smooth surfaces. The former can be used to analyze voluminous structures by representing surfaces with wire grids. In the application to time-domain analysis, the modeling by using thin wires has been employed throughout. The following description on the basic theory of NEC is extract from the program description [1].

The basic electric-field integral equation at the surface of a thin conducting wire in the axial direction is reduced to the following scalar equation (1) under the restriction of the boundary condition (2) in the axial direction. These equations are for a single angular frequency ω.

$$-\hat{\boldsymbol{s}}\cdot\boldsymbol{E}_{inc}(\boldsymbol{r}) = \frac{-j\eta}{4\pi k}\int_L I(s')(\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{s}}' - \frac{\partial^2}{\partial_s\partial_{s'}})g(\boldsymbol{r},\boldsymbol{r}')ds' \qquad (1)$$

$$\boldsymbol{E}_{scat}(\boldsymbol{r}) + \boldsymbol{E}_{inc}(\boldsymbol{r}) = 0\,, \qquad (2)$$

where

$$g(\boldsymbol{r},\boldsymbol{r}') = exp(-jk|\boldsymbol{r}-\boldsymbol{r}'|)/|\boldsymbol{r}-\boldsymbol{r}'|,$$

$$k = \omega\sqrt{\mu_0\varepsilon_0}\ ,\ \eta = \sqrt{\mu_0/\varepsilon_0}$$

s is the distance parameter along the wire axis at $\boldsymbol{r}$, and $\hat{s}$ is the unit vector tangent to the wire axis at $\boldsymbol{r}$. $\boldsymbol{E}_{inc}$ is the incident field and $\boldsymbol{E}_{scat}$ is the scattered field. The current I is represented by a filament on the wire axis.

Equation (1) is easily extended to lossy conductors by modifying the boundary condition from equation (2) to

$$\hat{s}\cdot[\,\boldsymbol{E}_{scat}(\boldsymbol{r}) + \boldsymbol{E}_{inc}(\boldsymbol{r})] = Z_L(s)I(s)\,, \qquad (3)$$

where $Z_L(s)$ is the impedance per-unit-length at s.

B. Numerical Solution by the Method of Moments

The integral equation (1) is solved by the method of moments [3]. This method applies to a general linear-operator equation,

$$Lf = e, \qquad (4)$$

where f is an unknown response, e is a known excitation, and L is a linear operator. The unknown function f may be expanded in a sum of basis functions, f_j, as

$$f \cong \sum_{j=1}^{N}\alpha_j \cdot f_j\,. \qquad (5)$$

A set of equations for the coefficients α_j are then obtained by taking the inner product of equation (4) with a set of weighting functions, $\{w_i\}$,

$$< w_i, Lf > = < w_i, e > \qquad i = 1,\ldots N. \qquad (6)$$

Due to the linearity of L, equation (5) substituted for f yields,

$$\sum_{j=1}^{N}\alpha_j < w_i, Lf_j > = < w_i, e > \qquad i = 1,\ldots,N. \qquad (7)$$

This equation can be written in matrix form as

$$[G][A] = [E], \qquad (8)$$

where $G_{ij} = <\omega_i, Lf_j>,\quad A_j = \alpha_j,\quad E_i = <\omega_i, e>$, and is easily solved.

In NEC-2, the weighting functions $\{w_i\}$ are chosen as a set of delta functions

$$\omega_i(\boldsymbol{r}) = \delta(\boldsymbol{r} - \boldsymbol{r}_i)\,, \qquad (9)$$

with $\{\boldsymbol{r}_i\}$ a set of points on the conducting surface. Wires are divided into short straight segments with a sample point at the center of each segment.

The basis functions or the current expansion functions in NEC-2 have the form

$$I_j = A_j + B_j \sin k(s - s_j) + C_j \cos k(s - s_j)\,, \qquad (10)$$

where s_j is the value of s at the center of segment j and Δ_j is the length of segment j. Of the three unknown constants A_j, B_j and C_j, two are eliminated by imposing continuity conditions on the current and charge at the segment ends. The remaining constant, related to the current amplitude, is determined by solving equation (8).

At a junction of two or more segments with unequal radii, the continuity of current is generalized to Kirchhoff's current law. The total charge in the vicinity of the junction is assumed to distribute itself on individual wires according to the wire radii, neglecting local coupling effects.

III. PREPARATION

A. Download and Compile

NEC-2 and related documents can officially be obtained either from Lawrence Livermore Laboratory or National Technical Service of the U. S. Government. They can also be downloaded from web sites, which are easily found by the keyword "numerical electromagnetics code" by using search engines. It is recommended to download source files because slight modification is necessary for our purpose. The source code is written in FORTRAN, and some knowledge on this language is required for the next step.

Some modification will usually be necessary to compile the code, and this procedure depends on the user's system. In the current version downloaded from the web as of October 1999, a file NEC2DPAR.INC, comprises only one line of the following may be missing.

```
PARAMETER(MAXSEG=5000, MAXMAT=5000)
```

This file must be in the same directory.

If the code is successfully compiled, then the format of the output needs to be modified. For our purpose, the complex amplitude of current on each segment only is necessary, therefore, disable the entire write commands except that of statement 68. This output will be multiplied with the Fourier transform of the input waveform of the source, either current or excitation field, in the time domain. If the complex amplitude of field is necessary, call SUBROUTINE NFPAT at statement 68, and print the values of desired field.

B. Modeling Guidelines

A conductor system to be analyzed by NEC-2 should be modeled by the composition of short cylindrical segments. A cylindrical segment is defined by the coordinates of its two end points and its radius. Modeling with cylindrical segments involves both geometrical and electrical factors. Geometrically, the segments should follow the paths of conductors as closely as possible, using a piece-wise linear fit on curves. The main electrical consideration is on the segment length ΔL relative to the wavelength λ, and $10^{-3}\lambda<\Delta L<0.1\lambda$ is recommended.

The radius of segment, a, should be much less than $\lambda/2\pi$ and $\Delta L/8$. For example, in the analysis of a high voltage measuring system [4], $a/\Delta L<1/120$ was necessary to obtain consistent results. If segments are electrically connected at their ends, the identical coordinates should be used. The angle of the intersection of segments should be as large as possible, for an acute angle may results in less accuracy. Abrupt changes of segment length should be also avoided.

NEC-2 allows lumped circuit elements to be incorporated into the model by simply defining the impedance of any given segments.

C. Application to Time-Domain Analysis

NEC-2 is a computer code in the frequency domain, therefore, to obtain the response of a system in the time domain, Fourier transform and inverse Fourier transform are used. For Fourier transform, FFT is employed. The flow of solution is shown in Fig.1.

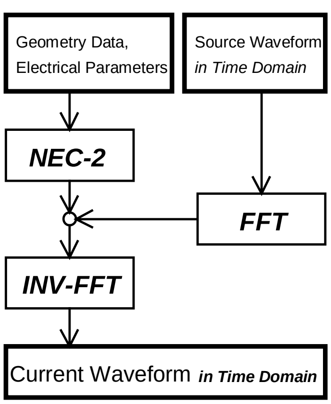


Fig. 1. Solution flow of transient analysis in the time domain using NEC-2 and FFT.

Careful considerations should be taken in determining the time step Δt and duration of analysis T (or the highest frequency f_{high} and the lowest frequency f_{low} in the analysis) in the Fourier transform of the input waveform of the source. The frequencies of the output must be coincided with the frequencies analyzed by NEC-2.

IV COMPUTATION

A. Example of Input Data

The sample input data deck to NEC-2, which is for an impulse voltage measuring system illustrated in Fig. 2, is shown in Table 1. For this example, the resistor divider of 3.3 m in height is divided into 6 segments of 0.55 m.

The input data deck must begin with comment lines ‘CM’. The comment lines are terminated by ‘CE’. A line starting with ‘GW’ represents a cylindrical straight wire. The number next to ‘GW’ is a tag number assigned to all segments of the wire. The one next to it is the number of segments into which the wire is divided. The decimal numbers next to them are the coordinates of the wire ends and the radius of the wire (x_1, y_1, z_1, x_2, y_2, z_2, a). Note that the unit is in meters.

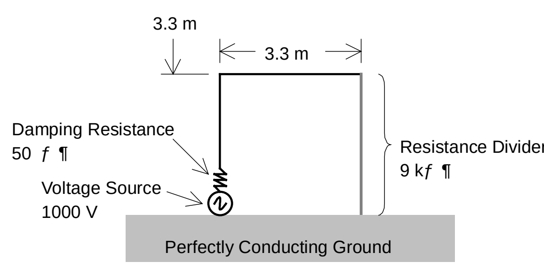


Fig. 2. An impulse voltage measuring system subject to analysis.

Table 1. Sample input data to NEC-2.

```
CM IMPULSE VOLTAGE MEASURING SYSTEM
CM N=512 Delta T=2.5ns
CE PERFECTLY CONDUCTING GROUND IS USED
GW 1  6    0.0   0.0   0.0   0.0   0.0   3.3  0.0025
GW 2  6    0.0   0.0   3.3   3.3   0.0   3.3  0.0025
GW 3  6    3.3   0.0   3.3   3.3   0.0   0.0  0.0025
GE 1
GN 1
LD 4  1  2  2   50.0
LD 4  3  1  6  1500.0
FR 0 256  0  0 7.813E-01 7.813E-01
EX 0  1  1 00  1000.0   0.0
PT 0  3  6
XO
```

The following two lines, 'GE 1' and 'GN 1', indicate a perfectly conducting ground exists at *z=0*, i.e. by these commands, images below ground are generated. The 9th and 10th lines beginning with 'LD' specify the impedance loading. The 9th line indicates that the 2nd segment of the set of segments whose tag number is 1 is loaded by resistance of 50Ω. Similarly, the 10th line indicates that each of the 1st segment through 6th segment of the set of segments having tag number of 3 is loaded by 1500Ω, respectively.

In the line starting with 'FR', the frequency range is specified as 0.7813 MHz to 200 MHz with the linear increment step of 0.7813 MHz. In the line of 'EX', the excitation for the structure is specified. In this case, a voltage source generating 1000 V is inserted into the 1st segment of the set of segments having tag number of 1. By the line beginning with 'PT', currents for the 6th segment of the set of segments whose tag number is 3 are printed. The last two commands, 'XQ' and 'EN', are commands of program execution and end, respectively.

B. Time-Domain Analysis

A waveform of voltage source in the time domain should be prepared independently, and it needs to be transformed into the frequency domain by FFT. The transformed source waveform with the computed result by NEC-2 are finally transformed back to the time domain again using inverse FFT.

The length of segments roughly determines f_{low} and f_{high}, that is, f_{low} should be higher than $10^{-3}c/\Delta L$=0.545 MHz, and f_{high} should be lower than $0.1c/\Delta L$= 54.5 MHz, if the recommended frequency range of $10^{-3}\lambda<\Delta L<0.1\lambda$ is kept. This frequency range corresponds to the duration of the analysis T=1.83 μs with the time interval Δt =18.3 ns.

If Δt =3 ns is desired to inspect the steeply rising part of a unit-step-response waveform of the measuring system, such a transient analysis cannot be performed within the recommended frequency range. However, from the authors' experience, analyses at a frequency range exceeding the recommended highest frequency usually produced satisfactory results in the time domain. Lower amplitudes of the frequency components of the input waveform in a higher frequency range may contribute to such results.

The recommended condition for the lowest frequency still needs to be satisfied since the accuracy of the computation at a low frequency is quite influential on the time-domain waveform of the result. After all, f_{high}=200 MHz and f_{low}= 0.7813 MHz are employed for the above analysis. The thin line in Fig. 3 is the input waveform of the voltage source, and the thick line is the computed current at the lowest segment of the divider in Fig. 2.

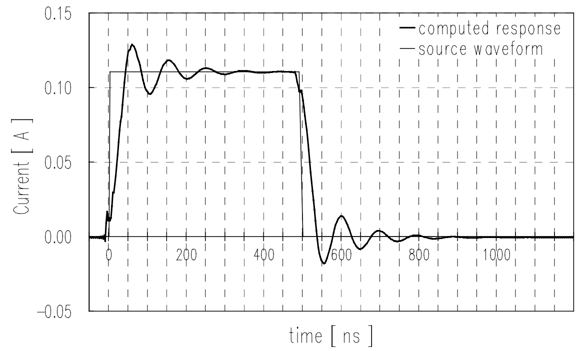


Fig. 3. Computed response in the time domain with the input source voltage waveform.

V. VALIDATION OF APPLICABILITY

Figure 4 shows the arrangement of an experiment on a vertical structure [5]. Step current is injected from the horizontal current lead wire into the structure, and induced voltage between the top of the structure and an insulated horizontal wire is measured. This measuring method is essentially the same as the direct method in the measurement of surge response characteristics of a transmission tower [6]. This experiment is numerically simulated by using NEC-2.

Figure 5 shows the computed and measured waveforms of the induced voltage at the top of the structure for cases of a single conductor and four parallel conductors. Figure 6 shows the measured and computed waveforms of the voltage. Not only the waveforms, but also the amplitudes are well reproduced by the computation. The experiment was carried out in

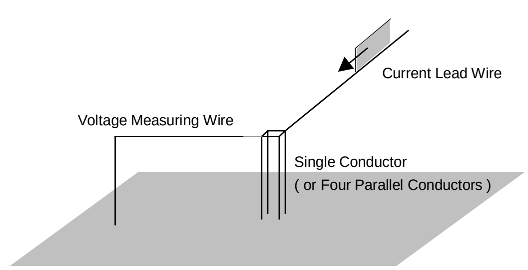
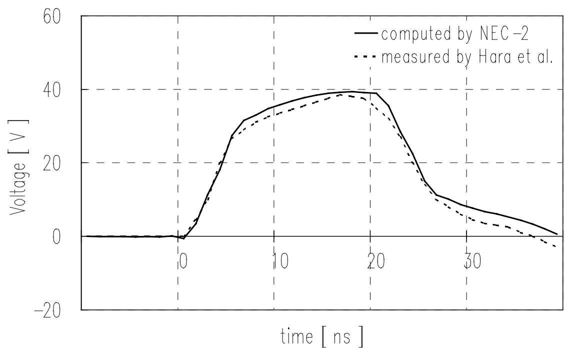
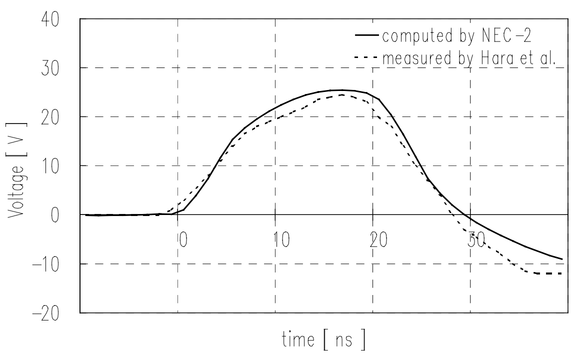


Fig. 4. A model analyzed by NEC-2, simulating an experiment on a 3-m high structure carried out by Hara et al. [5].



(a) A single vertical conductor.



(b) Four parallel conductors.

Fig. 5. Measured and computed waveforms of the voltage at the top of a single vertical conductor and four parallel conductors.

a laboratory, but the ceiling was not modeled in the numerical simulation. This might be the principal cause of the larger difference in the wave tails.

The difference of the peak voltage of the measured and computed waveforms is less than 5%. This small difference demonstrates that the numerical method is as effective as an experiment in investigating the lightning transients in metallic structures.

VI. OTHER APPLICATIONS

A. Tower Surge Response

The measurement of the voltages by the direct method by injecting a step-like current into a full-size transmission tower is a reliable method in evaluating the characteristics of the tower. However, current injection from a vertical wire is quite difficult, and arrangement of voltage measuring wires is restricted. High cost of the experiment prevents accumulation of data through this measuring method.

Measurement on a scale model can overcome such difficulties, but the measurement of the voltages is less reliable. The numerical simulation by NEC-2 is the most cost-effective and flexible, thereby this is most appropriate in investigating influences of various elements such as the inclination of the current injection wire and couplings between various wires. On the other hand, detailed modeling of the tower is difficult, and the reliability of the representative computed result had better be verified by comparison with similar experiments. This is not only because of the rough modeling, but also because the numerical analysis usually requires calculation in unfavorable frequency ranges as was explained in Section IV.

Taking advantage of the numerical method, the influence of the arrangement of the current injection wire relative to the voltage measuring wire is investigated [6]. Figure 6 shows the three cases of the arrangement of the wires in measuring the voltage at the top of a 120-m transmission tower. Fig. 7 shows the influence of the arrangement on the tower top voltage. In the measurement at a full-size tower, the measuring wires are arranged in similar ways as case (ii) or (iii); the tower top voltage is about 10 to 20% lower in these arrangements compared to the case of vertical current injection.

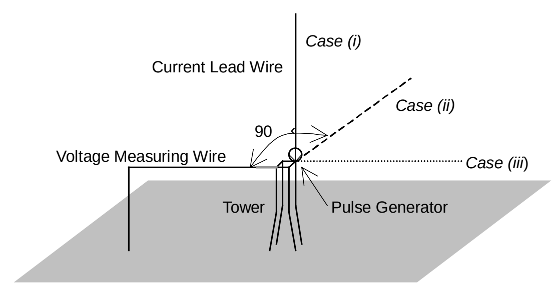


Fig. 6. A model simulating the measurement of tower surge response by the direct method, for the analysis by NEC-2.

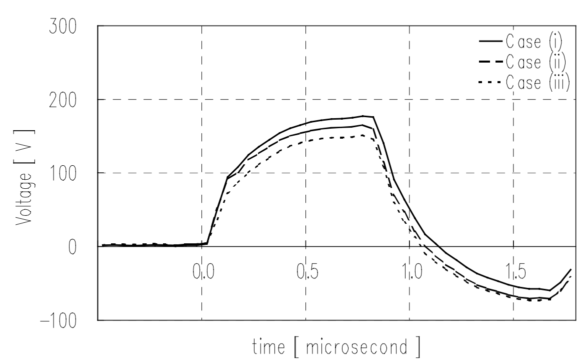


Fig. 7. Computed waveforms of voltages at the top of the tower [6].

B. Lightning Current Measured on Tall Structure

Lightning currents measured on tall structures such as CN tower in Toronto sometimes show features of reflection of traveling waves, and have been analyzed by modeling the tower by transmission lines [7]. The current can also be analyzed by NEC-2 by modeling the tower by thin wires, and by postulating the waveform of the source placed at the connecting point of the lightning channel and the tower [8].

CN tower is 550 m in height, and lightning current is measured at a point of 463 m. In the analysis by NEC-2, the lower 325 m is modeled by base-broadened three poles, and the upper part is by a thin cylinder of 175 m in length. A voltage source in series to a resistor of 400 Ω is placed at the top. The conductors of the model on perfectly conducting ground are divided into cylindrical segments of 25 m in length and 0.3 m in radius.

Figure 8 shows the measured [7] and the computed waveforms of lightning current. The postulated waveform of the voltage source is quite simple as shown in Fig. 9. It is obvious that the observed complex waveform seen in Fig. 8 results from successive reflection and refraction at discontinuous points such as the top, bottom and a bulge of the observation deck at 325 m. The increase of the current at about 4 μs is the reflection from the ground, traveling at the speed of light.

The computed waveform in Fig. 8 is obtained for the tower footing resistance of 30 Ω. Various values of footing resistance such as 50 Ω [7] or higher have been postulated in analyzing lightning current waveforms observed on tall structures when the tower is modeled by a transmission line. This tendency, that high footing impedance is initially observed by time-domain refractometry, even for a tower having low footing resistance evaluated at low frequency, was first reported for a model conical tower on a metal plane [9].

Based on the numerical electromagnetic analysis, this phenomenon is interpreted as the result of the distortion of the reflected current wave from the ground. Difference between the electromagnetic field associated with the downward current wave and reflected current wave from the ground is the principal cause. The apparent initial high footing impedance is observed at upper part of the tower only, and has nothing to do with the time-dependent characteristic of tower grounding.

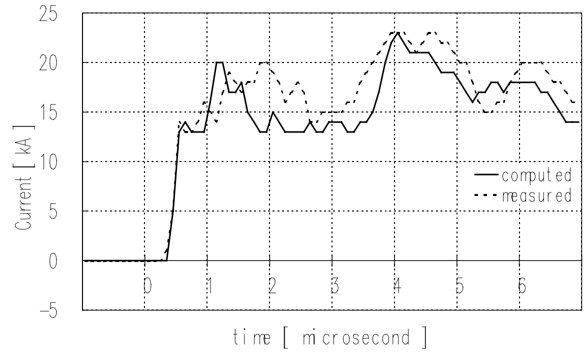


Fig. 8. Measured [7] and calculated waveforms of lightning current on CN tower.

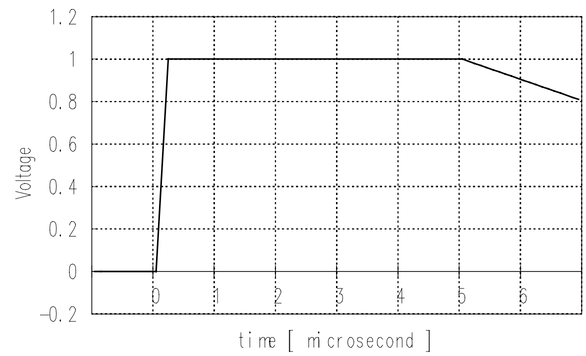


Fig. 9. Postulated waveform of the voltage source in the simulation of Fig. 8.

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