Rhythm-based Locomotion Control for Biped Robots

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Abstract. In the higher animals, periodic motion like locomotion is provoked by the rhythm which is generated in the central nervous system, and is corrected according to somatosensory and visual sensory information about posture and acting force. This rhythm-correction causes the higher animals to realize stable locomotion with robustness against disturbance. From these neurophysiological knowledges, we are considered "locomotion rhythm" being the essential nature for representing the global kinetic features of human biped locomotion, and we proposed "rhythm-based biped locomotion control" by defining the transition of link velocity as locomotion rhythm. In this paper, we summarize our rhythm-based biped locomotion control, highlighting rhythm generation and rhythm correction for securing coordination between motions on frontal plane and on sagittal plane.

1 Introduction

As the population ages with decreasing children, problems of a shortage of manpower and for keeping the quality of life of persons providing care for their aged family have arisen. The problems become serious from year to year in the world. Development of human-aid robots is anticipated as one solution to the problems. Such robots have a field of activity as wide as that of a man, and should have locomotion ability of a man to be put to practical use. Our living environments are designed and maintained presupposing biped locomotion, so biped locomotion is superior in compatibility with our living environment, and is suitable for locomotion of human-aid robots.

Locomotion control problem includes two subjects: design of reference and that of controller. Qualities of realized locomotion depend on properties of reference and the ability of controller. While other studies focused on designing controller with high ablility, we focused on design of proper reference for realizing locomotion with global kinetic features of human locomotion for the following reason. Biped locomotion is realized under the influence of interaction between a robot and the contact surface. Controller with high ability should be designed in due consideration of interaction between robot and the contact surface, but reliable model of the interaction is seldom obtained. So, we inferred that design of good reference is a preferred approach for realizing stable locomotion, and stable locomotion is realized by following the reference with high-gain servo control. Thus, a principle task in design of the suitable reference for realizing stable biped locomotion is to find control objectives reflecting general kinetic features of natural locomotion like a man. In the literature [1], control objectives found upon ZMP criterion or inverted-pendulum criterion have been used, but those only reflect specific kinetic features of biped locomotion, and too much load is put on actuators for attenuating disturbance during locomotion. As a result, few studies can realize stable biped locomotion in the real environment [2], [3].

Human locomotion is periodic with good coordination between motions on frontal plane and on sagittal plane. Periodic motion like locomotion, swimming of the higher animals are provoked to start by rhythm generated in the central nervous system in voluntary motion control scheme [4–8], and the rhythm is corrected according to somatosensory and visual sensory information about the posture and acting force. This correction function causes animals to realize stable locomotion with robustness against disturbance under the real environment. From those neurophysiological knowledge, "locomotion rhythm" depicts the essential nature for representing the global kinetic features of human locomotion, and coordination between motions on frontal plane and on sagittal plane is secured in accordance with the locomotion rhythm. Thus, we have proposed "rhythm-based biped locomotion control method" in which locomotion rhythm is defined by the transition of individual link velocity of each leg [9], [10]. In this paper, we refer to roll of rhythm played in realizing stable locomotion with coordination between motions on frontal plane and on sagittal plane, and summarize our rhythm-based biped locomotion control, highlighting rhythm generation and rhythm correction for securing coordination between the motions.

2 Voluntary Locomotion Control Scheme of the Higher Animals

Locomotion of the higher animals is controlled mainly by three nervous subsystems; the spinal cord, the brain stem and the cerebellum [4], as illustrated in Fig. 1. Some results of animal experiments using decerebrated cats presumed that a reference pattern for provoking locomotion is generated by central program that is innately incorporated into the central nervous system [5]. In the voluntary motion control scheme of the higher animals shown in Fig. 1, motor neurons are activated by the reference pattern, and the neurons actuate skeletal-muscle system. Further, the reference patterns are corrected by somatosensory and visual sensory feedback system according to the current posture and acting force from the contact surface. As a result, locomotion governed by regular rhythm is realized [6], [7].

It is not confirmed that same control scheme is applicable to human locomotion control. However, some clinical observations of fetuses and neonates support the existence of rhythm generating network in the human central nervous system [8]. On the basis of those knowledge about locomotion control scheme of the higher animals and a man, we can assert that the following control objectives should attain for realization of stable biped locomotion.

Control objectives for realizing stable biped locomotion

1. To generate a locomotion rhythm which suits the mechanism of robot.
2. To correct the locomotion rhythm in order to adopt to the changes of the robot and the physical contact surface.

In the following, locomotion rhythm depicts the essential nature for representing global kinetic features of biped locomotion, and the rhythm-based biped locomotion control method is introduces with the control objectives [9], [10].

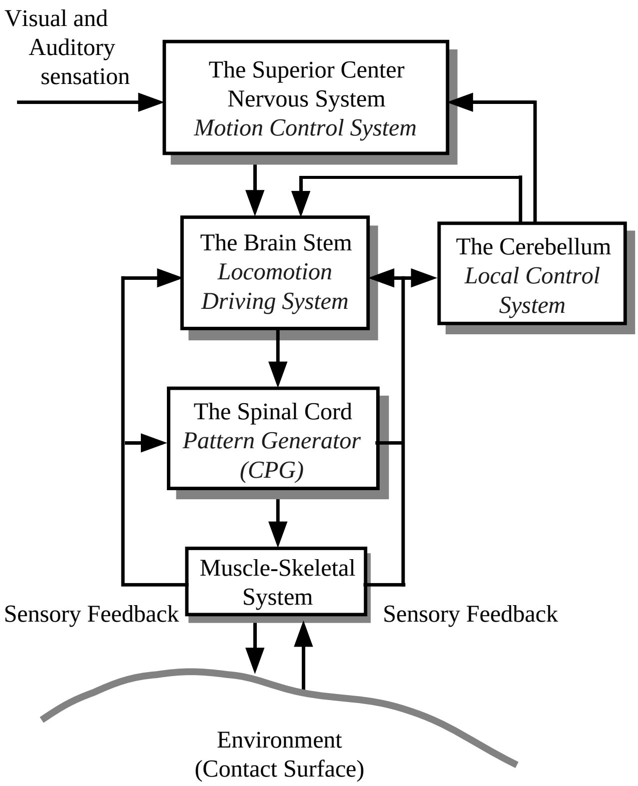


Fig. 1. Voluntary motion control scheme of the higher animals (K. Itoh [4])

3 Rhythm-based Locomotion Control for Biped Robots

For realizing stable biped locomotion, we have proposed the rhythm-based biped locomotion control in [9], [10]. Fig. 2 shows a diagram of the rhythm-based biped locomotion control system, which consists of trajectory generation part, inverse kinematics part, servo control part and rhythm correction part comprising two compliance controllers.

This scheme was modeled after information flow between the spinal cord and muscle-skeletal system shown in Fig. 1. The brain stem and the cerebellum build up two locomotion control subsystems; one is for controlling muscle tonus, the other for controlling the phasic relation between legs [6], [11]. But, each link of actual biped robots is drived by one actuator. Thus, the locomotion control system of biped robots is not necessarily divided into output torque control subsystem and phase control subsystem.

In our method, we regarded the transition of individual link velocity of each leg as locomotion rhythm, and designed rhythm pattern that consists of trajectory data of each link and some time data for providing output torque and output time of actuators [9], [10]. Such approach makes it possible to combine power control subsystem and phase control subsystem. And, output and phases are controlled by regulating output torque and output time of actuators based on the rhythm pattern with single control system.

In trajectory generation part, the time when the link passes certain points and acceleration/deceleration time are set on each basic trajectory on sagittal plane and frontal plane in advance. A rhythm pattern of 3-D locomotion trajectory data and time data is generated by comprising two basic trajectories with time data. Trajectory data making each link velocity change smoothly is generated from the rhythm pattern in real time by using trajectory generation algorithm [12]. The rhythm-based biped locomotion control system finds reference joint angles by solving an inverse kinematic equation using the trajectory data, and achieves trajectory following control using a high-gain positional servo controller. Compliance controllers on sagittal plane and frontal plane correct the reference rhythm by adjusting posture of supporting leg around pitch and roll axes of ankle joint to current state of motion.

3.1 Generation of Locomotion Rhythm

Characteristics of Locomotion Rhythm Characteristics of locomotion rhythm representing kinetic features of provoked biped locomotion are determined by basic trajectories on sagittal plane and frontal plane. Those trajectories are designed on the basis of observations of human locomotion. Locomotion on frontal plane can be treated as swing motion, which is characterized by maximum inclination angle $\omega_{\max}$

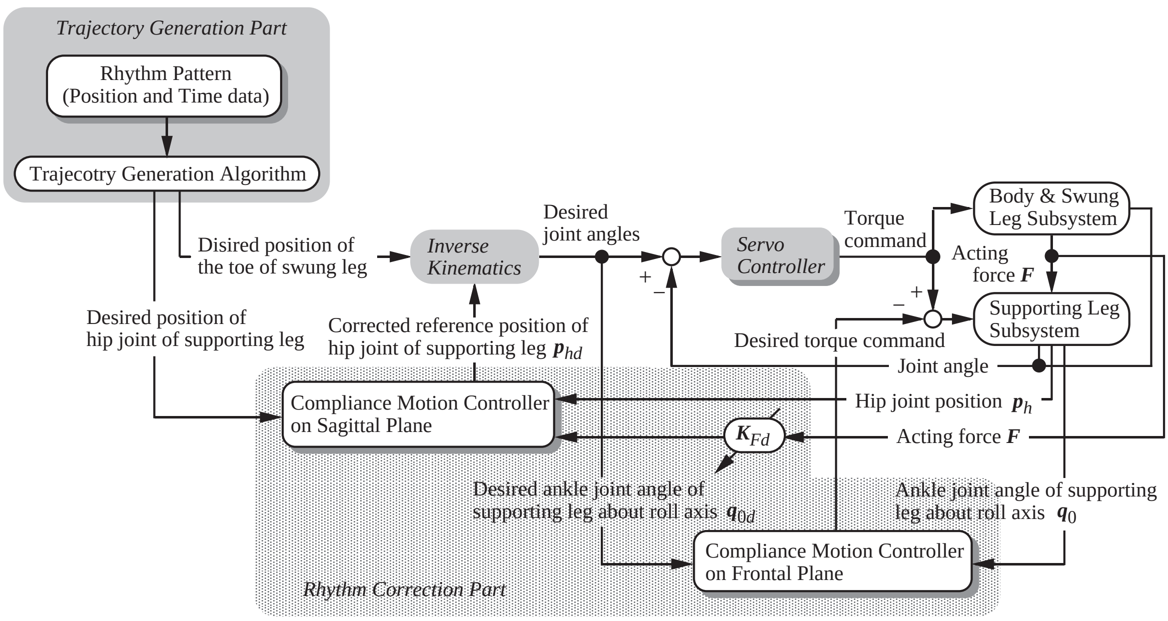


Fig. 2. Diagram of the rhythm-based biped locomotion control sytem

and its period. The period is naturally determined by specifying gait cycle. So, the maximum inclination angle is used as a design parameter of basic trajectory on frontal plane. Locomotion on sagittal plane consists of rotation of legs and translation of the body, and it is characterized by trajectories of hip joint and a foot of swung leg.

Fig. 3 shows basic trajectories on frontal plane and sagittal plane. In the figure, h_F and h_W represent heights of foot of the swung leg and hip joint measured along the supporting leg. d and h represent step length and initial height of hip joint. f and f' represent maximum heights of a foot of the swung leg and hip joint during locomotion, and are calculated as

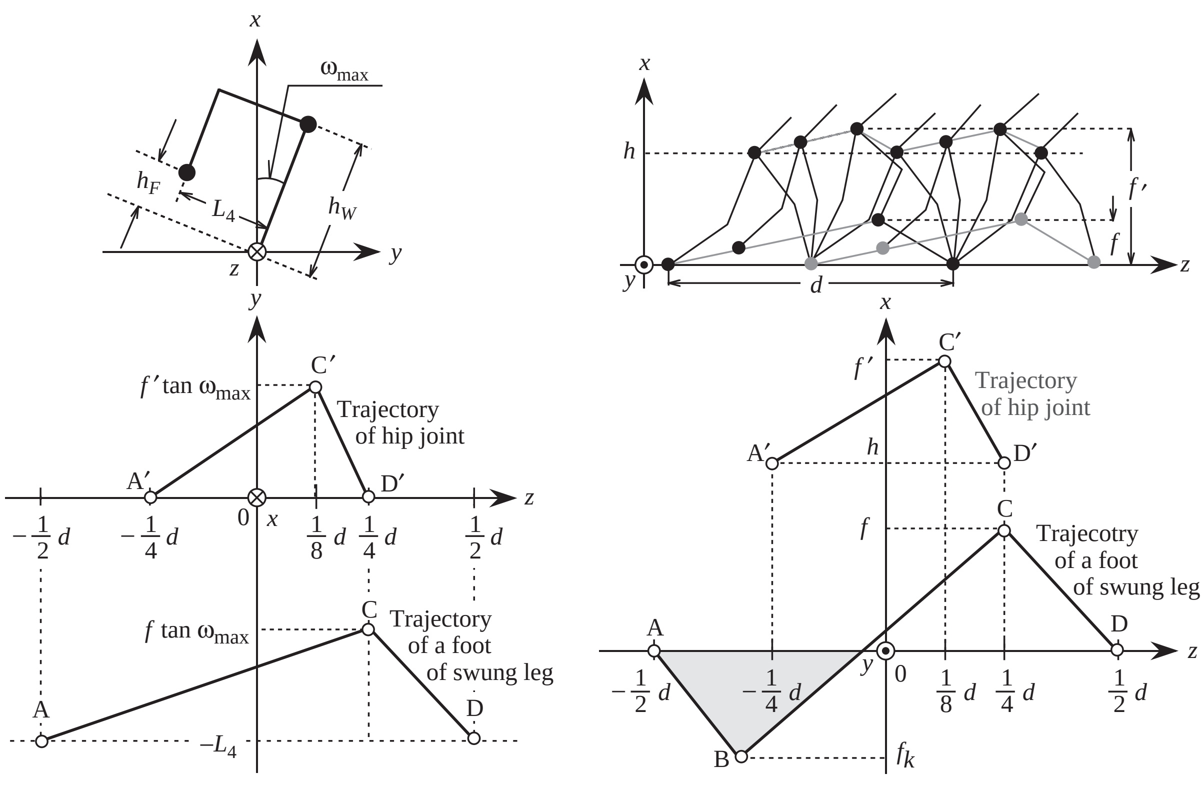
$$f = (h_F)_{\max} \cos \omega_{\max} + L_4 \sin \omega_{\max} \tag{1}$$

$$f' = (h_W)_{\max} \cos \omega_{\max} \tag{2}$$

where L_4 is the length of waist link.

During locomotion, kinetic energy of biped robot is lost due to the collision between their swung leg and contact surface. This energy loss should be appropriately compensated for preserving locomotion from being irregular. We utilized a kicking motion for the compensation. To realize a kicking motion, we design a basic trajectory of a foot of swung leg on sagittal plane as shown in Fig. 3 (b), in which f_k represents the depth of kicking motion and is a design parameter for determining kinetic effect of kicking motion. A region exists in the trajectory that the foot is placed under contact surface, and in which the error between desired position and realized position is always reduced by the servo controller. This causes the robot to push ground by its swung leg, and same kinetic effect of a kicking motion is achieved.

Rhythm Pattern Design Basic motion trajectories on frontal plane and sagittal plane shown in Fig. 3 are described statically by a function of step length while a rhythm pattern should be a function of time. The trajectories are transformed by forming a connection with gait cycle into dynamical locomotion trajectories with comprising positional data and time data, and the obtained 3-D dynamical locomotion pattern is regarded as a primitive rhythm pattern.



(a) Basic trajectory on frontal plane

(b) Basic trajectory on sagittal plane

Fig. 3. Basic trajectories

In the transformation, next time data are set on basic motion trajectories:

t_1 : Transit time of a foot of swung leg from point A to point B.
t_2 : Transit time of a foot of swung leg from point B to point C.
t_3 : Transit time of a foot of swung leg from point C to point D.
t_{acc} : Acceleration/deceleration time about points A, B, C, D in trajectory of a foot of swung leg and that about points A′, C′, D′ in trajectory of hip joint.

where points A and A′ are initial points in trajectory of a foot of swung leg and a hip joint, B is bottom point in trajectory of a foot of swung leg. Points C and C′ are top points in trajectory of a foot of swung leg and hip joint, points D and D′ are terminal points in each trajectory. Gait cycle is $t_1 + t_2 + t_3$ [sec]. Table 1 shows relations between basic motion trajectories and the time data.

The primitive rhythm pattern is followed with errors because the velocity of each link varies discontinuously about points A, B, C, D in trajectory of a foot of swung leg and points A′, C′, D′ in trajectory of hip joint. Locomotion trajectory data making each link velocity change smoothly are calculated for

Table 1. Relations between basic motion trajectories and the time data

(a) Relation on sagittal plane

Time	Point: Trajectory of a foot of swung leg	Point: Trajectory of a hip joint	Operation
0	A	A′	Start accerelating
t_{acc}			Stop accelerating
$t_1 - t_{acc}$			Start decelerating
t_1	B		Stop decelerating Start accelerating
$t_1 + t_{acc}$			Stop accelerating
$t_1 + t_2 - t_{acc}$			Start decelerating
$t_1 + t_2$	C	C′	Stop decelerating Start accelerating
$t_1 + t_2 + t_{acc}$			Stop accelerating
$t_1 + t_2 + t_3 - t_{acc}$			Start decelerating
$t_1 + t_2 + t_3$	D	D′	Stop decelerating Start accelerating

(b) Relation on frontal plane

Time	Point: Trajectory of a foot of swung leg	Point: Trajectory of a hip joint	Operation
0	A	A′	Start accelerating
t_{acc}			Stop accelerating
$t_1 + t_2 - t_{acc}$			Start decelerating
$t_1 + t_2$	C	C′	Stop decelerating Start accelerating
$t_1 + t_2 + t_{acc}$			Stop accelerating
$t_1 + t_2 + t_3 - t_{acc}$			Stop decelerating
$t_1 + t_2 + t_3$	D	D′	Stop decelerating Start accelerating

reducing the tracking error from the primitive rhythm pattern with trajectory generation algorithm [12], which is outlined in appendix.

3.2 Correction of Rhythm

In the rhythm-based biped locomotion control scheme, the stability of gait depends on reference rhythm compatibility with the robot mechanism. High-stability gait necessitates a reference rhythm being most suitable for the robot mechanism. Compatibility of the reference rhythm with the robot mechanism varies dynamically with the state of motion and physical features. So, control system should include rhythm compensation function that dynamically corrects the reference rhythm in order to suit the robot mechanism.

Biped robots often fall into unstable locomotion with the supporting leg unable to maintain contact under inertia force from the body on the supporting leg, because the mass of robot body is generally greater than that of the leg. Therefore, the robot must be controlled by correcting the reference rhythm using information about link positions and that about force acting on the robot to ensure that supporting leg contact is maintained. We focus on compliance function as reference rhythm correction based on position and force information, and introduce feedback control which corrects the reference rhythm using the mechanical impedance of the supporting leg so contact is maintained.

In designing rhythm-correction feedback control system, we put the following assumptions:

1) The supporting leg contacts the surface in four points; the tip of a toe, the heel, inside and outside points of the sole.
2) Static friction on the surface against translation is sufficient.
3) Static friction on the surface against rotation is very small.

Under these assumptions, to keep four-point contact of supporting leg with contact surface leads to realization of stable locomotion satisfying ZMP criterion [2].

Rhythm Correction on Sagittal Plane To design rhythm-correction feedback controller containing a compliance controller on sagittal plane, we divide the biped robot system into a supporting leg subsystem, body and swung leg subsystem as shown in Fig. 4. Compliance control is used for correcting the posture of the supporting leg about pitch axis of ankle joint against force acting from the body and swung leg subsystem so contact is maintained on sagittal plane.

Proper compliance characteristic ($\boldsymbol{M}_d$, $\boldsymbol{D}_d$, $\boldsymbol{K}_d$) around pitch axis of ankle joint can make possible motion of the supporting leg that corrects its posture so that contact is maintained on sagittal plane against force acting on it from the body and swung leg subsystem. We set up damping $\boldsymbol{D}_d$ and stiffness $\boldsymbol{K}_d$ so that the supporting leg demonstrates critical damping at upright position, attaching importance to the system's fast response. Desirable inertia is obtained from the inertia matrix of the supporting leg

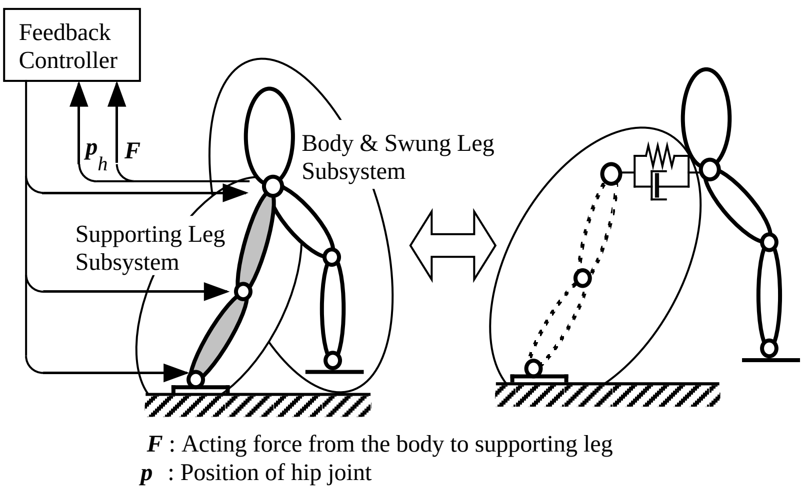


Fig. 4. Division of a biped robot into two subsystems

to prevent actuators from being overloaded. Feedback gain $\boldsymbol{K}_{Fd}$ is introduced to compensate for inertia determined by the state of motion alone. Adjusting inertia using feedback gain $\boldsymbol{K}_{Fd}$ approximates desirable compliance characteristic about pitch axis on ankle joint. Feedback gain $\boldsymbol{K}_{Fd}$ is analytically obtained from contact conditions, the equation of motion, and the compliance equation. See Refs. [13,14] for details on calculation of feedback gain $\boldsymbol{K}_{Fd}$.

Rhythm Correction on Frontal Plane Most biped robots have smaller number of DOF on frontal plane than that on sagittal plane, so that torque used for controlling motion on frontal plane is less than that for controlling motion on sagittal plane. So, interaction between motions on each plane easily causes the motion on frontal plane irregular. This suggests that locomotion rhythm should be corrected by controlling the motion on frontal plane in accordance with the motion on sagittal plane so that supporting leg contact on frontal plane may be kept by coordinating motion on frontal plane with that on sagittal plane

In designing rhythm correction feedback controller with compliance controller on frontal plane, we modeled the motion on frontal plane after an inverted pendulum shown in Fig. 5 and regarded that disturbance in motion on frontal plane is caused by an external force. In Fig. 5, M_f and r denote the mass of biped robot and the distance from the contact point to the center of gravity, respectively. The compliance controller corrects the roll axial torque generated at ankle joint of the supporting leg according to the external force so that supporting leg contact at inside and outside points of sole may be maintained.

The desired roll axial compliance characteristic (M_{df}, D_{df}, K_{df}), for realizing stable locomotion in a sense that ZMP criterion is satisfied, is obtained from contact conditions, the equation of motion, and the compliance equation, where M_{df}, D_{df}, and K_{df} are inertia, damping, and stiffness. Those characteristics are not derived uniquely, however. We set up inertia to be equal to total mass of a robot. Damping D_{df} and stiffness K_{df} are set up so that the supporting leg may demonstrate critical damping when the robot stand upright, attaching importance to the system's fast response.

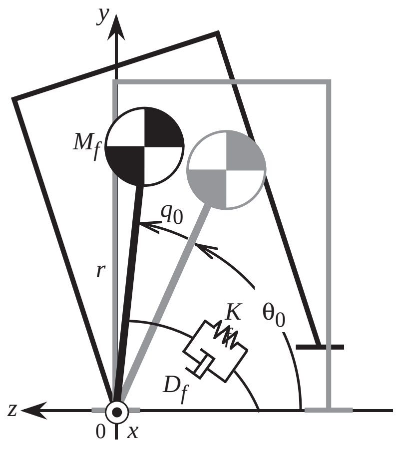


Fig. 5. Inverted pendulum model of motion of frontal plane

4 Simulations and Experiments

4.1 Simulations on Rhythm Correction

The effectiveness of the rhythm-correction feedback control in stabilizing locomotion was verified through locomotion simulations using a 5-link 5-joint model shown in Fig. 6. Basic motion trajectories used in simulations are shown in Fig. 7. Link parameters and trajectory parameters are shown in Table 2.

In simulations, vertical components f_a, f_b, f_c and f_d of reactions acting from the contact surface on supporting leg at the tip of toe, heel, inside and outside points of sole are calculated under the assumption

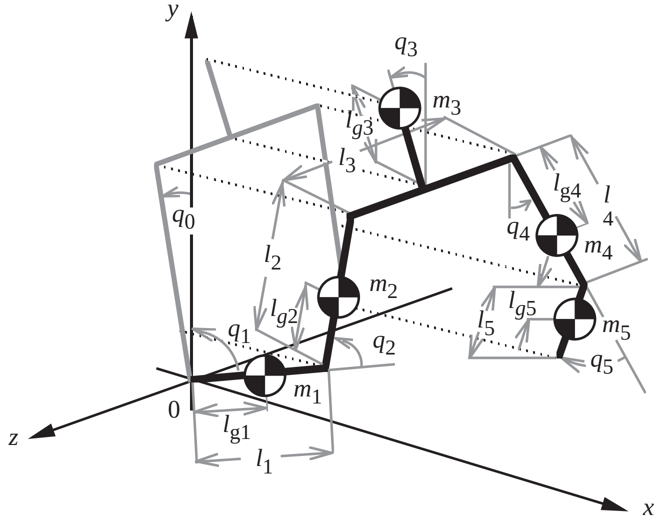
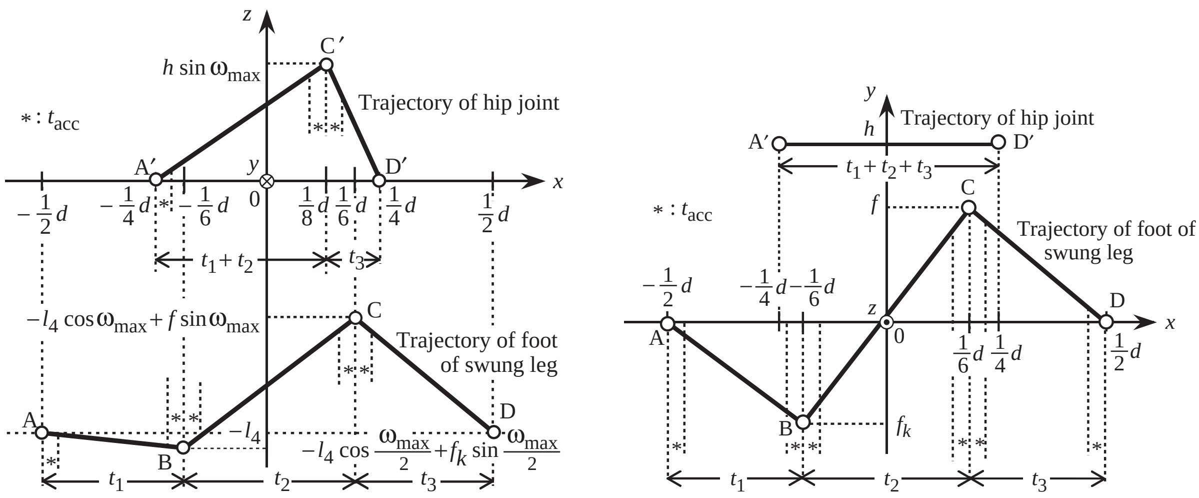


Fig. 6. Simulation model



(a) Basic trajectory on frontal plane

(b) Basic trajectory on sagittal plane

Fig. 7. Basic trajectories used in simulations

that supporting leg firmly contacts the contact surface. Some vertical components of reactions take on negative value in some cases in the simulation, which indicates that four-point contact of the supporting leg is broken at the moment when the components of reactions turned negative, and a polygon obtained by connecting each contact points on sole is reduced. This implies that the stability of locomotion decays in the sense that dynamical range of motion of a biped robot for satisfying ZMP criterion decreases.

Fig. 8 shows simulation results. In the figure (a), f_a is a vertical component of reaction acting on the tip of toe and f_b is that acting on heel. In the figure (b), f_c is a vertical component of reaction acting on inside point of sole and f_d is that acting on outside point of sole.

It can be found out from the figure that stable locomotion with maintaining four-point contact of supporting leg is realized by rhythm-correction feedback control while f_b, f_c and f_d turn negative if rhythm-correction feedback control is not executed. These results indicate that dynamic range of motion of the robot with satisfying ZMP criterion is enlarged. Therefore, it can be concluded on the basis of simulation results that rhythm-correction feedback control is effective for realizing stable locomotion.

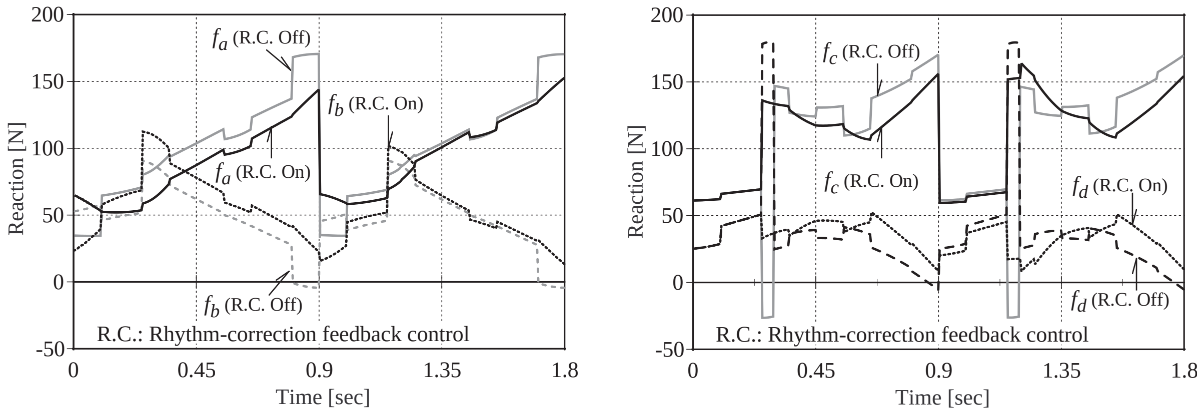
Table 2. Simulation parameters

(a) Model parameters

Link No. i	1	2	3	4	5
l_i [m]	0.206	0.206	0.251	0.206	0.206
l_{gi} [m]	0.103	0.103	0.100	0.103	0.103
m_i [kg]	2.223	2.223	6.223	2.223	3.112
I_{xi} [kg·m^2]	0.001	0.001	0.020	0.001	0.001
I_{yi} [kg·m^2]	0.031	0.031	0.132	0.031	0.045
I_{zi} [kg·m^2]	0.031	0.031	0.132	0.031	0.045

(b) Trajectory parameters

t_1 :	0.3	[sec],	t_2 :	0.3	[sec]
t_3 :	0.3	[sec],	t_{acc} :	0.05	[sec]
Gait cycle :	0.9	[sec],	Step length :	0.1	[m]
Maximum inclination angle on frontal plane $\omega_{\max}$:				7.0	[deg]
Initial position on hip joint of supporting leg h :				0.38	[m]
Top position f of a foot of swung leg :				0.03	[m]
Bottom position f_k of a foot of supporting leg :				0.03	[m]



(a) Time responses on sagittal plane (b) Time responses on frontal plane

Fig. 8. Time responses of vertical components of reactions

4.2 Experimental Locomotion

Experimens to verify the locomotive stability of rhythm-based locomotion control was conducted using a biped robot SR-4R in Kawaji Laboratory, as shown in Fig. 9. SR-4R consists of a 9-link, 8-DOF structure, with 2 DOF on the frontal plane and 6 DOF on the sagittal plane. DOF on the frontal plane is in the two ankles and that on the sagittal plane is in the hip, knee, and ankle joints of both legs.

An encoder measuring joint angle is fitted on each motor shaft, and an inclination angle sensor is fitted on the robot. Pressure sensors are on the soles of both feet.

Rhythm-correction feedback control system on frontal plane was not implemented in the experimental system. Therefore, it is assumed in experiments that frontal plane motion would be in accordance with reference rhythm by restraining interaction between motions on each plane through a high-gain positional

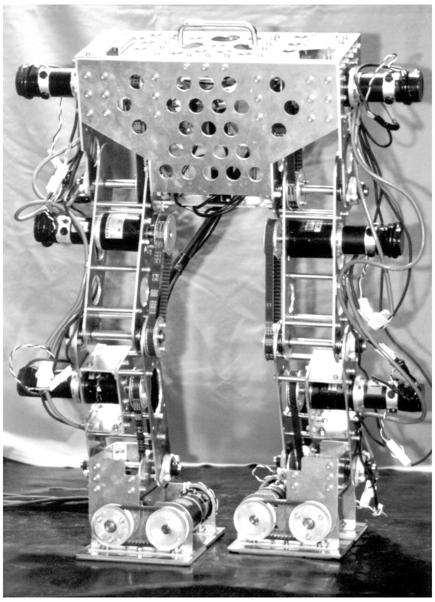


Fig. 9. Biped robot SR-4R

servo controller and a DC servomotor with a high-reduction-ratio reduction gear. Trajectory parameters used in experimetns are shown in Table 3.

Experimental results are shown in Figs.10, 11, and 12.

Fig. 10 shows the time responses of each joint angle, which is measured clockwise from upright. It is found out from the figure that knee joint and ankle joint of the supporting leg are extended before the leg lifts up from the contact surface and reference motion trajectory is well followed by each link. These results imply that locomotion with kicking motion is realized in accordance with the specified rhythm using rhythm-based biped locomotion control.

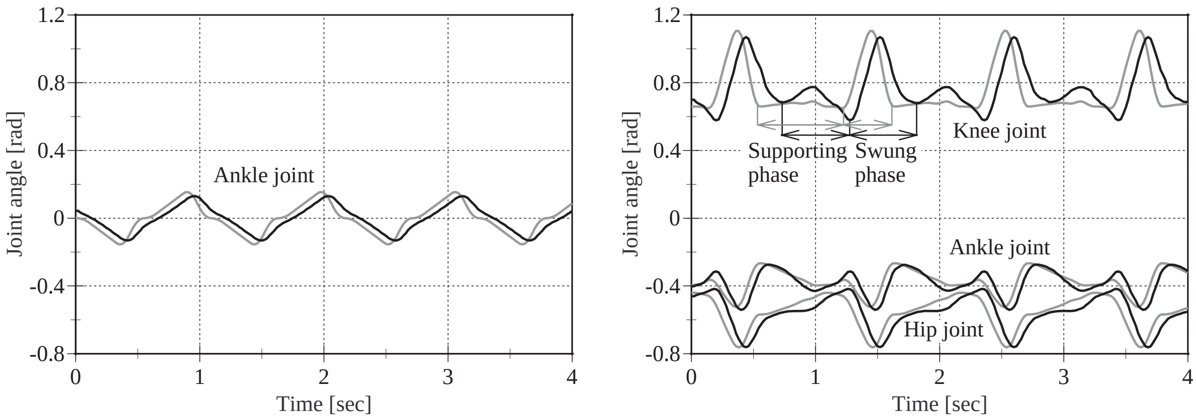
Fig. 11 shows the time responses of inclination angles of the body about its roll axis and its pitch axis. Motions on frontal plane and sagittal plane are well coordinated, and the coordination makes locomotion rhythm regular during walking.

Fig. 12 shows the output voltage of pressure sensors on the tip of toe and heel of the supporting leg. The figure represents the outputs in the sixth step and after, when locomotion had been steady. Either the toe or heel does not contact with the surface during most locomotion if rhythm-correction feedback control is not executed, making locomotion unstable. The supporting leg contacts with the surface at two points on the sagittal plane during most locomotion if the rhythm-correction feedback control is executed.

On the basis of the experimental results, we can say that the stable biped locomotion with satisfying ZMP criterion is realized in accordance with specified locomotion rhythm using rhythm-based locomotion control.

Table 3. Trajectory parameters used in experiments

t_1 :	0.23	[sec],	t_2 :	0.24	[sec],	t_3 :	0.23	[sec],	t_{acc} :	0.05	[sec]
$\omega_{\max}$:	12.0	[deg],	h :	0.36	[m],	f :	0.035	[m],	f_k :	0.02	[m]
	Gait cycle :			0.7	[sec],		Step length :			0.12	[m]



(a) Time responses of joint angles about roll axis

(b) Time responses of joint angles about pitch axis

— : Reference, — : Measured

Fig. 10. Time responses of joint angles during walking

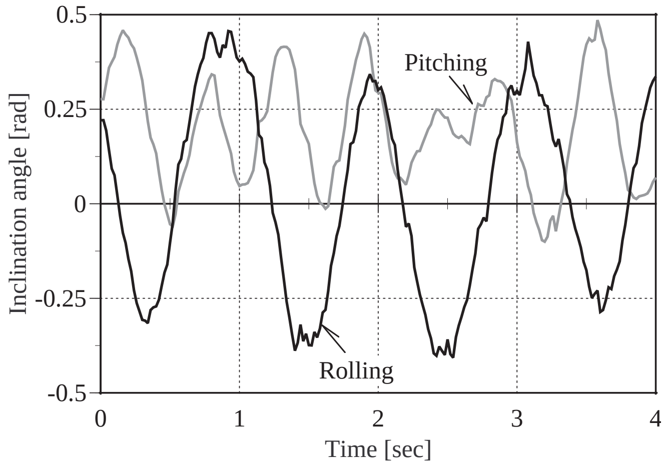
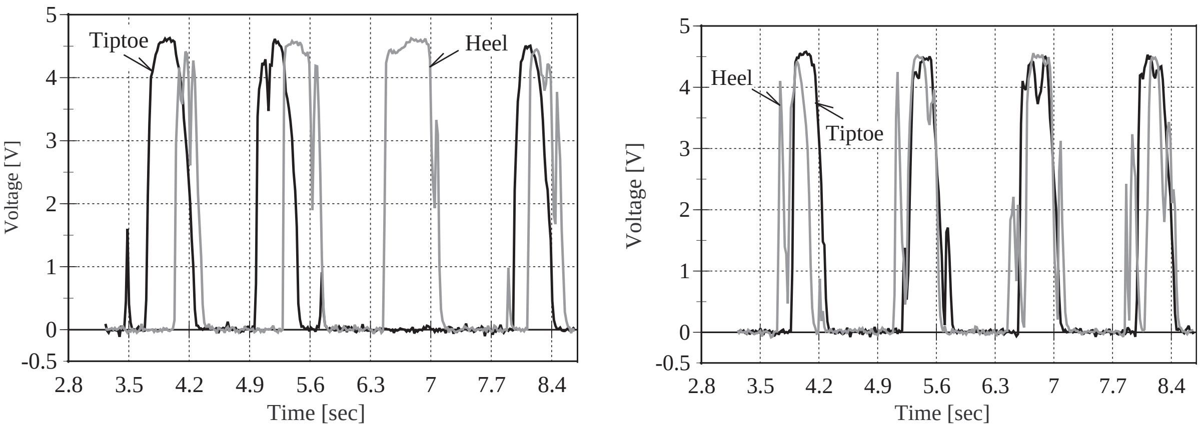


Fig. 11. Time responses of inclination angles of the body



(a) Rhythm-correction feedback control is inactive

(b) Rhythm-correction feedback control is active

Fig. 12. Output voltage of pressure sensors on the sole

5 Conclusion

We gave a summary of rhythm-based biped locomotion control, which is modeled after locomotion scheme of the higher animals. The locomotion rhythm is used as a characteristic representing kinetic features of motion globally.

We pointed out the importance of generating desired signals that represent global kinetic features of biped locomotion for realizing natural locomotion like a man, and regarded biped locomotion control problem as a generation problem of reference signals, while many other approaches aim to construct robust control systems against disturbance based on equation of motions of a biped robot. Then, we proposed rhythm-based biped locomotion control, and rhythm-correction feedback control for realizing stable locomotion according as current state of motion and physical conditions of the contact surface.

The effectiveness of the rhythm-based biped locomotion control in realizing stable locomotion with satisfying ZMP criterion was verified through some simulations and experiments. On the basis of the results, we can conclude that the rhythm-based biped locomotion is an effective control method for realizing stable biped locomotion in real environment.

A Trajectory Generation Algorithm

The proposed algorithm generates a continuous path which passes near vertices in the given trajectory by comprising linear trajectories with constant velocity and trajectories with constant acceleration [12].

Let $\boldsymbol{\xi}(t)$ be a state vector of a free linear system that is restricted in motion in a trajectory, and the vector is expressed as:

$$\boldsymbol{\xi}(t) = \begin{bmatrix} \boldsymbol{p}_d(t) & \boldsymbol{p}_d^{(1)}(t) & \ldots & \boldsymbol{p}_d^{(\mu)}(t) \end{bmatrix}^{\mathrm{T}} \tag{3}$$

where, $\boldsymbol{p}_d(t) \in \mathcal{R}^n$ is a position vector and $\boldsymbol{p}_d^{(\mu)}(t) = \dfrac{d^\mu \boldsymbol{p}_d}{dt^\mu}$. $\mathcal{R}$ denotes a set of real number. μ represents smoothness of the constrained trajectory.

The state equation and the output equation of the system is defined as follows:

$$\begin{cases} \dot{\boldsymbol{\xi}}(t) &= \boldsymbol{A}_r\, \boldsymbol{\xi}(t) \\ \boldsymbol{p}_d(t) &= \boldsymbol{C}_r\, \boldsymbol{\xi}(t) \end{cases} \tag{4}$$

where

$$\boldsymbol{A}_r = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_n & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_n & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{I}_n \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix} \in \mathcal{R}^{n(\mu+2)\times n(\mu+2)} \tag{5}$$

and $\boldsymbol{C}_r = [\,\boldsymbol{I}_n \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0}\,] \in \mathcal{R}^{n\times n(\mu+2)}$. From eq. (4), the position of the system at time $t \in [t_j, t_{j+1})$ is determined as follows:

$$\boldsymbol{p}_d(t) = \boldsymbol{C}_r e^{\boldsymbol{A}_r(t-t_j)} \boldsymbol{\xi}\,(t_j) \tag{6}$$

,and continuous path with initial condition $\boldsymbol{\xi}\,(t_j)$ can be obtained by connecting piecewice contiuous trajectories $\boldsymbol{p}_d(t)$ $(t \geqq t_j)$ with satisfying a following boundary condition:

$$\begin{bmatrix} \boldsymbol{C}_r\, \boldsymbol{\Phi}_j \\ \boldsymbol{C}_r\, \boldsymbol{A}_r\, \boldsymbol{\Phi}_j \\ \vdots \\ \boldsymbol{C}_r\, \boldsymbol{A}_r^\mu\, \boldsymbol{\Phi}_j \end{bmatrix} \boldsymbol{\xi}\,(t_j) = \begin{bmatrix} \boldsymbol{C}_r \\ \boldsymbol{C}_r\, \boldsymbol{A}_r \\ \vdots \\ \boldsymbol{C}_r\, \boldsymbol{A}_r^\mu \end{bmatrix} \boldsymbol{\xi}\,(t_{j+1})\,,\ \boldsymbol{\Phi}_j \equiv e^{\boldsymbol{A}_r(t_{j+1}-t_j)}. \tag{7}$$

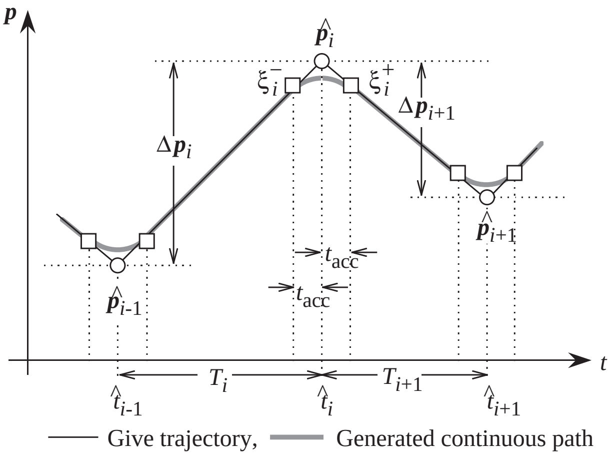


Fig. 13. Relation between given trajectory and generated continuous path

Let $\hat{\boldsymbol{P}}_i$ and $\hat{t}_i$ be the position vector of a vertex in given trajectory and the time when a constrained system reaches at the vertex. The state vector $\boldsymbol{\xi}\left(t_j\right)$ in eq. (3) at time t_j of the constrained system is determined from $\hat{\boldsymbol{P}}_i$ and $\hat{t}_i$ as follows:

$$\boldsymbol{\xi}\ \left(t_j\right) = \begin{cases} \boldsymbol{\xi}_i^-\ \left(\hat{t}_i - t_{\mathrm{acc}}\right) & \text{if } j = 2i+1 \\ \boldsymbol{\xi}_i^+\ \left(\hat{t}_i + t_{\mathrm{acc}}\right) & \text{if } j = 2i \end{cases} \tag{8}$$

where t_{acc} is the acceleration/deceleration time about the vertex in given trajectory. In eq. (8), $\boldsymbol{\xi}_i^-\ \left(\hat{t}_i - t_{\mathrm{acc}}\right)$ and $\boldsymbol{\xi}_i^+\ \left(\hat{t}_i + t_{\mathrm{acc}}\right)$ are calculated using Taylor's interpolation [16] with consideration of continuous velocity of the constrained system as

$$\boldsymbol{\xi}_i^-\ \left(\hat{t}_i - t_{\mathrm{acc}}\right) = \begin{bmatrix} \hat{\boldsymbol{P}}_i - \dfrac{t_{\mathrm{acc}}}{T_i}\boldsymbol{\Delta P}_i \\ \dfrac{\boldsymbol{\Delta P}_i}{T_i} \\ \dfrac{1}{2t_{\mathrm{acc}}}\left(\dfrac{\boldsymbol{\Delta P}_{i+1}}{T_{i+1}} - \dfrac{\boldsymbol{\Delta P}_i}{T_i}\right) \end{bmatrix}, \qquad \boldsymbol{\xi}_i^+\ \left(\hat{t}_i + t_{\mathrm{acc}}\right) = \begin{bmatrix} \hat{\boldsymbol{P}}_i + \dfrac{t_{\mathrm{acc}}}{T_i}\boldsymbol{\Delta P}_{i+1} \\ \dfrac{\boldsymbol{\Delta P}_{i+1}}{T_{i+1}} \\ 0 \end{bmatrix} \tag{9}$$

where $\boldsymbol{\Delta P}_i = \hat{\boldsymbol{P}}_i - \hat{\boldsymbol{P}}_{i-1}$, $T_i = \hat{t}_i - \hat{t}_{i-1}$. Fig. 13 illustrates the relation among $\boldsymbol{\xi}_i^-\ \left(\hat{t}_i - t_{\mathrm{acc}}\right)$, $\boldsymbol{\xi}_i^+\ \left(\hat{t}_i + t_{\mathrm{acc}}\right)$ and $\hat{\boldsymbol{P}}_i$.

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