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IEEE International Conference on Fuzzy Systems

Tutorial 2: Model based Fuzzy Logic Control: Overview and Perspectives

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Model Based Fuzzy Control: Overview and Perspectives

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Outline

- Brief review of fuzzy logic control
- T–S fuzzy model
- Representation theory of T-S fuzzy models
- Stability analysis based on common, piecewise, and fuzzy quadratic Lyapunov functions
- Stabilization based on common quadratic Lyapunov functions
- Stabilization based on piecewise quadratic Lyapunov functions
- Stabilization based on fuzzy quadratic Lyapunov functions
- Future perspectives

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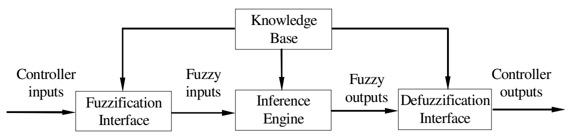
Fuzzy control systems: Structure



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Fuzzy control techniques: review

- Conventional fuzzy logic control
- Fuzzy PID
- Neuro-fuzzy control or fuzzy-neuro control
- Fuzzy sliding mode control
- Adaptive fuzzy control
- T-S model based control

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Conventional fuzzy logic control

- **Mamdani and Assilian's [199], [200]**
- **Heuristic and model free**
- **Applications**:
  warm water plant [137]
  heat exchanger [229]
  robot [10], [289], [314], [319],
  stirred tank reactor [146],
  traffic junction [237]
  steel furnace [153]
  aircraft [58], [161],
  network traffic management and congestion control [131], [169]
  bioprocesses [111]

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Conventional fuzzy logic control

- **The key idea for stability analysis and control design:** to regard a fuzzy controller as a nonlinear controller and to embed the problem of fuzzy control systems into conventional nonlinear system stability theory.
- **Typical approaches:**
  describing function approach [136]
  cell-state transition [132]
  Lure's system approach [59][208]
  Popov's theorem [91] circle criterion [226][244]
  conicity criterion [69]
- **Limitation:** Consistency in design and performance

Fuzzy PID

- **Direct-action type: [201]**
  within the conventional PID concept
  Mamdani fuzzy controller as a two input PI controller
- **Indirect-action type: [107][344]**
  "gain-scheduling" fuzzy PID
  controller gains change as operating condition varies
- **Fuzzy + PID**

Fuzzy PID

- **Advantages:**
  Nonlinear PID
  Better performance
- **Other topics:**
  Self-tuning fuzzy PID controllers [202][215]
  Optimal fuzzy PID control [113][270]
- **Limitation**
  Consistency in design and performance

Neuro-Fuzzy Control

- **Neural Networks:**
  learning capabilities and high computation efficiency
- **Fuzzy Systems:**
  expert knowledge representation
- **Both:**
  Store knowledge or data, robust with uncertainties
- **Fuzzy control augmented by neural networks**
  flexibility, data processing capability, and adaptability

Neuro-Fuzzy Control

- **Fuzzy control augmented by neural networks**
  process of fuzzy reasoning realized by neural networks
- **Advantage:** No information of the plants is required
- **Limitations:**
  (1) systematic analysis of stability
  (2) Learning convergence
- **Other topics:**
  tuning parameters in neuro-fuzzy controller via genetic algorithm [72], [209]
  self-organizing or adaptive neuro-fuzzy control [63], [177]

Fuzzy Sliding Mode Control

- **Fuzzy control similar to a modified sliding mode controller [234]**
- **Fuzzy control + Sliding mode control**
  taking advantages of both techniques
- **Fuzzy sliding surface:** elimination of chattering [94]
- **Fuzzy control + supervisory sliding mode control [80][206]**
- **Advantages:**
  (1) Easier stability analysis and control design
  (2) Robustness

Adaptive Fuzzy Control

- **Similar to neural networks [246]**
- **Fuzzy basis functions [301]**
- **Fuzzy systems [326] [336]**
- **Universal function approximators**
- **Key idea:**

  replacing the unknown nonlinear functions of the controlled systems with fuzzy systems

  linear regression with unknown parameters
- **Robust adaptive control**
- **Semi-global stability**

Motivation for model-based FLC

- Conventional fuzzy control lacks systematic guidelines
- Many nonlinear systems are linear in local regions
- Dynamic fuzzy models represent a large class of nonlinear systems
- There are many mature design techniques in conventional control field

Fuzzy model based control

- **Basic idea**: integration of conventional control theory and fuzzy logic concept

Fuzzy dynamic models or T-S model

$$R^l : If\ z_1\ is\ F_1^l ... AND\ z_q\ is\ F_q^l$$
$$Then\ Sx(t) = A_l x(t) + B_l u(t) + a_l$$
$$y(t) = C_l x(t)$$
$$l = 1,2,...,m$$

Definition

- The l-th fuzzy local dynamic model is a binary set defined as
- $FLDM_l := (\mu_l(x), (A_l, B_l, C_l, a_l))$,
- where (A_l, B_l, C_l, a_l) represents the crisp input-output relationship or dynamic properties of the system at a crisp point.

Using singleton fuzzifier, product fuzzy inference, centre-average defuzzifier

- Dynamic global fuzzy model

$$Sx(t) = A(\mu)x(t) + B(\mu)u(t) + a(\mu)$$
$$y(t) = C(\mu)x(t)$$
$$A(\mu) = \sum_{l=1}^{m} \mu_l A_l, B(\mu) = \sum_{l=1}^{m} \mu_l B_l,$$
$$C(\mu) = \sum_{l=1}^{m} \mu_l C_l, a(\mu) = \sum_{l=1}^{m} \mu_l a_l$$

Remarks

- The fuzzy dynamic model includes two kinds of knowledge
- The model has the structure of a two level control system
- Similar to the concept of ordinary piecewise linear approximation methods in nonlinear control

Stability analysis

$$x(t+1)=\sum_{l=1}^{m}\mu_l(z)A_l x(t)$$

- Common quadratic Lyapunov functions (CQLF)
- Piecewise quadratic Lyapunov functions (PQLF)
- Fuzzy quadratic Lyapunov functions (FQLF)

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Common Quadratic Lyapunov Functions

Lyapunov functions:

$$V(x)=x^T Px$$

Stability condition:

$$\Delta V(x)=x(t+1)^T Px(t+1)-x(t)^T Px(t)<0$$

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Common Quadratic Lyapunov Functions

Stability conditions (LMIs):

$$A_l^T PA_l - P<0 \qquad l\in L$$

Or

$$\begin{bmatrix}-X & XA_l^T\\ A_l X & -X\end{bmatrix}<0 \qquad l\in L$$

Remark: General conservative especially for highly nonlinear systems

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Example I- stability analysis

$$A_1=\begin{bmatrix}1 & 0.5\\ -0.3 & 0.8\end{bmatrix}, A_2=\begin{bmatrix}1 & 0.4875\\ -0.275 & 0.8\end{bmatrix},$$

$\dots$

$$A_6=\begin{bmatrix}1 & 0.4125\\ -0.125 & 0.8\end{bmatrix}, A_7=\begin{bmatrix}1 & 0.4\\ -0.1 & 0.8\end{bmatrix}$$

No CQLF can be found to confirm its stability!

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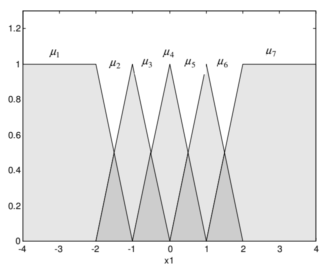
Membership functions



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Piecewise Quadratic Lyapunov Functions -I

Space partitions:

$$S_l = \{z | \mu_l(z) > \mu_i(z), \quad i = 1,2,...,m, \; i \neq l\}$$

System model in local region:

$$x(t+1) = (A_l + \Delta A_l(\mu))x(t) \qquad z(t) \in S_l$$

$$[\Delta A_l(\mu)]^T [\Delta A_l(\mu)] \leq E_{lA}^T E_{lA} \qquad l \in L$$

Piecewise Quadratic Lyapunov Functions -I

Lyapunov functions:

$$V(x) = x^T P_l x \qquad z \in S_l$$

Stability condition:

$$\Delta V(x) = x(t+1)^T P_l x(t+1) - x(t)^T P_j x(t) < 0$$

Piecewise Quadratic Lyapunov Functions -I

Stability condition:

$$\begin{bmatrix} A_l^T P_l A_l - P_l + E_{lA}^T E_{lA} & A_l^T P_l \\ P_l A_l & -(I - P_l) \end{bmatrix} < 0 \qquad l \in L$$

$$\begin{bmatrix} A_l^T P_j A_l - P_l + E_{lA}^T E_{lA} & A_l^T P_j \\ P_j A_l & -(I - P_j) \end{bmatrix} < 0 \qquad l, j \in \Omega$$

$$\Omega := \{(l, j) \mid z(t) \in S_l, \; z(t+1) \in S_j, \; \forall l, j \in L, l \neq j\}$$

Piecewise Quadratic Lyapunov Functions -I

Stability condition (alternative):

$$\begin{bmatrix} -X_l & X_l A_l^T & X_l E_{lA}^T \\ A_l X_l & -(X_l - I) & 0 \\ E_{lA} X_l & 0 & -I \end{bmatrix} < 0 \qquad l \in L$$

$$\begin{bmatrix} -X_l & X_l A_l^T & X_l E_{lA}^T \\ A_l X_l & -(X_j - I) & 0 \\ E_{lA} X_l & 0 & -I \end{bmatrix} < 0 \qquad l, j \in \Omega$$

Piecewise Quadratic Lyapunov Functions -II

Space partitions:

crisp (operating) and fuzzy (interpolation) regions

System model in local region:

$$x(t+1) = \sum_{k \in K(l)} \mu_k(z) \{A_k x(t)\}$$

$$z(t) \in S_l \qquad l \in \overline{L}$$

Piecewise Quadratic Lyapunov Functions -II

Lyapunov functions:

$$V(x) = x^T P_l x \qquad z \in S_l$$

Stability condition:

$$\Delta V(x) = x(t+1)^T P_l x(t+1) - x(t)^T P_j x(t) < 0$$

Piecewise Quadratic Lyapunov Functions -II

Stability condition:

$$A_k^T P_l A_k - P_l < 0 \qquad l \in \bar{L} \qquad k \in K(l)$$

$$A_k^T P_j A_k - P_l < 0 \qquad (l,j) \in \bar{\Omega} \qquad k \in K(l)$$

Piecewise Quadratic Lyapunov Functions -II

Stability condition (alternative):

$$\begin{bmatrix} -X_l & X_l A_k^T \\ A_k X_l & -X_l \end{bmatrix} < 0 \qquad l \in \bar{L} \qquad k \in K(l)$$

$$\begin{bmatrix} -X_l & X_l A_k^T \\ A_k X_l & -X_j \end{bmatrix} < 0 \qquad (l,j) \in \bar{\Omega} \qquad k \in K(l)$$

Example I- Stability analysis

$$A_1 = \begin{bmatrix} 1 & 0.5 \\ -0.3 & 0.8 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.4875 \\ -0.275 & 0.8 \end{bmatrix},$$

$$\ldots$$

$$A_6 = \begin{bmatrix} 1 & 0.4125 \\ -0.125 & 0.8 \end{bmatrix}, A_7 = \begin{bmatrix} 1 & 0.4 \\ -0.1 & 0.8 \end{bmatrix}$$

PQLF can be found to confirm its stability!

Example I- Stability analysis

$$P_1 = \begin{bmatrix} 0.0189 & 0.0063 \\ 0.0063 & 0.0330 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0.0185 & 0.0066 \\ 0.0066 & 0.0333 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.0183 & 0.0070 \\ 0.0070 & 0.0343 \end{bmatrix} \quad P_4 = \begin{bmatrix} 0.0179 & 0.0078 \\ 0.0078 & 0.0356 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0.0176 & 0.0086 \\ 0.0086 & 0.0376 \end{bmatrix} \quad P_6 = \begin{bmatrix} 0.0170 & 0.0090 \\ 0.0090 & 0.0403 \end{bmatrix}$$

$$P_7 = \begin{bmatrix} 0.0168 & 0.0094 \\ 0.0094 & 0.0427 \end{bmatrix}$$

Remarks

- PQLF –I based partition introduces uncertainty terms and thus leads to conservatism
- PQLF-II reduces to CQLF based approach if all the Lyapunov matrices are chosen to be the same
- PQLF based approaches are less conservative than those based on CQLFs, but computation is more demanding

Fuzzy Quadratic Lyapunov Functions

Lyapunov functions:

$$V(x) = \sum_{l=1}^{m} \mu_l(z) x^T P_l x$$

Stability condition:

$$\Delta V(x) = x(t+1)^T P_l x(t+1) - x(t)^T P_j x(t) < 0$$

Fuzzy Quadratic Lyapunov Functions

Stability condition:

$$A_l^T P_j A_l - P_l < 0 \qquad j, l \in L$$

Or

$$\begin{bmatrix} -X_l & X_l A_l^T \\ A_l X_l & -X_j \end{bmatrix} < 0 \qquad j, l \in L$$

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Remarks

- FQLF based approach reduces to CQLF based approach if all the Lyapunov matrices are chosen to be the same
- FQLF based approaches are less conservative than those based on CQLFs, but computation is more demanding
- Continuous time case leads to difficulty
- Stability analysis is much more involved with affine terms, [140], [141], [77], [128], and [295]

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Stabilization via CQLFs

Smooth controller:

$$C^l: \quad IF \quad z_1 \; is \; F_1^l \; and \ldots \; z_q \; is \; F_q^l$$
$$THEN \quad u_l(t) = -K_l x(t)$$
$$u(t) = \sum_{l=1}^{m} \mu_l u_l(t) = -\sum_{l=1}^{m} \mu_l K_l x(t) = -Kx(t)$$

Closed loop system:

$$x(t+1) = \sum_{j=1}^{m} \sum_{l=1}^{m} \mu_j \mu_l (A_l + B_l K_j) x(t)$$

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Stabilization condition

LMIs:

$$\begin{bmatrix} -X & XA_l^T + Q_j^T B_l^T \\ A_l X + B_l Q_j & -X \end{bmatrix} < 0 \quad l, j \in L$$

Controller gains:

$$K_l = Q_l X^{-1}$$

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Remarks

- Many improved approaches
- Reducing the number of LMIs
- Reducing the conservatism
- Or both
- [142], [192], [198], [267], [275], [286], [304]

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Stabilization via CQLFs

Switching controller:

$$u(t) = K_l x(t) \qquad z(t) \in S_l \qquad l \in L$$

Closed loop system:

$$x(t+1) = (A_l + \Delta A_l(\mu) + (B_l + \Delta B_l(\mu))K_l) x(t)$$

$$z(t) \in S_l \qquad l \in L$$

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Stabilization condition – switching controller

LMIs:

$$\begin{bmatrix} -X & XA_l^T + Q_l^T B_l^T & XE_{lA}^T & Q_l^T E_{lB}^T \\ A_l X + B_l Q_l & -(X-I) & 0 & 0 \\ E_{lA} X & 0 & -\frac{1}{2} I & 0 \\ E_{lB} Q_l & 0 & 0 & -\frac{1}{2} I \end{bmatrix} < 0 \quad l \in L$$

Controller gains:

$$K_l = Q_l X^{-1}$$

Remarks

- Switching controller approach has less number of LMIs
- Introduction of uncertainty terms might lead to conservatism
- Extension to H-infinity, H2 and other controller designs
- Extension to observer design, filter design
- Extension to time-delay systems and uncertain fuzzy systems

Examples

- Inverted pendulum
- Ball-beam system
- Bench-mark robust nonlinear control problem
- Backing-up control of truck-trailer
- MIMO coal pulverizer mill

Problems

- Tend to be conservative
- Common quadratic Lyapunov functions might not exist for highly nonlinear systems

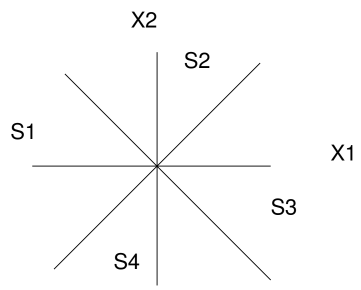
Example II – Stabilization

$$A_1 = \begin{bmatrix} 1 & 0.4 \\ -0.5 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.5 \\ -0.4 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

No stabilization solution can be found by CQLF based approaches!

Region Partitions



Stabilization via PQLFs

Space partitions:

$$S_l = \{z | \ \mu_l(z) > \mu_i(z), \quad i = 1,2,...,m,\ i \neq l\}$$

System model with switching controller in local region:

$$x(t+1) = (A_l + \Delta A_l(\mu) + (B_l + \Delta B_l(\mu))K_l)x(t) \qquad l \in L$$

Lyapunov functions:

$$V(x) = x^T X_l^{-1} x \qquad z \in S_l$$

Stabilization Conditions

$$\begin{bmatrix} -X_l & X_l A_l^T + Q_l^T B_l^T & X_l E_{lA}^T & Q_l^T E_{lB}^T \\ A_l X_l + B_l Q_l & -(X_l - I) & 0 & 0 \\ E_{lA} X_l & 0 & -\frac{1}{2} I & 0 \\ E_{lB} Q_l & 0 & 0 & -\frac{1}{2} I \end{bmatrix} < 0$$

$$l \in L$$

Stabilization Conditions-cont.

$$\begin{bmatrix} -X_l & X_l A_l^T + Q_l^T B_l^T & X_l E_{lA}^T & Q_l^T E_{lB}^T \\ A_l X_l + B_l Q_l & -(X_j - I) & 0 & 0 \\ E_{lA} X_l & 0 & -\frac{1}{2} I & 0 \\ E_{lB} Q_l & 0 & 0 & -\frac{1}{2} I \end{bmatrix} < 0 \qquad l, j \in \Omega$$

Controller gains:

$$K_l = Q_l X_l^{-1}, \quad l \in L$$

Stabilization via PQLFs

Space partitions:

crisp (operating) and fuzzy (interpolation) regions

System model with switching controller in local region:

$$x(t+1) = \sum_{k \in K(l)} \mu_k(z(t))(A_k + B_k K_l)x(t)$$

$$z(t) \in S_l \qquad l \in L$$

Stabilization Conditions

$$\begin{bmatrix} -X_l & X_l A_k^T + Q_l^T B_k^T \\ A_k X_l + B_k Q_l & -X_l \end{bmatrix} < 0 \qquad l \in \overline{L} \qquad k \in K(l)$$

$$\begin{bmatrix} -X_l & X_l A_k^T + Q_l^T B_k^T \\ A_k X_l + B_k Q_l & -X_j \end{bmatrix} < 0 \qquad (l, j) \in \Omega \quad k \in K(l)$$

Controller gains:

$$K_l = Q_l X_l^{-1}, \quad l \in \overline{L}$$

Remarks

- Improved approaches to reducing number of LMIs or conservatism
- Extension to H-infinity, H2 and other controller designs
- Extension to observer design, filter design
- Extension to time-delay systems and uncertain fuzzy systems

New Piecewise Lyapunov Functions
Continuous time Case

Lyapunov function:

$$V(x)=\begin{cases} x^T P_l x & x\in S_l, l\in L_0 \\ \bar{x}^T \bar{P}_l \bar{x} & x\in S_l, l\in L_1 \end{cases}$$

$$P_l = F_l^T T F_l, \quad l\in L_0$$

$$\bar{P}_l = \bar{F}_l^T T \bar{F}_l, \quad l\in L_1$$

Boundary condition:

$$\bar{F}_l \bar{x} = \bar{F}_j \bar{x}, \quad x\in S_l \cap S_j$$

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New Piecewise Lyapunov Functions
Continuous time Case

- It is difficult to develop controller design approaches via LMI techniques due to the special structure of the Lyapunov matrices
- BMI based approaches have been developed

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Example II – Cont.

$$A_1=\begin{bmatrix} 1 & 0.4 \\ -0.5 & 1 \end{bmatrix}, A_2=\begin{bmatrix} 1 & 0.5 \\ -0.4 & 1 \end{bmatrix},$$

$$B_1=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_2=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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Example II: Solution with PQLFs

$$P_1=\begin{bmatrix} 1.9607 & -0.8576 \\ -0.8576 & 3.2186 \end{bmatrix}, P_2=\begin{bmatrix} 3.2186 & 0.8576 \\ 0.8576 & 1.9607 \end{bmatrix}$$

$$K_1=[0.4038 \quad -1.0444], K_2=[-1.0444 \quad -0.4038]$$

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Example III - Henon map

$$\begin{cases} x_1(t+1) = -x_1^2(t)+0.3x_2(t)+1.4+u(t)+0.01\sin(2\pi t) \\ x_2(t+1)=x_1(t) \end{cases}$$

$$\begin{cases} e_1(t+1) = -e_1^2(t)+0.3e_2(t)+u(t)+0.01\sin(2\pi t) \\ e_2(t+1)=e_1(t) \end{cases}$$

$$e_1(t)=x_1(t) \qquad e_2(t)=x_2(t)+14/3$$

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Fuzzy model

R^1: *If* e_1 *is* F_1

Then $e(t+1)=A_1e(t)+B_1u(t)+D_1v(t)$

R^2: *If* e_1 *is* F_2

Then $e(t+1)=A_2e(t)+B_2u(t)+D_2v(t)$

$$F_1(e_1(t))=\frac{1}{2}\left(1-\frac{e_1(t)}{d}\right) \qquad F_2(e_1(t))=\frac{1}{2}\left(1+\frac{e_1(t)}{d}\right)$$

$$d=30$$

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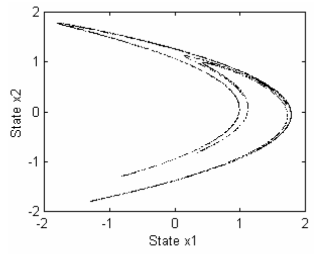
Henon map - Parameters

$$A_1 = \begin{bmatrix} d & 0.3 \\ 1 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} -d & 0.3 \\ 1 & 0 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad D_1 = D_2 = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}$$

$$v(t) = \sin(2\pi t)$$

Open Loop Response



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Solutions

$$P_1 = \begin{bmatrix} 0.0317 & -0.0144 \\ -0.0144 & 1.9243 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 0.0315 & 0.0567 \\ 0.0567 & 8.4147 \end{bmatrix}$$

$$K_1 = [-30.0246 \quad -0.3] \qquad K_2 = [29.9762 \quad -0.3]$$

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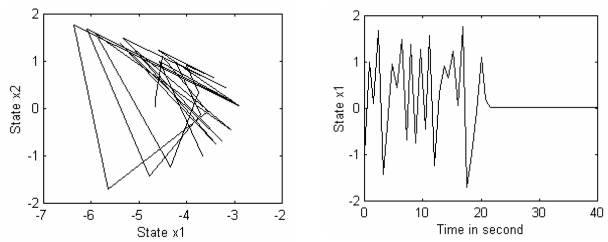
Closed loop responses



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Stabilization via FQLFs

Smooth controller:

$$u(t) = \sum_{l=1}^{m} \mu_l K_l x(t)$$

Closed loop system:

$$x(t+1) = \sum_{j=1}^{m} \sum_{l=1}^{m} \mu_j \mu_l (A_l + B_l K_j) x(t)$$

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Stabilization via FQLFs

Lyapunov functions:

$$V(x) = \sum_{l=1}^{m} \mu_l(z) x^T X_l^{-1} x$$

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Stabilization conditions

LMIs:

$$\begin{bmatrix} X_l - G_j^T - G_j & G_j^T A_l^T + Q_j^T B_l^T \\ A_l G_j + B_l Q_j & -X_i \end{bmatrix} < 0 \qquad i, j, l \in L$$

Controller gains:

$$K_j = Q_j G_j^{-1}, \quad j \in L$$

Remarks

- Improved approaches to reducing number of LMIs or conservatism
- Extension to H-infinity and H2 limited
- Extension to observer design limited

Stabilization via FQLFs :Continuous time case

$$V(x) = \sum_{l=1}^{m} \mu_l(x) x^T P_l x$$

$$\dot{\mu} \rightarrow \dot{x} \rightarrow difficulty$$

Universal fuzzy controllers

$$\dot{x}(t) = f(x(t), u(t))$$

$$u(t) = \hat{g}(x(t), \mu(t))$$

$$\hat{g}(x(t), \mu(t)) = \sum_{l=1}^{m} \mu_l(t) g_l(x(t))$$

$$\dot{x}(t) = f(x(t), \hat{g}(x(t)))$$

Preliminary Results

- Fuzzy dynamic models with affine terms are universal function approximator
- Fuzzy controllers are universal controllers for a class of nonlinear systems

Challenges

- What kind of nonlinear systems can be represented by FDM? How to determine FDM?
- Fuzzy controller design: piecewise Lyapunov function or fuzzy Lyapunov function, *other techniques*
- How to use as much information of FDM as possible?
- Universal fuzzy controllers
- Adaptive control based on FDM

Challenges

- Whether can fuzzy controllers, which are designed to stabilize T-S fuzzy models, stabilize the original nonlinear systems? How to achieve it if possible?
- Time-delay nonlinear systems
- Hybrid control systems
- Stochastic systems
- Applications

Conclusions

- Conventional control theory can be combined with fuzzy logic control
- Constructive designs are possible with guaranteed performance for FLC

THANKS!

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T−S FUZZY MODEL AND UNIVERSAL FUNCTION APPROXIMATION

- **General Result**: Able to approximate any continuous nonlinear function on any compact set of interest.
- Let $f(x)$ be continuous on compact set X, for any $\varepsilon > 0$,
  - there exists a standard (i.e., Mamdani) fuzzy system
    $$F(x) = \sum_{l=1}^{m} \mu_l(x) a_l$$
  - there exists an affine T-S fuzzy system
    $$F(x) = \sum_{l=1}^{m} \mu_l(x)(a_l + A_l x)$$

  such that
  $$\max_{x \in X} | f(x) - F(x) | < \varepsilon$$

T−S FUZZY MODEL AND UNIVERSAL FUNCTION APPROXIMATION

- **Particular Result for control**: Able to approximate any continuous nonlinear function with zero as its equilibrium point.
- Let $f(x)$ be continuous differentiable on compact set X with $f(0)=0$, there exists a linear T-S system $(i.e., a_l \equiv 0)$
  $$F(x) = \sum_{l=1}^{m} \mu_l(x) A_l x$$
  such that
  $$\max_{x \in X} | f(x) - F(x) | < \varepsilon$$

T−S FUZZY MODEL AND UNIVERSAL FUNCTION APPROXIMATION

- **Why**?
  - The reason is that, when $f(x)$ continuous differentiable on compact set X with $f(0)=0$, there exists a continuous matrix function $G(x)$ as

$$g_{ij}(x_1,\ldots,x_n) = \begin{cases} \dfrac{f_i(0,\ldots,0,x_j,x_{j+1},\ldots,x_n) - f_i(0,\ldots,0,0,x_{j+1},\ldots,x_n)}{x_j} & \text{if } x_j \neq 0 \\ \dfrac{\partial f_i(0,\ldots,0,0,x_{j+1},\ldots,x_n)}{\partial x_j} & \text{if } x_j = 0 \end{cases}$$

such that $f(x) = G(x)x$. Using a Mamdani fuzzy system to approximate $G(x)$ leads to the above result.

Affine nonlinear control systems and T-S fuzzy control systems

- Recall dynamic (affine) T-S fuzzy model

$$Sx(t) = A(\mu)x(t) + B(\mu)u(t) + a(\mu)$$
$$y(t) = C(\mu)x(t)$$
$$A(\mu) = \sum_{l=1}^{m} \mu_l(x)A_l, B(\mu) = \sum_{l=1}^{m} \mu_l(x)B_l,$$
$$C(\mu) = \sum_{l=1}^{m} \mu_l(x)C_l, a(\mu) = \sum_{l=1}^{m} \mu_l(x)a_l$$

here assume $z = x$, other cases will consider later.

- **Question 1**: What nonlinear systems can be represented by above T-S fuzzy control systems?

Affine nonlinear control systems and T-S fuzzy control systems

- **Answer to Question 1:** Let

$$Sx(t) = N[x(t), u(t)]$$

be a nonlinear system with $x(t) \in X$ (compact set) and zero as its equilibrium point. Then it can be approximated to any degree of accuracy by an affine T-S fuzzy model if and only if it is an affine nonlinear system. That is,

$$Sx(t) = N[x(t), u(t)] = f[x(t)] + g[x(t)]u(t)$$

Affine nonlinear control systems and T-S fuzzy control systems

- **Why**?
  - $f(x)$ can be approximated by an affine T-S fuzzy system

$$f_A(x) = \sum_{l=1}^{m} \mu_l(x)(a_l + A_l x) = a(\mu) + A(\mu)x$$

  - $g(x)$ can be approximated by a standard fuzzy system

$$B(\mu) = \sum_{l=1}^{m} \mu_l(x)B_l$$

Therefore

$$Sx(t) = f[x(t)] + g[x(t)]u(t)$$

can be approximated by

$$Sx(t) = a(\mu) + A(\mu)x(t) + B(\mu)u(t)$$

Affine nonlinear control systems and T-S fuzzy control systems

- **Questions needed to be further addressed**
  - **Question 2**. Most existing stability analysis and stabilization methods assume that fuzzy control systems are

$$Sx(t) = A(\mu)x(t) + B(\mu)u(t)$$
$$A(\mu) = \sum_{l=1}^{m} \mu_l(x)A_l, \ B(\mu) = \sum_{l=1}^{m} \mu_l(x)B_l$$

That is, $a(x) = 0$

Then the question is what nonlinear systems can be represented by such fuzzy control systems (called linear T-S fuzzy control systems from now on)?

Affine nonlinear control systems and T-S fuzzy control systems

- **Problems needed to be further addressed**
  - **Question 3**. Given a nonlinear observation function $y(t) = H[x(t)]$, whether it can be approximated by fuzzy observation function

$$y(t) = C(\mu)x(t) \quad C(\mu) = \sum_{l=1}^{m} \mu_l(x)C_l$$

to any degree of accuracy?

Affine nonlinear control systems and T-S fuzzy control systems

- **Answer to Question 2:** Let

$$Sx(t) = N[x(t), u(t)] = f[x(t)] + g[x(t)]u(t)$$

be an affine nonlinear system with $x(t) \in X$ (compact set) and zero as its equilibrium point. If $f(x)$ continuous differentiable, then it can be approximated to any degree of accuracy by a linear T-S fuzzy model as

$$Sx(t) = A(\mu)x(t) + B(\mu)u(t)$$
$$A(\mu) = \sum_{l=1}^{m} \mu_l(x)A_l, \ B(\mu) = \sum_{l=1}^{m} \mu_l(x)B_l$$

Affine nonlinear control systems and T-S fuzzy control systems

- **Answer to Question 3:** Let nonlinear observation function $y(t) = H[x(t)]$ be continuous differentiable with zero as its equilibrium point. then it can be approximated to any degree of accuracy by a linear T-S fuzzy observation model as

$$y(t) = C(\mu)x(t) \quad C(\mu) = \sum_{l=1}^{m} \mu_l(x)C_l$$

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Affine nonlinear control systems and T-S fuzzy control systems

- **Why**?
  - $f(x)$ can be approximated by a linear T-S fuzzy system

$$\sum_{l=1}^{m} \mu_l(x)A_l x = A(\mu)x$$

  - $g(x)$ can be approximated by a standard fuzzy system

$$B(\mu) = \sum_{l=1}^{m} \mu_l(x)B_l$$

  - $H(x)$ can be approximated by a linear T-S fuzzy system

$$\sum_{l=1}^{m} \mu_l(x)C_l x = C(\mu)x$$

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Affine nonlinear control systems and T-S fuzzy control systems

- **Summarize Answers to Questions 2 and 3:** Let

$$Sx(t) = N[x(t), u(t)] \quad y(t) = H[x(t)]$$

be a continuous differentiable nonlinear system. Then it can be approximated to any accuracy by a linear T-S fuzzy control system

$$Sx(t) = A(\mu)x(t) + B(\mu)u(t) \quad y(t) = C(\mu)x(t)$$

$$A(\mu) = \sum_{l=1}^{m} \mu_l(x)A_l,\ B(\mu) = \sum_{l=1}^{m} \mu_l(x)B_l,\ C(\mu) = \sum_{l=1}^{m} \mu_l(x)C_l$$

if and only if it is an affine nonlinear system. That is,

$$N[x(t), u(t)] = f[x(t)] + g[x(t)]u(t)$$

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Affine nonlinear control systems and T-S fuzzy control systems

- **Summarize Answers to Questions 2 and 3:**
  - Another representation:

$$\begin{aligned} Sx(t) &= f[x(t)] + g[x(t)]u(t) \\ &= [A(\mu) + \Delta A(\mu)]x(t) + [B(\mu) + \Delta B(\mu)]u(t) \\ y(t) &= H[x(t)] = [C(\mu) + \Delta C(\mu)]x(t) \end{aligned}$$

$$|\Delta A(\mu)| \le \varepsilon E_A,\ |\Delta B(\mu)| \le \varepsilon E_B,\ |\Delta C(\mu)| \le \varepsilon E_C$$

  - That is, the original nonlinear system can be stabilizable if

$$Sx(t) = A(\mu)x(t) + B(\mu)u(t) \quad y(t) = C(\mu)x(t)$$

is robust stabilizable under uncertainty $\Delta A(\mu), \Delta B(\mu), \Delta C(\mu)$

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Affine nonlinear control systems and T-S fuzzy control systems

Remarks:

- If a given nonlinear system is continuous differentiable, then the representation capabilities of simpler linear T-S fuzzy systems are the same as affine T-S systems
- This means that more commonly existing methods for stabilization based on linear T-S fuzzy systems are enough in most applications when a considered nonlinear system is an affine one.

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Affine nonlinear control systems and T-S fuzzy control systems

Remarks:

- On the other hand, affine T-S fuzzy systems are useful
  - If a given nonlinear system is only continuous, as some continuous only nonlinear affine systems can not be approximated by linear T-S fuzzy systems. For example:

$$\dot{x}(t) = \sqrt{|x(t)|} + u(t)$$

  - Affine T-S models require less local systems as the approximation accuracy of affine T-S systems are 3rd order whereas linear T-S systems are 2nd order (related to the width of fuzzy partitions)

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General nonlinear control systems and T-S fuzzy control systems

- As shown before that linear or affine T-S fuzzy systems are enough and only enough to represent affine nonlinear control systems.
- For a general (non-affine) nonlinear system, more general T-S model are needed. That is, the following question needs to be solved
  - **Question 4**. If a given nonlinear control system is not affine one, how to represent it by fuzzy control system model?

General nonlinear control systems and T-S fuzzy control systems

- **Answers to Questions 4.** Let

$$Sx(t) = N[x(t), u(t)]$$

be a continuous differentiable nonlinear system on $[x(t), u(t)] \in X \times U$ (compact set) with zero as its equilibrium point. Then it can be approximated to any degree of accuracy by a general T-S fuzzy control system

$$Sx(t) = A(\mu)x(t) + B(\mu)u(t)$$

$$A(\mu) = \sum_{l=1}^{m} \mu_l(z)A_l,\ B(\mu) = \sum_{l=1}^{m} \mu_l(z)B_l$$

where $z = (x, u)$

General nonlinear control systems and T-S fuzzy control systems

- **Why**? $N(x,u)$ can be approximated by a linear T-S fuzzy system

$$\sum_{l=1}^{m} \mu_l(x,u)[A_l x + B_l u] = A(\mu)x + B(\mu)u$$

- The rule base of such general T-S fuzzy control system can be written as

$$R^l: If\ x_1\ is\ F_1^l ...AND\ x_q\ is\ F_q^l, u_1\ is\ U_1^l ...AND\ u_m\ is\ U_m^l,$$

$$Then\ Sx(t) = A_l x(t) + B_l u(t) \qquad l = 1,2,...,m$$

where $\mu_l(x,u) = \prod_{i=1}^{q} F_i^l(x_i) \prod_{j=1}^{m} U_j^l(u_j)$

General nonlinear control systems and T-S fuzzy control systems

Remarks

- So far it is shown that it is not difficult to represent a general nonlinear control system by a T-S fuzzy control system
- However, the control design becomes much complicated and this is generally speaking still an open problem

THANKS!
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Questions