Evolutionary Multi-Objective Optimization (EMO)

Fundamentals, State-of-the-art Methodologies, and Future Challenges

Kalyanmoy Deb

Deva Raj Chair Professor

Indian Institute of Technology Kanpur

Kanpur, PIN 208016, INDIA

deb@iitk.ac.in

http://www.iitk.ac.in/kangal/deb.htm

Currently a Finnish Distinguished Professor at Helsinki School of Economics

Overview of Tutorial

- Part A: Introduction to EMO
  - Introduction to multi-objective optimization
  - Main classical methods
  - Philosophy of evolutionary methods
  - Early non-elitist EMO methods
  - Efficient elitist EMO methods
- Part B: Applications of EMO
  - Decision-making
  - *Innovization*: Innovation through EMO
  - Aiding in other problem-solving tasks
- Part C: Advanced EMO and future challenges
- Conclusions

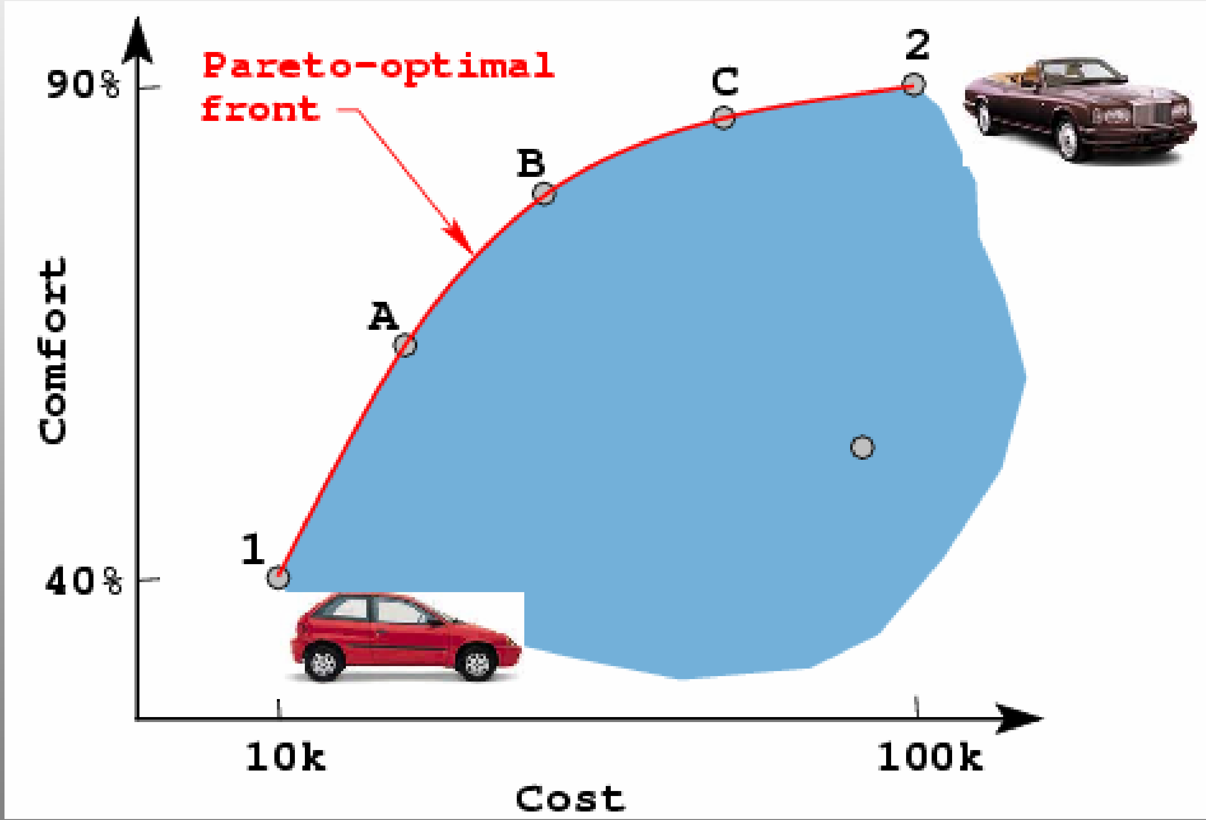
EMO Books (Since 2001)

Part A: Introduction to EMO

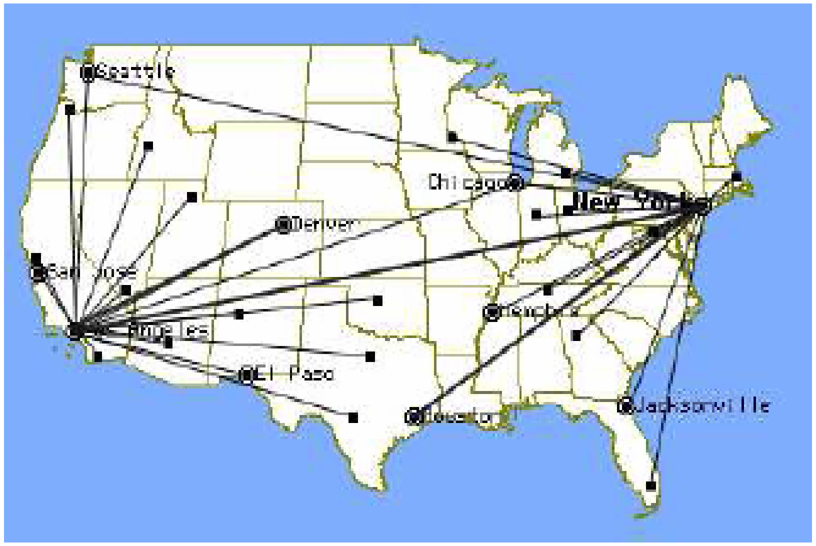
- Multi-objective optimization
- Definitions and theory
- Classical methods
- Difficulties with classical methods
- Early EMO methodologies
- State-of-the-art EMO
- Constraint handling in EMO

Multi-Objective Optimization

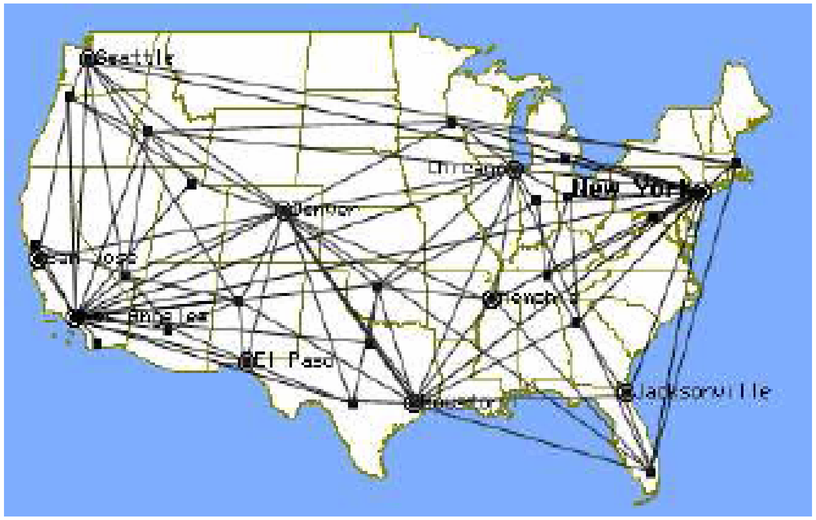
- Really need no introduction



More Examples



A cheaper but inconvenient flight



A convenient but expensive flight

As a Mathematical Programming Problem

- Multiple objectives, constraints, and variables

$$
\begin{aligned}
\text{Min/Max} \quad & (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})) \\
\text{Subject to} \quad & g_j(\mathbf{x}) \geq 0 \\
& h_k(\mathbf{x}) = 0 \\
& \mathbf{x}^{(L)} \leq \mathbf{x} \leq \mathbf{x}^{(U)}
\end{aligned}
$$

Optimality Condition

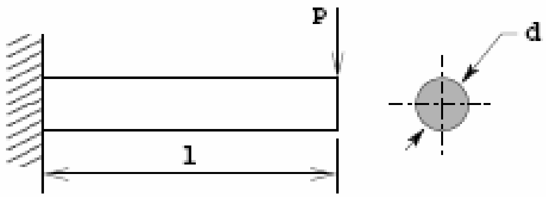
Fritz-John Necessary Condition:

Solution x^* satisfy

1. $\sum_{m=1}^{M} \lambda_m \nabla f_m(x^*) - \sum_{j=1}^{J} u_j \nabla g_j(x^*) = 0,$ and
2. $u_j g_j(x^*) = 0$ for all $j = 1, 2, 3, \ldots, J$
3. $u_j \geq 0$, $\lambda_j \geq 0$, for all j and $\lambda_j > 0$ for at least one j

- u_j's are Lagrange multipliers
- A necessary condition
- To use above conditions requires differentiable objectives and constraints

An Engineering Example



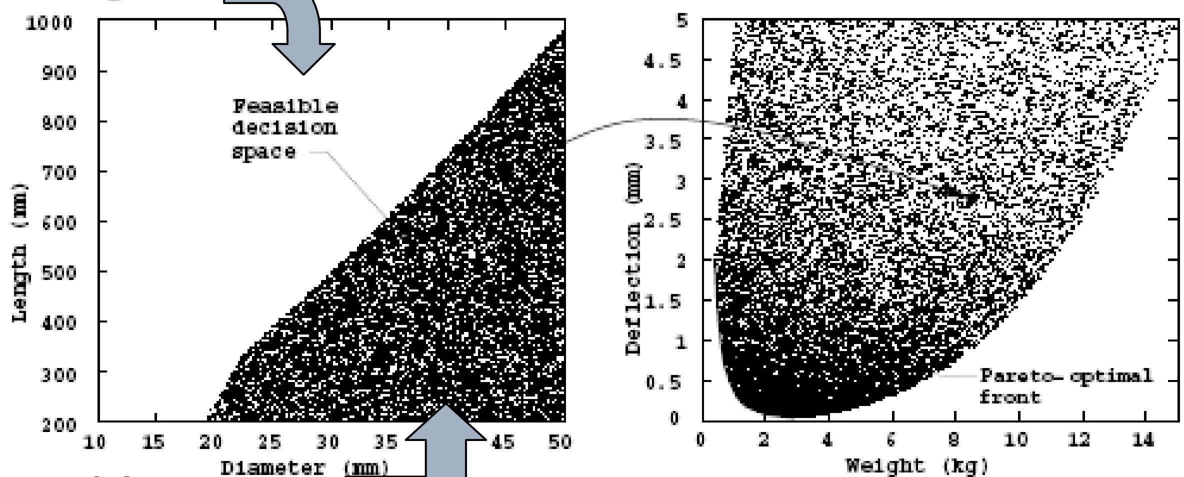
$$\text{Minimize} \quad f_1(d,l) = \rho \frac{\pi d^2}{4} l$$

$$\text{Minimize} \quad f_2(d,l) = \delta = \frac{64 P l^3}{3 E \pi d^4}$$

$$\text{subject to} \quad \sigma_{\max} \leq S_y$$

$$\delta \leq \delta_{\max}$$

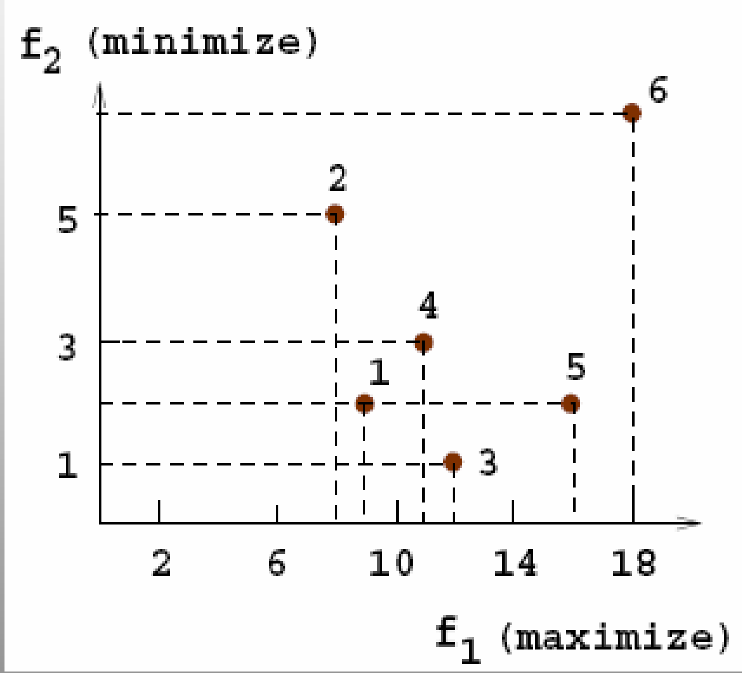
Infesaible region



Feasible region

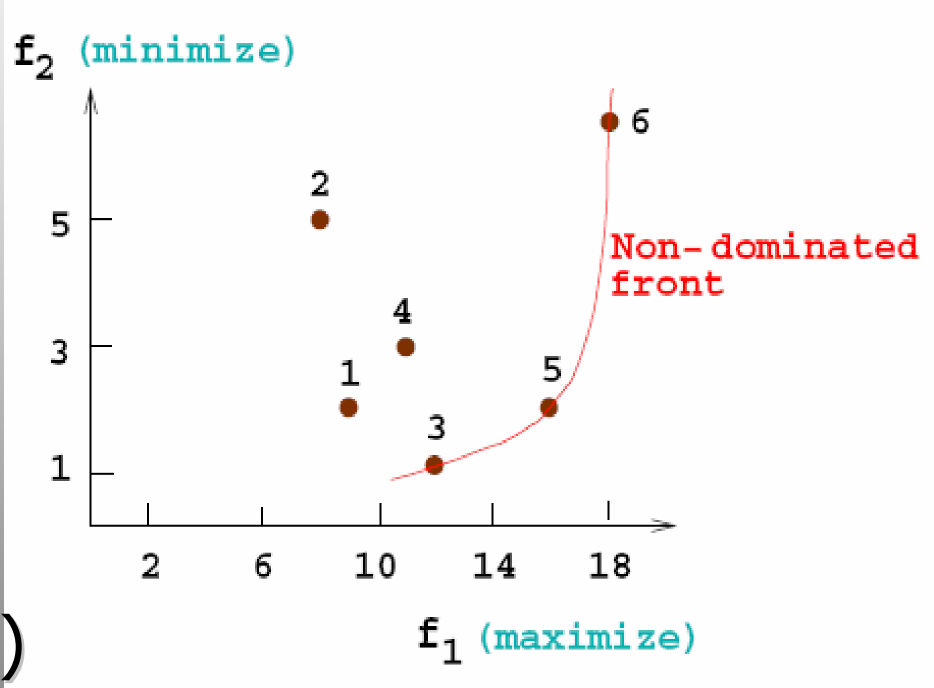
Which Solutions are Optimal?

- Relates to the concept of domination
- $x^{(1)}$ dominates $x^{(2)}$, if
  - $x^{(1)}$ is no worse than $x^{(2)}$ in all objectives
  - $x^{(1)}$ is strictly better than $x^{(2)}$ in at least one objective
- Examples:
  - 3 dominates 2
  - 3 does not dominate 5



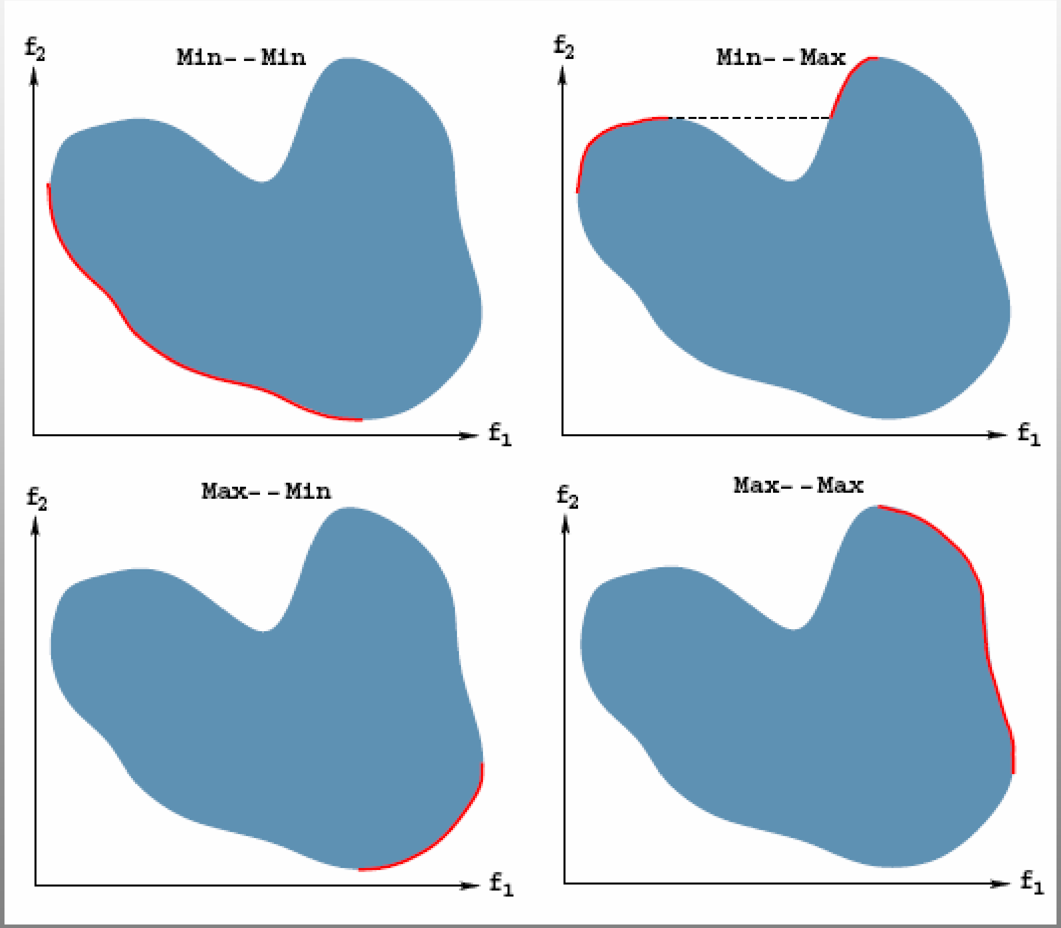
Pareto-Optimal Solutions

- P'=Non-dominated(P)
  - Solutions which are not dominated by any member of the set P
- O(N log N) algorithms exist
- Pareto-Optimal set = Non-dominated(S)
- A number of solutions are optimal



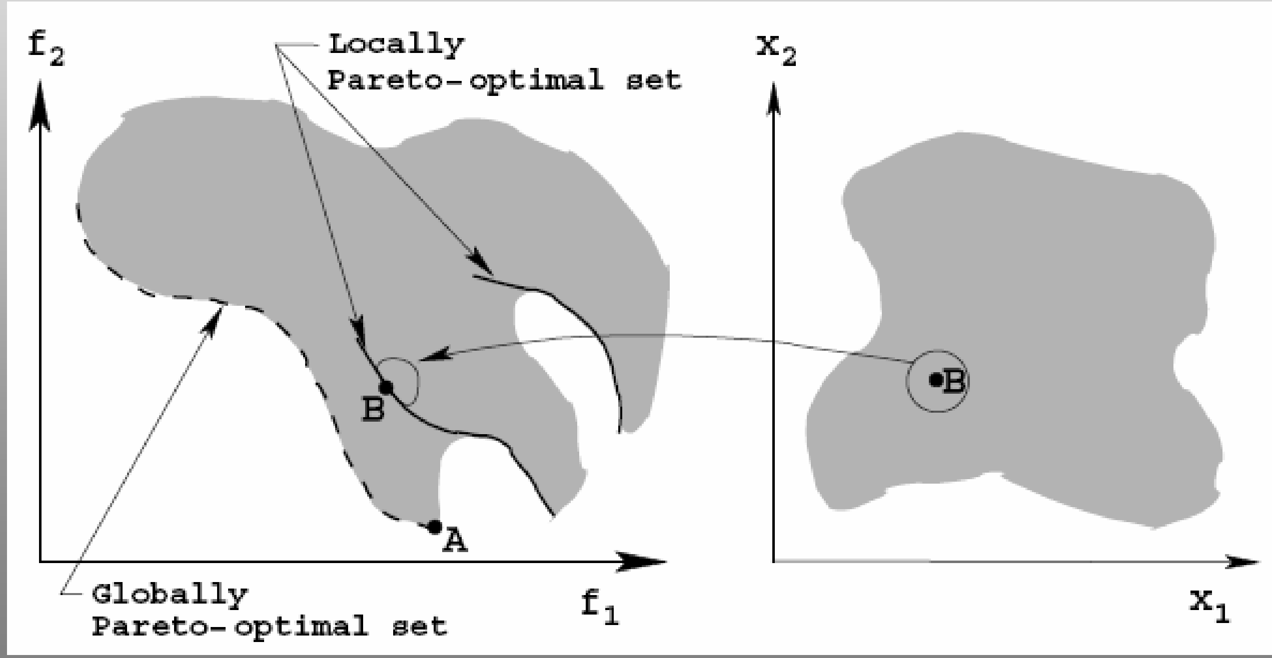
Pareto-Optimal Fronts

- Depends on the type of objectives
- Always on the boundary of feasible region
- Higher dimensional Pareto-optimal front with more objectives



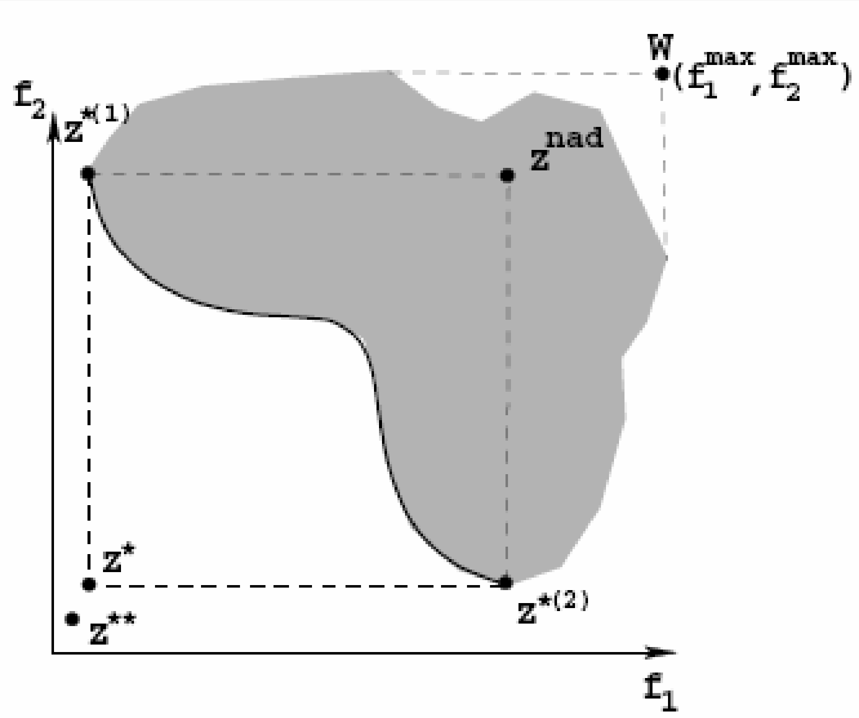
Local Versus Global Pareto-Optimal Fronts

Local Pareto-optimal Front: Domination check is restricted within a neighborhood (in decision space) of P



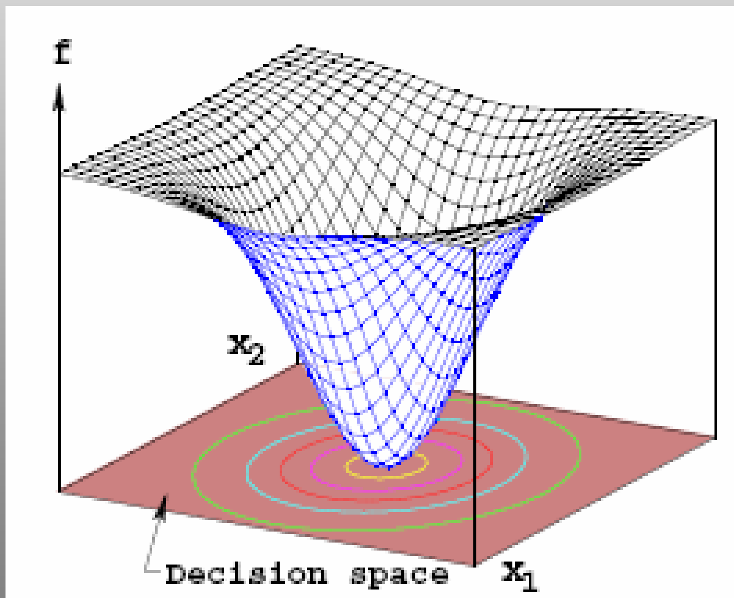
Some Terminologies

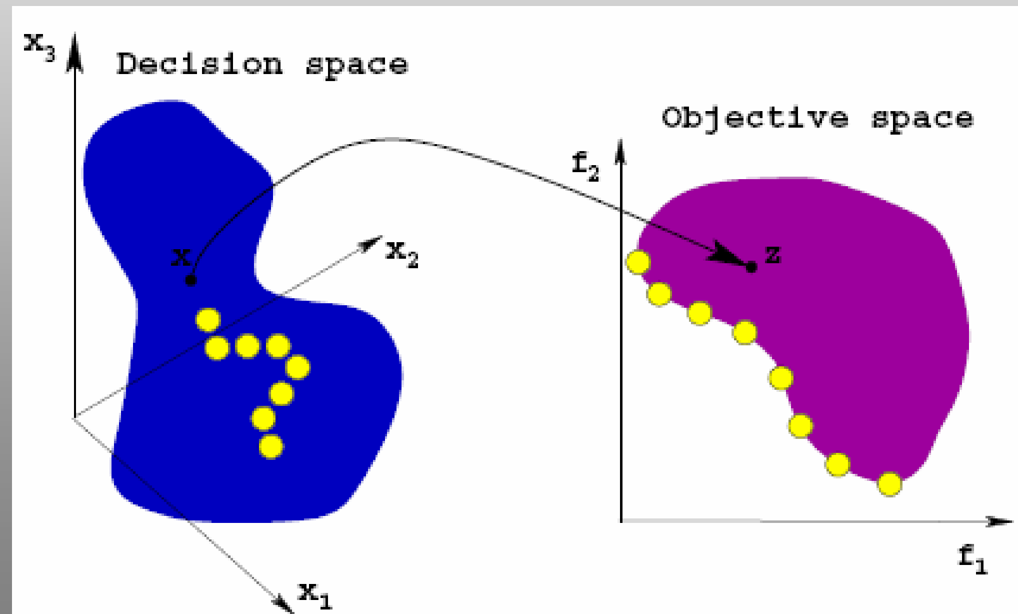
- Ideal point (z^*)
  - nonexistent, lower bound on Pareto-optimal set
- Utopian point (z^{**})
  - nonexistent
- Nadir point (z^{nad})
  - Upper bound on Pareto-optimal set
- Normalization: $$f_i^{\text{norm}} = \frac{f_i - z_i^*}{z_i^{\text{nad}} - z_i^*}$$



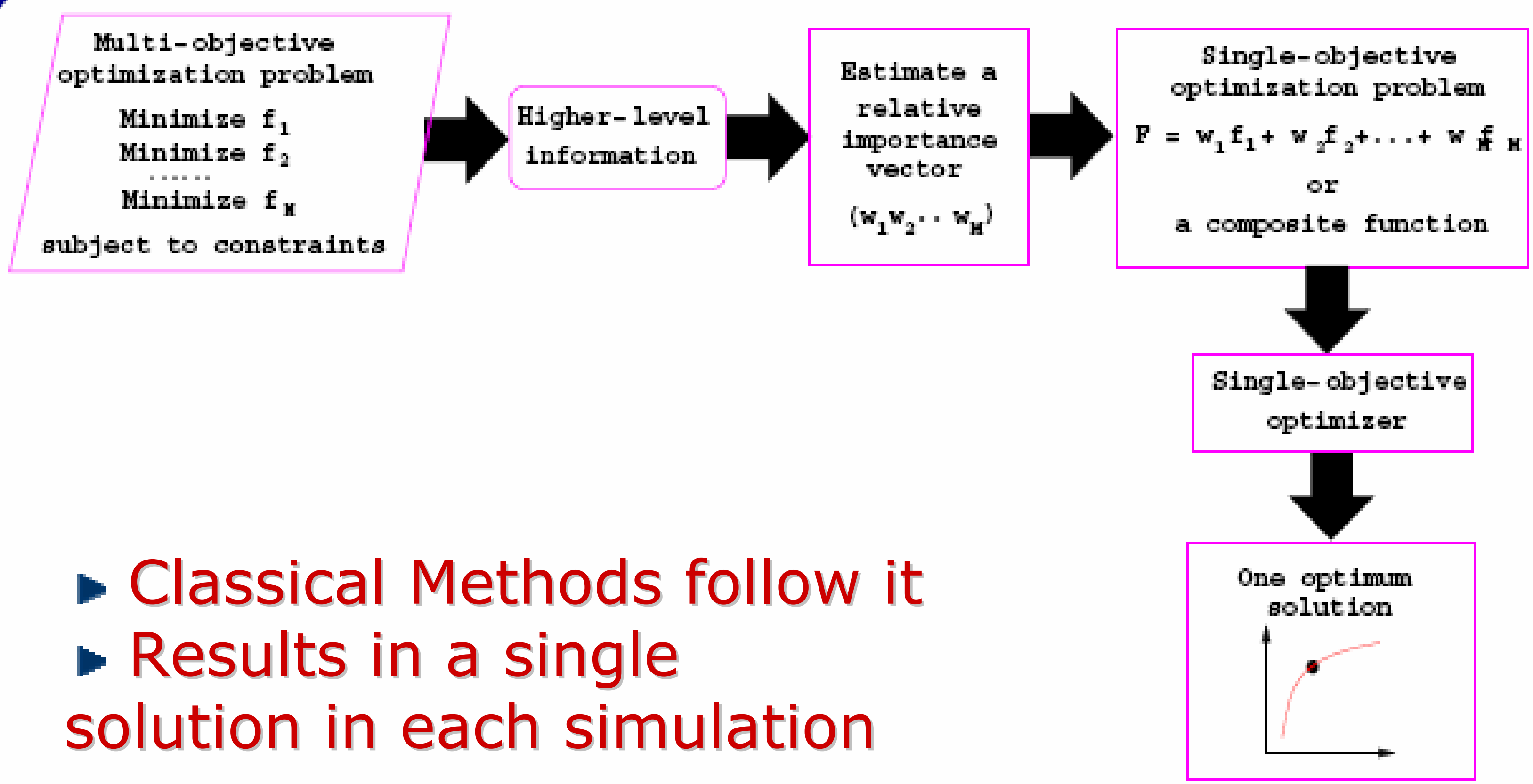
Differences with Single-Objective Optimization

- One optimum versus multiple optima
- Requires search and decision-making
- Two spaces of interest, instead of one





Preference-Based Methods



- Classical Methods follow it
- Results in a single solution in each simulation

Classical Approaches

Miettinen (1999):

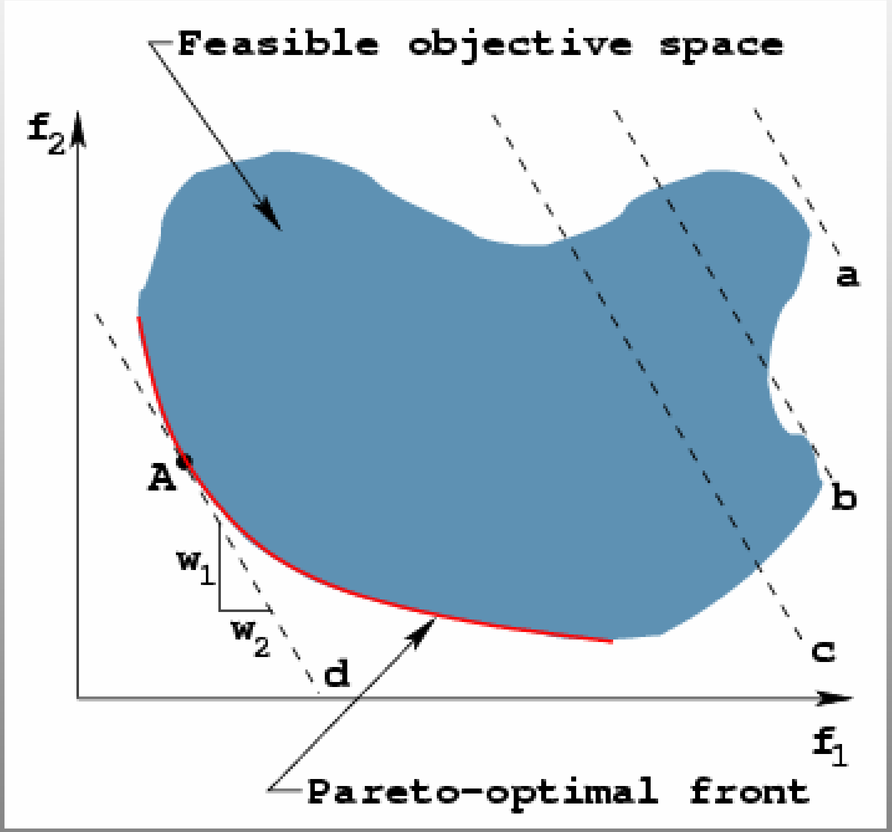
- No Preference methods (heuristic-based)
- Posteriori methods (generating solutions) discussed later
- A-priori methods (one preferred solution)
- Interactive methods (involving a decision-maker)

Classical Approach: Weighted Sum Method

- Construct a weighted sum of objectives and optimize

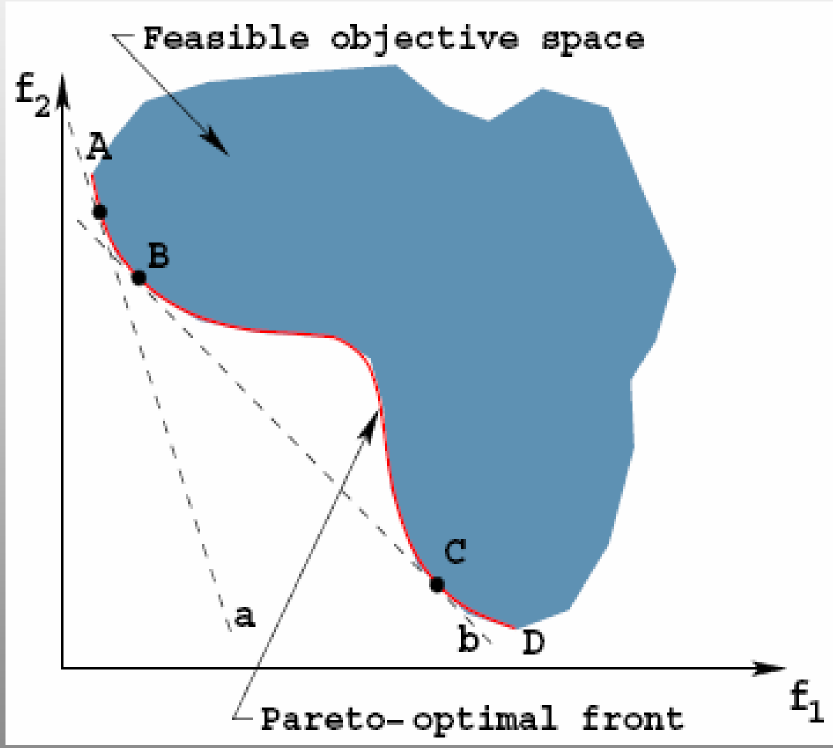
$$F(x) = \sum_{i=1}^{M} w_i f_i(x)$$

- User supplies weight vector *w*



Difficulties with Weighted-Sum Method

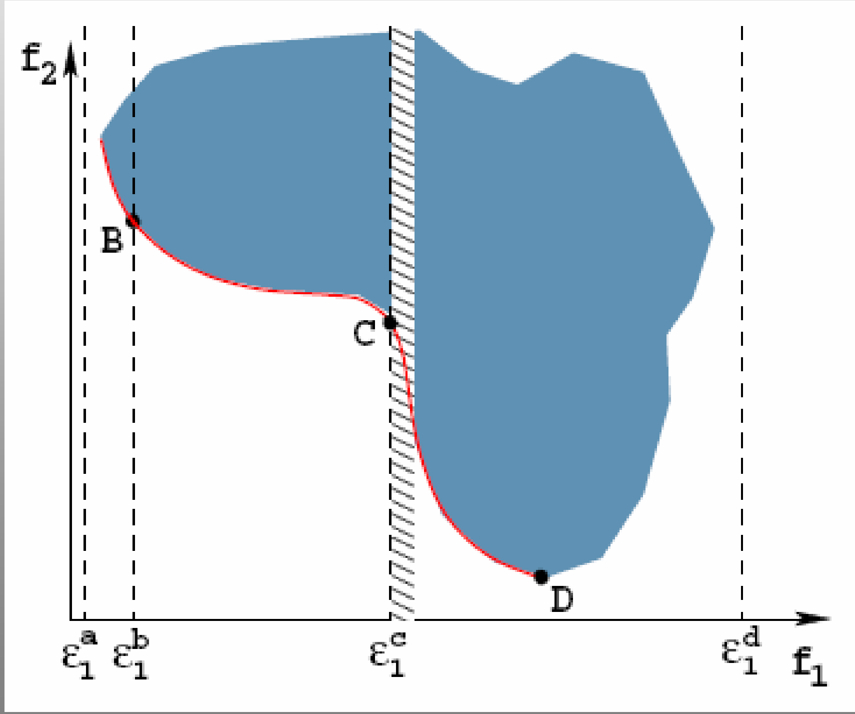
- Need to know **w**
- Non-uniformity in Pareto-optimal solutions
- Inability to find some Pareto-optimal solutions (those in non-convex region)
- However, a solution of this approach is always Pareto-optimal



ε-Constraint Method

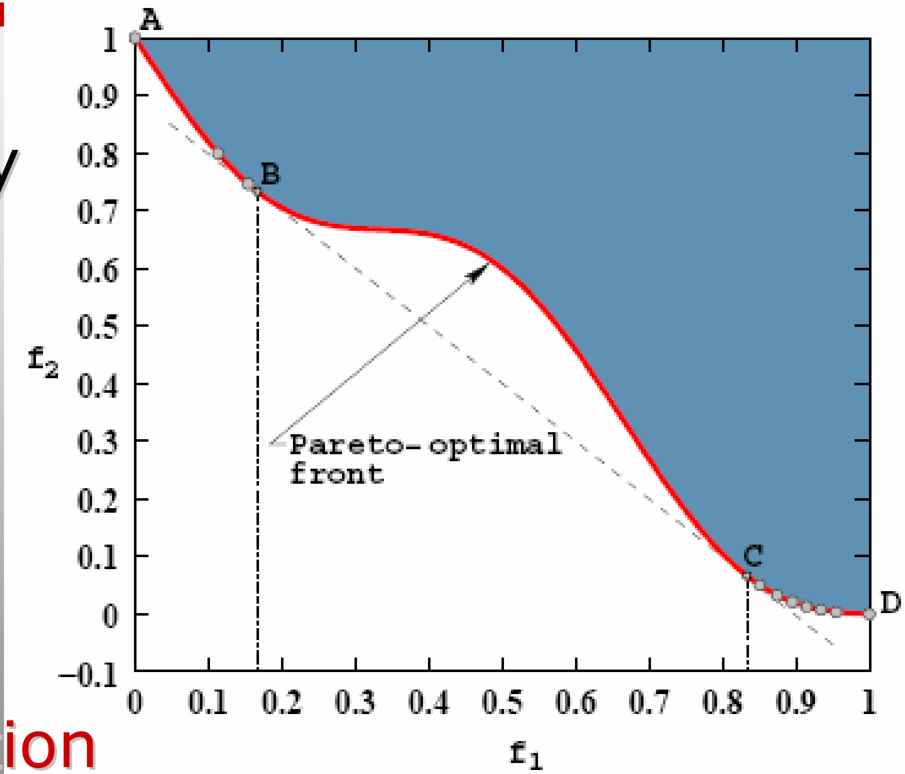
- Constrain all but one objective
- Need to know relevant **ε** vectors
- Non-uniformity in Pareto-optimal solutions
- However, any Pareto-optimal solution can be found with this approach

$$\begin{aligned} \text{Minimize} \quad & f_\mu(\mathbf{x}), \\ \text{subject to} \quad & f_m(\mathbf{x}) \le \epsilon_m, \; m \ne \mu; \end{aligned}$$



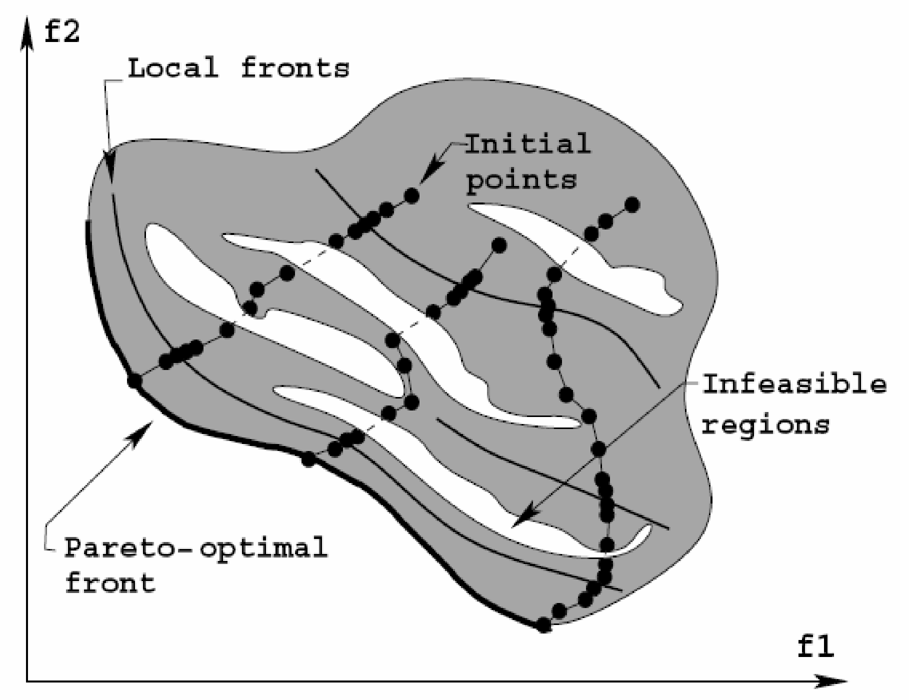
Difficulties with Most Classical Approaches

- Need to run a single-objective optimizer many times
- Expect a lot of problem knowledge
- Even then, good distribution is not guaranteed
- Multi-objective optimization as an application of single-objective optimization



Classical Generating Methods

- One-at-a-time and repeat
- Population approaches
  - Timmel's method
  - Schaffler's method
- Absence of parallel search is a drawback
- EMO finds multiple solutions with an implicit parallel search

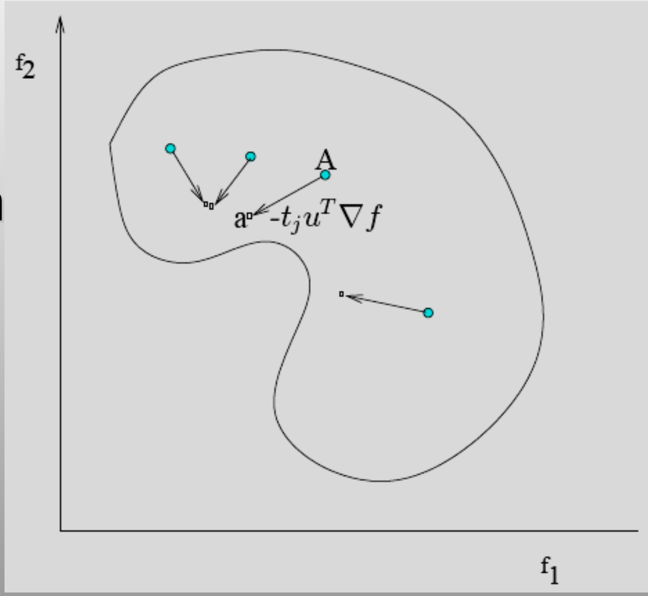


Classical Methods for Finding Multiple Points

- Timmel's (1980) population approach (in German)
  - Start with a population of solutions $A^{(t)}$
  - From each point *i*, move a step $s^{(t)}$ in direction

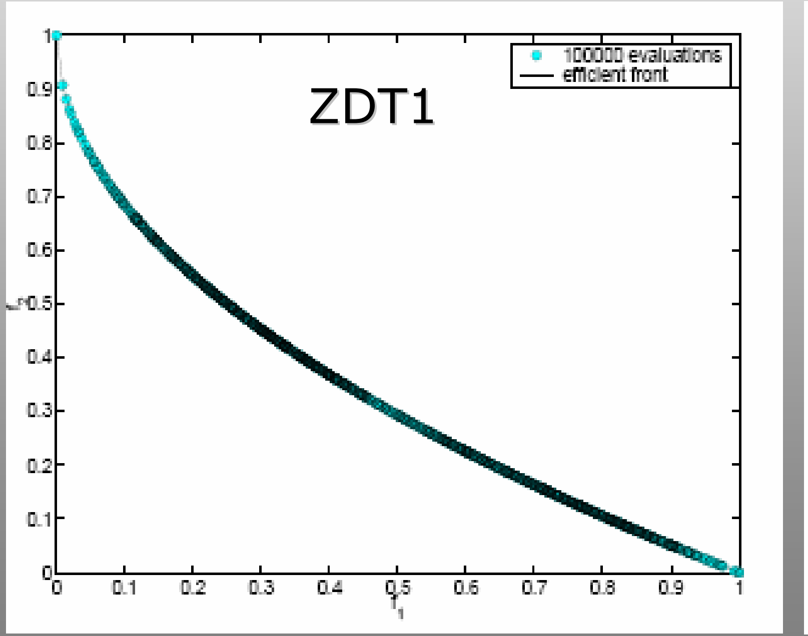
$$d_i^{(t)} = -\sum_{j=1}^{M} u_j \nabla f(x_i)$$

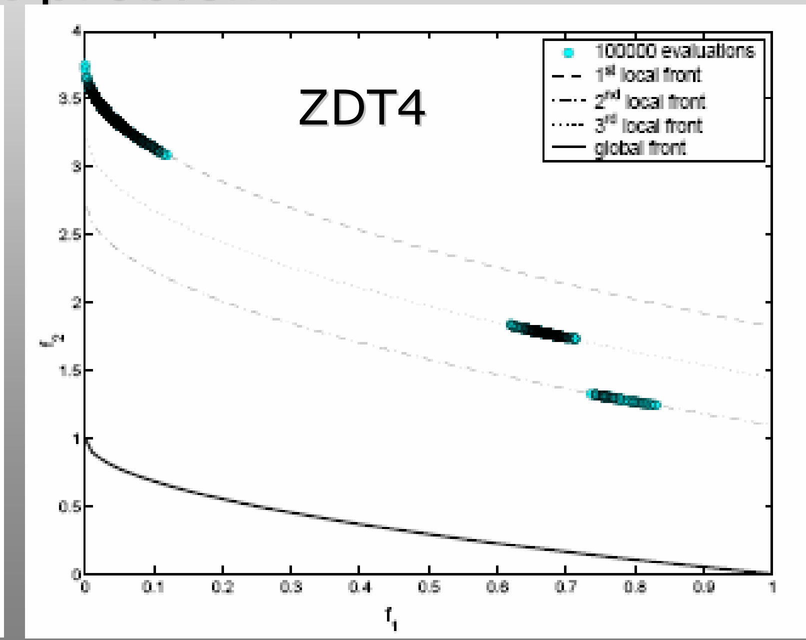
    u_j is a random number
  - Keep non-dominated solutions from $A^{(t)} \cup A^{(t+1)}$ in $A^{(t+1)}$
- Asymptotic convergence proof for a suitable $s^{(t)}$ sequence
- Need to choose a proper step length



Timmel's Population Approach

- Shukla and Deb (EMO, 2005)
- 100,000 evaluations
- Works on simpler problems, gets stuck at a local P-O frontier on a difficult problem





A Stochastic Method

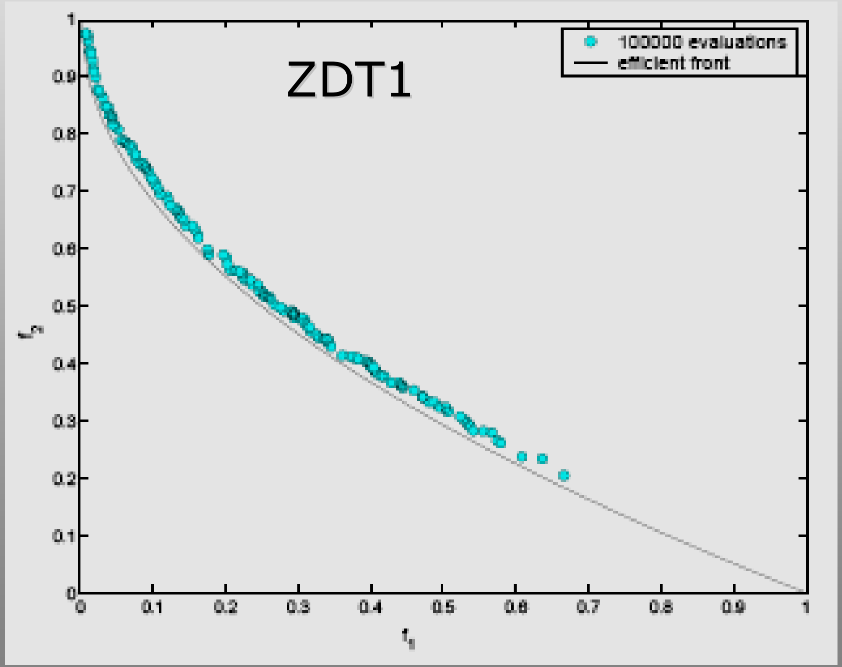
- Schaffler et al. (2002) JOTA article
  - Start with a point $x^{(t)}$
  - Find a descent direction $q\left(x^{(t)}\right)=\sum_{i=1}^{M}\alpha_i \nabla f_i\left(x^{(t)}\right)$ which solves

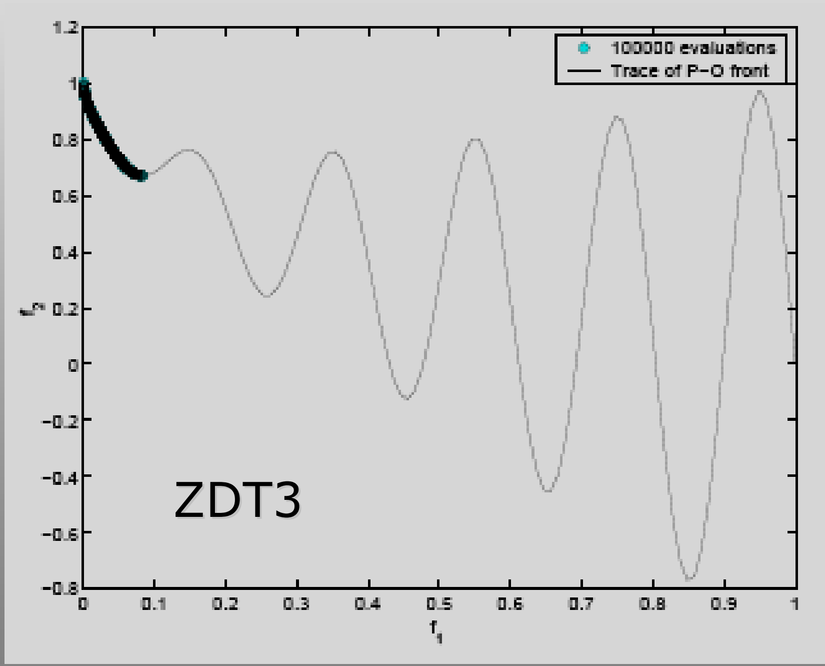
$$\min_{\alpha} \left\| \sum_{i=1}^{M} \alpha_i \nabla f_i\left(x^{(t)}\right) \right\| \quad \text{such that} \quad \sum_{i=1}^{M} \alpha_i = 1$$

  - Update the point: $dx^{(t)} = -qx^{(t)}dt + \varepsilon dB^{(t)}$
  - $B^{(t)}$ is a *n*-dimensional Browinian motion
- A numerical iterative algorithm suggested
- Reaches the P-O frontier and then spreads

Schaffler et al.'s Point Approach

- Shukla and Deb (EMO, 2005)
- 100,000 evaluations
- Slower than TPM





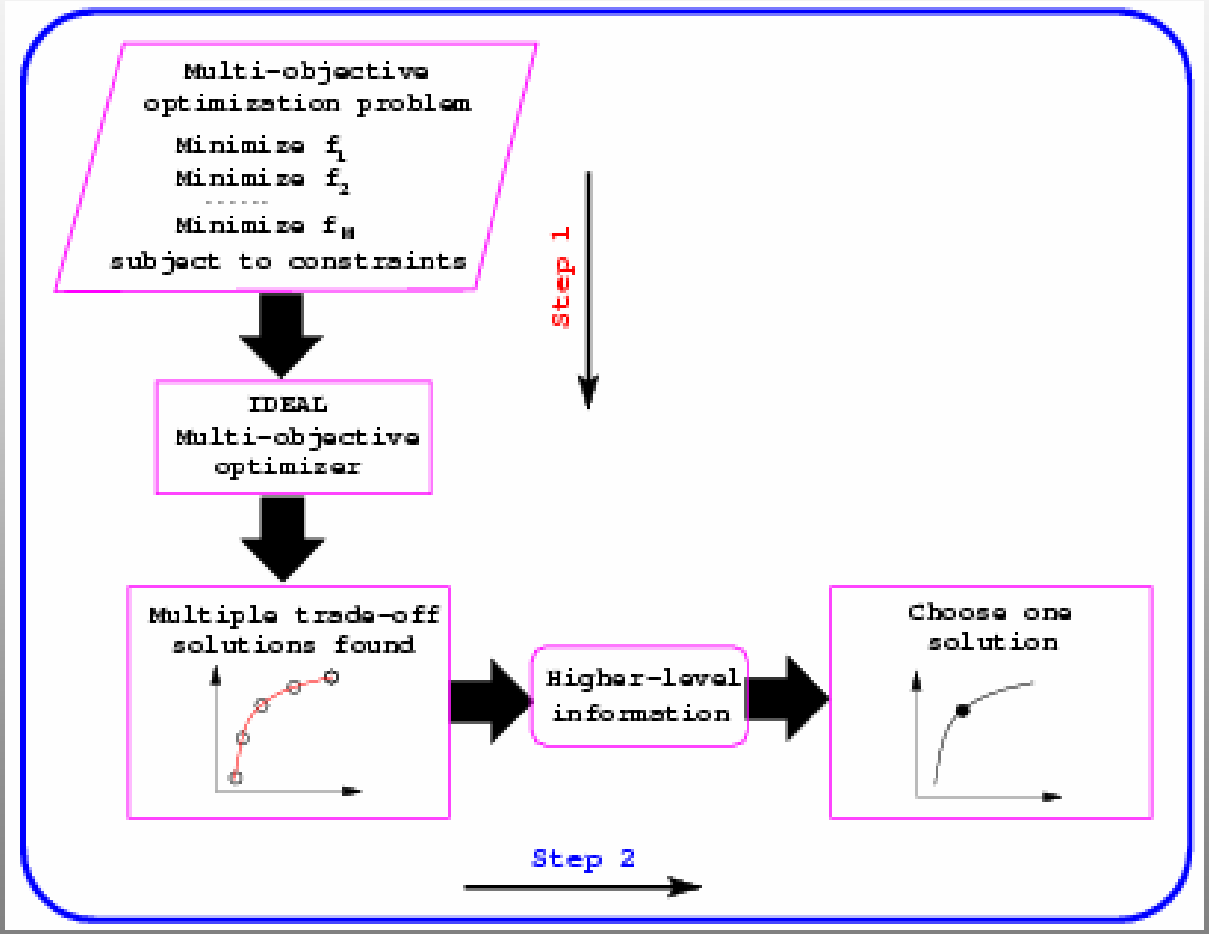
Ideal Multi-Objective Optimization

Step 1 :

Find a set of Pareto-optimal solutions

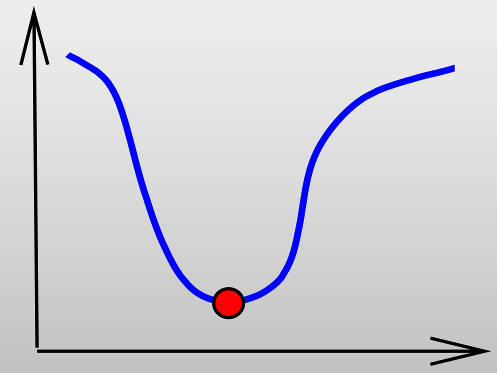
Step 2 :

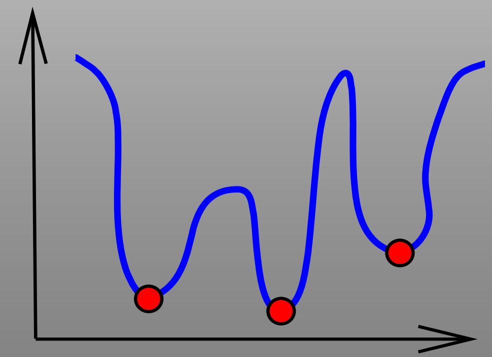
Choose one from the set



A More Holistic Approach for Optimization

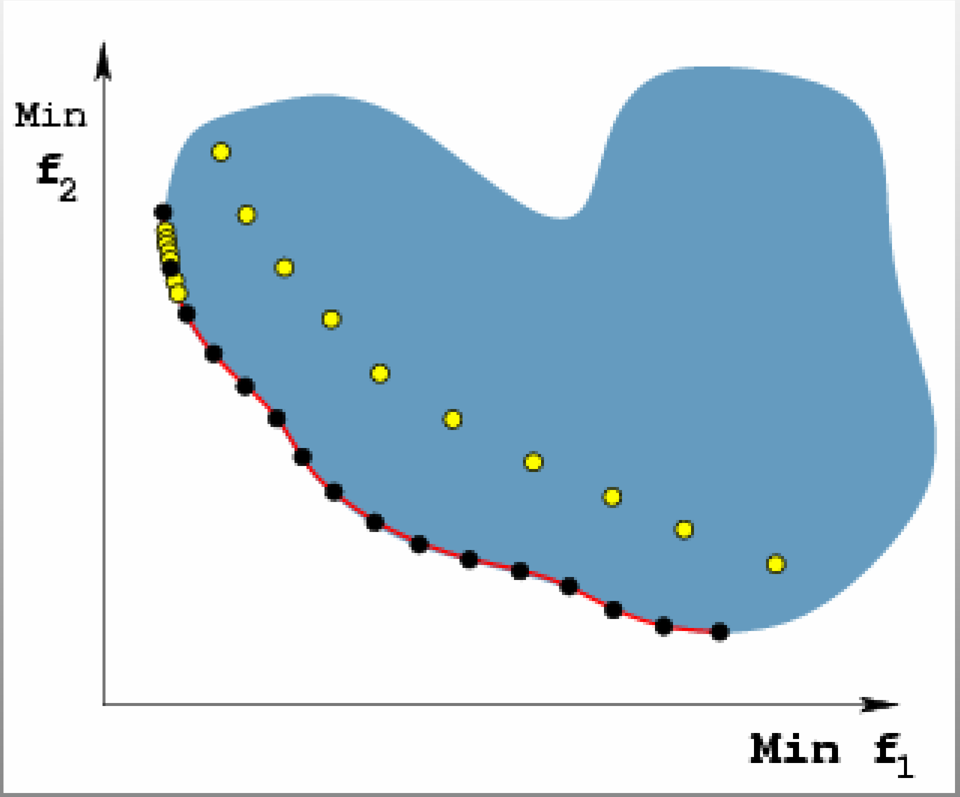
- Decision-making becomes easier and less subjective
- Single-objective optimization is a degenerate case of multi-objective optimization
  - Step 1 finds a single solution
  - No need for Step 2
- Multi-modal optimization possible
- Demonstrate an omni-optimizer later





Two Goals in Ideal Multi-Objective Optimization

- Converge to the Pareto-optimal front
- Maintain as diverse a distribution as possible

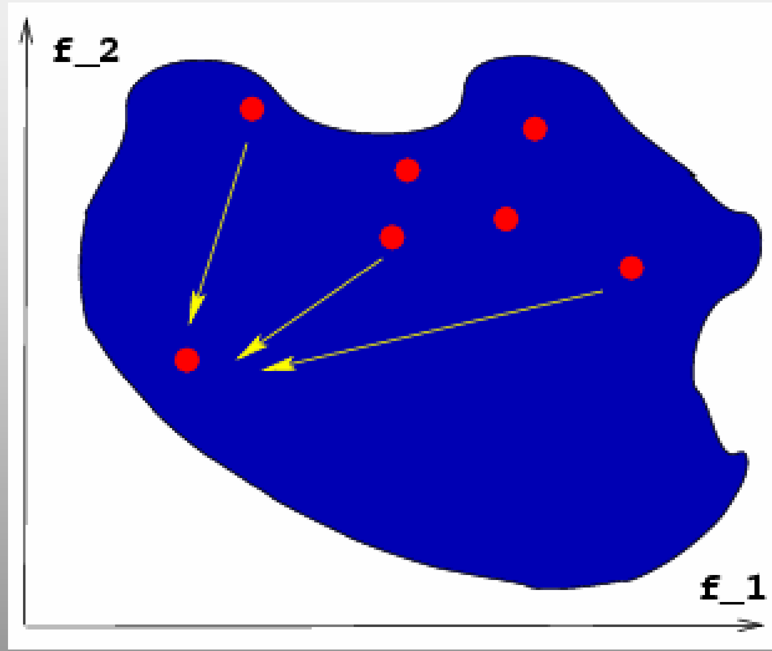


Evolutionary Multi-Objective Optimization (EMO)

- Principle:
  - Find multiple Pareto-optimal solutions simultaneously
- Two main reasons:
  - Help in choosing a particular solution
  - Unveil salient optimality properties of solutions
  - Assist in other problem solving

Why Use Evolutionary Algorithms?

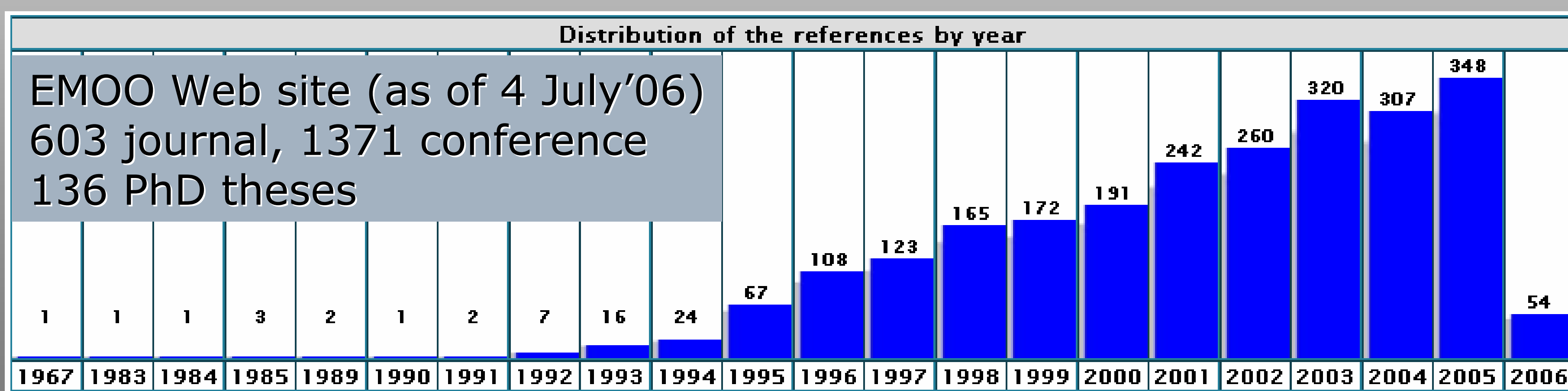
- Population approach suits well to find multiple solutions
- Niche-preservation methods can be exploited to find diverse solutions
- Implicit parallelism helps provide a parallel search



- Multiple applications of classical methods do not constitute a parallel search

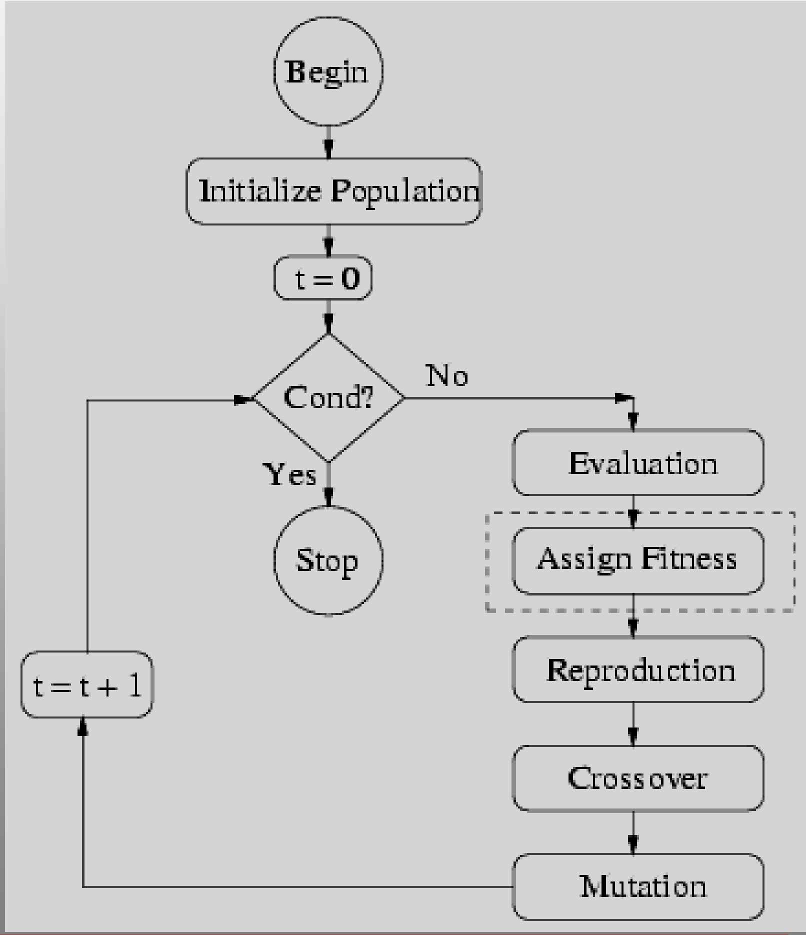
History of Evolutionary Multi-Objective Optimization (EMO)

- Early penalty-based approaches
- VEGA (1984)
- Goldberg's (1989) suggestion
- MOGA, NSGA, NPGA (1993-95) used Goldberg's suggestion
- Elitist EMO (SPEA, NSGA-II, PAES, MOMGA etc.) (1998 -- Present)



What to Change in a Simple GA?

- Modify the fitness computation
- Emphasize non-dominated solutions for convergence
- Emphasize less-crowded solutions for diversity



Identifying Non-Dominated Set

Step 1 Set $i = 1$ and create an empty set P'.

Step 2 For a solution $j \in P$ (but $j \neq i$), check if solution j dominates solution i. If yes, go to Step 4.

Step 3 If more solutions are left in P, increment j by one and go to Step 2; otherwise, set $P' = P' \cup \{i\}$.

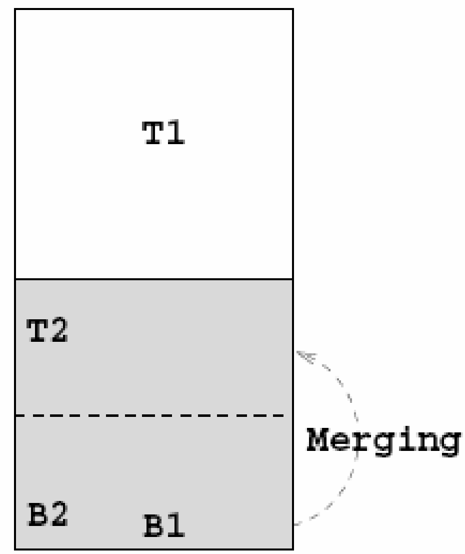
Step 4 Increment i by one. If $i \leq N$, go to Step 2; otherwise stop and declare P' as the non-dominated set.

- $O(MN^2)$ computational complexity

Kung et al.'s (1975) Approach

Step 1 Sort the population in descending order of importance of f_1

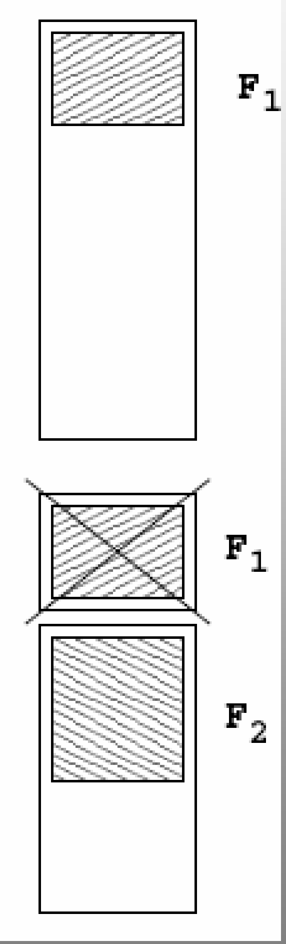
Step 2, Front(P) If $|P| = 1$, return P as the output of **Front(P)**. Otherwise, $T = \textbf{Front}(P^{(1)} - - P^{(|P|/2)})$ and $B = \textbf{Front}(P^{(|P|/2+1)} - - P^{(|P|)})$. If the i-th solution of B is not dominated by any solution of T, create a merged set $M = T \cup \{i\}$. Return M as the output of **Front(P)**.



$O\left(N(\log N)^{M-2}\right)$ for $M \geq 4$ and $O(N \log N)$ for $M = 2$ and 3

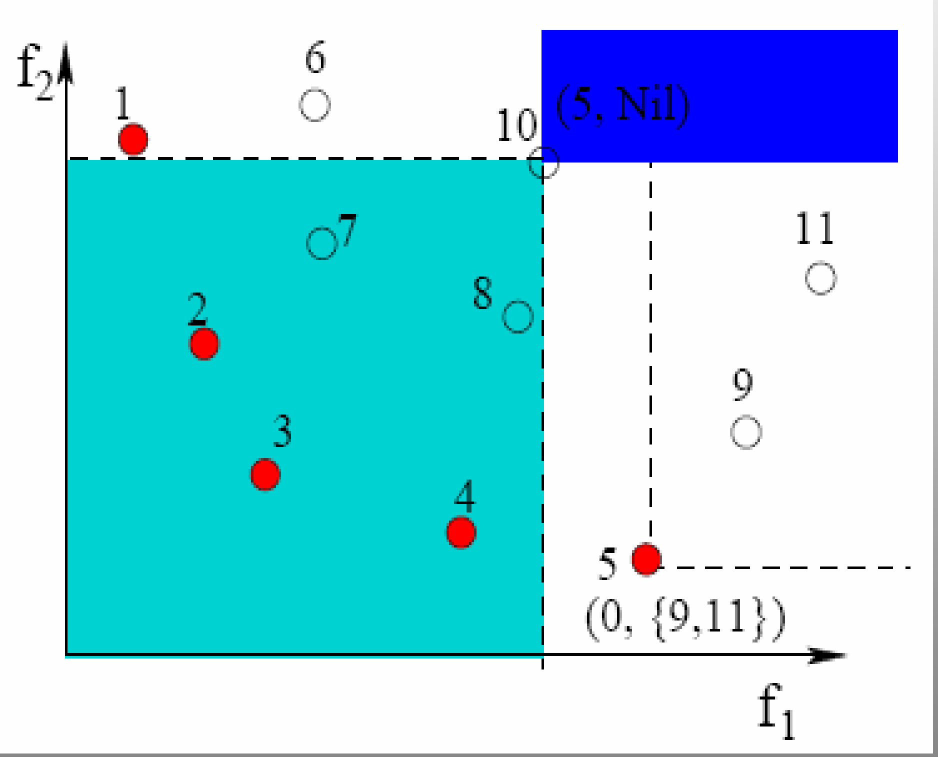
Non-Dominated Sorting: A Naive Approach

- Identify the best non-dominated set
- Discard them from population
- Identify the next-best non-dominated set
- Continue till all solutions are classified



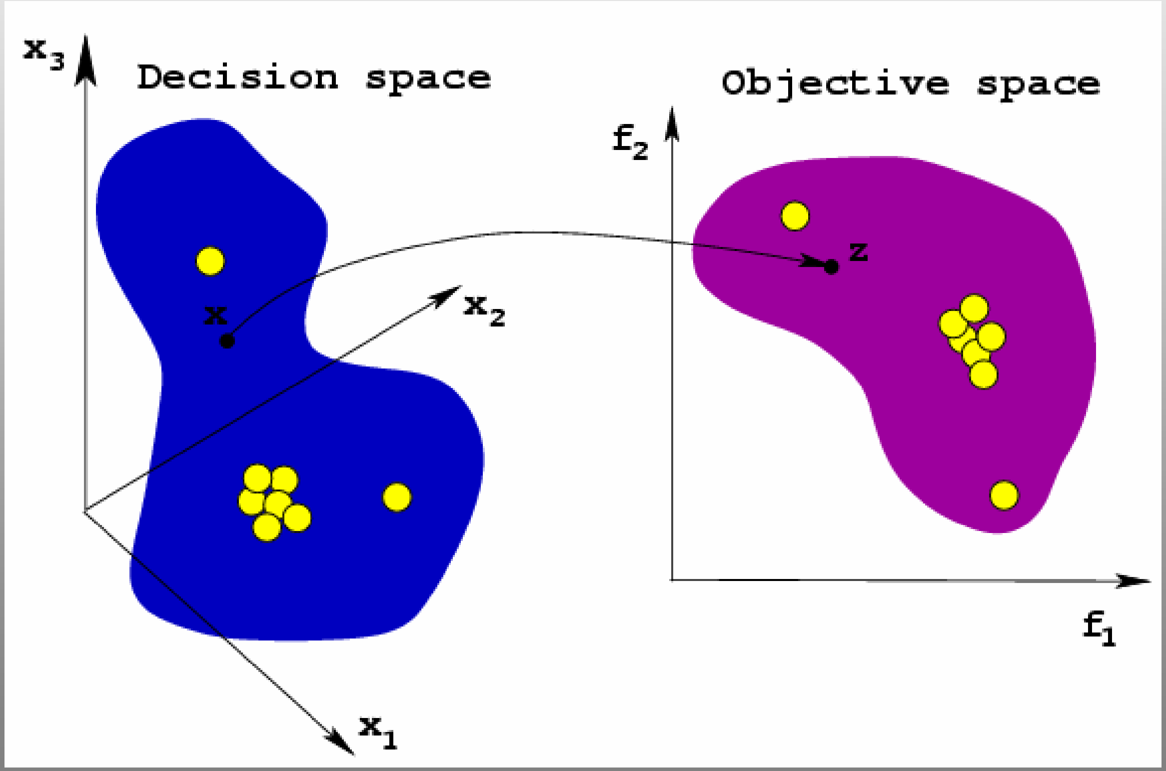
A Fast Non-Dominated Sorting

- Calculate (n_i, S_i) for each solution i
- n_i: Number of solutions dominating i
- S_i: Set of solutions dominated by I
- Follow an iterative procedure
- A faster procedure later in Lecture L6



Which are Less-Crowded Solutions?

- Crowding can be in decision variable space or in objective space



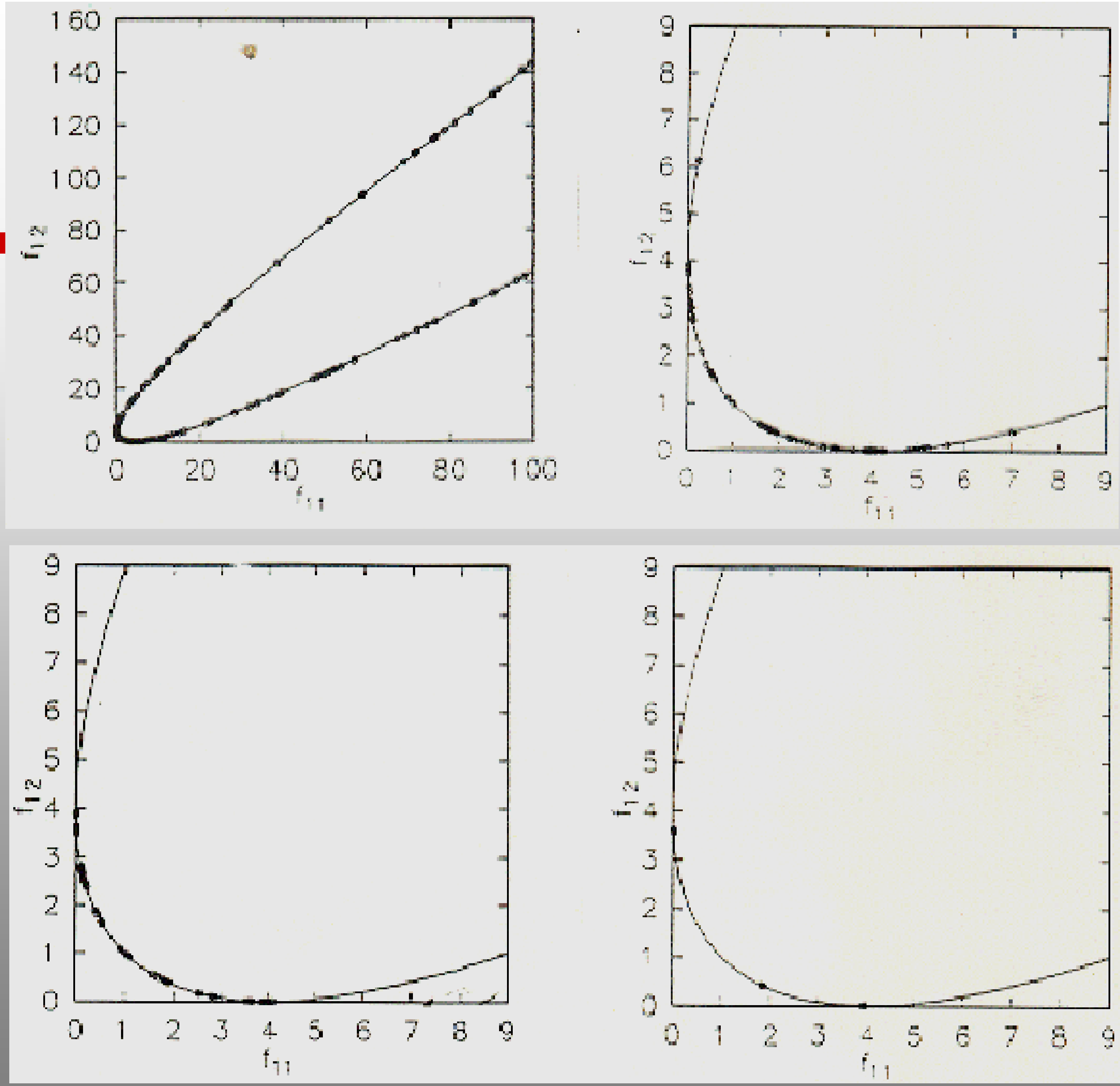
Non-Elitist EMO Procedures

- Vector evaluated GA (VEGA) (Schaffer, 1984)
- Vector optimized EA (VOES) (Kursawe, 1990)
- Weight based GA (WBGA) (Hajela and Lin, 1993)
- Multiple objective GA (MOGA) (Fonseca and Fleming, 1993)
- Non-dominated sorting GA (NSGA) (Srinivas and Deb, 1994)
- Niched Pareto GA (NPGA) (Horn et al., 1994)
- Predator-prey ES (Laumanns et al., 1998)
- Other methods: Distributed sharing GA, neighborhood constrained GA, Nash GA etc.

Schaffer's (1984) Vector-Evaluated GA (VEGA)

- Divide population into M equal blocks
- Each block is reproduced with one objective function
- Complete population participates in crossover and mutation
- Bias towards to individual best objective solutions
- A non-dominated selection: Non-dominated solutions are assigned more copies
- Mate selection: Two distant (in parameter space) solutions are mated
- Both necessary aspects missing in one algorithm

VEGA Results

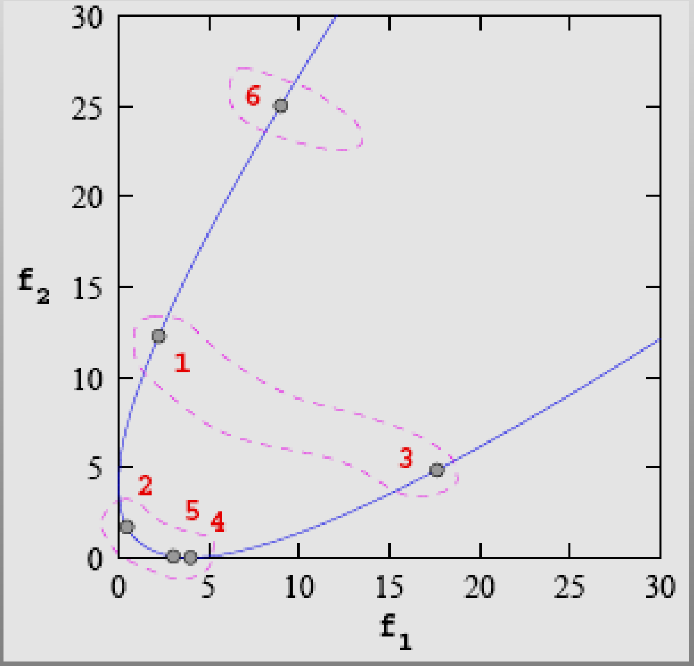


Srinivas and Deb's (1995) Non-dominated Sorting GA (NSGA)

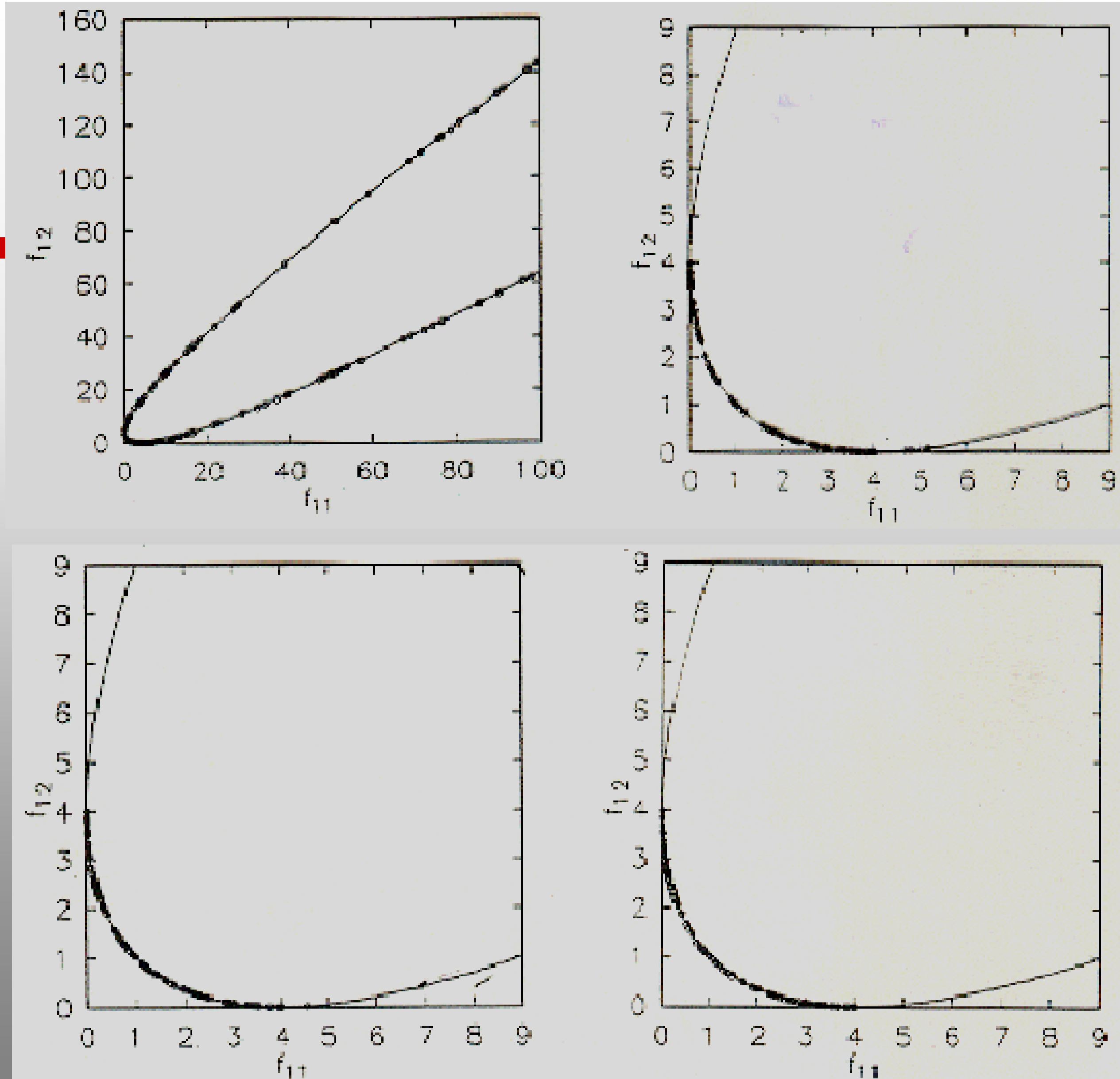
- Niching in parameter space
- Non-dominated solutions are emphasized
- Diversity among them is maintained

Example:
$f_1(x)=x^2$
$f_2(x)=(x-2)^2$

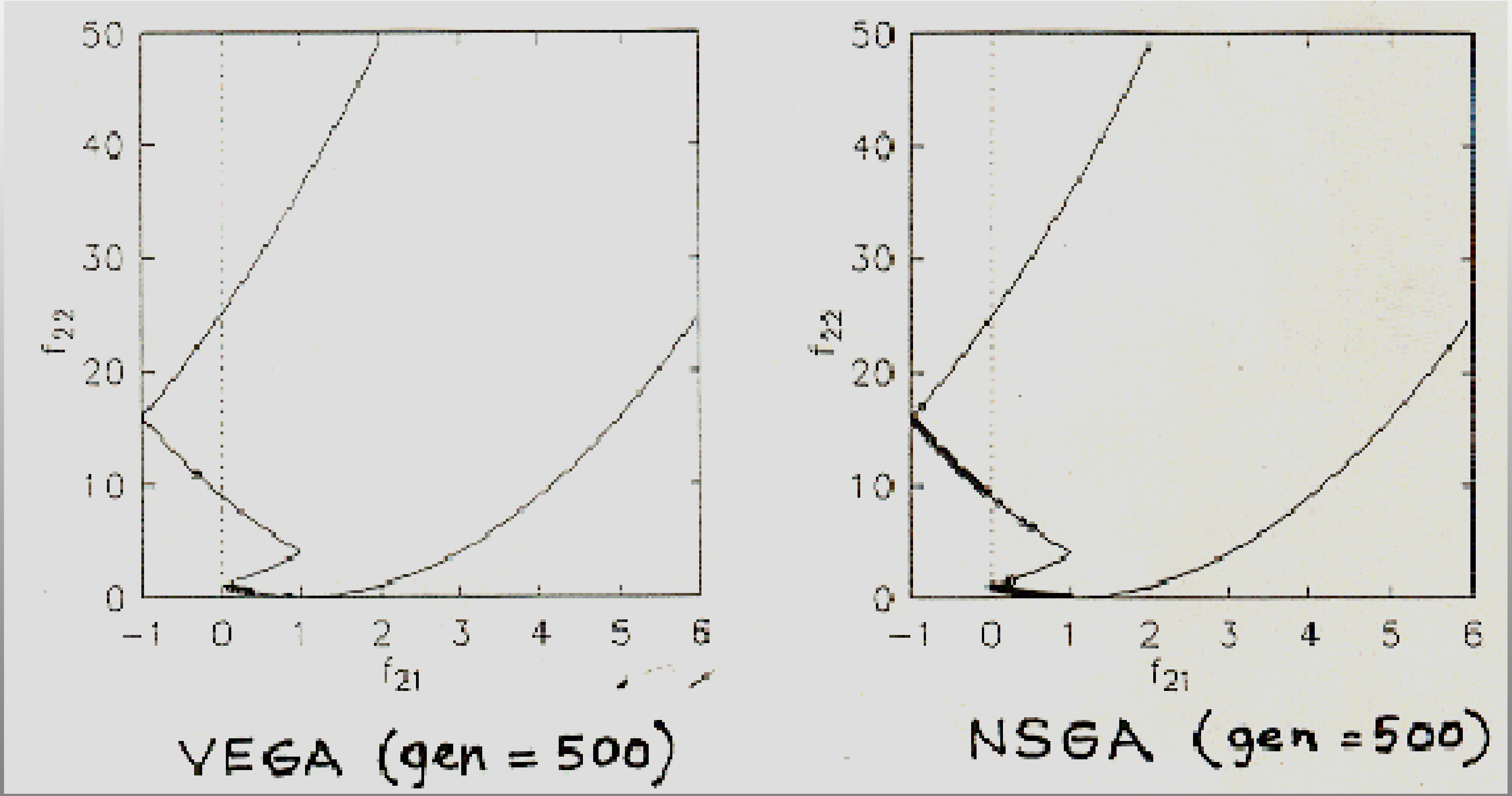
x	f_1	f_2	Front	Fitness before	Fitness after
−1.50	2.25	12.25	2	3.00	3.00
0.70	0.49	1.69	1	6.00	6.00
4.20	17.64	4.84	2	3.00	3.00
2.00	4.00	0.00	1	6.00	3.43
1.75	3.06	0.06	1	6.00	3.43
−3.00	9.00	25.00	3	2.00	2.00



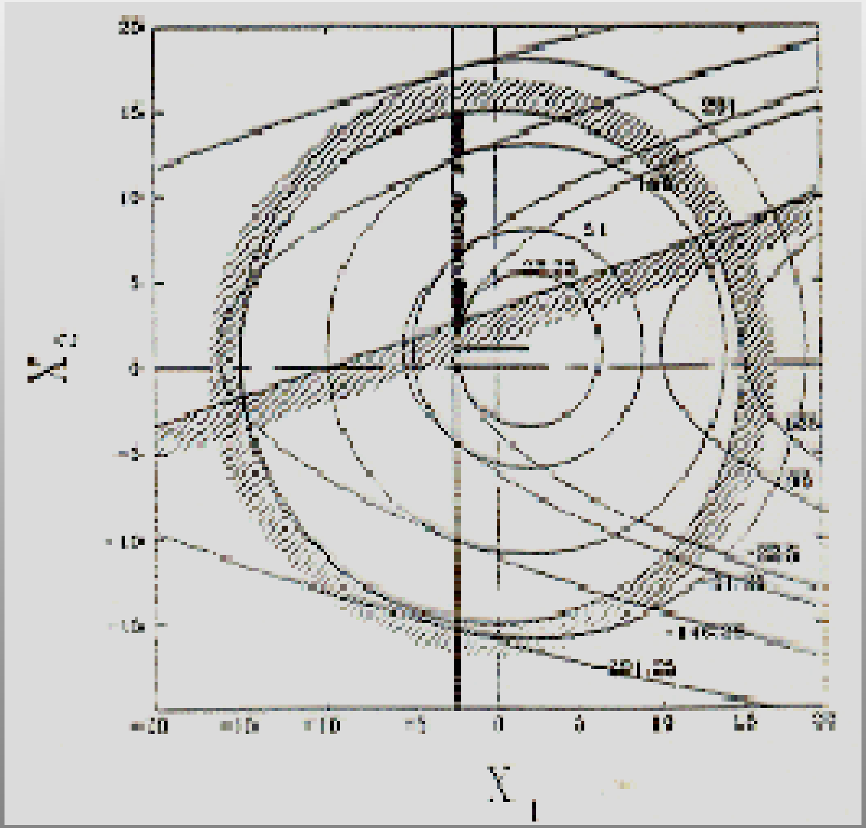
NSGA Results

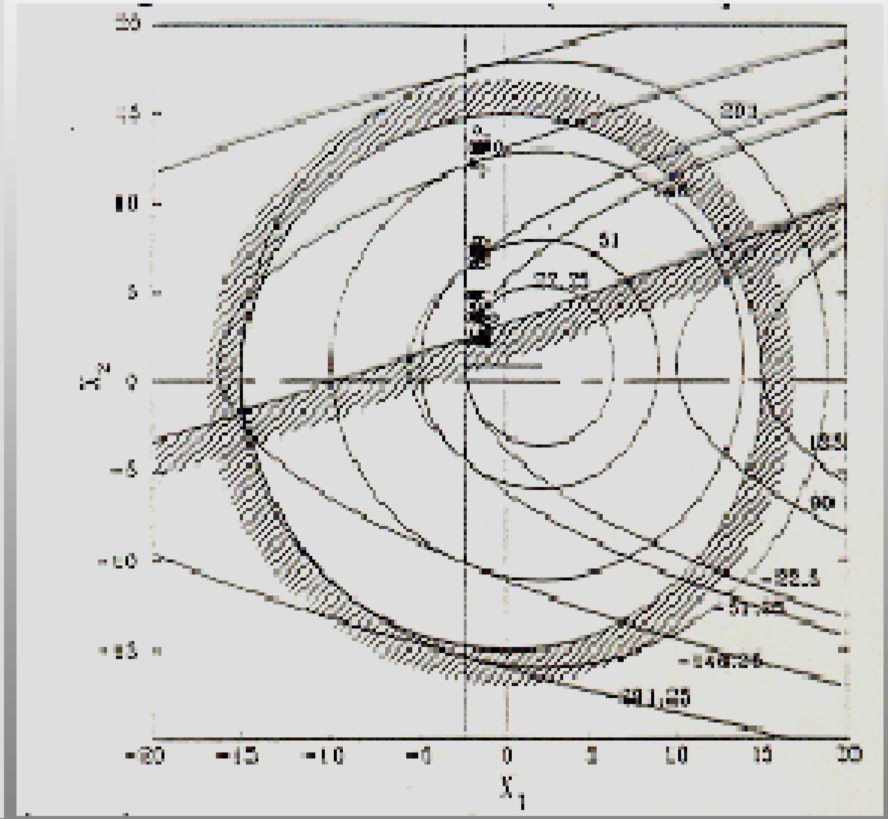


Another Test Problem



NSGA and VEGA





Fonseca and Fleming's (1993) Multi-Objective GA (MOGA)

- Count the number of dominated solutions (say n)
- Fitness: F=n+1
- A fitness ranking adjustment
- Niching in fitness space
- Rest all are similar to NSGA

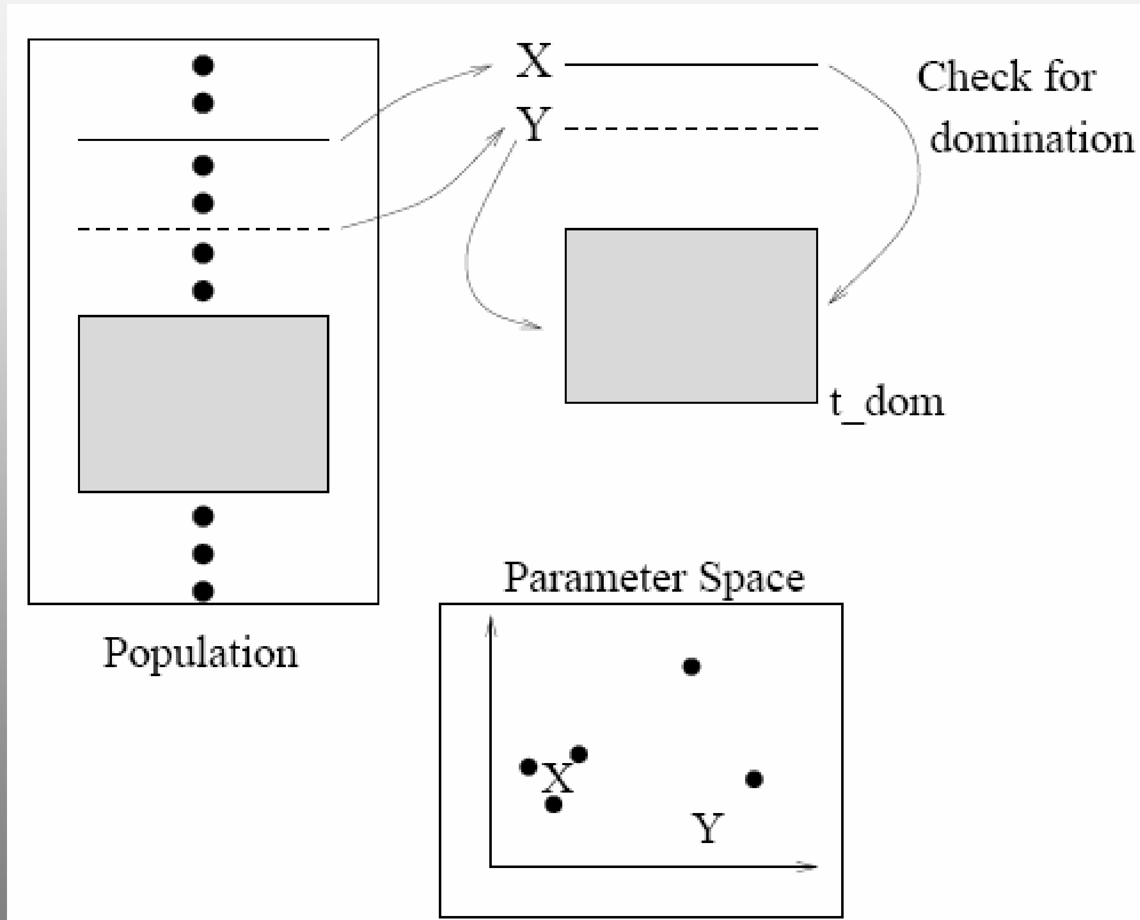
Example:

	F	Asgn.	Fit.
1	2	3	2.5
2	1	6	5.0
3	2	2	2.5
4	1	5	5.0
5	1	4	5.0
6	3	1	1.0

Horn et al.'s (1995) Niched Pareto GA (NPGA)

- Tournament within a small subpopulation (t_{dom})
- If one dominated and other non-dominated, select second
- If both non-dominated or both dominated, choose the one with smaller niche count in the subpopulation
- Algorithm depends on t_{dom}
- Nevertheless, it has both necessary components

NPGA (cont.)


X
Y
Check for
domination
t_dom
Population
Parameter Space
X
Y

Shortcomings of Non-Elitist EMO Procedures

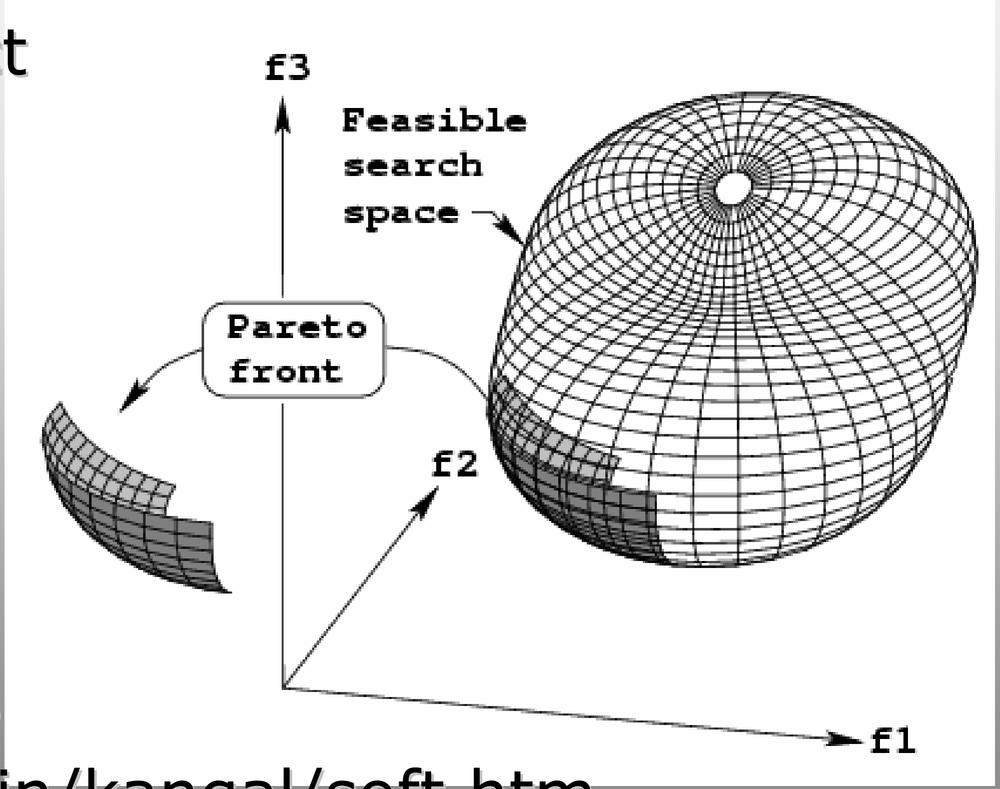
- Elite-preservation is missing
- Elite-preservation is important for proper convergence in single-objective EAs
- Same is true in EMO procedures
- Three tasks
  - Elite preservation
  - Progress towards the Pareto-optimal front
  - Maintain diversity among solutions

Elitist EMOs

- Distance-based Pareto GA (DPGA) (Osyczka and Kundu, 1995)
- Thermodynamical GA (TDGA) (Kita et al., 1996)
- Strength Pareto EA (SPEA) (Zitzler and Thiele, 1998)
- Non-dominated sorting GA-II (NSGA-II) (Deb et al., 1999)
- Pareto-archived ES (PAES) (Knowles and Corne, 1999)
- Multi-objective Messy GA (MOMGA) (Veldhuizen and Lamont, 1999)
- Other methods: Pareto-converging GA, multi-objective micro-GA, elitist MOGA with co-evolutionary sharing

Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II)

- NSGA-II can extract Pareto-optimal frontier
- And find a well-distributed set of solutions
- Adopted by iSIGHT and ModeFrontier
- Code downloadable

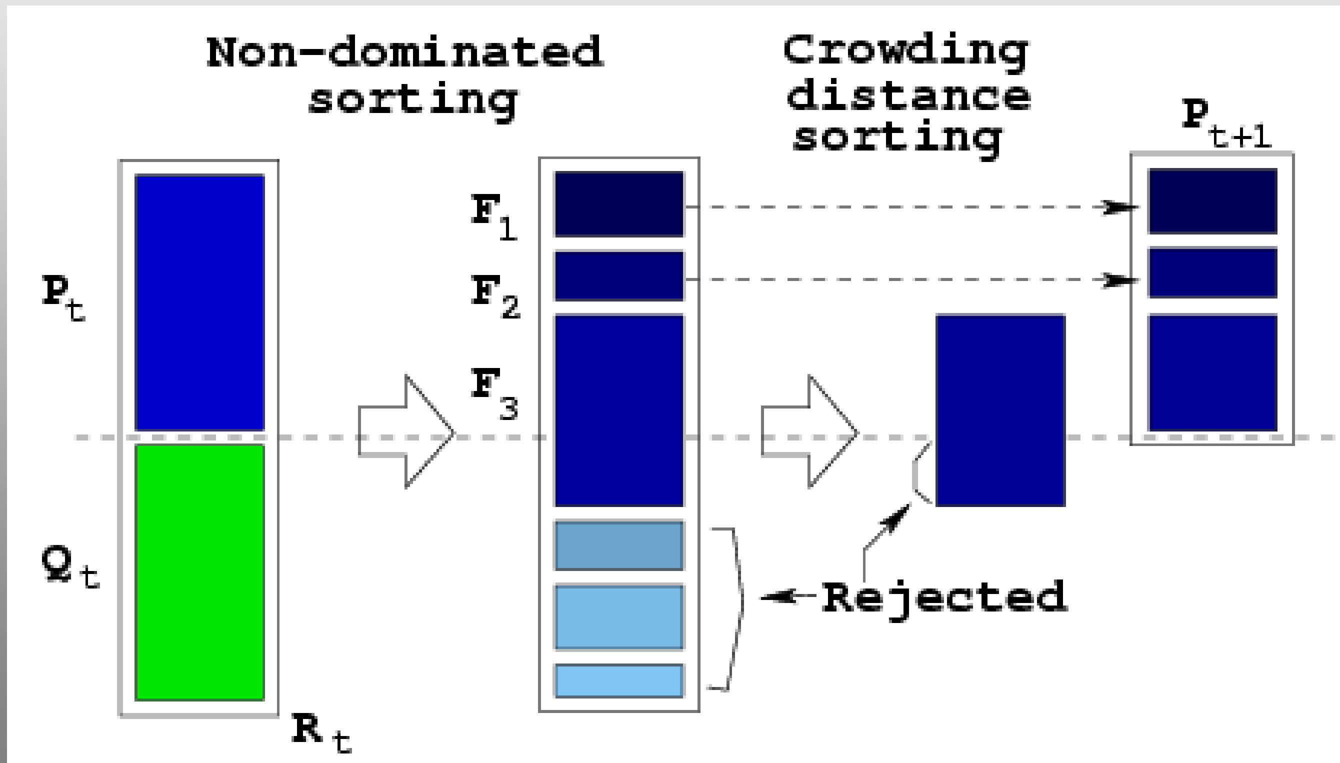


http://www.iitk.ac.in/kangal/soft.htm

IEEE TEC paper awarded 'Fast Breaking Paper in Engg. by ISI Web of Sc.

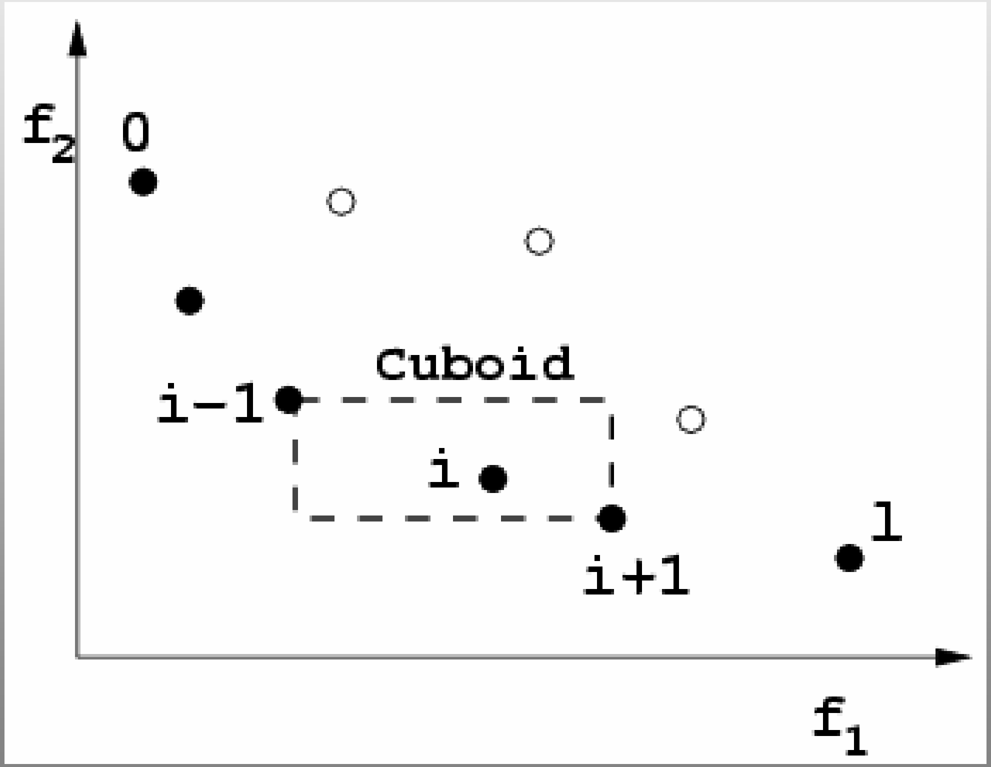
NSGA-II Procedure

Elites are preserved
Non-dominated solutions are emphasized



NSGA-II (cont.)

Diversity is maintained



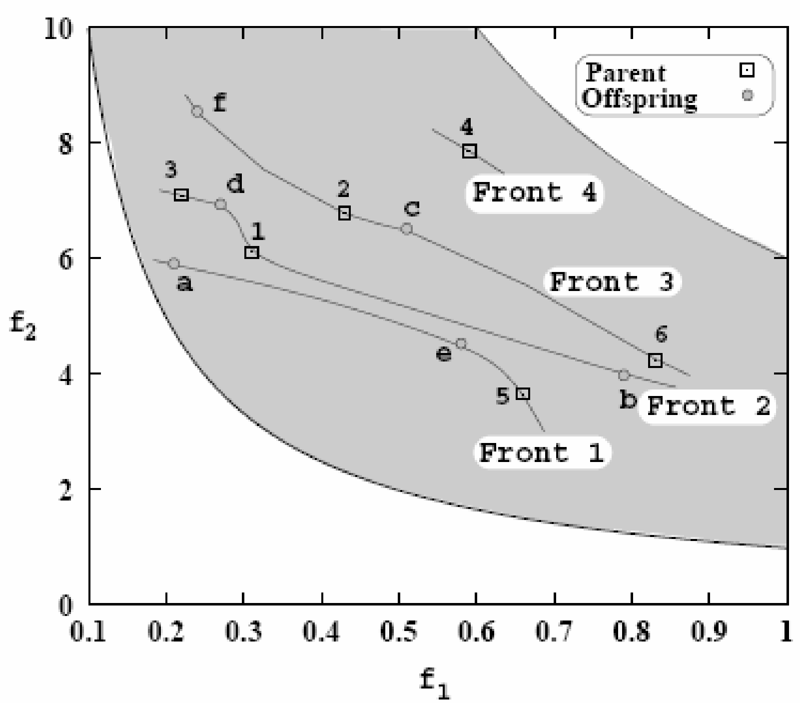
Overall Complexity
$O(N \log^{M-1} N)$

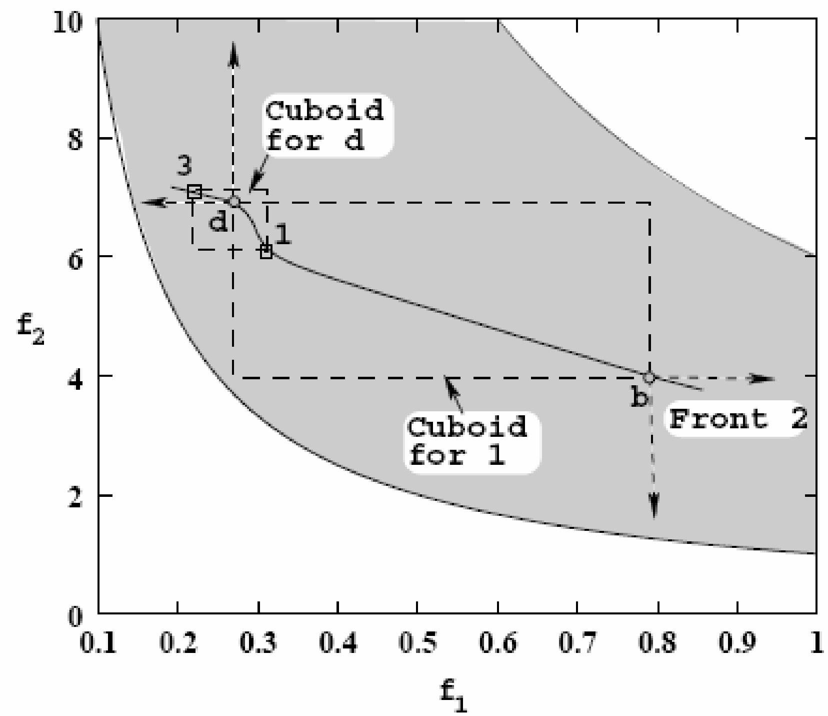
Improve diversity by

- k-mean clustering
- Euclidean distance measure
- Other techniques

One Iteration of NSGA-II

- Six parents and six offspring
- Parents after one iteration: (a,3,1,e,5,b)





NSGA-II on Test Problems

(Min) $f_1(x) = x_1$

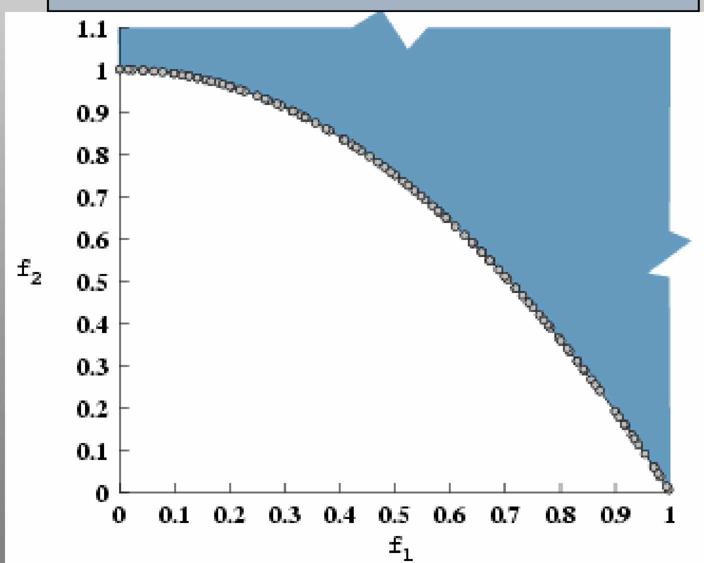
(Min) $f_2(x) = g\left[1 - \left(\frac{f_1}{g}\right)^2\right]$

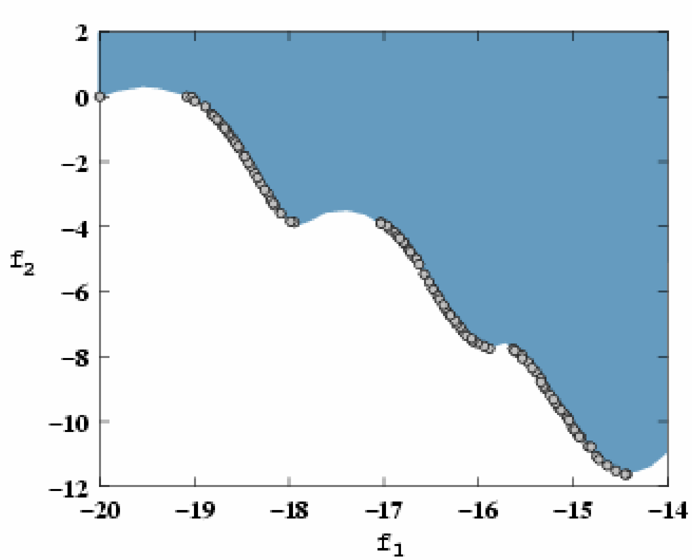
Where $g(x) = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i$

(Min) $f_1(x) = x_1$

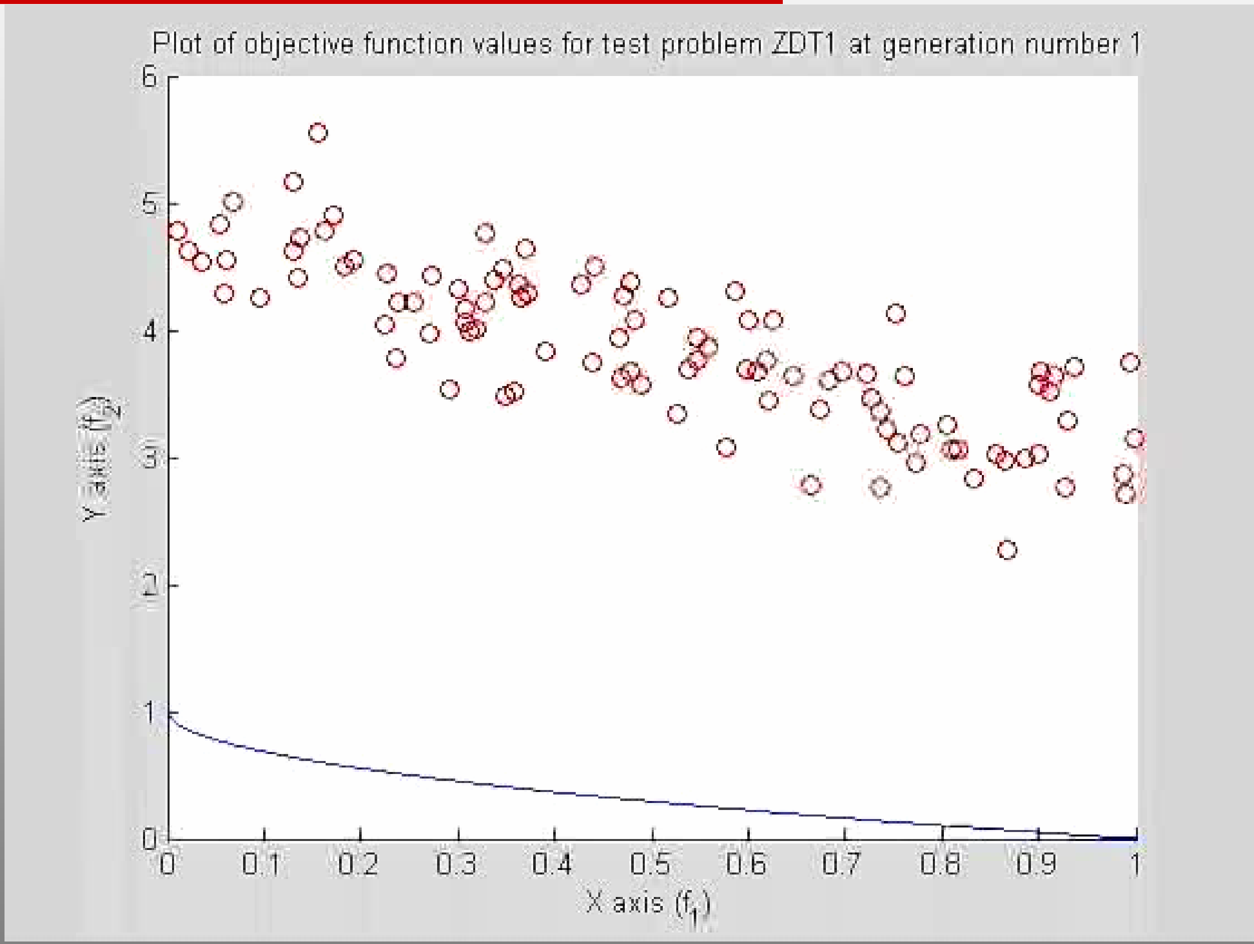
(Min) $f_2(x) = g\left[1 - \sqrt{\frac{f_1}{g}} - \frac{f_1}{g}\sin(10\pi f_1)\right]$

Where $g(x) = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i$

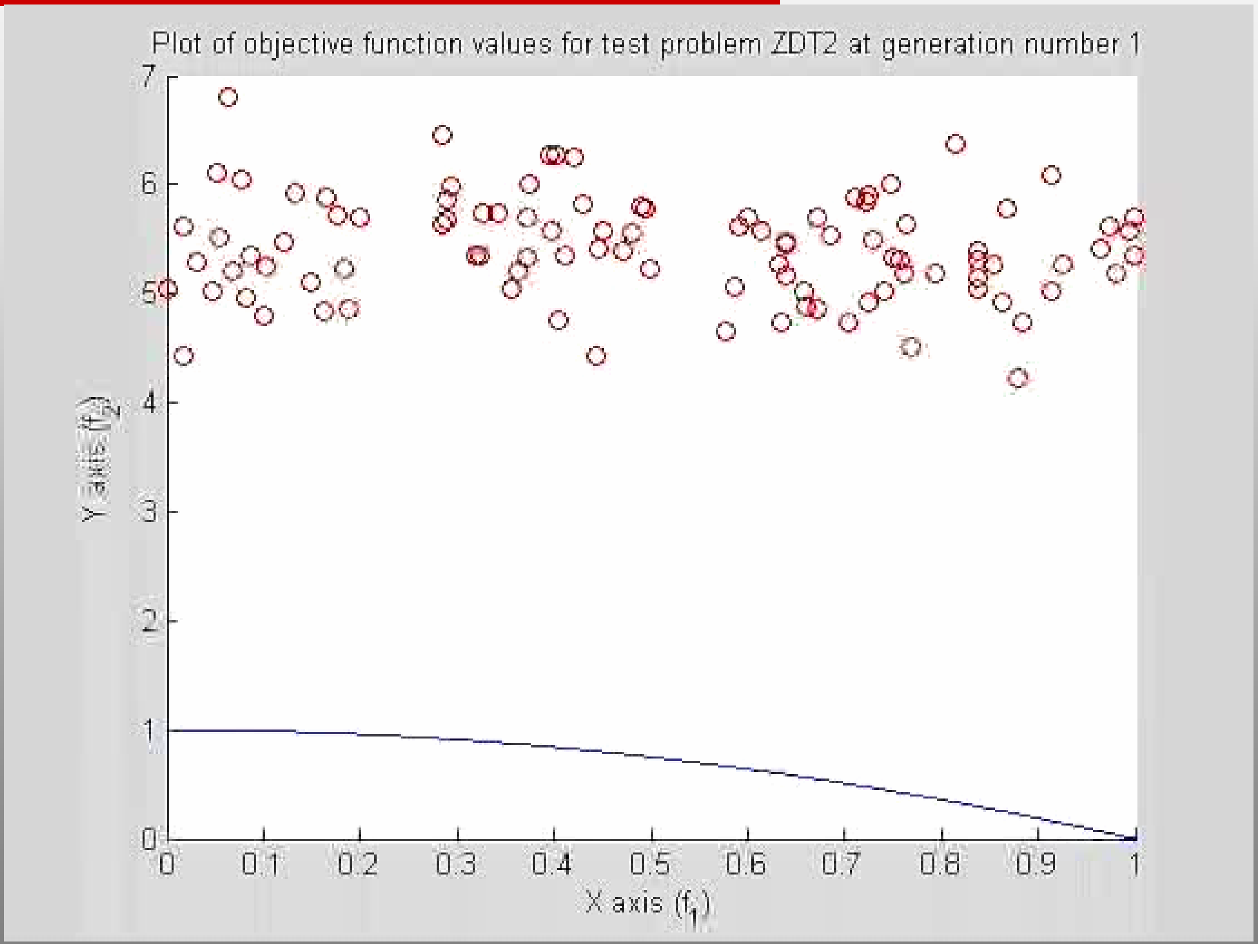




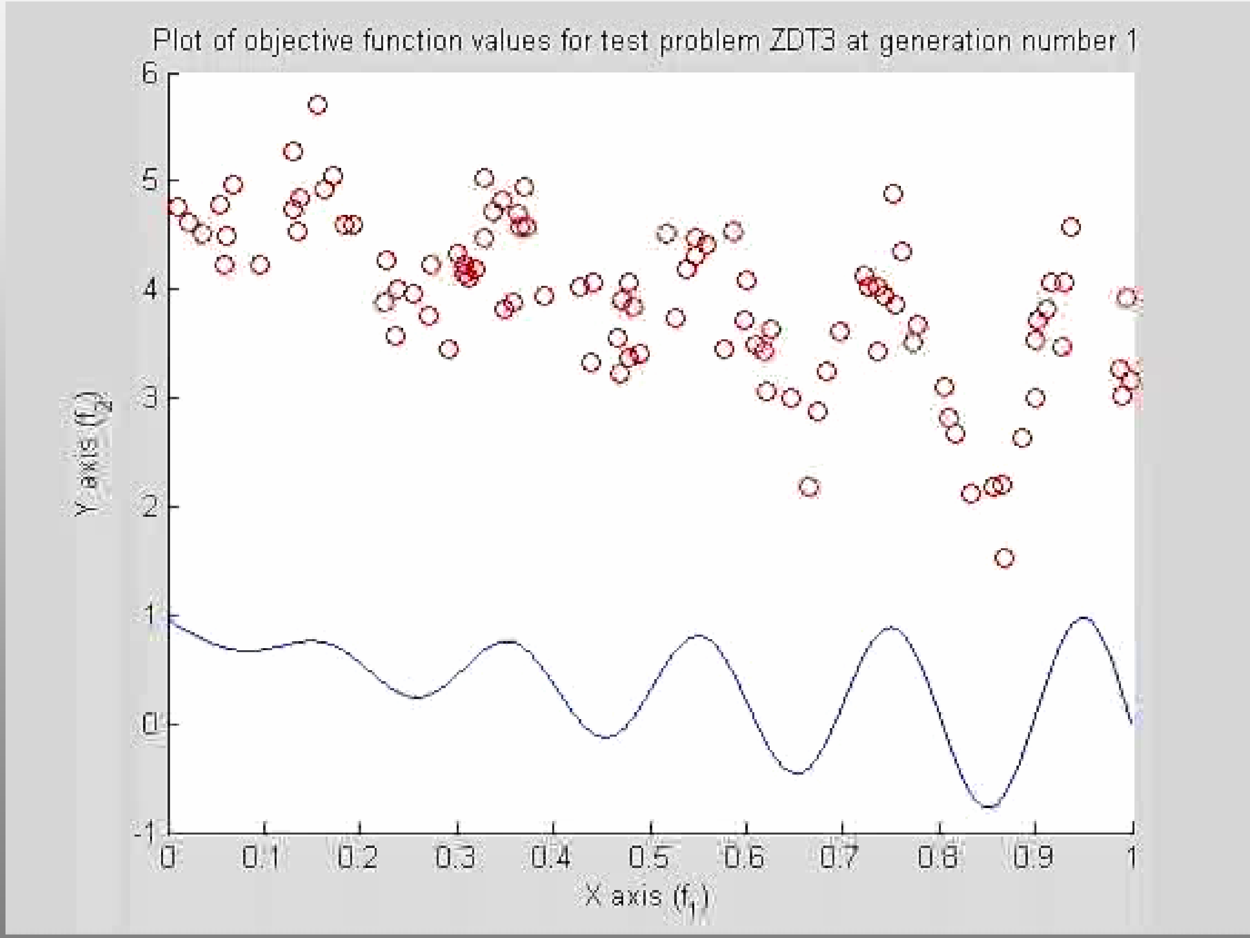
Simulation on ZDT1



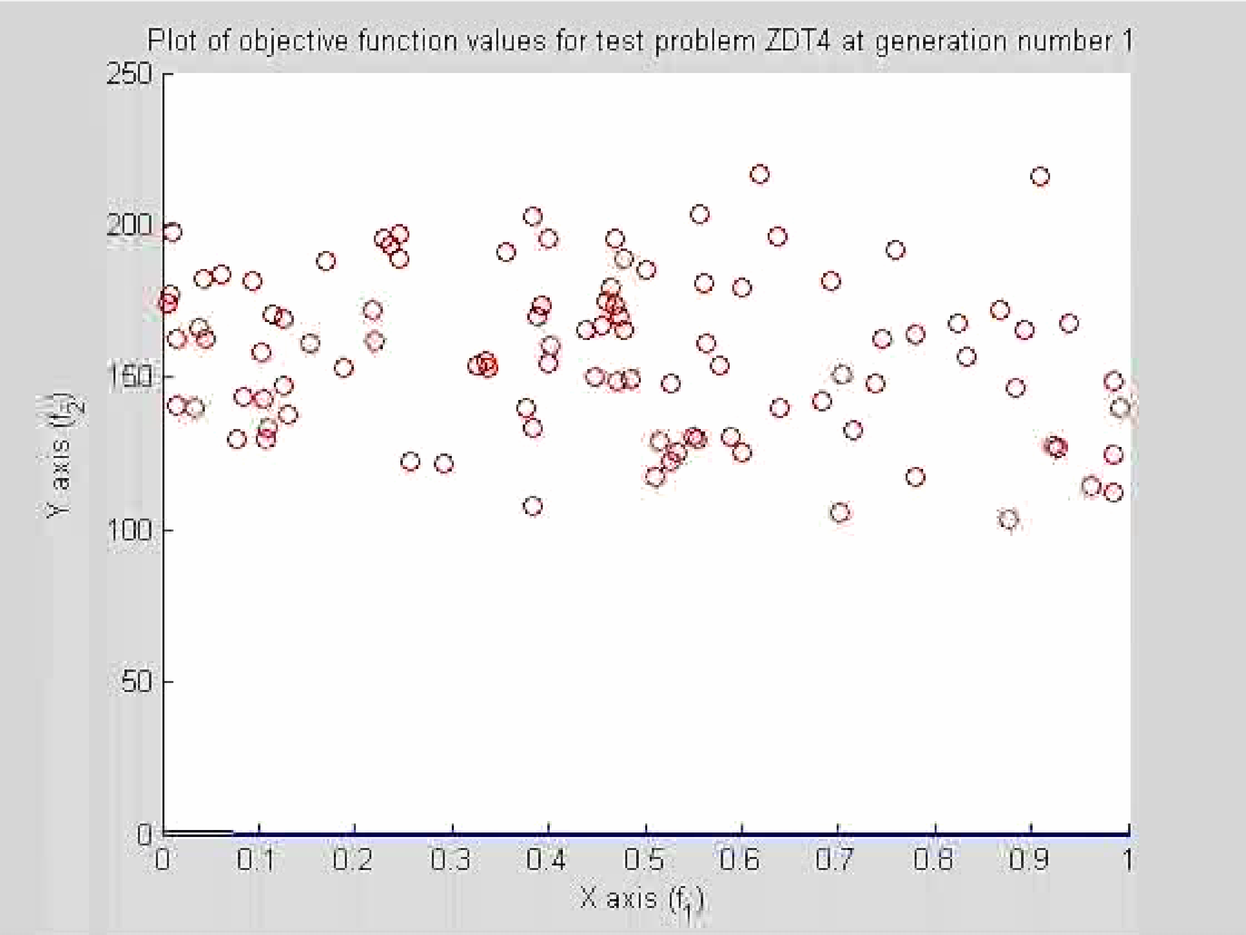
Simulation on ZDT2



Simulation on ZDT3



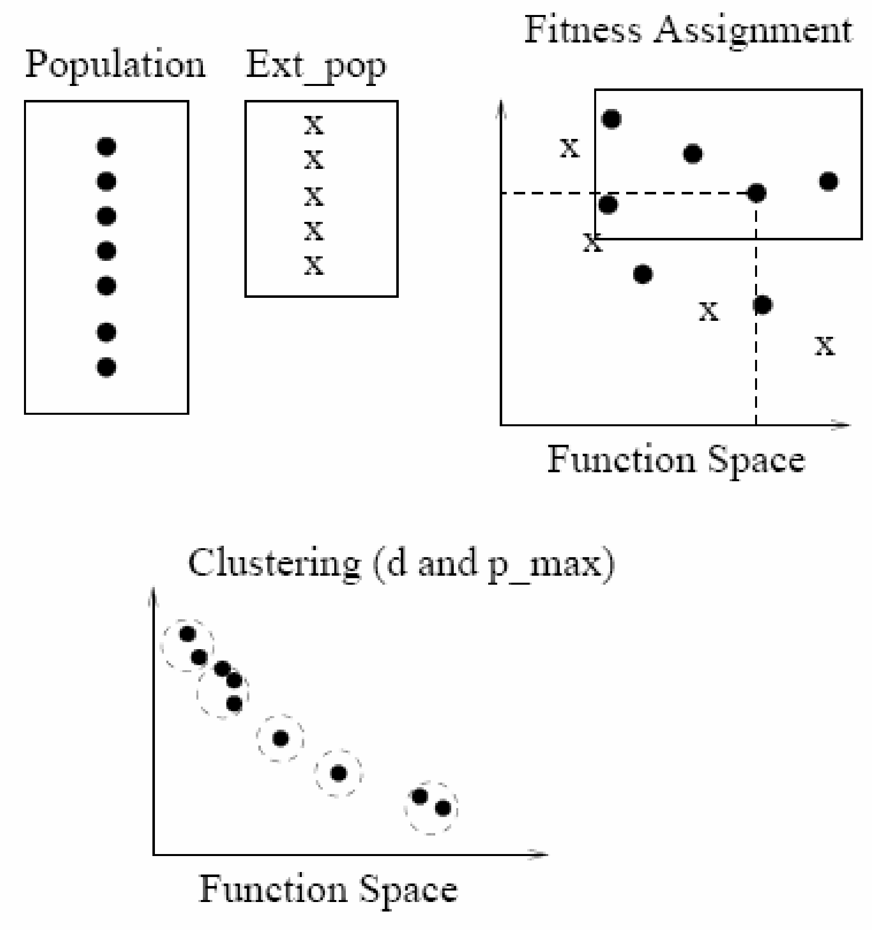
Simulation on ZDT4



Zitzler and Thiele's (1999) Strength Pareto EA (SPEA)

- Stores non-dominated solutions externally
- Pareto-dominance to assign fitness
- External members: Assign number of dominated solutions in population (smaller -> better)
- Population members: Assign sum of fitness of external dominating members (smaller->better)
- Tournament selection and recombination applied to combined population
- A clustering technique to maintain diversity in updated external population, when size increases a limit

Fitness Assignment and Clustering in SPEA



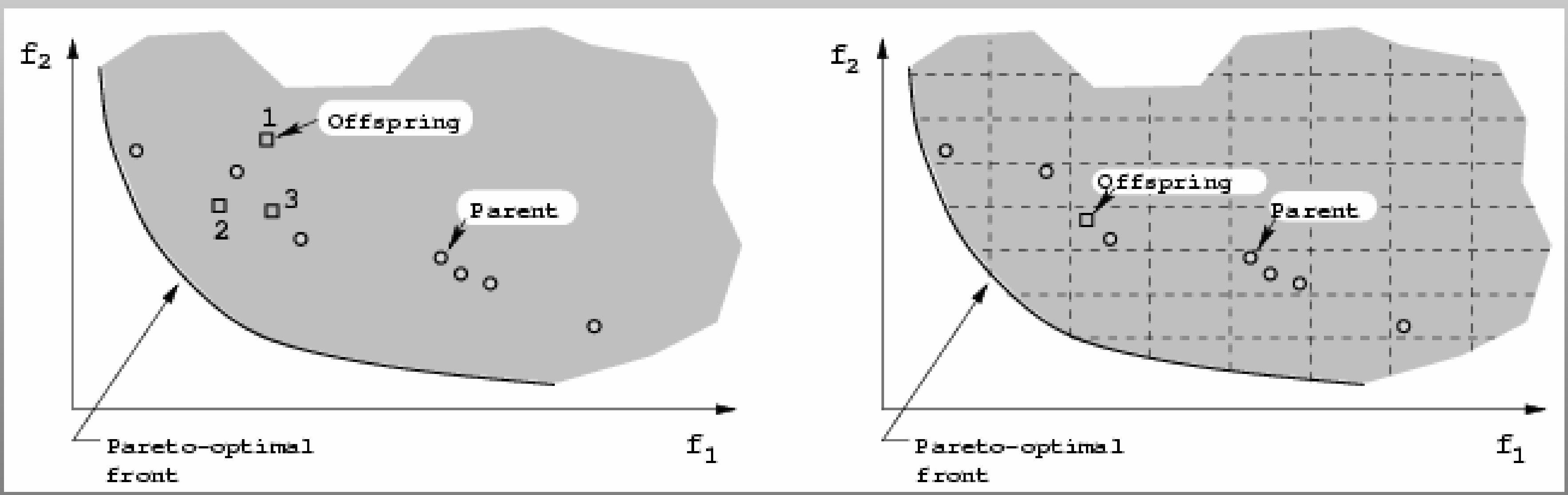
- SPEA:
  - $O(MN^3)$ operation
- SPEA2
  - Improved clustering for not losing boundary points

$$k=\sqrt{N+\bar{N}}$$

  - Fixed archive size
  - Mating within archive members
  - Dominating points use different fitness

Pareto Archived ES (PAES)

- A point-by-point approach
- Parent p_t and child c_t are compared with an external archive A_t
- Child can enter the archive and can become a parent



Comparative Results: Convergence

Algorithm		SCH	FON	POL	KUR
NSGA-II		0.003391	0.001931	0.015553	0.028964
		0	0	0.000001	0.000018
SPEA		0.003403	0.125692	0.037812	0.045617
		0	0.000038	0.000088	0.00005
PAES		0.001313	0.151263	0.030864	0.057323
		0.000003	0.000905	0.000431	0.011989

Algorithm	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6
NSGA-II	0.033482	0.072391	0.114500	0.513053	0.296564
	0.004750	0.031689	0.007940	0.118460	0.013135
SPEA	0.001799	0.001339	0.047517	7.340299	0.221138
	0.000001	0	0.000047	6.572516	0.000449
PAES	0.082085	0.126276	0.023872	0.854816	0.085469
	0.008679	0.036877	0.00001	0.527238	0.006664

IEEE TEC (2002) By Deb et al.

Comparative Results: Diversity

Algorithm		SCH	FON	POL	KUR
NSGA-II		0.477899	0.378065	0.452150	0.411477
		0.003471	0.000639	0.002868	0.000992
SPEA		1.021110	0.792352	0.972783	0.852990
		0.004372	0.005546	0.008475	0.002619
PAES		1.063288	1.162528	1.020007	1.079838
		0.002868	0.008945	0	0.013772

Algorithm	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6
NSGA-II	0.390307	0.430776	0.738540	0.702612	0.668025
	0.001876	0.004721	0.019706	0.064648	0.009923
SPEA	0.784525	0.755148	0.672938	0.798463	0.849389
	0.004440	0.004521	0.003587	0.014616	0.002713
PAES	1.229794	1.165942	0.789920	0.870458	1.153052
	0.004839	0.007682	0.001653	0.101399	0.003916

Constrained Handling

- Penalty function approach

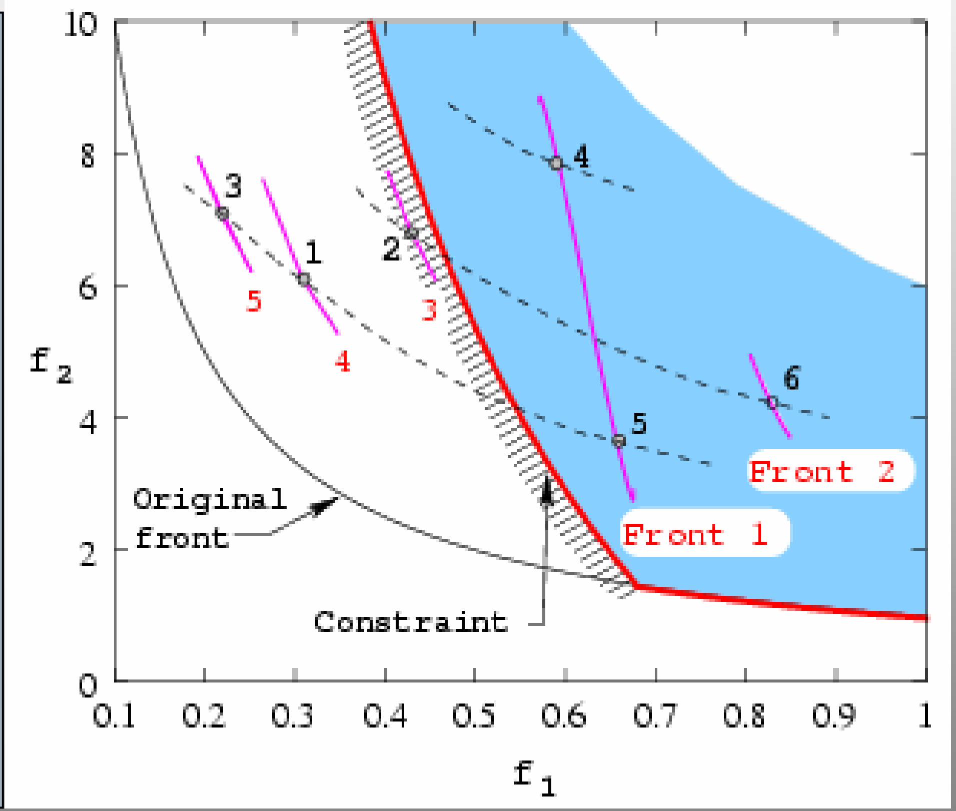
$$F_m = f_m + R_m \Omega \left(\vec{g} \right)$$

- Explicit procedures to handle infeasible solutions
  - Jimenez's approach
  - Ray-Tang-Seow's approach
- Modified definition of domination
  - Fonseca and Fleming's approach
  - Deb et al.'s approach

Constraint-Domination Principle

A solution *i* constraint-dominates a solution *j*, if any is true:

1. *i* is feasible and *j* is not
2. *i* and *j* are both infeasible, but *i* has a smaller overall constraint violation
3. *i* and *j* are feasible and *i* dominates *j*



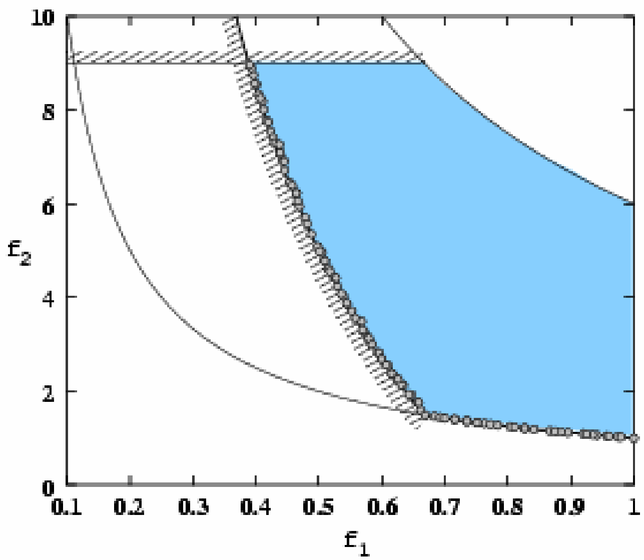
Constrained NSGA-II Simulation Results

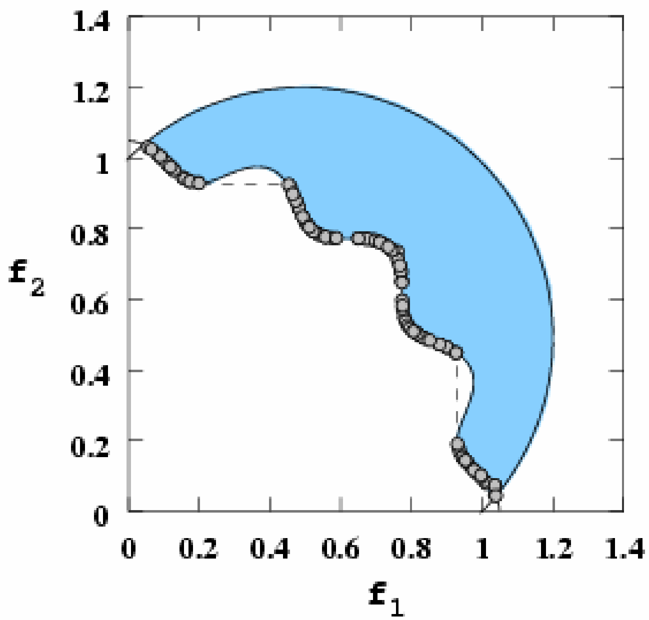
Minimize $f_1(x) = x_1$ $\quad f_2(x) = \dfrac{1+x_2}{x_1}$

Where
$$x_2 + 9x_1 \geq 6$$
$$-x_2 + 9x_1 \geq 1$$

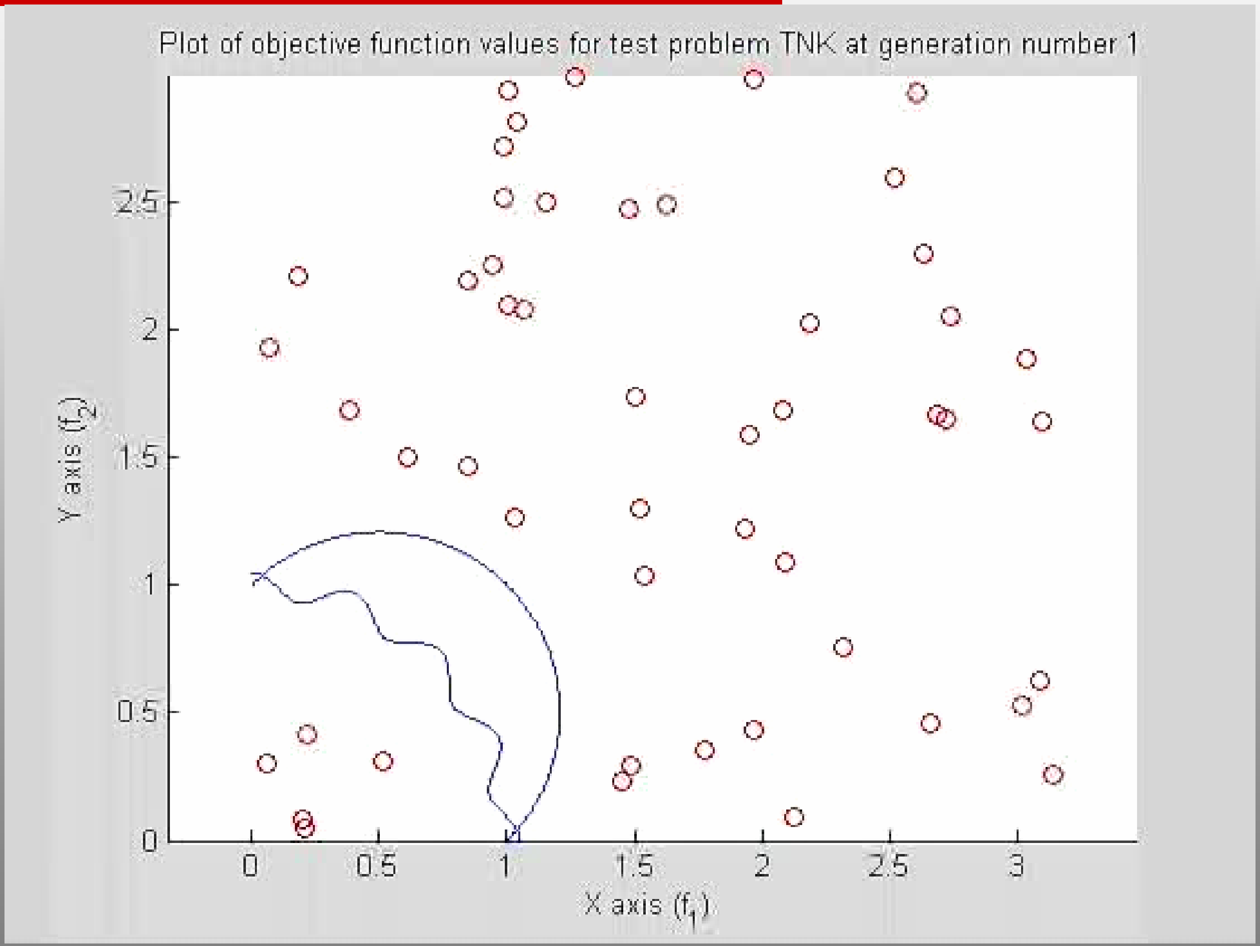
Minimize $f_1(x) = x_1$ $\quad f_2(x) = x_2$

Where
$$x_1^2 + x_2^2 - 1 - \frac{1}{10}\cos\left(16\tan^{-1}\frac{x_1}{x_2}\right) \geq 0$$
$$(x_1 - 0.5)^2 + (x_2 - 0.5)^2 \leq 0.5$$

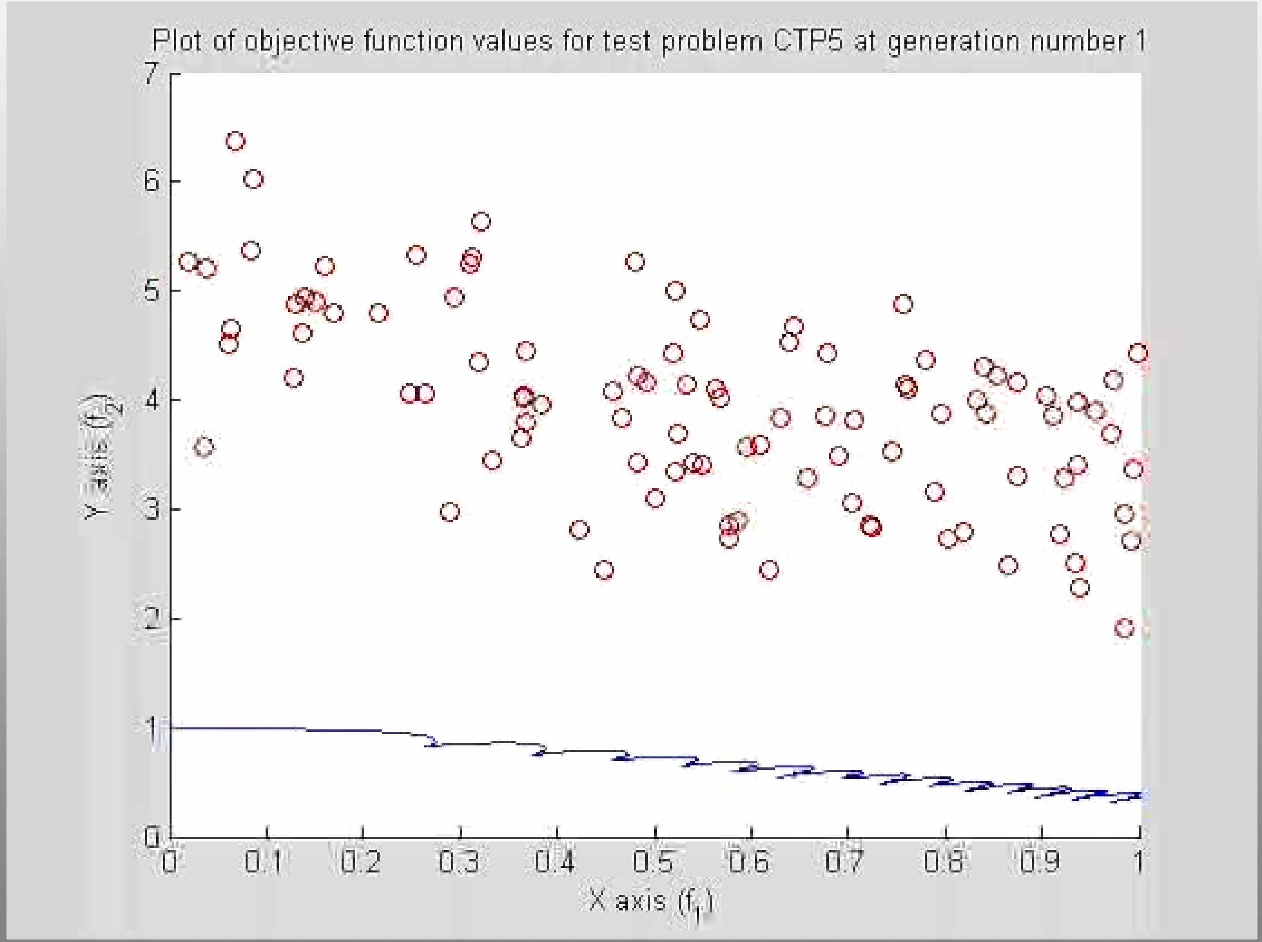




Simulation on TNK



Simulation on CTP5

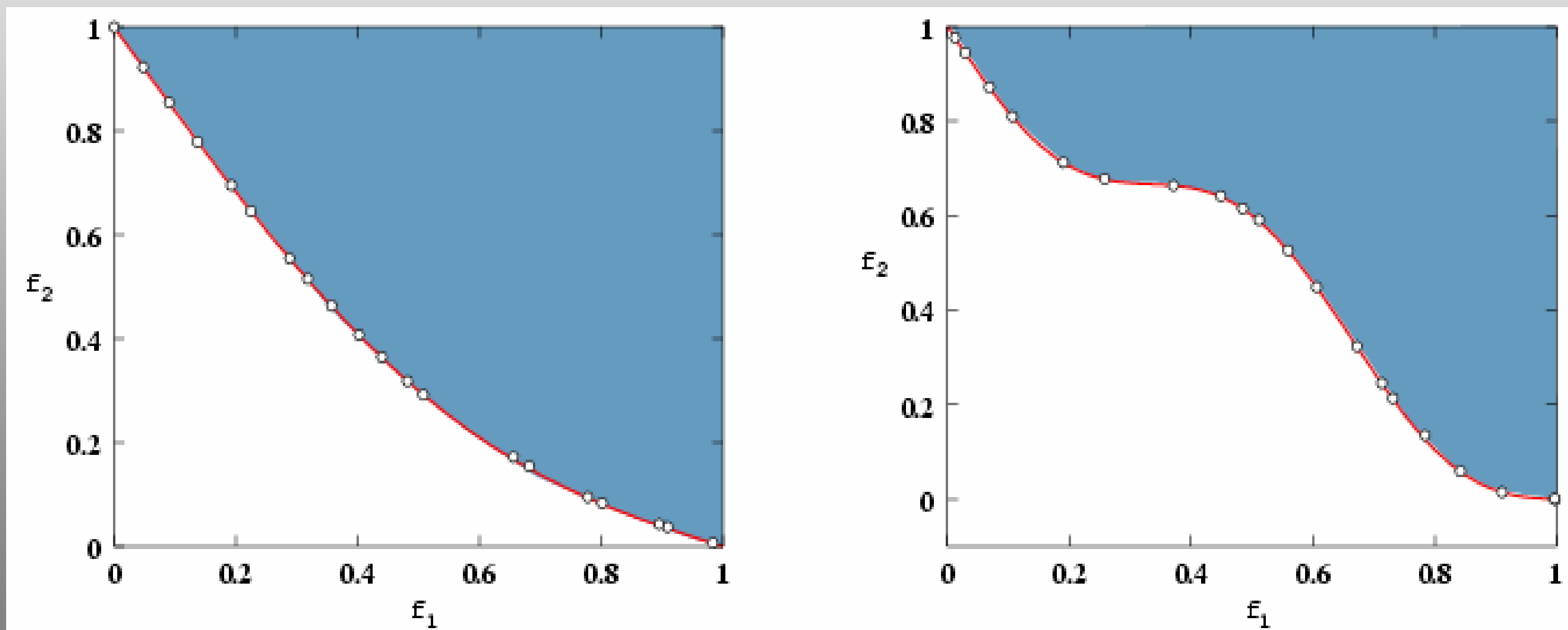


Achieving Confidence in NSGA-II Solutions

- NSGA-II is a numerical method
- Verify with ε-constraint method
- Verify the extreme solutions
- Verify by other means (NBI, NC, etc.)
- Verify by lower-dimensional solutions
- Cluster the frontier
  - Check to see if they are KKT points
  - Norm of gradient expression is close to zero

Advantages of EMO

- Shape of Pareto-optimal front is not a matter
  - Discontinuity, disconnectedness, nonconvexity etc.
- No need to rediscover important common properties



Shortcomings of EMO

- No proof of convergence in a finite time
- Diversity preservation prohibits such a proof
- Can never tell proximity to Pareto-optimal front
- Defining diversity in higher dimensions difficult
- Large number of points to represent a large-dimensional Pareto-optimal front

Conclusions of Part A

- EMO procedures can find multiple Pareto-optimal solutions in one simulation run
  - Parallel search
  - Computationally faster approach than classical generating methods
- An optimal front provides an idea of objective interactions
  - Decision-making better and easier
  - Not possible before
- Part B discusses scope of EMO application

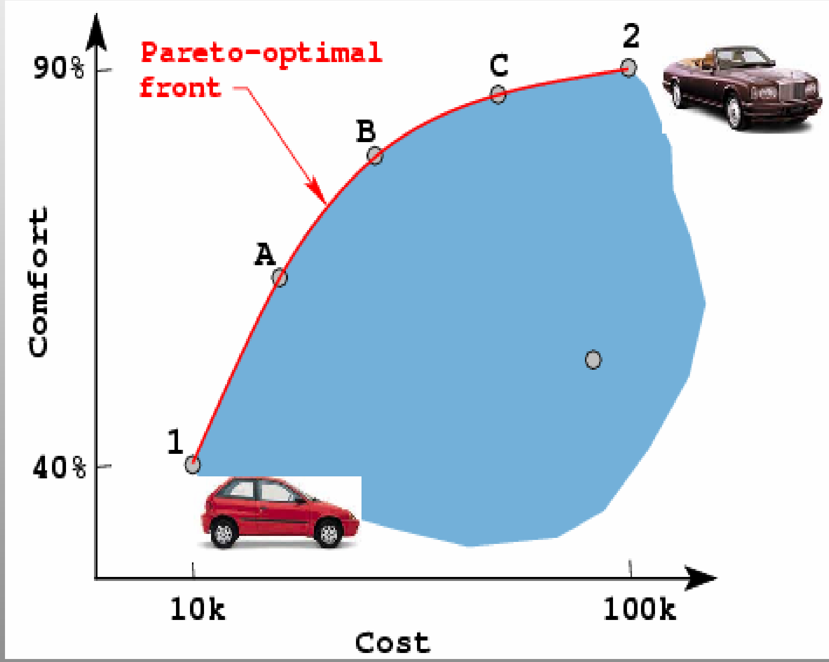
Part B: Application Studies in EMO

- EMO Applications in three directions
  - Better decision-making
  - Unveiling common principles
  - Solving other optimization problems

Look for a new book
on different uses of EMO
(Springer, December, 2007)

Decision-Making Easier

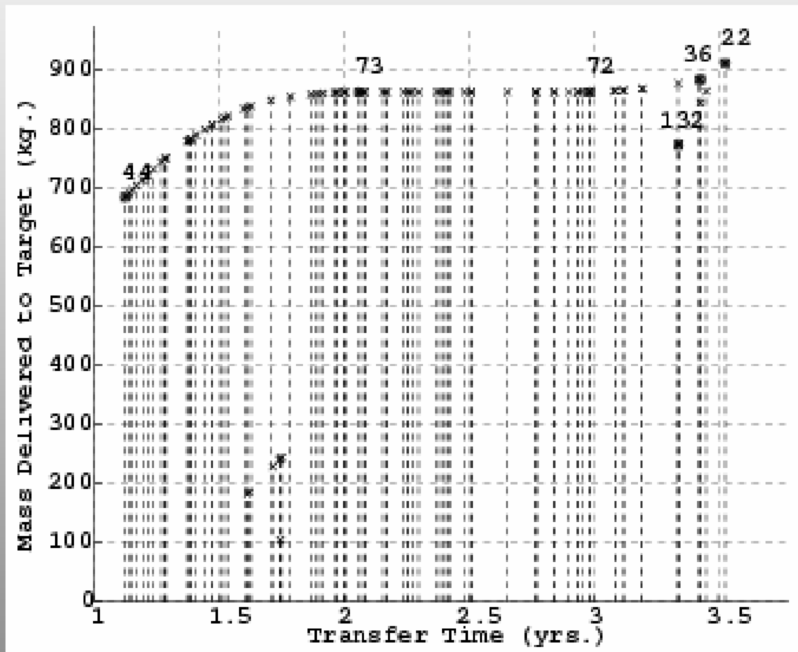
- Existence of multiple trade-off solutions
  - Provide trade-off information
  - A better idea of the nature of Pareto-optimal front
  - An idea of range of solutions
- Weighted scheme with 70-30
  - Always wonder what if
  - 69-31 or 71-29?

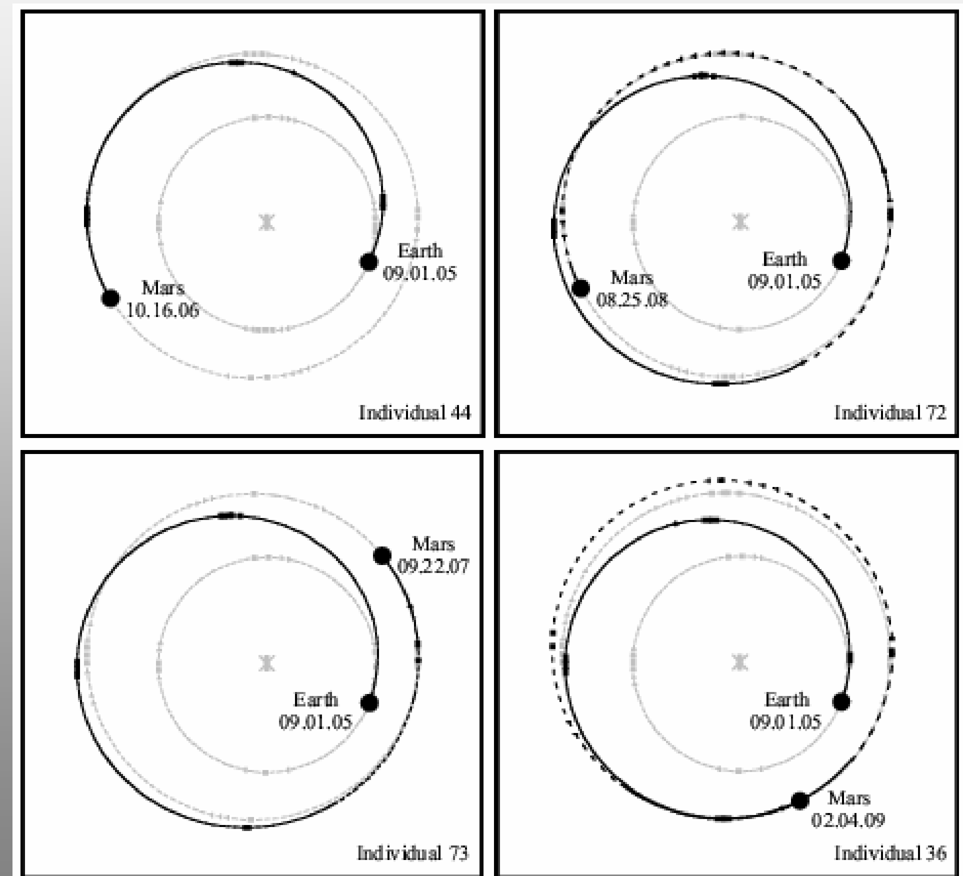


For a Better Decision-Making

- Spacecraft trajectory optimization (Coverstone-Carroll et al. (2000) with JPL Pasadena)
- Three objectives for inter-planetary trajectory design
  - Minimize time of flight
  - Maximize payload delivered at destination
  - Maximize heliocentric revolutions around the Sun
- NSGA invoked with SEPTOP software for evaluation

Earth-Mars Rendezvous



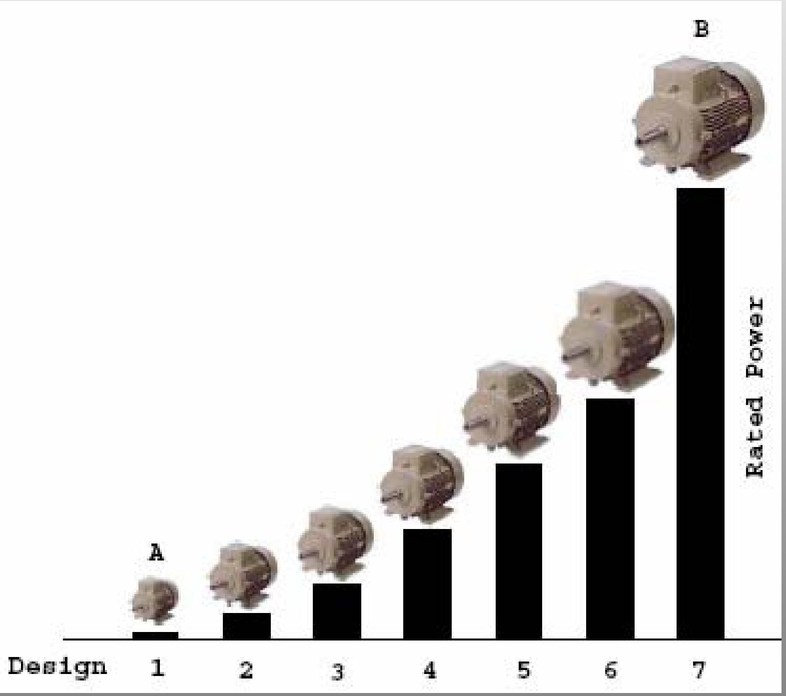


Innovization:

Discovery of Innovative design principles through optimization

- Understand important design principles in a routine design scenario

- Example: Electric motor design with varying ratings, say 1 to 10 kW
  - Each will vary in size and power
  - Armature size, number of turns etc.
- How do solutions vary?
  - Any common principles!



Single versus Multiple Objectives

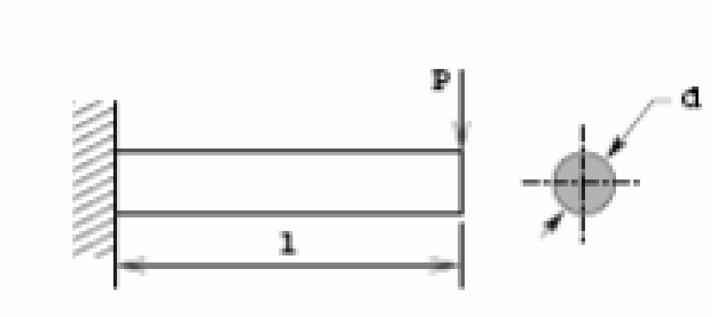
- Say, a cantilever beam design for minimum weight

Minimize $f_1(d,l) = \rho \frac{\pi d^2}{4} l$

subject to $\frac{32Pl}{\pi d^3} \leq S_y$

$0.01 \leq d \leq 0.05$

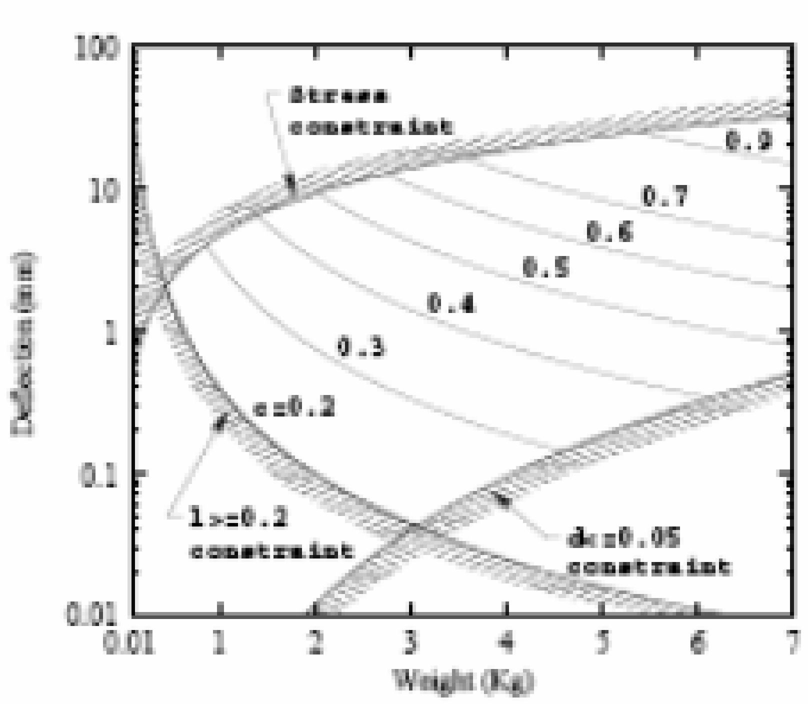
$0.2 \leq l \leq 1.0$



- Optimal design:
  - d=18.94 mm, l=200 mm, defl. = 2 mm
- Want defl.=1 mm, what design?
- Redo optimization with a constraint
- Turns out: d=22.52 mm, l=200 mm

Knowledge Discovery

- Minimize (weight,defl.)
- Try if *l*=*c* is true
- Innovization:
  - Set l = c = 0.2m to be optimal
  - Range of d: (18.94,50) mm
- Knowledge discovery!
- How do systemize the procedure?



Innovization Procedure

- Choose two or more conflicting objectives (e.g., size and power)
  - Usually, a small sized solution is less powered
- Obtain Pareto-optimal solutions using an EMO
- Investigate for any common properties manually or automatically
- Why would there be common properties?
  - Recall, Pareto-optimal solutions are all optimal!

In Search of Common Optimality Properties

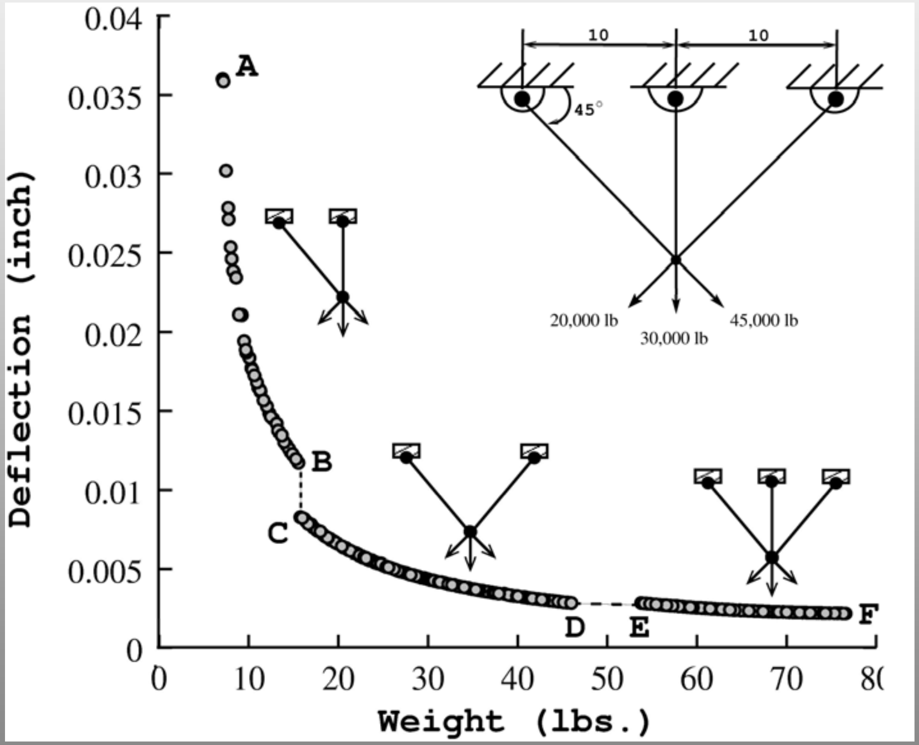
Fritz-John Necessary Condition:

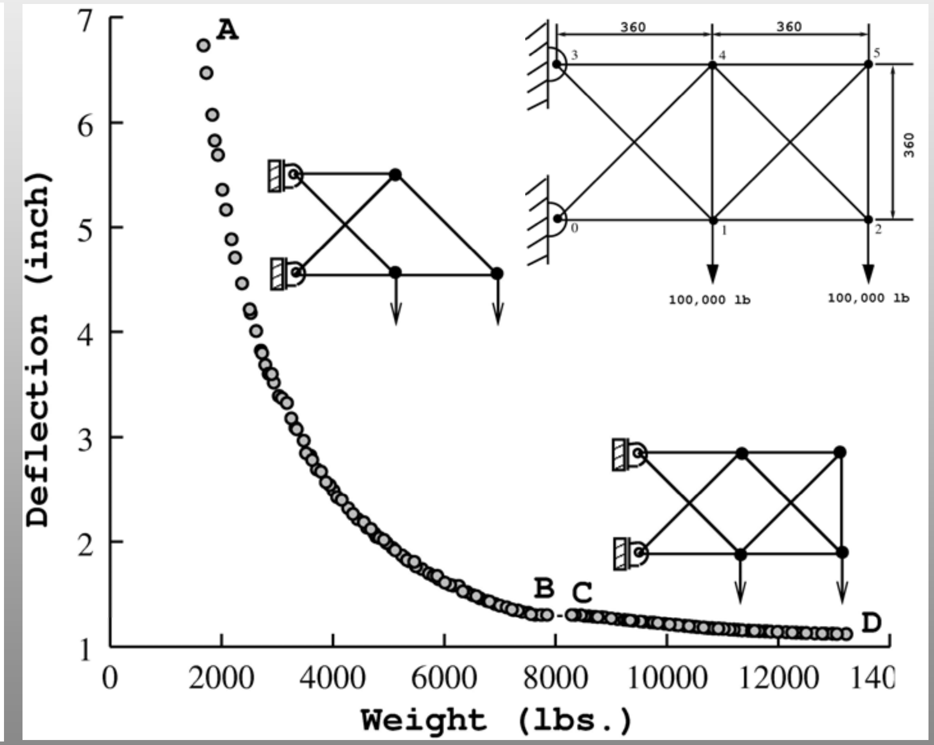
Solution x^* satisfy

1. $\sum_{m=1}^{M} \lambda_m \nabla f_m(x^*) - \sum_{j=1}^{J} u_j \nabla g_j(x^*) = 0,$ and
2. $u_j g_j(x^*) = 0$ for all $j = 1, 2, 3, \ldots\ldots, J$
3. $u_j \geq 0$, $\lambda_j \geq 0$, for all j and $\lambda_j > 0$ for at least one j

- To use above conditions requires differentiable objectives and constraints
- Yet, it lurks existence of some properties among Pareto-optimal solutions

Revealing Salient Insights: Truss Structure Design

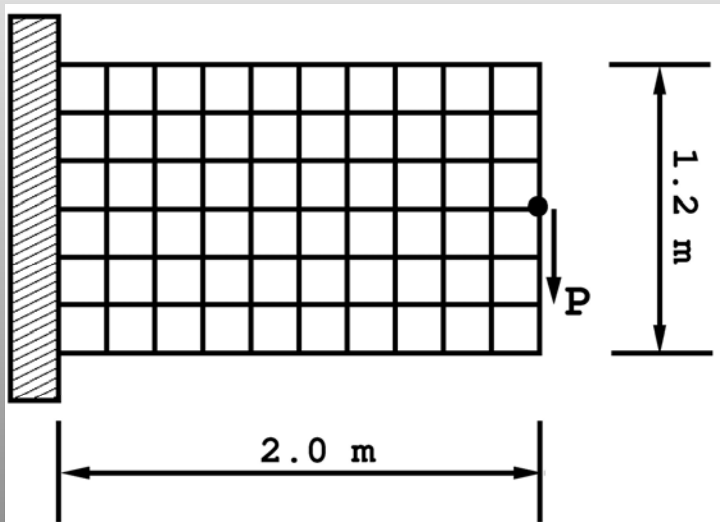




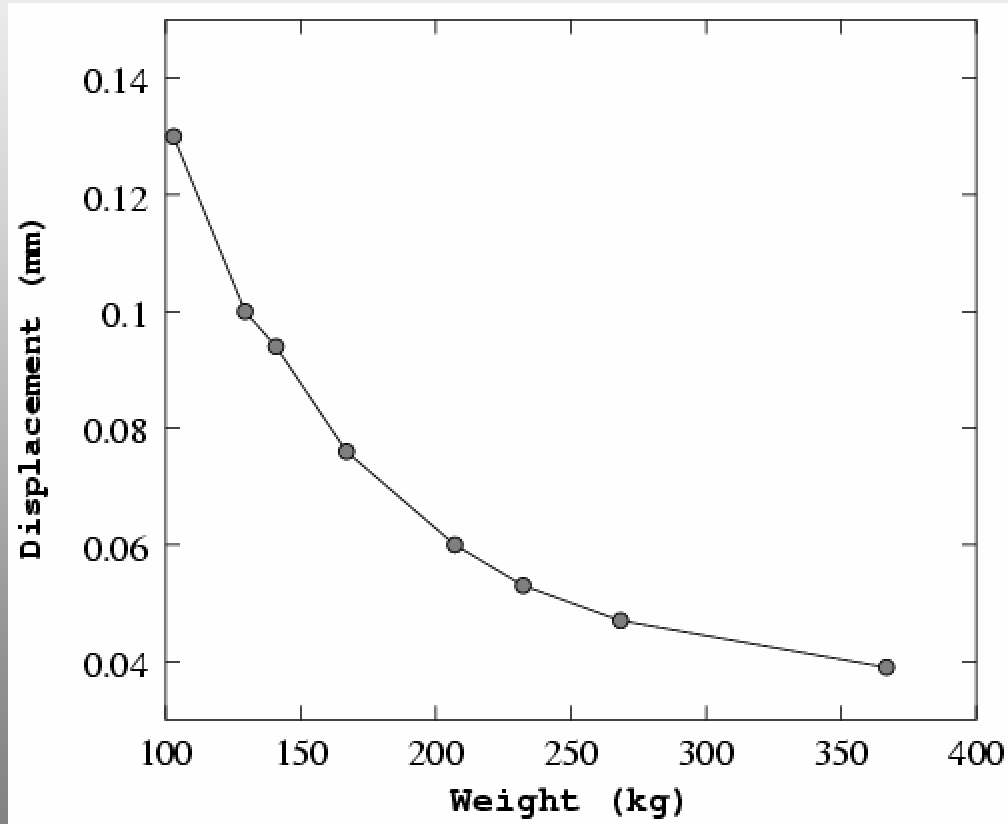
(Deb, Khan and Jindal, 2000)

Revealing Salient Insights:
A Cantilever Plate Design

Base Plate



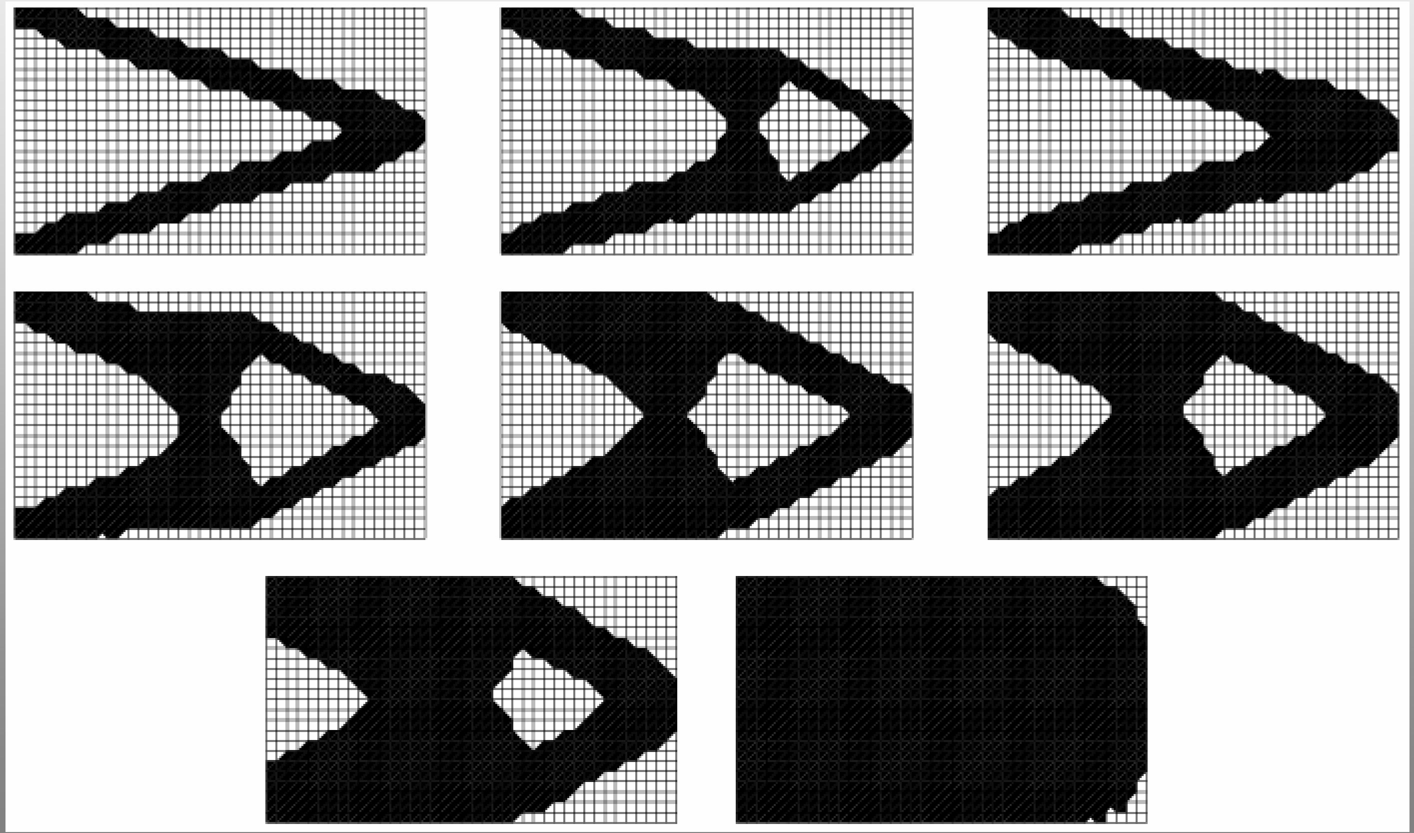
Eight trade-off solutions are chosen



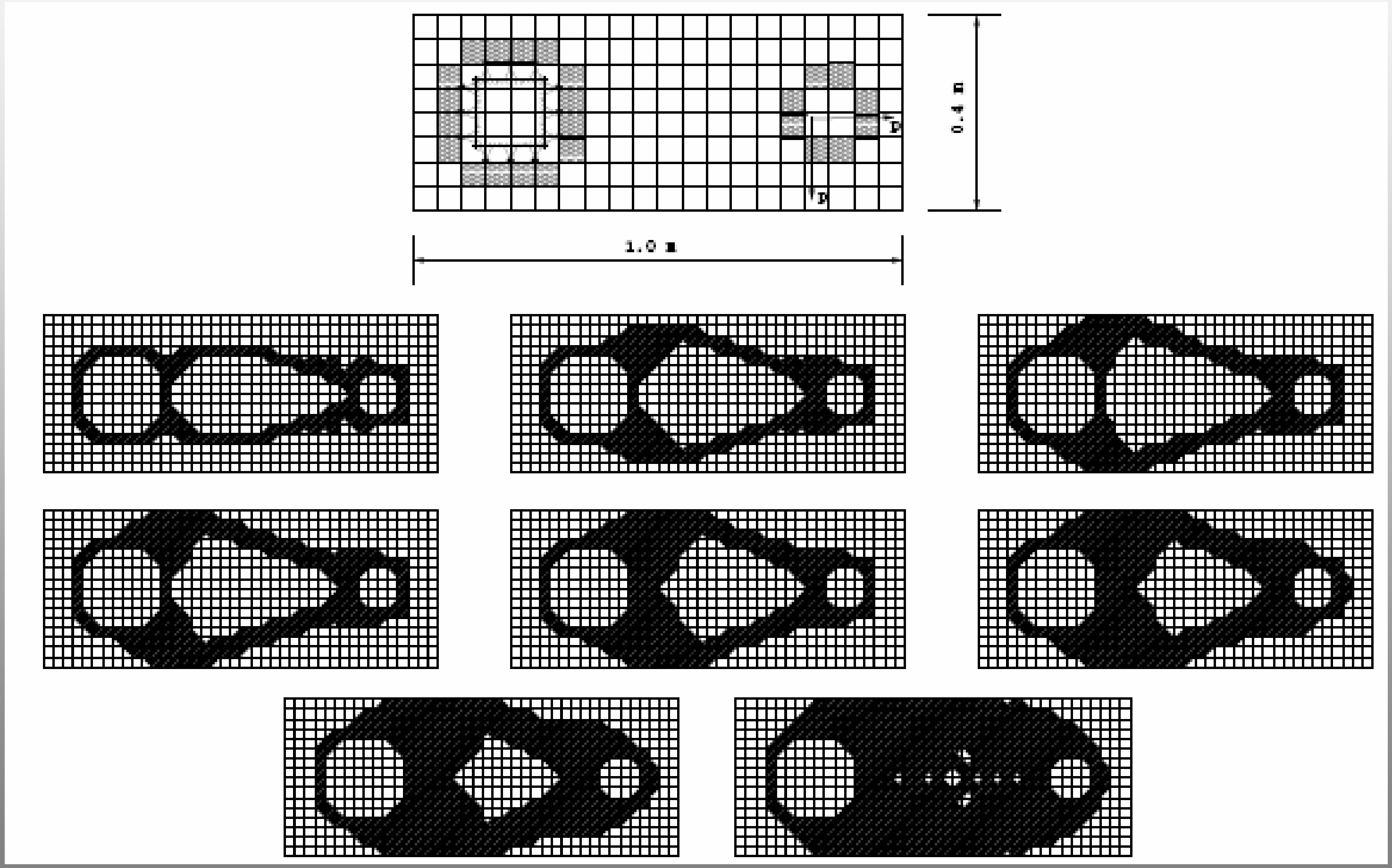
(Deb and Chaudhuri, 2003)

Trade-Off Solutions

- Symmetry in solutions about mid-plane, discovery of stiffener



A Connecting Rod



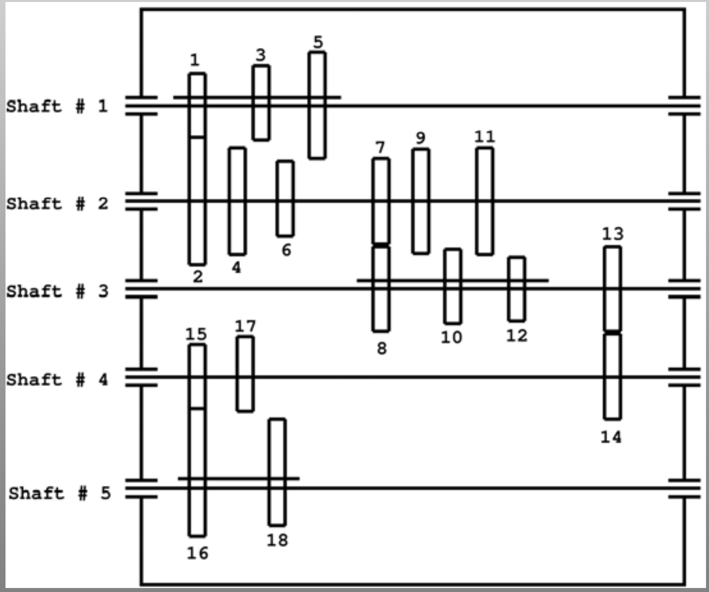
Innovized Principles

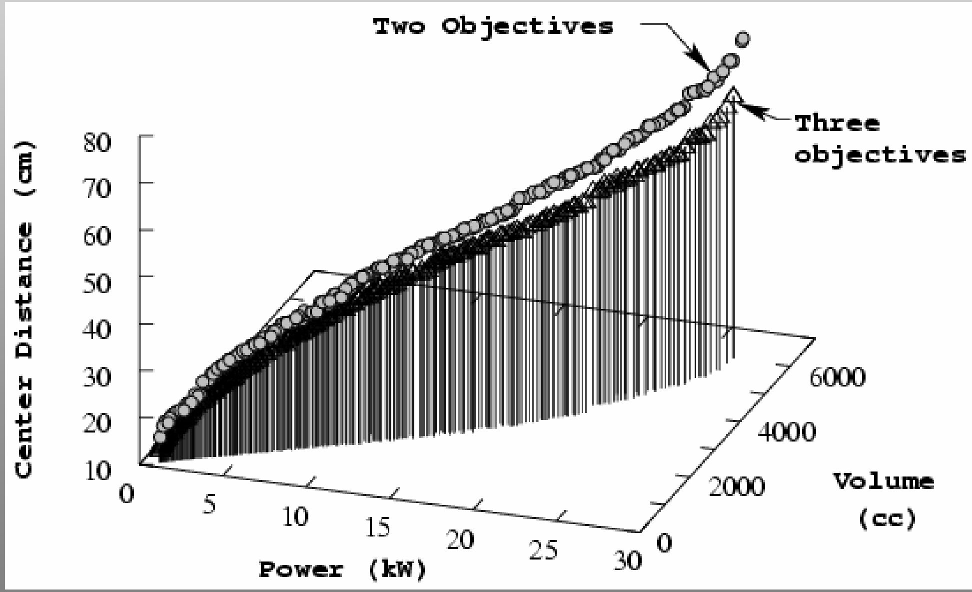
- Mid-line symmetry
- Straight arms to reach load is minimum-weight strategy
- Two ways to increase stiffness
  - Thickening of arms
  - Use of a stiffener
  - Additional stiffening by a combination
- Chamfering of corners helpful

Gear-box Design

- A multi-spindle gear-box design (Deb and Jain, 2003)
- 28 variables (integer, discrete, real-valued)
- 101 non-linear constraints

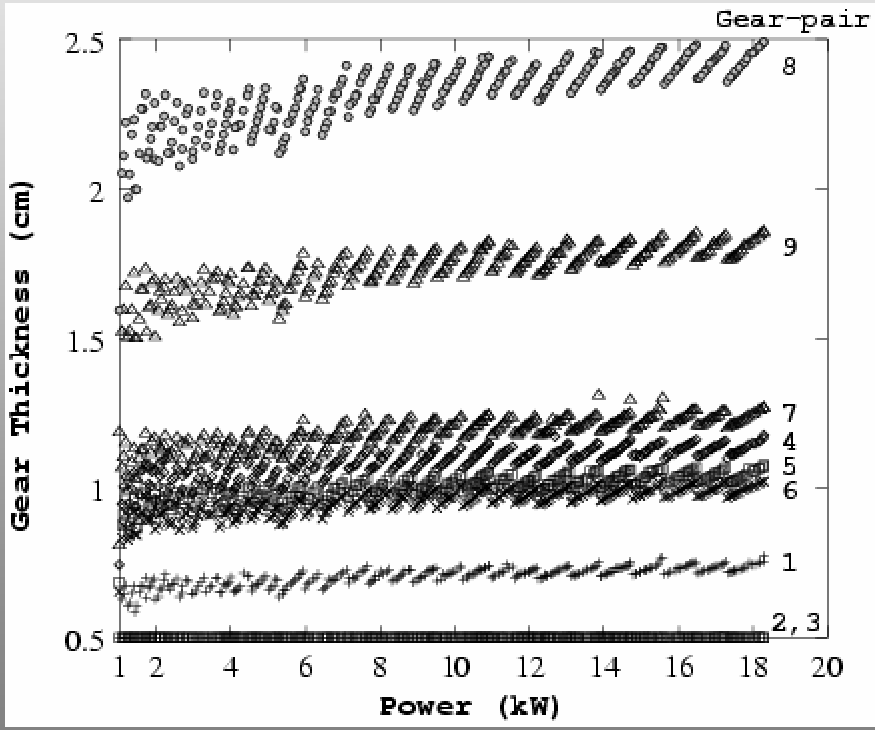
- Important insights obtained
  (larger module for more power)

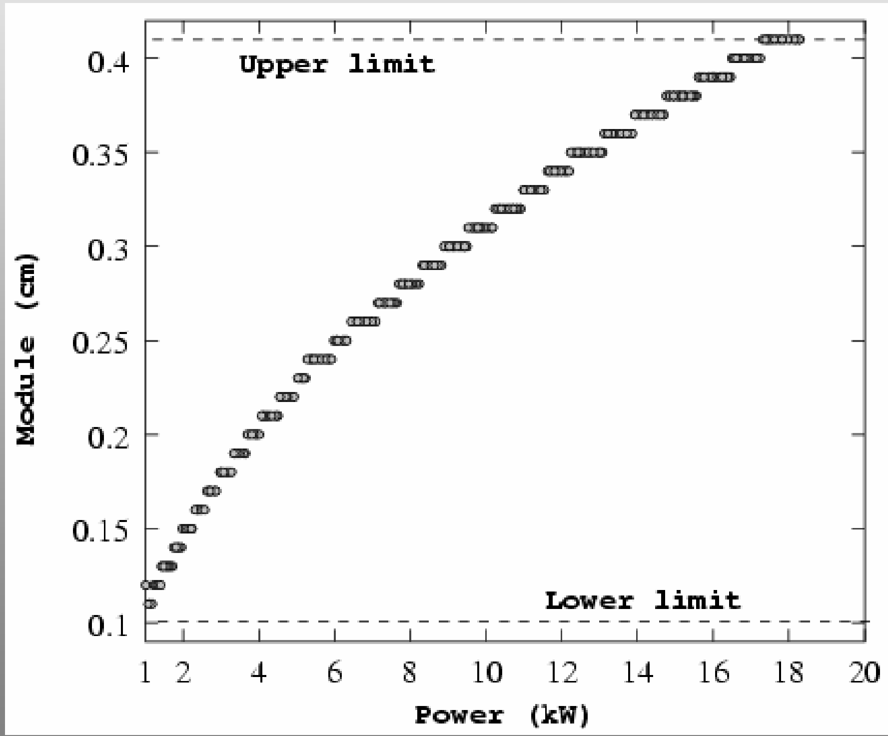




Innovized Principles

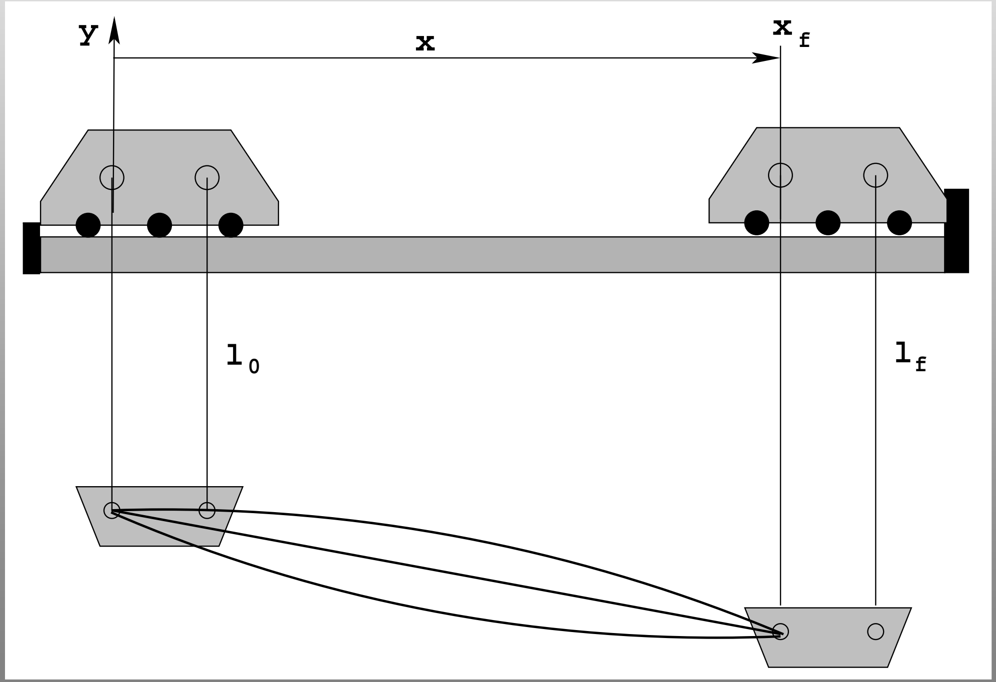
- Module varies proportional to square-root of power
- Keep other 27 variables more or less the same

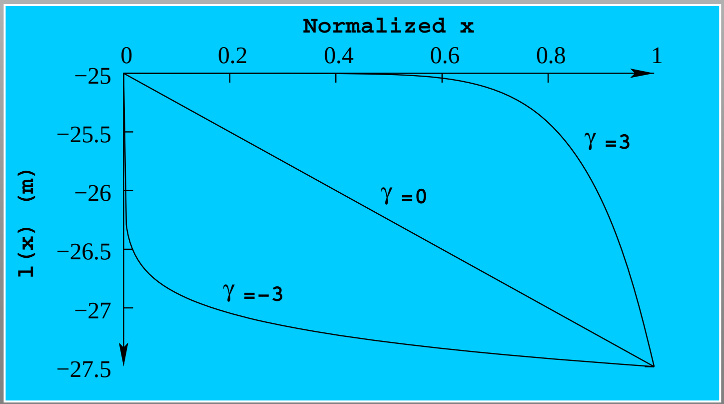




Overhead Crane Maneuvering

- Minimize time of operation
- Minimize operating energy

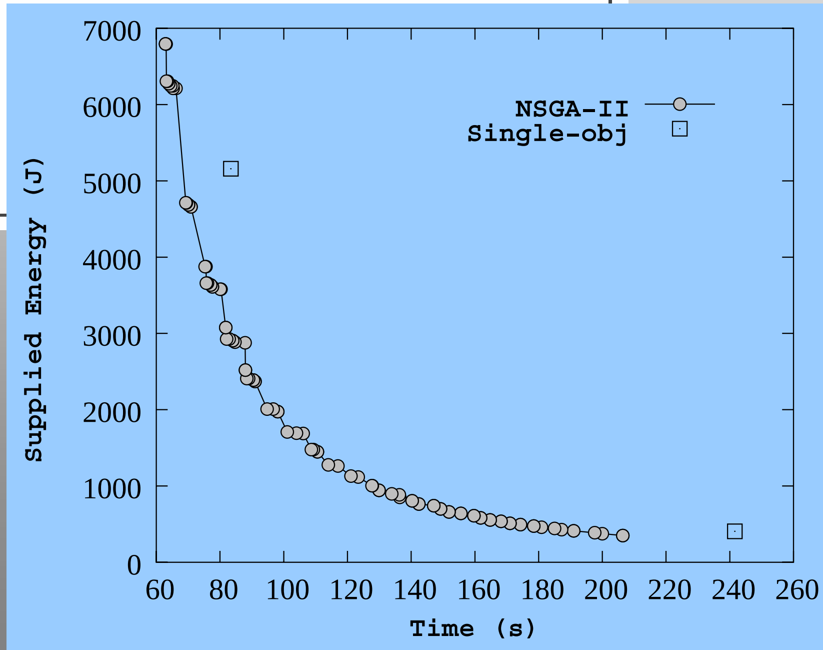




Simulation Results

γ	Force (N)	Pattern
2.98	149.26	111111110010001000000000000000000000000000000000000
3.00	187.76	11111111001000100000000000000000000000000
3.00	238.56	111111110010001000000000000000000
3.00	327.86	11111011001001100000000000
2.98	427.93	111110110010011000000
3.00	544.94	11111111001000100
2.87	629.62	111111110010001
2.88	818.99	110111010010001

- NSGA-II finds trade-off and interesting properties



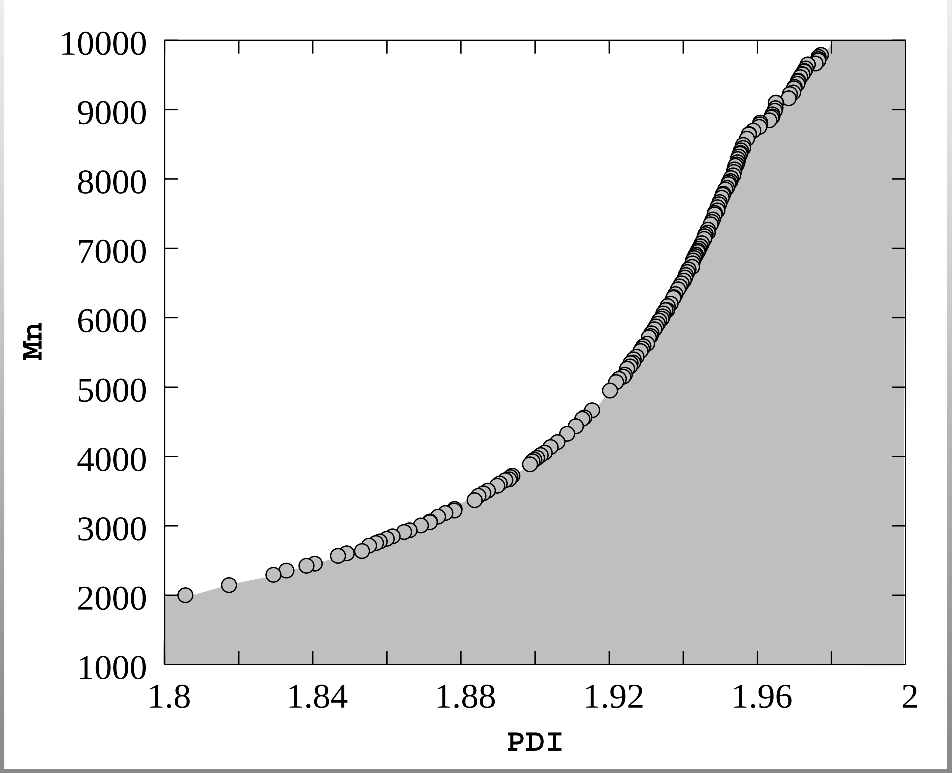
Innovative Principles

- For optimum operation:
  - Lower load suddenly at the end
  - Spend energy only at the beginning
- Fast unloading demands more energy
  - For fixed θ, E ∞ l
  - Delay lowering for saving energy

$$E = mg(1 - \cos\theta)l$$

Epoxy Polymerization

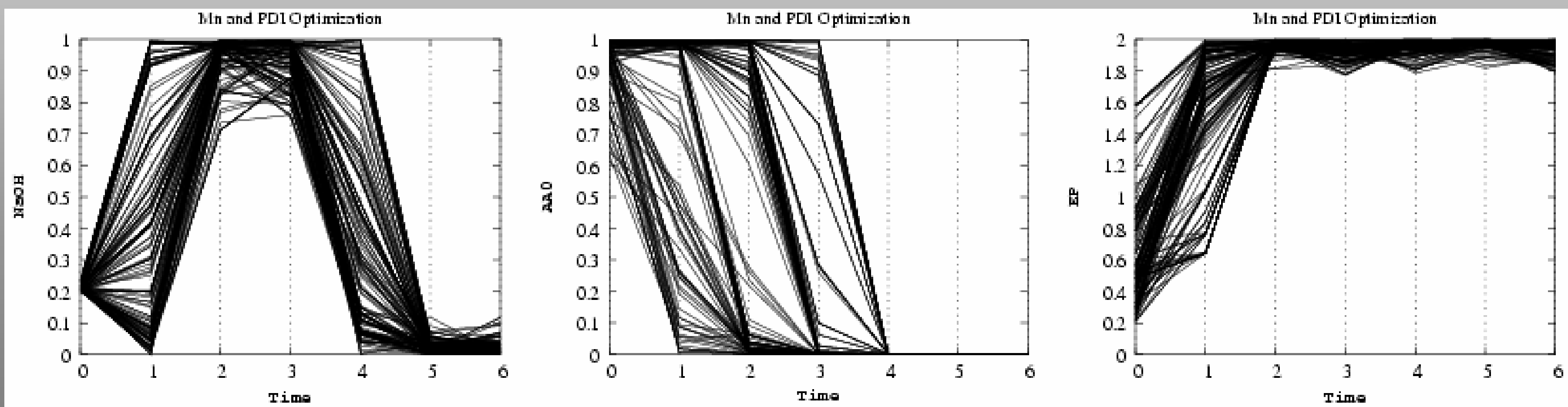
- Three ingredients added hourly
- 54 ODEs solved for a 7-hour simulation
- Maximize chain length (Mn)
- Minimize polydispersity index (PDI)
- Total 3x7 or 21 variables
- (Deb et al., 2004)



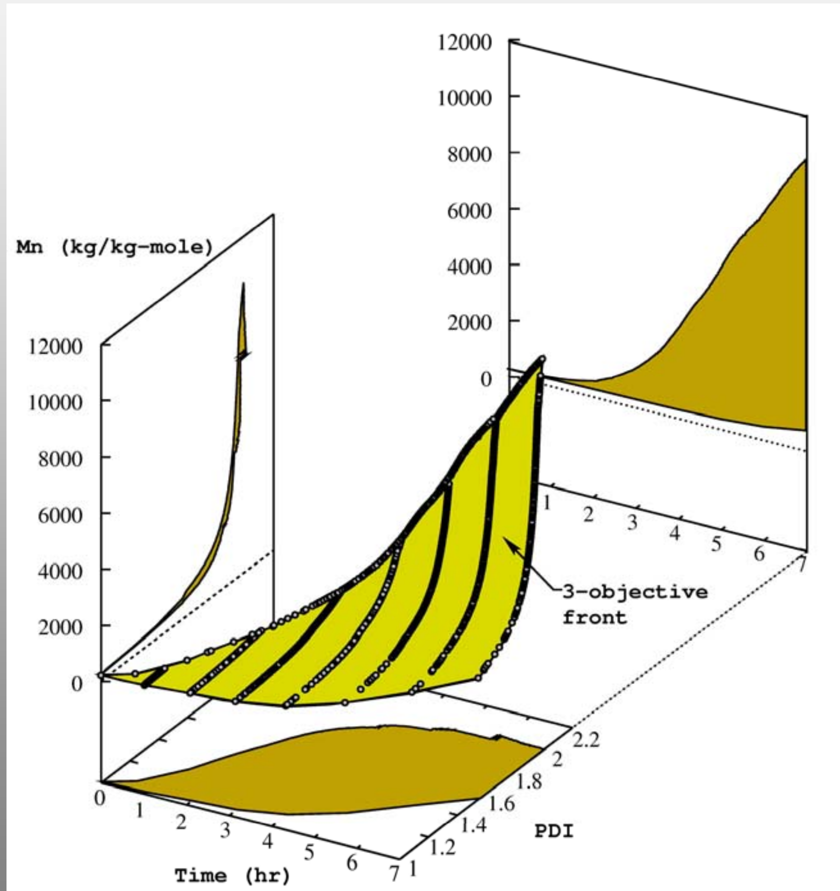
A non-convex frontier

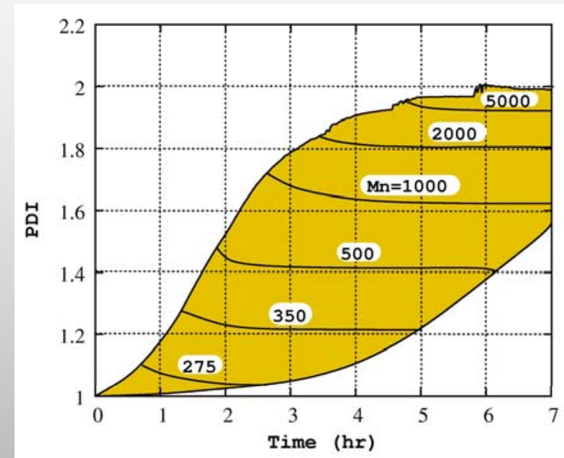
Epoxy Polymerization (cont.)

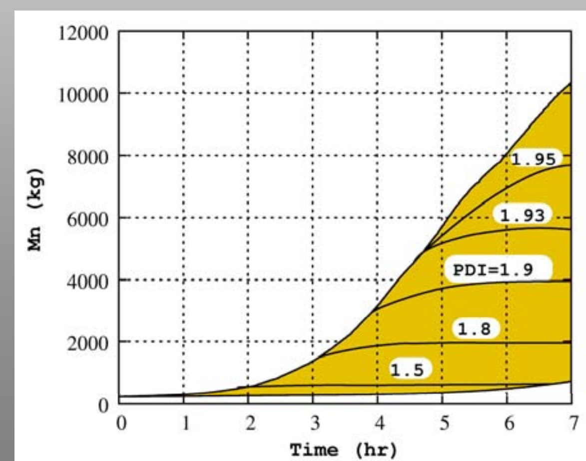
- Some patterns emerge among obtained solutions
- Chemical significance unveiled



Innovized Principles: *An Optimal Operating Chart*

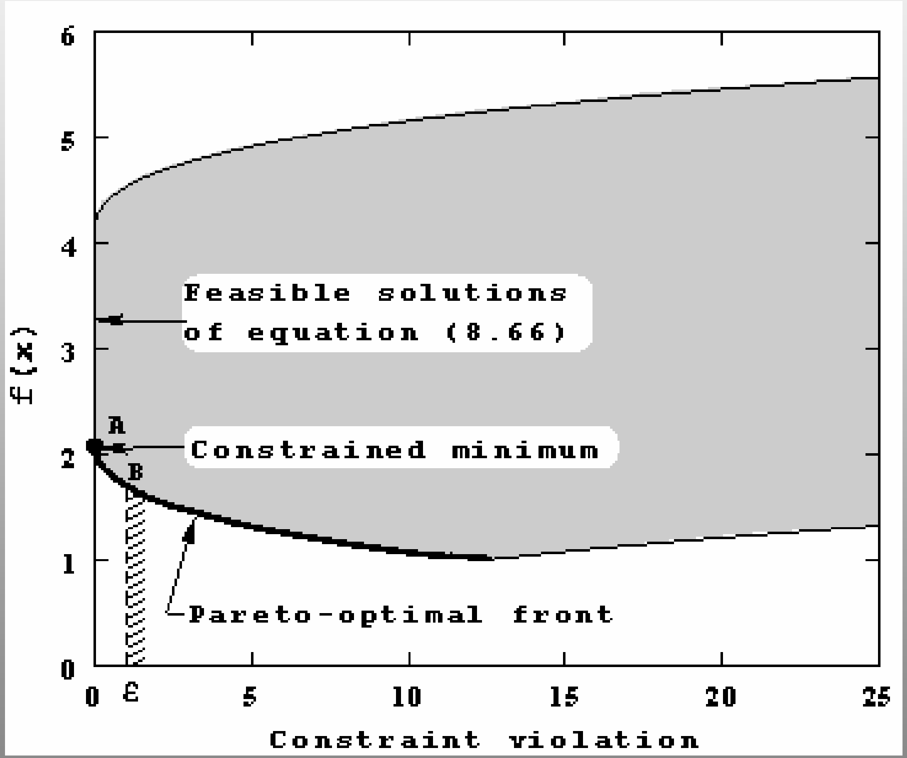






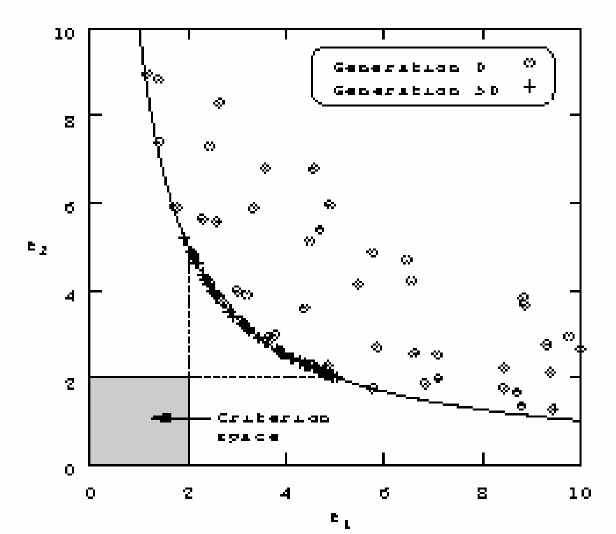
Multi-Objectivity in Other Optimization Tasks

- Constrained handling
  - Constraint violations as additional objectives
- Find partial front near zero-CV
- May provide a flexible search



Goal Programming and Others

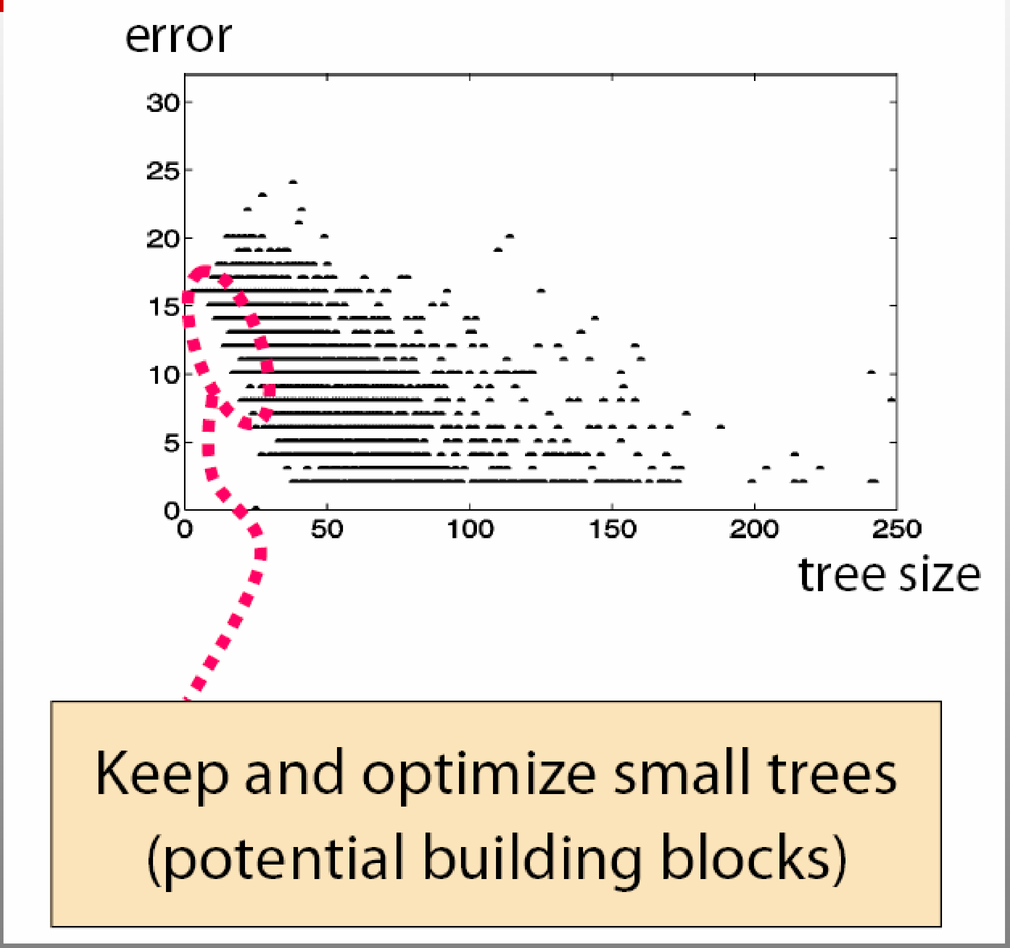
- Goal programming to find multiple solutions
- Avoids fixing a weight vector (Deb, 2001)



- Reducing the chance of getting trapped in local optima (Knowles et al., 2001)
- Use secondary objectives for maintaining diversity (Abbass and Deb, 2003, Jensen, 2003)

Reducing Bloating in GP

- Bleuler et al., (2001)
- Find small-sized programs with small error
- Minimization of Size of Program as second objective



Conclusions of Part B

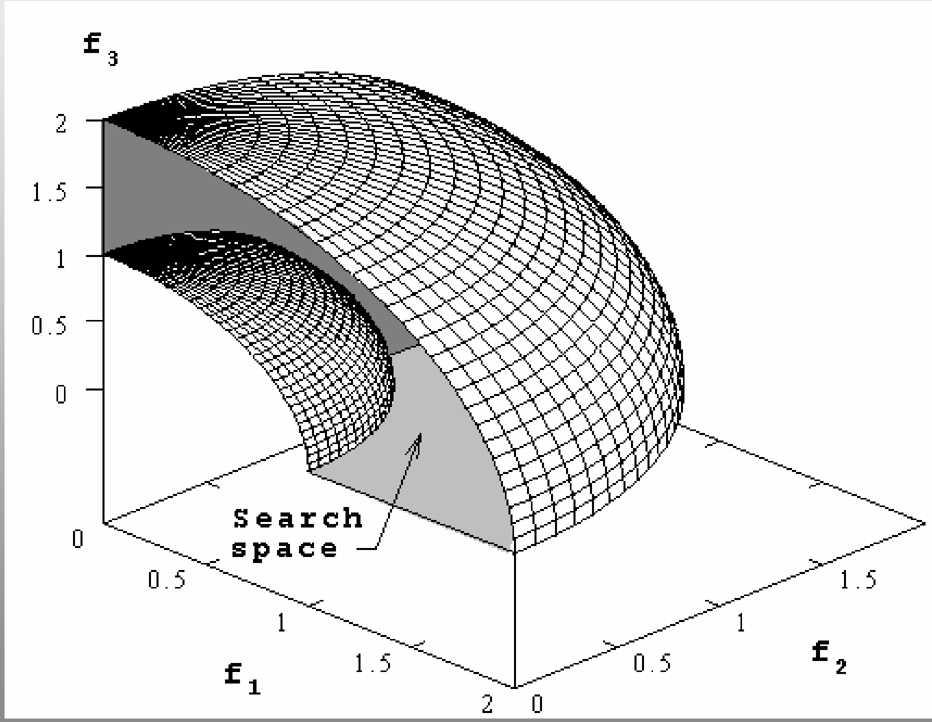
- Two kinds of applications
  - For decision making
  - For learning the problem at hand
- Other optimization problem-solving
  - Constraint handling
  - Introducing diversity
  - Clustering (smaller intra-distance, larger inter-distance)
- Part C discusses advanced issues

Part C: Advanced Studies in EMO

- Scalable test problem design
- ε-domination based EMO techniques
- Finding a partial frontier
- Distributed computing of the Pareto-optimal frontier
- Performance metrics and comparative studies
- Dynamic EMO
- Robustness and Reliability based EMO
- Interactive EMO

Scalable Test Problems

Deb et al. (CEC-2002)



- **Step 1** Define Pareto-optimal front mathematically
- **Step 2** Build the objective search space using it
- **Step 3** Map variable space to objective space

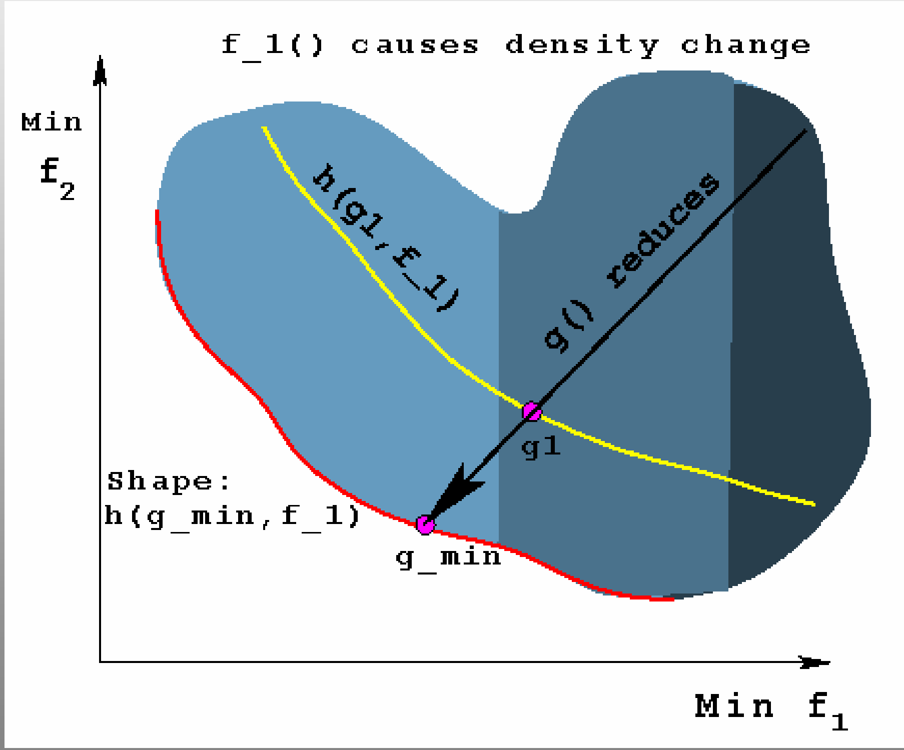
- Scalable DTLZ problems suggested

Two-Objective Test Problems

- Pareto-optimal front is controllable and known
- ZDT problems:

$$\text{Min.} \quad f_1(\mathbf{x}) = f_1(\mathbf{x}_I),$$
$$\text{Min.} \quad f_2(\mathbf{x}) = g(\mathbf{x}_{II})h(f_1, g).$$

- Choose $f_1()$, $g()$ and $h()$ to introduce various difficulties



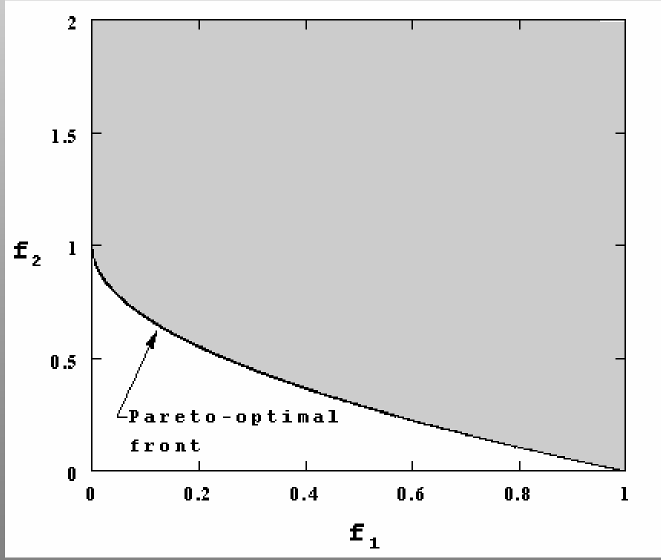
Zitzler-Deb-Thiele's Test Problems

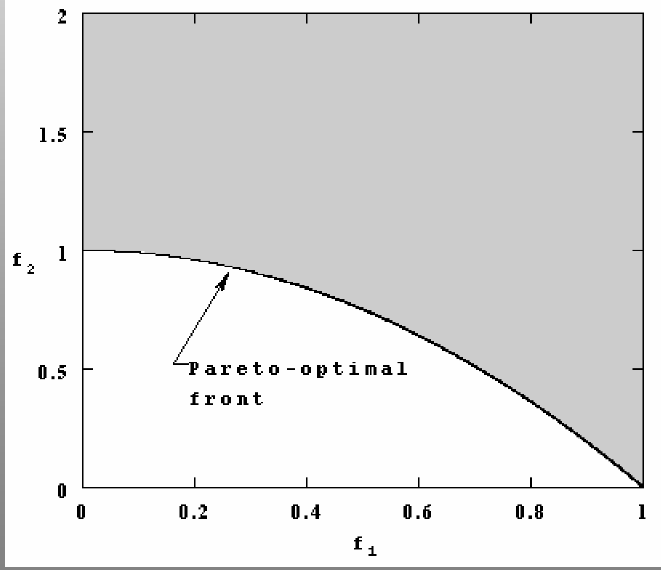
ZDT1

$$\begin{aligned} f_1(\mathbf{x}) &= x_1, \\ g(\mathbf{x}) &= 1+\tfrac{9}{n-1}\sum_{i=2}^{n} x_i, \\ h(f_1,g) &= 1-\sqrt{f_1/g}. \end{aligned}$$

ZDT2

$$\begin{aligned} f_1(\mathbf{x}) &= x_1, \\ g(\mathbf{x}) &= 1+\tfrac{9}{n-1}\sum_{i=2}^{n} x_i, \\ h(f_1,g) &= 1-(f_1/g)^2. \end{aligned}$$

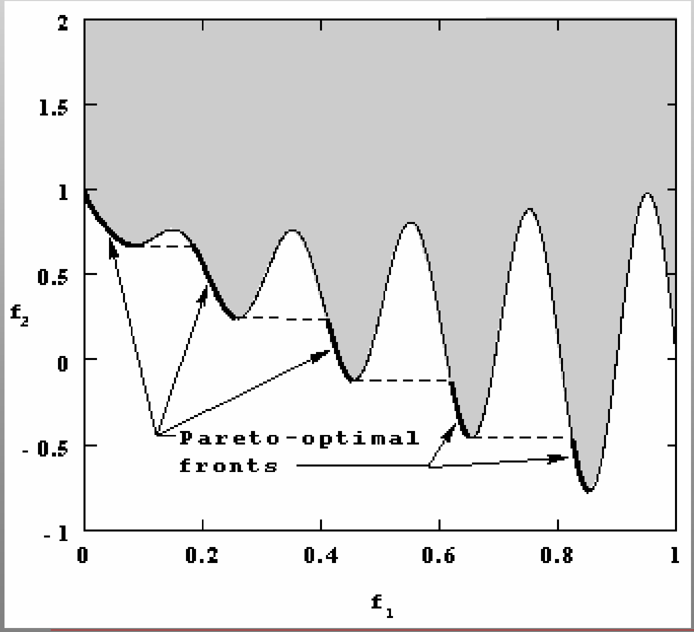




Zitzler-Deb-Thiele's Test Problems

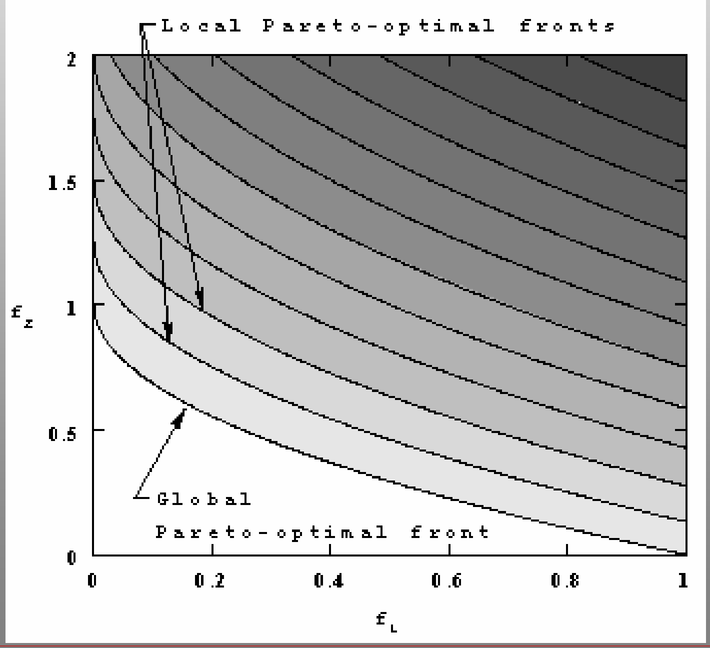
ZDT3

$$f_1 = x_1,$$
$$g = 1 + \tfrac{9}{n-1}\sum_{i=2}^{n} x_i,$$
$$h = 1 - \sqrt{f_1/g} - (f_1/g)\sin(10\pi f_1).$$



ZDT4

$$f_1 = x_1,$$
$$g = 10n - 9 + \sum_{i=2}^{n} (x_i^2 - 10\cos(4\pi x_i)),$$
$$h = 1 - \sqrt{f_1/g}.$$



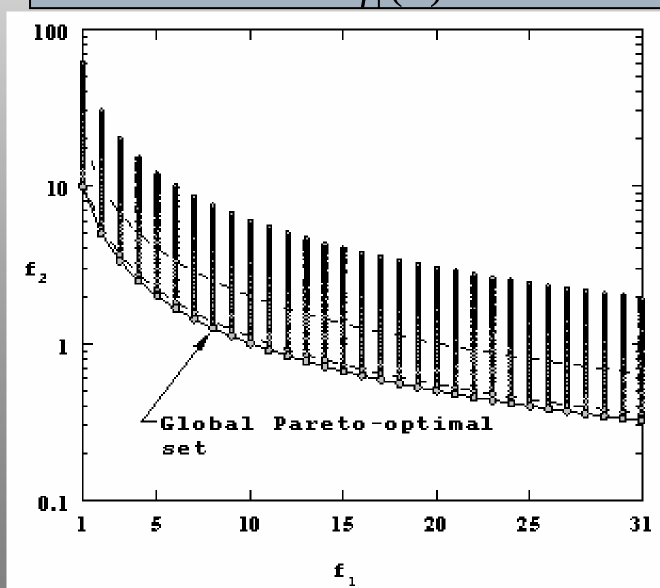
Zitzler-Deb-Thiele's Test Problems

ZDT5

$$f_1 = 1 + u(x_1),\ g = \sum_{i=2}^{11} v(u(x_i))$$

$$v = \begin{cases} 2 + u(x_i) & \text{if } u(x_i) < 5, \\ 1 & \text{if } u(x_i) = 5, \end{cases}$$

$$h = 1/f_1(\mathbf{x})$$

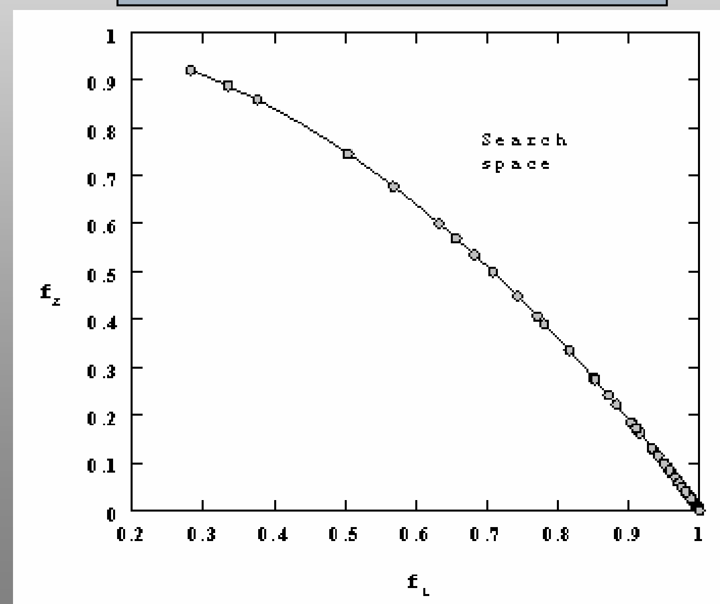


ZDT6

$$f_1 = 1 - \exp(-4x_1)\sin^6(6\pi x_1),$$

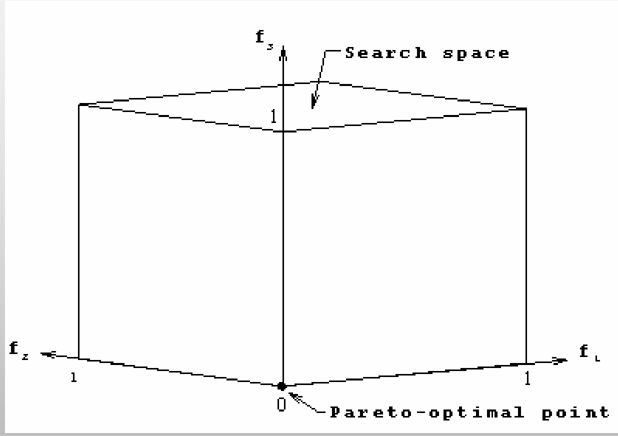
$$g = 1 + 9\left[(\textstyle\sum_{i=2}^{10} x_i)/9\right]^{0.25},$$

$$h = 1 - (f_1/g)^2.$$

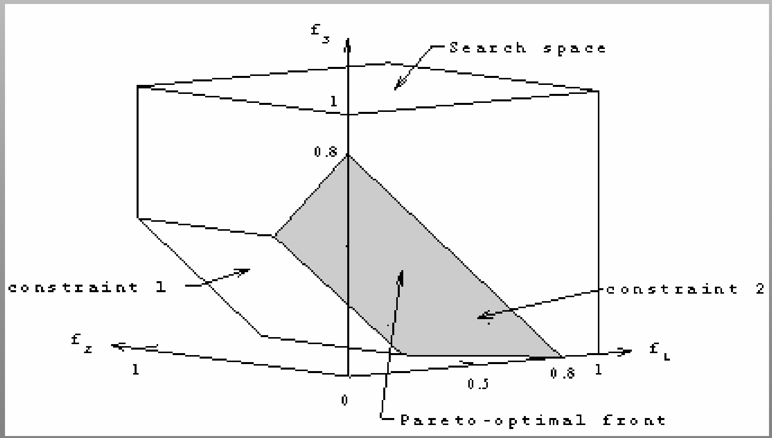


Constraint Surface Approach

- Step 1: Define a rectangular hyper-box
- Step 2: Chop off regions using constraints



- Adv: Easy to construct
- Disadv: Difficult to define Pareto-optimal front



Constrained Test Problem Generator

- Some test problems in Veldhuizen (1999)
- More controllable test problems are called for

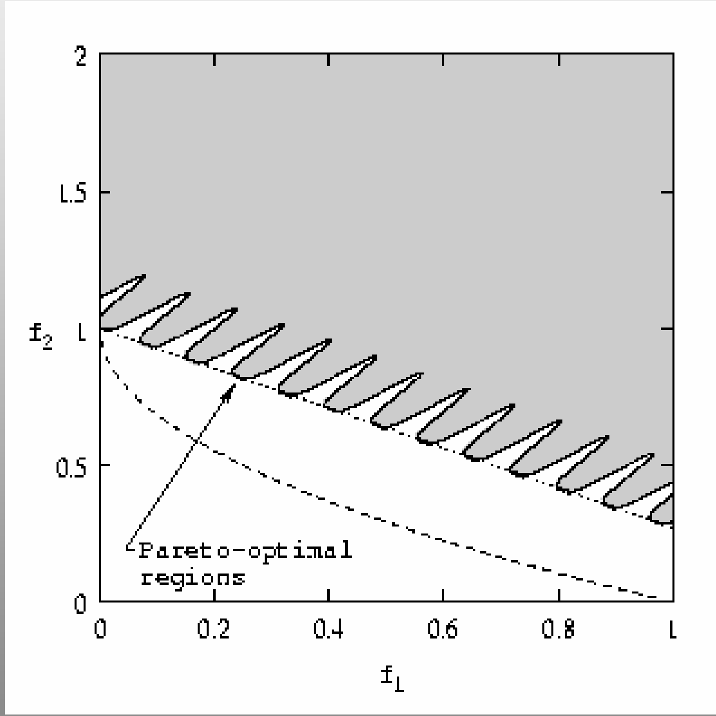
Minimize $$f_1(\mathbf{x}) = x_1$$

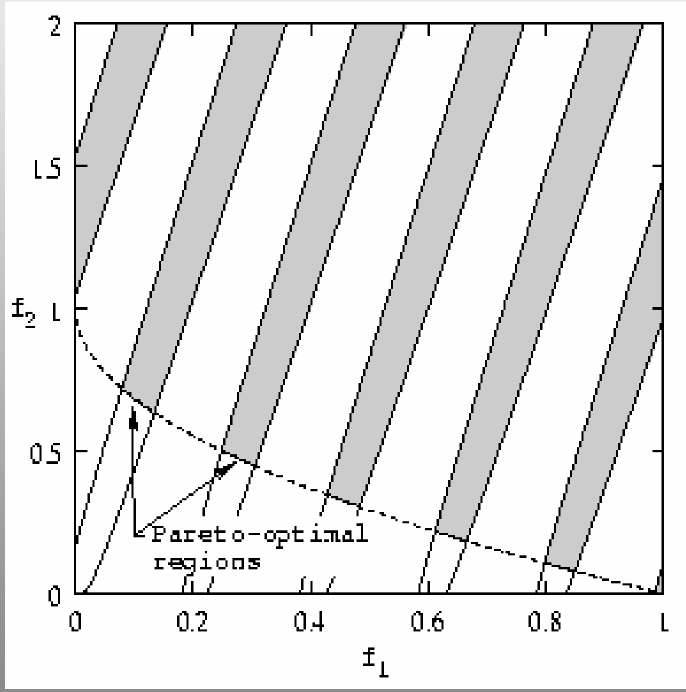
Minimize $$f_2(\mathbf{x}) = g(\mathbf{x})\left(1 - \frac{f_1(\mathbf{x})}{g(\mathbf{x})}\right)$$

Subject to $$c(\mathbf{x}) \equiv \cos(\theta)(f_2(\mathbf{x}) - e) - \sin(\theta) f_1(\mathbf{x}) \geq a\left|\sin\left(b\pi\left(\sin(\theta)(f_2(\mathbf{x}) - e) + \cos(\theta) f_1(\mathbf{x})\right)^c\right)\right|^d$$

Various Parameter Settings

CTP2: $\theta = -0.2\pi,\ a = 2,\ b = 10,\ c = 1,\ d = 6,\ e = 1$

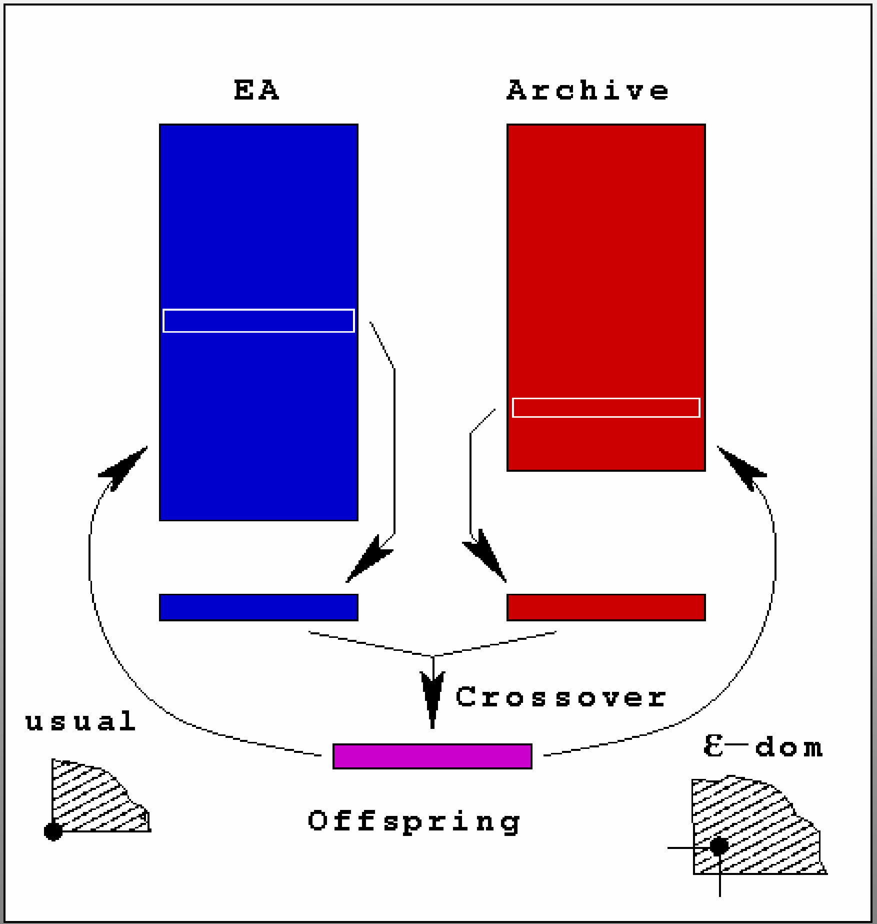




CTP7: $\theta = -0.05\pi,\ a = 40,\ b = 5,\ c = 1,\ d = 6,\ e = 0$

ε-MOEA: Using ε-Dominance

- EA and archive populations evolve
- One EA and one archive member are mated
- Archive update using ε-dominance
- EA update using usual dominance

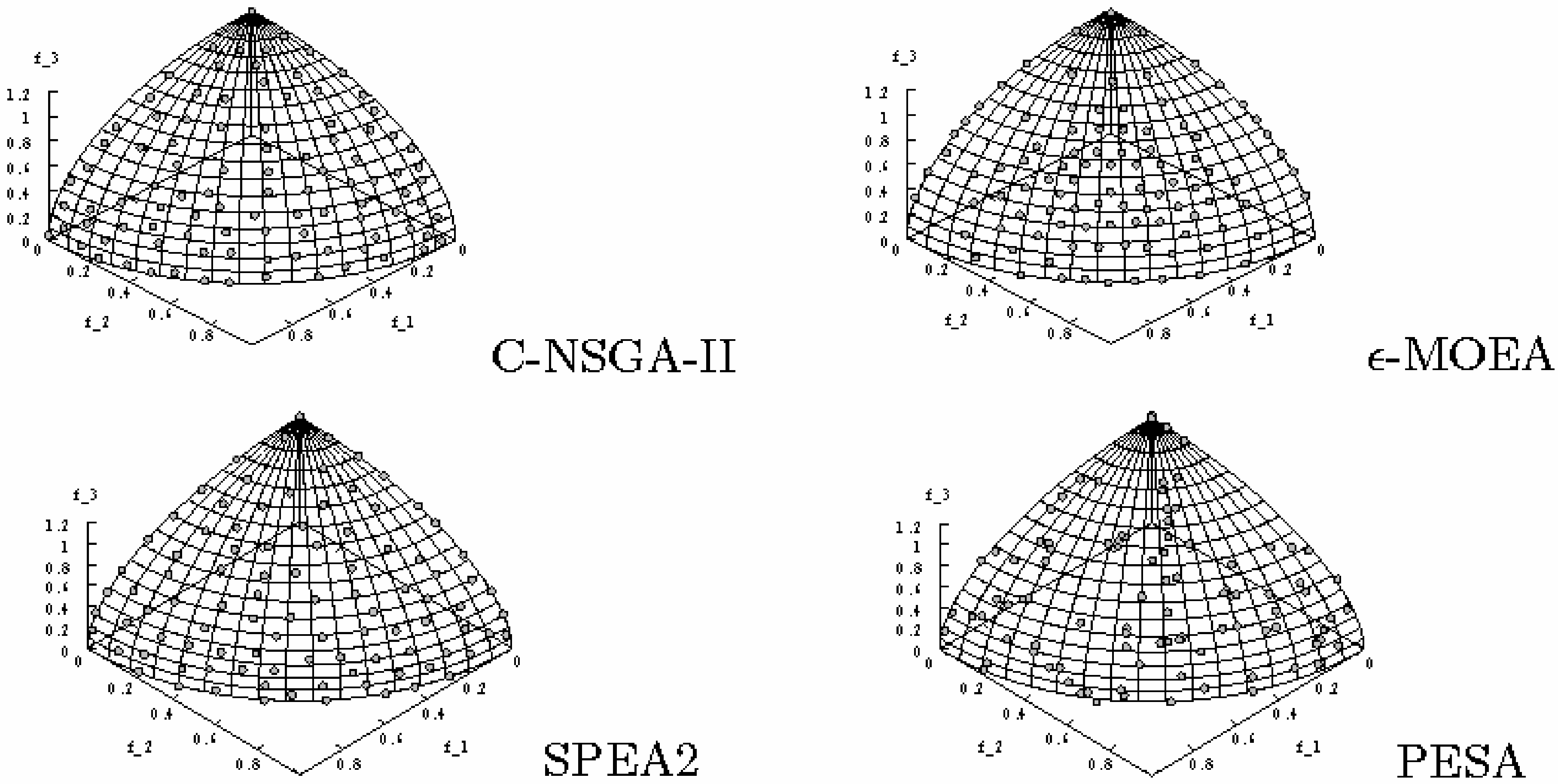


Deb, Mohan & Mishra (ECJ-2005)

Comparative Study on Three-Obj. DTLZ Functions

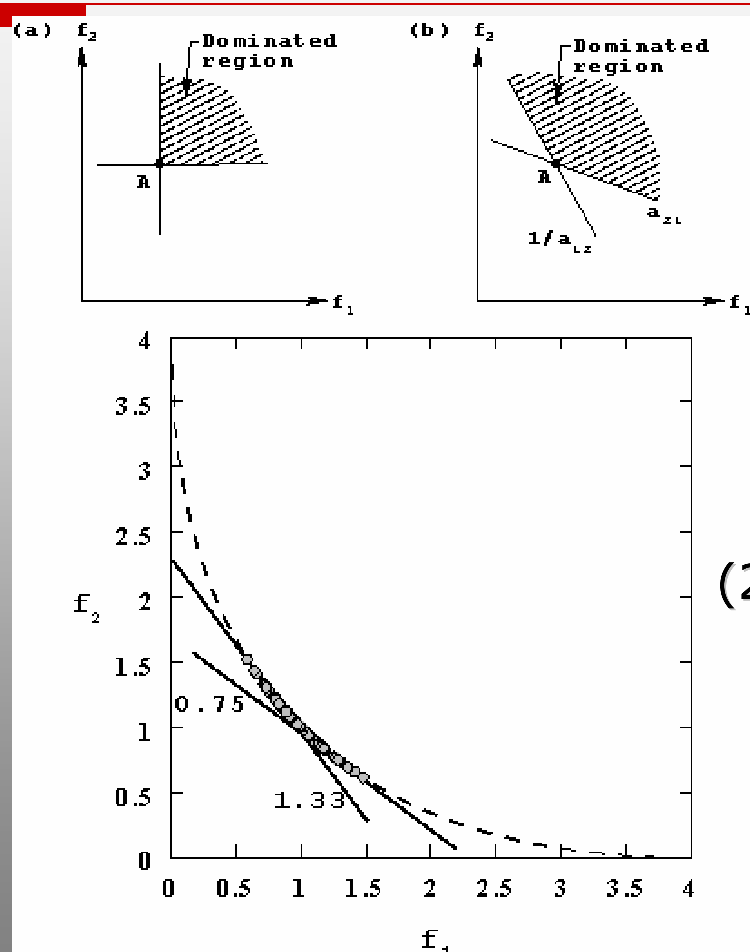
EMO	Convergence measure Average	Convergence measure Std. Dev.	Sparsity Average	Sparsity Std. Dev.	Time (sec) Average	Time (sec) Std. Dev.
DTLZ2						
NSGA-II	0.0137186	0.0020145	0.931111	0.0124474	17.16	0.196
C-NSGA-II	0.0107455	0.0008424	**0.999778**	0.0004968	7837.42	81.254
PESA	**0.0106292**	0.0025483	0.945778	0.0309657	88.01	12.901
SPEA2	0.0126622	0.0009540	0.998889	0.0007855	2164.42	19.858
ϵ-MOEA	0.0108443	0.0002823	0.999104	0.0009316	**2.01**	0.032
DTLZ3						
NSGA-II	0.0149156	0.01028	0.839228	0.02961	136.45	31.080
C-NSGA-II	0.0202315	0.00898	0.995521	0.00613	24046.03	4690.032
PESA	0.0130633	0.00449	0.722296	0.02785	89.49	12.527
SPEA2	0.0122429	0.00194	**0.999771**	0.00031	9080.81	963.723
ϵ-MOEA	**0.0122190**	0.00223	0.993207	0.00974	**9.42**	2.180
DTLZ5						
NSGA-II	0.00208342	11.976e-05	0.953778	0.00992	11.49	0.036
C-NSGA-II	0.00256138	30.905e-05	0.996667	0.00314	1689.16	81.365
PESA	0.00094626	11.427e-05	0.772110	0.02269	53.27	11.836
SPEA2	0.00197846	16.437e-05	**1.000000**	0.00000	633.60	14.082
ϵ-MOEA	**0.000953623**	4.892e-05	0.980867	0.01279	**1.45**	0.051

Test Problem DTLZ2



Finding a Partial Pareto Frontier

- Using a DM's preference (not a solution but a region)
- Guided domination principle: Biased niching approach
- Weighted domination approach

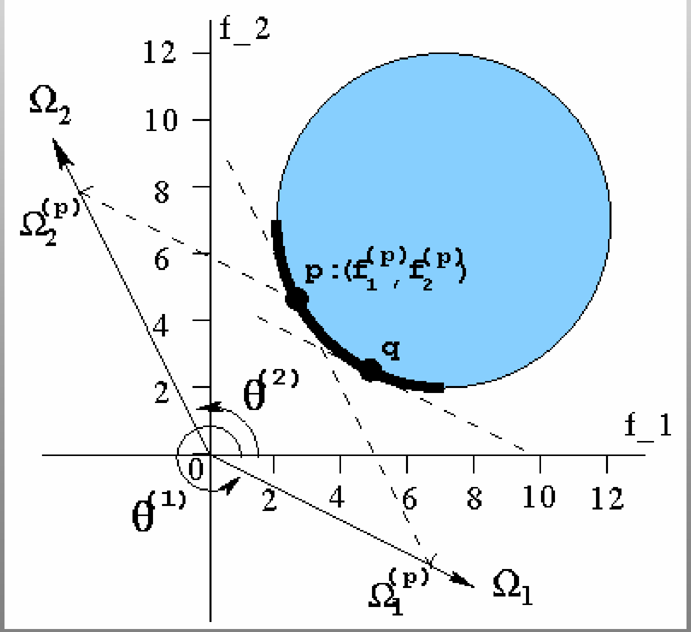


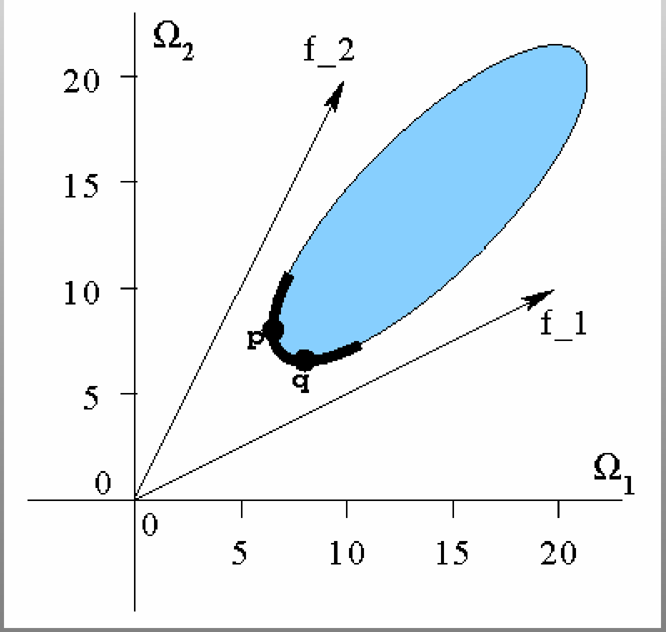
Deb (2001)

Distributed Computing of Pareto-Optimal Set

Deb, Zope & Jain (EMO-2003)

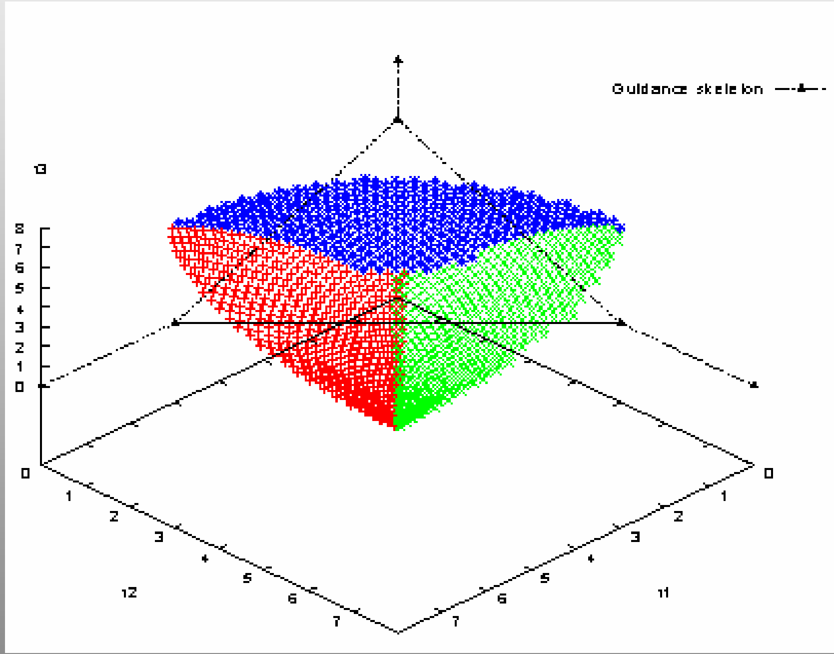
- Guided domination concept to search different parts of Pareto-optimal region
- Distributed computing of different parts



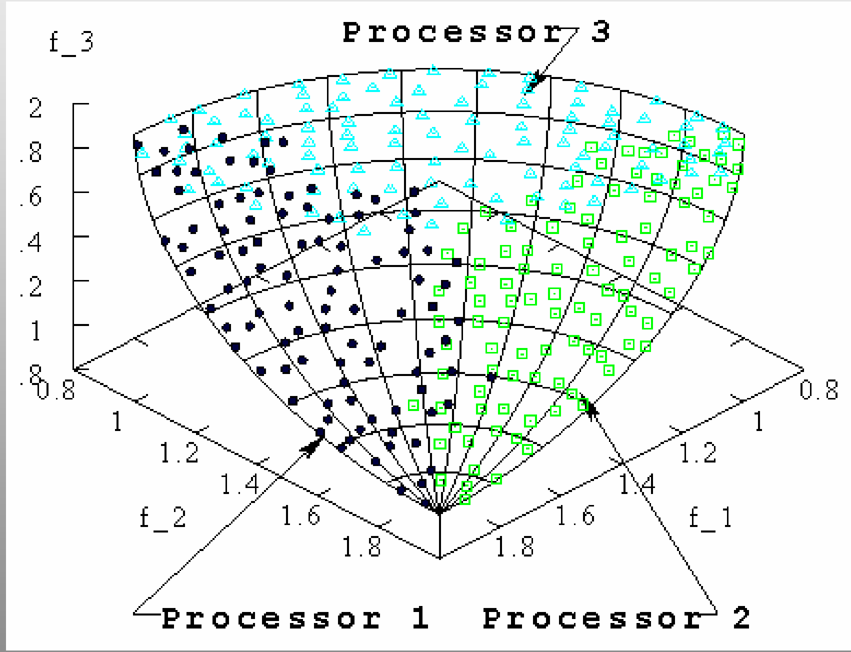


Distributed computing: A Three-Objective Problem

- Spatial computing, not temporal



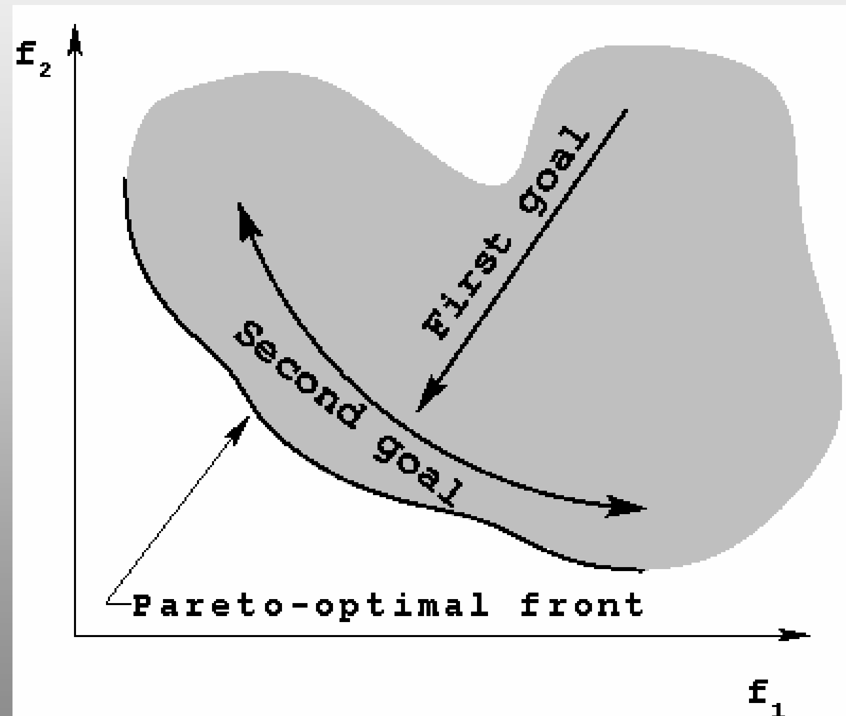
Theory



NSGA-II Simulations

Performance Metrics

- A recent study by Zitzler et al. suggests at least metrics
- Two essential metrics (functionally)
  - Convergence measure
  - Diversity measure



Metrics for Convergence

- Error ratio:

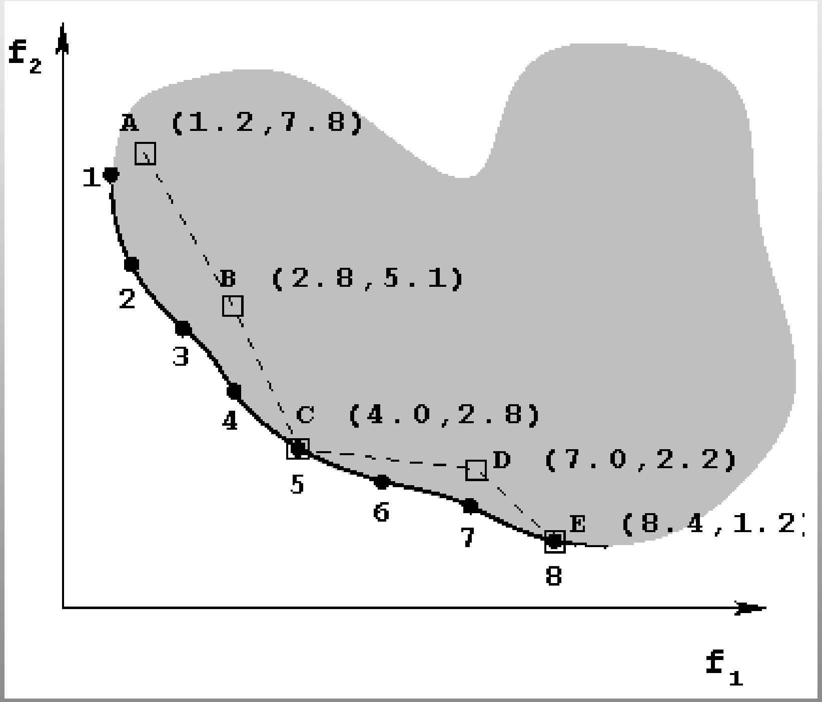
$$ER = \frac{\sum_{i=1}^{|Q|} e_i}{|Q|}$$

- Set Coverage:

$$C(A,B) = \frac{|\{b \in B \mid \exists a \in A : a \preceq b\}|}{|B|}$$

- Generational distance:

$$GD = \frac{\left(\sum_{i=1}^{|Q|} d_i^{\,p}\right)^{1/p}}{|Q|}$$



Metrics for Diversity

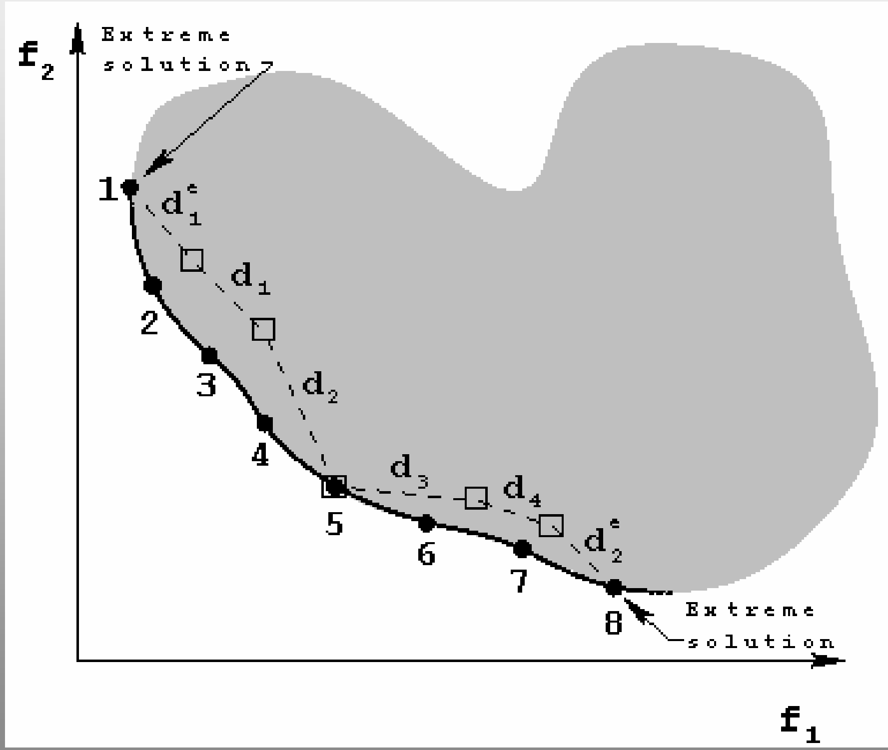
- Spacing:

$$S = \sqrt{\frac{1}{|Q|}\sum_{i=1}^{|Q|}(d_i - \bar{d})^2}$$

- Spread:

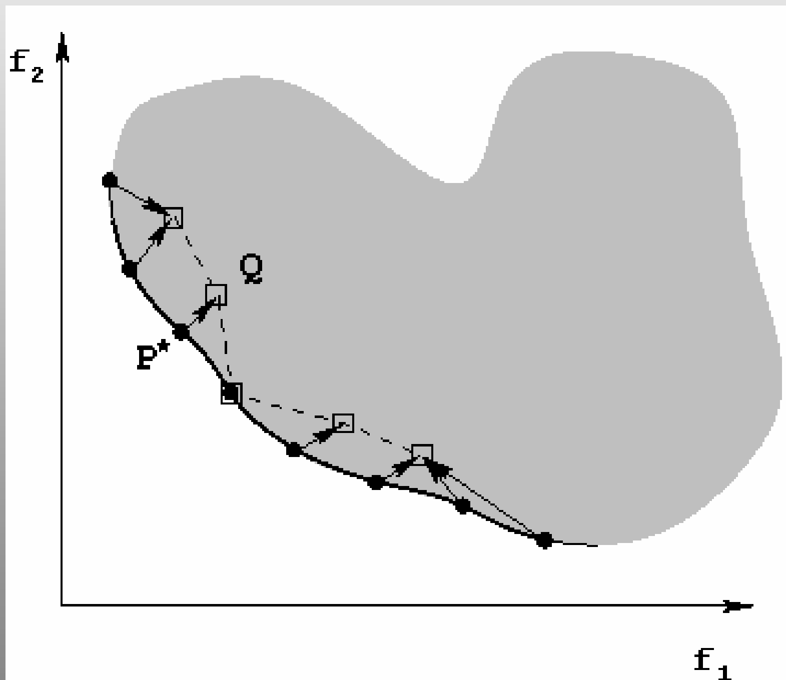
$$\Delta = \frac{\sum_{m=1}^{M} d_m^e + \sum_{i=1}^{|Q|} |d_i - \bar{d}|}{\sum_{m=1}^{M} d_m^e + |Q|\bar{d}}$$

- Chi-square like deviation measure

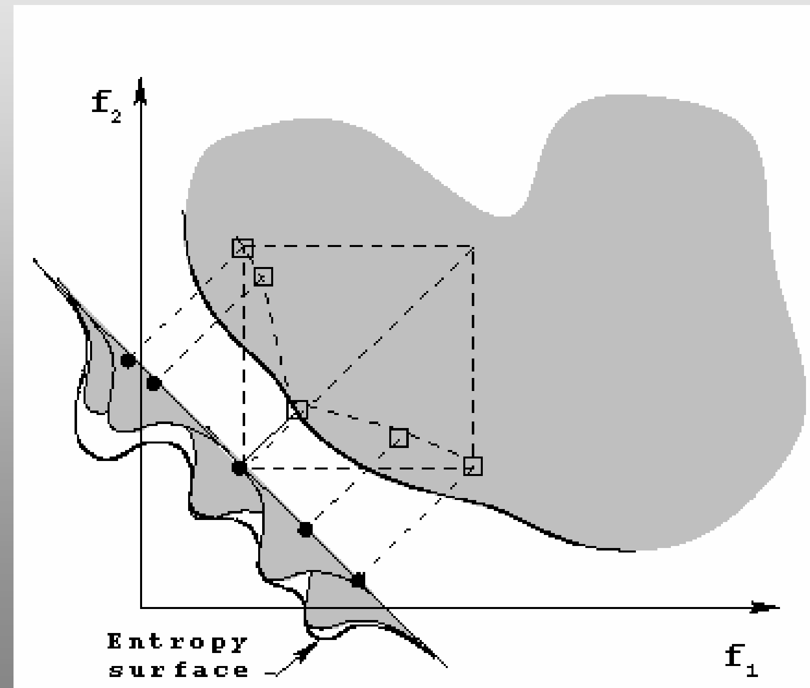


Metrics for Diversity (Cont.)

Distance from P*

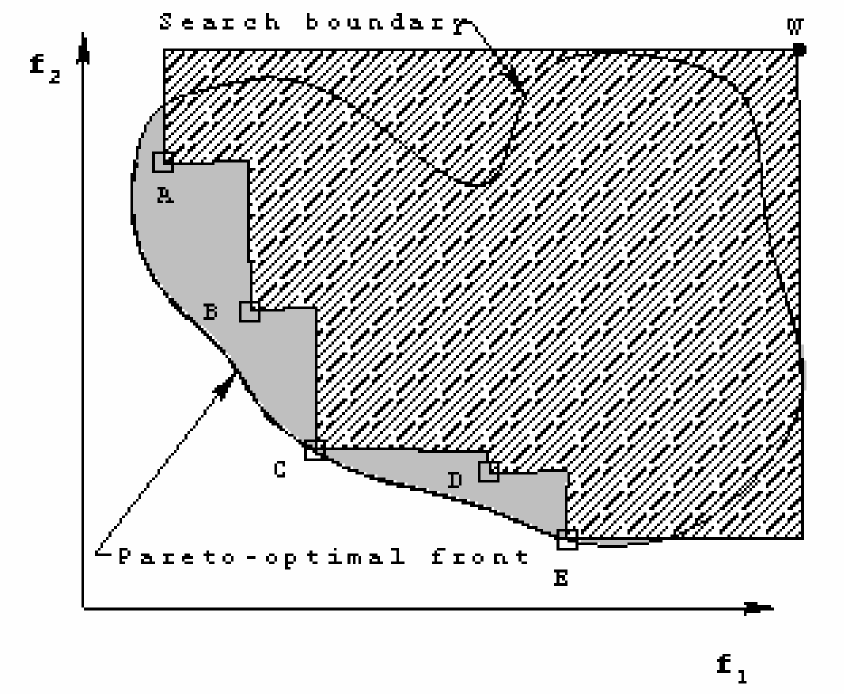


Entropy Measure

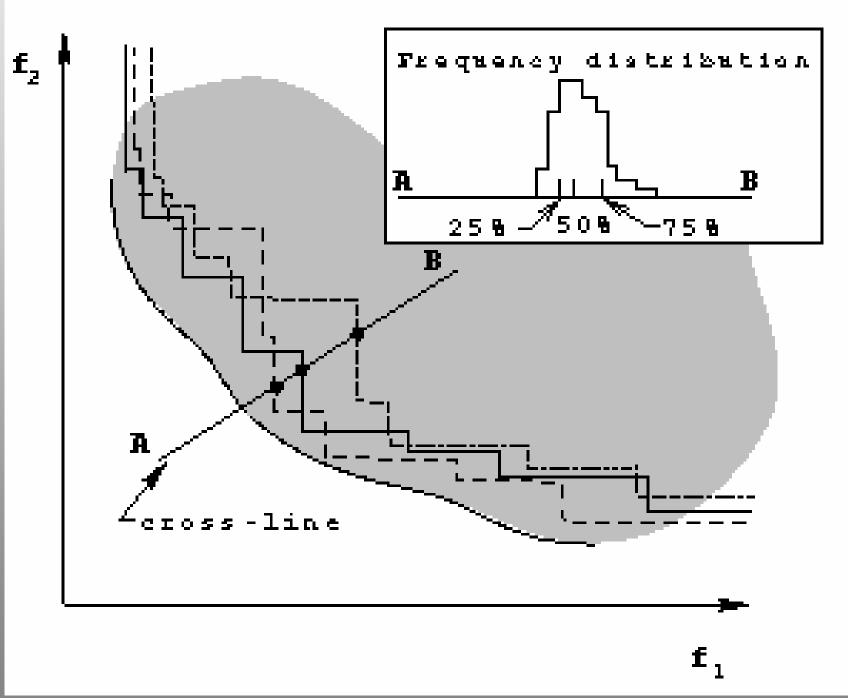


Metrics for Convergence and Diversity

Hypervolume



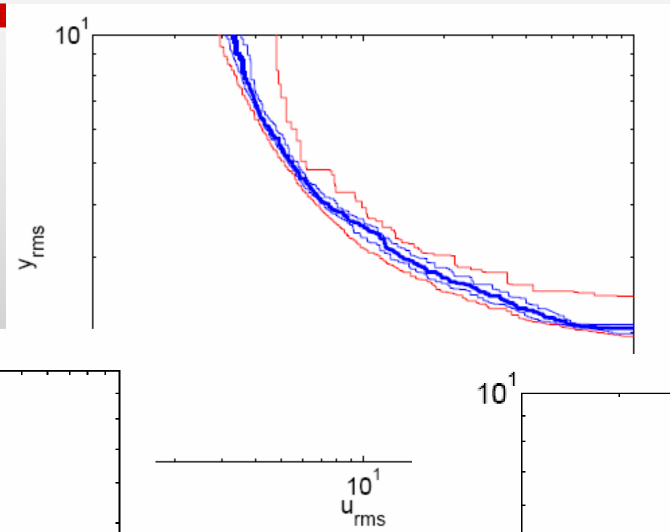
Attainment Surface Method

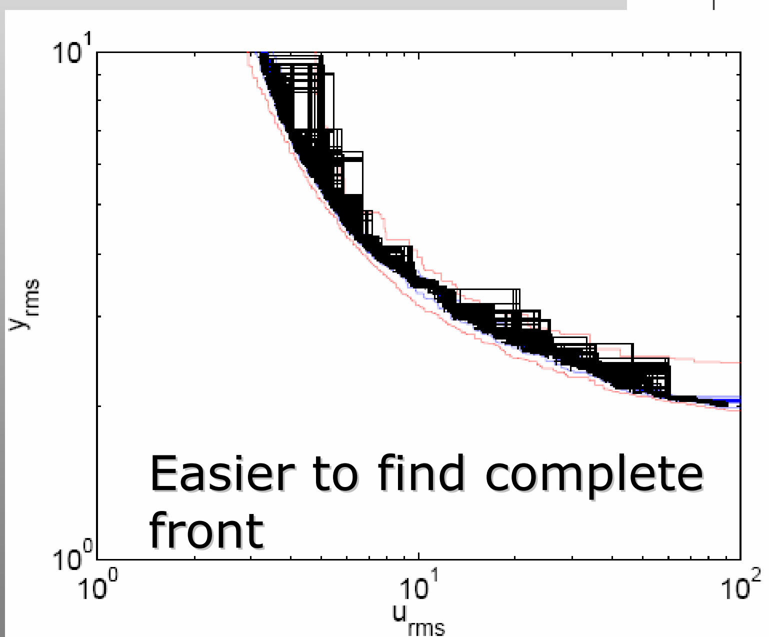


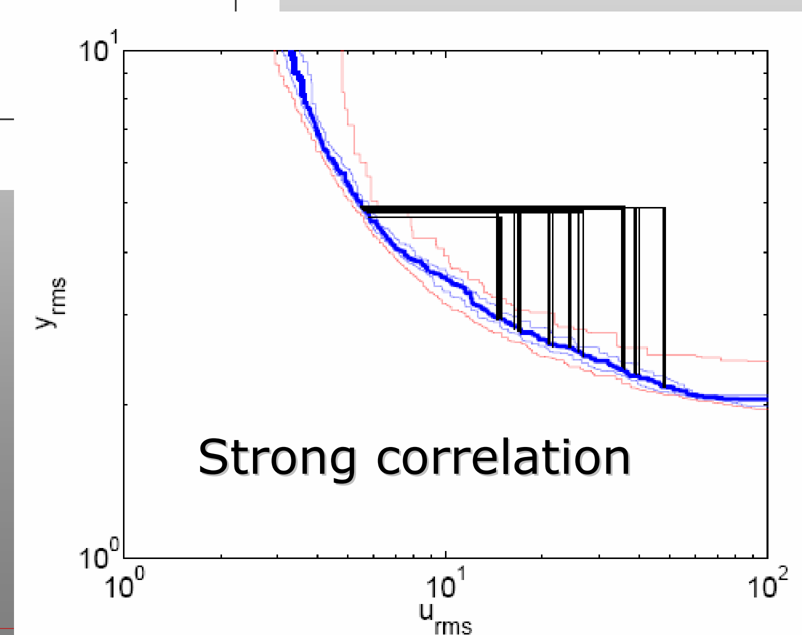
Higher-order Attainment Surfaces

- Positive and negative relationships

Taken from Carlos Fonseca



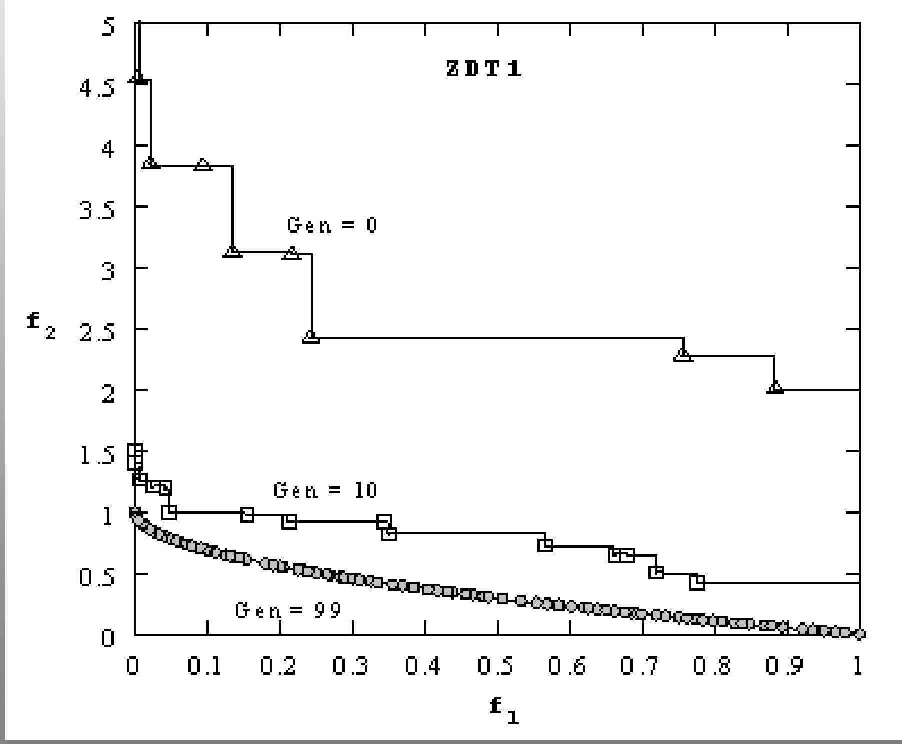


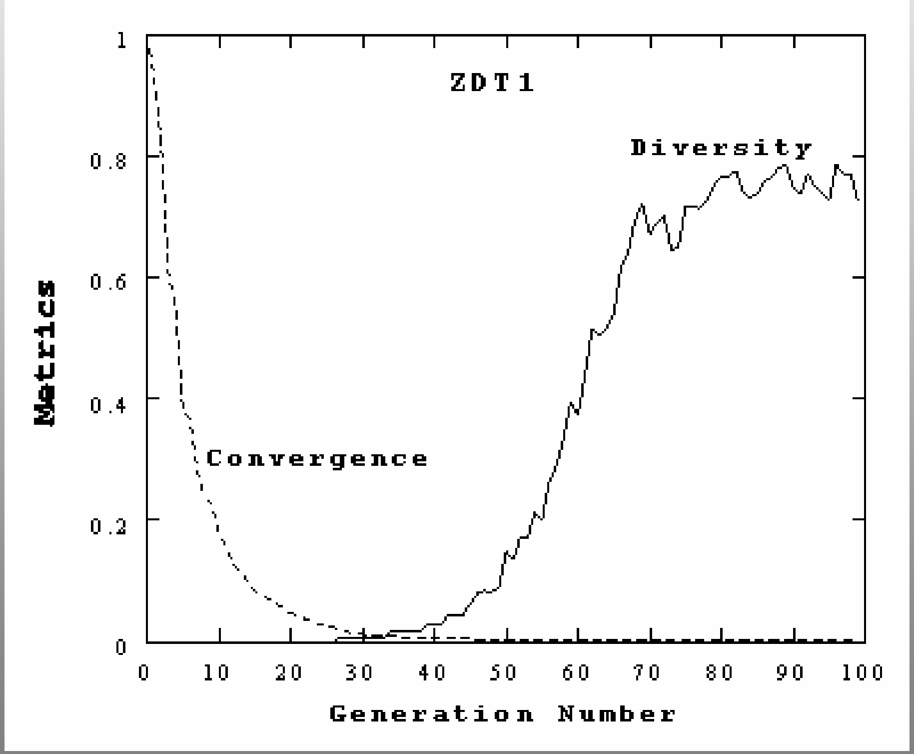


Running Metrics

Deb and Jain
(SEAL-2002)

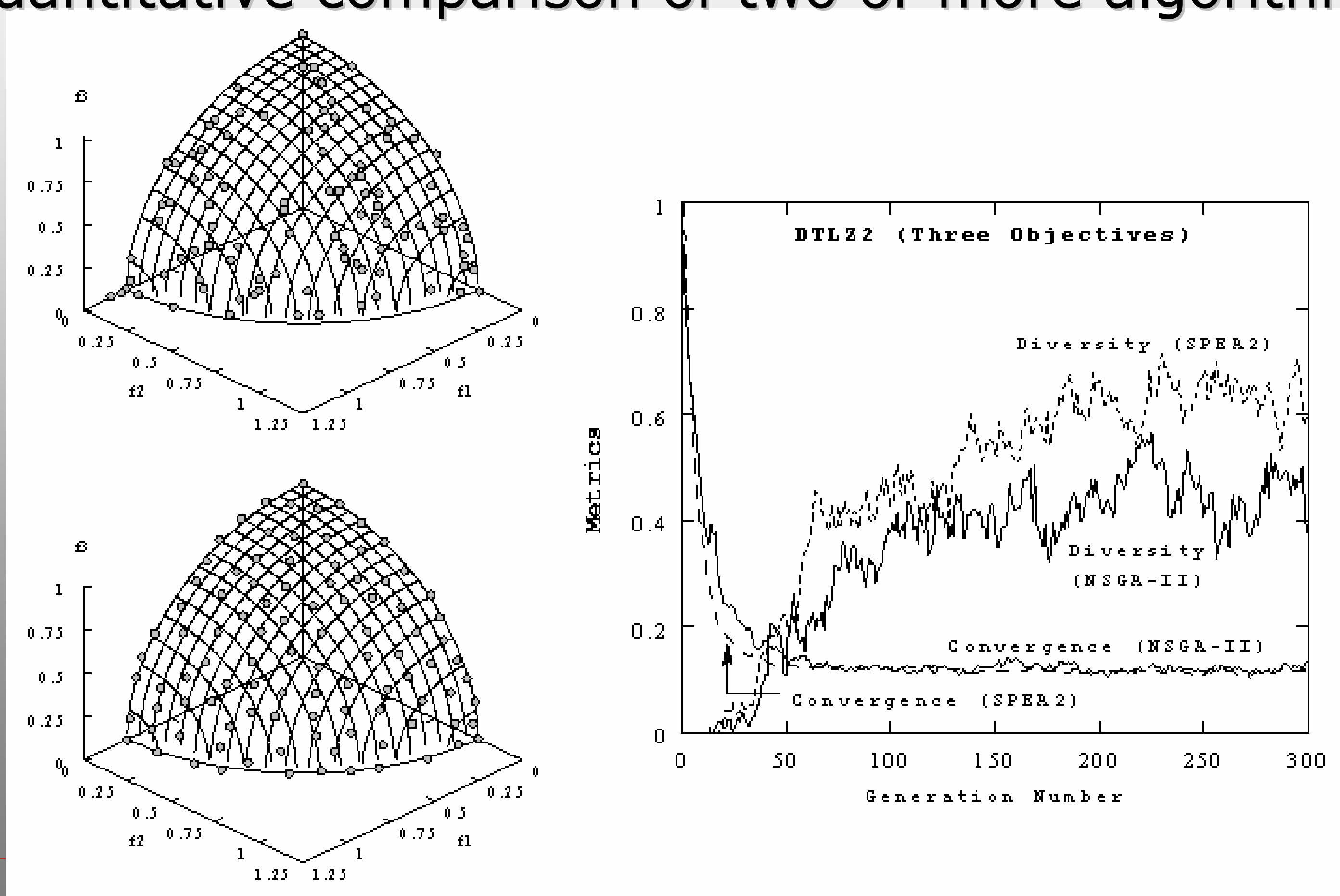
- Performance metric changes with generation





Running Metrics (cont.)

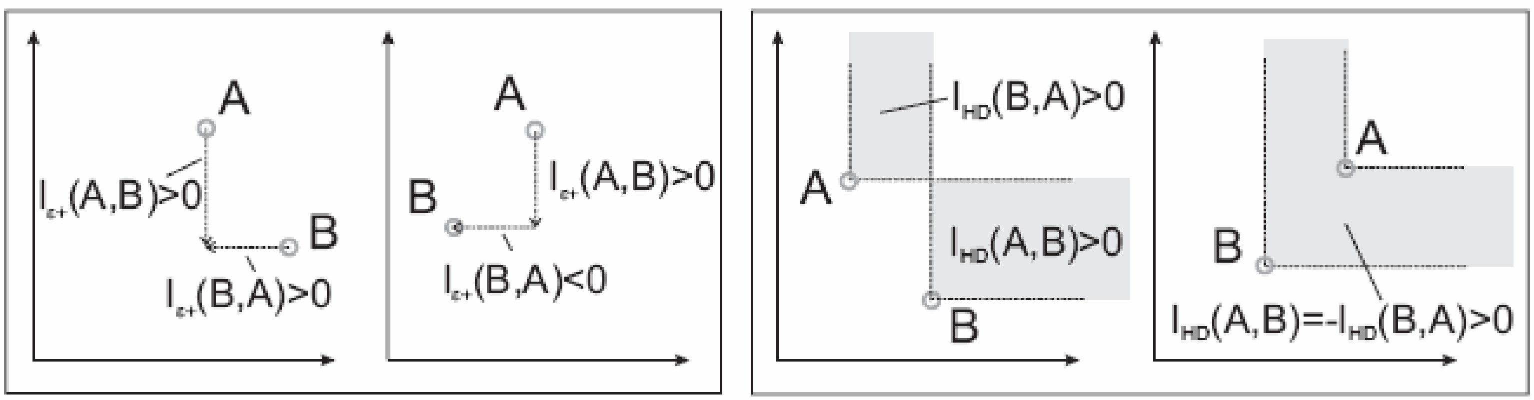
- Quantitative comparison of two or more algorithms



Indicator-Based EMO

- Zitzler et al. (2004)
- Move point A so as to make weakly-dominated with B
- Positive if reduction in hypervolume
- Negative, otherwise

$$I_{\epsilon^+}(A,B) = \min_\epsilon \left\{\forall \boldsymbol{x}^2 \in B\ \exists \boldsymbol{x}^1 \in A\ :\ f_i(\boldsymbol{x}^1) - \epsilon \leq f_i(\boldsymbol{x}^2) \text{ for } i \in \{1,\ldots,n\}\right\}$$



IBEA (cont.)

- Develop a single-objective EA with indicator functions

$$F(\boldsymbol{x}^1) = \sum_{\boldsymbol{x}^2 \in P \setminus \{\boldsymbol{x}^1\}} -e^{-I(\{\boldsymbol{x}^2\},\{\boldsymbol{x}^1\})/\kappa}$$

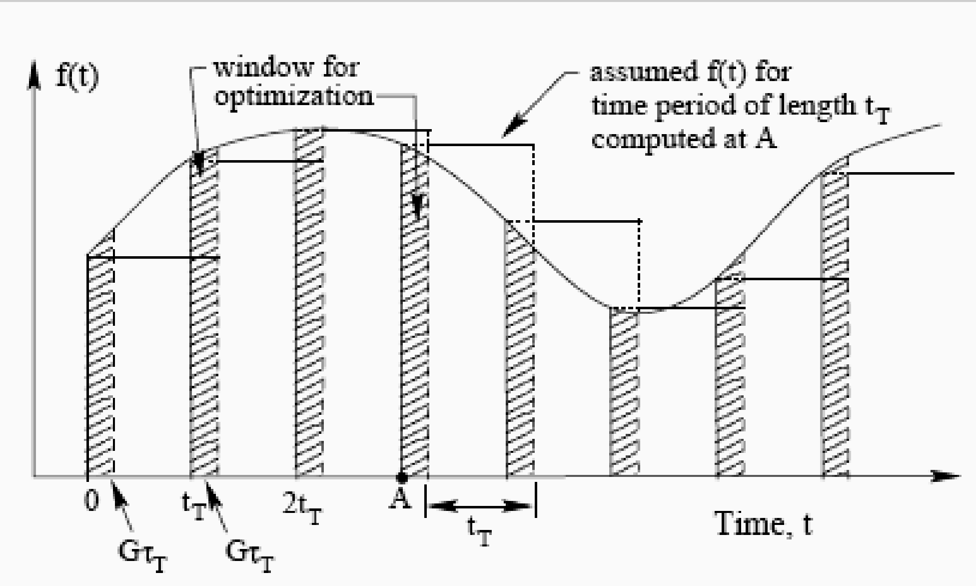
Theorem 1. *Let I be a binary quality indicator. If I is dominance preserving, then it holds that $\boldsymbol{x}^1 \succ \boldsymbol{x}^2 \Rightarrow F(\boldsymbol{x}^1) > F(\boldsymbol{x}^2)$.*

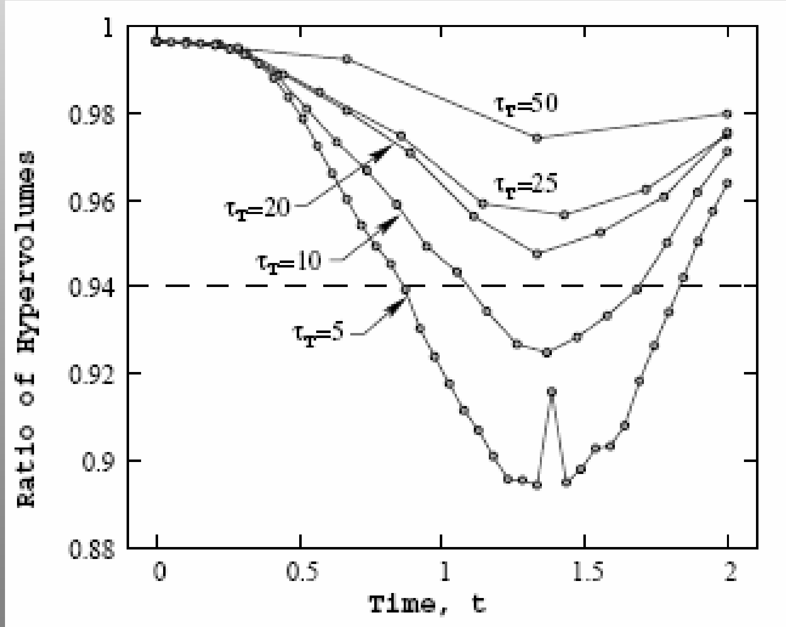
- Smaller the fitness F, worse is the solution
- Some niche preservation is needed
- Better solutions reported

Dynamic Optimization

Deb, Rao, Karthik
(EMO 2007)

- Assume a statis in problem for a time step
- Find a critical frequency of change
- FDA2 test problem

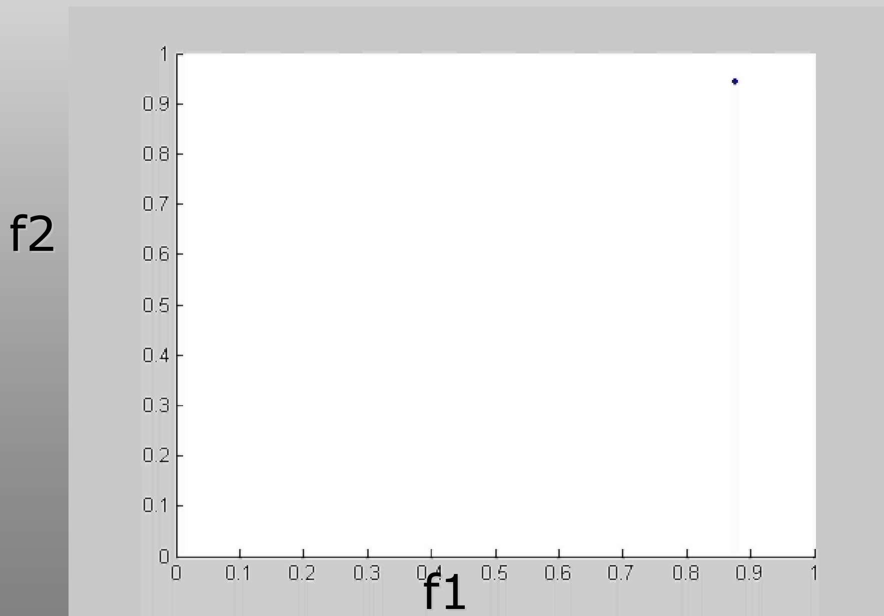


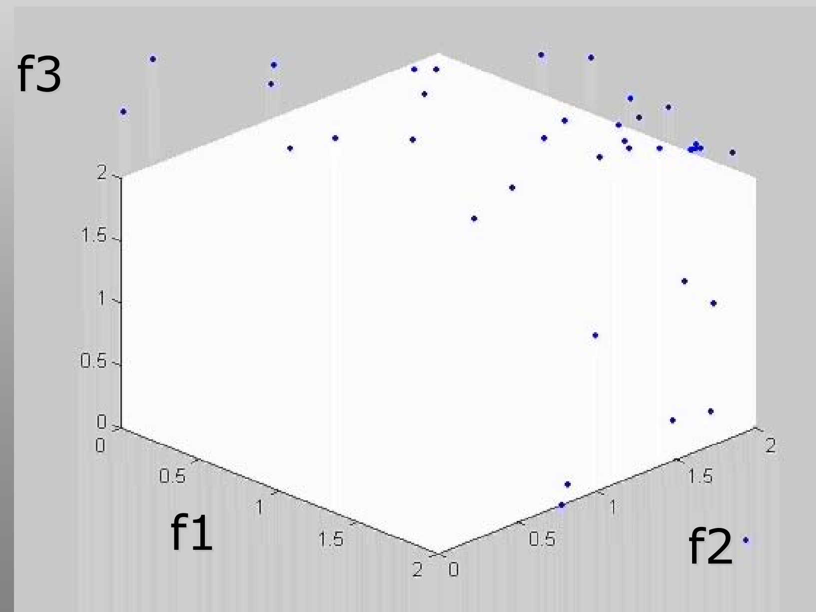


NSGA-II Simulations

- Problems change as NSGA-II runs
- Elitism is eliminated from NSGA-II

Simulations





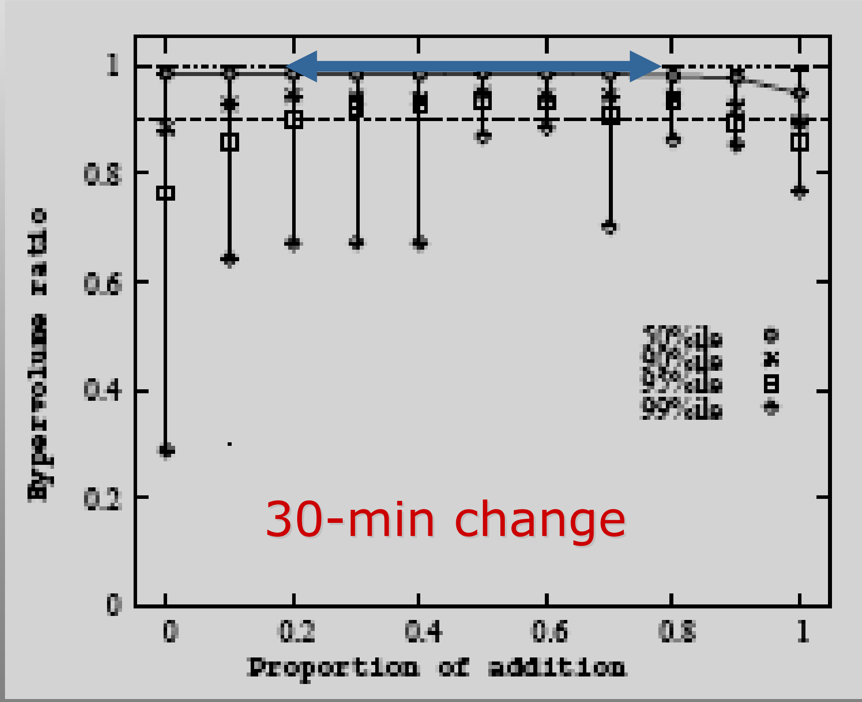
Hydro-Thermal Power Dispatch

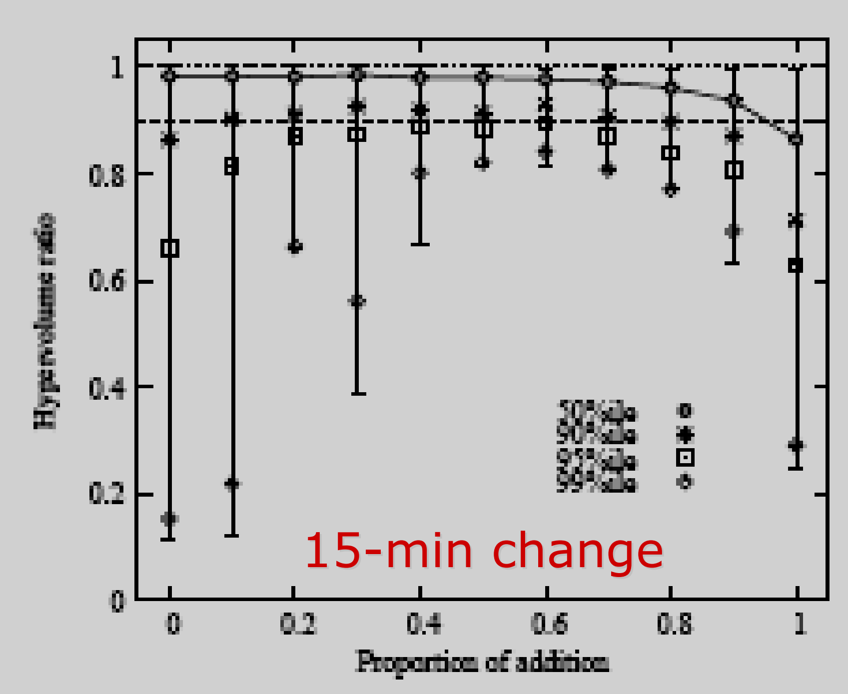
$$
\begin{aligned}
&\text{Minimize} && f_1(\mathrm{x}) = \textstyle\sum_{m=1}^{M}\sum_{s=1}^{N_s} t_m[a_s + b_s P^{\mathrm{s}}{}_{sm} + c_s (P^{\mathrm{s}}{}_{sm})^2 + |d_s \sin(e_s(P^{\mathrm{s}}{}_{s,\min} - P^{\mathrm{s}}{}_{sm}))|],\\
&\text{Minimize} && f_2(\mathrm{x}) = \sum_{m=1}^{M}\sum_{s=1}^{N_s} t_m[\alpha_s + \beta_s P^{\mathrm{s}}{}_{sm} + \gamma_s (P^{\mathrm{s}}{}_{sm})^2 + \eta_s \exp(\delta_s P^{\mathrm{s}}{}_{sm})],\\
&\text{subject to} && \textstyle\sum_{s=1}^{N_s} P^{\mathrm{s}}{}_{sm} + \sum_{h=1}^{N_h} P^{\mathrm{h}}{}_{hm} - P_{Dm} - P_{Lm} = 0, \quad m = 1, 2 \ldots, M,\\
& && \textstyle\sum_{m=1}^{M} t_m (a_{0h} + a_{1h} P^{\mathrm{h}}{}_{hm} + a_{2h} (P^{\mathrm{h}}{}_{hm})^2) - W_h = 0, \quad h = 1, 2, \ldots, N_h.\\
& && P^{\mathrm{h}}{}_{h,\min} \le P_{hm} \le P^{\mathrm{h}}{}_{h,\max}, \quad h = 1, 2, \ldots, N_h, m = 1, 2, \ldots, M,\\
& && P^{\mathrm{s}}{}_{s,\min} \le P_{sm} \le P^{\mathrm{s}}{}_{s,\max}, \quad s = 1, 2, \ldots, N_s, m = 1, 2, \ldots, M.
\end{aligned}
$$

- Minimize Cost and NOx emission
- Power balance and water head limits
- Dynamic due to change in power demand with time

Dynamic
Hydro-Thermal Power Scheduling

- Addition of random or mutated points at changes
- 30-min change found satisfactory

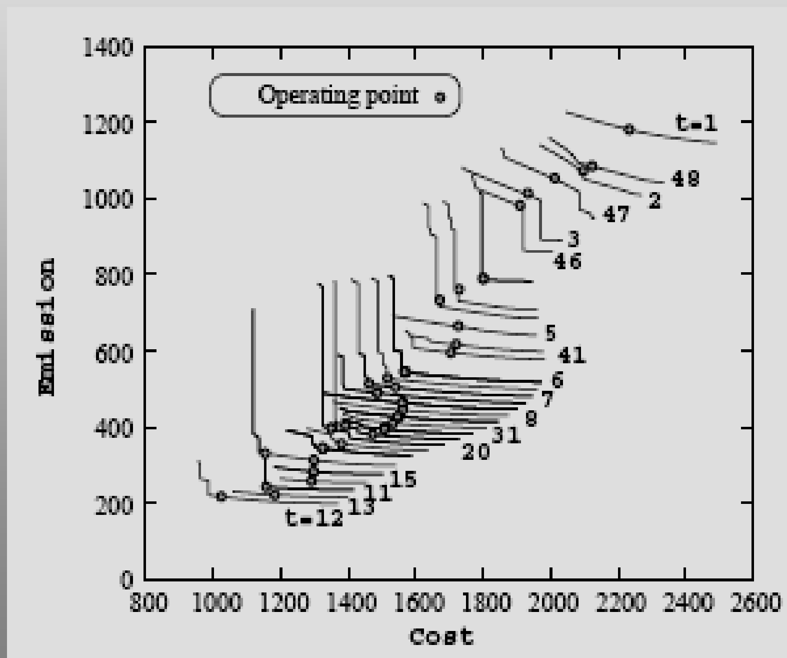


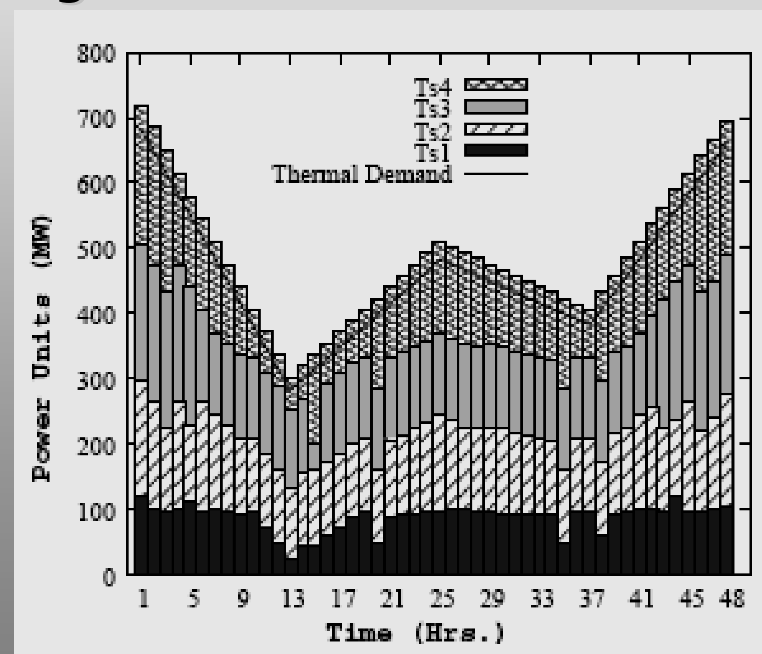


Dynamic EMO with Decision-Making

- Needs a fast decision-making
- Use an automatic procedure
  - Utility function, pseudo-weight etc.

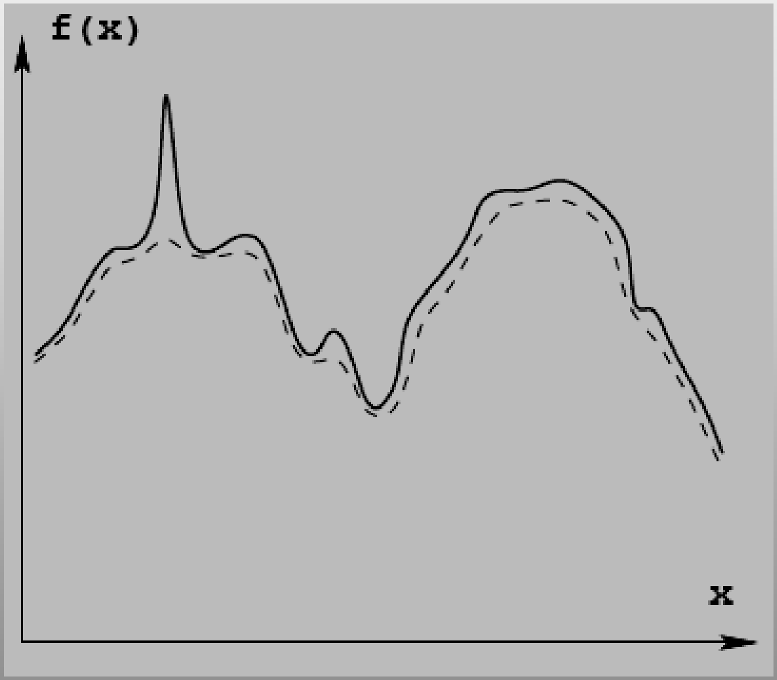
Case	Cost	Emission
50-50%	74239.07	25314.44
100-0%	69354.73	27689.08
0-100%	87196.50	23916.09





Robust Optimization
Handling uncertainties in variables

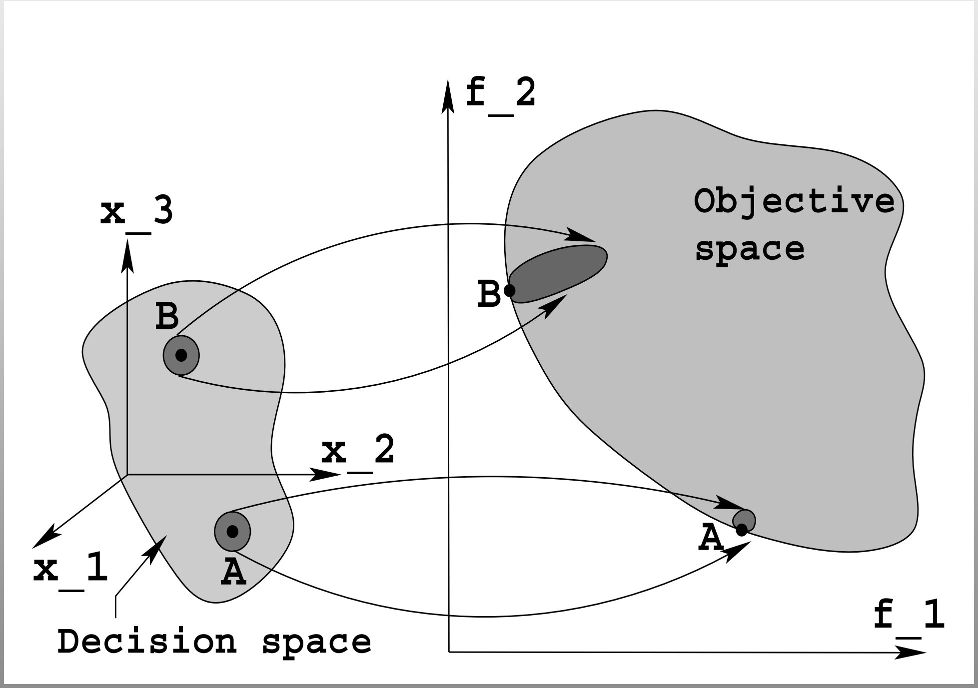
- Parameters are uncertain and sensitive to implementation
  - Tolerances in manufacturing
  - Material properties are uncertain
  - Loading is uncertain
- Who wants a sensitive optimum solution?
- Single-objective robust EAs exist



Deb and Gupta (EMO 2005)

Multi-Objective Robust Solutions

- Solutions are averaged in δ-neighborhood
- Not all Pareto-optimal points may be robust
- A is robust, but B is not
- Decision-makers will be interested in knowing robust part of the front



Multi-Objective Robust Solutions of Type I and II

- Similar to single-objective robust solution of type I

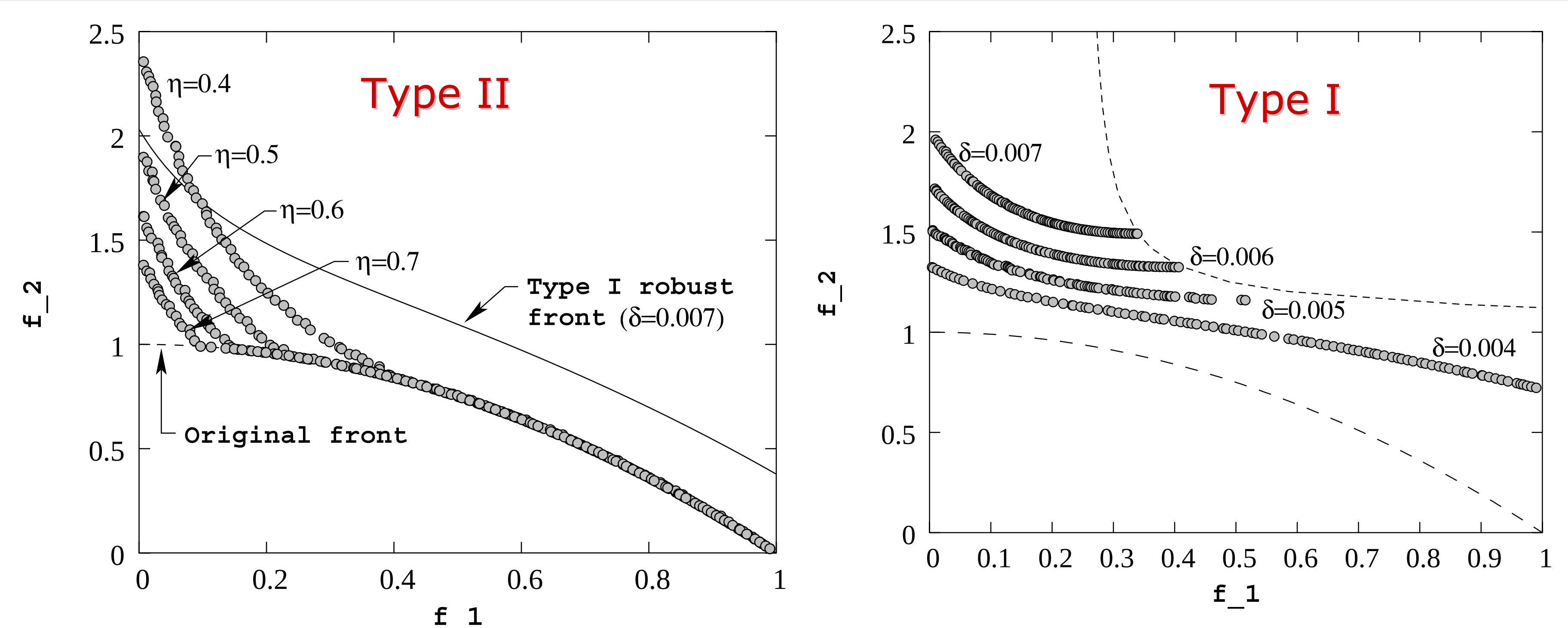
$$\begin{array}{ll} \textit{Minimize} & (f_1^{\text{eff}}(\mathbf{x}), f_2^{\text{eff}}(\mathbf{x}), \ldots, f_M^{\text{eff}}(\mathbf{x})), \\ \textit{subject to} & \mathbf{x} \in \mathcal{S}, \end{array}$$

- Type II

$$\begin{array}{ll} \textit{Minimize} & \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})), \\ \textit{subject to} & \frac{\|\mathbf{f}^{\text{p}}(\mathbf{x}) - \mathbf{f}(\mathbf{x})\|}{\|\mathbf{f}(\mathbf{x})\|} \leq \eta, \\ & \mathbf{x} \in \mathcal{S}. \end{array}$$

Robust Frontier for Two Objectives

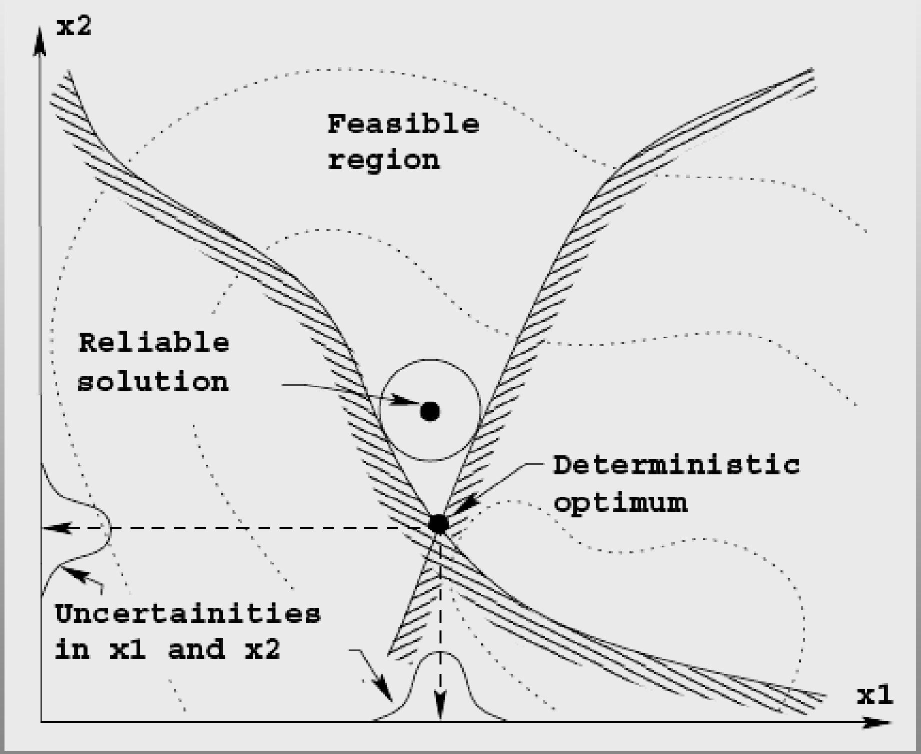
- Identify robust region
- Allows a control on desired robustness



Reliability-Based Optimization:
Making designs safe against failures

- Deterministic optimum is not usually reliable
- Reliable solution is an interior point
- Chance constraints with a given reliability

Minimize $\mu_f + k\sigma_f$
Subject to $Pr(g_j(x) \geq 0) \geq \beta_j$
β_j is user-supplied

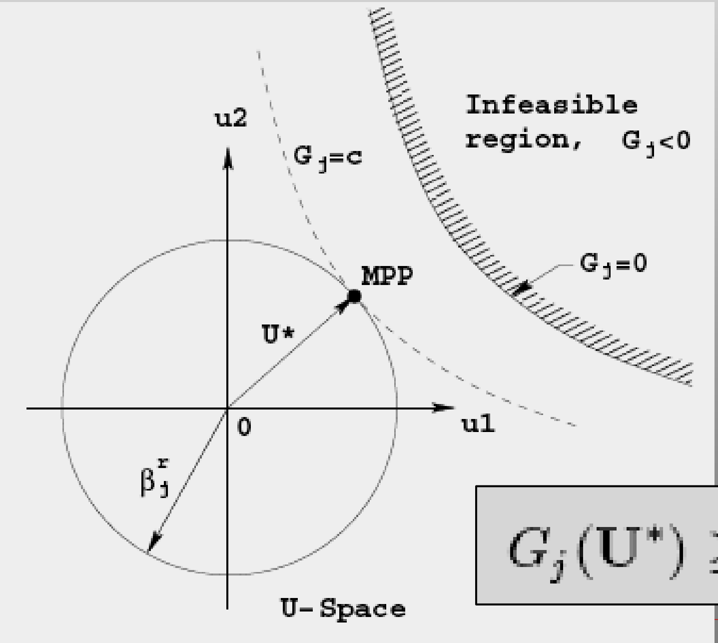


Deb et al. (EMO 2005)

Statistical Procedure: *Check if a solution is reliable*

- PMA approach

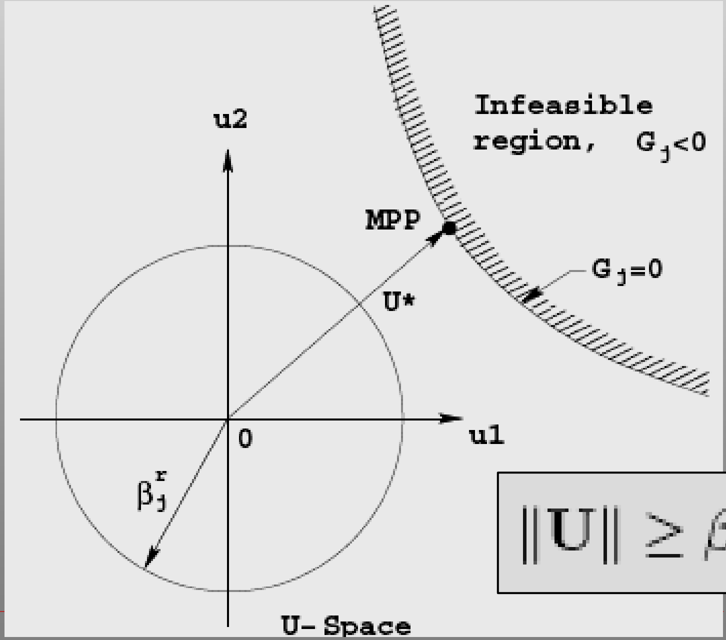
$$\begin{aligned} &\text{Minimize} && G_j(\mathbf{U}), \\ &\text{Subject to} && \|U\| = \beta_j^r, \end{aligned}$$



$$G_j(\mathbf{U}^*) \geq 0.$$

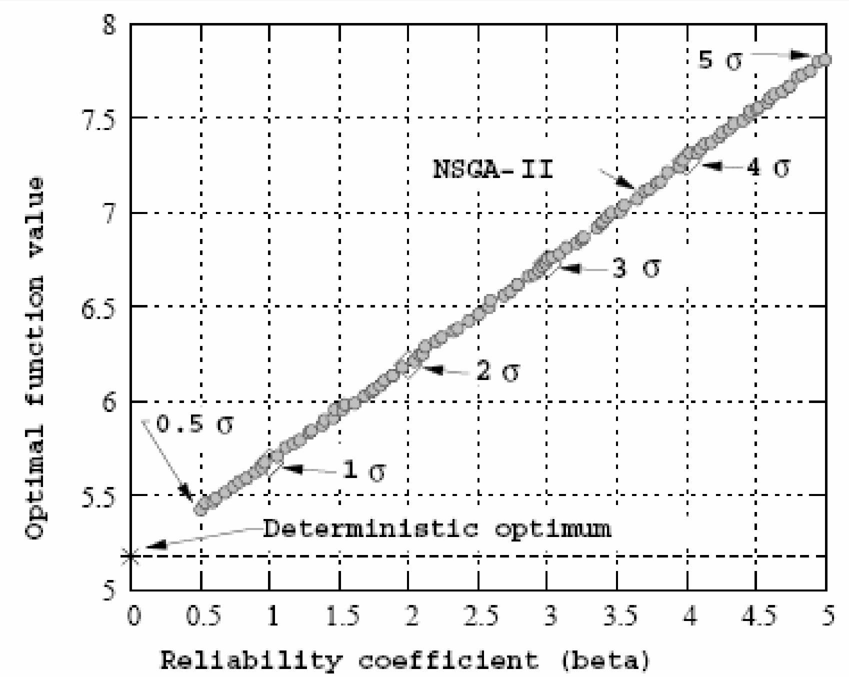
- RIA approach

$$\begin{aligned} &\text{Minimize} && \|\mathbf{U}\|, \\ &\text{Subject to} && G_j(\mathbf{U}) = 0. \end{aligned}$$

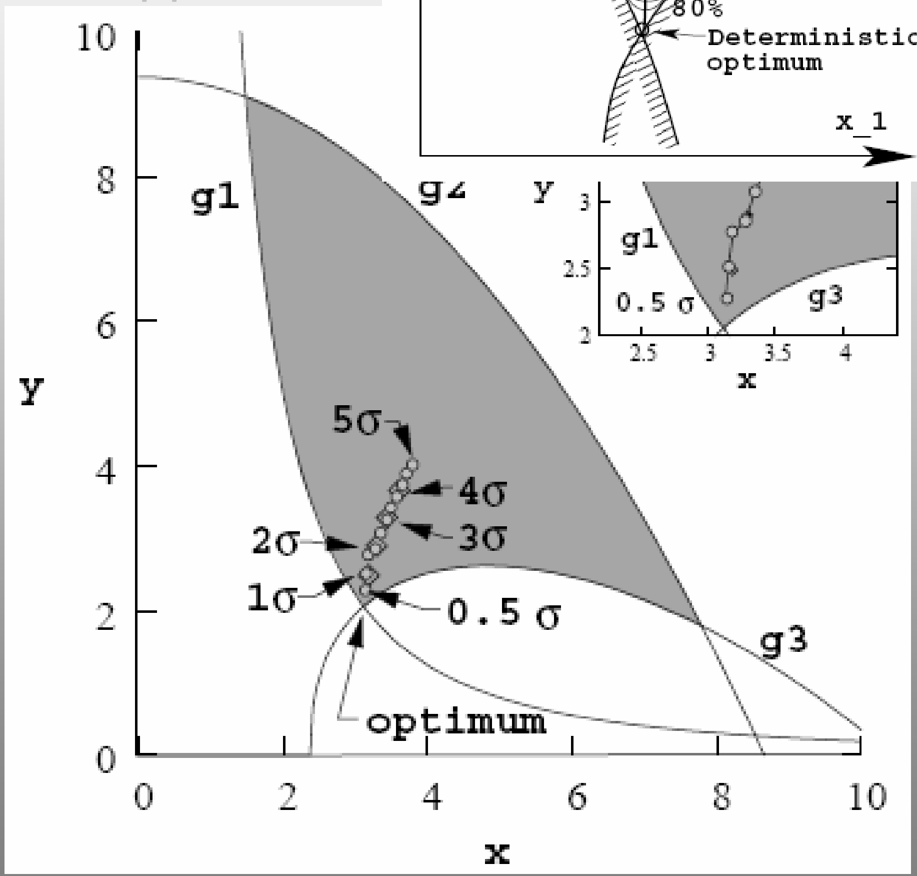


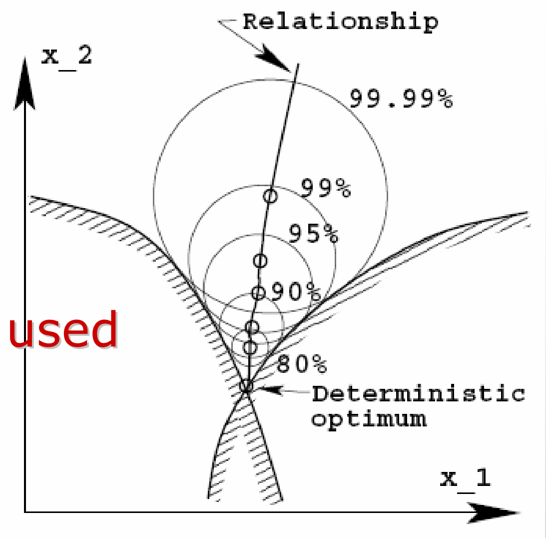
$$\|\mathbf{U}\| \geq \beta_j^r.$$

Multiple Reliability Solutions: *Get a better insight*



RIA approach is used

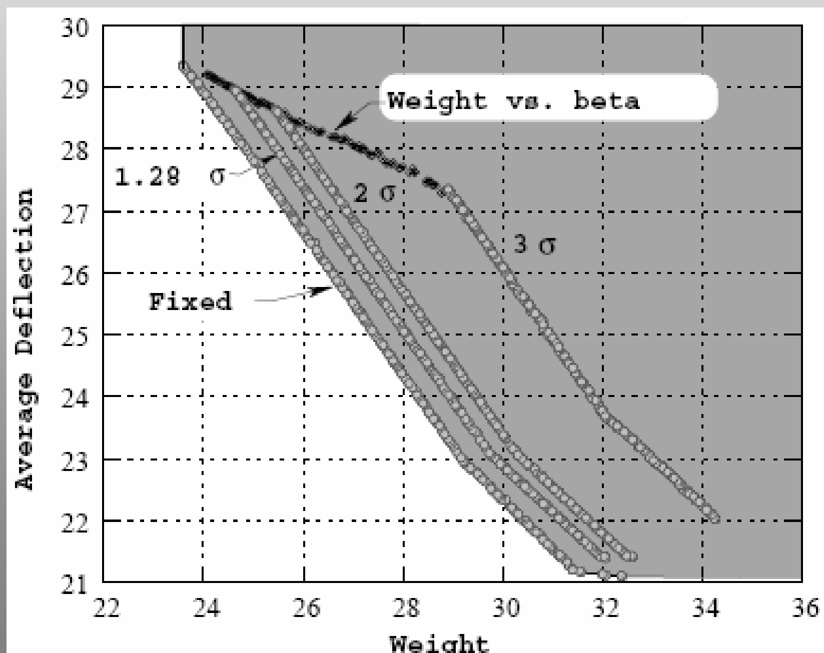


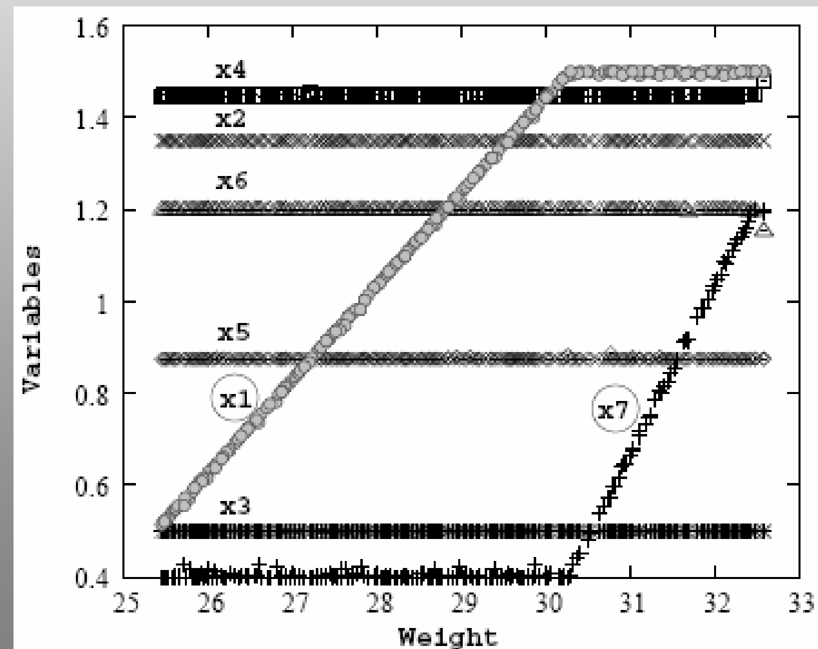


$$
\begin{aligned}
\text{Maximize} \quad & x + y, \\
\text{Subject to} \quad & g_1(x,y) \equiv \tfrac{1}{20}x^2 y - 1 \geq 0, \\
& g_2(x,y) \equiv \tfrac{1}{30}(x+y-5)^2 + \tfrac{1}{120}(x-y-12)^2 - 1 \geq 0, \\
& g_3(x,y) \equiv \tfrac{80}{x^2+8y+5} - 1 \geq 0, \\
& 0 \leq x, y \leq 10.
\end{aligned}
$$

Multi-Objective Reliability-Based Optimization

- Reliable fronts show rate of movement
- What remains unchanged and what gets changed!





Handling Many Objectives

- Identify redundant objectives
- EMO+PCA in iterations

10-objective DTLZ5 problem

Iter.1

Iter. 1 : PCA-1 (58.83 % variance)	f_7 f_{10}
PCA-2 (28.26 % variance)	f_1
PCA-3 (06.53 % variance)	f_8
PCA-4 (03.27 % variance)	f_8

	f_1	f_7	f_8	f_{10}
f_1	+	-	+	-
f_7	-	+	+	-
f_8	+	+	+	-
f_{10}	-	-	-	+

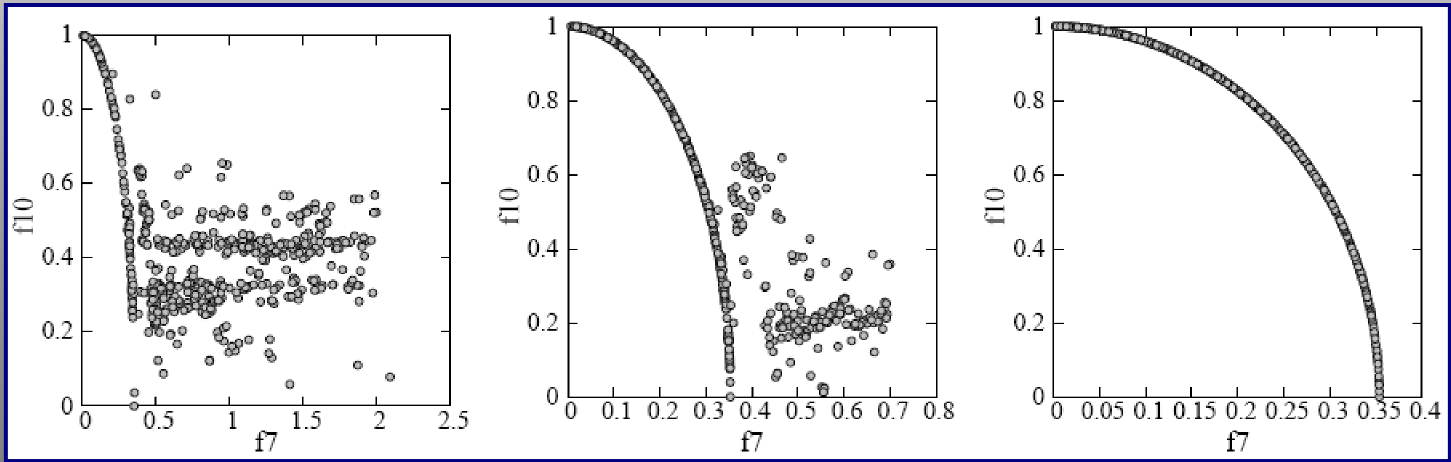
Iter.2

Iter. 2 : PCA-1 (94.58 % variance)	f_7 f_{10}
PCA-2 (4.28 % variance)	f_8

	f_7	f_8	f_{10}
f_7	+	+	-
f_8	+	+	-
f_{10}	-	-	+

	e1:0.9458	e2:0.0428	
f_7	+0.543	-0.275	c7=0.5253
f_8	+0.457	+0.672	c8=0.4610
	PCA1	PCA2	

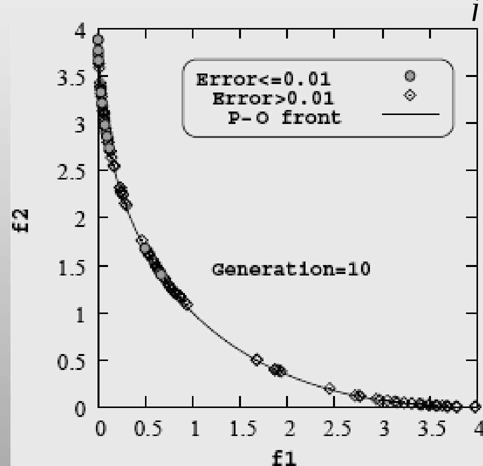
Saxena and Deb (CEC-2006, EMO-2007, CEC-2007)

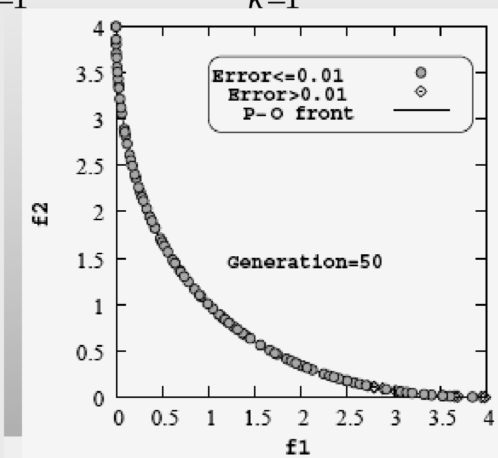


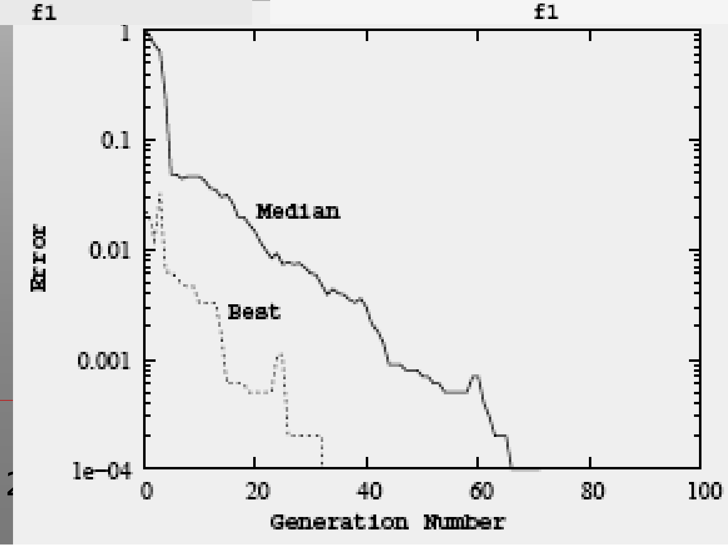
EAs with Theoretical Confidence
(Deb et al., CEC 2007)

- EA solution(s) improved with local search (classical or hill-climbing)
- If derivative exists, verify the solution to be a KKT point
- For every point, calculate a norm stating extent of KKT condition satisfaction

$$\lambda_i X \nabla f_i = \sum_{j=1}^{J} \mu_j \nabla g_j + \sum_{k=1}^{K} \mu_k \nabla h_k$$







Norm can be used as termination criteria

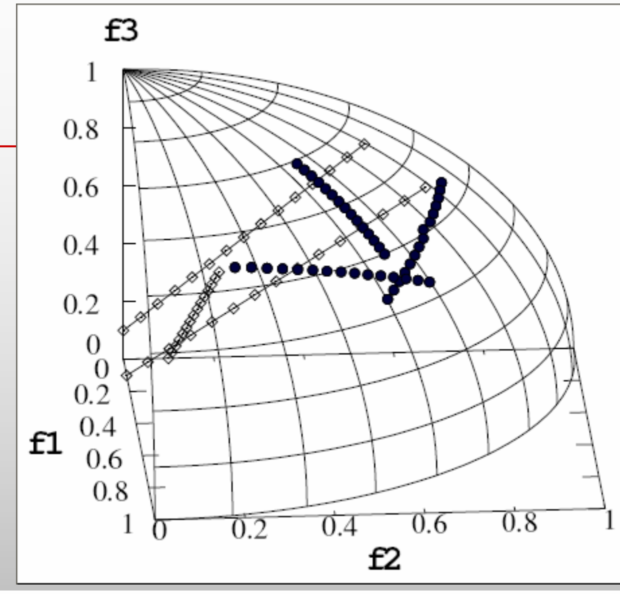
EMO and Decision-Making

- Finding a P-O set (using EMO) is half the story
- How to choose one solution (MCDM)
- First EMO, then MCDM
- EMO+MCDM all along
  - Use where multiple, repetitive applications are sought
  - Use where, instead of a point, a trade-off region is sought
  - Use for finding points with specific properties (nadir point, knee point, etc.)
  - Use for robust, reliable or other fronts
- Use EMO for an idea of the front, then decision-making (I-MODE)

More forthcoming through a Springer book in Early 2008, derived from Dagstuhl seminars (2004, 2006)

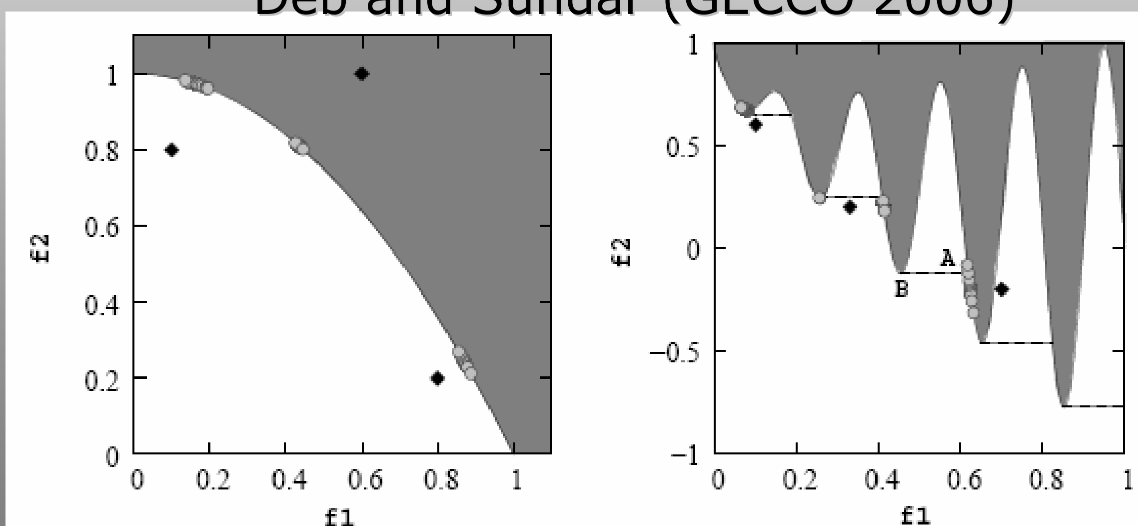
Making Decisions: Current Focus

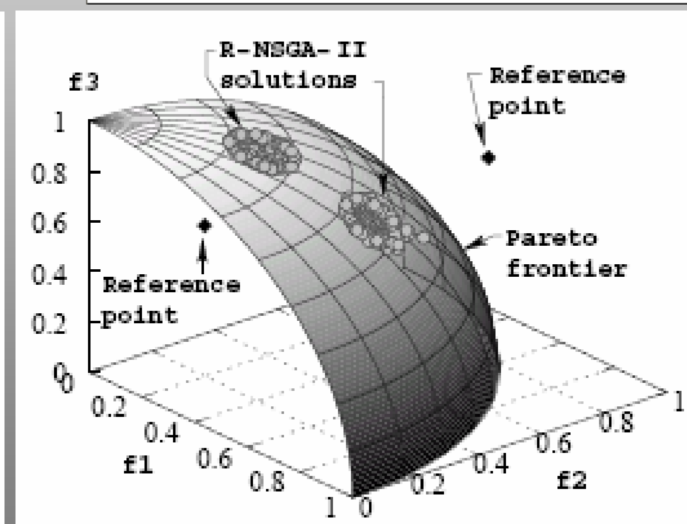
Deb and Kumar (GECCO-2007)



- Ranking based on closeness to each reference point or a reference direction

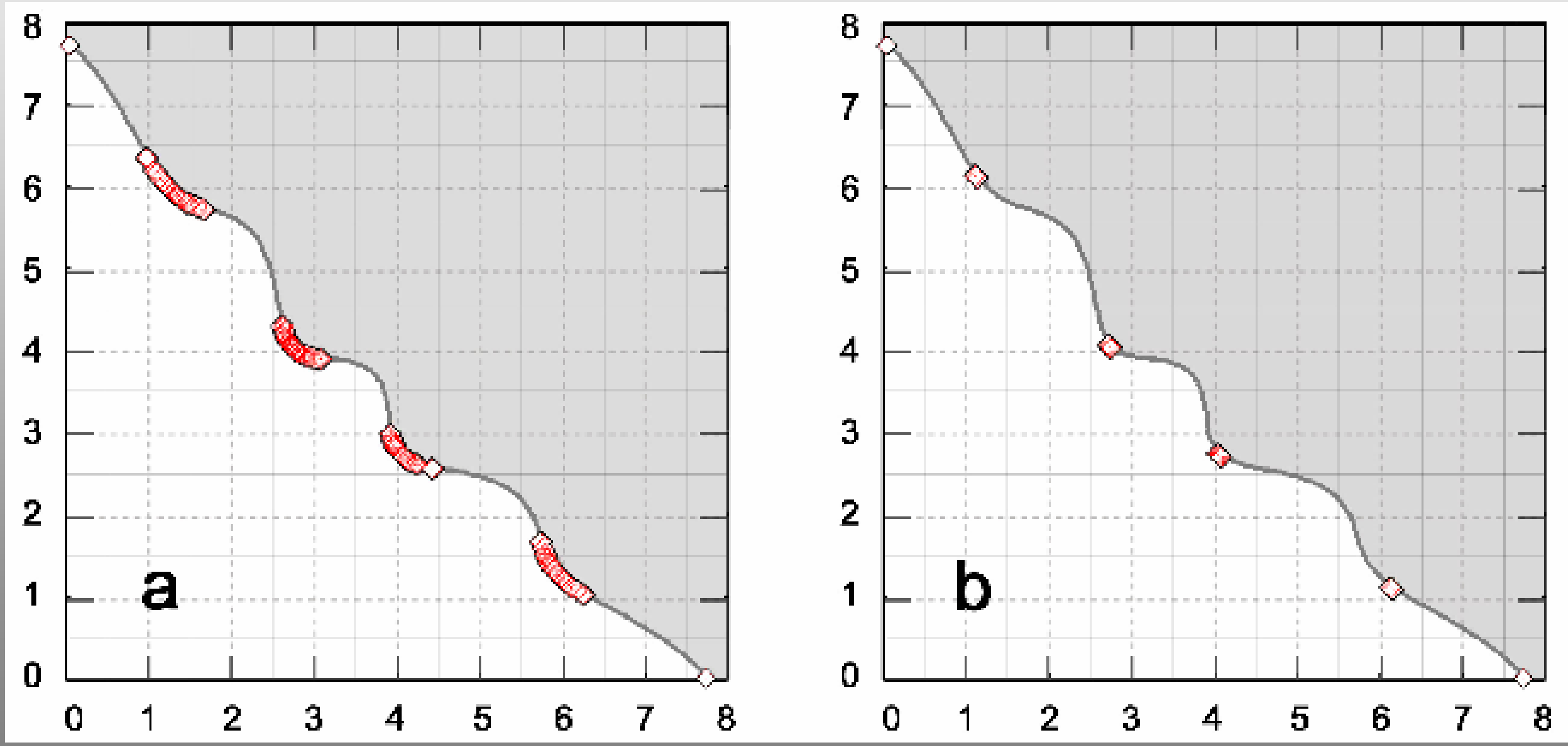
Deb and Sundar (GECCO 2006)





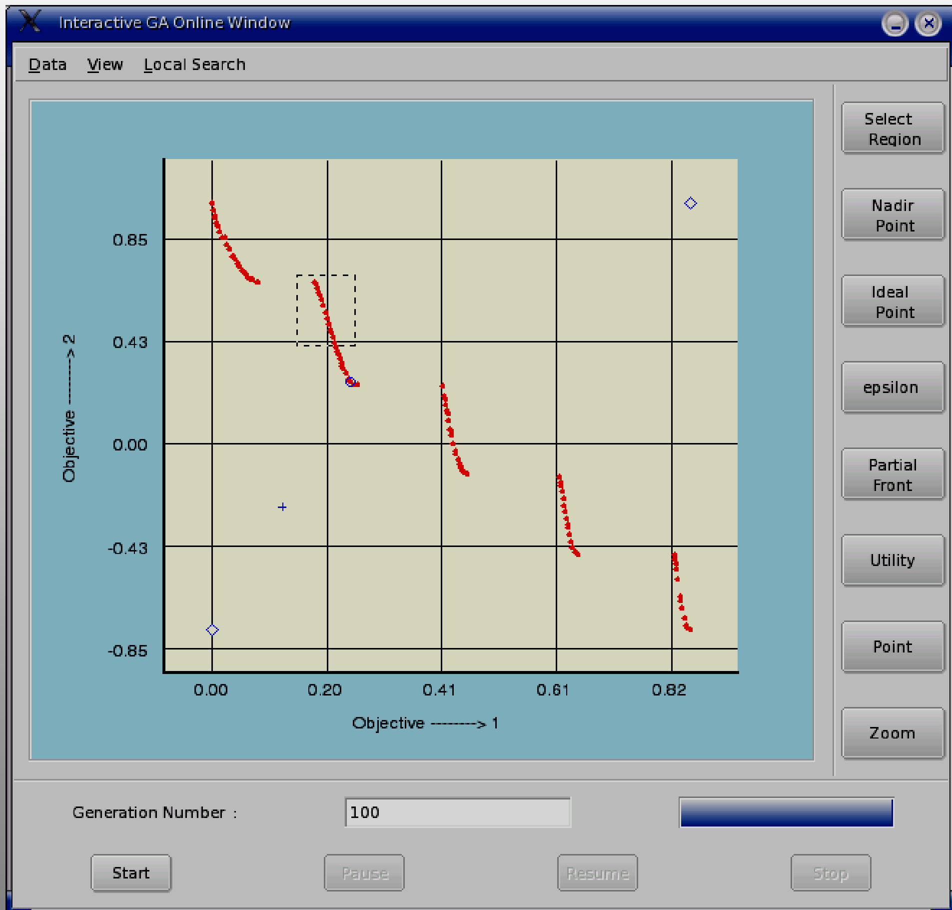
Finding Knee Solutions
(Branke et al., 2004)

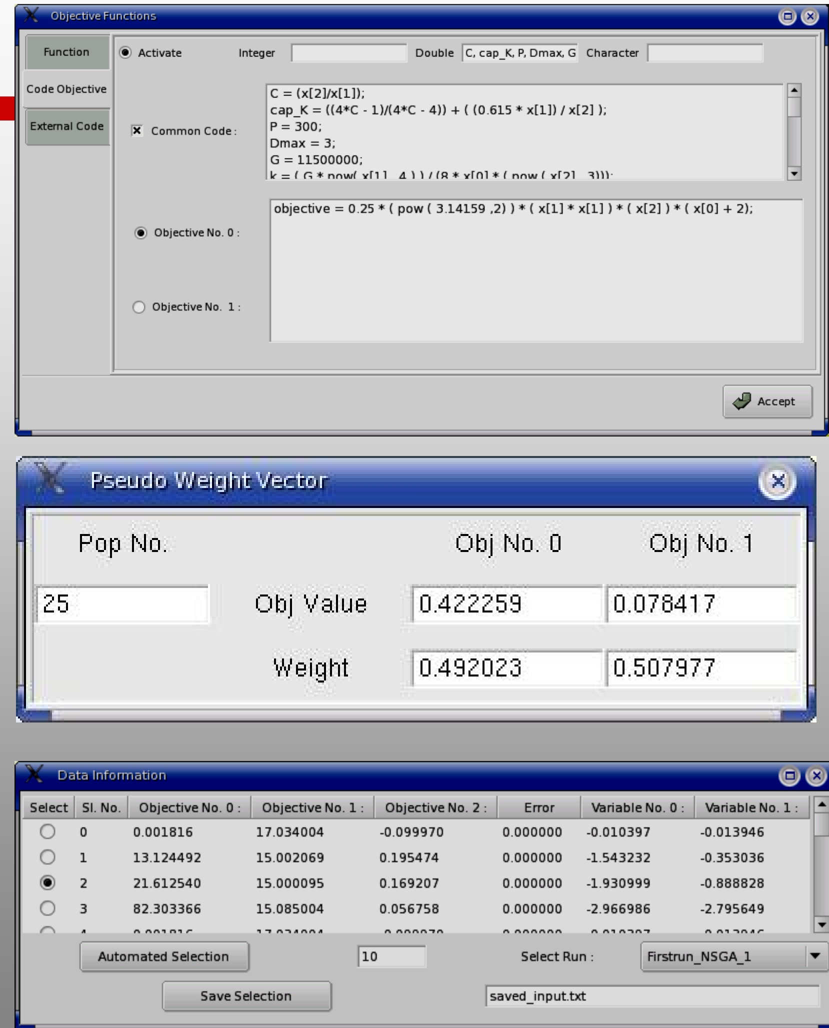
- Find only the knee or near-knee solutions



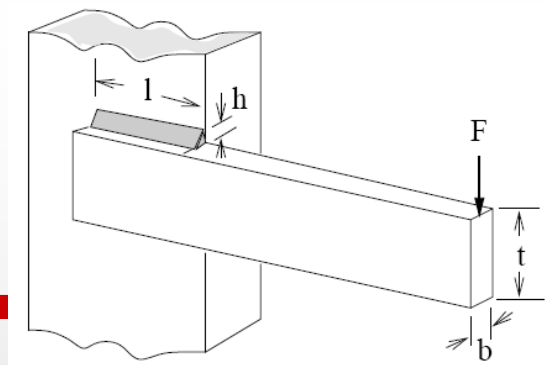
I-MODE Software Developed at KanGAL

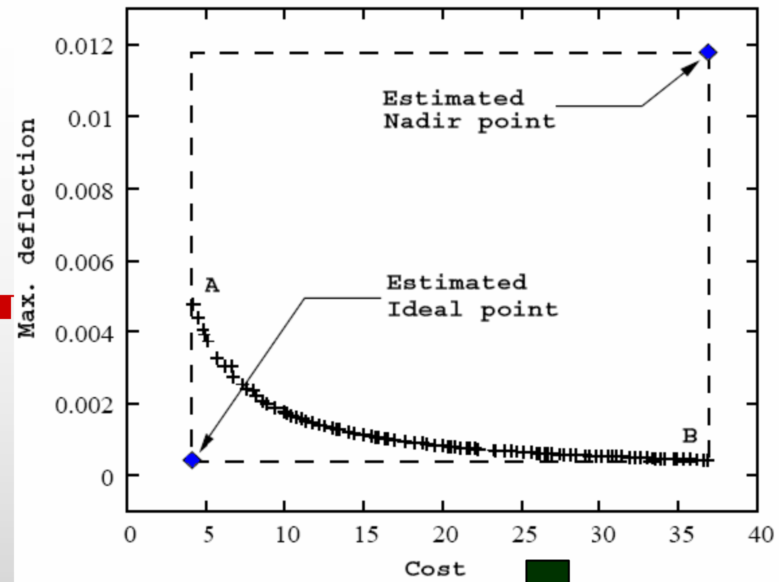
Deb and Chaudhuri, EMO-07

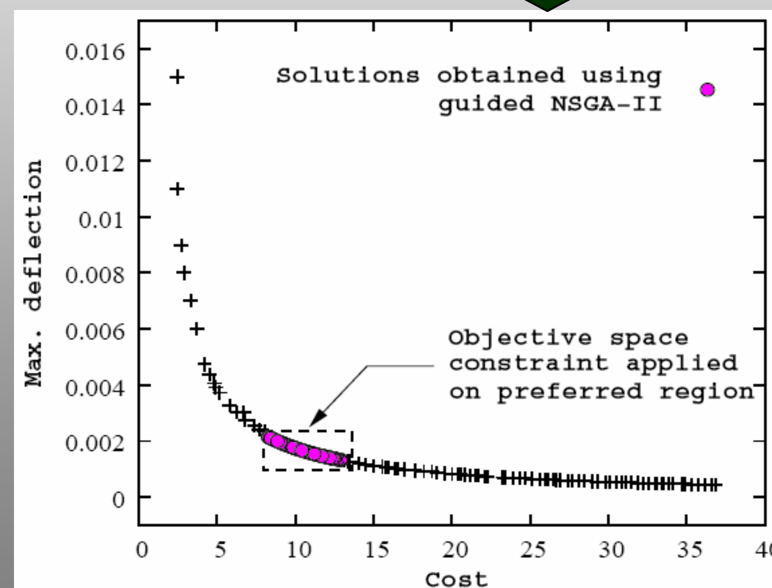


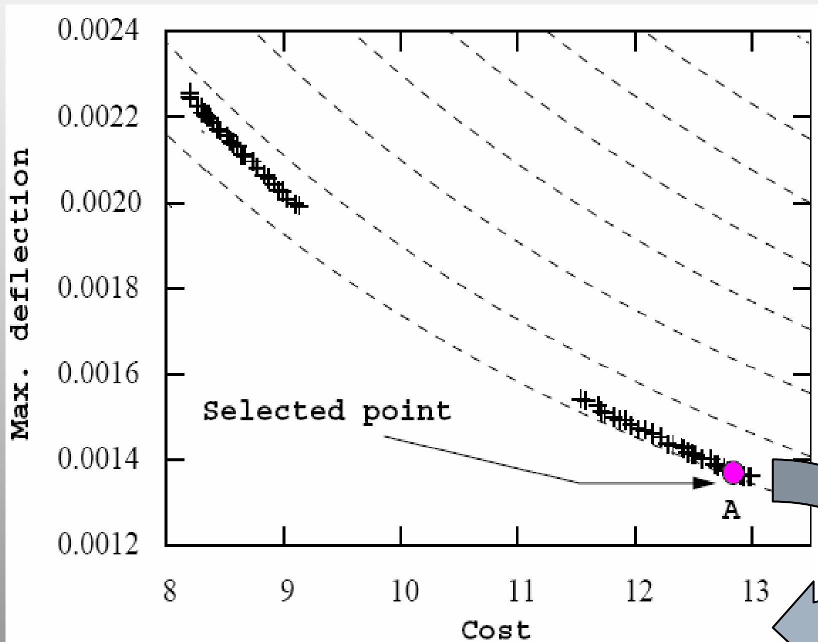


Example:









Decision Variables				Objective Values	
h (in)	*l* (in)	*t* (in)	*b* (in)	Cost	Max. Deflection (in)
0.917	1.009	9.856	1.672	12.838	0.001371

Conclusions of Part C

- EMO is a fast-growing field of research and application
- Practical applications and challenges surfacing
- EMO+MCDM, EMO+Math optimization
- Commercial softwares available
  - iSIGHT and modeFrontier
- Computer codes freely downloadable

Regular EMO Activities

- A dedicated two-yearly conference (EMO):
  EMO-01 (Zurich),
  EMO-03 (Faro),
  EMO-05 (Guanajuato),
  EMO-07 (Sendai)
- Next one in Nantes, France (EMO-09)
- Other major EA conferences (EMO tracks)
- Special issues of journals
- 150+ PhD theses so far since 1993

www.emo09.org