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### Tutorial 4: Rule Base Compression in Fuzzy Systems



## **Rule Base Compression in Fuzzy Systems Tutorial 4**

**IEEE International Conference  
on Fuzzy Systems  
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## **Tutorial Sections**

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2. Basic Types of Fuzzy Rule Based Systems
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## **Tutorial on Rule Base Compression in Fuzzy Systems**

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## Preface

*Doing research is a great adventure  
As any adventure sometimes it is hard  
You may feel alone and with no idea where to go  
But if you have courage and press onwards  
You will eventually stand where no one has stood  
And see the world as no one has seen it  
There can be no better feeling than this!*  
Adaptation from 'Introduction to Research', Tom Addis (2004)

Complexity has always been an important attribute in research methods. Even nowadays, in the age of supercomputers capable of performing computations at a very high speed, there are still open problems in many research areas which are due to computational complexity, and in particular, to the lack of adequate methods for managing this complexity. For example, some large scale problems in cosmology and genetics are still unsolvable within a reasonable time with the computational power available at present. In the case of threat from terrorism or natural disasters, the amount of information that is relevant to the decision making process can be so large that it may not be possible to process this information reliably within a reasonable time even with the help of the fastest computers.

We could possibly wait for computer technology to become more powerful to hopefully cope with the current challenges of complexity. Unfortunately, this process may take too long, and if and when it has been finally completed, the expected result may still be quite unreliable. The reason for this is that complexity as an attribute of the world that we live in has not only a quantitative dimension in terms of number and scale but also a qualitative dimension in terms of uncertainty and ambiguity. And until we learn how to deal with the qualitative aspects of complexity, we may never be in the position to solve some of the current problems however advanced the computer technology is. Also, we must bear in mind that efficiency of computer software usually has a much greater impact on computational times associated with large scale problems than the speed of computer hardware on which the software is running.

So, if we want to tackle successfully the current challenges of complexity, we need to be able to develop and implement efficient and intelligent computational algorithms. These algorithms must be capable of not only dealing with the quantitative aspects of complexity on the basis of their efficiency but also with its qualitative aspects by means of their intelligence. In this context, fuzzy systems in the form of rule bases are possibly the best tool available for accounting qualitative aspects of complexity such as the uncertainty of the environment. However, the processes of fuzzification, inference and defuzzification usually make these systems suffer from some quantitative aspects of complexity such as the large number of fuzzy rules and associated operations on the fuzzy membership functions of the inputs and the outputs.

The focus of this work is on the management of complexity in fuzzy rule based systems. This problem has been pushed from a marginal location into the mainstream of fuzzy logic research in recent years. The reason for this move is due to the ever more increasing demand for using fuzzy logic not only for small scale domestic purposes but also in large scale industrial and other applications. As a result, many papers and books on fuzzy logic have started to discuss the complexity aspects of the proposed methods in separate paragraphs or even whole sections. Moreover, a number of specific methods for complexity reduction in fuzzy systems have already been developed and used.

This work is based on the idea of managing complexity rather than only reducing it. In this context, management is viewed as a group of activities such as perception, understanding, and analysis of complexity with the intention of simplifying it formally in a universal and systematic way. This type of approach is quite different from most of the existing complexity reduction methods, which are usually characterised by a limited application scope and empirical nature. In fact, many of these methods simplify the complexity in fuzzy systems by actually ignoring it without adequate justification and with the hope that the resultant simplified system would behave similarly to the original complex one.

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Alexander Gegov

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## Abbreviations

SRB – single rule base  
IRB – identity rule base  
TRB – transpose rule base  
PRB – permutation rule base  
MRB – multiple rule base  
FF – feedforward  
FB – feedback  
CADR – conjunctive antecedents in disjunctive rules  
DADR – disjunctive antecedents in disjunctive rules  
CACR – conjunctive antecedents in conjunctive rules  
DACR – disjunctive antecedents in conjunctive rules  
CON – conjunctive  
DIS – disjunctive  
MIMO – multiple input multiple output  
MISO – multiple input single output  
MO – multiple output  
SO – single output  
IFS – initial fuzzy system  
RFS – reduced fuzzy system  
ERB – equivalent rule base  
I/T/P RB – identity / transpose / permutation rule base  
P-T RB – permutation-transpose rule base  
FRB – feedback rule base  
ARBN – arbitrary rule base network  
CRBN – canonical rule base network  
MRBO – multiple rule base output  
SRBO – single rule base output  
CS – conventional system  
AS – aggregated system  
SS – sorted system  
FS – filtered system  
HS – hierarchical system  
EO – elementary operations

# 1 Introduction

## 1.1 Quantitative and Qualitative Complexity

Complexity is a common attribute of the world that we live in. Although the recent advances in different technologies have made our life easier in many aspects, these advances also bring new challenges that add to the overall complexity in this world. A typical example in this respect is global warming which is a direct consequence of our habits to consume products that require the use of large amounts of energy. Without a doubt, global warming has already become a very complex environmental problem that requires urgent and coordinated actions by international institutions if we want to save our world for future generations.

In general, complexity can be characterised by two main aspects – quantitative and qualitative. The quantitative aspect usually has to do with concepts such as number and scale. For example, if we look at the Internet, it has been growing at an enormous rate as a result of which the number of web pages has increased dramatically over the recent years. Apart from that, more and more geographic areas are acquiring access to the Internet almost every day and this has contributed significantly to the overall enlargement of its scale. The qualitative aspect is usually related to concepts such as uncertainty and ambiguity. If we take again the example with the Internet, it is obvious that as the level of its quantitative complexity increases in terms of the number of web pages and scale, it becomes more difficult to understand and interpret its behaviour, i.e. its level of qualitative complexity also increases.

## 1.2 Time and Safety Critical Implications

In order to be able to cope with the challenges of complexity, we need to identify its possible implications. In this respect, two of the most important implications are time critical and safety critical problems.

In the case of a time critical problem, we need to find a solution to the problem within a limited period of time – otherwise the solution may become irrelevant. For example, an industrial robot that classifies and assembles different particles according to their size and shape has to do these tasks in a way that matches the speed at which the conveyor carrying these particles is moving. Otherwise, the robot would be late for the corresponding assembly operations and the whole manufacturing process may have to be stopped temporarily so that the particles affected by the slow operations of the robot can be rearranged accordingly. Obviously, an increased level of the quantitative and the qualitative complexity in terms of the number of particles carried by the conveyor and the uncertainty in their size or shape, respectively, would make this time critical problem more difficult.

In the case of a safety critical problem, if we do not find a solution to this problem then this could lead to loss of human life. For example, an on-board aircraft collision avoidance system is supposed to detect other aircrafts within a certain radius, determine their location and speed, and if necessary, instruct the pilot to change course. Otherwise, the aircraft would continue to follow the preset course, which may lead to a collision with another aircraft. Here again, an increased level of the quantitative and the qualitative complexity in terms of the number of other closely flying aircrafts and some uncertainty in their location or speed would obviously make this safety critical problem quite difficult.

### 1.3 Speed and Intelligence of Computers

Having looked at the main aspects of complexity and its implications, it would be interesting to see how we could possibly minimise the undesirable impact of this complexity. As the world today is becoming more dominated by computers that assist people in their everyday activities, it would be reasonable to assume that computer hardware and software are the main factors determining the potential impact of complexity on these activities. What really matters in this case is the relevant attributes of computer hardware and software such as speed and intelligence.

The speed of computers is usually associated with the clock frequency at which the basic arithmetic operations are performed at the hardware level and the algorithmic efficiency at the software level which represents the amount of computations as a function of the size of the problem to be solved. Obviously, the speed of computers can affect directly the quantitative aspects of complexity by reducing the time that it takes to carry out a certain amount of computations.

As opposed to speed, intelligence in computers is still something quite unclear and hard to quantify but by this term we usually mean the ability of computer hardware and software to behave in a way that resembles intelligent human beings, e.g. to be able to learn and reason. In this case, there is a clear link with the qualitative aspects of complexity because learning and reasoning are especially helpful in an environment that is characterised by uncertainty, i.e. where the ability to see 'invisible' objects and to predict 'unexpected' events could make a big difference.

### 1.4 Past and Current Research in Fuzzy Logic

Research in the field of fuzzy logic has gone a long way since the idea about fuzzy sets was first introduced in the mid 60's of the 20<sup>th</sup> century by Lotfi Zadeh. Since then, we have witnessed a number of trends most of which have lasted for about a decade.

The late 60's and the 70's were characterised mainly by theoretical works that helped fuzzy logic establish itself as a separate discipline alongside deterministic mathematics and statistics. The main drawback of this period was the highly abstract nature of research, which made many applied scientists ignore fuzzy logic before even making themselves familiar with it to some extent. It was not unusual at that time to hear statements describing fuzzy logic as a totally useless theory that would never work in practice.

However, in the 80's, scientists from Japan started to implement fuzzy logic in a number of domestic appliances such as vacuum cleaners, refrigerators and cookers. Later on, in the 90's, we were able to witness the first successful industrial applications of fuzzy logic. But despite the big number of successful applications of fuzzy logic in this period, the criticism against fuzzy logic did not stop – it has even got bigger in the recent years. This time, the main object of attack coming from both outside and inside of the fuzzy academic community has been the empirical nature of fuzzy logic.

As a whole, the main focus of the criticism on fuzzy logic has been on its inability to behave as a systematic science. A scientific method is usually expected to be applicable for solving a particular problem with a guaranteed success. Unfortunately, that is not the case for most of the known fuzzy logic methods, which are empirical and therefore not quite reliable in terms of the expected results.

### 1.5 Complexity Issues in Fuzzy Systems

Fuzzy systems are usually good at capturing the qualitative aspect of complexity by means of their linguistic modeling and approximate reasoning capabilities. However, this comes with a price because the associated fuzzy operations in the fuzzification, the inference and the

defuzzification stages increase the level of quantitative complexity of the problem. This increase becomes even more embarrassing as the number of inputs in the fuzzy system gets bigger because the amount of these operations is a function of the number of rules which, on its turn, depends on the number of inputs.

Apart from the increased level of their quantitative complexity, the transparency and interpretability of fuzzy systems tends to deteriorate as the number of fuzzy rules increases. In this case, it is harder to observe and explain what is happening in the system. In other words, although the qualitative complexity of the environment is usually well accounted for by the fuzzy system, the system itself appears to generate 'new' qualitative complexity by its existence. So, it turns out that the problem has been actually moved from the environment that the fuzzy system is supposed to model, to the system itself.

There has been a steadily growing interest in complexity issues of fuzzy systems in recent years [1, 3, 4, 13, 14, 15, 18, 19, 24, 36, 40, 42, 44, 62, 69, 75, 77]. This is due to the fact that fuzzy systems have become more widely used in applications of a larger scale as a result of which the associated complexity becomes more apparent. For example, many recent papers and books on fuzzy logic discuss the complexity aspects of the proposed fuzzy methods in separate sections or even chapters. However, the focus of these discussions is usually on quantitative complexity whereas qualitative complexity is often ignored.

## 1.6 From Reduction to Management of Complexity

It is not surprising that research in fuzzy systems has been focused mainly on quantitative complexity issues. After all, it is normal to expect current research methodologies to be strongly affected by the dominant profit orientated values in our society and the associated material targets such as improved efficiency and increased productivity. That is why most of the known methods dealing with complexity in fuzzy systems are aimed primarily at reducing the time for the completion of the required computations.

This work preaches a different philosophy. It argues that we have to change our narrow minded and conservative way of thinking only within 'the box'. Our duty as academics is to open new horizons and suggest viable alternatives to the existing 'status quo' in our subjects as well as in the world in general. Trying to break this 'status quo' would be the best way of not only discovering new knowledge but also of demonstrating a broad, progressive and independent attitude.

As far as fuzzy systems are concerned, we have to start addressing the complexity issues in these systems from a different angle. The idea of reducing complexity by actually ignoring it in a semi-mechanistic way may suit the market needs but it does not speak well about academia. Before reducing complexity, we need to be able to identify, analyse and understand it properly. This is what planned complexity management is all about and this work argues that such an approach must replace the current approach of chaotic complexity reduction.



## 10 Conclusion

### 10.1 Formal Approach for Rule Base Compression

This work treats in detail complexity management aspects in fuzzy systems. In particular, Sections 4-9 are dedicated to different techniques for complexity management. However, there is a common feature uniting most of these techniques in that complexity management in fuzzy systems is usually implemented by compression of the rule base.

In the case of formal presentation, the integer table of a rule base is compressed by a Boolean matrix or binary relation. In formal manipulation, the Boolean matrices or binary relations of two rule bases are compressed into a single Boolean matrix or binary relation by merging. Formal transformation is an extension of formal manipulation whereby the Boolean matrices or binary relations describing a MRB system are compressed into a single Boolean matrix or binary relation describing the equivalent SRB system. And finally, in the case of formal simplification, the Boolean matrix or binary relation of a SRB system is compressed by aggregation of inconsistent rules or filtration of non-monotonic rules.

In this context, formal presentation, manipulation and transformation techniques deal mainly with qualitative aspects of complexity whereby the fuzzy rule bases become more transparent and easier for interpretation. At the same time, formal simplification techniques are focused predominantly on quantitative aspects of complexity whereby the amount of on-line operations and the associated computational times in fuzzy systems are reduced.

### 10.2 Theoretical Significance of Rule Base Compression

The formal approach of fuzzy rule base compression introduced in this work has a big theoretical significance. It makes good use of mathematics and its power to formalise and facilitate a particular course of action as well as to justify and guarantee the result from this action. In this context, a number of algorithms and numerous examples are given for illustration of the underlying theoretical concepts.

Another interesting feature of the formal approach used is that it leads to non-lossy compression of the rule base. In particular, the compression of the IFS is done in a way which allows its reconstruction, if necessary, and ensures that the solution is uncompromised, i.e. the defuzzified output from the RFS is the same as the one from the IFS. In this sense, the approach is very suitable for both time-critical and safety-critical applications.

Also, this formal approach undoubtedly improves the important attributes of computer speed and intelligence. These attributes are crucial for the successful treatment of time-critical and safety-critical applications of fuzzy systems characterised by quantitative and qualitative complexity.

### 10.3 Application Framework for Rule Base Compression

The formal approach of fuzzy rule base compression described in this work is applicable to a wide range of fuzzy systems. It would not be an overstatement to say that it can be applied almost universally to any type of fuzzy system irrespective of the number of its inputs, rules, etc. In this context, the algorithm below describes briefly an application framework for the formal approach.

**Algorithm 10.1**

Off-line

1. Present formally a fuzzy system.
2. For a MRB system, go to step 3; for a SRB system, go to step 5.
3. Manipulate formally the constituent SRB systems of the MRB system.
4. Transform formally the MRB system into an equivalent SRB system.
5. For a MO SRB system, go to step 6; for a SO SRB system, go to step 7.
6. Convert the MO SRB system into an equivalent collection of SO SRB systems.
7. For each SO SRB system, sort the inconsistent rules in groups.
8. For each SO SRB system, aggregate the inconsistent rules from each group.
9. For each SO SRB system, sort the non-monotonic rules in groups.

On-line

1. For each SO SRB system, apply the fuzzification stage.
2. For each SO SRB system, apply the application substage of the inference stage.
3. For each SO SRB system, filter the non-monotonic rules from each group.
4. For each SO SRB system, apply the implication substage of the inference stage.
5. For each SO SRB system, apply the aggregation substage of the inference stage.
6. For each SO SRB system, apply the defuzzification stage.

It is obvious from Algorithm 10.1 that almost all formal compression techniques from this work are applied in off-line steps 1-9. The only exception is the filtration of non-monotonic rules, which is applied in on-line step 3. The remaining on-line steps, i.e. steps 1-2 and 4-6, are occupied by standard processes in fuzzy systems such as fuzzification, application, implication, aggregation and defuzzification. This algorithm shows that the formal approach for rule base compression fits very well within the established general application framework for fuzzy systems.

The on-line steps in Algorithm 10.1 reflect only one simulation cycle of a fuzzy system. In the case of more simulation cycles, all on-line steps must be applied for each new cycle. In this case, the computations for each SO SRB system may be done in parallel, which would reduce the overall computational time.

**10.4 Future Directions for Related Research**

The formal approach of fuzzy rule base compression presented in this work is expected to encourage and stimulate new research in fuzzy systems and related areas. This expectation is based on the fact that this formal approach has a natural overlap with some recent research trends in other types of complex systems, e.g. deterministic and probabilistic systems.

One possibility in this respect would be the extension of fuzzy systems to fuzzy networks, which has already been initiated by the techniques of formal transformation of a MRB system into an equivalent SRB system. In this context, MRB systems can be viewed as networks whose nodes are in the form of SRB systems and whose connections are of FF or FB type.

Another possibility would be the extension of fuzzy systems to fuzzy multi-agent systems. This has also been initiated by the techniques of formal transformation of fuzzy systems whereby the transformation process exhibits features similar to the ones of multi-agent systems. In this context, MRB systems can be viewed as multi-agent systems whose agents are in the form of SRB systems and whose migration is in the form of its relocation during the transformation process.

A third possibility would be the development of a new type of adaptive fuzzy systems by means of formal simplification techniques. In this case, the aggregation of inconsistent rules and the filtration of non-monotonic rules can be viewed as an off-line and on-line adaptation of the associated rule base, respectively.