

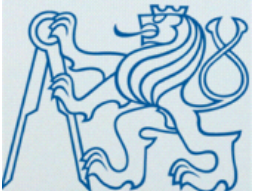
3D Reconstruction from Photographs

Principles & Applications

T o m a s P a j d l a

with contributions from

Z. Kukelova, M. Bujnak, A. Torii, T. Schilling, J. Heller,
C. Albl, ...



Czech Technical University Prague

Center for Machine Perception





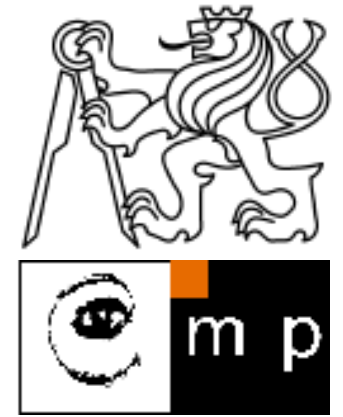
T O M A S P A J D L A

Center for Machine Perception

Department of Cybernetics

Czech Technical University Prague

pajdla@cmp.felk.cvut.cz



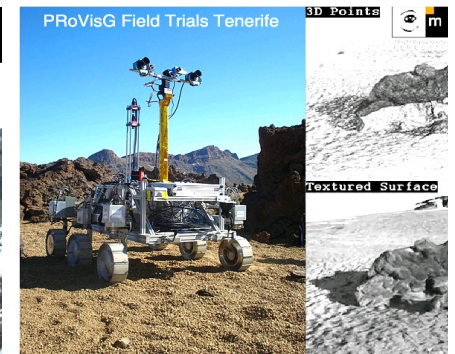
Research

3D

Geometry

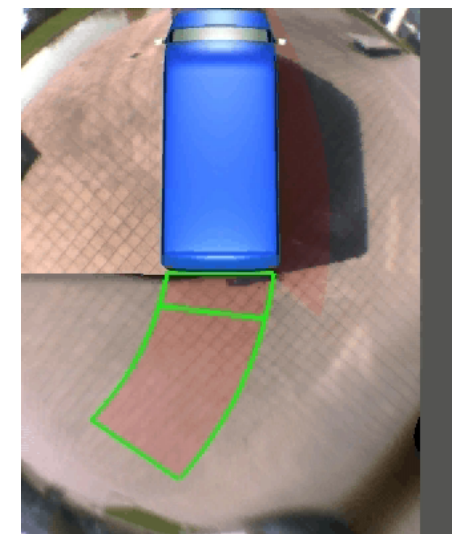
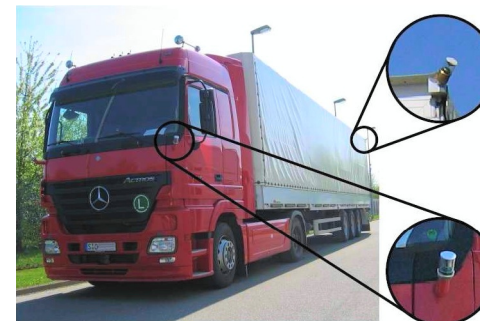
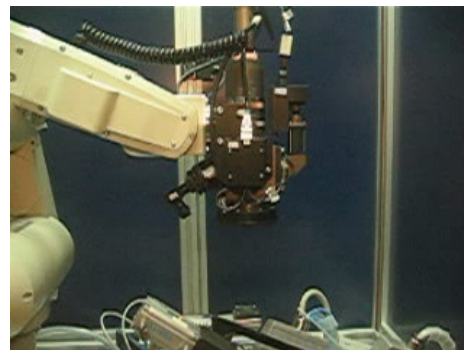
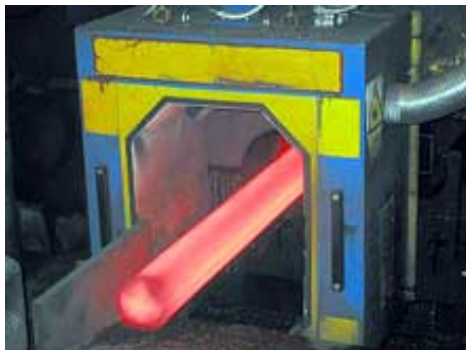
Algebra

Optimization



Applications

Neovision s.r.o., Daimler, Siemens, ...





3 D RECONSTRUCTION FROM PHOTOGRAPHS

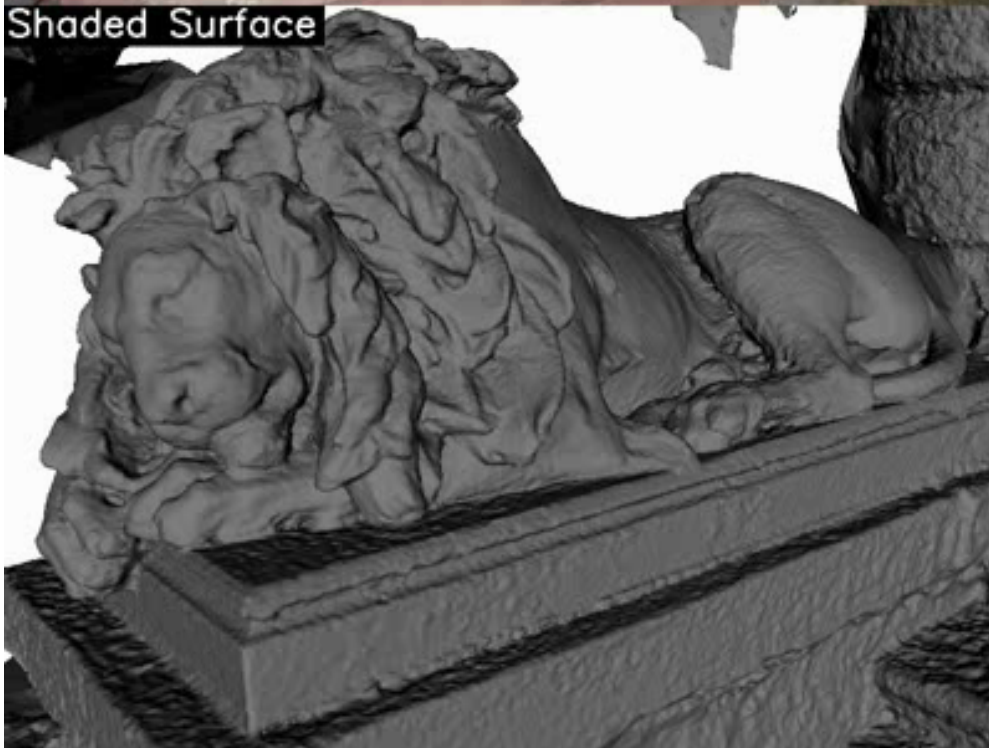
Original Image



3D Points



Shaded Surface



Textured Surface





Automatic 3D Reconstruction of Sternberg data-set.
Sternberg data-set: 324 (3056 x 2296) images.



(c) 12.4.2011, Michal Jancosek, jancom1@cmp.felk.cvut.cz





ASTRIUM

AN EADS COMPANY



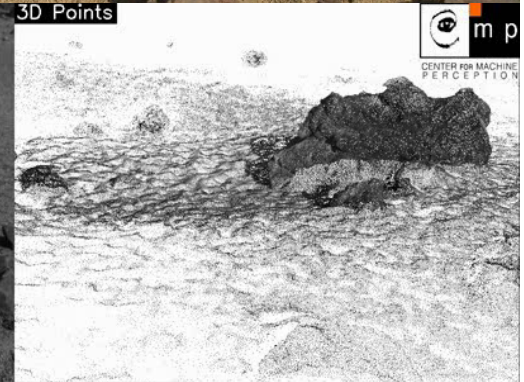
PRoVisG Field Trials Tenerife



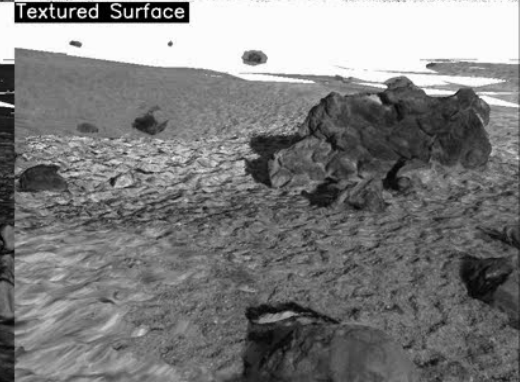
Original Image



Shaded Surface



3D Points



Textured Surface



3D reconstruction pipeline

Welcome to CMP SfM Web ...

ptak.felk.cvut.cz/sfmservice/

cmp CMP Lit N Net News OC Pro Rev Trav XtrA Other bookmarks

switch datatype
case { pta
if RANGAC
msg(1,
(Model
else
msg(1,
(Model
end
msg(1,
end
case
ut
if RANGAC
msg(1,
(Model
end
else
msg(1,
(Model
end
msg(1,
(Model
end

CMP SfM WebService

CMP SfM Web Service

User login

Username:

Password:

[Log in](#)

Menu

- [SfM Web Service](#)
- [Authors](#)
- [Gallery](#)

What?

CMP SfM Web Service provides a remote access to the 3D reconstruction systems developed in [Center for Machine Perception](#). The access to the system is granted on request by email to [Tomas Pajdla <pajdla@cmp.felk.cvut.cz>](mailto:pajdla@cmp.felk.cvut.cz).

Why?

We provide the access to the service to our partners and to people in the Computer Vision community to make it easier to use our codes. There is no need to install any code on a client's computer and all the computations are performed on our dedicated computing cluster. Further, it makes it easier to compare the results of different methods to ours based on the same data.

News

- 17/03/11 - You can now use [▶ icon](#) in 'Datasets table' to run SfM (No XML file needed).
- 06/01/11 - New web interface (version Tokyo).

Input Images

Output Model

300 users
2000 scenes
5000 rec's
See the proceedings ...

Start Present Microsoft PowerPoint - [...] Welcome to CMP SfM ... EN 54 cpu 20:28

OUTLINE

1. Key elements of 3D reconstruction
 1. Camera relative pose computation
 2. From point clouds to surfaces
2. Applications

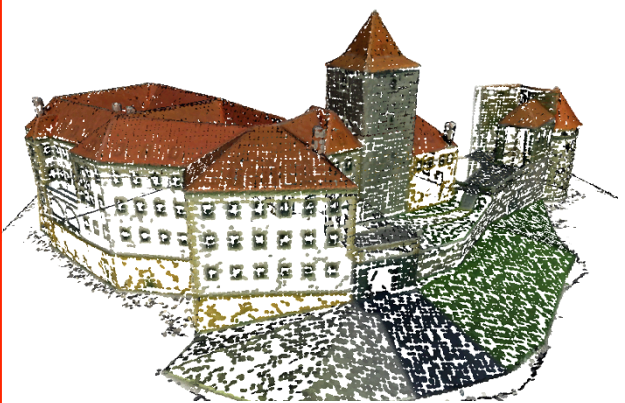
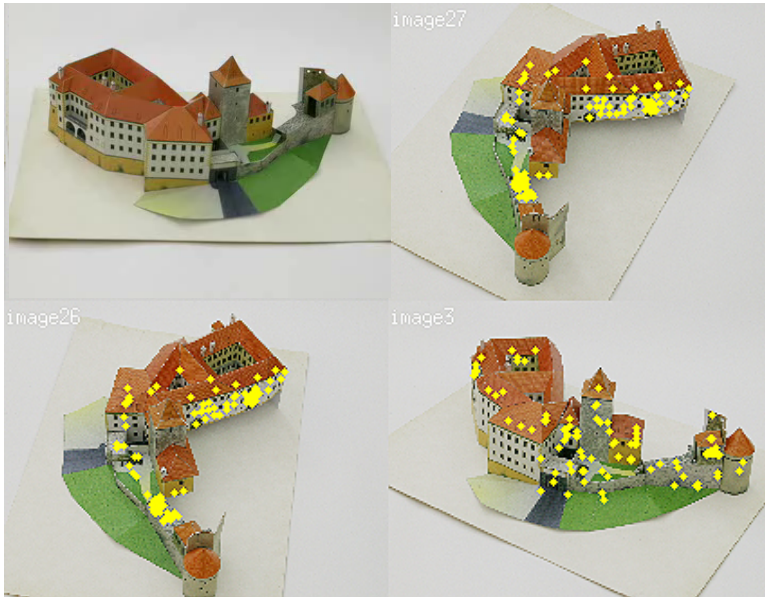


Key elements of 3D RECONSTRUCTION

3D Reconstruction from Photographs

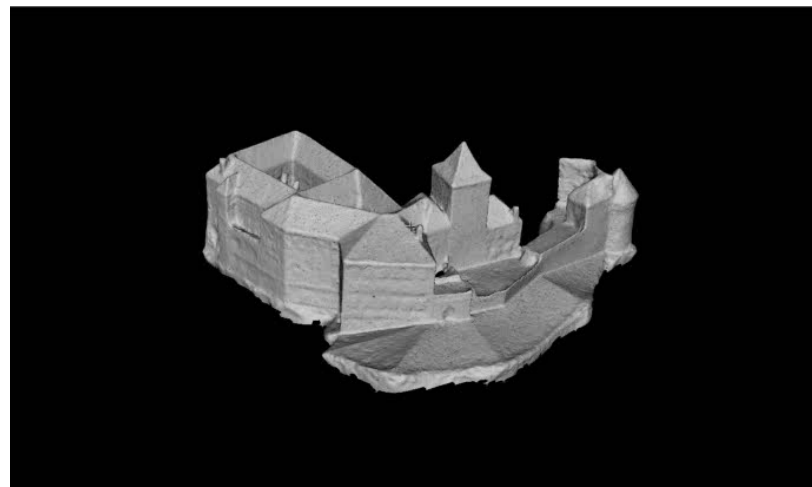
1. SFM: Images & EXIFs

→ Cameras poses → Sparse point cloud



2. MVS:

→ 3D Surface → Texture





RELATIVE CAMERA POSE

PROBLEM

SOLUTIONS

2004(6-12)

An old important problem (Grunnert 1841 ...) with new solutions:

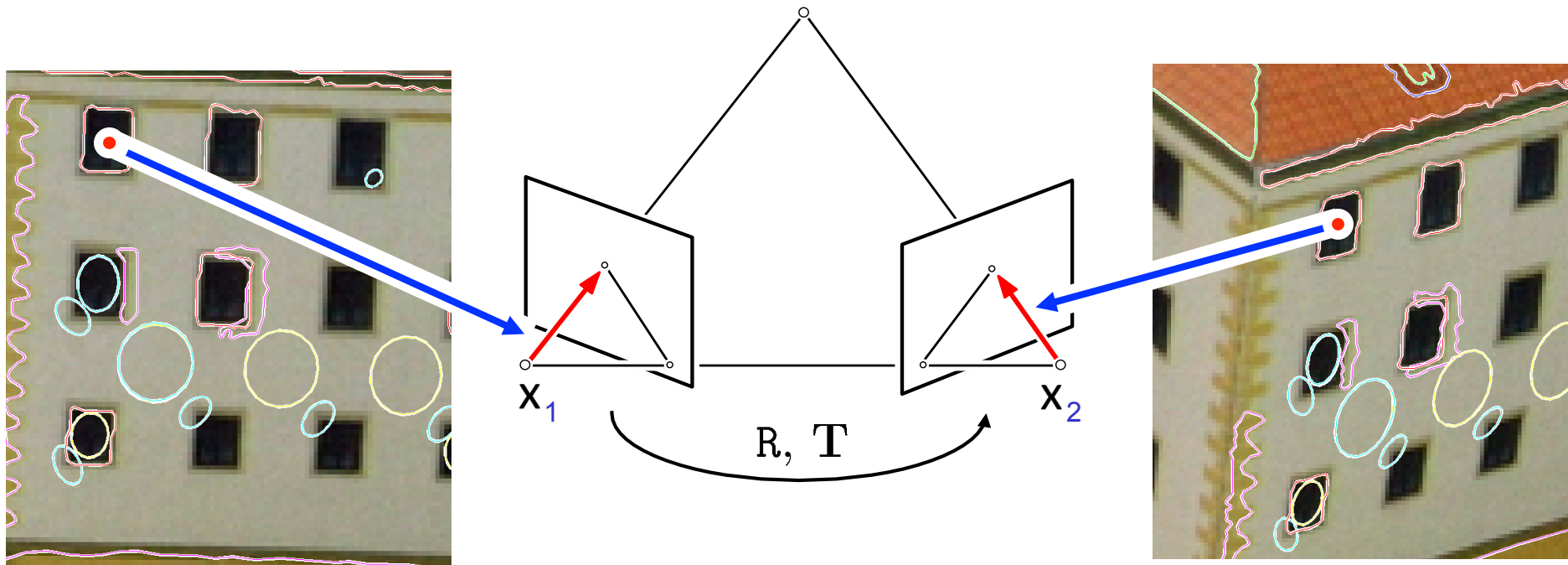
D. Nister. *An efficient solution to the five-point relative pose*
IEEE PAMI, 2004.

H. Stewenius, C. Engels, and D. Nister. *Recent developments on direct relative orientation*
ISPRS J. of Photogrammetry and Remote Sensing, 2006.

Z. Kukelova, M. Bujnak, T. Pajdla
Polynomial eigenvalue solution to the 5-pt and 6-pt relative pose problems
BMVC 2008

H. Li, R. Hartley. *An Efficient Hidden Variable Approach to Minimal-Case Camera Motion Estimation*
PAMI 2012

RELATIVE CAMERA POSE PROBLEM



$$\mathbf{x}_2^\top \mathbf{F} \mathbf{x}_1 = 0$$

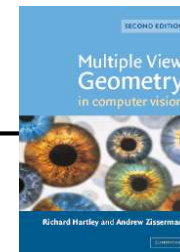
$$\det \mathbf{F} = 0$$

$$2\mathbf{F}\mathbf{F}^\top\mathbf{F} - \text{trace}(\mathbf{F}\mathbf{F}^\top)\mathbf{F} = 0$$

Algebraic equations

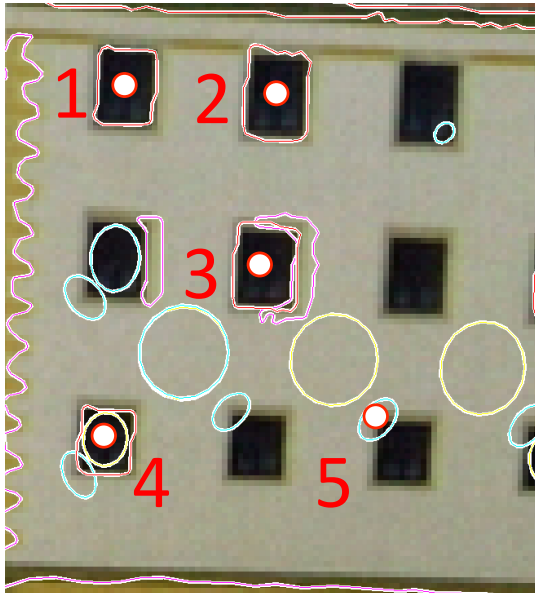
solve

\mathbf{F}

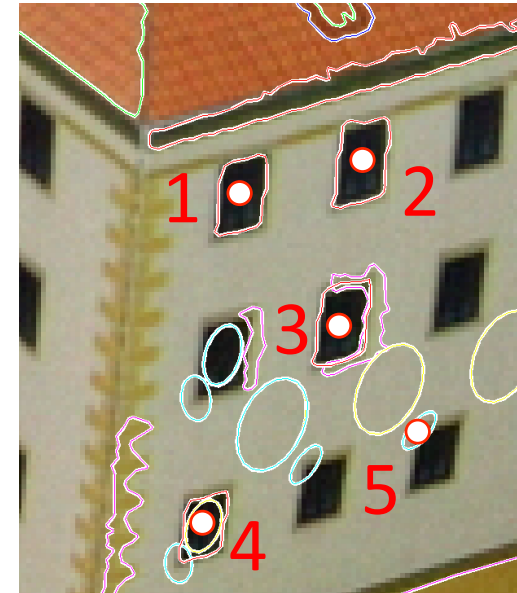


$\mathbf{R}, \begin{bmatrix} \mathbf{T} \\ \|\mathbf{T}\| \end{bmatrix}$

MINIMAL PROBLEMS & RANSAC



5 (correct) matches
are sufficient
to compute F
(but there are many
incorrect tentative
matches)

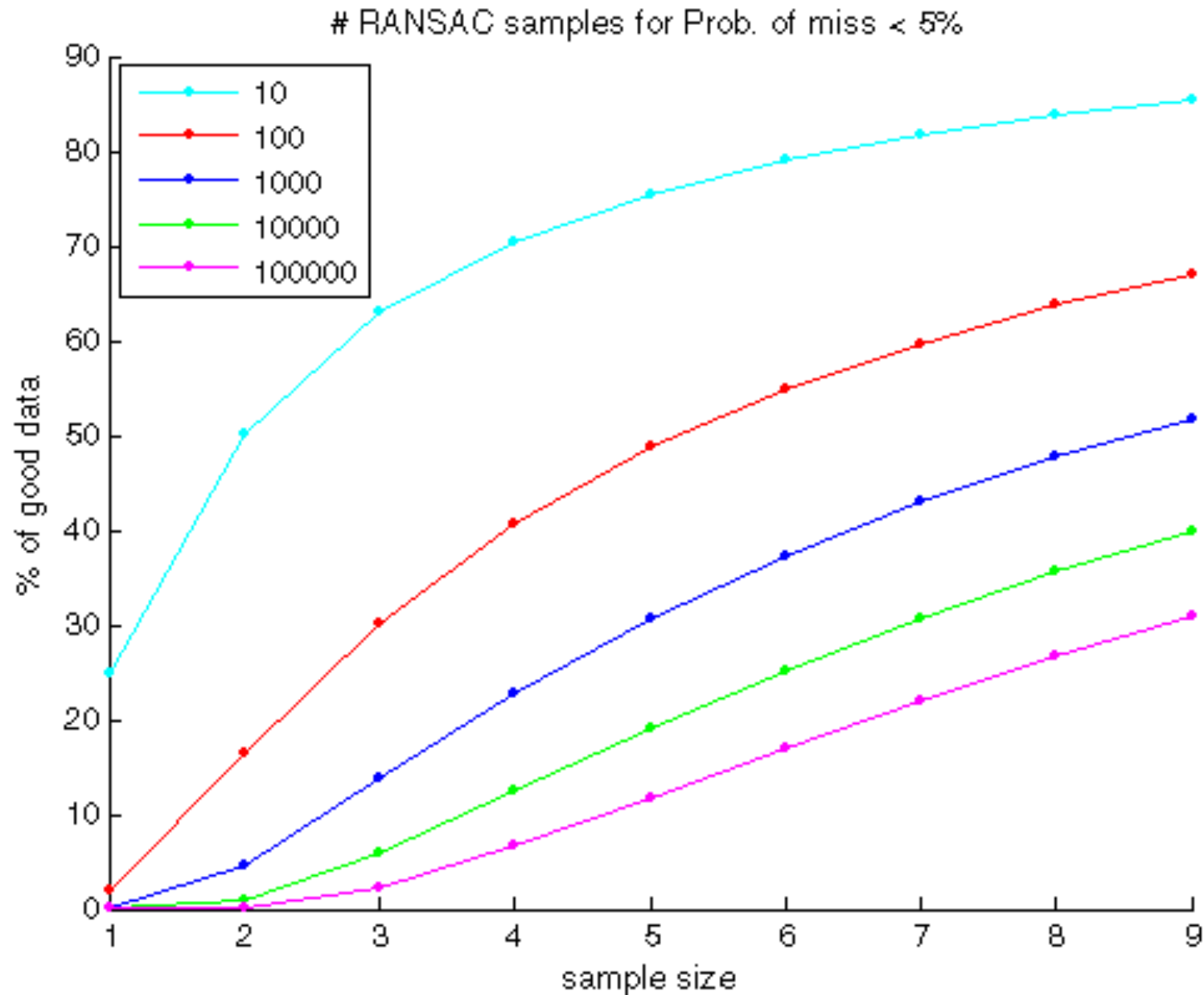


RANSAC (RANDOM SAMPLING CONSENSUS)

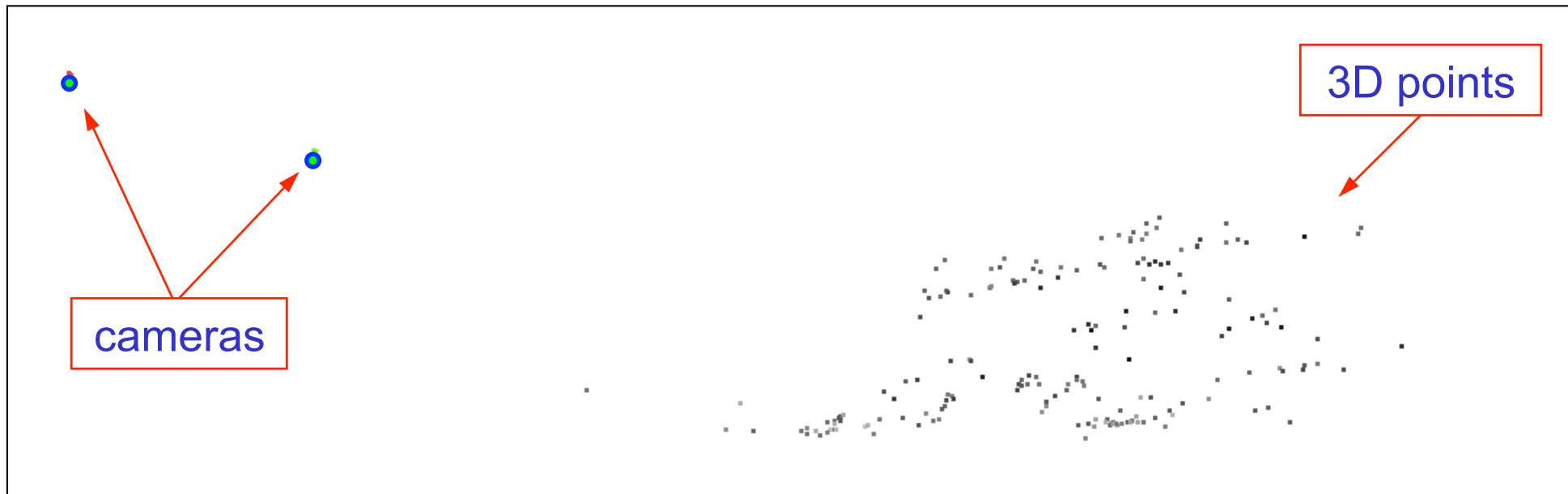
1. Generate random 5-tuples of matches
2. Compute F by solving $\mathbf{x}_2^\top \mathbf{F} \mathbf{x}_1 = 0$ (not so trivial)
3. Count the number of good matches

Return the largest set of good matches

of RANSAC SAMPLES for PROB. of MISS < 5%



SPARSE MATCHES between TWO VIEWS



FISH-EYE IMAGE SEQUENCE



Features
Tracks
Matches



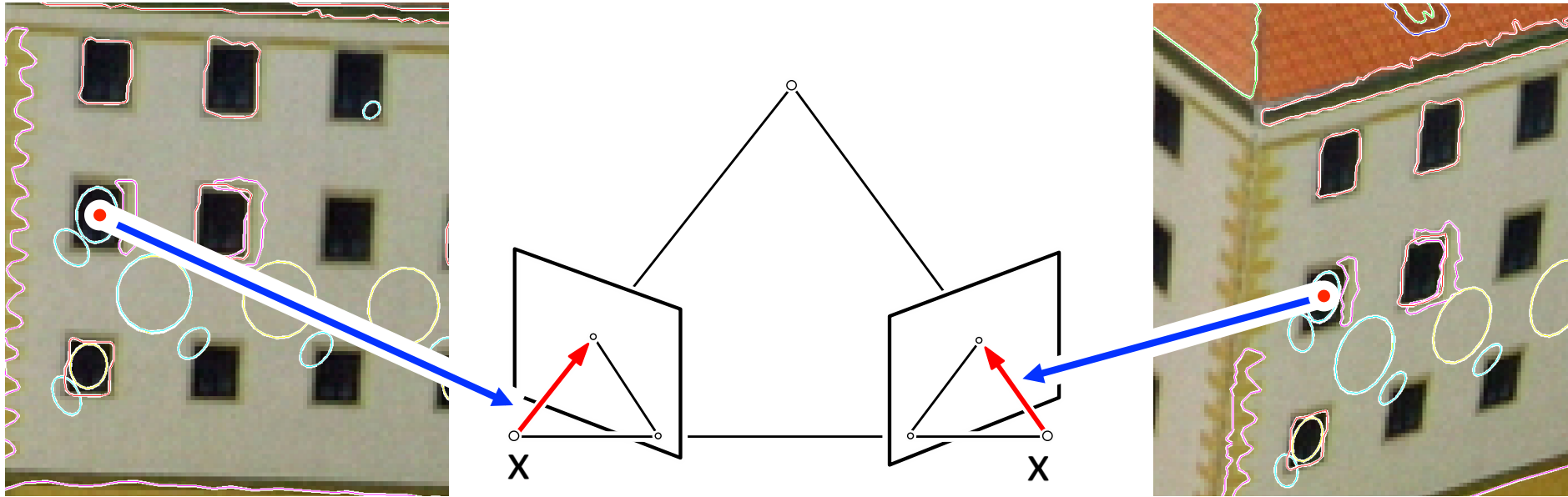
RELATIVE CAMERA POSE

PROBLEM

ESSENTIAL (F) MATRIX

COMPUTATION

FORMULATION



$$\left. \begin{aligned} \mathbf{x}_2^\top \mathbf{F} \mathbf{x}_1 &= 0 \\ \det \mathbf{F} &= 0 \\ 2\mathbf{F}\mathbf{F}^\top \mathbf{F} - \text{trace}(\mathbf{F}\mathbf{F}^\top)\mathbf{F} &= 0 \end{aligned} \right\} \text{Algebraic equations}$$

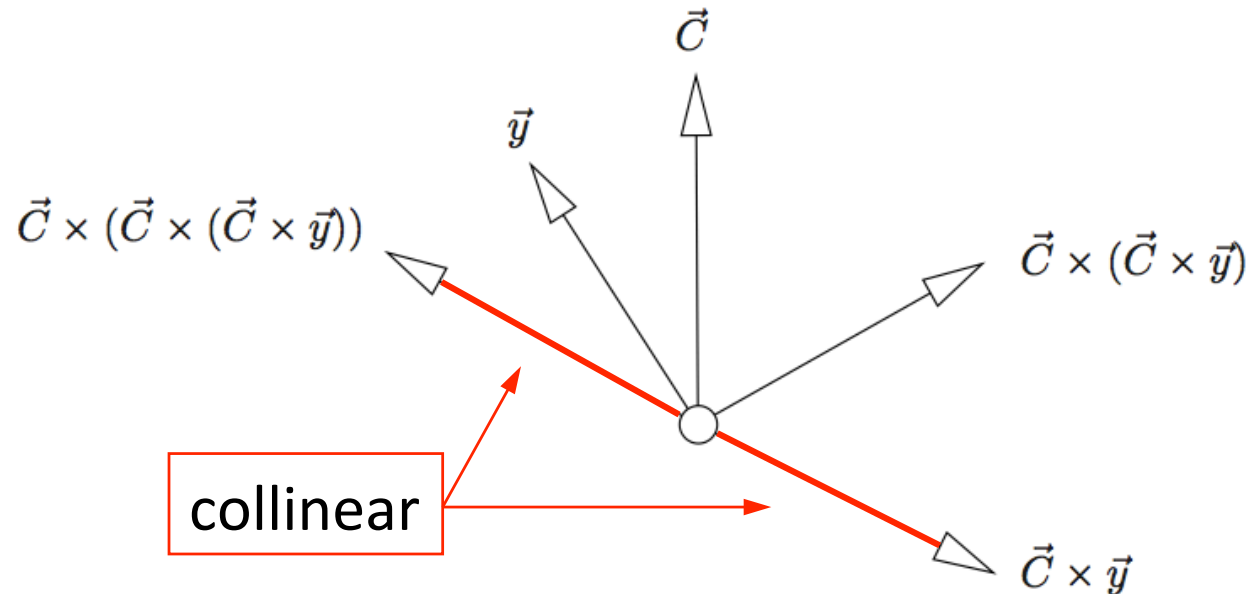
EQUATIONS

$\mathbf{x}_2^\top \mathbf{F} \mathbf{x}_1 = 0$... Epipolar constraint (1 eq per match)


$\det \mathbf{F} = 0$... Fundamental matrix (1 eq)

$2 \mathbf{F} \mathbf{F}^\top \mathbf{F} - \text{trace}(\mathbf{F} \mathbf{F}^\top) \mathbf{F} = 0$... Essential matrix (9 eqns)

$$\mathbf{F} = \mathbf{R} \left[\vec{\mathbf{C}}_{\epsilon_1} \right]_{\times}$$



UNKNOWN S

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$


9 unknowns but only 8 have to be found (F up to scale)

→ we need at least 8 independent equations

EQUATIONS

$$\det F = 0$$

1 equation, degree 3

$$2FF^T F - \text{trace}(FF^T)F = 0$$

9 equations, degree 3

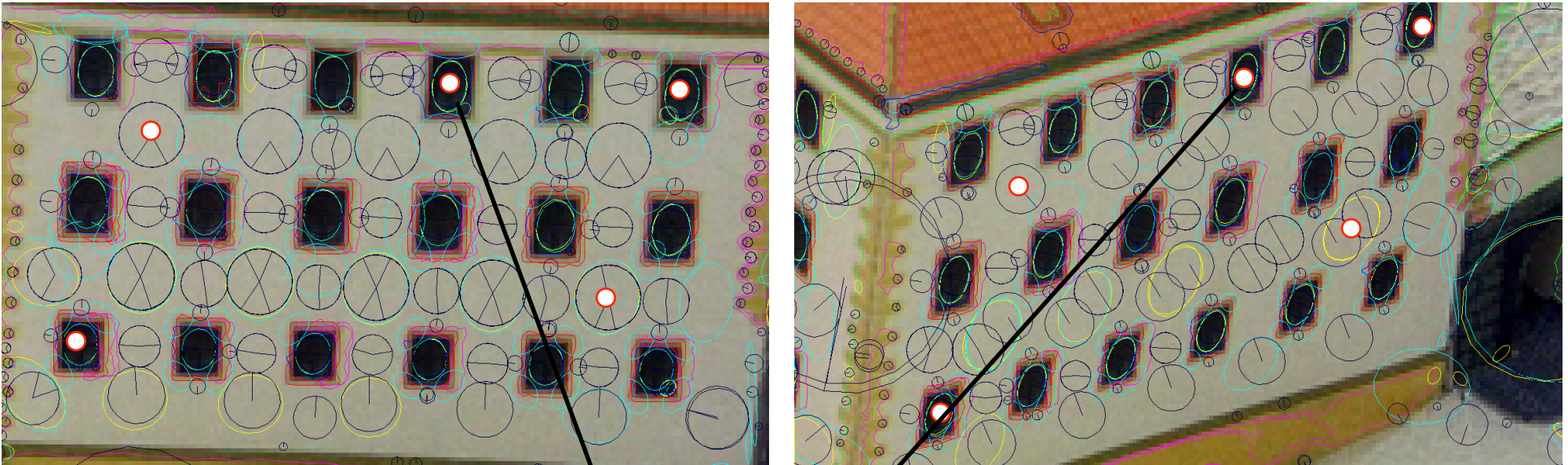


10 equations but only 3 “independent”

$$8 = 3 + 5$$

→ 5 more equations needed

5 EQUATIONS from image points



$$\mathbf{x}_2^T \mathbf{F} \mathbf{x}_1 = 0$$

5 linear equations:

$$\bullet f_{11} + \bullet f_{12} + \bullet f_{13} + \bullet f_{21} + \bullet f_{22} + \bullet f_{23} + \bullet f_{13} + \bullet f_{23} + \bullet f_{31} + \bullet f_{32} + \bullet f_{33} = 0$$

ELIMINATING UNKNOWNNS

5 linear equations:

$$\bullet f_{11} + \bullet f_{12} + \bullet f_{13} + \bullet f_{21} + \bullet f_{22} + \bullet f_{23} + \bullet f_{13} + \bullet f_{23} + \bullet f_{31} + \bullet f_{32} + \bullet f_{33} = 0$$

can be written in a matrix form

$$\underbrace{\begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix}}_A \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = 0$$

ELIMINATING UNKNOWNNS

$$\underbrace{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}}_A \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = 0$$

A 5×9 matrix \rightarrow it has a 4 dimensional nullspace

$$A N = 0$$

$$N = \boxed{x} N_1 + \boxed{y} N_2 + \boxed{z} N_3 + \boxed{w} N_4$$

known

new unknowns

ELIMINATING UNKNOWNNS

$$F \sim x N_1 + y N_2 + z N_3 + w N_4 \quad \dots 4 \text{ unknowns}$$

F is up to scale \rightarrow choose a representative by setting $w = 1$

$$F := x N_1 + y N_2 + z N_3 + N_4$$


3 unknowns x, y, z

 substitute

$$\left. \begin{array}{l} \det F = 0 \\ 2 F F^T F - \text{trace}(F F^T) F = 0 \end{array} \right\} 10 \text{ 3}^{\text{rd}} \text{ order equations in } 3 \text{ unknowns } x, y, z$$

SOLVING IT

$$\left. \begin{array}{l} \det F = 0 \\ 2FF^T F - \text{trace}(FF^T)F = 0 \end{array} \right\} \begin{array}{l} 10 \text{ 3}^{\text{rd}} \text{ order equations in} \\ 3 \text{ unknowns } (x, y, z) \end{array}$$

?



How?



ESSENTIAL (F) MATRIX

COMPUTATION

THE FASTEST (available) SOLUTION

Idea (1567 μ s)

Z. Kukelova, M. Bujnak, T. Pajdla

Polynomial eigenvalue solution to the 5-pt and 6-pt relative pose problems

BMVC 2008

Efficient implementation (34 μ s)

H. Li, R. Hartley.

An Efficient Hidden Variable Approach to Minimal-Case Camera Motion Estimation

PAMI 2012

1 UNKNOWN → EIGENVALUES

1 equation, 1 variable → companion matrix → eigenvalues

$$f(x) = x^3 + 4x^2 + x - 6 = -6 + 1x + 4x^2 + 1x^3$$

$$M_x = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 0 & -1 \\ 0 & 1 & -4 \end{bmatrix}$$

... a simple rule

>> e=eig(M_x)

$$e = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$$

$$x_1 = 1, x_2 = -2, x_3 = -3$$

EIGENVALUES SOLVE ALG EQNS

$$\gg \lambda = \text{eig}\left(\begin{bmatrix} 0 & 0 & 6 \\ 1 & 0 & -1 \\ 0 & 1 & -4 \end{bmatrix}\right)$$

Numerical implementation

$$\begin{bmatrix} -\lambda & 0 & 6 \\ 1 & -\lambda & -1 \\ 0 & 1 & -4 - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

“Hide” λ

$$6x_3 - \lambda x_1 = 0$$

$$x_1 - x_3 - \lambda x_2 = 0$$

$$x_2 - 4x_3 - \lambda x_3 = 0$$



linear bilinear (linear in λ)

Eigenvalues solve a special system of algebraic eqns via the hidden variable method

POLYNOMIAL EIGENVALUE SOLUTION


m equations, n variables

$$f_1(x, y) = 25xy - 15x - 20y + 12$$

$$f_2(x, y) = x^2 + y^2 - 1$$

“Hide” y (the “hidden variable method”)

$$\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} y^2 + \begin{bmatrix} 0 & 25 & 20 \\ 0 & 0 & 0 \end{bmatrix} y + \begin{bmatrix} 0 & -15 & 12 \\ 1 & 0 & -1 \end{bmatrix} \right) \begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2 × 3 matrix → ∞ sols !!! → 

POLYNOMIAL EIGENVALUE SOLUTION

$$f_1(x, y) = 25xy - 15x - 20y + 12$$

$$f_2(x, y) = x^2 + y^2 - 1$$

“Hide” y (the “hidden variable method”)

$$\left(\begin{array}{c|c|c|c} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} & y^2 & + & \begin{bmatrix} 0 & 25 & -20 \\ 0 & 0 & 0 \\ 25 & -20 & 0 \end{bmatrix} & y & + & \begin{bmatrix} 0 & -15 & 12 \\ 1 & 0 & -1 \\ -15 & 12 & 0 \end{bmatrix} \end{array} \right) \begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \text{add } x f_1(x, y) = 25x^2y - 15x^2 - 20xy + 12x$$

C_2

C_1

C_0

```
>> [v, y] = polyeig(C0, C1, C2)
```


POLYNOMIAL EIGENVALUE PROBLEM

Quadratic eigenvalue problem

$$(\lambda^2 A + \lambda B + C) \mathbf{x} = 0$$

... can be rewritten as

$$\begin{aligned} \lambda^2 A \mathbf{x} + \lambda B \mathbf{x} + C \mathbf{x} &= 0 \\ \lambda A (\lambda \mathbf{x}) + \lambda B \mathbf{x} + C \mathbf{x} &= 0 \\ \lambda A \mathbf{y} + \lambda B \mathbf{x} + C \mathbf{x} &= 0 \end{aligned}$$

$\mathbf{y} = \lambda \mathbf{x}$

$$\left(\lambda \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} + \begin{bmatrix} 0 & C \\ -I & 0 \end{bmatrix} \right) \begin{bmatrix} \mathbf{y} \\ \mathbf{x} \end{bmatrix} = 0$$

Generalized eigenvalue problem

... generalization of

$$M \mathbf{x} = \lambda \mathbf{x}$$



$$(M - \lambda I) \mathbf{x} = 0$$

Higher order PolyEigs:

A. Wallack, I. Z. Emiris, D. Manocha.

MARS - A Maple-Matlab-C

Resultant-based Solver.

6-pt UNKNOWN f CAMERA RELATIVE POSE

Epipolar constraint $\mathbf{x}'_j{}^\top \mathbf{F} \mathbf{x}_j = 0, j = 1, \dots, 6$

9 unknowns – 6 correspondences = 3 dimensional nullspace

$$\mathbf{F} = x \mathbf{F}_1 + y \mathbf{F}_2 + \mathbf{F}_3$$

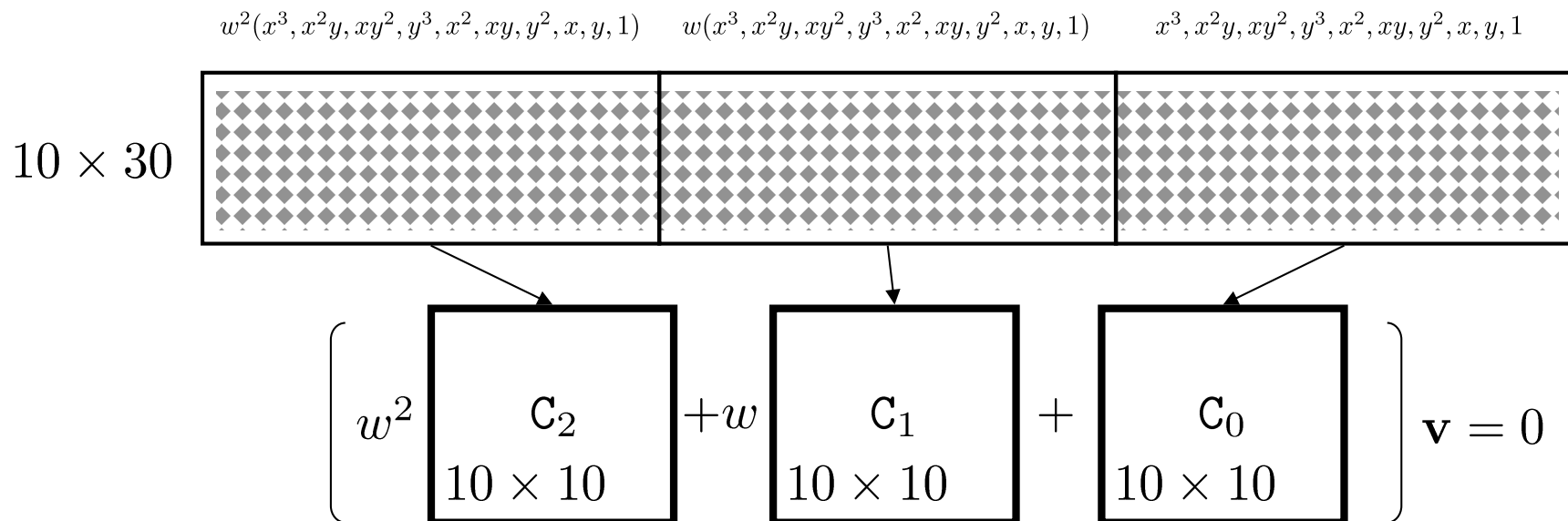
Fundamental matrix is singular $\det(\mathbf{F}) = 0$

Transfer \mathbf{F} to the essential matrix $\mathbf{E} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{F} \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Constraints on \mathbf{E} $2 \mathbf{E} \mathbf{E}^\top \mathbf{E} - \text{trace}(\mathbf{E} \mathbf{E}^\top) \mathbf{E} = 0$

6-pt UNKNOWN f CAMERA RELATIVE POSE

- 10 x 3rd and 5th order polynomial equations in 3 variables x, y and $w = f^{-2}$ with 30 monomials



- Quadratic polynomial eigenvalue problem (in MATLAB)

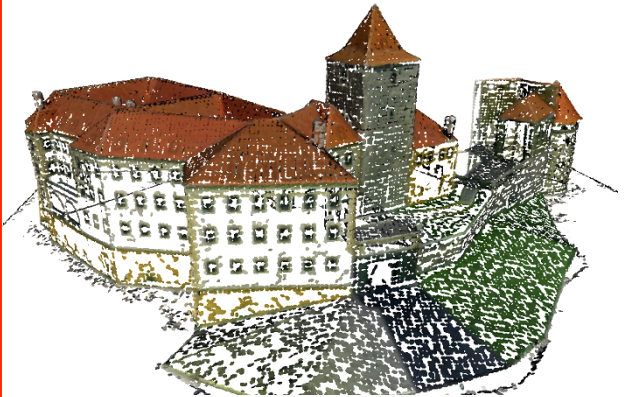
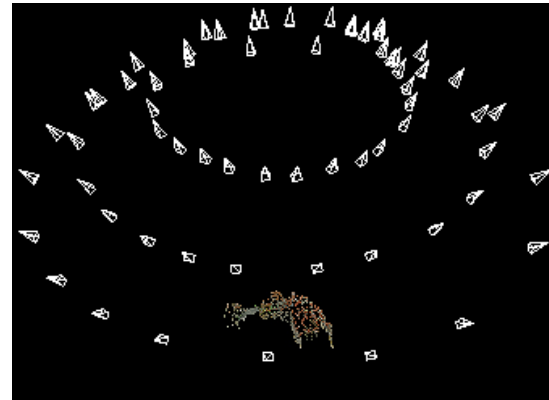
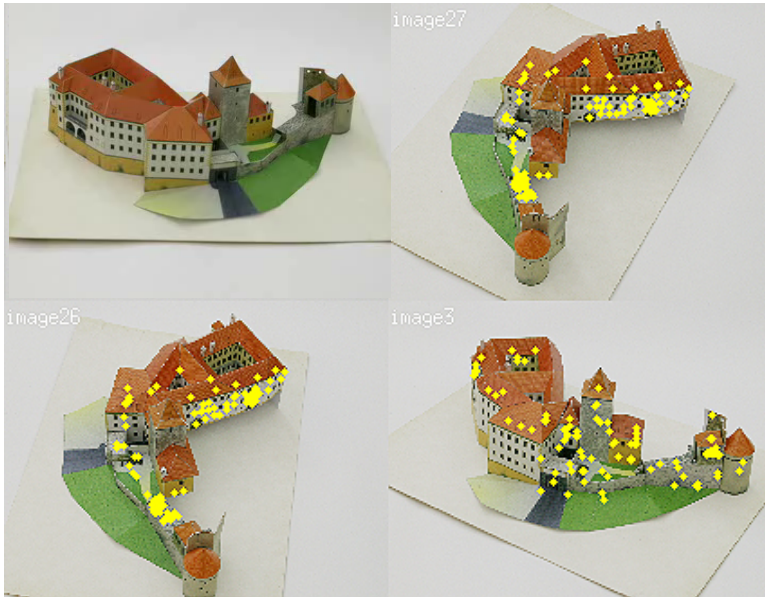
$$[v, w] = \text{polyeig}(\mathbf{C}_0, \mathbf{C}_1, \mathbf{C}_2)$$

3D Reconstruction from Photographs

1. SFM: Images & EXIFs

→ Cameras poses

→ Sparse point cloud



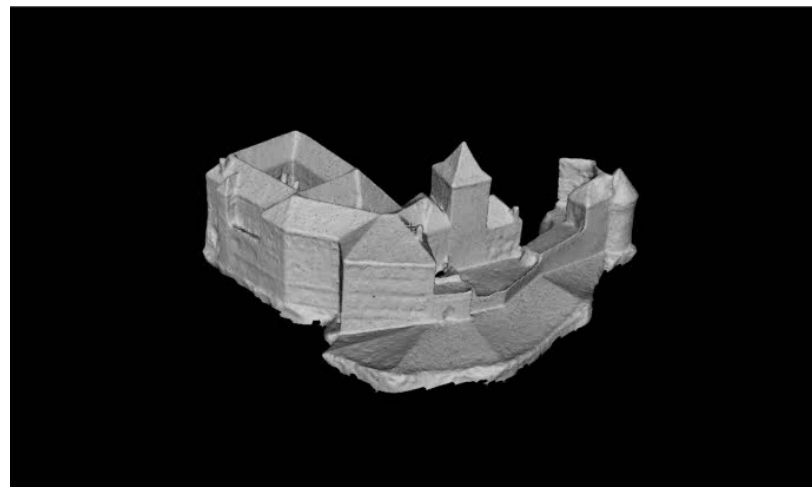
2. MVS:

→

3D Surface

→

Texture





FROM POINT CLOUDS TO SURFACES

MVS – MULTIVIEW STEREO

1. Making a point cloud



Plane sweeping

- Known camera poses and calibration (from SFM)
- Corrected perspective images
- Camera neighborhood graph by intersecting view cones
- Sweeping planes with multiple orientations
- Cross correlation (GPU)

MVS – MULTIVIEW STEREO

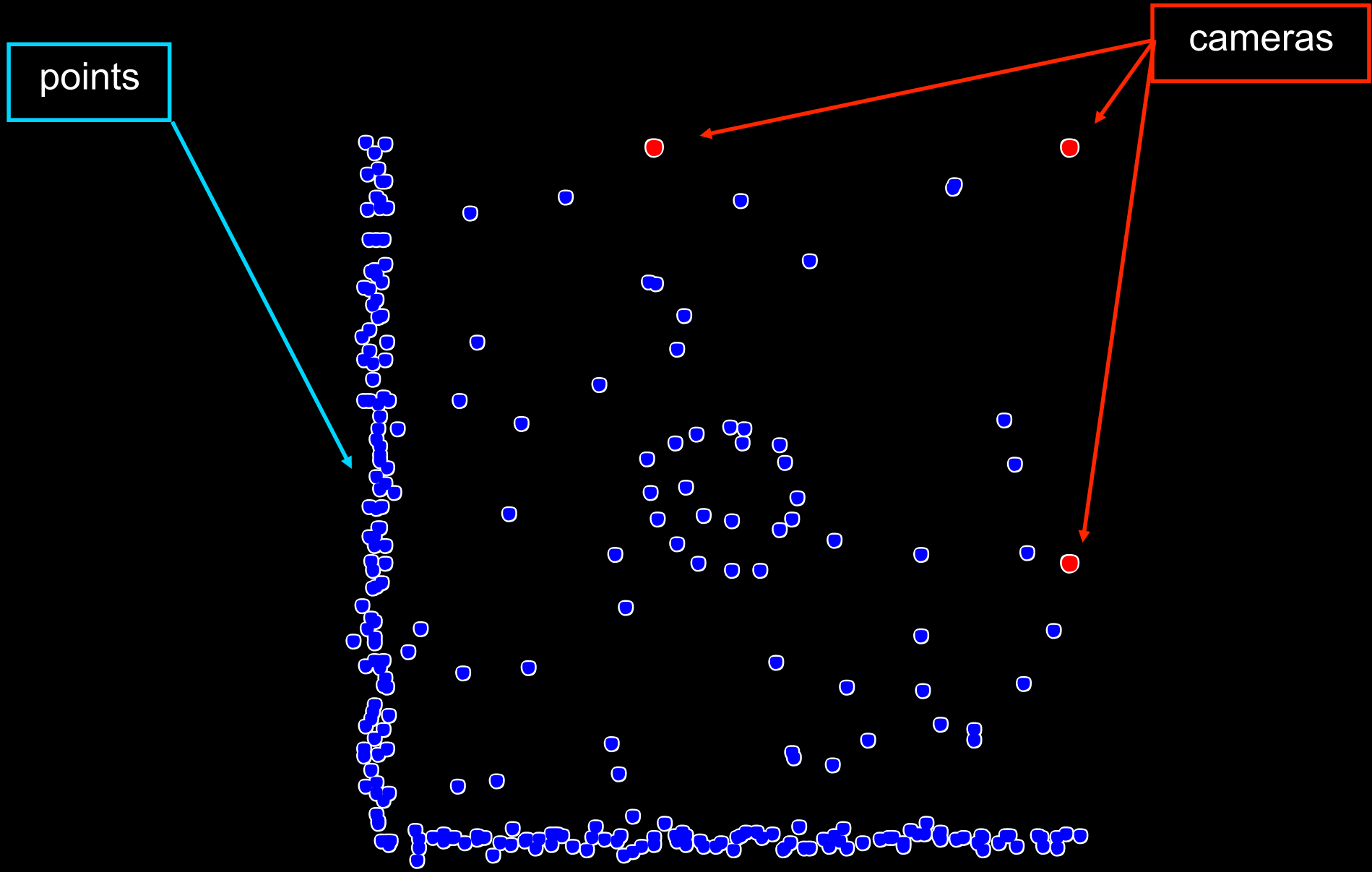
2. Surface from a point cloud



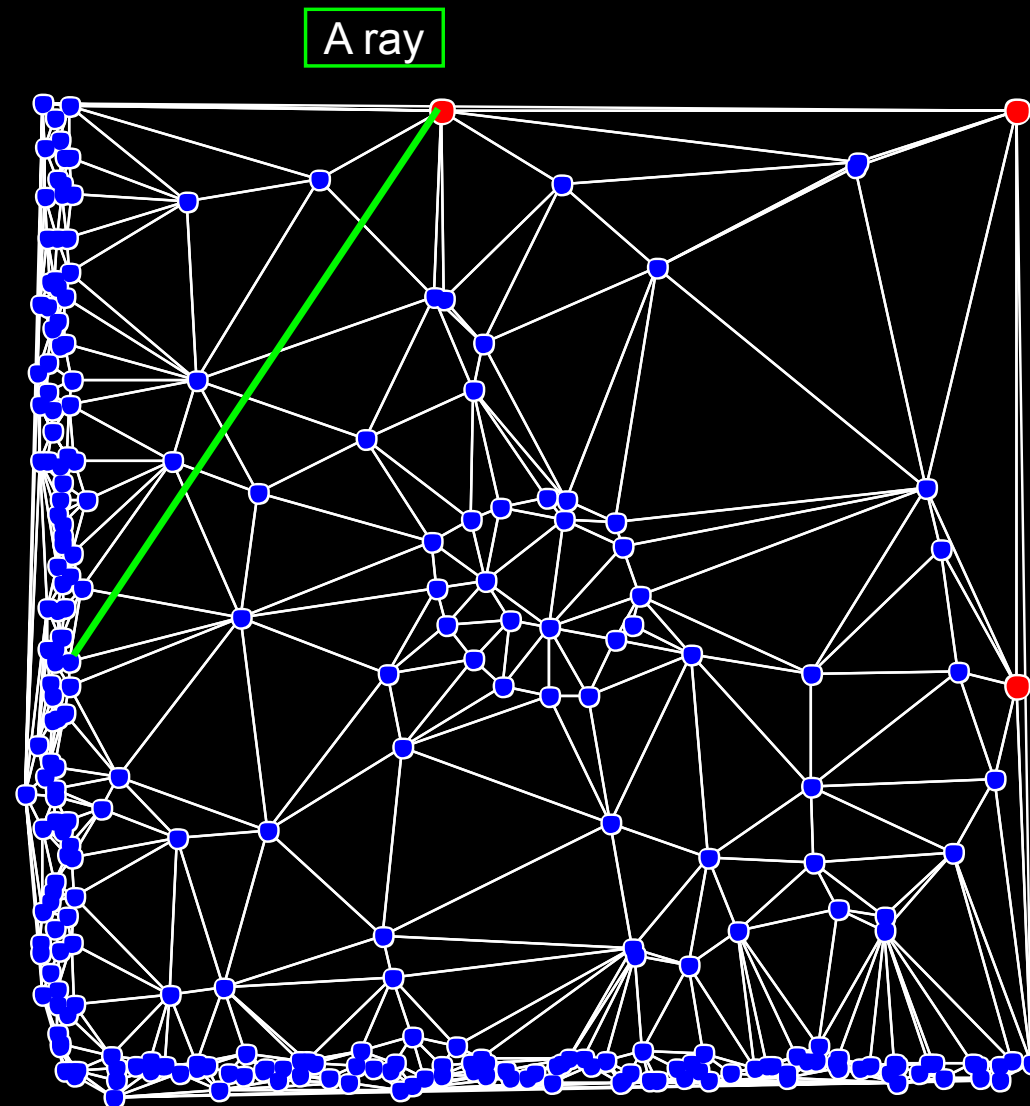
Surface by st-cut of a graph

- Point clustering reduces the number of points
- Based on visibility, i.e. does not use images (which makes it faster)
- Semi-global = space partitioned into boxes, which are solved separately but visibility constraints are shared between the neighboring boxes
- can solve 1k (2-3 MPix) images in an hour (Capturing Reality impl.)

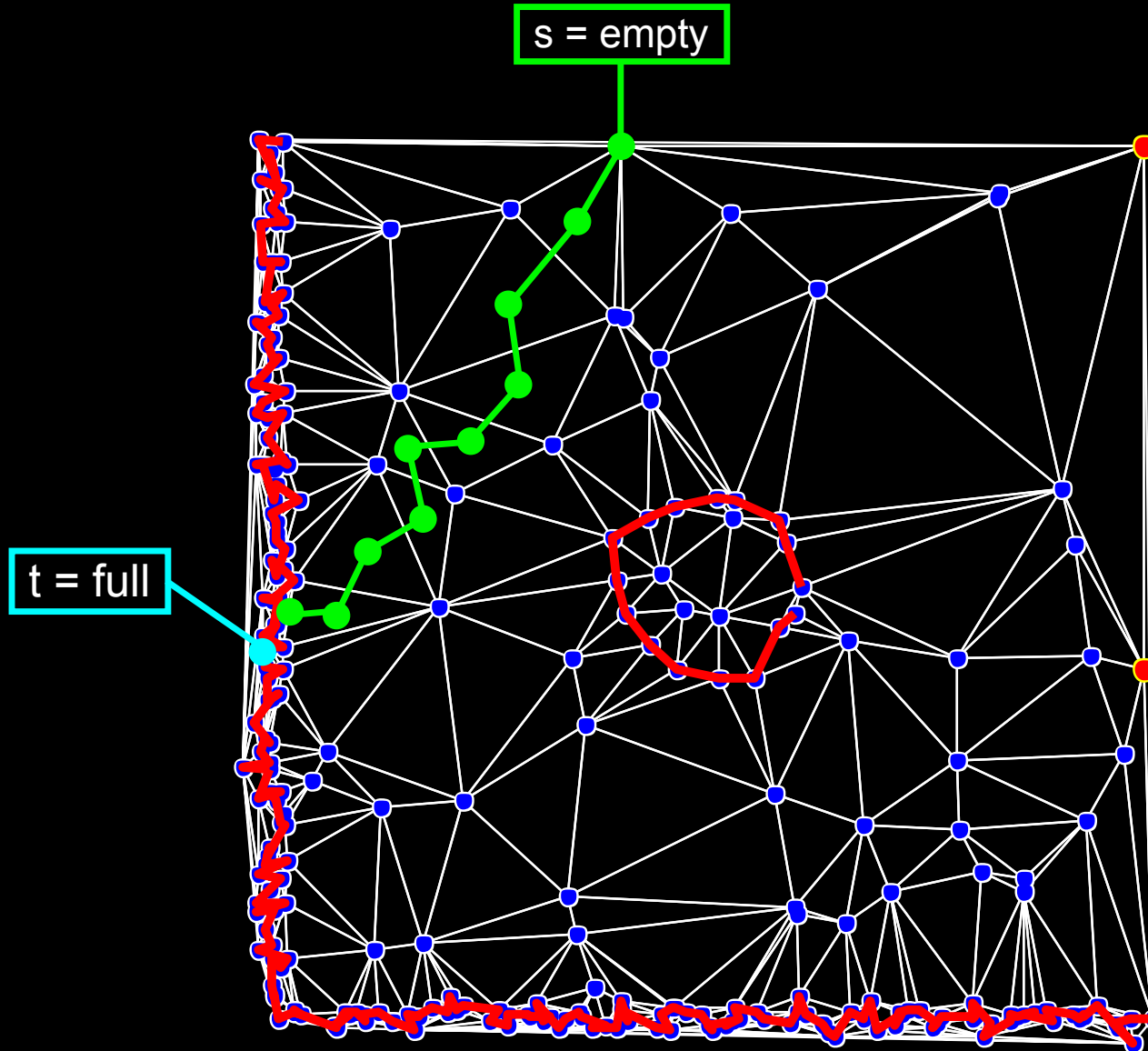
Cameras from SFM and points from plane sweeping



Weighting the dual graph (Delaunay tetrahedralization)



Surface = **s-t cut** through the dual graph



MVS – MULTIVIEW STEREO

2. Surface from a point cloud

P. Labatut, J.-P. Pons, R. Keriven. (→ Accute3D)

Efficient Multi-View Reconstruction of Large-Scale Scenes using Interest Points, Delaunay Triangulation and Graph Cuts.

ICCV 2007

M. Jancosek, T. Pajdla. (CMPMVS → Capturing Reality)

Multi-View Reconstruction Preserving Weakly-Supported Surfaces.

CVPR 2011

[Binary available for CMPMVS](#)

<http://ptak.felk.cvut.cz/sfmservice/websfm.pl?menu=cmpmvs>



Applications of 3D RECONSTRUCTION

Miraikan -- National Museum of Emerging Science and Innovation (Nippon Kaikan) | IAPR MVA2015 | Venue

https://www.google.com/maps/place/Miraikan+--+National+Museum+of+Emerging+Science+and+Innovation+(Nippon+Ka...)

Miraikan -- National Museum of Emerging Science and Innovation

Miraikan -- National Museum of Emerging Science and Innovation +81 3-3570-9151

4.3 ★★★★★ 96 reviews

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Museum of Maritime Science 船の科学館

南極観測船宗谷

Tokyo Wangan Police Station 東京湾岸警察署

青海客船ターミナル

theSOHO

the Canteen

合同庁舎

日本学生支援機構 (独立行政法人) 東京国際交流館

国際研究交流大学 村東京国際交流館

フジテレビジョン

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東京電力(株)青島支所

海上保安庁海洋

Google Earth

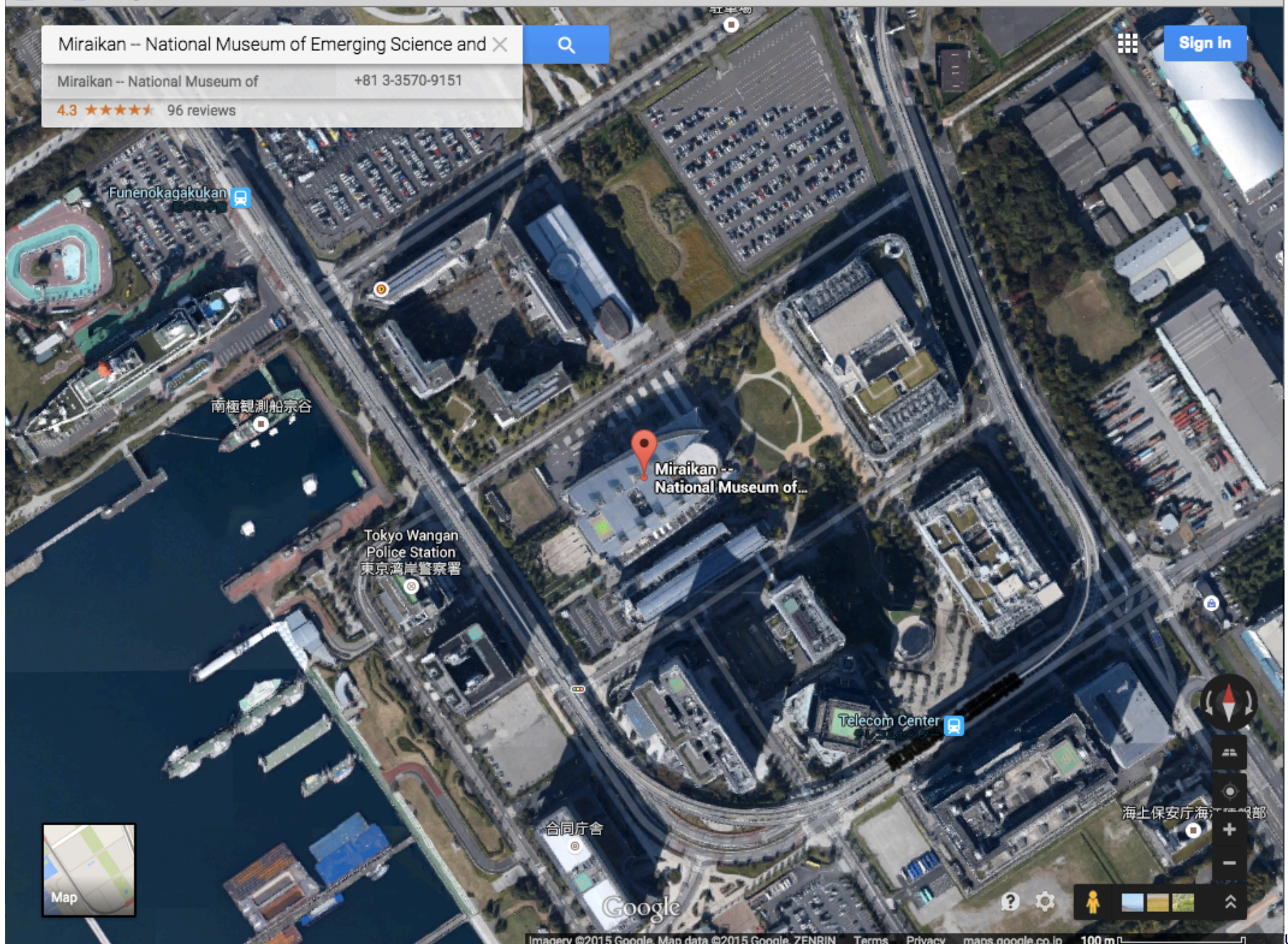
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4.3 ★★★★★ 96 reviews

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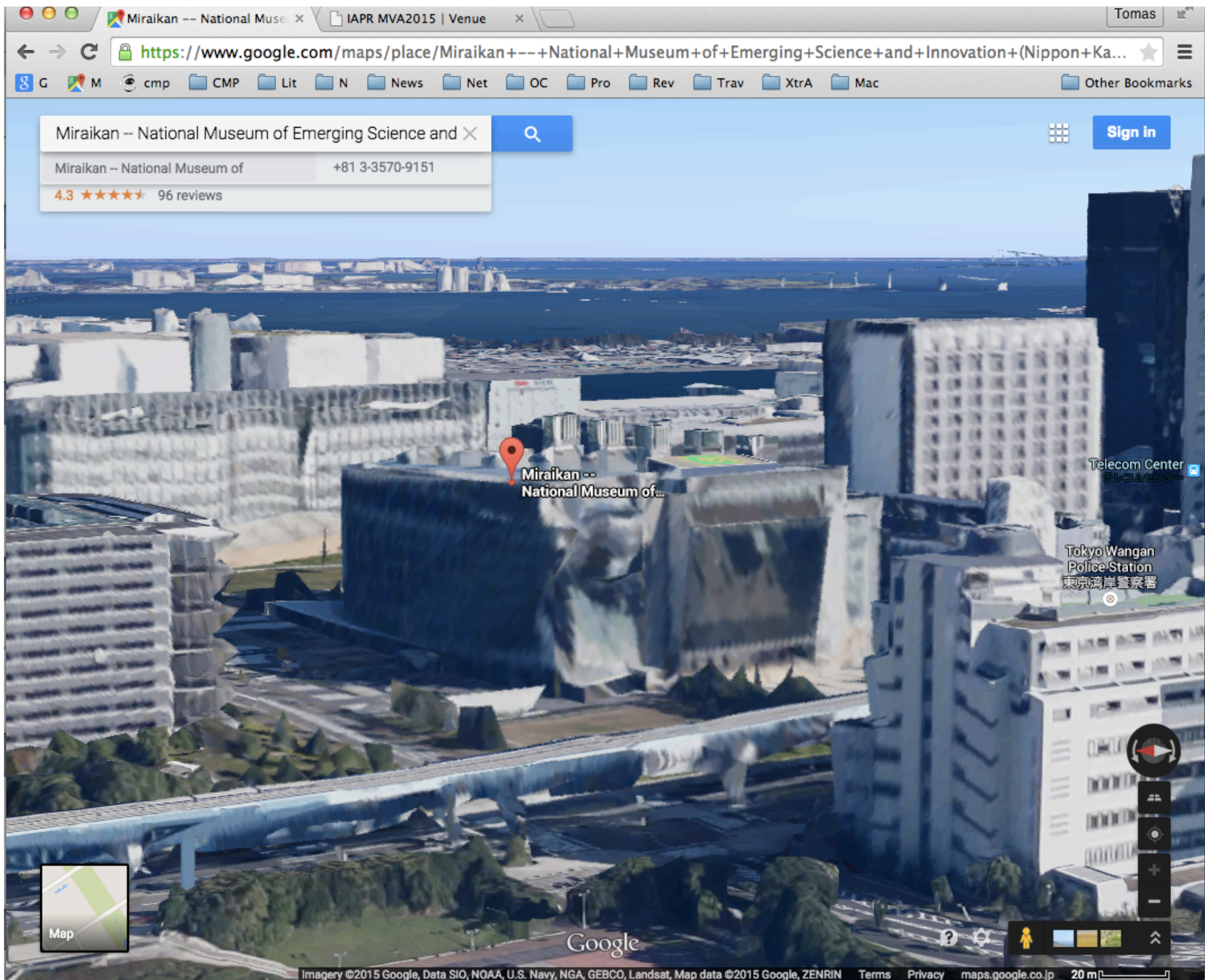


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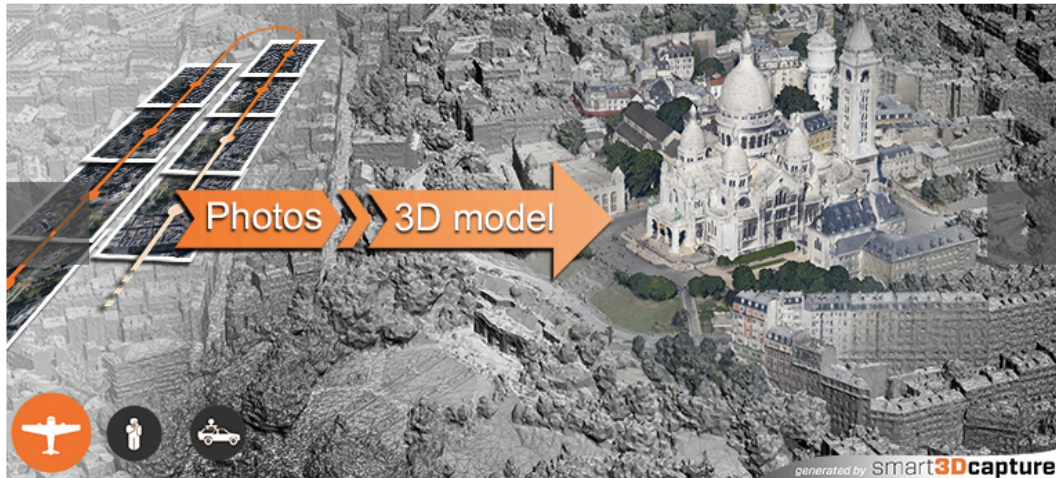






Bentley Systems has acquired Acute3D
 Read the official press release

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News



BENTLEY SYSTEMS HAS ACQUIRED ACUTE3D
 February 10th, 2015



SMART3DCAPTURE V3.2 IS RELEASED
 December 23rd, 2014



TECHNIDRONE TO TRAIN ALL PILOTS ON SMART3DCAPTURE
 November 28th, 2014



ACUTE3D PARTNERSHIP WITH ALTIGATOR
 November 28th, 2014



ACUTE3D RECRUITS
 October 22nd, 2014



Founded 2011
 Renaud Keriven (co-founder)
 Formerly at IMAGINE Lab
 École des Ponts ParisTech



Jean-Philippe Pons (co-founder)
 Formerly at Centre Sci. et Tech. du
 Bâtiment in Sophia Antipolis



Accuracy + Efficiency



Founded 2011, Spin-off of EPFL Lausanne
Christoph Strecha (CEO, co-founder)
Formerly EPFL Lausanne

WE ARE HAPPY TO INTRODUCE REALITYCAPTURE

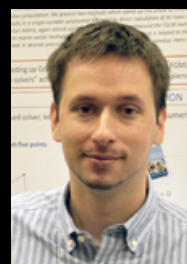
The state-of-the-art software which automatically extracts beautiful and accurate 3D models from a set of ordinary images and / or laser-scans.



Detailed 3D model of more than 19 million triangles reconstructed automatically from 263x 21MPx images in less than two hours.
Photography: Pato Safko



Founded 2014
Michal Jancosek (co-founder)
Formerly at CTU in Prague
PhD student of T. Pajdla



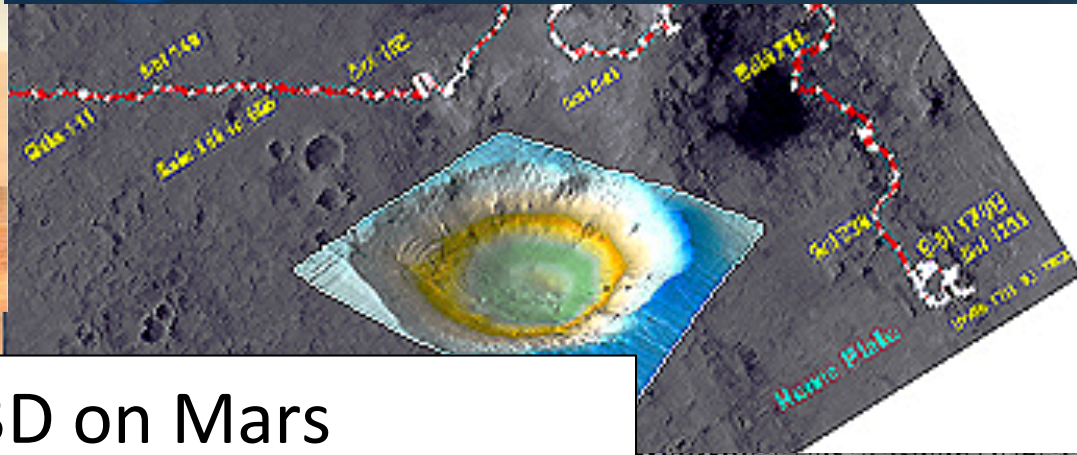
Martin Bujnak (co-founder)
Formerly at CTU in Prague
PhD student of T. Pajdla



PROVISG



Jet Propulsion Laboratory
California Institute of Technology



3D on Mars
CVVT Workshop @ CVPR 2015

