# Soft Computing for Sensor and Algorithm Fusion

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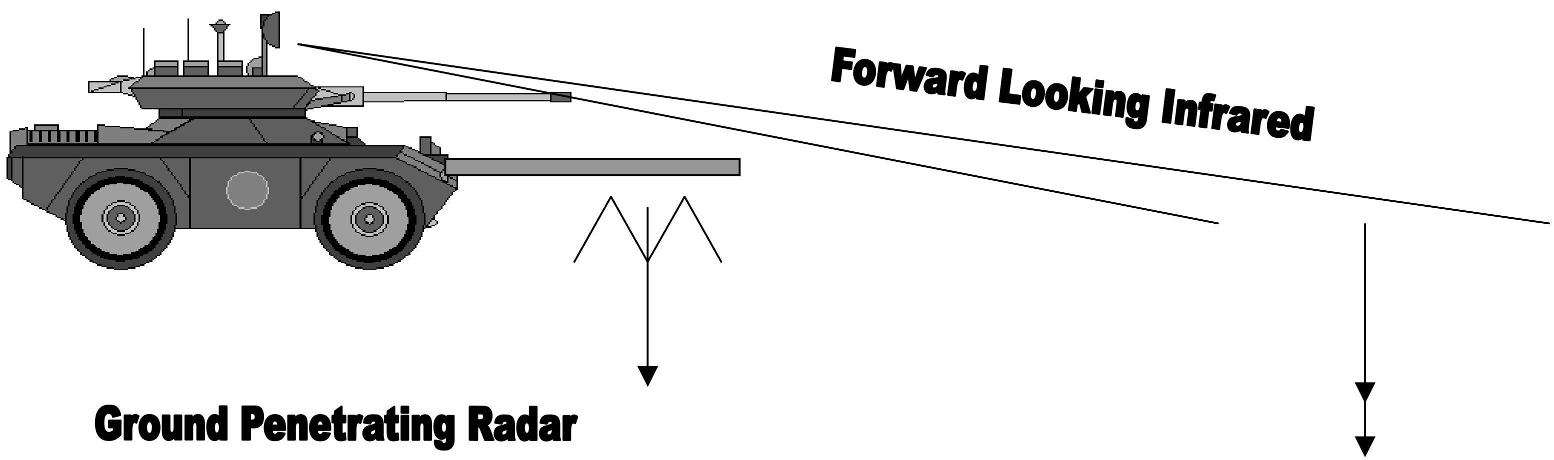
Work done with many colleagues and students at

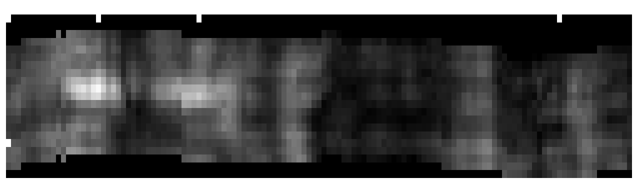
the University of Missouri and elsewhere

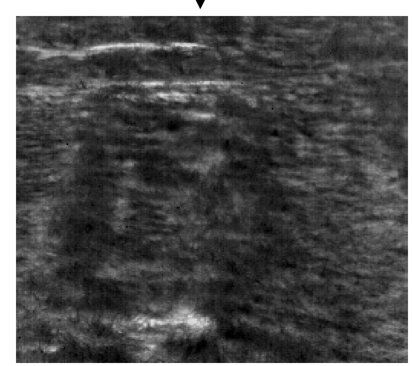
# Introduction

- **Complex problem domains like Landmine Detection, ATR, Handwriting Recognition, etc**
  - **Need multiple features, algorithms, sensors to resolve ambiguity**
- **Uncertainty abounds in these situations**
- **We Need Models and Calculi to Manage Uncertainty in Computational Systems for Algorithm/Sensor Fusion**
- **Here, we discuss Fuzzy Set Theory and Fuzzy Logic as the underlying fusion technology**

LADAR Range Image





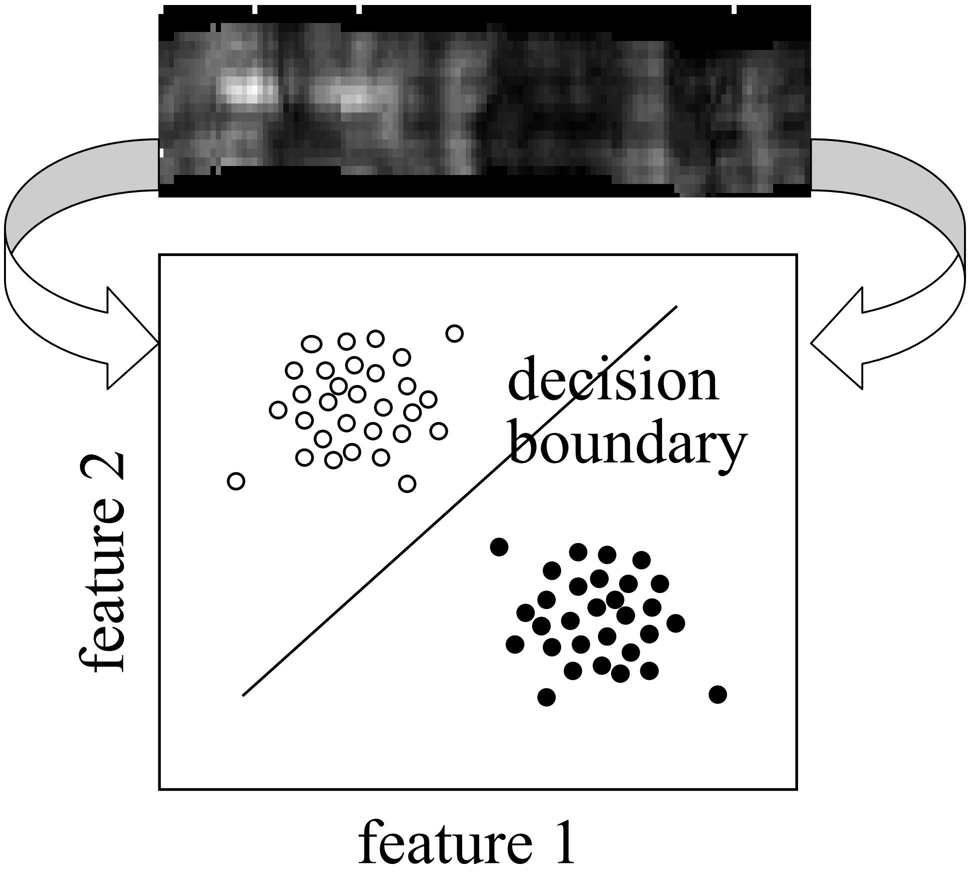
Humanitarian Demining

# Pattern Recognition

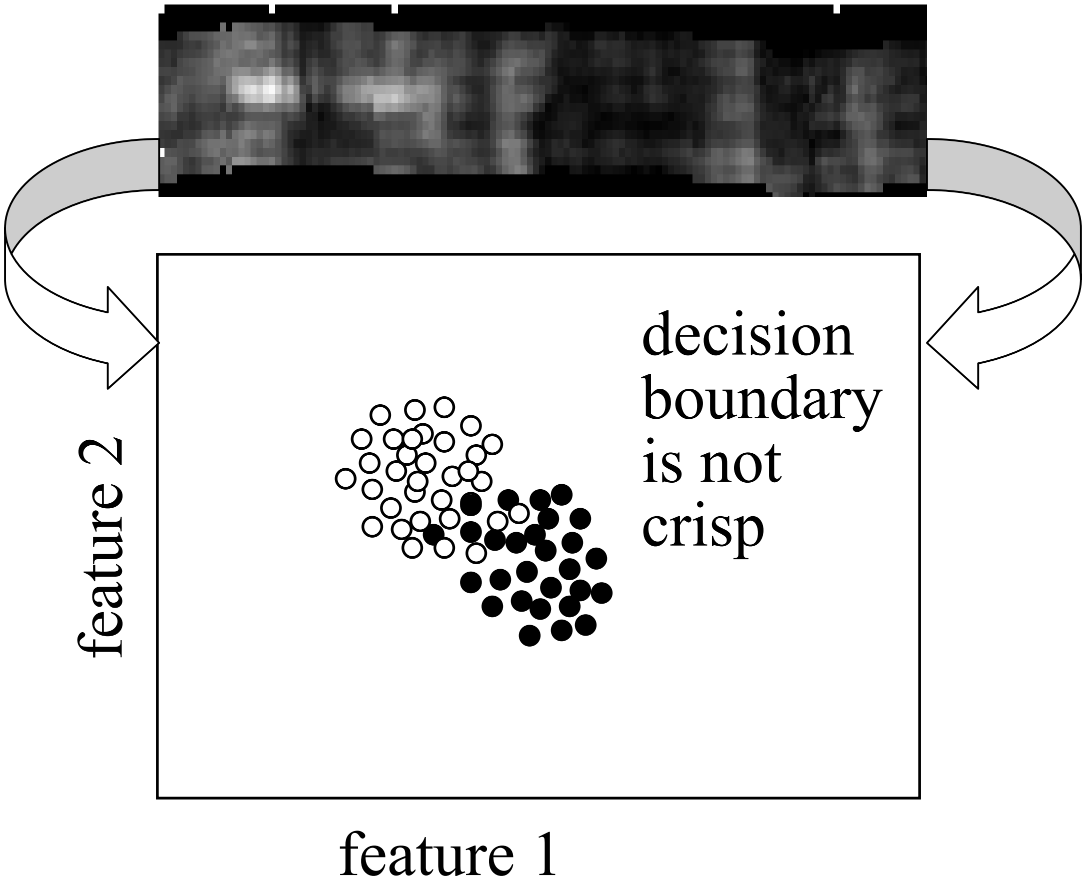
- **Find Meaningful Associations Among Representations of Objects**
  - **Assign Class Labels to New (Unknown) Samples**
- **Supervised Learning**
  - **Have Labeled Training Data to "Learn"the Assignment Function**
    - **Bayes Decision Theory,**
    - **MLP Neural Networks, …**
- **Unsupervised Learning**
  - **Find "Natural" Groups Within a Complex DataSet**
    - **Clustering,**
    - **SOFM, etc**

# How We'd Like the "World" to Be

**Nice Crisp Boundaries between classes**



# More Likely How It Is!



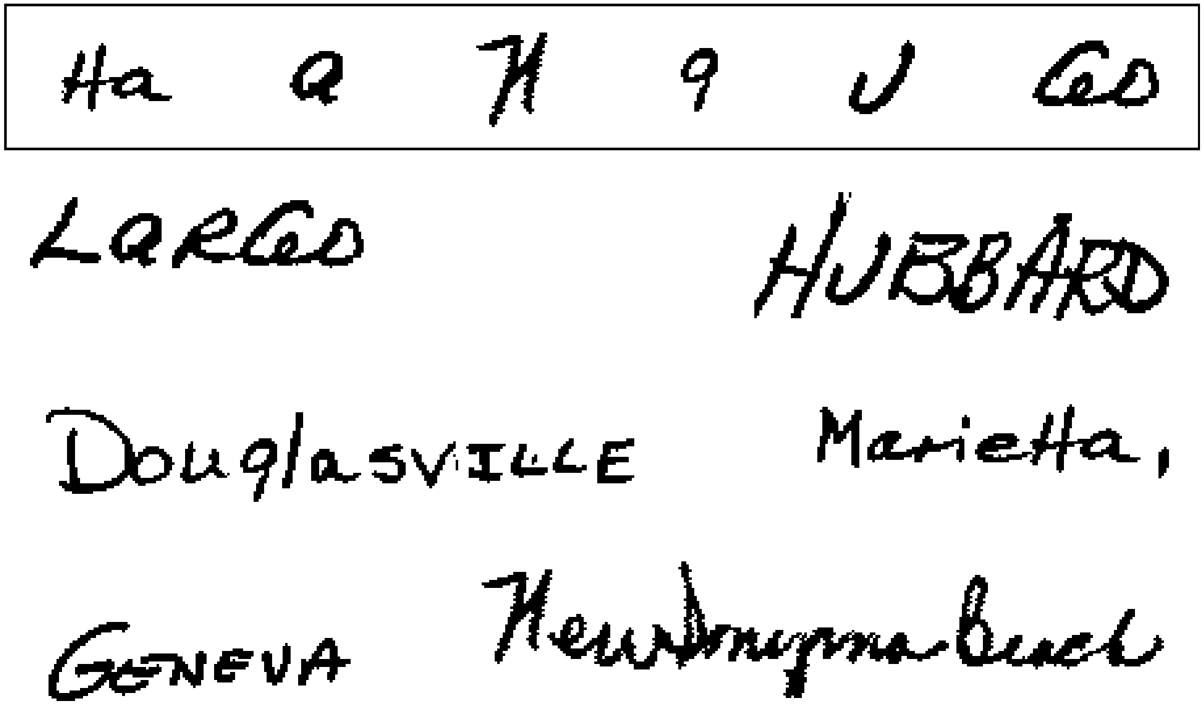

**Need to Model and Manage the Uncertainty in the Data**

# Class Membership Ambiguity

- To What Chacter Classes Do the Following Cursive Examples Belong?

- Can We Make a Precise Assignment Here for Use in a Handwritten Word Recognition System?

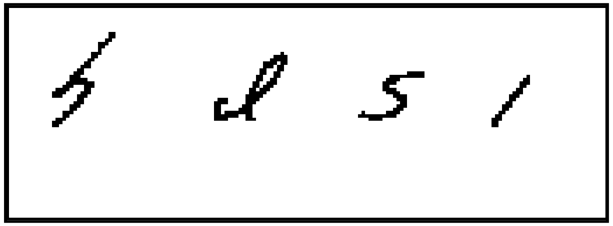
# Class Membership Ambiguity



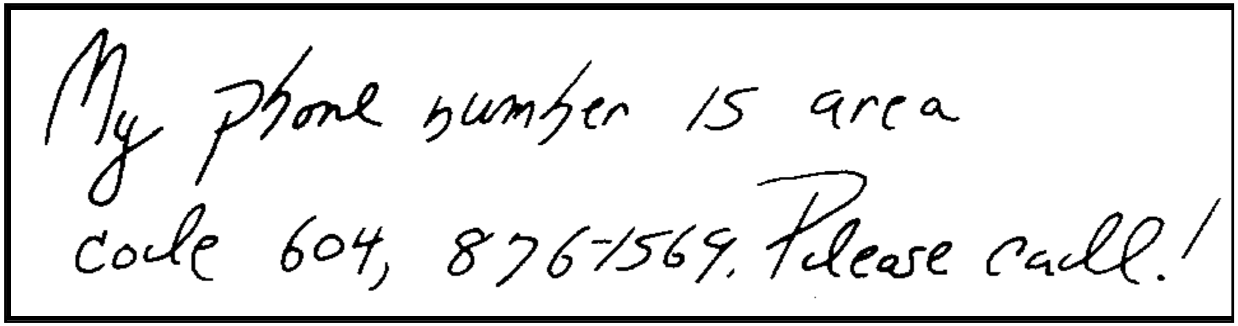

- **Context is Needed to Resolve Ambiguity**

# Another Look at Handwriting Recognition

**Pixel Patterns**



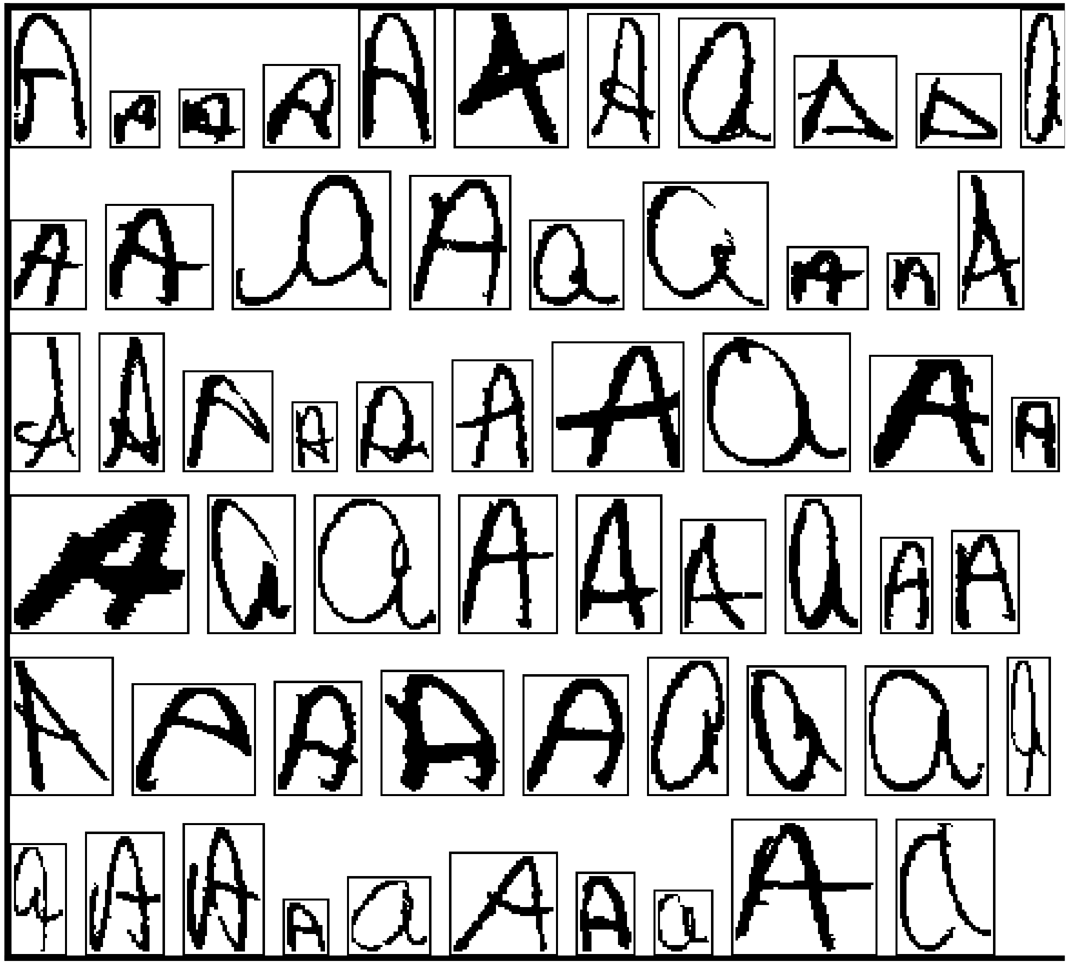
**Assignment is not Unique**




**Need to Allow Multiple Hypotheses for Character Class Assignment**

**Principle of Least Committment**

# Uncertainty in Class "A" Definition



All are perfectly good A's

# Fuzzy Set Theoretic Solutions Support the Two Principles of David Marr for the Design of Intelligent (Vision) Algorithms:

## Principle of Least Commitment

**Don't do something that may later have to be undone**

**"If the Principle of Least Commitment has to be disobeyed, one is either doing something wrong or something very difficult"**

# Marr's Second Principle

## Principle of Graceful Degradation

**Degrading the data will not prevent the delivery of at least some of the answer**

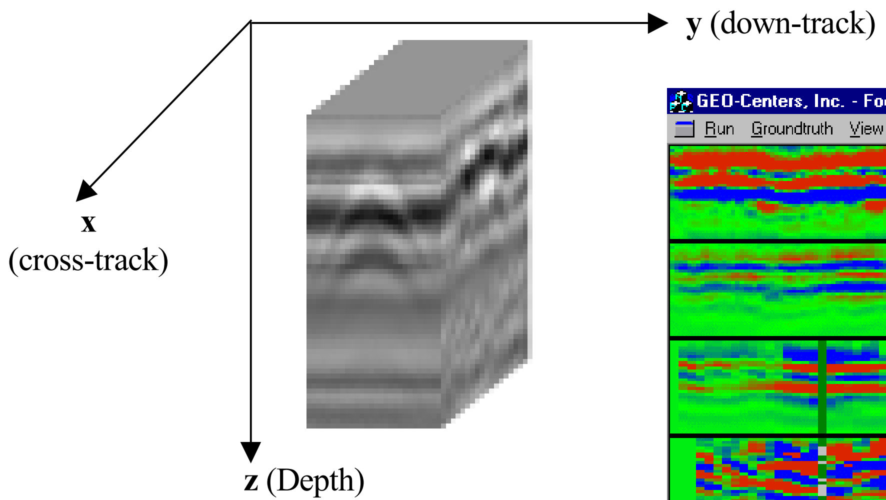
**Algorithms Should Be Robust**

**Processes should possess some degree of Continuity**

# Fuzzy Confidence in Pattern Recognition

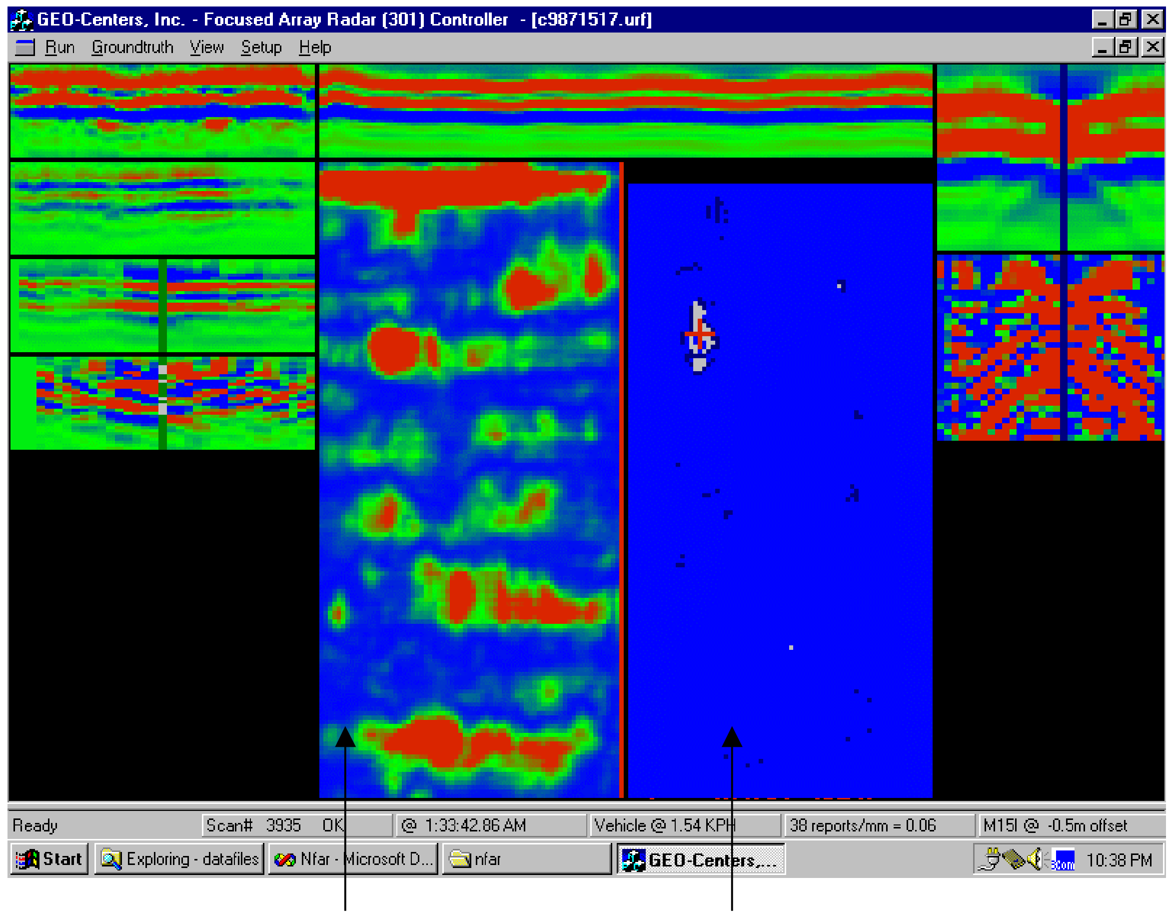
- Many (most) pattern recognition algorithms can be "softened"
- Advantage comes from combination of multiple features
- Example in Land Mine Detection using 3-D volumetric GPR data
  - Fuzzy prototypes for mines/background developed
    - Based on idea that a hyperbola in radar should exist for idea mine
  - Confidence assigned in 3 space, aggregated and projected to surface

# Simple Fuzzy Set Approach to Landmine Detection




**Three dimensional Ground Penetrating Radar**

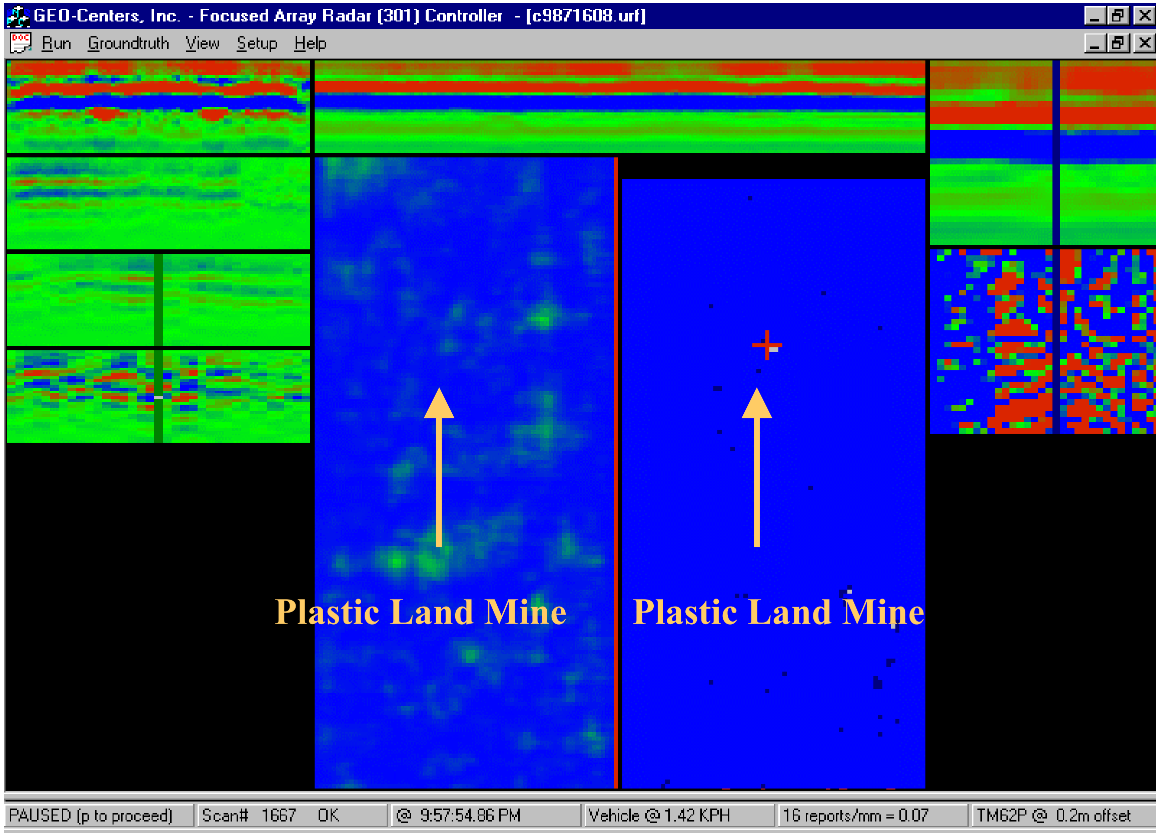
## DECREASE False Alarms




OLD ALGORITHM MU ALGORITHM

# Simple Fuzzy Set Approach to Landmine Detection

## Increased Detections (Verified through Blind Tests)




**OLD ALGORITHM** **MU ALGORITHM**

# Information Fusion

- **Wald gave a definition of information fusion**
  - **"Information Fusion is a formal framework in which are expressed means and tools for the alliance of data originating from different sources".**

- **The *different* sources of information**
  - **Different Sensor Systems**
  - **Different classifiers**
    - **Statistical**
    - **Deterministic**
    - **Fuzzy**
  - **Different features for the same classifier**
  - **Human Intelligence**

# What Fusion methods will we discuss?

- **Fuzzy Integrals**
  - Non-linear combination of information source confidence with (possibly subjective) estimates of worth of subsets of the information sources
  - Will consider numeric and linguistic (higher order) fusion
- **Fuzzy Logic**
  - Rule-based approaches, attempting to model human decision making

# Fuzzy Integrals

- Sources of information in a set X (sensors, features, algorithms, etc.)
- Worth of sources comes from a Fuzzy Measure: $g: 2^X \rightarrow [0,1]$
  - $g(\phi) = 0$ and $g(X) = 1$
  - $g(A) \leq g(B)$ if $A \subseteq B$
  - If $\{A_i\}$ is an increasing sequence of subsets of X, then

$$\lim_{i \to \infty} g(A_i) = g\left( \bigcup_{i=1}^{\infty} A_i \right)$$

# Fuzzy Integrals

- A fuzzy measure g is called a Sugeno measure ($g_\lambda$-fuzzy measure) if additionally:

  For all $A, B \subseteq X$ with $A \cap B = \phi$,

$$g(A \cup B) = g(A) + g(B) + \lambda \cdot g(A) \cdot g(B) \text{ for some } \lambda > -1$$

- For any Sugeno fuzzy measure $\lambda$ can be uniquely determined for a finite set X by solving

$$1 + \lambda = \prod_{i=1}^{n} \left(1 + \lambda g^i\right)$$

- where $X = \{x_1, \ldots, x_n\}$ and $g^i = g(\{x_i\})$ interpreted as the (possibly subjective) importance of the single information source $x_i$ in determining the evaluation of a class hypothesis

# Where do the fuzzy measures come from?

- Heuristic assignment
  - easier for $\lambda$-measures: Only need to specify "worth" of each source
- Direct training of the densities
  - Reward/punishment approach
  - Genetic algorithms
- Optimize entire measure for Choquet integral via Quadratic Programming – more on this later

# The Sugeno Fuzzy Integral

- Let X be a finite set of information sources, $h:X\to[0,1]$ a partial evaluation function, and $g:2^X\to[0,1]$ a fuzzy measure

- The Sugeno fuzzy integral of h with respect to g is

$$\int_S h \circ g = \sup_{\alpha\in[0,1]} \left\{\min(\alpha, g(H_\alpha)\right\}$$

where $H_\alpha = \left\{x \middle| h(x) \geq \alpha\right\}$

- The Best Pessimistic Agreement between evidence and the worth of the evidence

# The Sugeno Fuzzy Integral

- **Since X is finite, re-order X so that** $\mathrm{h}(\mathrm{x}_1) \geq \mathrm{h}(\mathrm{x}_2) \geq \cdots \geq \mathrm{h}(\mathrm{x}_\mathrm{n})$
- **Then the Sugeno Fuzzy Integral is**

$$\int_S h \circ g = \bigvee_{i=1}^{n} \big( h(x_i) \wedge g(H_i) \big)$$

**where** $H_i = \{x_1, \cdots, x_i\}$

- **The measures of these n subsets can be found recursively for λ-measures**
- **Problem: Not a true extension of Lebesgue integral (if measure is additive)**

# Choquet Fuzzy Integral

- Let X={$x_1$,…,$x_n$}, *g* be a fuzzy measure and *h*: X→[0, 1]

- Finite Choquet Integral

$$\int_c h \circ g = \sum_{i=1}^{n} g(X_i)\left[h(x_i) - h(x_{i+1})\right]$$

or

$$\int_c h \circ g = \sum_{i=1}^{n} h(x_i)\left[g(X_i) - g(X_{i-1})\right]$$

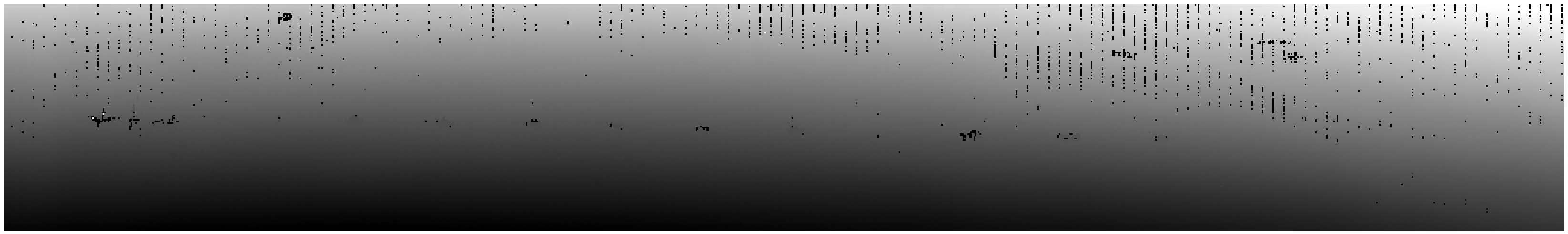
- where $h(x_{n+1}) = 0$, $g(X_0) = 0$, $h(x_1) \geq h(x_2) \geq \ldots \geq h(x_n)$ and $X_i = \{x_1, \ldots, x_i\}$

- Looks like a linear combination, but depends on the sort

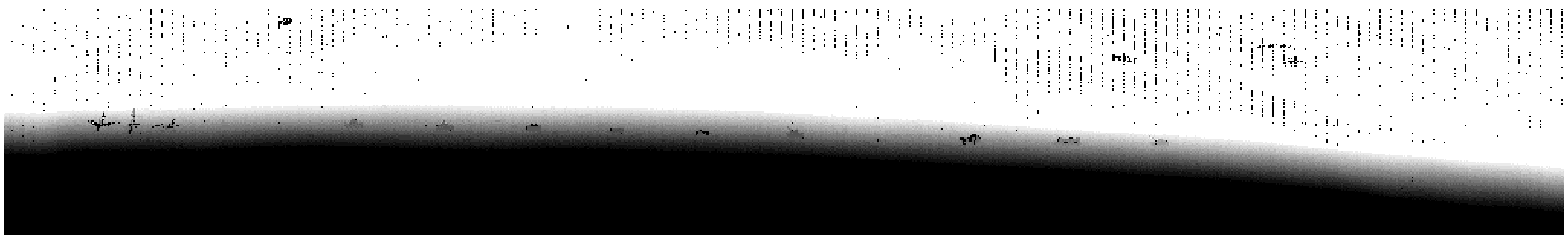
# Fuzzy Integrals for Image Processing

- Both Sugeno and Choquet fuzzy integrals used for non-linear image filtering
- Can implement morphological filters, all linear and order statistic filters, all linear combination of order statistic filters, etc.
- Used instead of means and variances for “size-contrast” filters
- Used to segmentation and to fuse multiple detectors

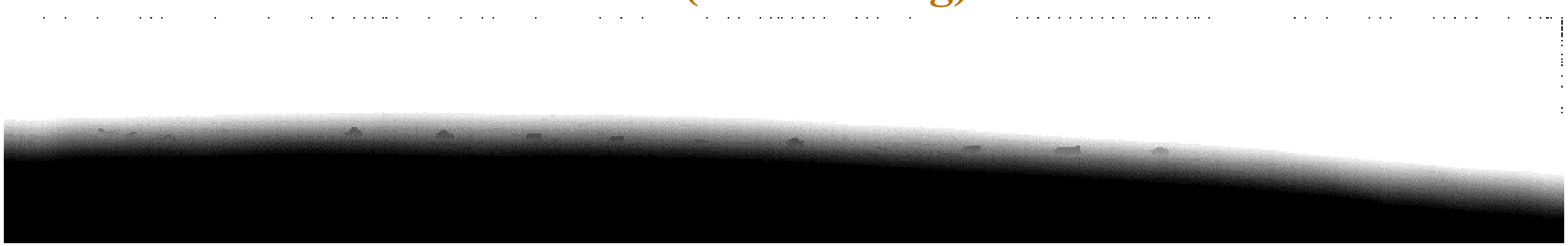
# Choquet Integrals in LADAR ATR



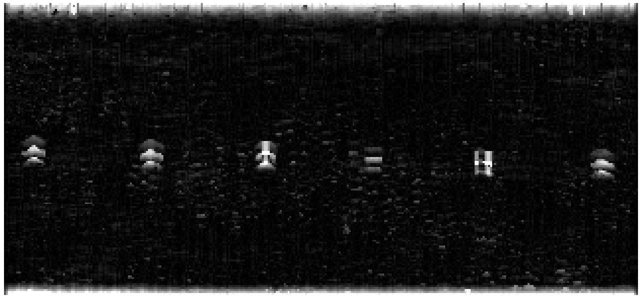
Original



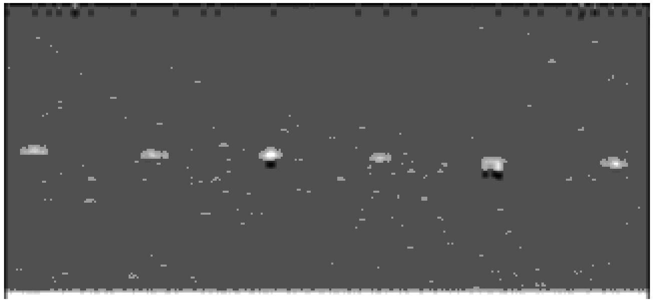
Scaled (for Viewing)



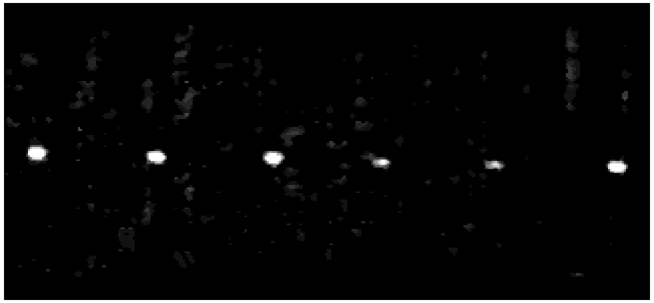
Preprocessed by Choquet Filter



Detector 1

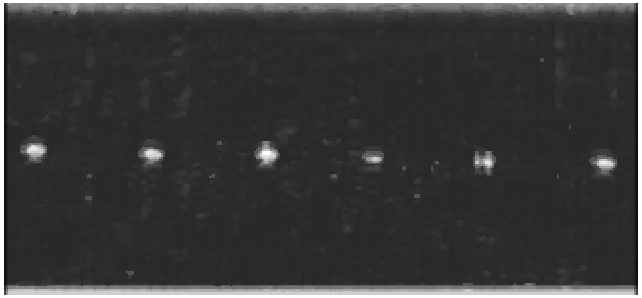


Detector 2

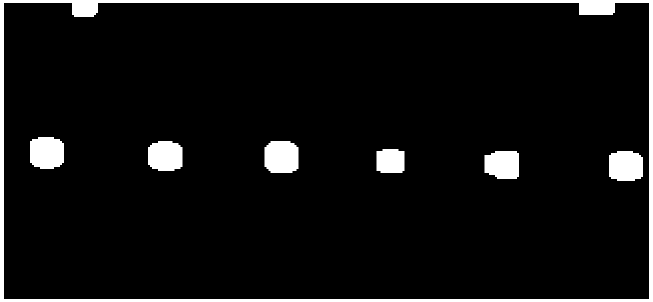


Detector 3

Keep detections up but lower false alarms



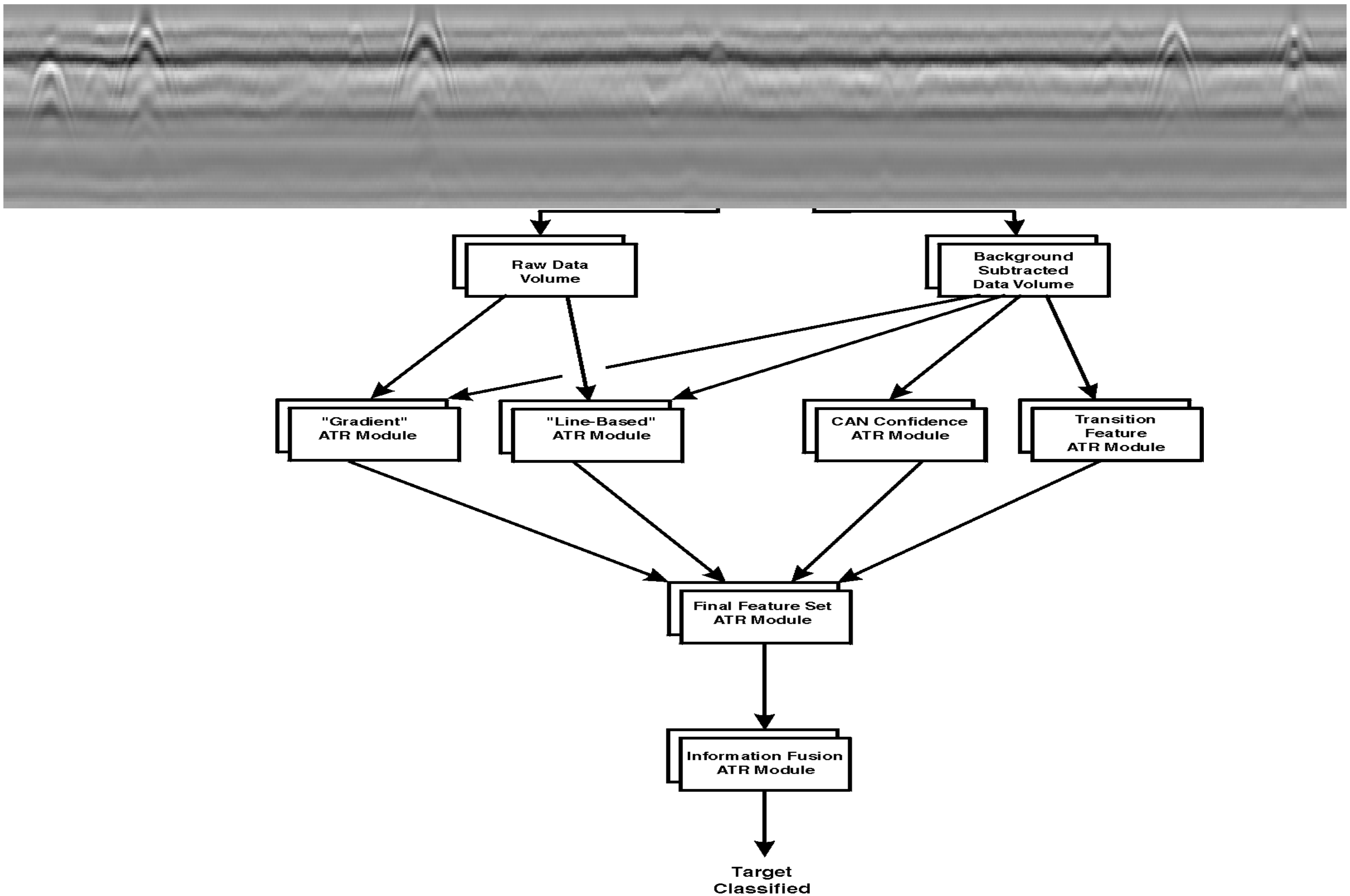
Choquet Fusion



Thresholded and Dilated

# Information Fusion for Demining

## Fusing Outputs of Multiple Algorithms on GPR



# Numeric Fusion on MU Signature Library

## Improvement given by Algorithm Fusion with Choquet and Sugeno Fuzzy Integrals

**Reduction in False Alarms over Line-Based ATR (One Basic ATR)**

| DETRANGE | LINE | CHOQ | SUG | CHOQ_RED | SUG_RED |
|---|---|---|---|---|---|
| 206-207 | 82 | 97 | ND | -18% | NA |
| 187-193 | 42 | 27 | 27 | 36% | 36% |
| 183-186 | 40 | 22 | 24 | 45% | 40% |
| 181-183 | 32 | 22 | 24 | 32% | 26% |
| AVERAGE REDUCTION IN FALSE ALARM | | | | 24% | 34% |

ND = No detections in that range

# Numeric Fusion on MU Signature Library

## Reduction in False Alarms over Gradient-Based ATR

(Another Basic Detection Algorithm)

| DETRANGE | GRAD | CHOQ | SUG | CHOQ_RED | SUG_RED |
|---|---|---|---|---|---|
| 206-207 | 82 | 97 | ND | -18% | NA |
| 201-203 | 64 | 67 | 42 | -4% | 35% |
| 198-199 | 57 | 45 | 36 | 22% | 37% |
| 195-197 | 54 | 37 | ND | 31% | NA |
| 191-193 | 52 | ND | 27 | NA | 48% |
| 187-189 | 50 | 27 | ND | 46% | NA |
| 181-183 | 50 | 22 | 18 | 56% | 64% |
| AVERAGE REDUCTION IN FALSE ALARMS | | | | 22% | 46% |

ND = No detections in that range

# Minimum Classification Error for Information Fusion

## Hot off the Press!

Mendez-Vazquez, A., Gader, P., Keller, J., and Chamberlin, K., "Minimum classification error training for Choquet integrals with applications to landmine detection",

*IEEE Transactions on Fuzzy Systems*, in Press, 2007

# Choquet Integral for Fusion

- **Potential Problem when used for classification**
  - **Fusion of evidence for each class is treated independently**
  - **Measures are learned for each class**
    - **Provide target values for each intgral**
  - **Class-Integral values are not directly comparable**
  - **Similar to problem with HMMs**
- **Like with HMMs, use discriminative training**
  - **Train all integrals simultaneously**

# The Choquet Integral

- **For this development, use the alternate formulation**
- **Let the original sources of information $\{x_1, \ldots, x_n\}$ be reordered into $\{x_{(1)}, \ldots, x_{(n)}\}$ such that**

$$0 \le f(x_{(1)}) \le \cdots \le f(x_{(n)})$$

- **Let $A_{(i)}$ be defined as** $A_{(i)} = \{x_{(i)}, \ldots, x_{(n)}\}$

- **Then**

$$C_g(f) = \sum_{i=1}^{n} g(A_{(i)}) \cdot (f(x_{(i)}) - f(x_{(i-1)}))$$

# Least Squared Error (LSE)

- **Error Function depends on knowing desired outputs**

$$E^2 = \frac{1}{2}\left( \sum_{x \in \omega_1} (C_g(f(x)) - \alpha_1)^2 + \sum_{x \in \omega_2} (C_g(f(x)) - \alpha_2)^2 \right)$$

- **Use Quadratic Programming to find the measure**
- **Or use Gradient Descent for Sugeno $\lambda$-measures**
  - **Need some of the derivatives to follow**

# Minimum Classification Error

- **In minimum classification error (MCE):**
  - **We do not consider cost functions that used fixed desired outputs**
  - **W depend on a function of differences of Choquet integral in the dissimilarity measure**

$$d_i(x) = -C_{g_i}(f_i(x)) + \max_{j \neq i}\{C_{g_j}(f_j(x))\}$$

*Note that under this function, smaller values of* $d_i(x)$ *give higher confidence that* $\omega$ *belongs to the class i.*

# Minimum classification error (Cont)

- **Use a smooth, monotonically increasing function that is differentiable almost in everywhere**

$$l_i(x)=\begin{cases}\dfrac{1}{1+e^{(-\alpha d_i(x))}}, & d_i(x)>0\\ 0, & d_i(x)\le 0\end{cases}$$

# Final Cost function

- The final cost function to be minimized looks like

$$E = \sum_{x \in C_1} l_1(x) + \ldots + \sum_{x \in C_n} l_n(x)$$

- Use Gradient Descent

# Derivative of the cost function

- **Differentiating the cost function with respect to the densities we have**

$$\frac{\partial E}{\partial g_i^j} = \sum_{x \in C_1} l_1(x)(1 - l_1(x)) \frac{\partial d_1(x)}{\partial g_1^j} + \ldots + \sum_{x \in C_n} l_n(x)(1 - l_n(x)) \frac{\partial d_n(x)}{\partial g_n^j}$$

**where $g_i^j$ represent the *j*th density for class *i*.**

- The term $\frac{\partial d_n(x)}{\partial g_n^j}$ is equal to

$$\frac{\partial d_n(x)}{\partial g_n^j} = \begin{cases} \frac{\partial C_{g_k}\left(f_k(x)\right)}{\partial g_i^j} & \text{if k} = \text{i} \\ \frac{\partial C_{g_i}\left(f_i(x)\right)}{\partial g_i^j} & \text{if k} \neq \text{i and } C_{g_i}\left(f_i(x)\right) = \max\limits_{s \neq k}\left\{C_{g_s}\left(f_s(x)\right)\right\} \\ 0 & \text{if k} \neq \text{i and } C_{g_i}\left(f_i(x)\right) \neq \max\limits_{s \neq k}\left\{C_{g_s}\left(f_s(x)\right)\right\} \end{cases}$$

# Problem: What is $\dfrac{\partial C_{g_k}\left(f_k(x)\right)}{\partial g_i^j}$ ?

- **This is simply**

$$\frac{\partial C_g(f)}{\partial g_j} = \sum_{i=1}^{n} \frac{\partial g\left(A_{(i)}\right)}{\partial g_j}\left(f\left(x_{(i)}\right) - f\left(x_{(i-1)}\right)\right)$$

- **Hence, we still need to know**

$$\frac{\partial g\left(A_{(i)}\right)}{\partial g_j}$$

# The derivative of $\dfrac{\partial g\left(A_{(i)}\right)}{\partial g_j}$

- We know that

$$g(A_{(i)}) = g\left(\left\{x_{(i)}\right\} \cup A_{(i+1)}\right) = g_{(i)} + g(A_{(i+1)}) + \lambda g_{(i)} g(A_{(i+1)})$$

- Thus

$$\frac{\partial g\left(A_{(i)}\right)}{\partial g_j} = \begin{cases} 1 + \lambda g\left(A_{(i)}\right) + g_{(i)} g\left(A_{(i+1)}\right) \dfrac{\partial \lambda}{\partial g_j} + \left(1 + \lambda g_{(i)}\right) \dfrac{\partial g\left(A_{(i+1)}\right)}{\partial g_j}, \text{if } (\mathrm{i}) \neq n, (i) = j \\ g_{(i)} g\left(A_{(i+1)}\right) \dfrac{\partial \lambda}{\partial g_j} + \left(1 + \lambda g_{(i)}\right) \dfrac{\partial g\left(A_{(i+1)}\right)}{\partial g_j}, \quad \text{if } (\mathrm{i}) \neq n, (i) \neq j \\ 1 \quad \text{if } (\mathrm{i}) = n,\ j = n \\ 0 \quad \text{if } (\mathrm{i}) \neq n,\ j \neq n \end{cases}$$

## The derivative $\dfrac{\partial \lambda}{\partial g_j}$

- **Using the fact that the Sugeno λ-measure has the property** $$(\lambda + 1) = \prod_{i=1}^{n} (1 + \lambda g_i)$$

- **We have** (using implicit differentiation)

$$\frac{\partial \lambda}{\partial g_j} = \frac{\lambda^2 + \lambda}{\left(1 + g_j \lambda \left[1 - (\lambda + 1) \sum_{i=1}^{n} \frac{g_i}{1 + g_j \lambda}\right]\right)}$$

# Gradient Descent for the MCE

- **With the previous results, we implement a Gradient Descent algorithm for the MCE**

$$\vec{g} = \vec{g} + \alpha \nabla E$$

- **Does not need complex formulations**
  - **Required for non-linear optimization**

# Experiments in Landmine Detection

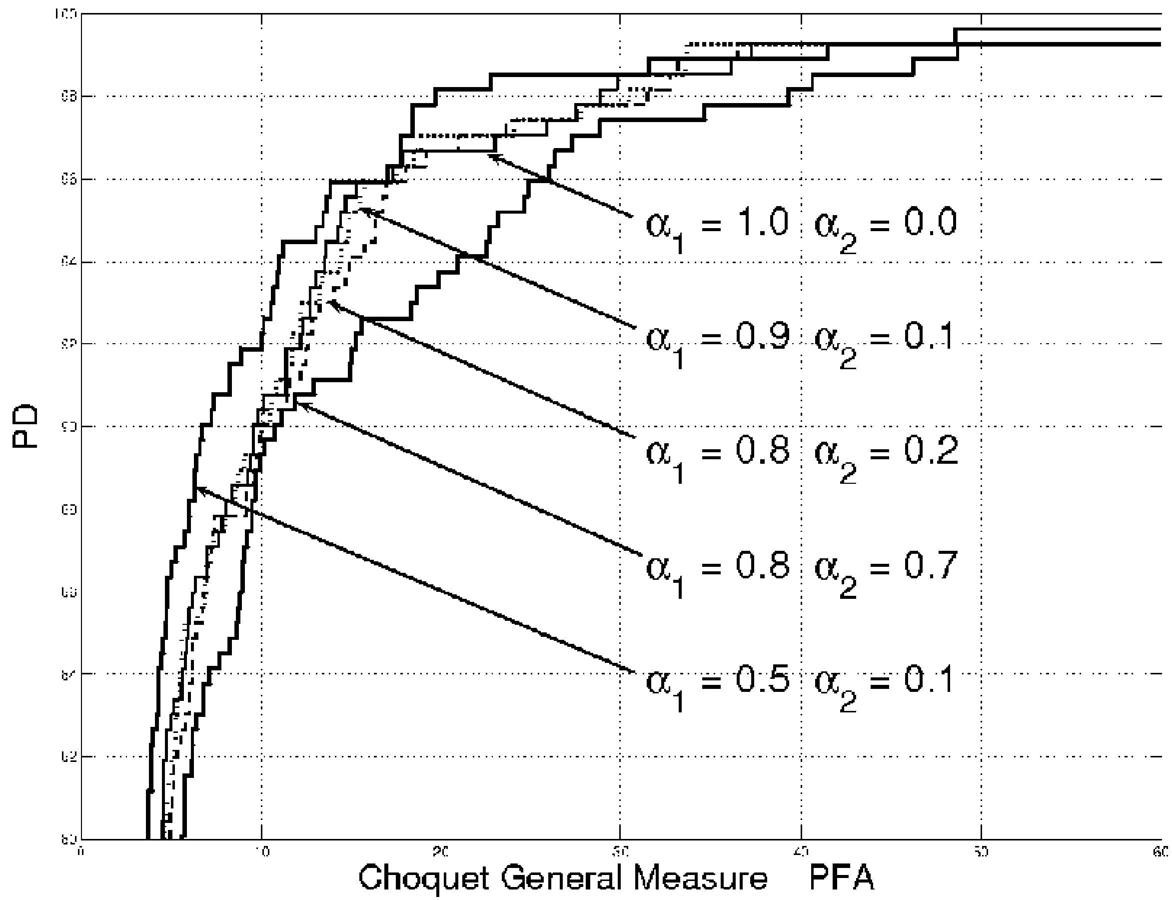
- **The Least Squared Error(LSE) and MCE training methods were applied to a two-class algorithm fusion problem in landmine detection**
  - **The landmine detection problem involved processing Ground Penetrating Radar (GPR) sensor returns**

- **The data set contained 2422 8-dimensional samples of confidence values from each of the eight detection algorithms. The data set contained 271 mines samples and 2151 non-mine samples.**

# Experimental Results

## Least squared error sensitive to desired outputs

- ROC curves showing the sensitivity to desired outputs for **LSE** in the full measure. $\alpha_1$ and $\alpha_2$ represent the desired outputs for mine and non-mines.



# Advantages of MCE

- MCE is not sensitive to desired outputs, as is LSE
- MCE is, in general, better than the individual detectors, with a range of improvement between 0.44% and 65.07%
- On this data, improvement of MCE over LSE training
  - Ranges between 11.06% and 37.51%
- It is possible for the Sugeno λ-measure and the general measure trained with LSE to be as good as the one trained by MCE. For this to happen, it is necessary to have a set of correct desired outputs!

# Experiments as a Classifier

- The MCE training was also applied to the Iris and Breast Cancer data with results in
- K. Xu, Z. Wang, P.-A. Heng, and K.-S. Leung, "Classification by nonlinear integral projections," IEEE Transactions on Fuzzy Systems, ol. 11, no. 2, pp. 187–2001, 2003

| Method | Iris Data(%) | Breast Cancer Data(%) |
|---|---|---|
| Linear | 2 | 29 |
| Quadratic | 2.7 | 34.4 |
| Nearestneighbor | 4 | 34 |
| Bayes independent | 6.7 | 28.2 |
| Bayes quadratic | 16 | 34.4 |
| Neuronal net | 3.3 | 28.5 |
| PVM rule | 4 | 22.9 |
| QUAD | 3.3 | 31.5 |
| CLMS | 4 | 27.1 |
| HLMS | 4.7 | 22.6 |
| WCIPP | 4 | 26.2 |
| **MCE** | **4** | **22.73** |

- But, sometimes numbers aren't enough
  - Confidence is High
  - IR sensor is Weak
  - Mine location is Around (1.5, 3.7)
- Need Higher Order Constructs
  - Type II fuzzy sets and linguistic vectors
  - Fuzzy logic rules

Story from the Local Newspaper

Doctors Make Decisions from This type of Information

Shouldn't Computers?

# Facing up to flu season

As the American Lung Association kicks off a month-long flu campaign this week, predictions are the flu season will be especially severe this year. How to tell what ails you:

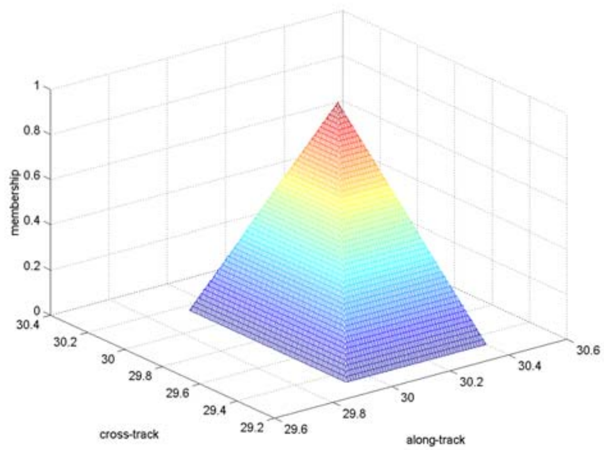
| SYMPTOMS | COLD | FLU | PNEUMONIA |
|---|---|---|---|
| FEVER | rare | 100.4°-104°, sudden onset, lasts 3-4 days | may or may not be high |
| ACHES/PAINS | slight | usual, often severe | occasional |
| FATIGUE/ WEAKNESS | quite mild | extreme | may occur |
| RUNNY/ STUFFY NOSE | common | sometimes | not characteristic |
| SNEEZING | usual | sometimes | not characteristic |
| SORE THROAT | common | sometimes | not characteristic |
| CHEST PAIN, COUGH | mild to moderate | can become severe | frequent/ may be severe |

Source: American Lung Association

*Stories by Leslie Ansley*

# Linguistic Sensor Fusion

- What if Entities to be Fused Don't Lend Themselves to Numeric Representations?
  - Position Estimates - Commonly Use Halos or Windows
  - Confidence May Not be Known Exactly
    - varies among classifiers/sensors




- Generalize These Values to Fuzzy Sets
  - Linguistic Vectors
    - Generalize halos and confidence intervals

# Linguistic Choquet Fuzzy Integral

- So, How Do We Fuse Linguistic Vectors of Sensor Output?
- Both Position Confidence and Detection Confidence
- Extend the Choquet Integral to These Vectors
- Based on Extension Principle and Decomposition Theorem of Fuzzy Set Theory
  - Plus interval arithmetic if we use fuzzy numbers - nice fuzzy subsets of reals

# Background

- **Extension Principle (How to extend functions from numbers to fuzzy sets)**

$$\mu_Z(y) = \sup_{x_1 \otimes x_2 = y} \min(\mu_A(x_1), \mu_B(x_2))$$

- **For fuzzy numbers (normal convex fuzzy subsets of the reals)**
  - **Use interval arithmetic and the decomposition theory:**

$$A \otimes B = \bigcup_{\alpha \in [0,1]} [A \otimes B]_\alpha$$

$$= \bigcup_{\alpha \in [0,1]} [A]_\alpha \otimes [B]_\alpha$$

- **Linguistic Vectors**
  - **Vectors of Non-interactive Fuzzy Numbers**

# Linguistic Choquet Fuzzy Integral

- Extend regular Choquet integral to a linguistic Choquet fuzzy integral [Grabisch94b]:
- Let $h$: X→$\mathfrak{I}$([0, 1]) where $\mathfrak{I}$([0, 1]) is a fuzzy power set of [0, 1], and

$$[h_i]_\alpha = \left[\underline{[h_i]_\alpha}, \overline{[h_i]_\alpha}\right] \quad \text{for } 1 \le i \le n \text{ and } 0 \le \alpha \le 1.$$

- Then

$$\int_c h \circ g = \bigcup_{\alpha \in [0,1]} \left[\int_c h \circ g\right]_\alpha$$

where

$$\left[\int_c h \circ g\right]_\alpha = \left[\int_c \underline{[h]_\alpha} \circ g, \int_c \overline{[h]_\alpha} \circ g\right]$$

# Linguistic Choquet Fuzzy Integral

- **How to Compute Densities?** **Two properties:**
  - **Property 1.** **The more uncertain a sensor is, the less important it is for the fusion**
  - **Property 2.** **The further away the sensor's detection location is from the true location in training data, the less important that sensor is for the fusion**

# Linguistic Choquet Fuzzy Integral

- **How to Compute Densities?** **Two properties:**

  **Property 1. The more uncertain a sensor is, the less important it is for the fusion**

  – **Use U-uncertainty measure[Klir95] to compute density of fuzzy number $A_j$ in a particular dimension.**

$$g_u^j = 1 - \frac{\sum_{i=2}^{p} (\alpha_i - \alpha_{i+1}) \log_2 \left| [A_j]_{\alpha_i} \right|}{\log_2 \left( \left| \sup(A_j) \right| \right)}$$

# Linguistic Choquet Fuzzy Integral

- **Property 2.** **The further away the sensor's detection location is from the true location in training data, the less important that sensor is for the fusion**

  – **Know that** $$\left\{ [A_j]_{\alpha_1} \subseteq [A_j]_{\alpha_2} \subseteq \cdots \subseteq [A_j]_{\alpha_p} \right\}$$

  – **The distance between** $[A_j]_{\alpha_i}$ **and** $[\beta]_{\alpha_i}$ **is**

$$d_{A_j}(\alpha_i) = \max\left( \left| \underline{[A_j]_{\alpha_i}} - \beta \right|, \left| \overline{[A_j]_{\alpha_i}} - \beta \right| \right)$$

  – **where** $\beta$ **represents the actual location in a particular dimension**

# Linguistic Choquet Fuzzy Integral

- The densities of fuzzy number *j* in a particular dimension relating to the distance are:

$$g_d^j = \sum_{i=1}^{p} (\alpha_i - \alpha_{i+1}) P\left( \left[ A_j \right]_{\alpha_i} \right) \qquad \text{(Generalized Expected Value)}$$

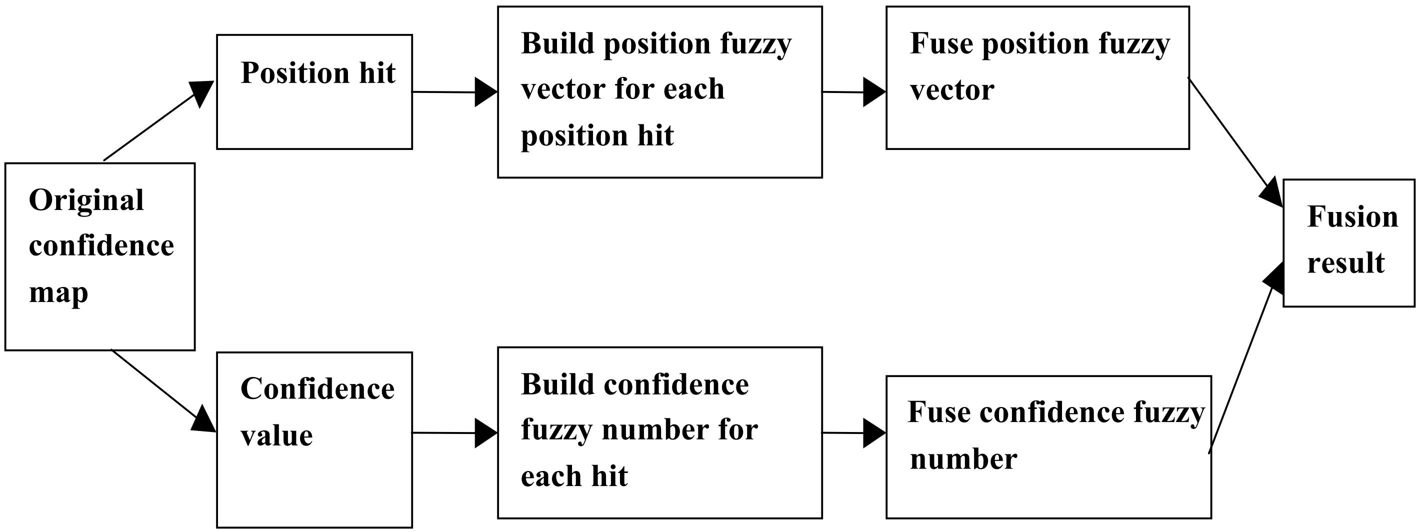
where

$$P\left( \left[ A_j \right]_{\alpha_i} \right) = 1 - \frac{d_{A_j}(\alpha_i)}{\sum_k d_{A_k}(\alpha_i)}$$

- The final densities of fuzzy number $A_j$ in a particular dimension relating to both properties are:

$$g^j = g_u^j \times g_d^j$$

# Linguistic Fusion of Real Algorithm Results

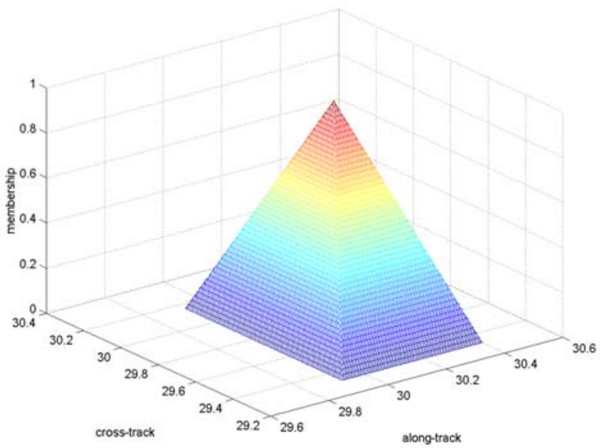


# Linguistic Fusion Experiment

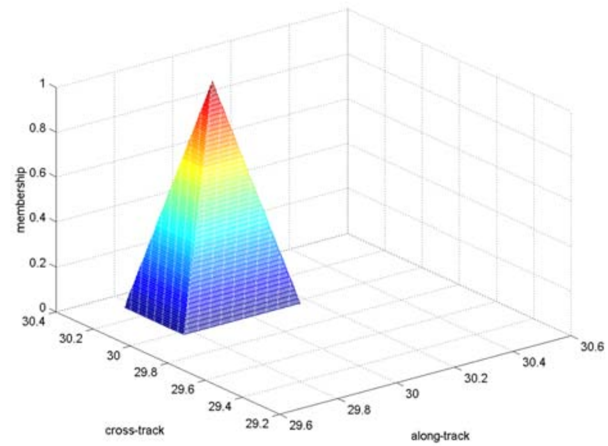
- Fuse the information from the Fuzzy ATR, the HMM and the CMSNN
  - Generated from GPR data at Aberdeen site
- Threshold confidences from three algorithms
- Use the centroid of each connected component to be the hit position ($x$,$y$)
- Fuzzify each dimension of each hit location:
  - 5x5 window with center at ($x$,$y$), average column-wise to get vector in along-track and average row-wise to get vector in cross-track
  - Build normal convex fuzzy sets for these vectors
- Build the mask with the center of mask at the current hit position
- Fuse locations inside the mask

# Example of Position Fusion

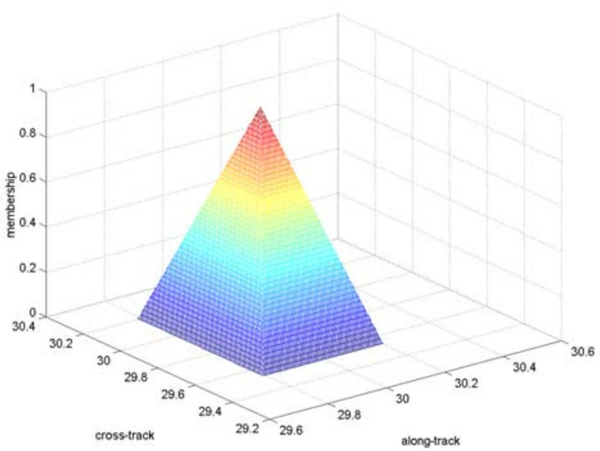
Sensor 1

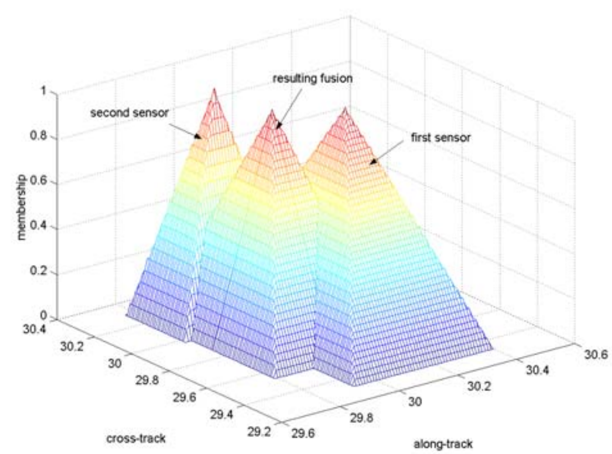



Sensor 2



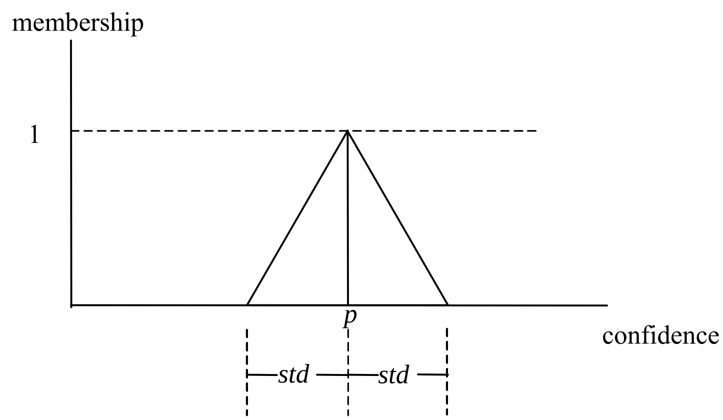

Fusion result



# Linguistic Fusion

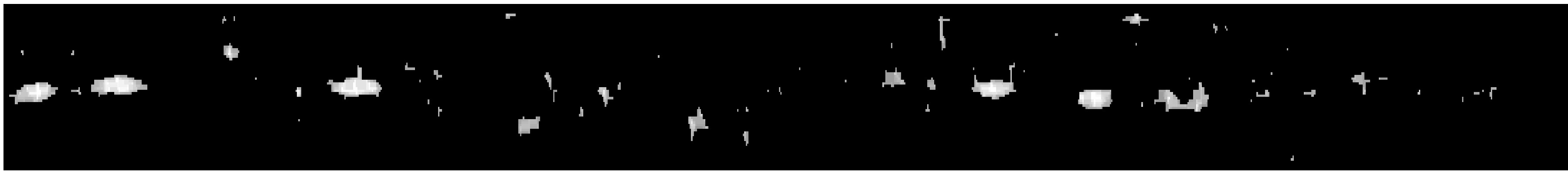
- **Build the confidence fuzzy number**
  - **Pick the maximum confidence *p* in corresponding 5x5 window**
  - **Compute standard deviation *std* of the confidence in that neighborhood**



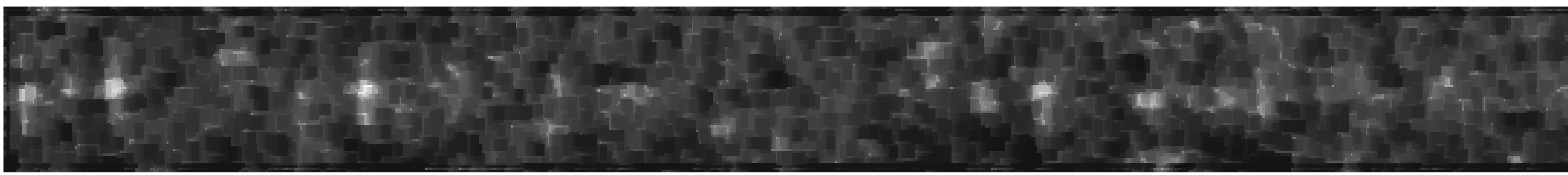

- **Fuse confidences corresponding to the position linguistic vectors that are combined at a particular "position"**

# Linguistic Fusion

- **Original confidence map from three algorithms on GPR data on Mine Lane collected at a US site**



Fuzzy ATR



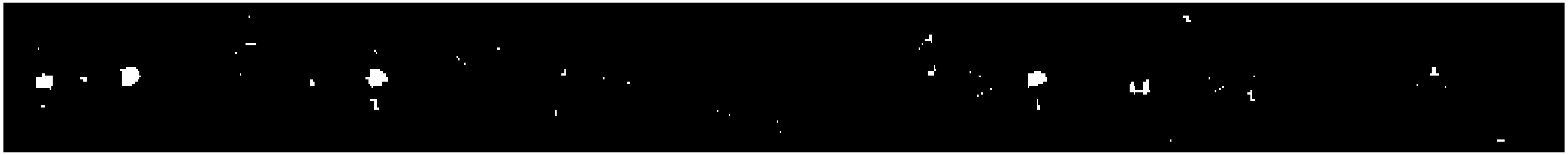
HMM

CMSNN

# Linguistic Fusion

- **After Thresholding: The Fuzzy ATR has 6 detections and 27 false alarms, The HMM has 5 detections and 10 false alarms, and The CMSNN has 6 detections and 6 false alarms**

Fuzzy ATR

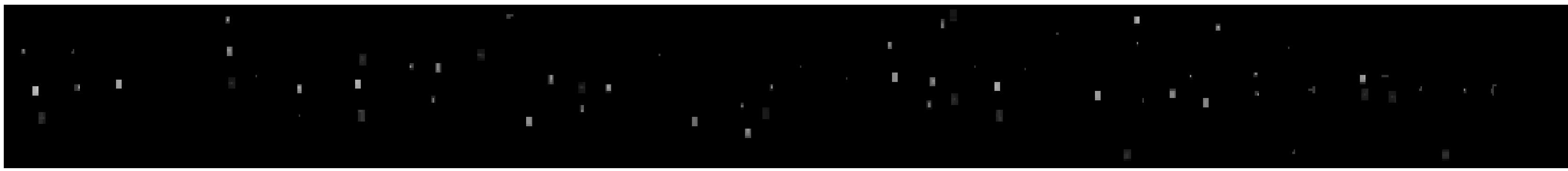


HMM

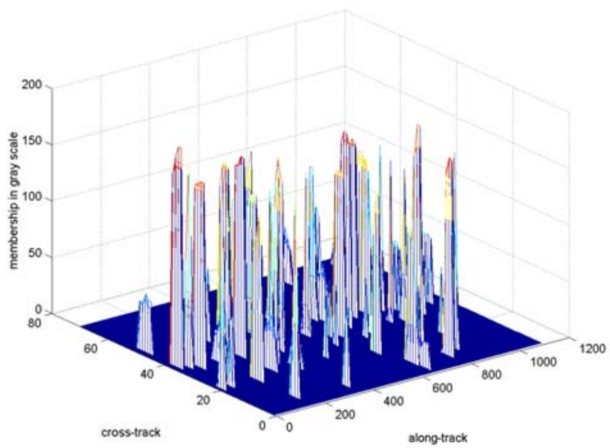
CMSNN

# Linguistic Fusion

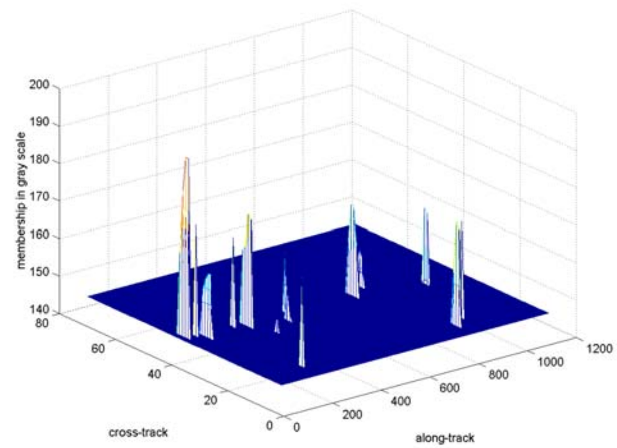
- After Fusion



After Thresholding: 6 detections and 2 false alarms



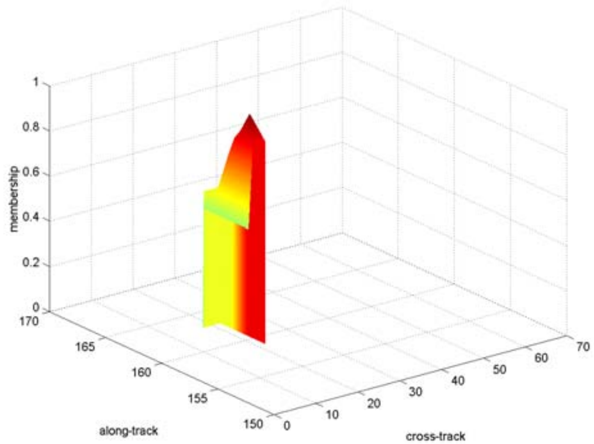

Before thresholding



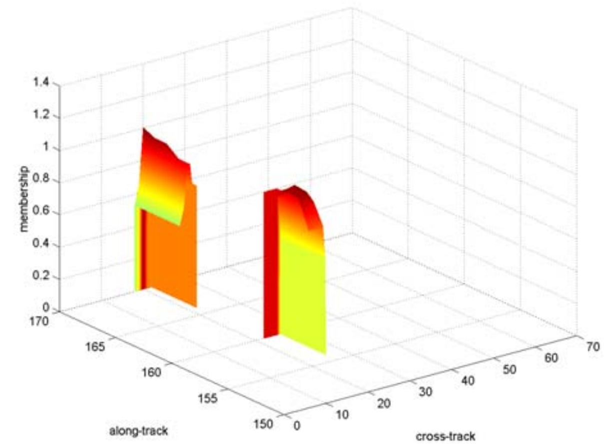

After thresholding

# Linguistic Fusion

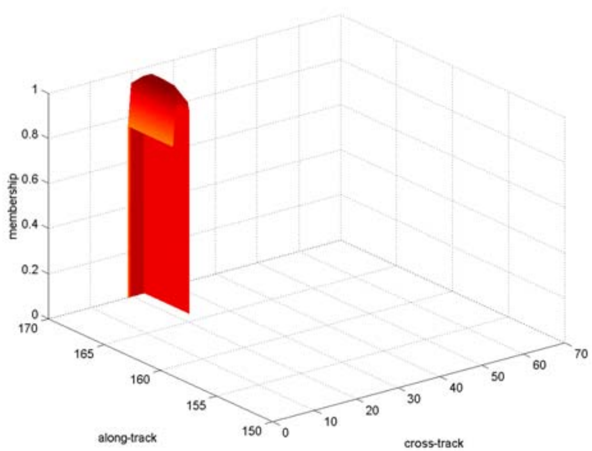
- Example of false alarm eliminated by fusion



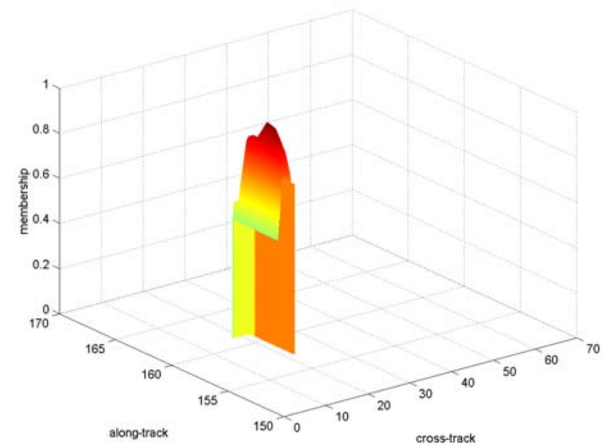

Position from Fuzzy ATR




Position from HMM

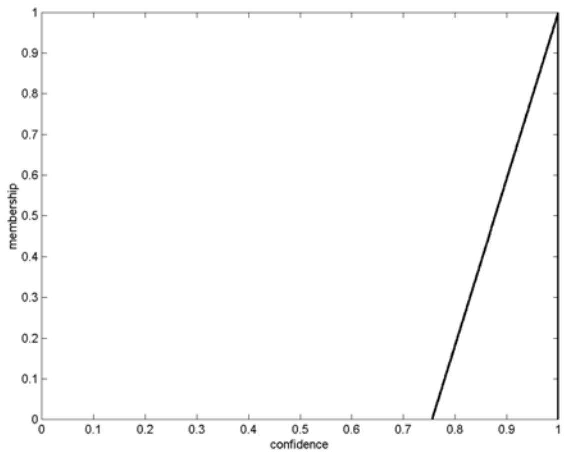



Position from CMSNN

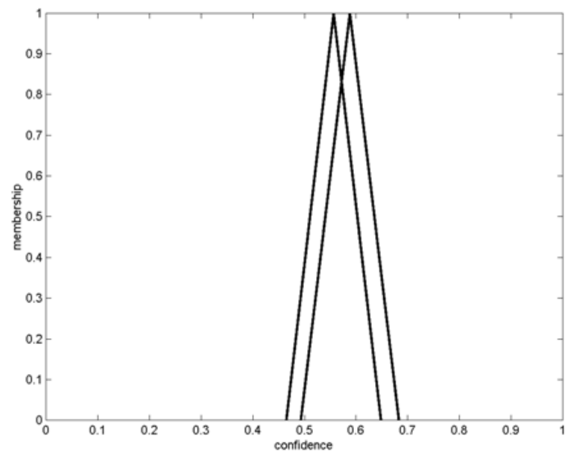



Position fusion result

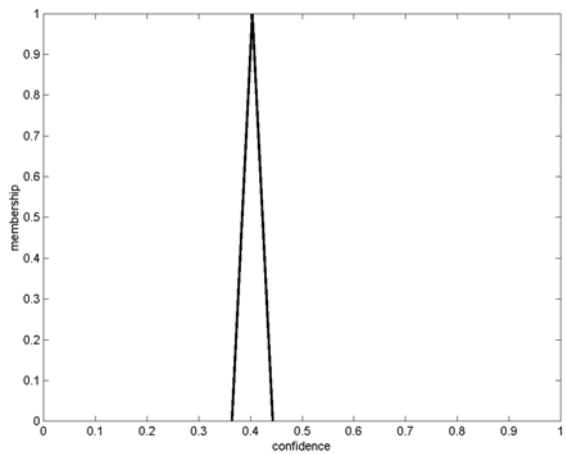
# Linguistic Fusion



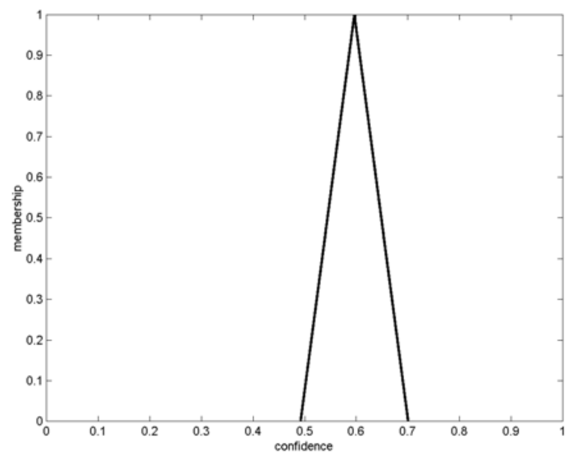

Confidence from Fuzzy ATR




Confidence from HMM

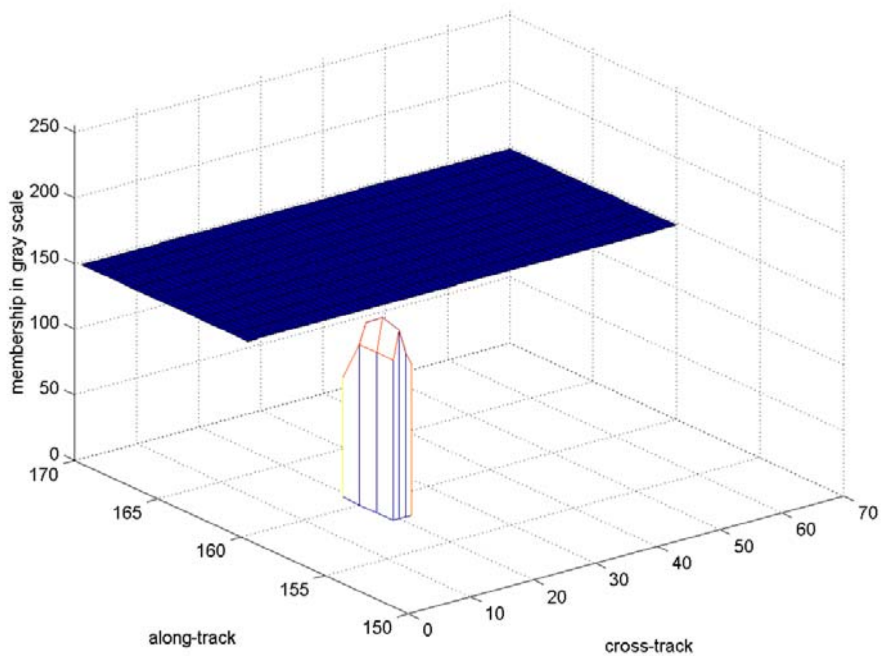



Confidence from CMSNN




Confidence fusion result

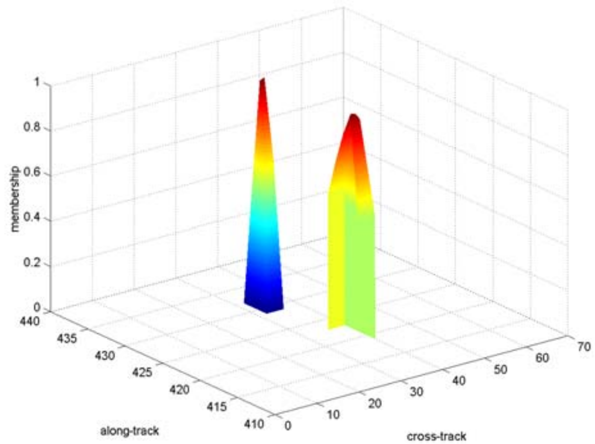
# Linguistic Fusion



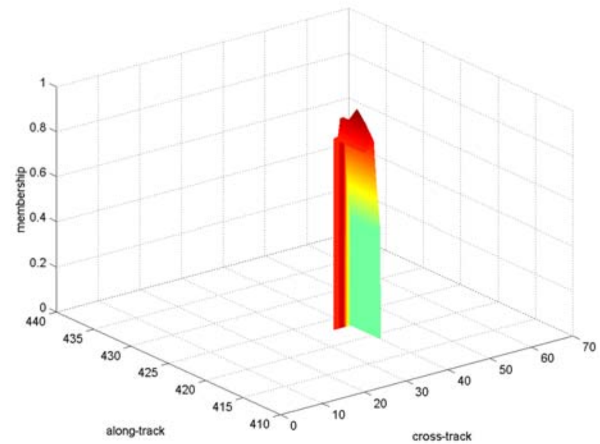

**After multiplying the peak of confidence fusion with the membership of position fusion**

# Linguistic Fusion

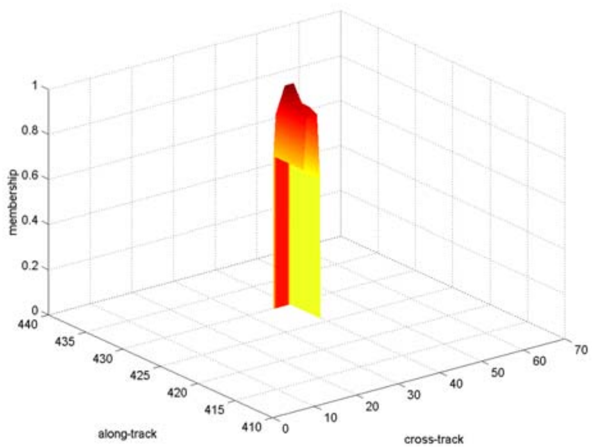
- Example of mine that fusion can detect while one algorithm cannot




Position from Fuzzy ATR

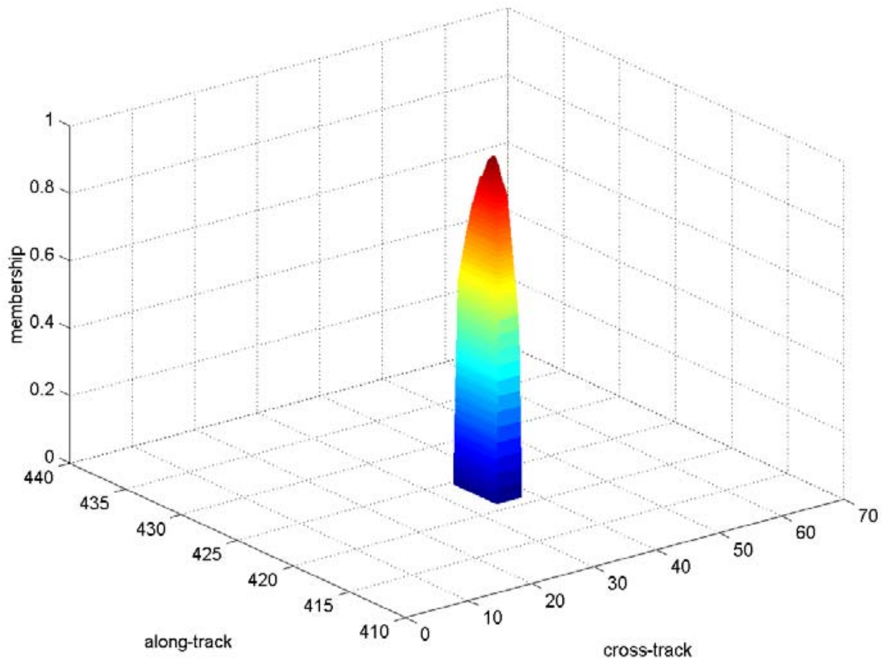



Position from HMM hit location at (38,419). Considered as noise by HMM when scoring because there are only 2 pixels above threshold.



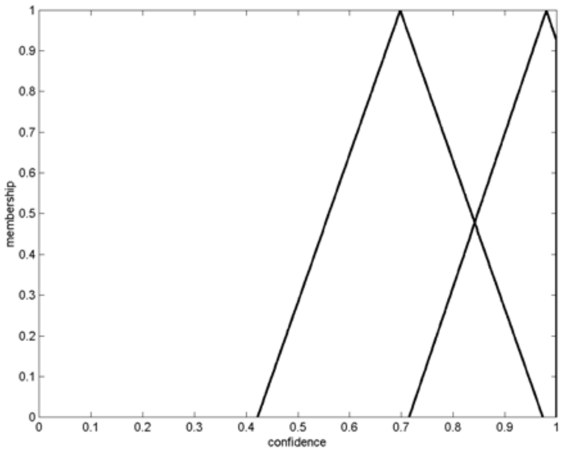

Position from CMSNN

# Linguistic Fusion

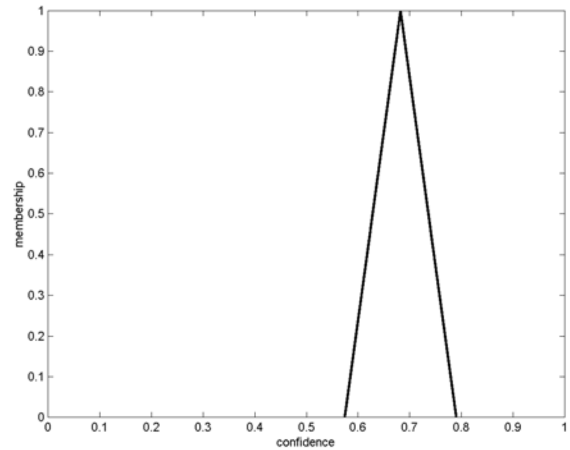



Position fusion result

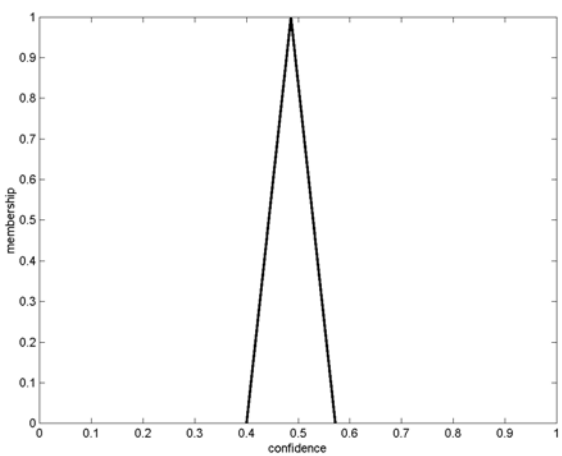
# Linguistic Fusion



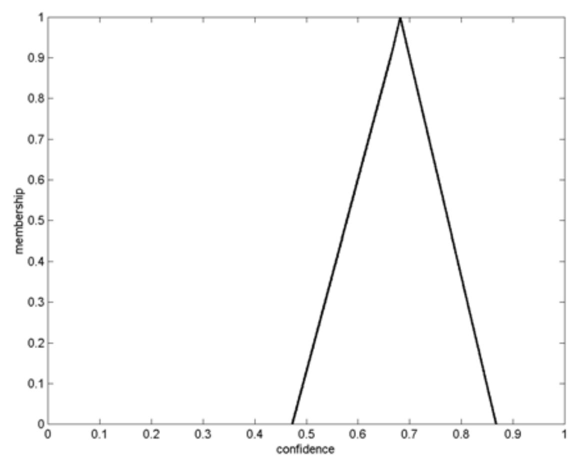

**Confidence from Fuzzy ATR**




**Confidence from HMM**

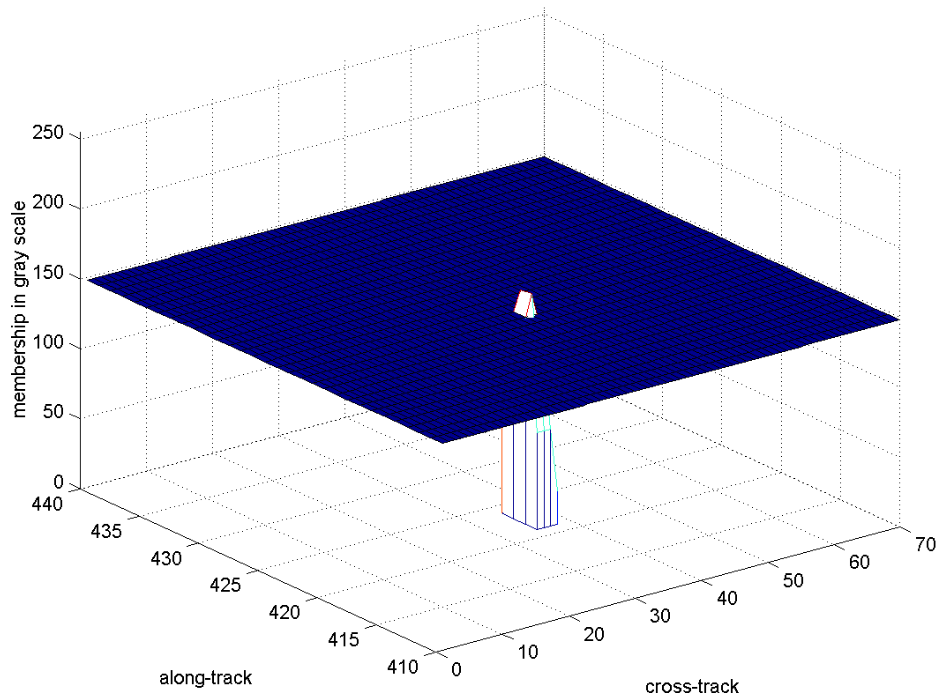



**Confidence from CMSNN**




**Confidence fusion result**

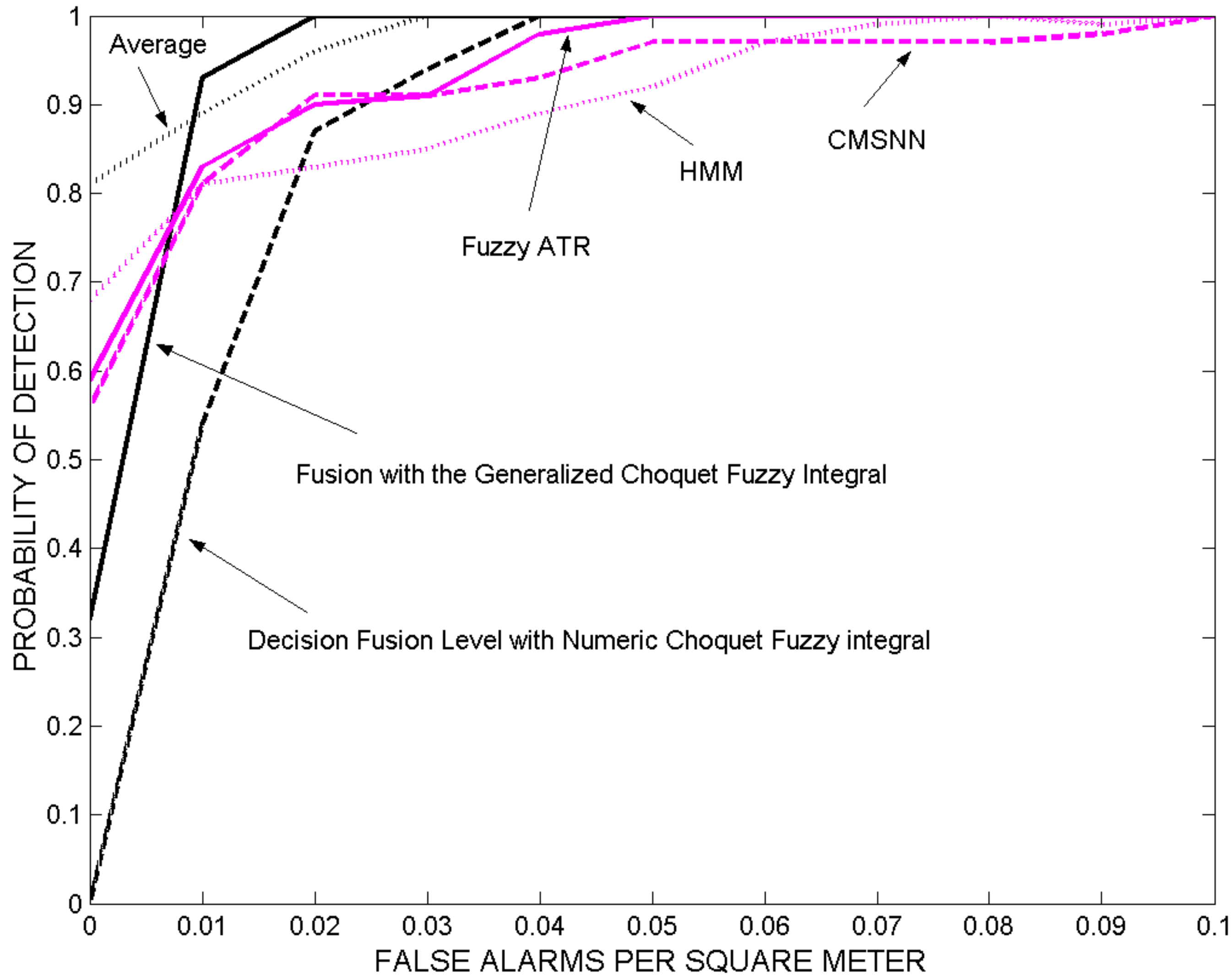
# Linguistic Fusion




**After multiplying the peak of confidence fusion with the membership of position fusion**

# Linguistic Fusion - The Payoff

- Run on ten lanes (16 passes) from US Army temperate site



# Predictive Sensor Fusion

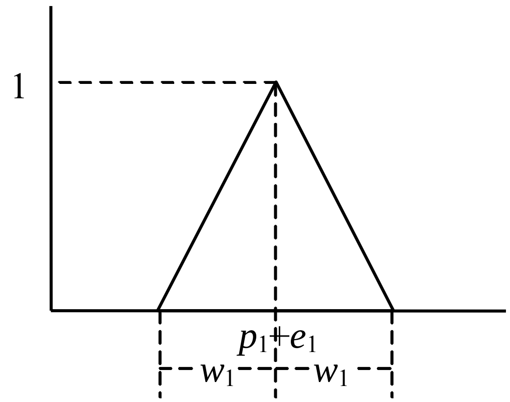
- Two Questions:
  - Can we predict the value added by fusing multiple sensor system outputs together?
  - Can we specify the needed characteristics of a new sensor system to add to an existing suite to a gain a desired improvement in performance?
- Approach
  - Use Monte Carlo simulations based on actual data from 2 GPRs to study effect of adding a third senor

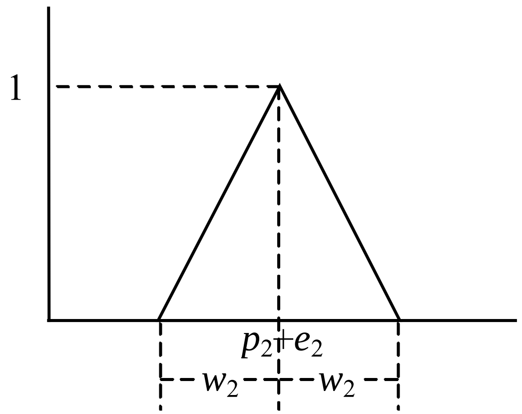
# Experimental Setup

- **Summary results from [Rotondo98] on the position accuracy of GPR detection systems for two contractors during the Advanced Technology Demonstrations at Aberdeen**
  - **We chose the on-road information generated at Aberdeen**
  - **The data is in terms of the position error over the lanes modeled by a gaussian density function.**
  - **Hence, we have the expected performance of these two sensor/algorithm suites in terms of mean and standard deviation of position error in two dimensions.**

# Predictive Fusion Experiment

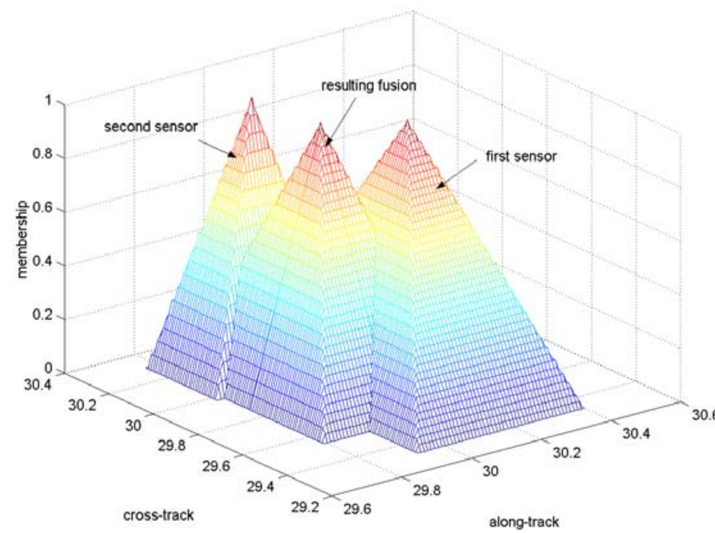
- **Chose position resolution from GPR on on-road generated at Aberdeen**
- **Generate error position ($e_1$,$e_2$) in along-track and cross-track**
- **Assume the actual location is ($p_1$,$p_2$). The detection location is at ($p_1+e_1$,$p_2+e_2$)**
- **Fuzzify each dimension of each location using Guassian distributions, e.g., N(0.1,$\sigma_1$) or N(0.21, $\sigma_1$) etc.**



# Predictive Fusion Experiment

- An example of the linguistic position fusion of the 2 sensor systems



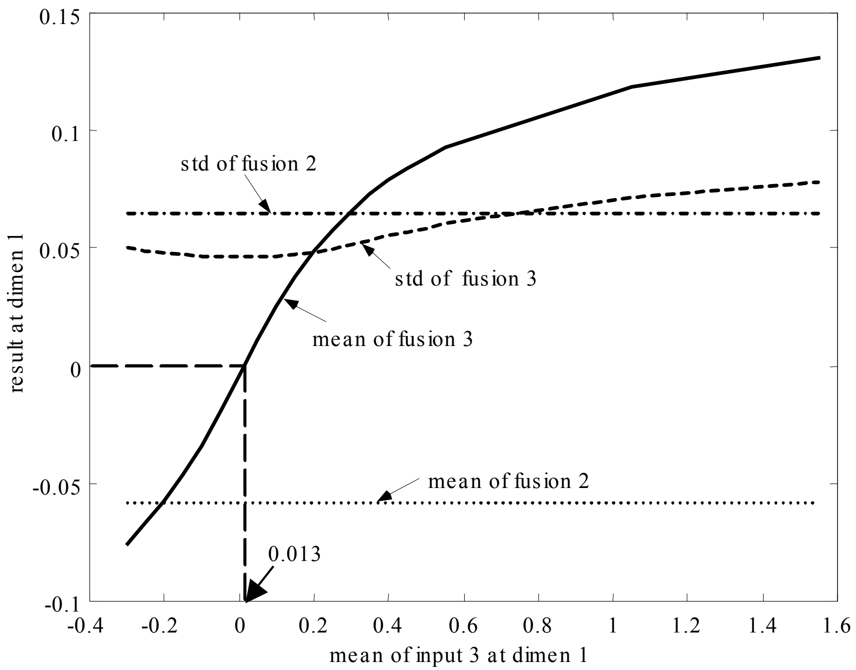

- Defuzzy each fuzzy number by picking the midpoint of the core. Also, compute the radius of the support to represent the width of that fuzzy number
- Generate 2000 trials according to a given Guassian distribution.
- Collect the error and width, then compute mean and standard deviation

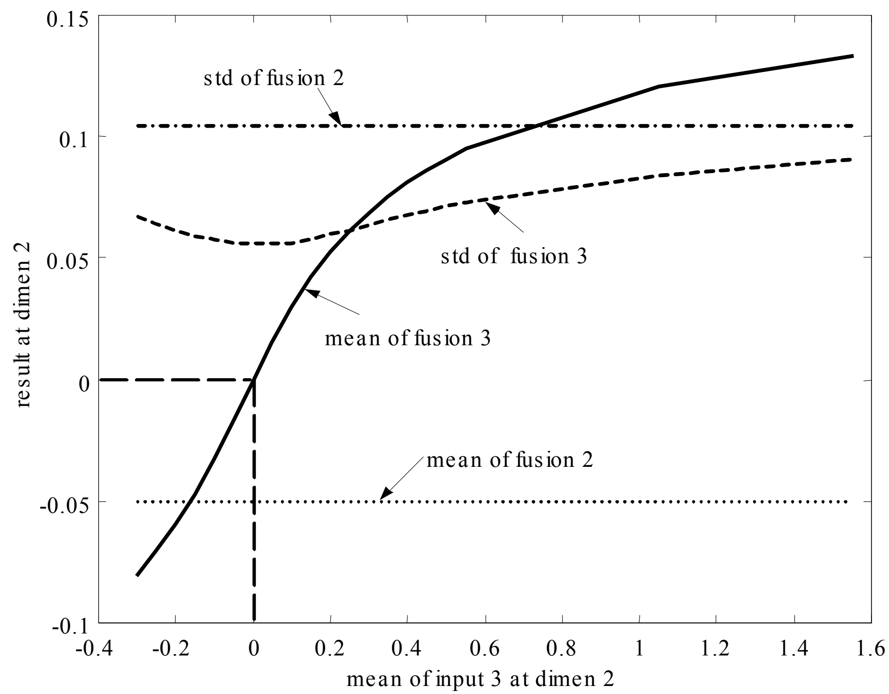
# Adding a "Third" System

- **Changing error mean of hypothetical sensor system 3**

| Sensor | Along-track | | Cross-track | |
|---|---|---|---|---|
| | Position resolution (mean(m),std (m)) | Width (mean(m),std (m)) | Position resolution (mean(m),std (m)) | Width (mean(m),std( m)) |
| 1 | (0.1,0.12) | (0.12,0) | (-0.06,0.21) | (0.21,0) |
| 2 | (-0.1,0.1) | (0.1,0) | (0.04,0.08) | (0.08,0) |
| 3 | (*x*,0.1) | (0.1,0) | (*x*,0.1) | (0.1,0) |

# Predictive Fusion Results



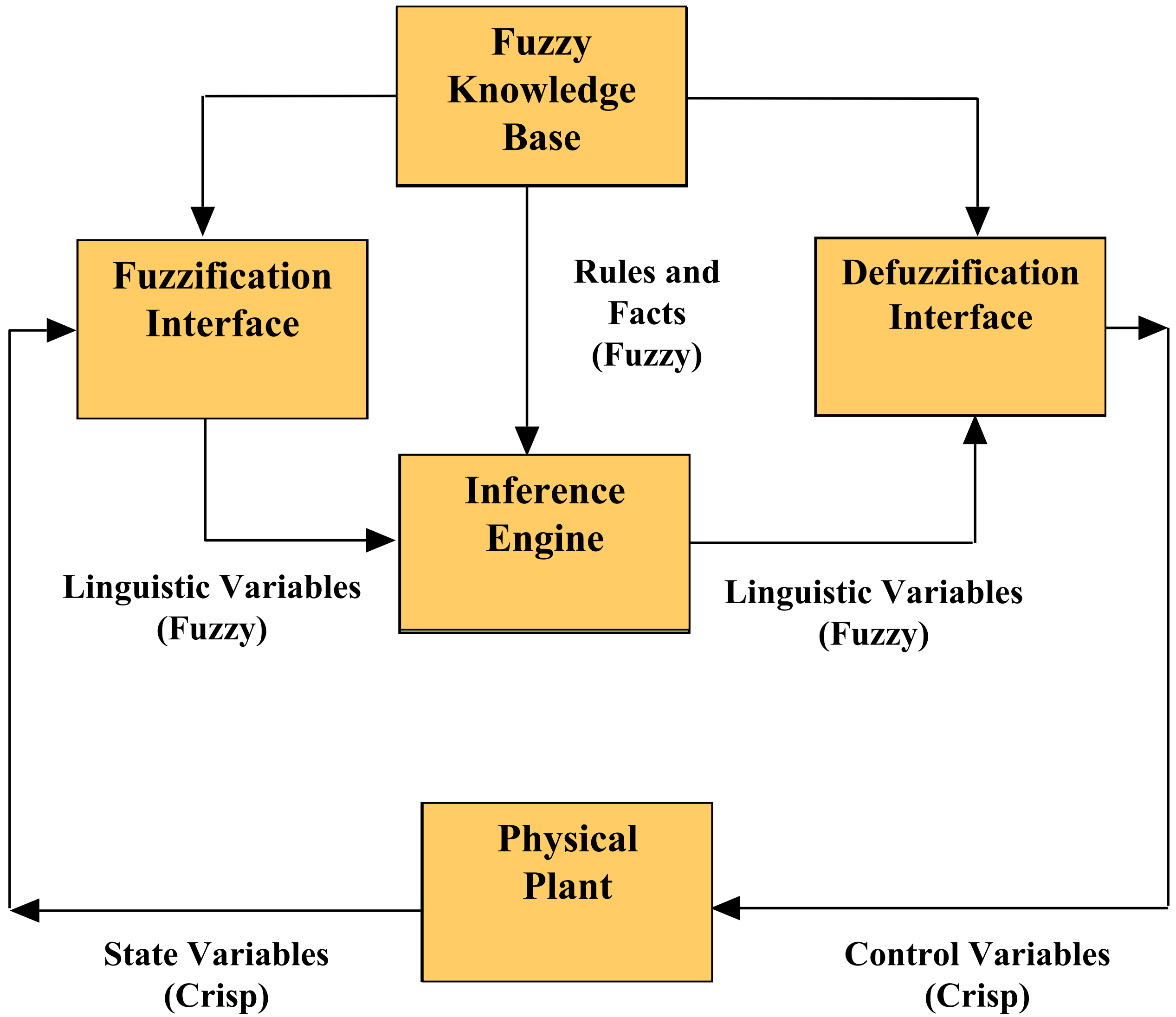
# Fuzzy Logic Systems

- Rule-based approaches to combine evidence
- Rules used by "intelligent" systems may not contain

  PRECISE PREDICATES

- Propositions modeled via fuzzy set theory
  - Simple propositions, conjunctions, disjunctions, implication
- Rules of inference generalized to accommodate extensions from binary to continuous

# Basic Configuration of a Fuzzy Logic Control System



# Knowledge Base contains the Translations of Linguistic Propositions into Fuzzy Logic Constructs

- **Simple Propositions** "X is A"

- **Compound Propositions**

| | |
|---|---|
| **Conjunctions:** | "X is A **AND** Y is B" |
| **Disjunctions:** | "X is A **OR** Y is B" |
| **Implications(Rules):** | "**IF** X is A **THEN** Y is B" |

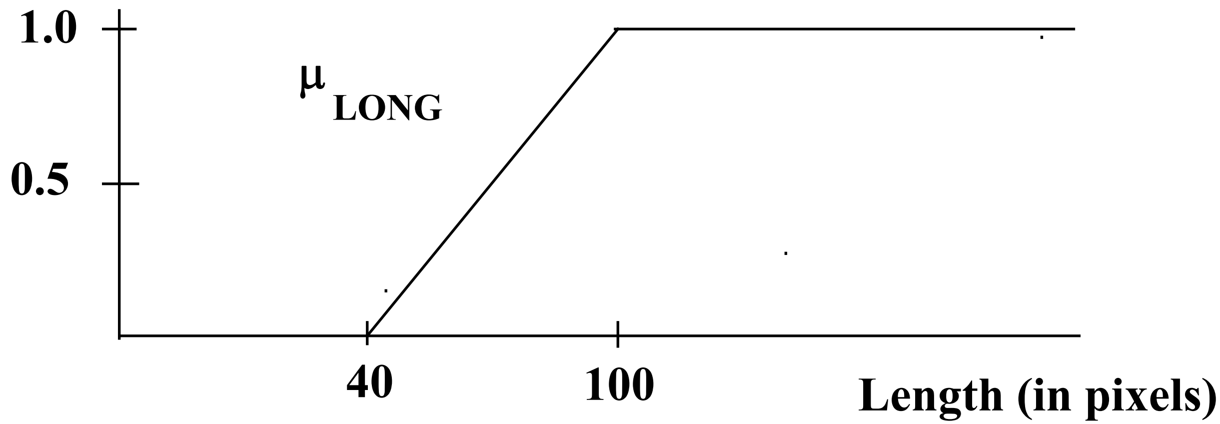
- **Plus, current knowledge of system parameters and choices**

# Knowledge Base

- **Propositions are modeled by Possibility Distributions**

**"Region is LONG" induces a fuzzy variable -LENGTH whose value is LONG**

**LONG is then described by a fuzzy set over an appropriate domain of discourse**



# Knowledge Base
# Compound Propositions

**X is NOT A : Replace membership function with that of $A^c$**

$$\mu_X(u) = \mu_{A^c}(u) \quad \{= 1 - \mu_A(u)\}$$

**Conjunctions and Disjunctions (Induces fuzzy set on pair (X,Y)):**

**Use your favorite Intersection and Union Operator**

**X is A AND Y is B:** $\mu_{(X,Y)}(u,v) = \mu_A(u) \wedge \mu_B(v)$

**X is A OR Y is B:** $\mu_{(X,Y)}(u,v) = \mu_A(u) \vee \mu_B(v)$

# Knowledge Base
# Implication

**Implication is more difficult: IF X is A THEN Y is B**

**From Classical Logic: "A implies B" equivalent to "Not A OR B"**

**Direct Translation:**

$$\mu_{(X,Y)}(u,v) = \mu_{A^c}(u) \vee \mu_B(v)$$

$$= (1 - \mu_A(u)) \vee \mu_B(v)$$

$$= (1 - \mu_A(u) + \mu_B(v)) \wedge 1$$

**compatible with Lukasiewicz multivalued logic**

# Knowledge Base
# Implication

<u>**There are many other translation schemes**</u>

**Most Common for Control:**

**Correlation Min:** $\mu_{(X,Y)}(u,v) = \mu_A(u) \wedge \mu_B(v)$

**Correlation Product:** $\mu_{(X,Y)}(u,v) = \mu_A(u) \times \mu_B(v)$

**Why? Because they are Simple and Fast!**

# Fuzzy Logic Inference

- So, Now You've Got

  P1: IF X is A Then Y is B

  P2: X is A′

- Want to Derive Value for Y: (Y is B′)

  Generalized Modus Ponens

  Actually, Many Ways to Do This

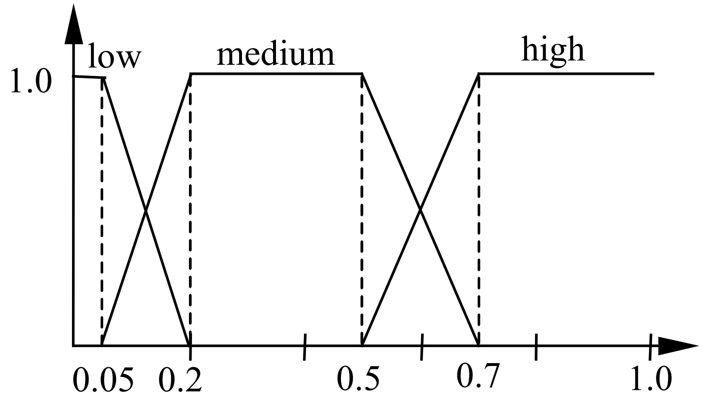
- Interesting to Note that if all Domains are finite, this looks like Matrix-Vector Multiplication with Sum replaced by Union and Product Replaced by Intersection

# Ingredients of a Fuzzy Logic System

- **Linguistic Variables: Angle, Δangle, GPR Confidence, Aspect Ratio, etc.**

  **- Problem Dependent**

- **Membership Functions to Model the Meaning of Variable Values**




**Typical Membership Functions**

# More Ingredients

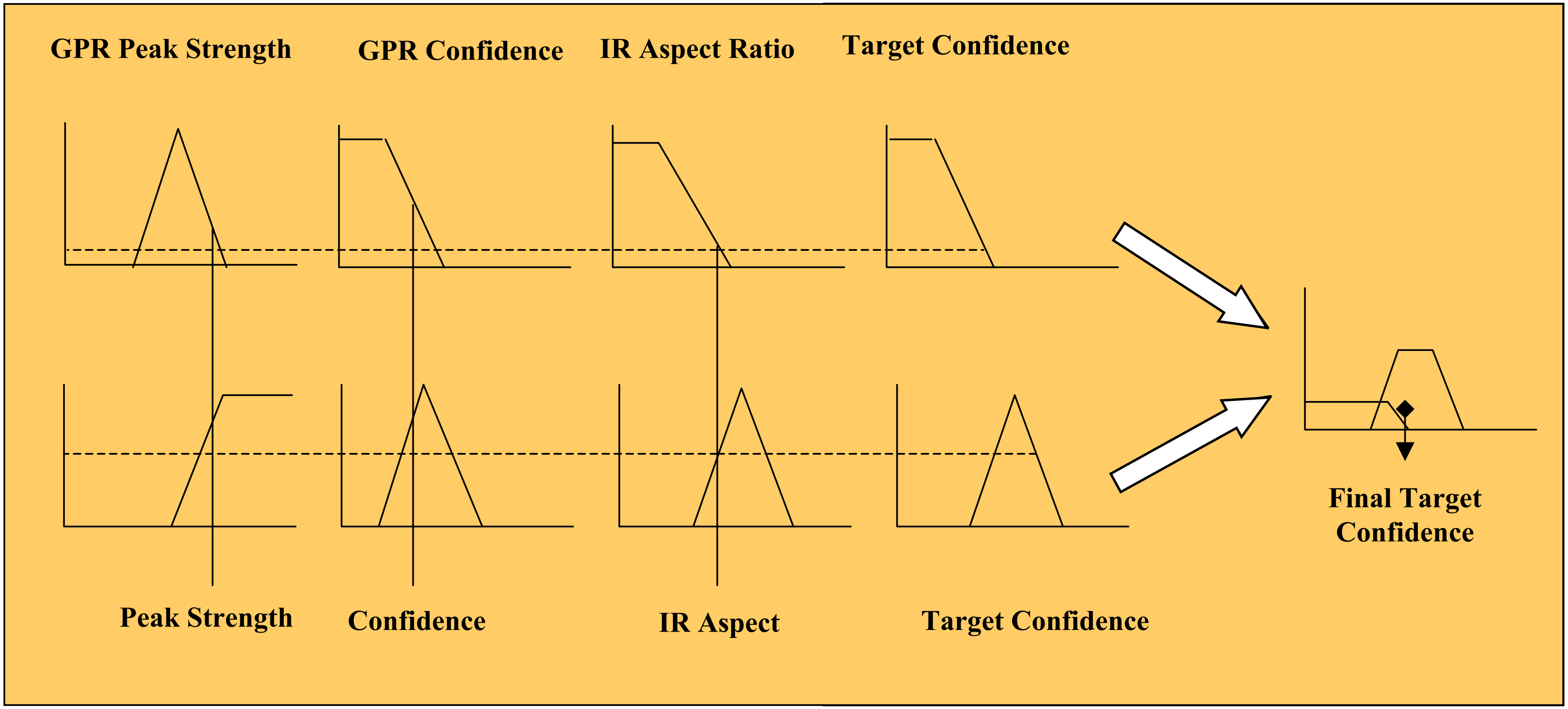
- Add a cup full of rules (the more the merrier)

IF GPR peak strength is *MEDIUM* AND
GPR confidence is *LOW* AND
ASPECT RATIO of the IR is *NOT_MINE_LIKE*
THEN
Overall_Target_Confidence is *LOW*

IF GPR peak strength is *HIGH* AND
GPR confidence is *MEDIUM* AND
ASPECT RATIO of the IR is *MEDIUM_MINE_LIKE*
THEN
Overall_Target_Confidence is *RATHER HIGH*

# How's Inference Really Done?

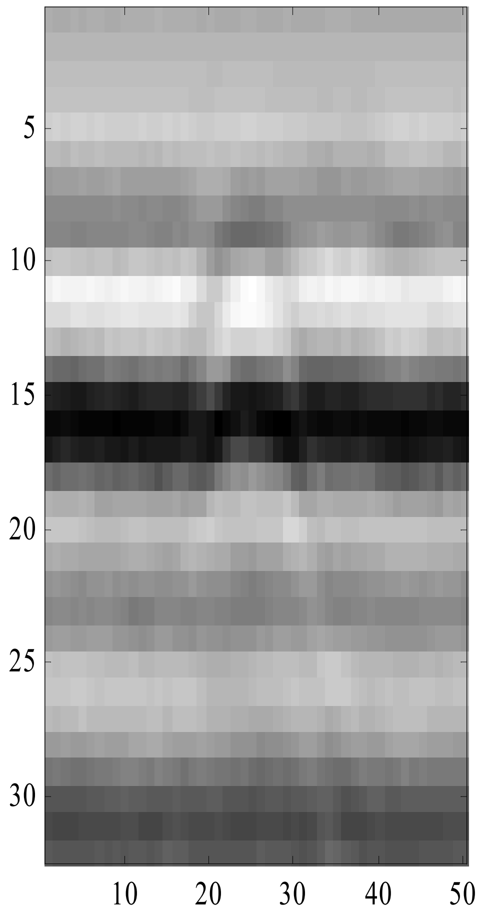
- All Rules Fire to Some Degree - Produce Output Fuzzy Sets
- Output Fuzzy Sets Are "Aggregated" - Usually Added; Sometimes Maxed
- Output Value is Computed From All Outputs - Defuzzification (UGH!!!!)

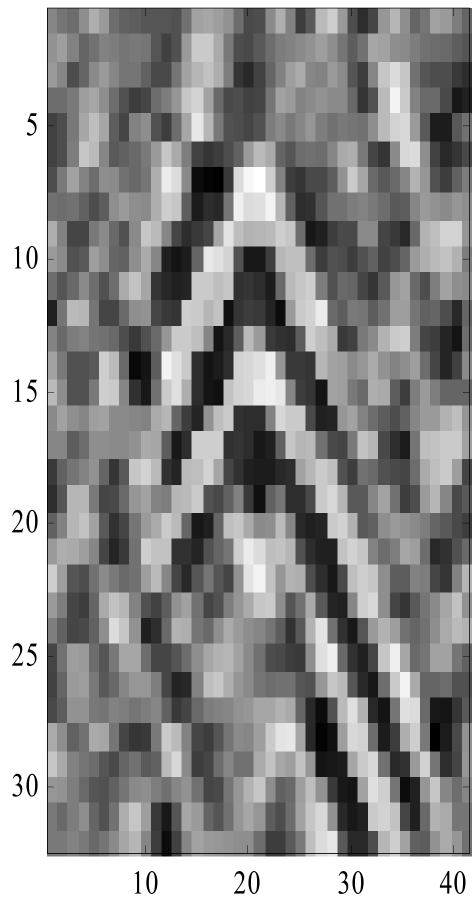


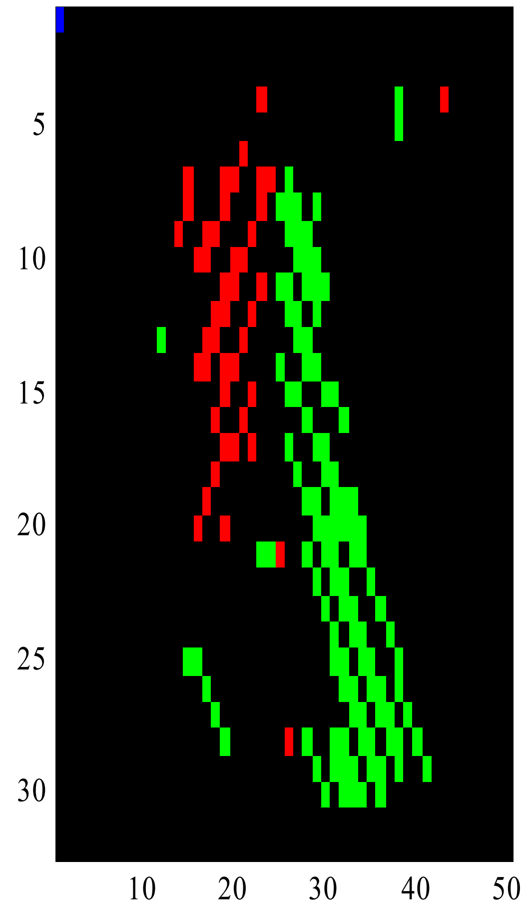
# Example - Describing a GPR Signature

## Plastic Mine

RAW GPR ENERGY LEADING & TRAILING EDGES



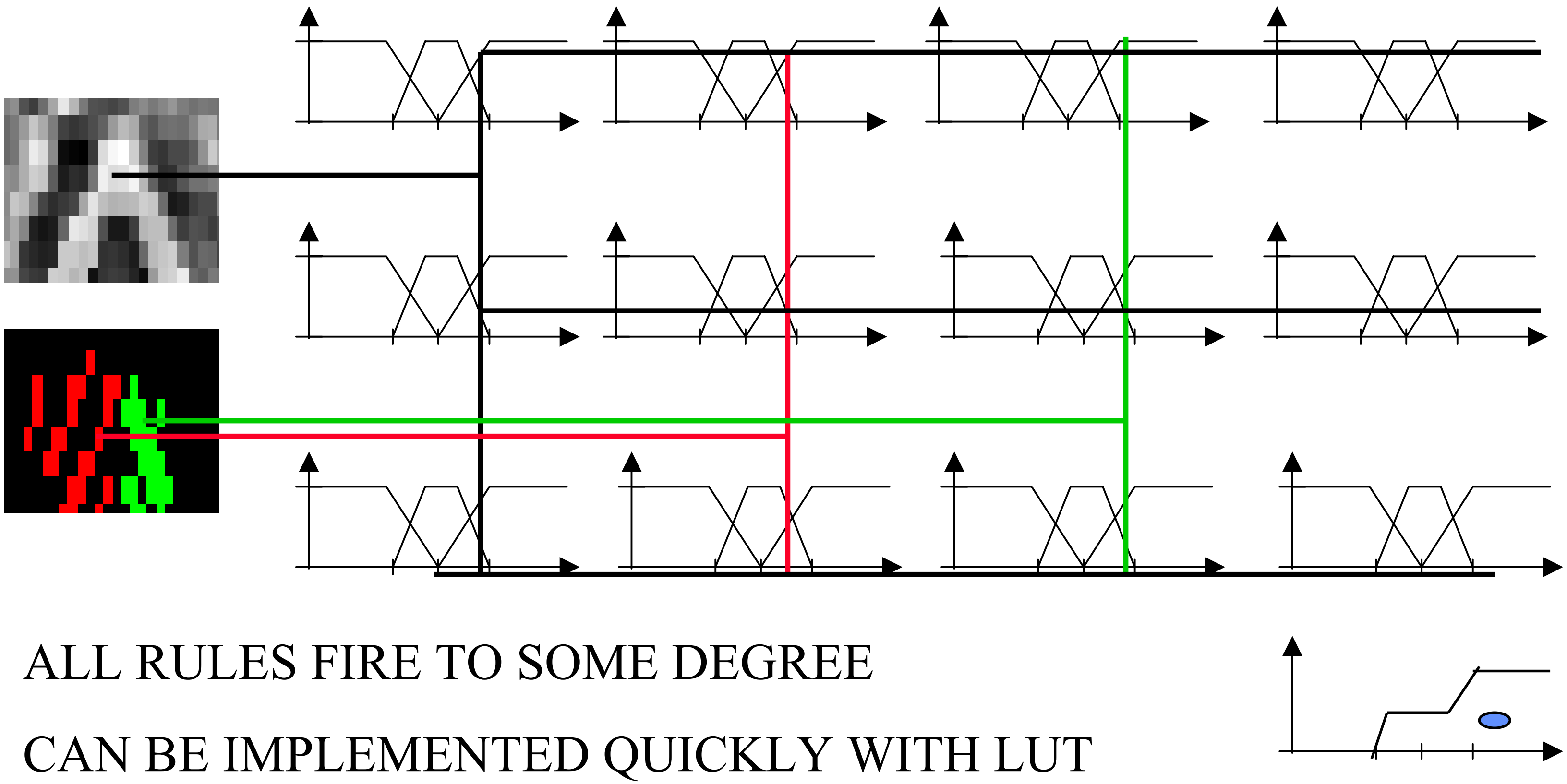



# Example - Describing a GPR Signature

| | | | | |
|---|---|---|---|---|
| R1: | If | Energy is | HIGH | AND |
| | | Leading Edge is | STRONG | AND |
| | | Trailing Edge is | STRONG | |
| | THEN | | | |
| | | Mine Confidence is HIGH | | |
| R2: | If | Energy is | HIGH | AND |
| | | Leading Edge is | MEDIUM | AND |
| | | Trailing Edge is | STRONG | |
| | THEN | | | |
| | | Mine Confidence is MEDIUM | | |
| R3: | If | Energy is | HIGH | AND |
| | | Leading Edge is | WEAK | AND |
| | | Trailing Edge is | WEAK | |
| | THEN | | | |
| | | Mine Confidence is LOW | | |

# Example - Describing a GPR Signature (Mine)

ENERGY LEADING EDGE TRAILING EDGE

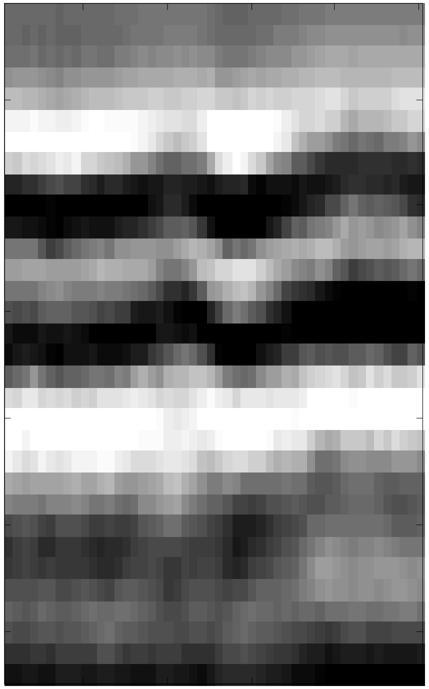


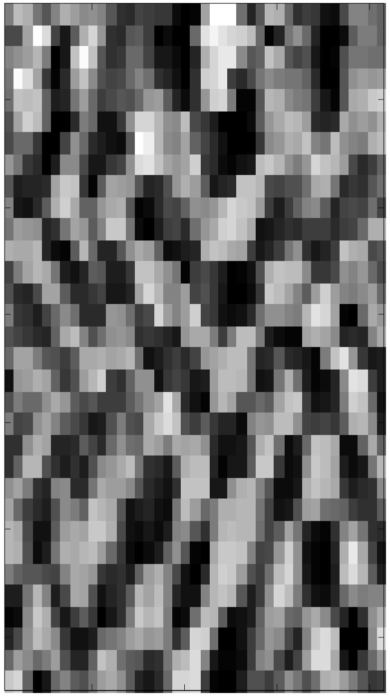
ALL RULES FIRE TO SOME DEGREE

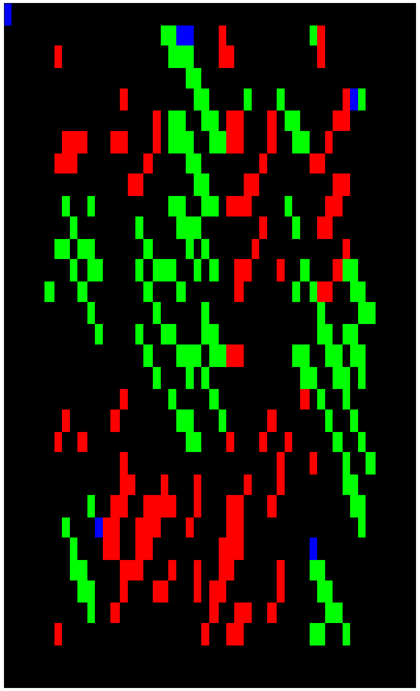
CAN BE IMPLEMENTED QUICKLY WITH LUT

# Example - Describing a GPR Signature

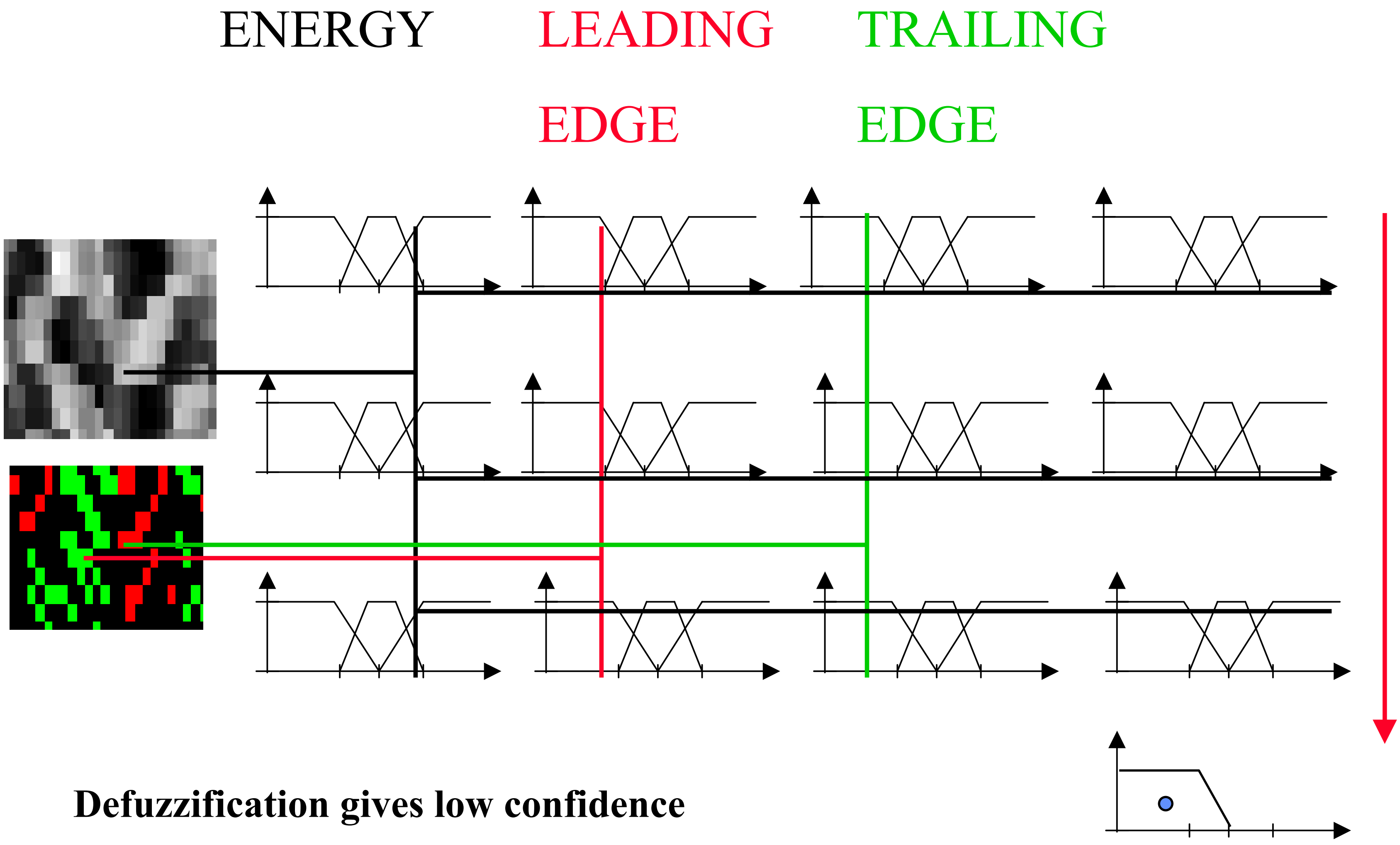
## Clutter







# Example - Describing a GPR Signature (Clutter)




**Defuzzification gives low confidence**

# Fuzzy Rule Bases for Image Processing

**Rules use Pixel Neighborhood Characteristics to "select" strength of different enhancement/smoothing operators in combination**

**Original (with Noise)**

**Median Filter**

**Fuzzy Rule Base**

**Which one do you like best?**

Lena

Noisy

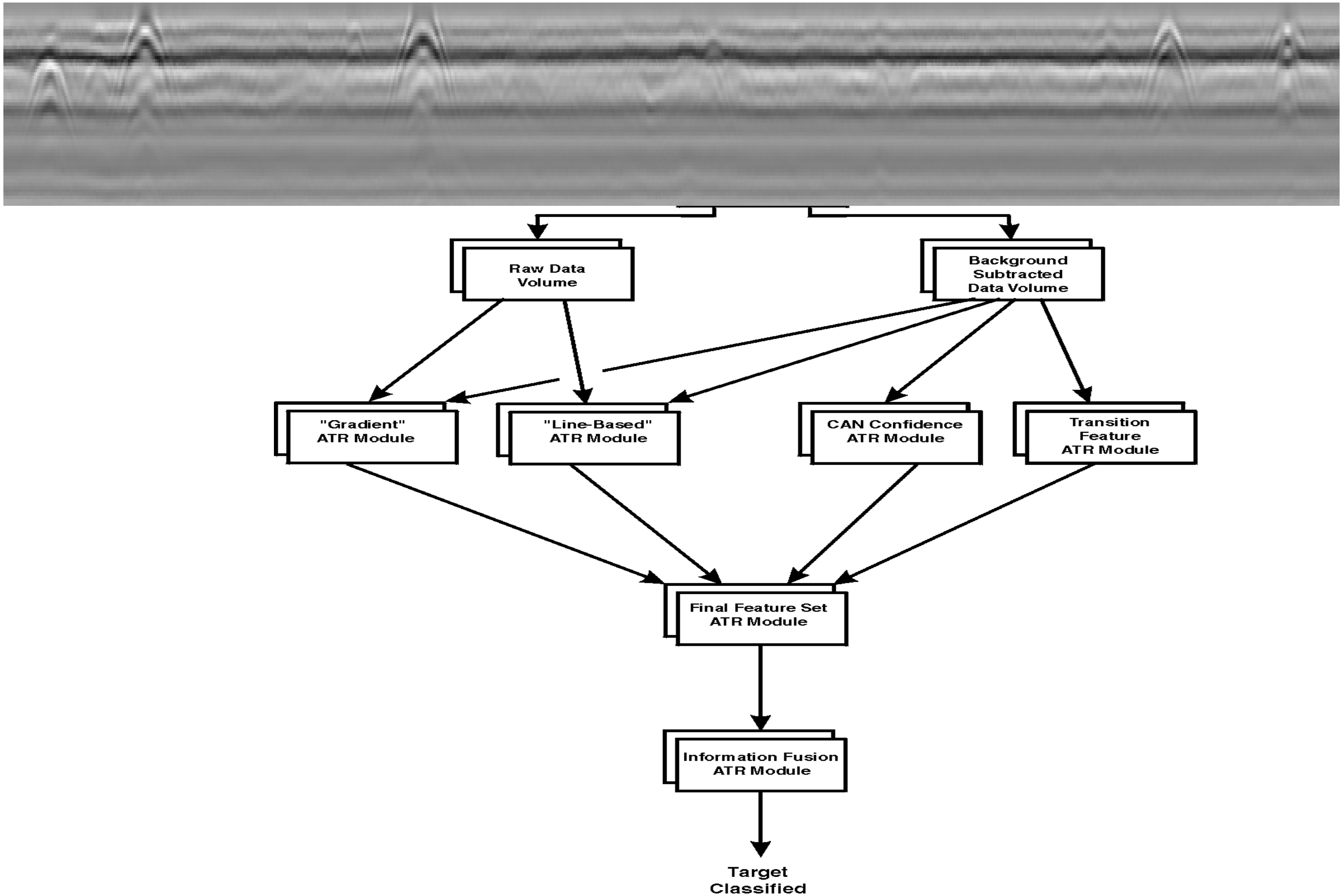
Median

Saint-Marc

Fuzzy Rules

# Information Fusion

Fusing Outputs of Multiple Algorithms on GPR
Bruce Nelson of Geo-Centers Made the Rule Base



# Multiple Algorithm Fusion Results for Blind Tests

- Fuzzy Integrals and Fuzzy Logic Rule Base
  - Developed on Signature Library
  - Fused confidence outputs from several sources
  - (3 primary detectors and 2 secondary algorithms)
- Taken to the field with Geo-Centers Mobile Landmine Detection System
- Target Reports submitted to US Army

# Information Fusion - Rule Base

## Joint with Bruce Nelson - GeoCenters

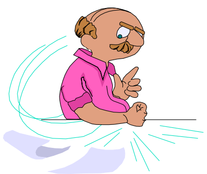
| | GRAD ATR | LINE ATR | CAN CONF | TRANS CONF | BLOB LENGTH | SYSTEM CONF |
|---|---|---|---|---|---|---|
| 1 | L | L | L | L | | L |
| 2 | L | L | L | | | L |
| 3 | L | L | | L | | L |
| 4 | L | | L | L | | L |
| 5 | | L | L | L | | L |
| 6 | L | H | M | M | | ML |
| 7 | M | H | M | L | | ML |
| 8 | | | L | M | | ML |
| 9 | | | L | H | | ML |
| 10 | | | L | L | L | ML |
| 11 | | L | | L | L | ML |
| 12 | L | | | L | L | ML |
| 13 | | L | L | | L | ML |
| 14 | L | | L | | L | ML |
| 15 | L | L | | | L | ML |
| 16 | | | M | H | | M |
| 17 | | | M | M | | M |
| 18 | | | L | L | | ML |
| 19 | M | M | H | L | | M |
| 20 | M | H | M | L | | M |
| 21 | H | M | M | L | | M |
| 22 | | | M | L | | MH |
| 23 | H | H | H | | | MH |
| 24 | H | H | | H | | MH |
| 25 | H | | H | H | | MH |
| 26 | | H | H | H | | MH |
| 27 | | | H | L | | MH |
| 28 | | | H | M | | MH |
| 29 | | H | H | L | | MH |
| 30 | H | | H | L | | MH |
| 31 | H | H | | L | | H |
| 32 | H | H | H | H | | H |
| 33 | M | H | L | M | | ML |
| | | | | | | |

# Blind Test Results at U.S. Army Site

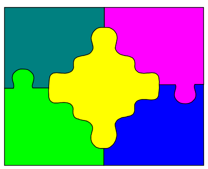
## Blind Test Results (Scored by U.S. Army)

| Soil Type | Method | Fuzzy Rules | | Choquet Integal | | Sugeno Integral | |
|---|---|---|---|---|---|---|---|
| | | PD | FAR | PD | FAR | PD | FAR |
| DIRT | AVERAGE | 75% | 0.05 | 74% | 0.06 | 71% | 0.06 |
| DIRT | MEDIAN | 73% | 0.05 | 75% | 0.06 | 74% | 0.06 |
| GRAVEL | AVERAGE | 94% | 0.03 | 93% | 0.01 | 93% | 0.02 |
| GRAVEL | MEDIAN | 93% | 0.03 | 91% | 0.01 | 92% | 0.02 |
| OFFRD | AVERAGE | 83% | 0.03 | 77% | 0.02 | 80% | 0.02 |
| OFFRD | MEDIAN | 82% | 0.02 | 77% | 0.02 | 82% | 0.02 |
| ALL | AVERAGE | 83% | 0.04 | 80% | 0.04 | 80% | 0.04 |
| ALL | MEDIAN | 86% | 0.03 | 78% | 0.02 | 82% | 0.02 |

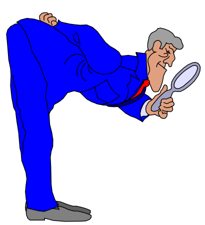
# Conclusions



Fuzzy Set Theory and Fuzzy Logic are serious tools for complex, ill-defined decision making problems like fusion

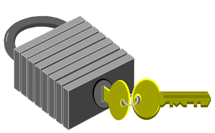


Not competitive, but complementary to traditional methods



Many Open Questions

Good for research activities



Keep an open mind -- Need a Big Bag of Tools

Have Fun!