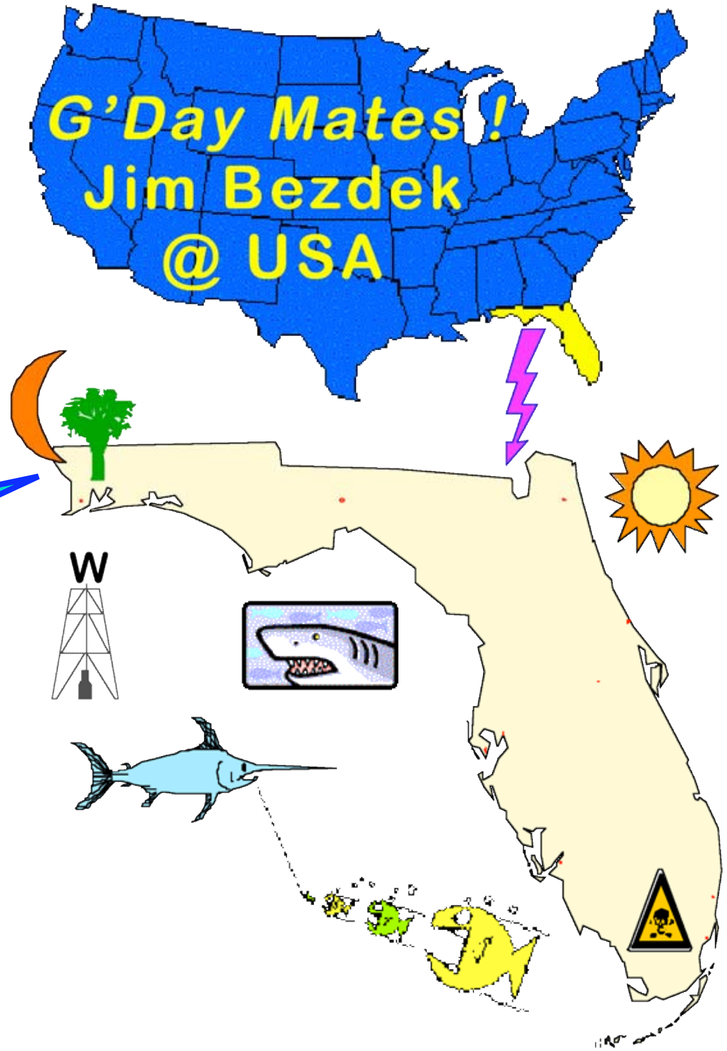
I'm from ...

University of West Florida

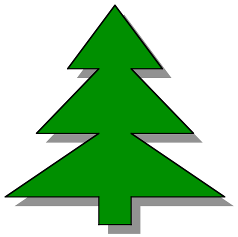


Pensacola

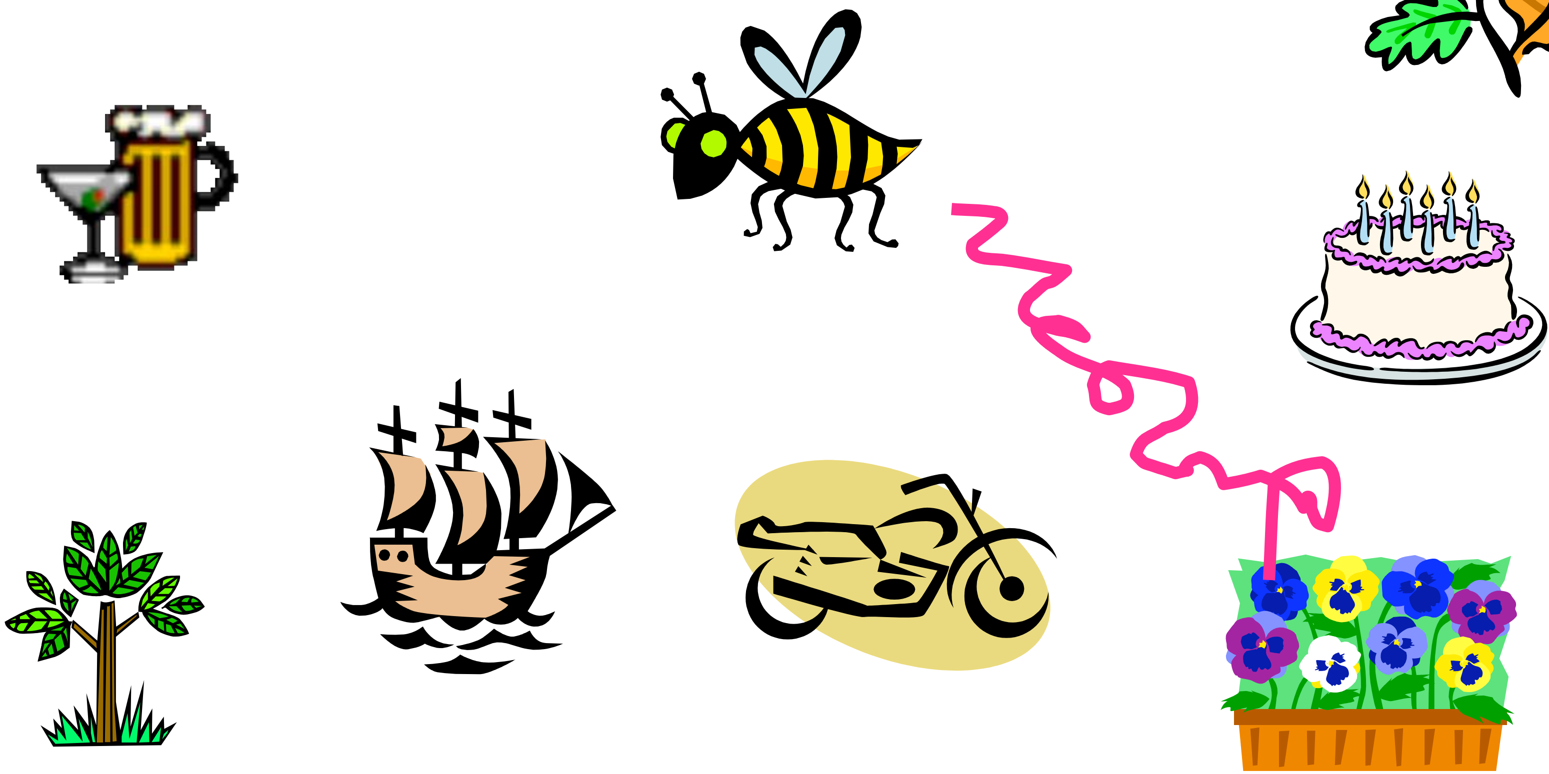
Pensacola before Hurricane IVAN


Pensacola
after 9/16/04

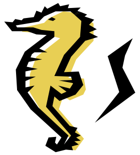
What IS Pattern Recognition ?



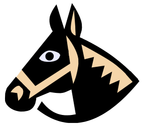
Where will this bee go?



Pattern Recognition

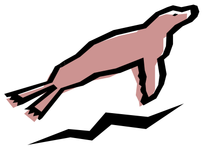


Numerical Pattern Recognition

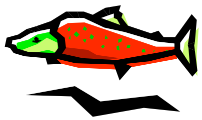


Feature Analysis

Similarity : Distances and Norms



Label Vectors and Partitions



Cluster Analysis

Classifier Design

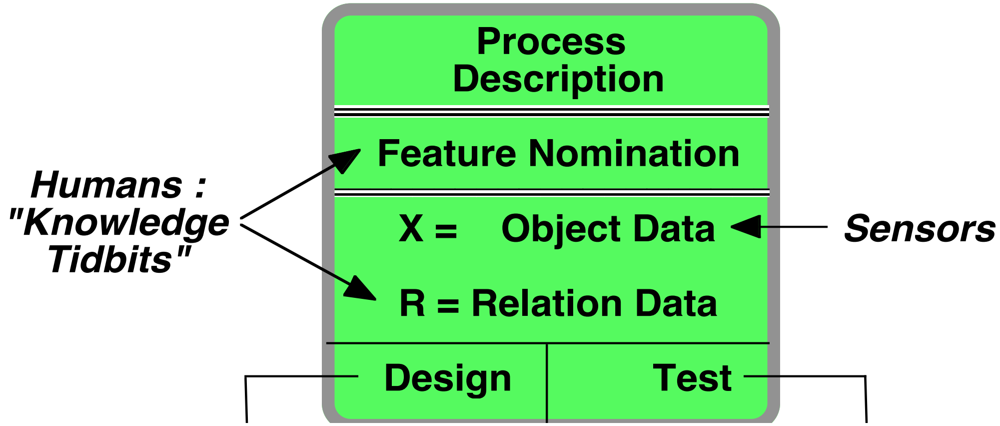
Numerical Pattern Recognition

Process Description

Feature Analysis

Cluster Analysis

Classifier Design



2 kinds of data for Pattern Recognition

Objects $O = \{o_1, \ldots, o_n\}$: o_i = i-th *physical* object

Object Data $X = \{x_1, \ldots, x_n\}$: x_i = *feature vector* for o_i

x_{ji} = j-th (*measured*) feature of x_i : $1 \leq j \leq p$

Relational data $R = [r_{ij}]$ = *relationship*(o_i, o_j)) or (x_i, x_j)

s_{ij} = *similarity* (o_i, o_j) or (x_i, x_j)

d_{ij} = *dissimilarity* (o_i, o_j) or (x_i, x_j)

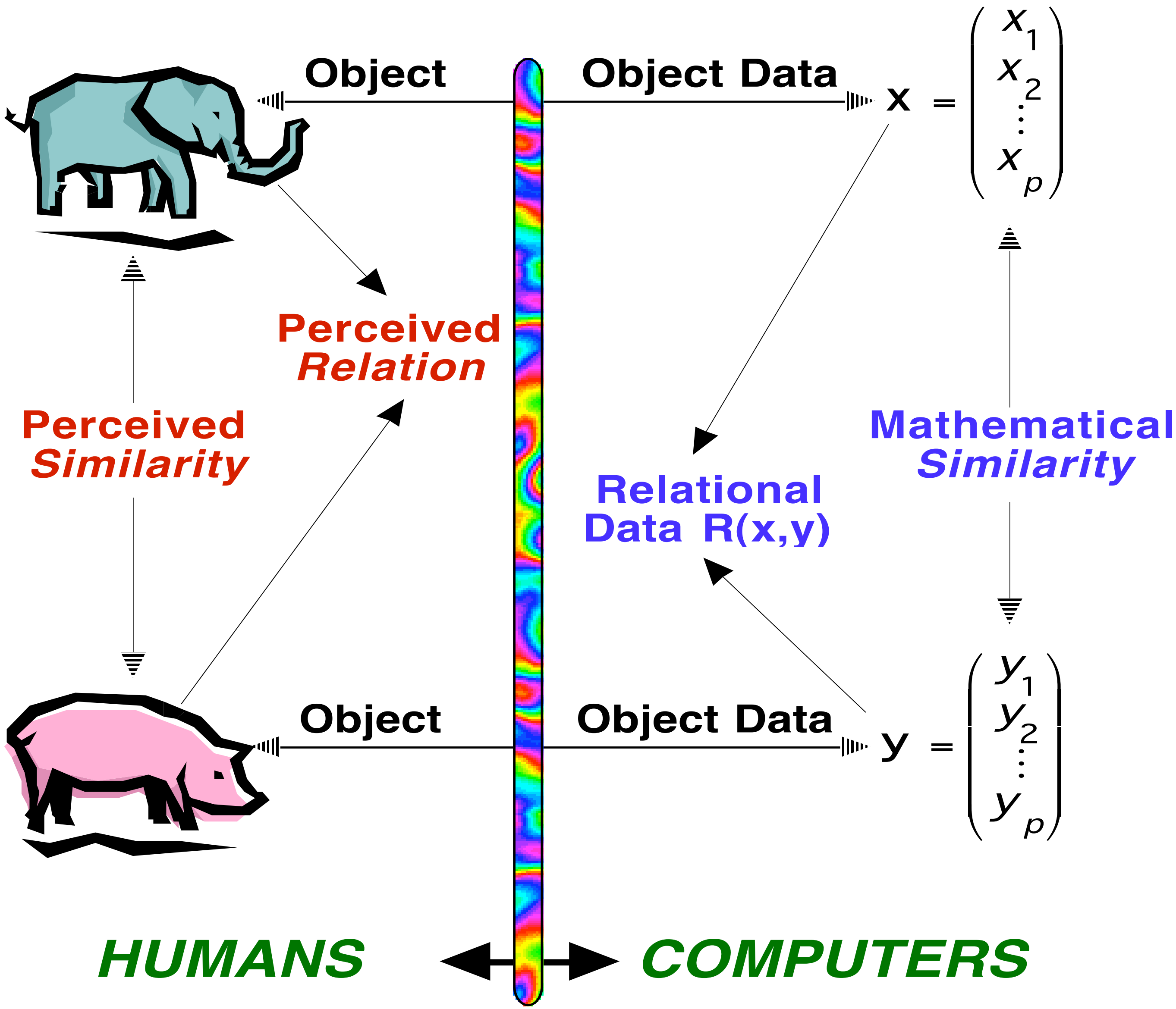
Tyically *(R = D)*

(i) $d_{ij} \geq 0$ $1 \leq i \neq j \leq n$

(ii) $d_{ii} = 0$ $1 \leq i \leq n$

(iii) $d_{ij} = d_{ji}$ $1 \leq i \neq j \leq n$

When $X \rightarrow D$, $d_{ij} = \|x_i - x_j\|_E$, D is "Euclidean"


Object
Object Data
x =
x_1
x_2
x_p
Perceived Relation
Perceived Similarity
Relational Data R(x,y)
Mathematical Similarity
Object
Object Data
y =
y_1
y_2
y_p
HUMANS
COMPUTERS

Measuring (Dis)Similarity : The 5 Good Norms

Mahalonobis

< , > Norms

Diagonal

$$\delta_{M^{-1}} = \|x - v\|_{M^{-1}} = \sqrt{(x - v)^T M^{-1} (x - v)}$$

$$\delta_{D^{-1}} = \|x - v\|_{D^{-1}} = \sqrt{(x - v)^T D^{-1} (x - v)}$$

Euclidean

$$\delta_2 = \|x - v\|_{I^{-1}} = \sqrt{(x - v)^T I^{-1} (x - v)}$$

City Block

Minkowski Norms

Sup or Max

$$\delta_1 = \|x - v\|_1 = \sum_{j=1}^{p} |x_j - v_j|$$

$$\delta_\infty = \|x - v\|_\infty = \max_{j=1}^{p} \{|x_j - v_j|\}$$

Parameters of the Statistical Norms

Mean Vector

$$\bar{v} = \sum_{k=1}^{n} x_k / n$$

Covariance Matrix

$$M = [m_{ij}] = \frac{\sum_{k=1}^{n} (x_k - \bar{v})(x_k - \bar{v})^T}{n}$$

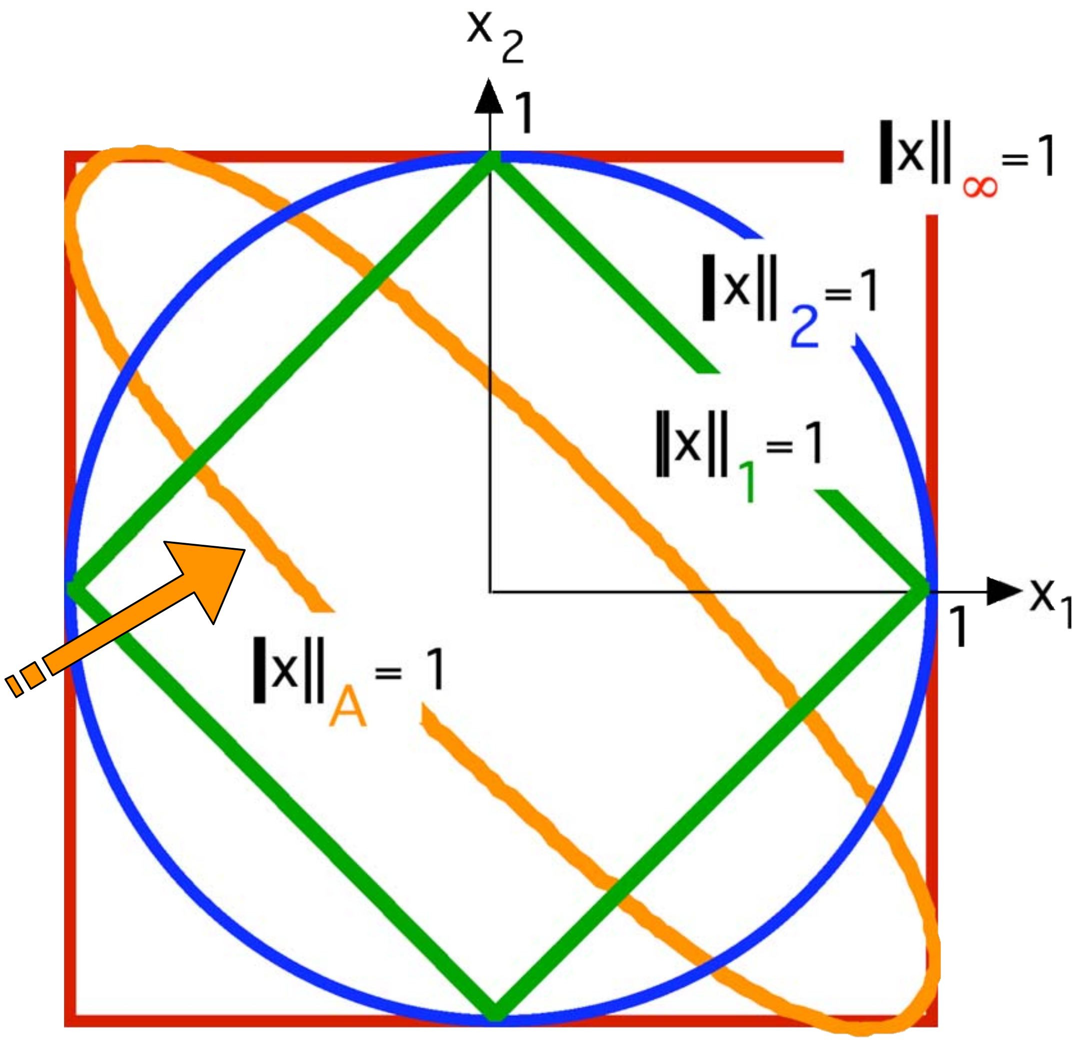
Variance (Matrix)

$$D = \begin{bmatrix} m_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & m_{pp} \end{bmatrix} = \begin{bmatrix} s_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_p^2 \end{bmatrix}$$

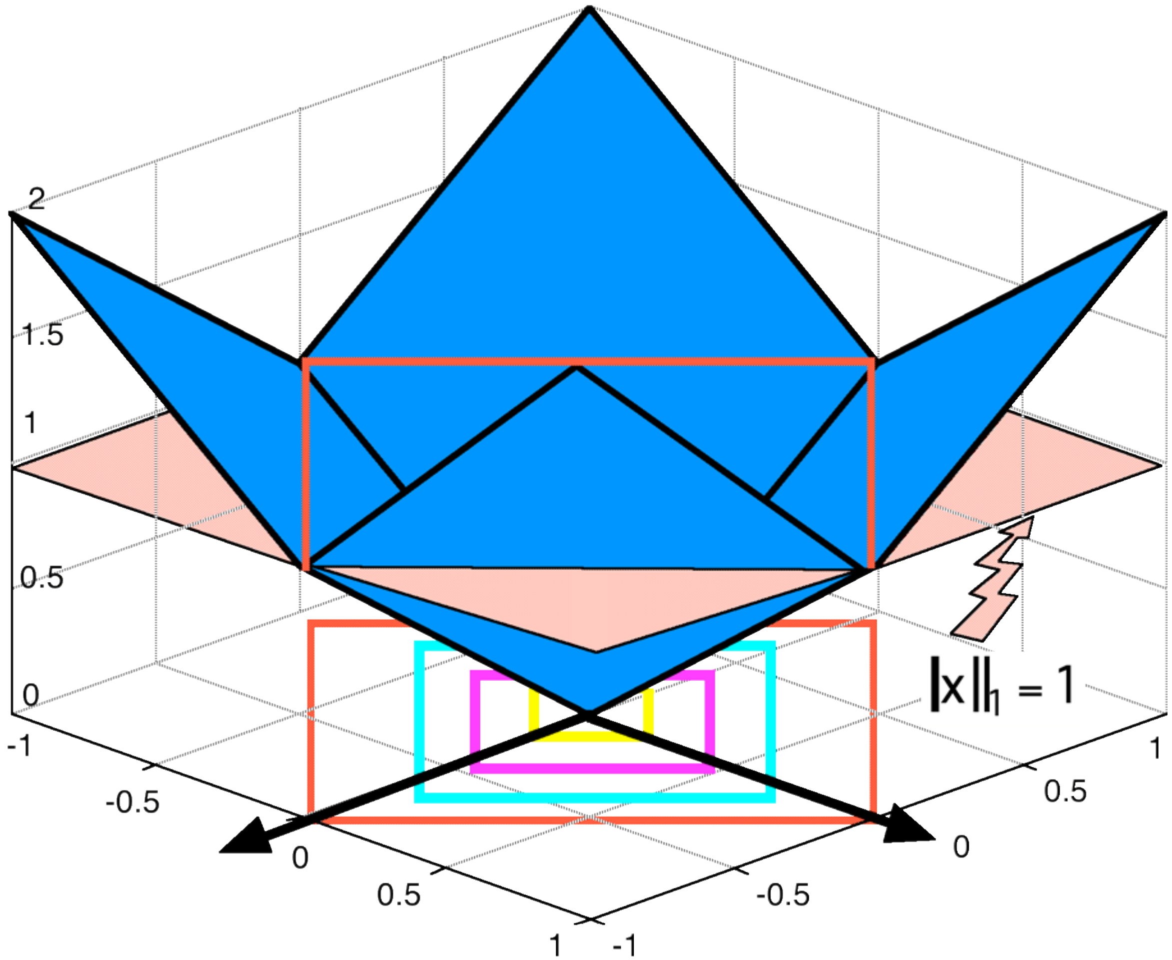
Level Sets and Open balls of Norms in R^2

Inner Product Norms
induce elliptical balls
via eigenstructure of *A*

$$\langle x, x \rangle_A = \|x\|_A^2 = x^T A x$$

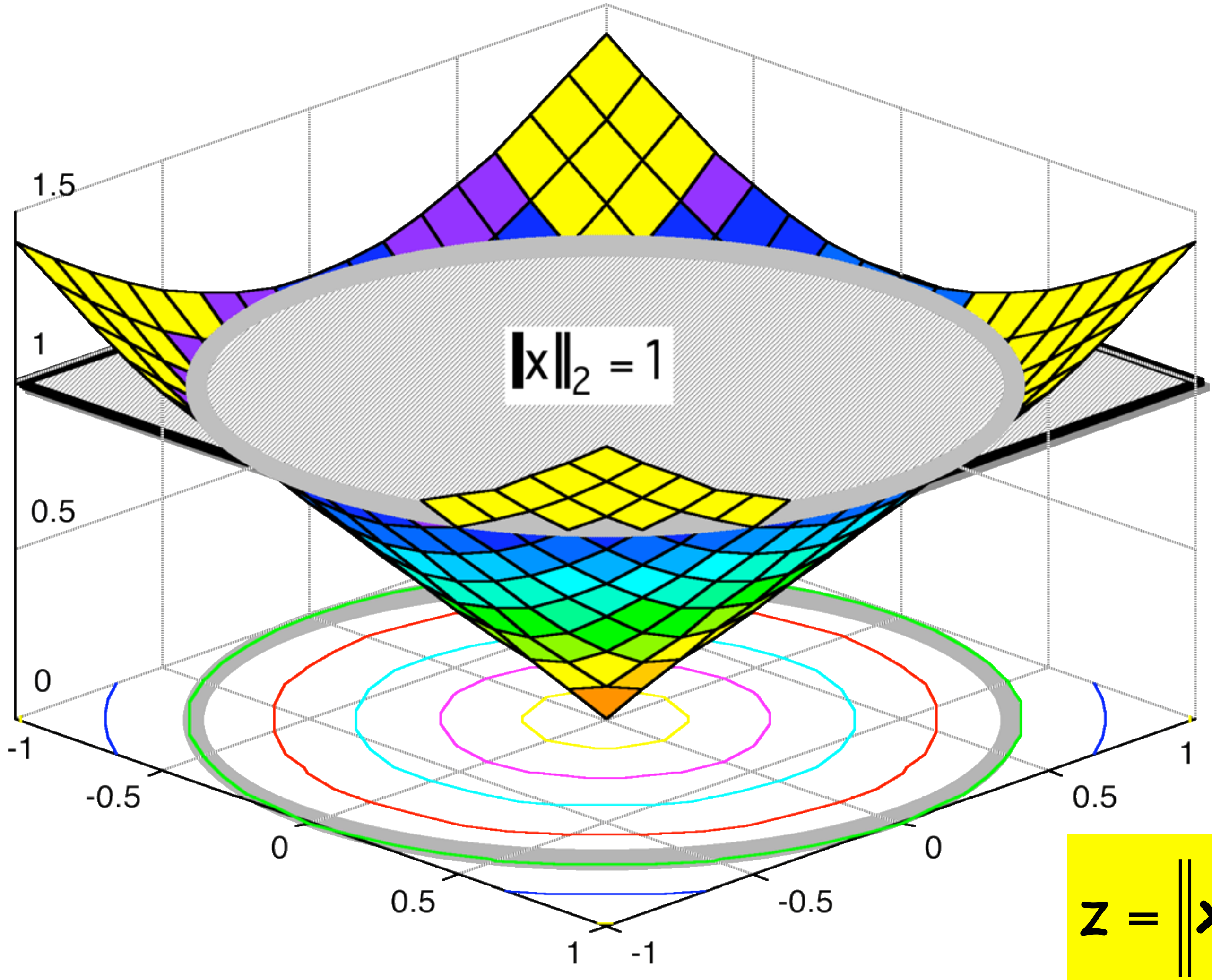


Level Sets and Open Balls of 1-norm



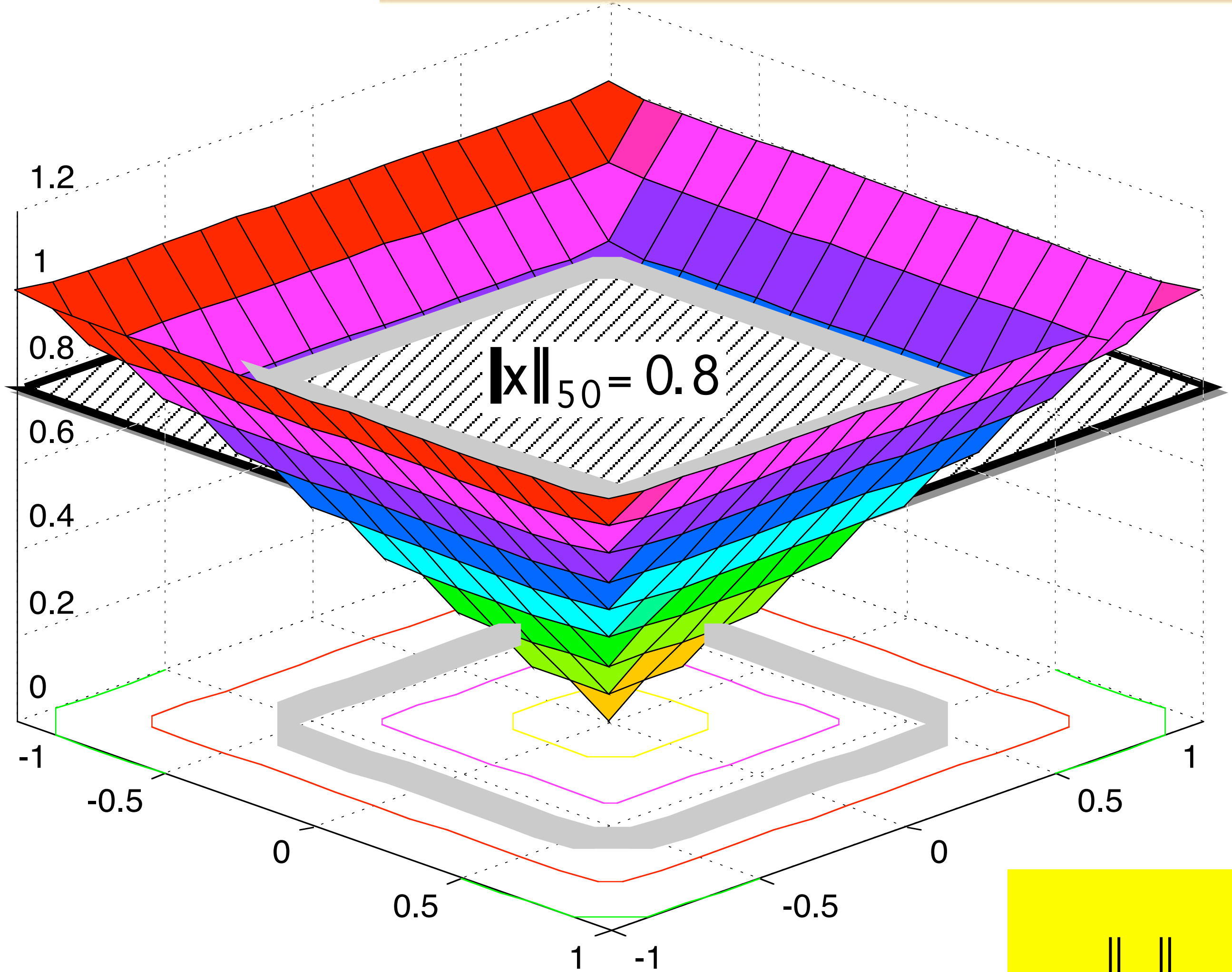
$$z = \|x\|_1 = |x| + |y|$$

Level Sets and Open Balls of 2-norm



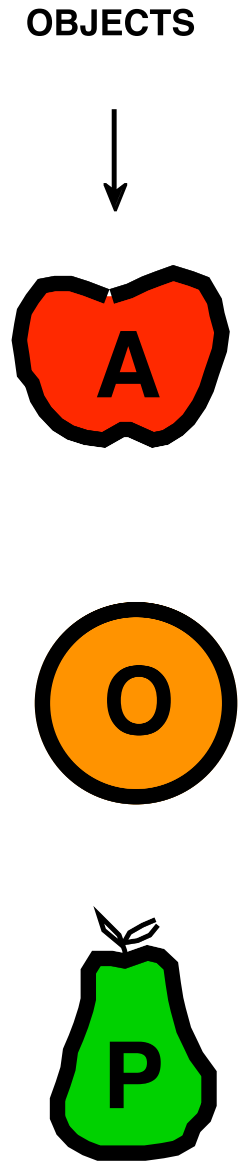
$$z = \|\mathbf{x}\|_2 = \sqrt{x^2 + y^2}$$

Level Sets and Open Balls of 50-norm



$$z = \|x\|_{50} = \left(x^{50} + y^{50}\right)^{\frac{1}{50}}$$

Feature Analysis



Feature Nomination : Issues

? What are the problems to solve ?

? What variables are important ?

? Which variables *can be* measured ?

? What data analysis will be done ?

? Are *more* features better than *less* ?

⇓ ⇓ ⇓

Propose and collect raw (object) data

Feature Extraction : Why?

Find $\phi : \underbrace{P(\Re^p)}_{X} \mapsto \underbrace{P(\Re^q)}_{Y=\phi[X]}$ to:

1. Increase dimension : p < q

Functional link networks
Support Vector Machines

2. Decrease dimension : p > q

Decrease time and space
Eliminate “redundant” features
Assess tendencies visually (q=2)

3. Y should have the “same information” as X for :

Clustering and Classification
Prediction and Control

Feature Extraction : How?

1. Functions

$$f : \underbrace{\Re^p}_{X} \mapsto \underbrace{\Re^q}_{Y=f(X)}$$

Linear

Axial ON Projection
Derived ON Projection (PCA)
Derived non-OG Projection

Non-Linear

3. Use your 

2. Algorithms

$$A : \underbrace{P(\Re^p)}_{X} \mapsto \underbrace{P(\Re^q)}_{Y=A[X]}$$

Sammon's Algorithm
FFBP Neural Nets
SVMs (in theory !)

Feature Analysis Example : Iris Data

Each object vector represents an Iris flower

n = 150 vectors

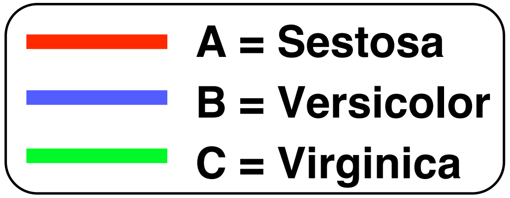
p = 4 features

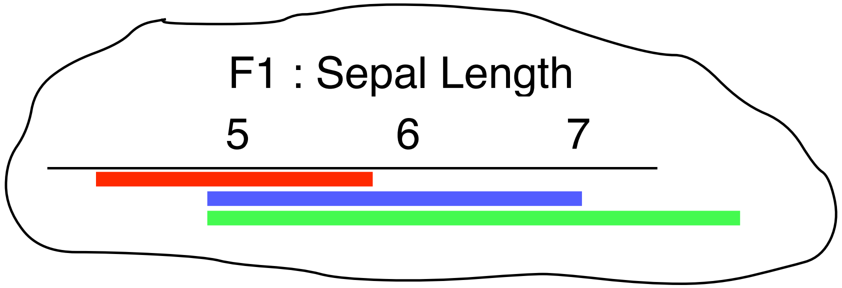
c = 3 *physical* subspecies

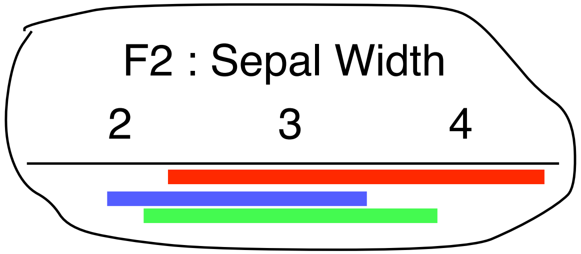
(n_i = 50 vectors each)

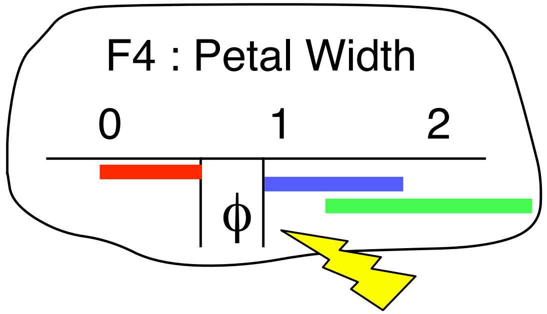
$$\mathbf{x}_k = \begin{pmatrix} x_{1k} \leftarrow \text{sepal length} \\ x_{2k} \leftarrow \text{sepal width} \\ x_{3k} \leftarrow \text{petal length} \\ x_{4k} \leftarrow \text{petal width} \end{pmatrix}$$

Visual Examination of Feature Ranges

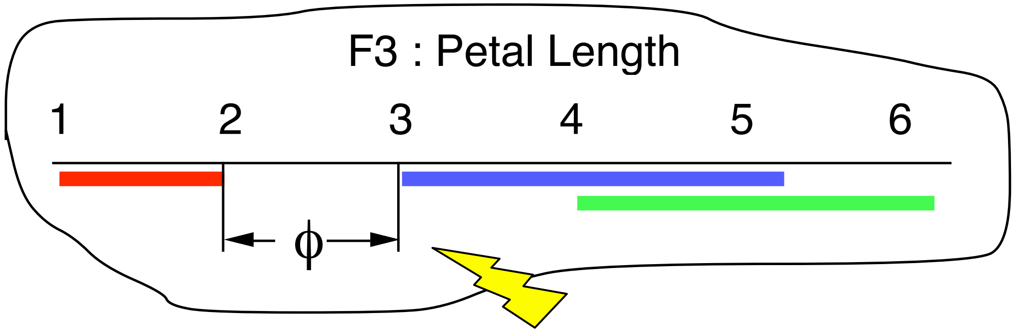








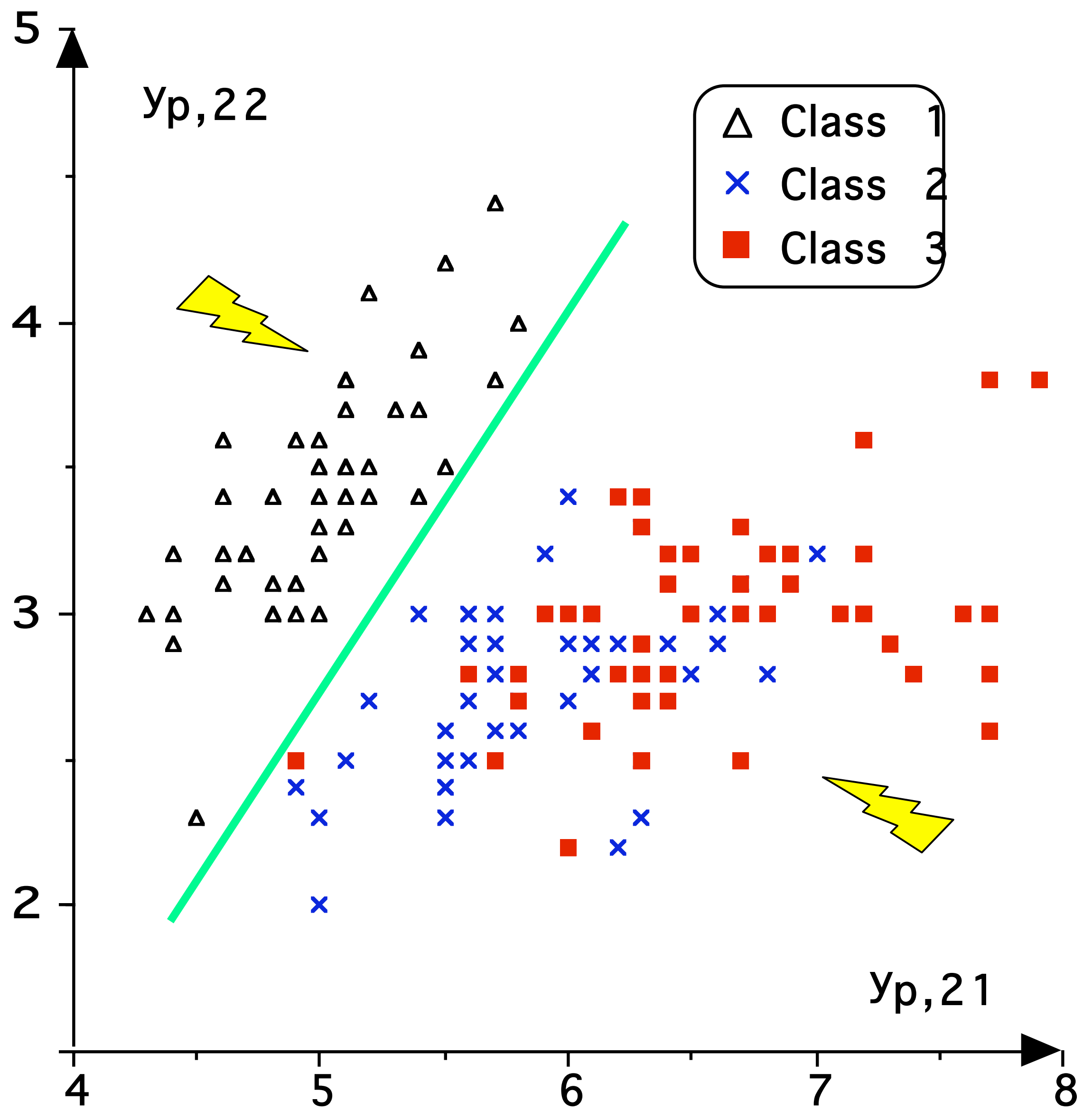
F3 or F4 separates A from BUC



Linear Selection (Projection)

F1 vs F2

(F1, F2) *linearly* separates A from B∪C
------and------
suggests that Iris has only c = 2 *geometric* clusters



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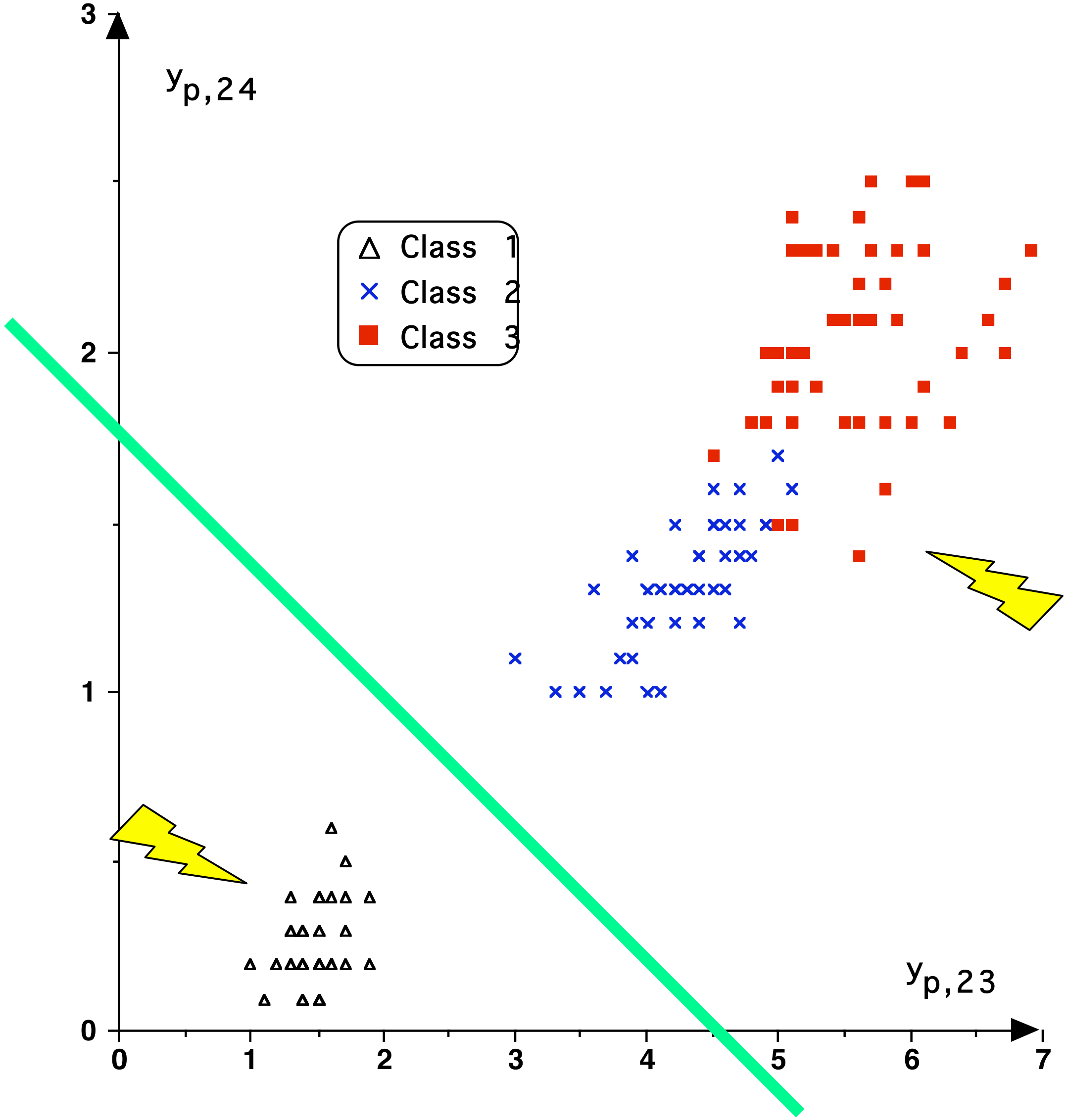
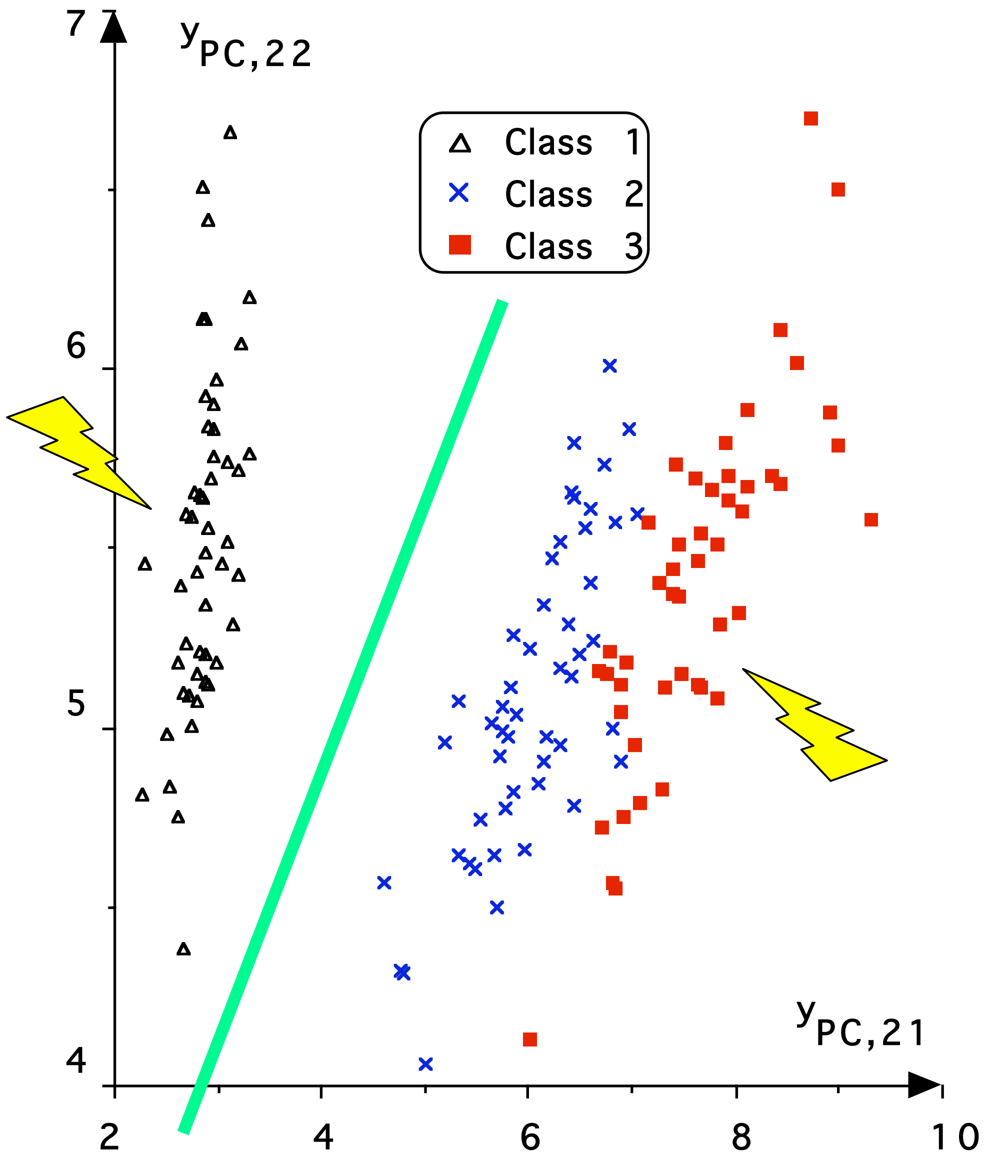


Linear Extraction (PCA Projection)

First 2 Principle Components (y_1, y_2)

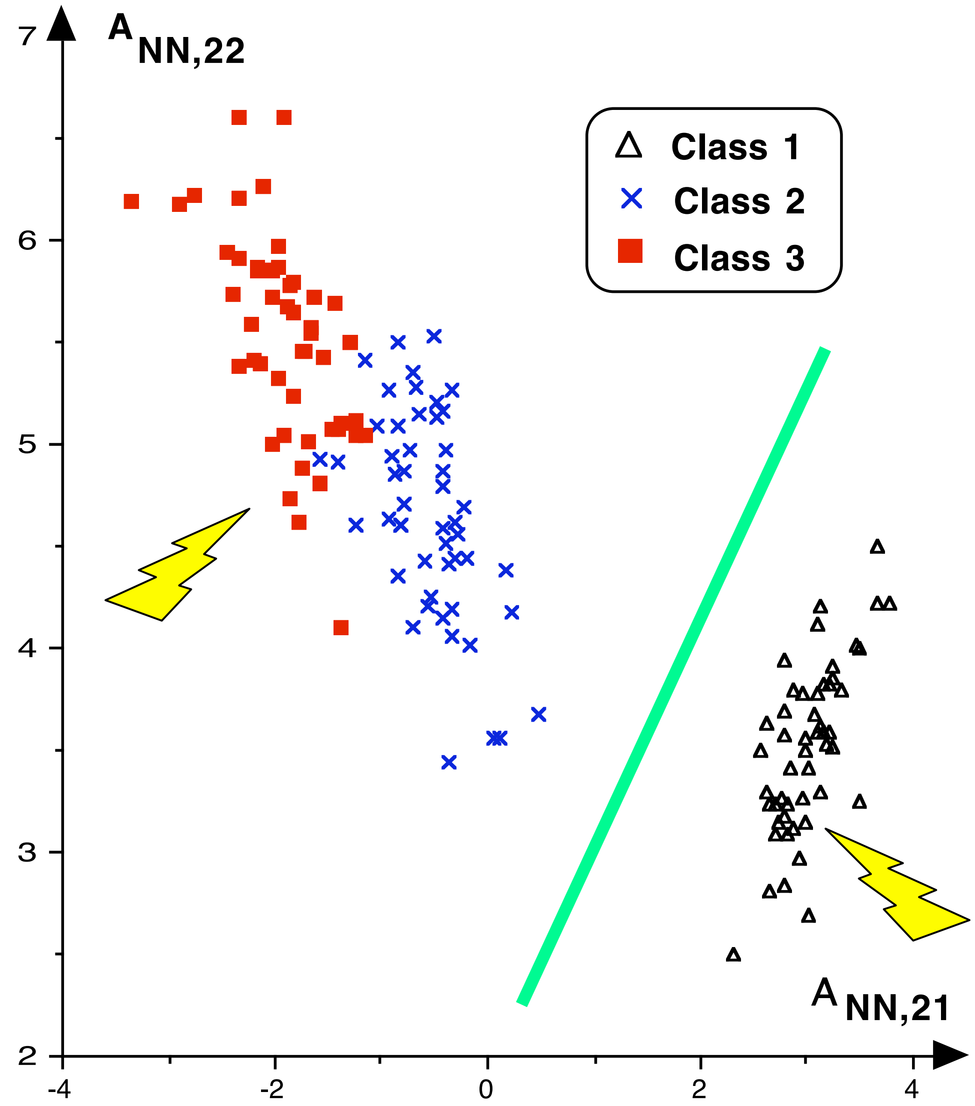
(y1, y2) *linearly* separates A from B∪C
------and------
suggests that Iris has only c = 2 *geometric* clusters



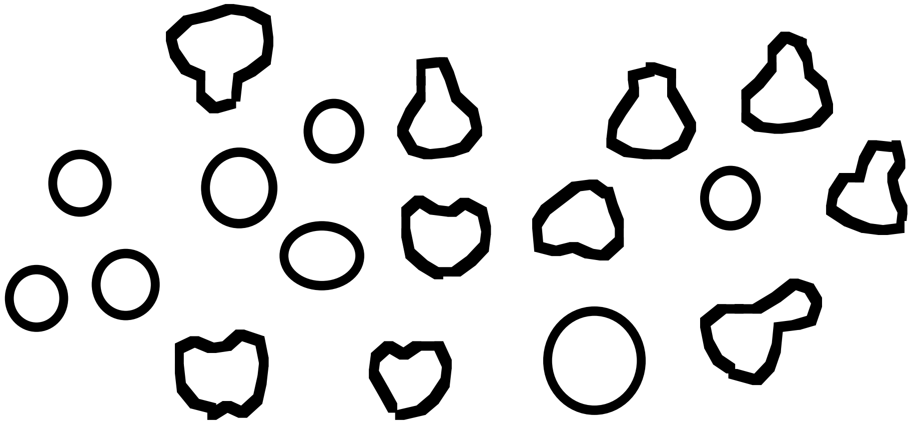
NL Extraction by 4:2:4 FFBP NN

Extracted feature pair (NN_{21}, NN_{22})

(NN_{21}, NN_{22})
linearly
separates A
from B∪C
------and------
suggests that Iris
has only c = 2
geometric clusters



Label the similar objects

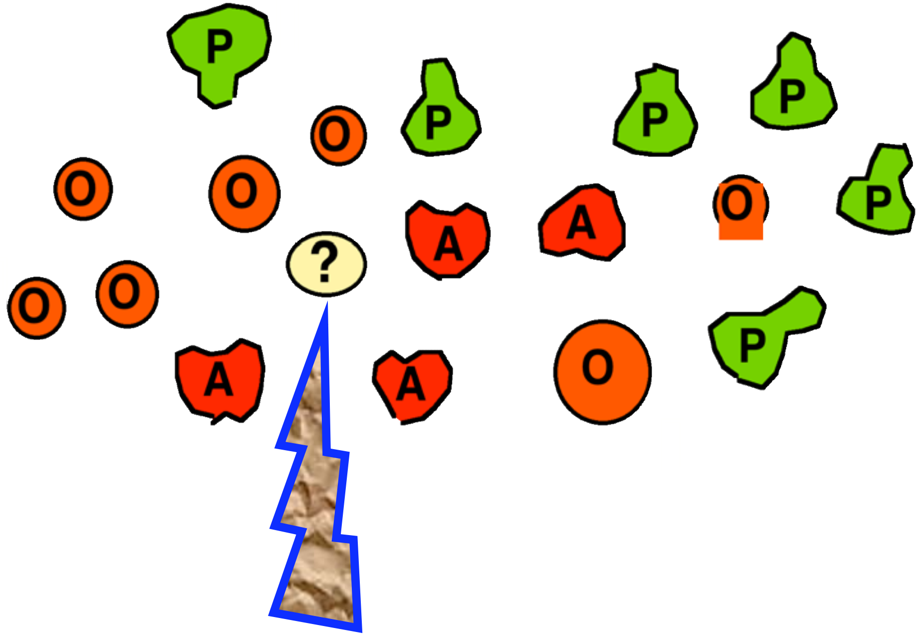


What is Clustering ?

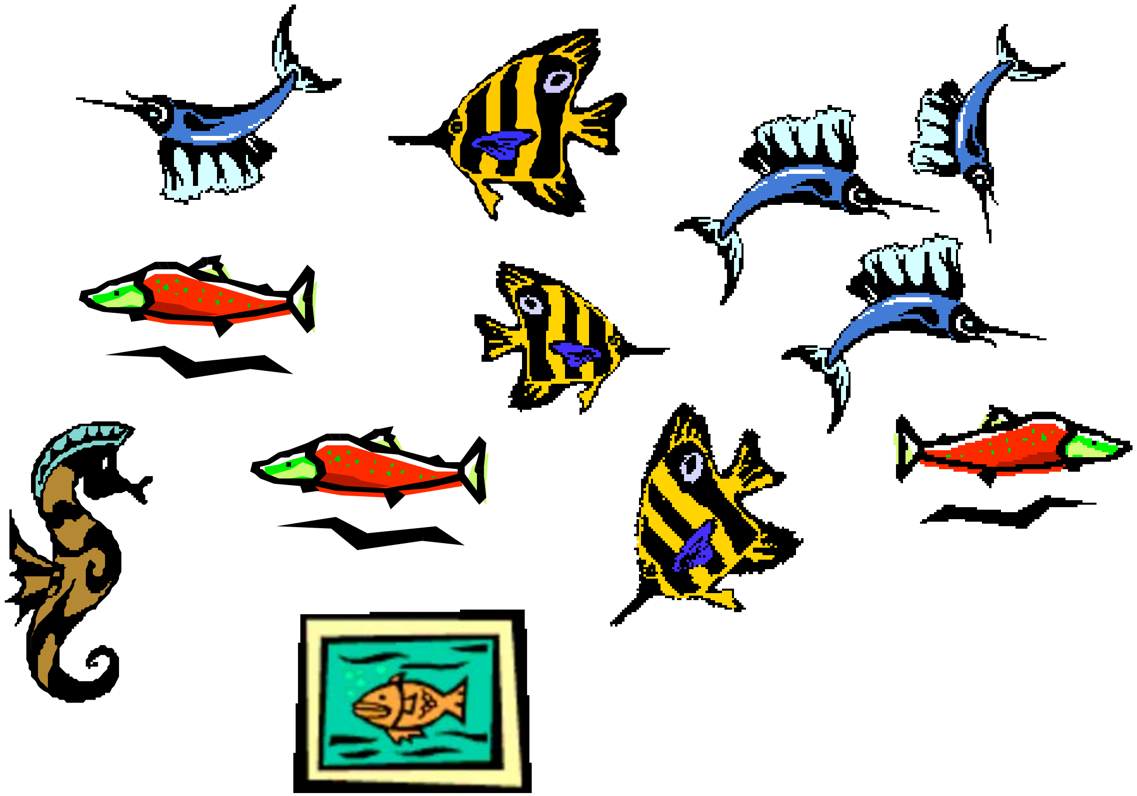
(Unsupervised Learning}

What is "similar"?

How many groups (c) ?



Data sets usually contain *mixtures* of objects

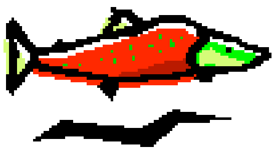


Clustering groups and labels objects

Easy for Humans !

Hard for computers !

Similarity Mistakes



What is "similar"?

Confusing Objects

Wrong features?

Mystery Data (Noise)

Incomplete data

Non-linear boundaries

How many groups (c)?

What kind of labels?

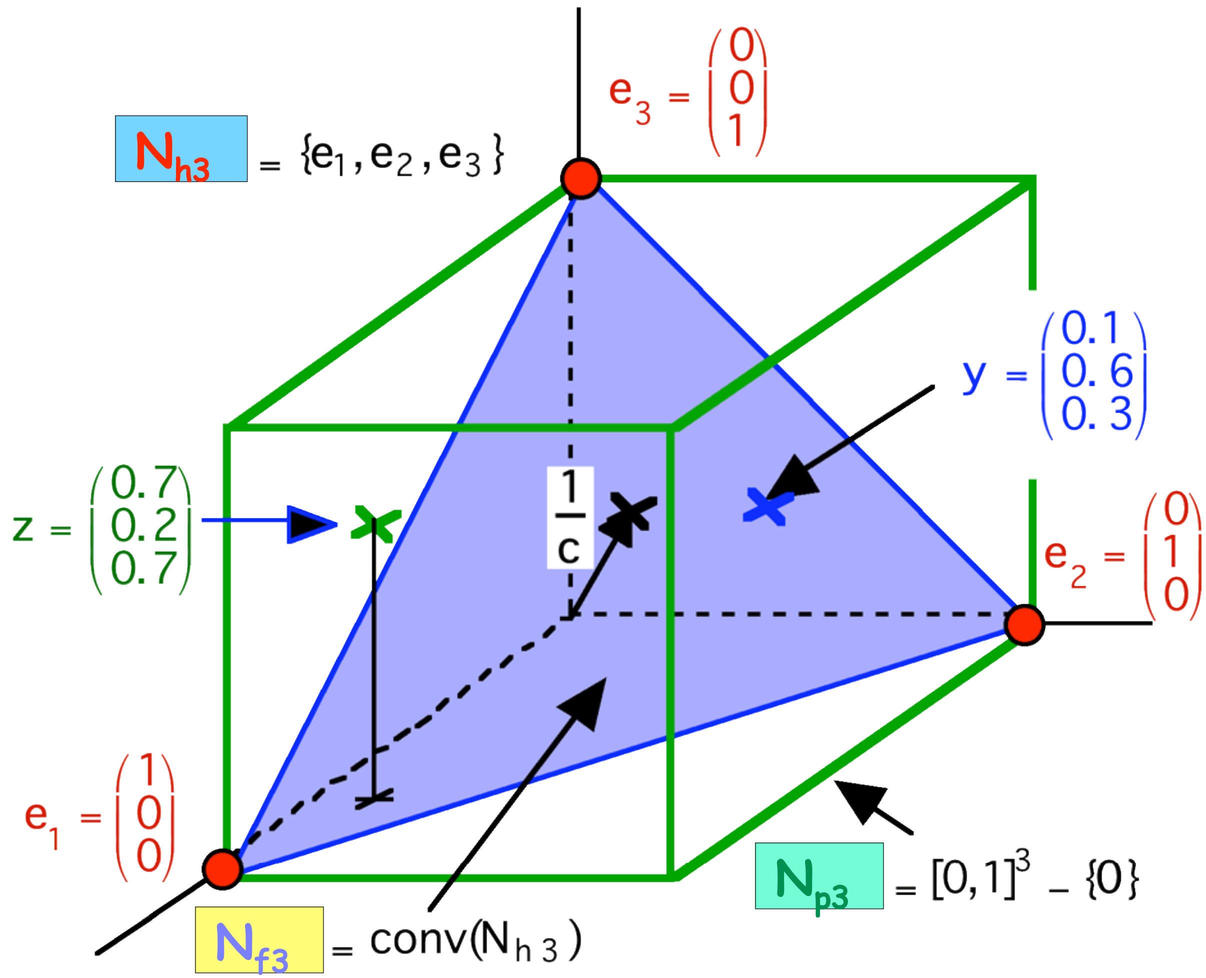
Yipes !

Unipolar Label Vectors @ c = 3

Crisp
= Vertices

Fuzzy or
Probabilistic
= Triangle

Possibilistic
= Cube-{θ}



Partition Matrices

row i ➠ M'ship of all o_k's in cluster i

o_1 o_k o_n *Objects*

$$U = \begin{bmatrix} u_{11} & \cdots & u_{1k} & \cdots & u_{1n} \\ \vdots & & \vdots & & \vdots \\ u_{i1} & \cdots & u_{ik} & \cdots & u_{in} \\ \vdots & & \vdots & & \vdots \\ u_{c1} & \cdots & u_{ck} & \cdots & u_{cn} \end{bmatrix}$$

clusters

col j ➠ M'ship of o_k in each cluster

Membership Functions

$u_i : O \rightarrow [0,1]$

$u_i(o_k) = u_{ik}$ = M'ship of o_k in cluster i

	Crisp	Fuzzy/Prob	Possibilistic
Row sums	$\sum_{k} u_{ik} > 0$	same	same
Col sums	$\sum_{i} u_{ik} = 1$	same	$\sum_{i=1}^{c} u_{ik} \leq c$
M'ships	$u_{ik} \in \{0,1\}$	$u_{ik} \in [0,1]$	same
Set Name	M_{hcn} $\subset$	M_{fcn} $\subset$	M_{pcn}
Example	$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & .07 & 0 & .44 \\ 0 & .91 & 0 & .06 \\ 0 & .02 & 1 & .50 \end{matrix}$	$\begin{matrix} 1 & .07 & 1 & .44 \\ 0 & .91 & 0 & .52 \\ 0 & .02 & 1 & .38 \end{matrix}$

Take a 2nd 

Partition Set Theory

Non Degenerate — Degenerate (0 rows)

$$\underbrace{\left(0 < \sum_{k=1}^{n} u_{ik}\right)}_{M_{hcn} \subset M_{fcn}} \rightarrow \underbrace{\left(0 \le \sum_{k=1}^{n} u_{ik}\right)}_{M_{hcn0} \subset M_{fcn0}}$$

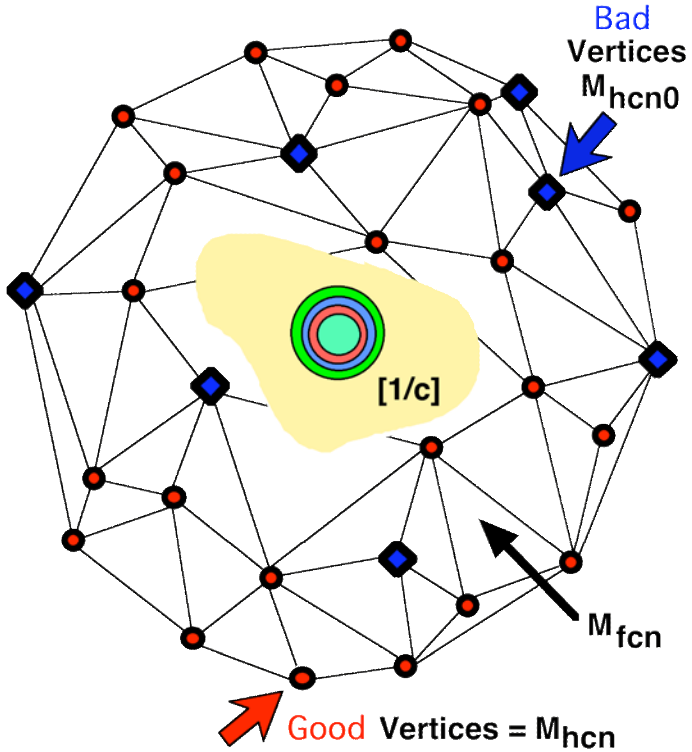
M_{fcn}=conv(M_{fcn0})=convex polytope

Each face of M_{fcn0} looks like N_{fc}

U =[1/c] = centroid of M_{fcn}

$$\left|M_{hcn}\right| = \left(\frac{1}{c!}\right)\sum_{j=1}^{c}\binom{c}{j}(-1)^{c-j} j^{n} \approx \left(\frac{c^{n}}{c!}\right)$$

dim(M_{fcn})=n(c-1)



Hardening Partitions

Defuzzification (Deprobabilization = Bayes Rule for $U \in M_{fcn}$!)

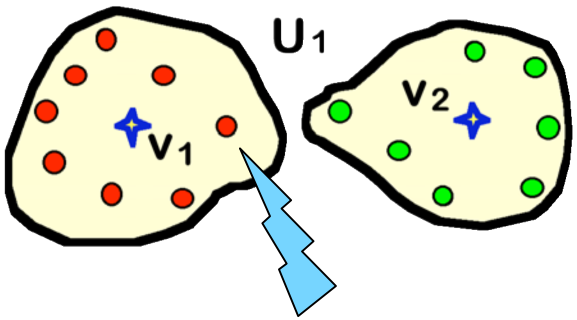
$$U = \begin{bmatrix} 1 & .07 & .6 & .5 \\ 0 & .81 & .2 & 0 \\ 0 & .12 & .7 & .5 \end{bmatrix} \in M_{pcn} \qquad H_1(U) = U_{MM} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \in M_{hcn}$$

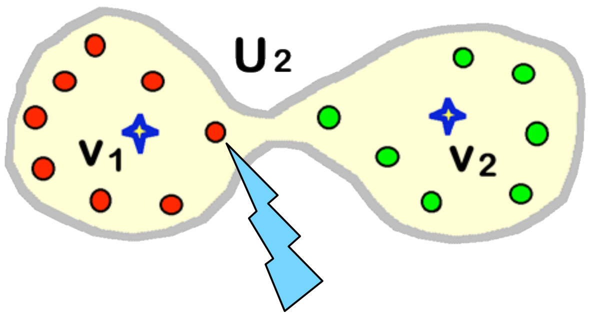
resolve ties randomly

α -cut thresholding (0< α <1) filters chosen levels of m'ship

$$H_{.9}(U) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \in M_{hcn} \qquad H_{.8}(U) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \in M_{hcn}$$

Clusters represented by partition matrix (U) *or* prototypes $V = \{v_k\}$





$$U_c = \begin{bmatrix} 1 & 1 & \ldots & 1 & 0 & 0 & \ldots 0 \\ 0 & 0 & \ldots & 0 & 1 & 1 & \ldots 1 \end{bmatrix}$$

$$U_f = \begin{bmatrix} .9 & 1 & \ldots & .6 & .3 & 0 & .2 & \ldots & 0 \\ .1 & 0 & \ldots & .4 & .7 & 1 & .8 & \ldots & 1 \end{bmatrix}$$

Crisp : hard boundaries

Fuzzy : soft boundaries

4 types of clustering models

U only models

V only models

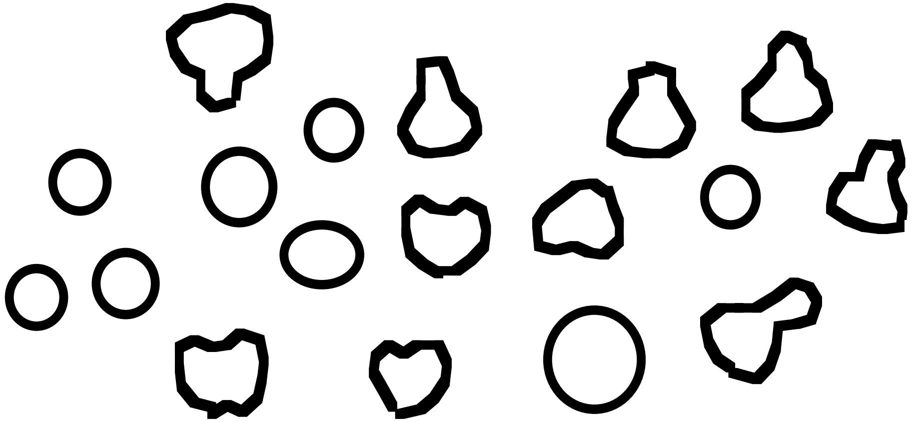
(U, V, +) models

(U, V) models

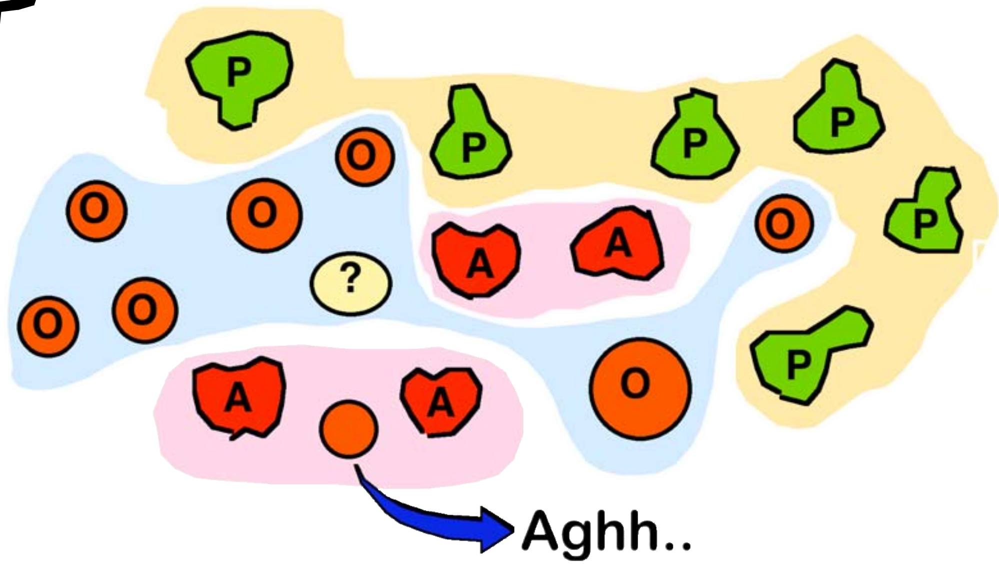
What is Classification ?

(Supervised Learning)

Find regions that contain similar objects

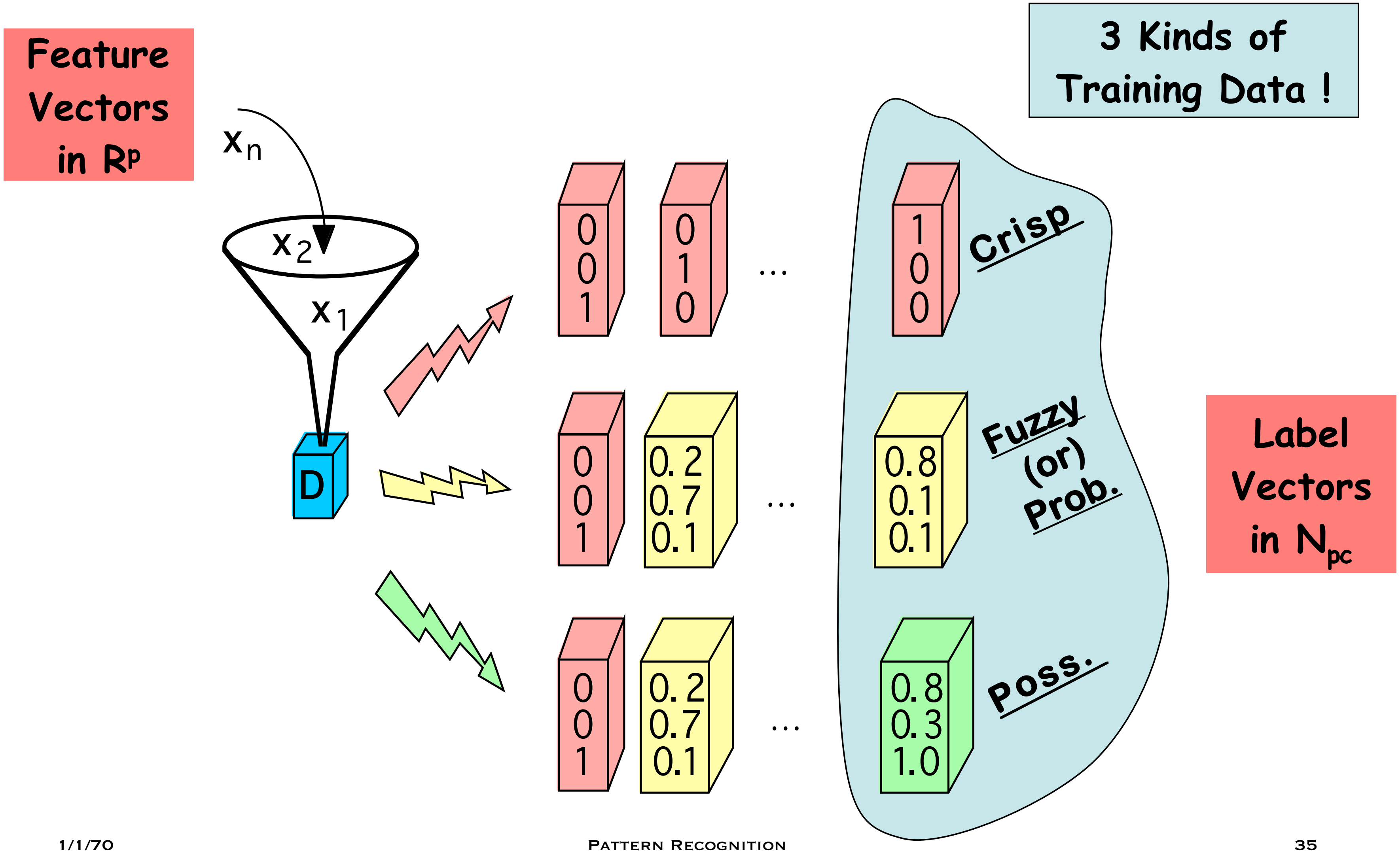


What are " (decision) regions"?



Classifier Functions

$D : \mathfrak{R}^p \mapsto N_{pc} = [0, 1]^c$



Classifier Training

1 Choose a family of models $\mathcal{F}$ = { D_1 (θ) … D_i (θ) … }

2 Training Data X_t = { $x_{t,1}$ … $x_{t,k}$ … $x_{t,n}$ }

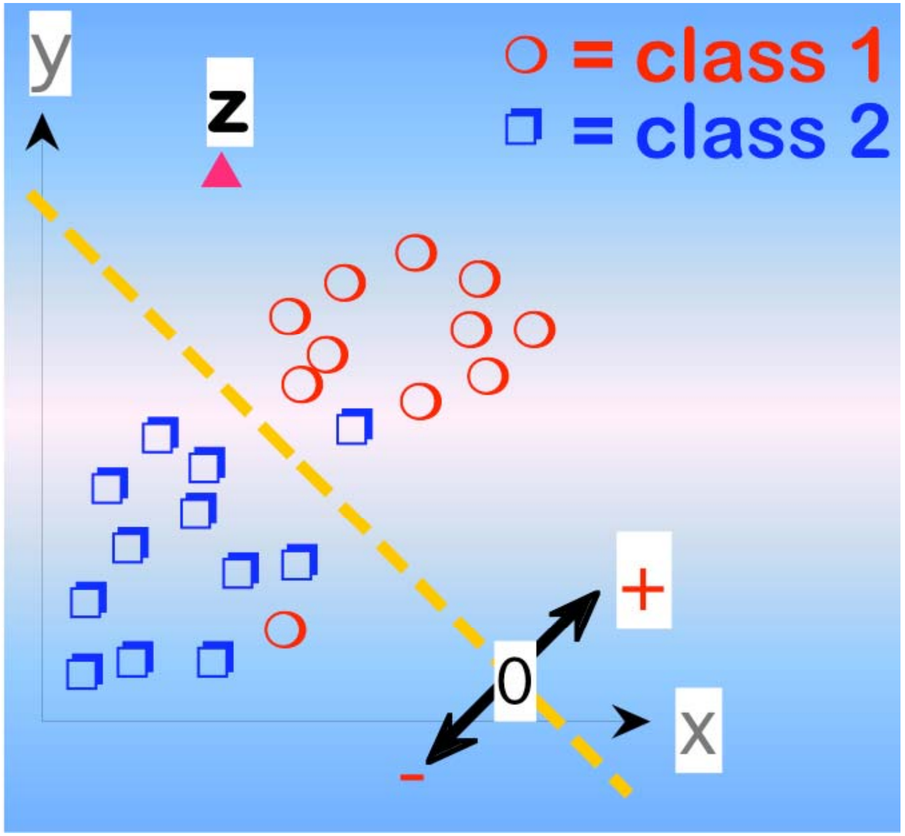
3 Training Labels U_t =

1	.4	.7
0	.6	.5

4 Algorithm A "looks for" an optimal (θ*) of D

5 Use (X, U_X, A) to find (θ*) of D ("learning")

$$\underbrace{d_{HP}(x) = \left\langle x, \vec{w} \right\rangle + \alpha}_{\text{(Linear) HP Model}} + \underbrace{X_t}_{\text{Data}} + \underbrace{U_t}_{\text{Labels}} + \underbrace{\text{Hyperplane}}_{\text{Algorithm}}$$



$$\theta = \left[\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \alpha \right]$$

$$\underbrace{d_{HP}(x) = x + y - 1}_{\text{(e.g. SVM) Classifier}}$$

$$\theta^* = \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, -1 \right]$$

D^*_{HP} *crisply* labels *every* point in R^p

Classifier Function

$$D^*_{HP}(x) = \begin{cases} e_1 = (1 \quad 0)^T & ; d_{HP}(x) > \alpha^* \quad \Rightarrow x \in 1 \\ e_1 \text{ or } e_2 & ; d_{HP}(x) = \alpha^* \quad \Rightarrow \text{Tie} \\ e_2 = (0 \quad 1)^T & ; d_{HP}(x) < \alpha^* \quad \Rightarrow x \in 2 \end{cases}$$

Decision (or Discriminant) function

Mistakes can be costly : a doctor treats patient x with A or B

Use $(A|x \in 1) \Rightarrow$ cures x

Use $(B|x \in 1) \Rightarrow$ no cure, no harm

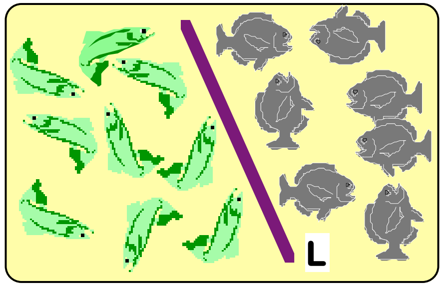
Use $(B|x \in 2) \Rightarrow$ cures x

Use $(A|x \in 2) \Rightarrow$  - a cure?

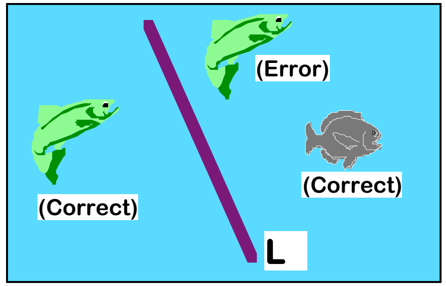
Patient x *wants* D^*_{HP} to be *soft*

$$D^*_{HP}(x) = \begin{cases} \begin{pmatrix} 0.95 \\ 0.05 \end{pmatrix} \Rightarrow \begin{pmatrix} \text{use} \\ A \end{pmatrix} \\ \begin{pmatrix} 0.53 \\ 0.47 \end{pmatrix} \Rightarrow \begin{pmatrix} \text{more} \\ \text{tests} \end{pmatrix} \end{cases}$$

Non-Adaptive
Off Line Training



Non-Adaptive
On Line Operation



3 Kinds of Training and Operation

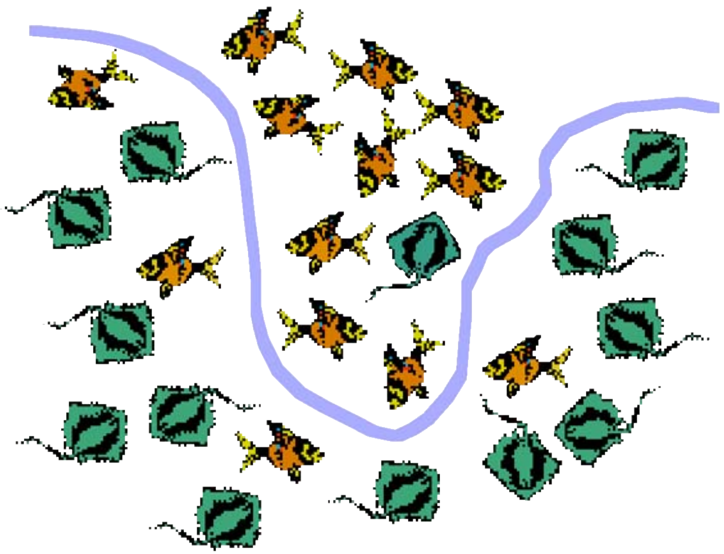
3 Data Sets needed to Train and Test a Classifier

Training $X_{tr} = \{x_{tr,1} \dots x_{tr,n}\}$ with labels $u_{tr} \in N_{pc}$

Validation $X_v = \{x_{va,1} \dots x_{va,s}\}$ with labels $u_{va} \in N_{hc}$

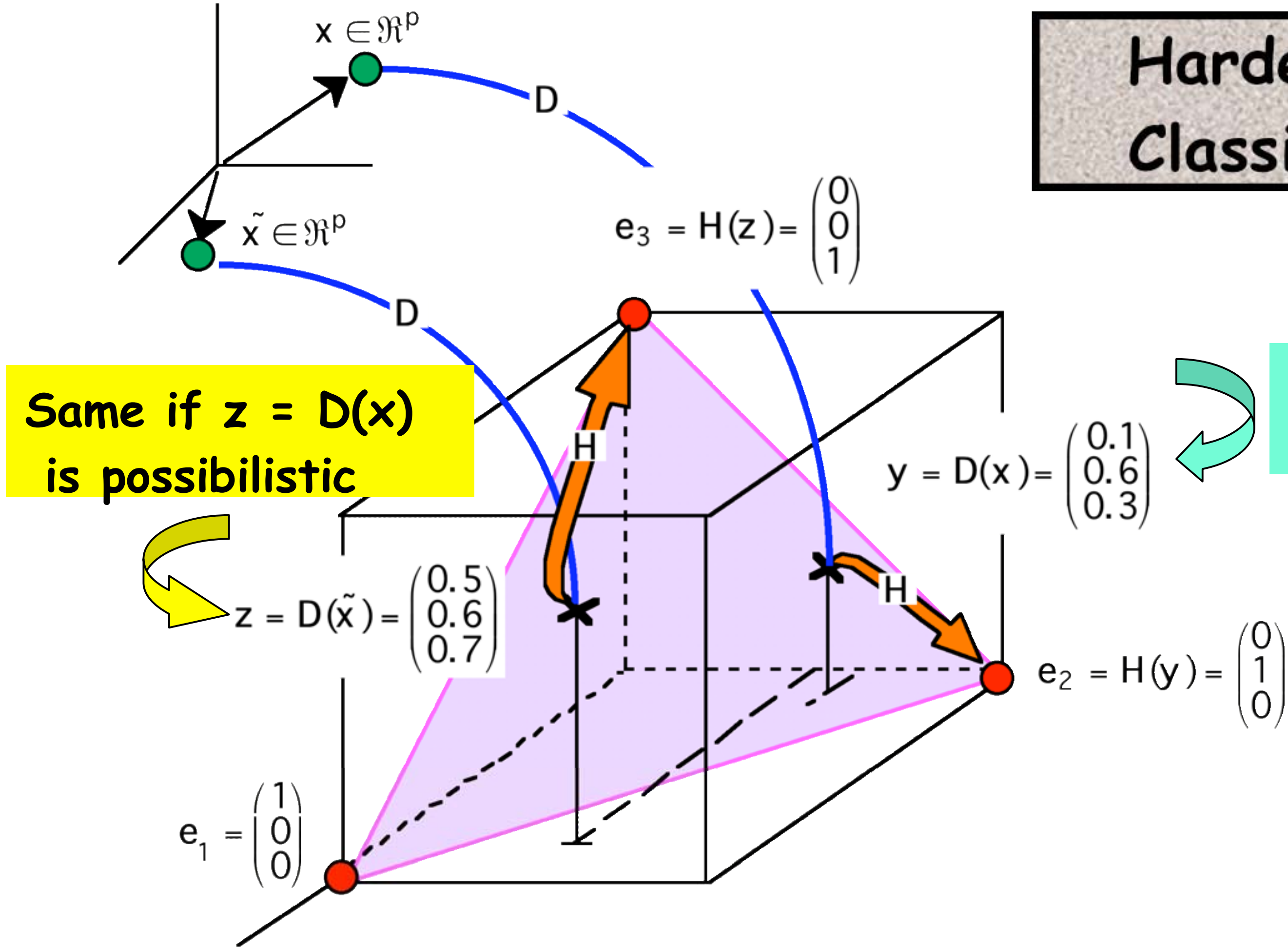
Test $X_t = \{x_{te,1} \dots x_{te,q}\}$ with labels $u_{te} \in N_{hc}$

Data can be uncooperative !



- Non-Linear Clusters
- Noise & Outliers
- Mixtures of Points
- High Dimensional
- Wrong Features

Hardening SOFT Classifier Labels



y = D(x) = fuzzy
(or probabilistic)

$y \in i$
$\Updownarrow$
$y_i > y_k$

(or)

$$H(y) = e_i \Leftrightarrow \left\| y - e_i \right\| = \min_k \left\{ \left\| y - e_k \right\| \right\}$$

Error rate estimation

$D(X)$ = classifier learned by applying algorithm A to X

$U_{D(X)}$ = (possibly) soft labels given to X by classifier D

$H(U_{D(X)})$ = crisp labels given to X by hardening $U_{D(X)}$

U_X = (known) crisp labels of data set X (X_{va} or X_{te})

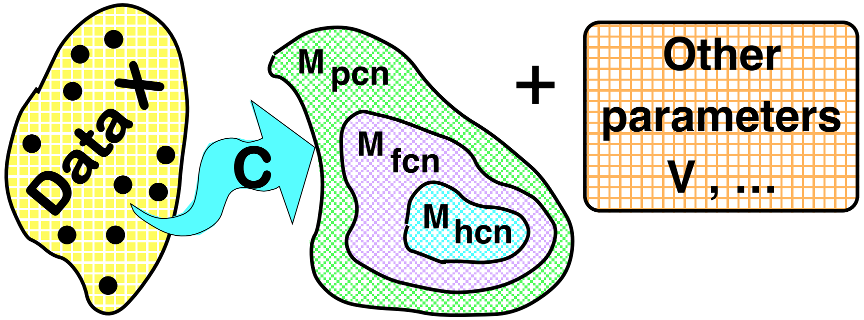
Empirical Error Rate (generalization error)

$$E_D(X) = \left(\frac{\#\ \text{wrong}}{\#\ \text{tried}} \right) = \frac{\left\| U_X - H(U_{D(X)}) \right\|}{2\,|X|}$$

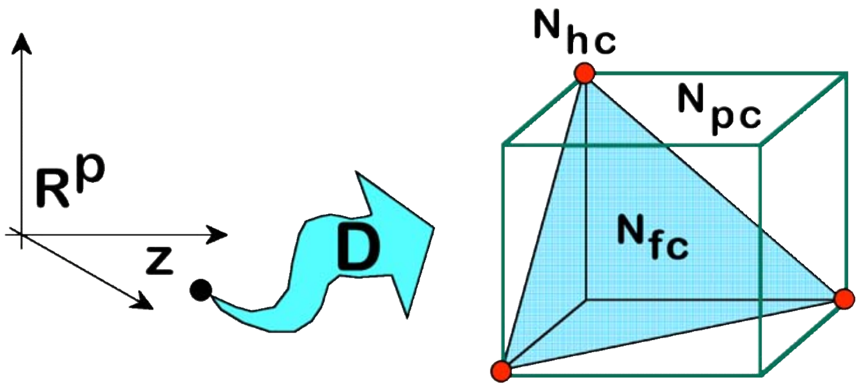
Minimize E_D iteratively by learning with X_{tr} and then computing $E_D(X_{va})$

Estimate E_D ONCE by computing $E_D(X_{te})$

Clustering vs Classification : Summary



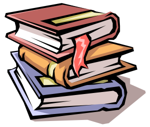
Clustering labels *only* the submitted data



Classifiers can label *all* the data in their domain space

¡ Building a PR system - the *big*

Process Description	Feature Analysis	Cluster Analysis	Classifier Design
Model Type	**Operation**	**Criterion**	**Model Basis**
Numerical Rule-Based ⋮ Syntactic	Extraction Filtering ⋮ 2D-Displays	Dissimilarity Connectivity ⋮ Frequency	Deterministic Fuzzy ⋮ Statistical
Data Needed	**Model + Algorithm**	**Model + Algorithm**	**Model + Algorithm**
Type Structure Nomination Collection ⋮ Sensors	Projection PCA/LDA Sammon SOMs ⋮ VAT-coVAT	c Means SAHN Entropy GMD ⋮ ART NN	FFBP NN 1-np rules K-nn rules SVMs ⋮ Bayes Quad.



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