Conservation laws of optimization criteria for segmentation and time prediction of locomotion trajectories

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While some characteristics of human locomotion are dictated by the dynamics and settings, the motor system has freedom in selecting other characteristics. This is the case for **global timing** (motion durations) of trajectories of locomotion, whose relation to **geometry** (path shapes) we hereby examine.

We focus our study on the timing of full body center of mass trajectories during human locomotion. While geometric speed profiles of motion trajectories are well modelled by movement optimization models (e.g. minimization of acceleration or jerk) and by assuming geometric invariance, global human motion properties are less well understood. The phenomenon of isochrony, which characterizes human motion, may arise from affine invariance (the mixed geometry model, Bennequin et al. 2009). However, current models are yet unable to successfully predict the temporal duration of a walking trajectory given its geometric shape and scale.

We examine motion durations by applying an optimization based approach. Previous studies based on similar approaches either used movement duration as a preselected parameter, or optimized performance with respect to total duration combined with other costs. Huh (2012) presented an interesting viewpoint based on Noether's theorem. He asserted that the relations between movement duration and amplitude depend on the magnitude of an intensive quantity of optimal motion, termed drive $D = \frac{\partial A}{\partial t}$ $where\ A$ is the total cost and the drive is conserved both within and across different optimal motions. For the minimum jerk model the drive $D = \dddot{r}^2 - 2\,\ddot{r}\,\ddddot{r} + 2\,\dot{r}\,\overset{\cdot\cdot\cdot}{\ddot{r}}$ is known to be theoretically conserved (Polyakov et al. 2009, Meirovitch 2014).

We use Noether's theorem to extract several conserved quantities for different optimization criteria. Each symmetry of an optimization criterion predicts a conserved quantity. For instance, the drive parameter arises from time symmetry and Euclidean invariance but depends on the optimization criteria (minimum acceleration, jerk or snap etc.) being used. Such criteria also have rotational symmetries which result in additional conservation laws. We present new theoretic formulations of conservation laws for well-studied Euclidean models as well as for non-Euclidean optimization criteria. We present new numeric methods for calculation of conservation laws for both Euclidean and non-Euclidean models, overcoming the need for extracting high numerical derivatives.

We recently conducted a thorough experimentation of drive conservation in the context of hand motions and concluded that drive is not conserved across different human drawing movements. Following these findings we present a new computational algorithm suggesting how conserved quantities and specifically drive can be used as a basis for examining trajectory optimization and segmentation.

For recorded center of mass trajectories during human locomotion along different geometric paths, we describe our examination of the extent to which conserved quantities of optimization models do explain global motion timing and geometry. Finally, we implement the new optimization based segmentation algorithm in order to extract the optimal motion segments composing the different locomotion patterns. We also describe how these new approaches might prove beneficial for the planning of gait trajectories for humanoid robots.