Qualitative Relational Mapping

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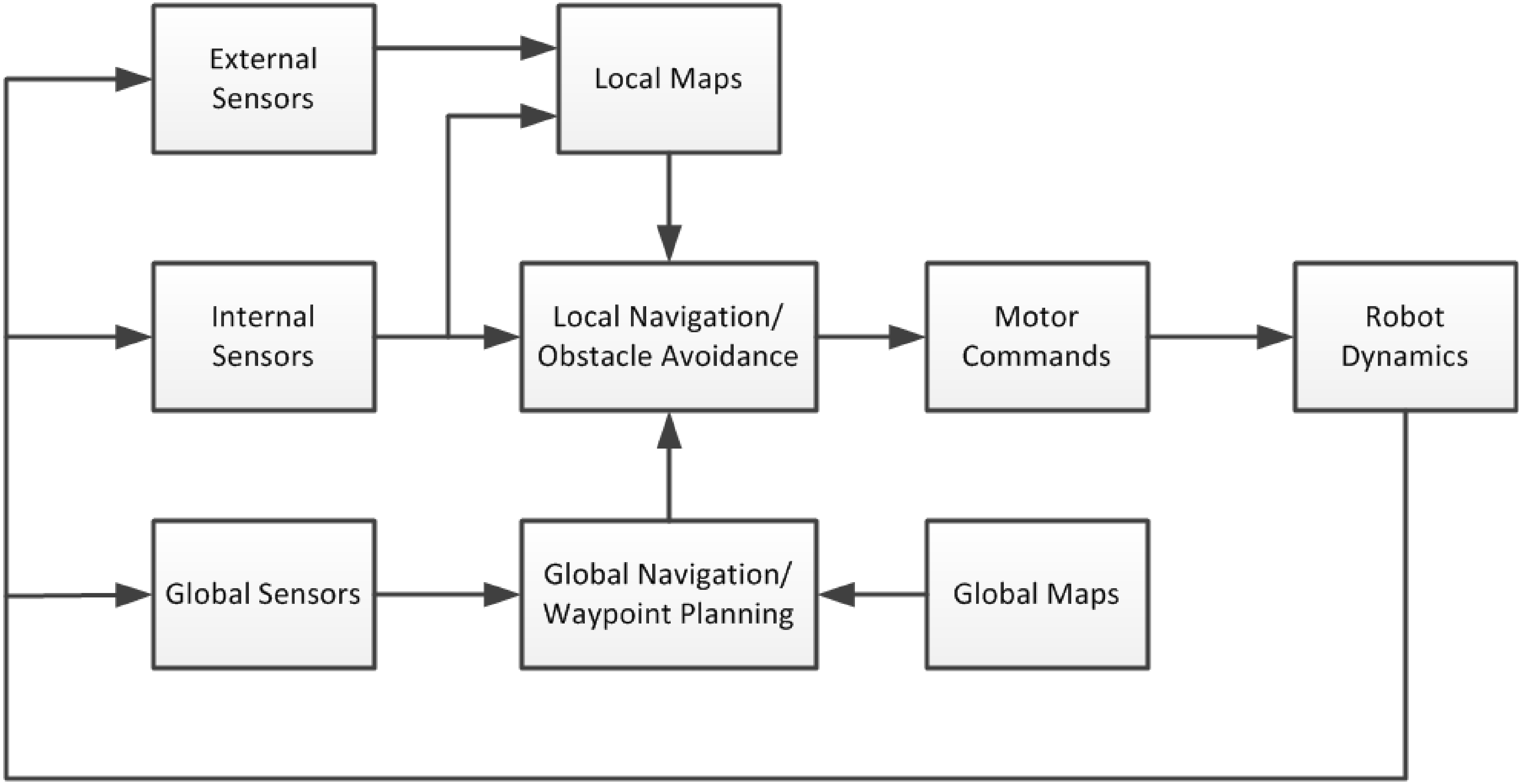
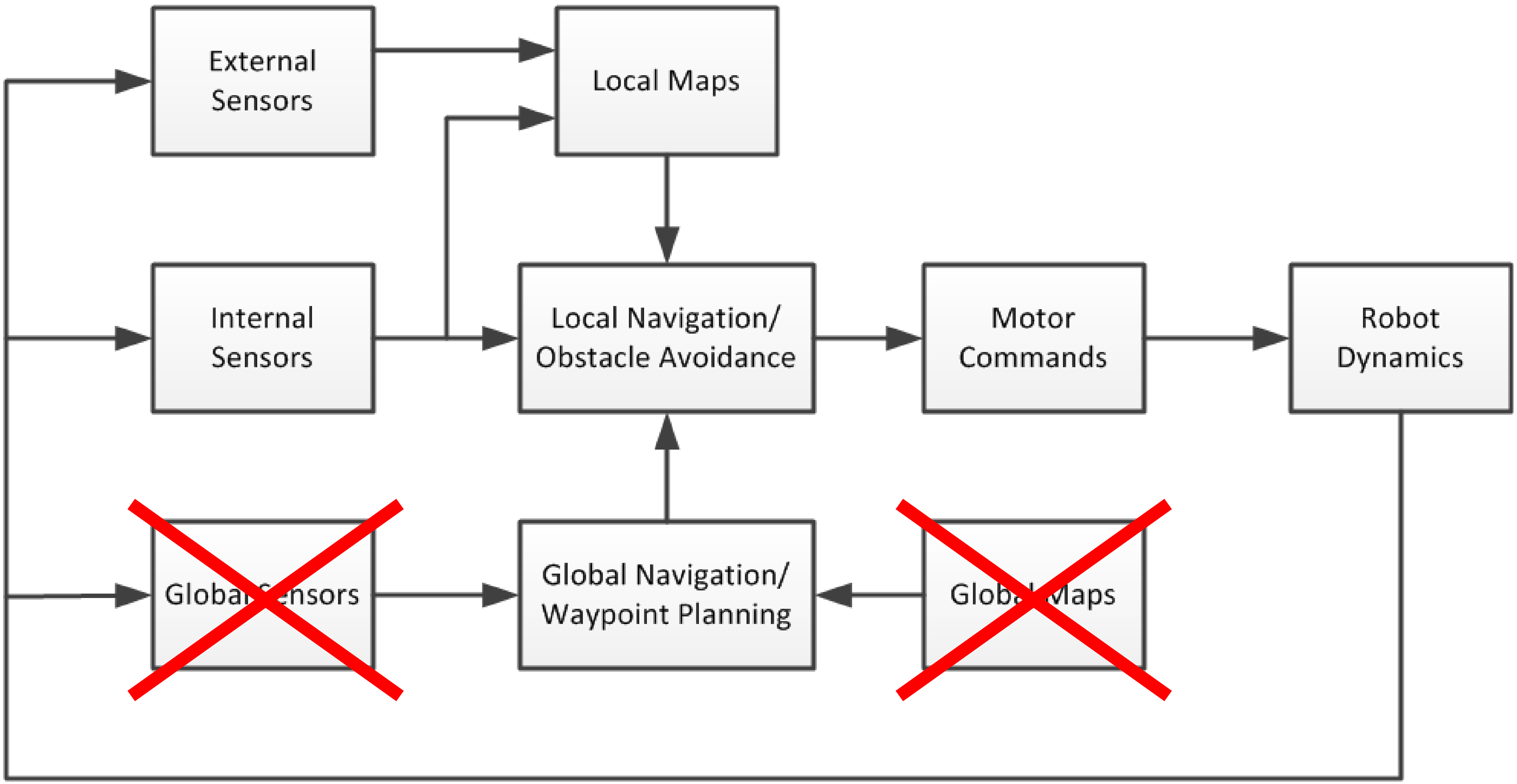
Motivation and Problem Statement

- How can we enable long-term autonomy for a robot operating in an unstructured, large scale space without a known global reference frame?
  - Required for exploration of outer planets and moons as time delay is too long for remote control
    - Complex coordination of multiple vehicles
    - Dynamic environments
    - Vehicle lifetimes may be short
  - Possible terrestrial applications when GNSS is unavailable: underwater, in urban disaster areas, etc
  - Martian exploration acts as a motivating problem as we know the challenges of operating semi-autonomous robots there

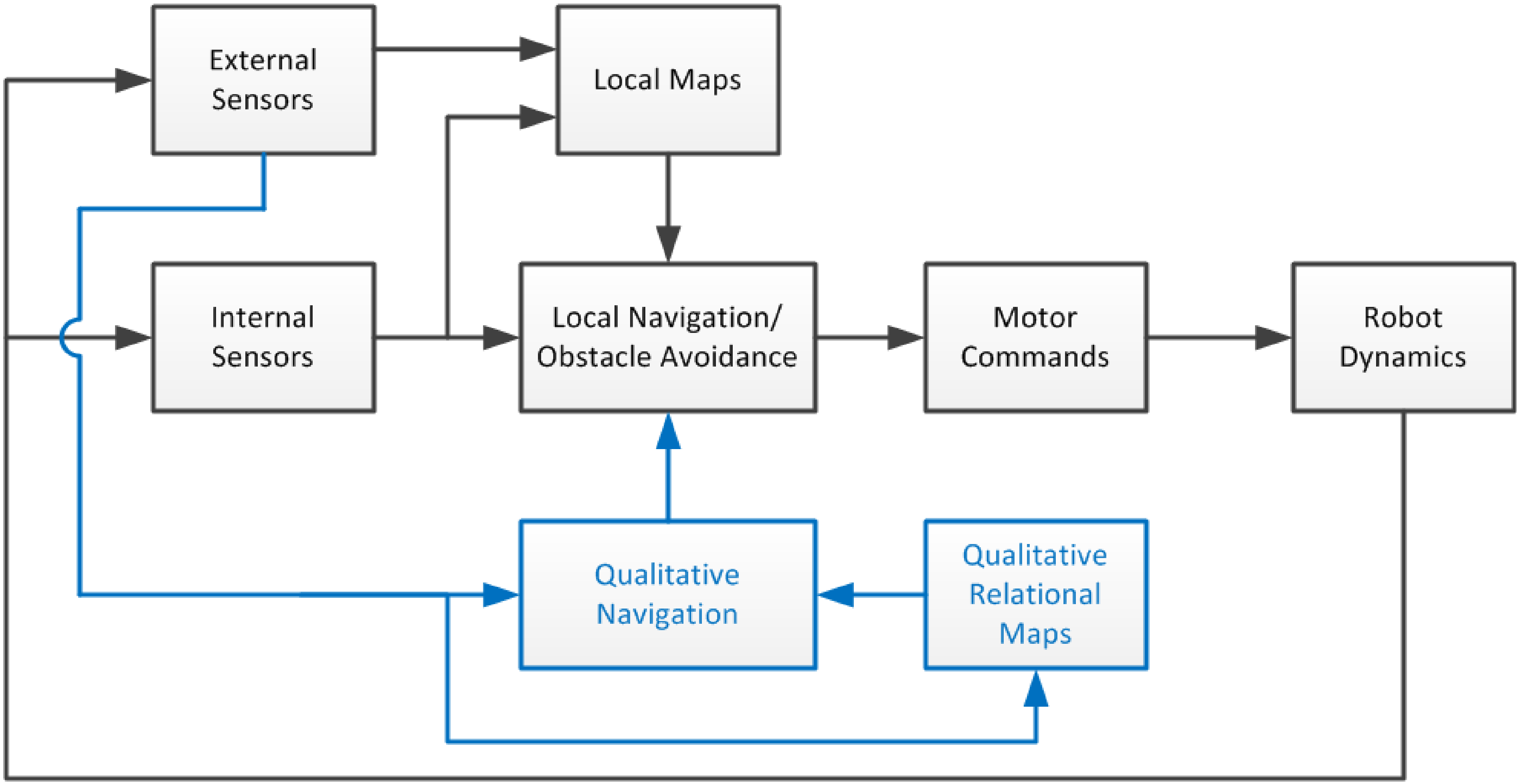
Common Components of Robotic Navigation



Common Components of Robotic Navigation



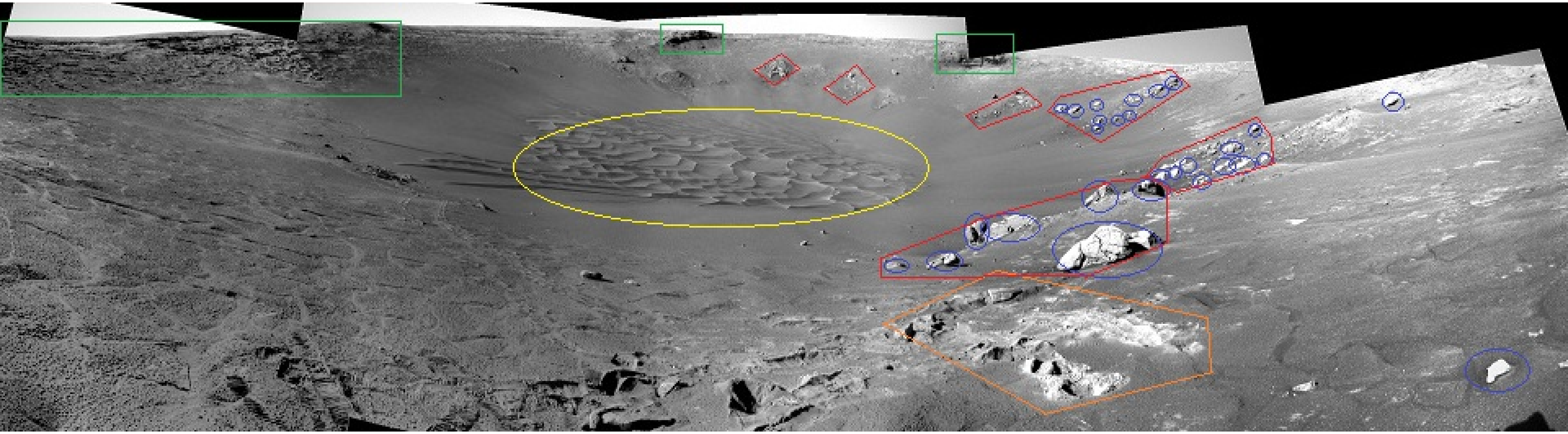
Common Components of Robotic Navigation



Qualitative Relational Mapping

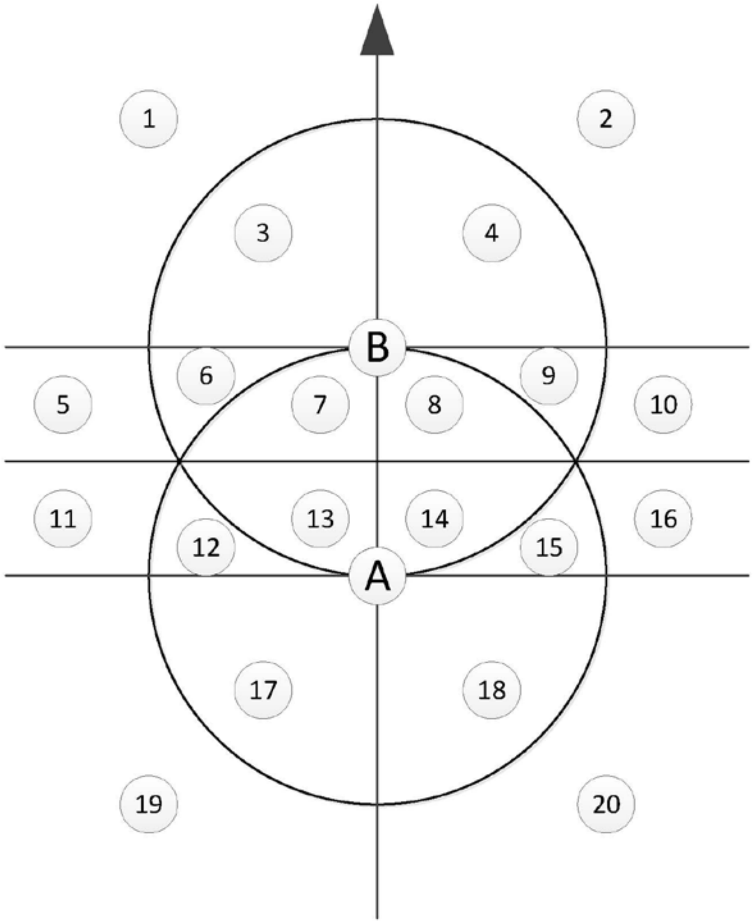
- Extract visually distinctive landmarks from camera images
- Represent landmark locations using discrete qualitative statements
- Maintain relative position and orientation of landmarks rather than global positions

210⁰ Panorama From Opportunity on Sol 270



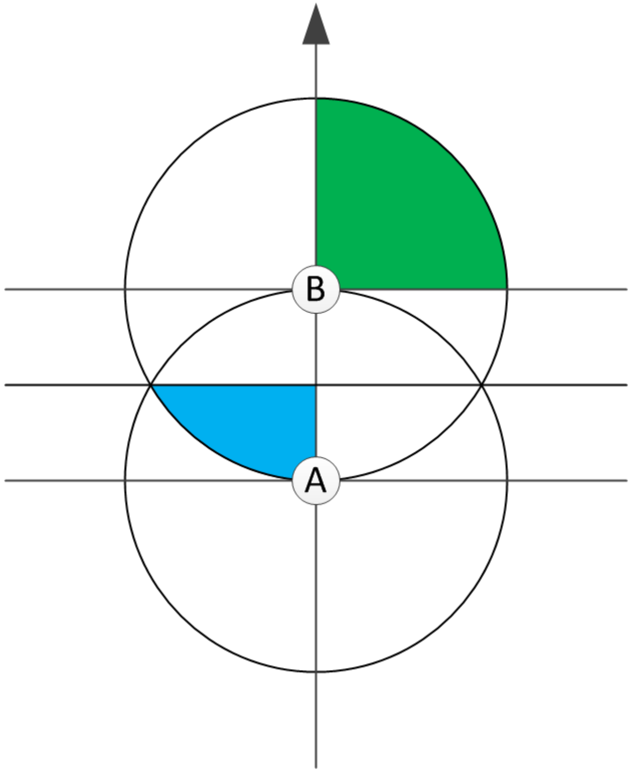
Qualitative States: The Extended Double Cross

- The position of a landmark can be specified qualitatively in relation to other landmarks.
  - Define the triple **AB:C** to be the relation of point C with respect to the vector from A to B
  - Split space around AB using qualitative statements
    - Left/Right of AB
    - Front/Back of A
    - Front/Back of B
    - Closer to A/Closer to B
    - Closer/Further to A than |AB|
    - Closer/Further to B than |AB|



Qualitative State Permutation Operators

- Given relationship **AB:C**, we would like to reason about different views of the same landmark triple
  - The inverse **BA:C**



AB:C

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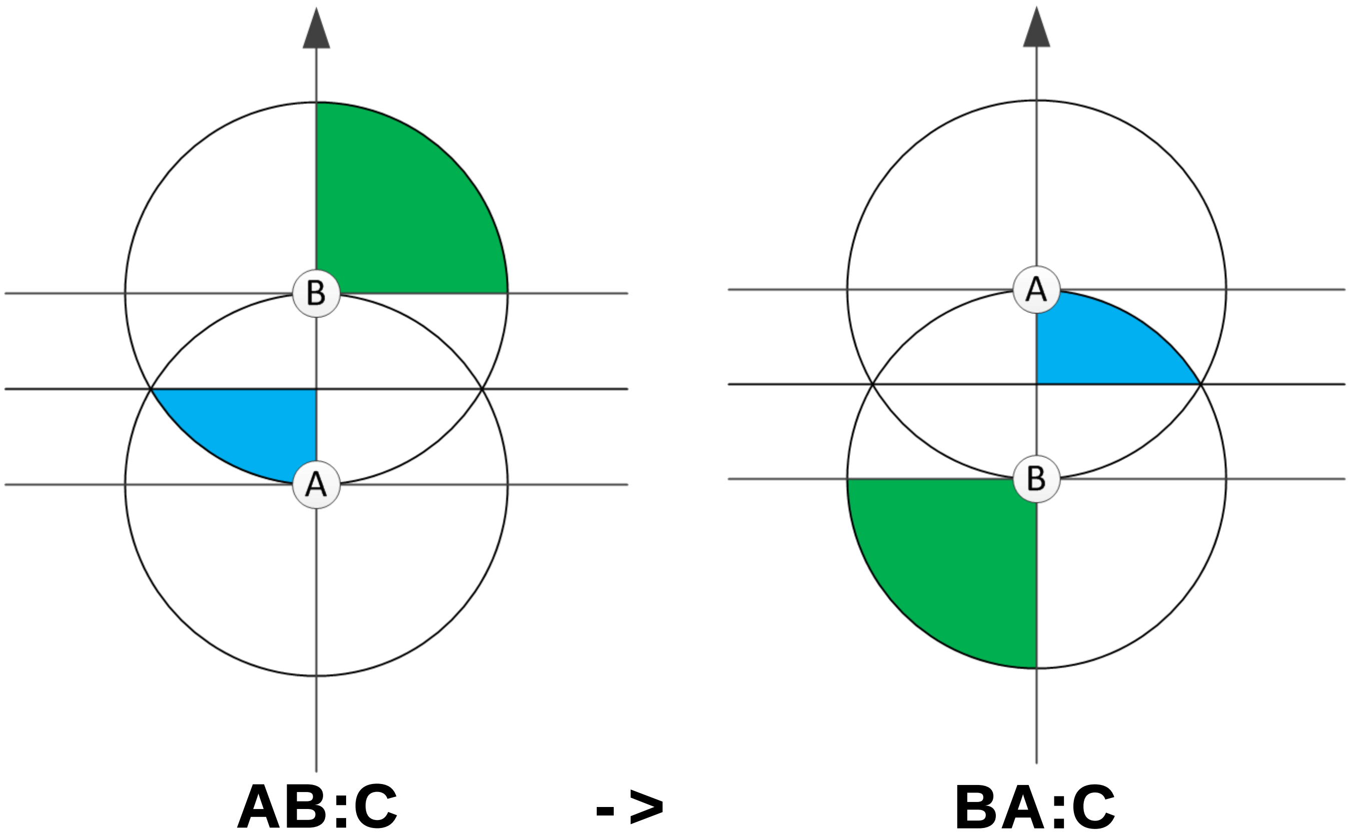
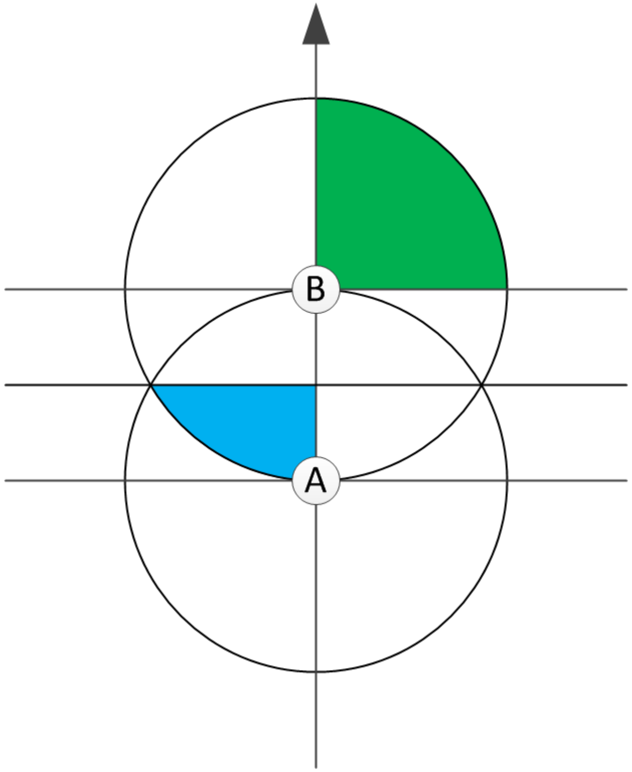


Qualitative State Permutation Operators

- Given relationship **AB:C**, we would like to reason about different views of the same landmark triple
  - The left-shifted permutation **BC:A**



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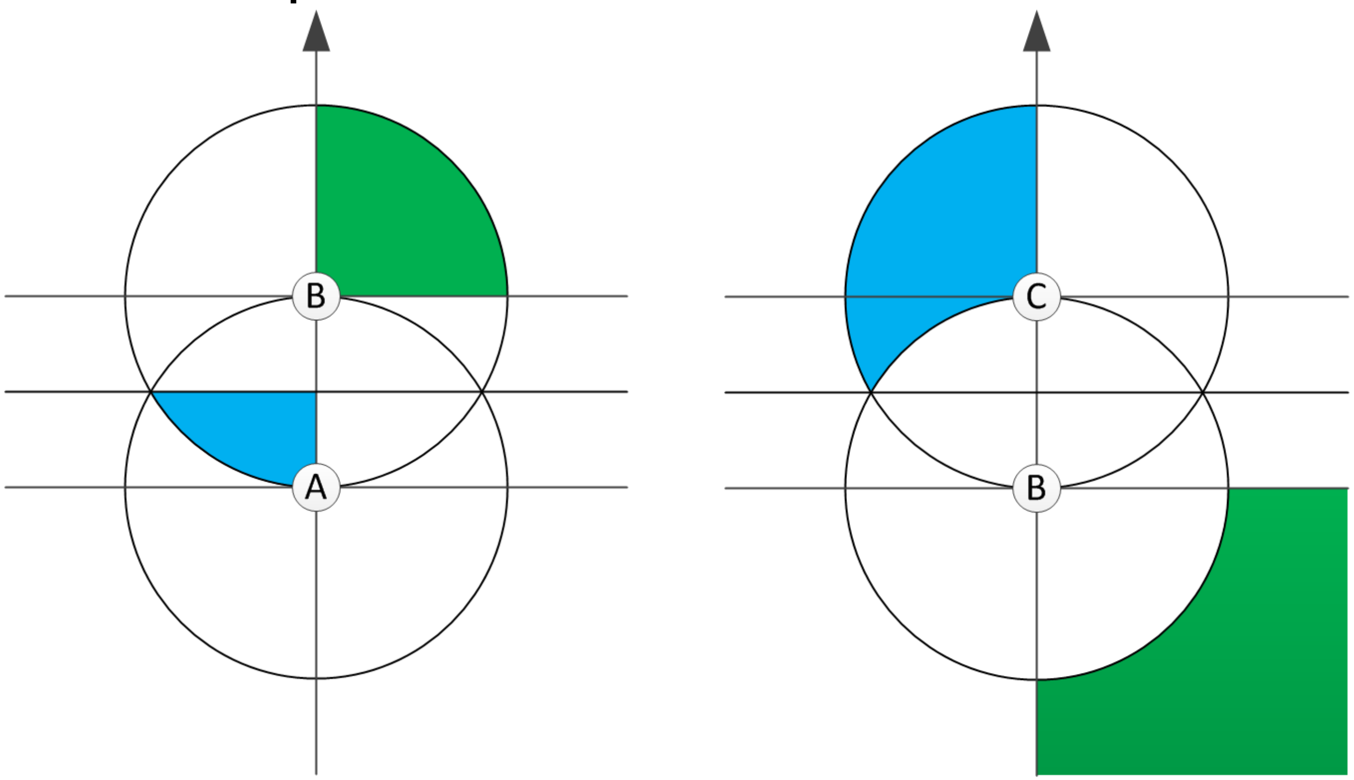
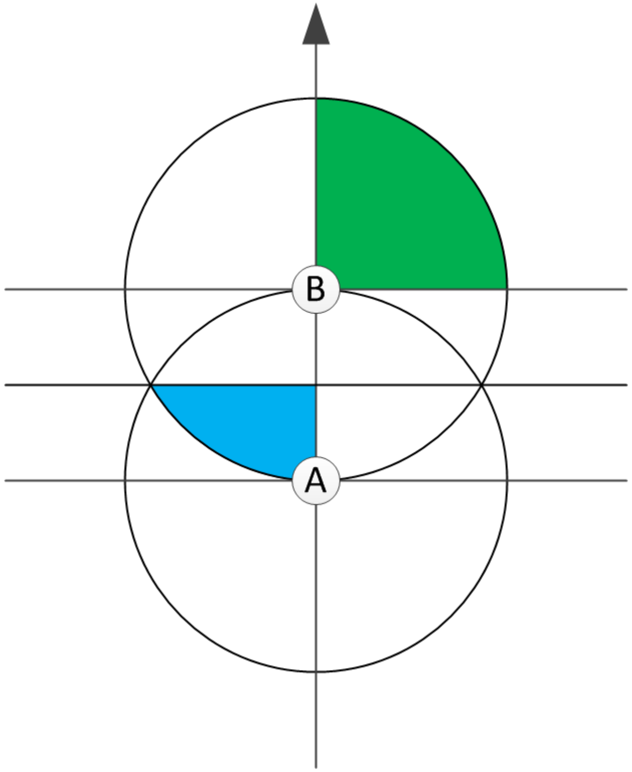


AB:C -> **BC:A**

Qualitative State Permutation Operators

- Given relationship **AB:C**, we would like to reason about different views of the same landmark triple
  - The right-shifted permutation **CA:B**



AB:C

Qualitative State Permutation Operators

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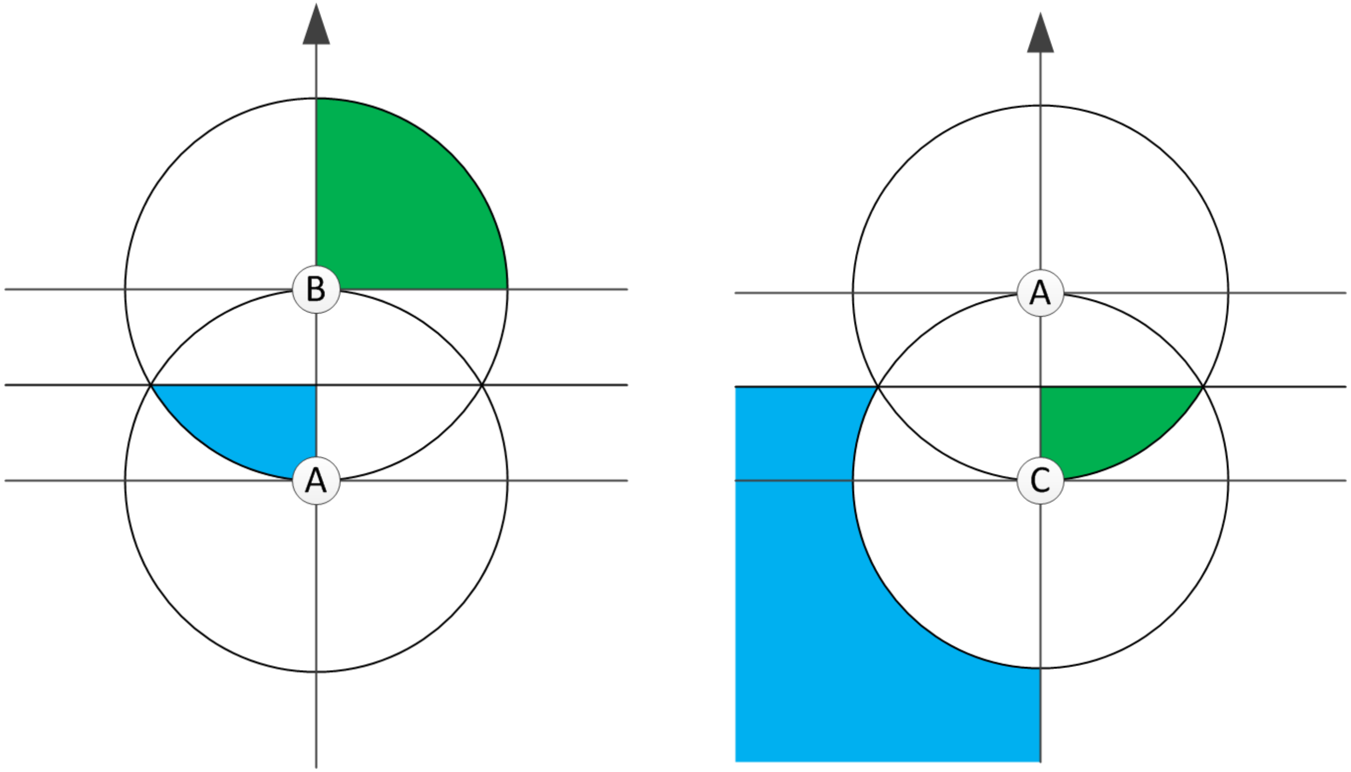
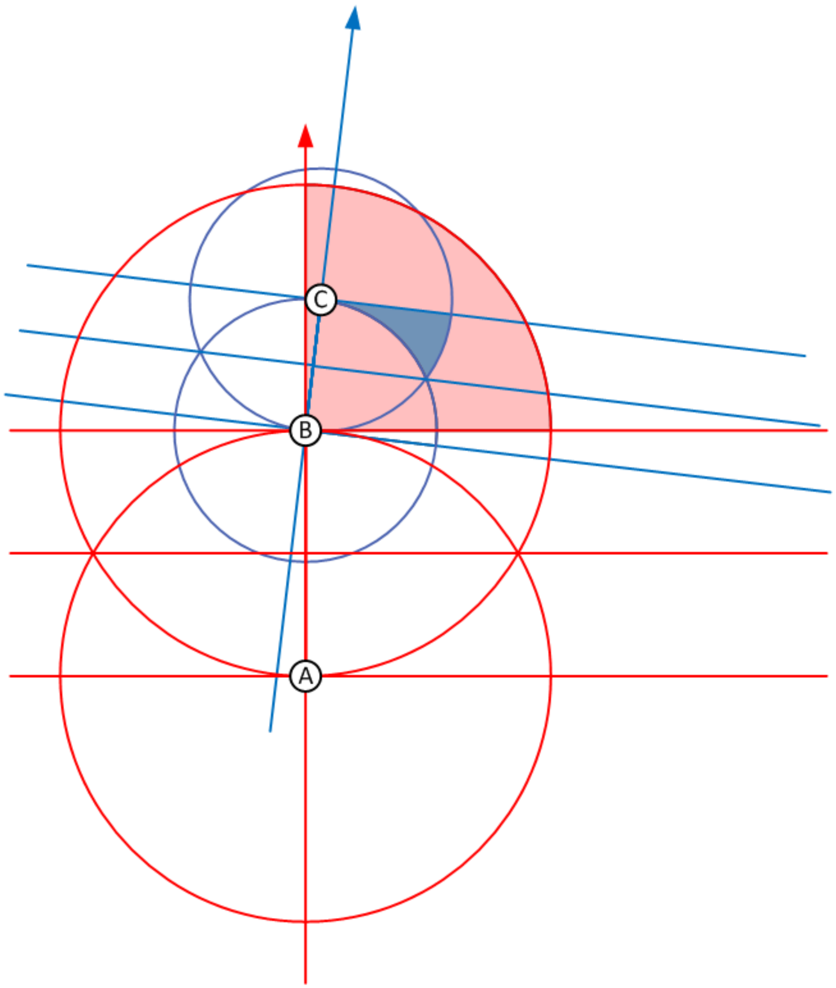


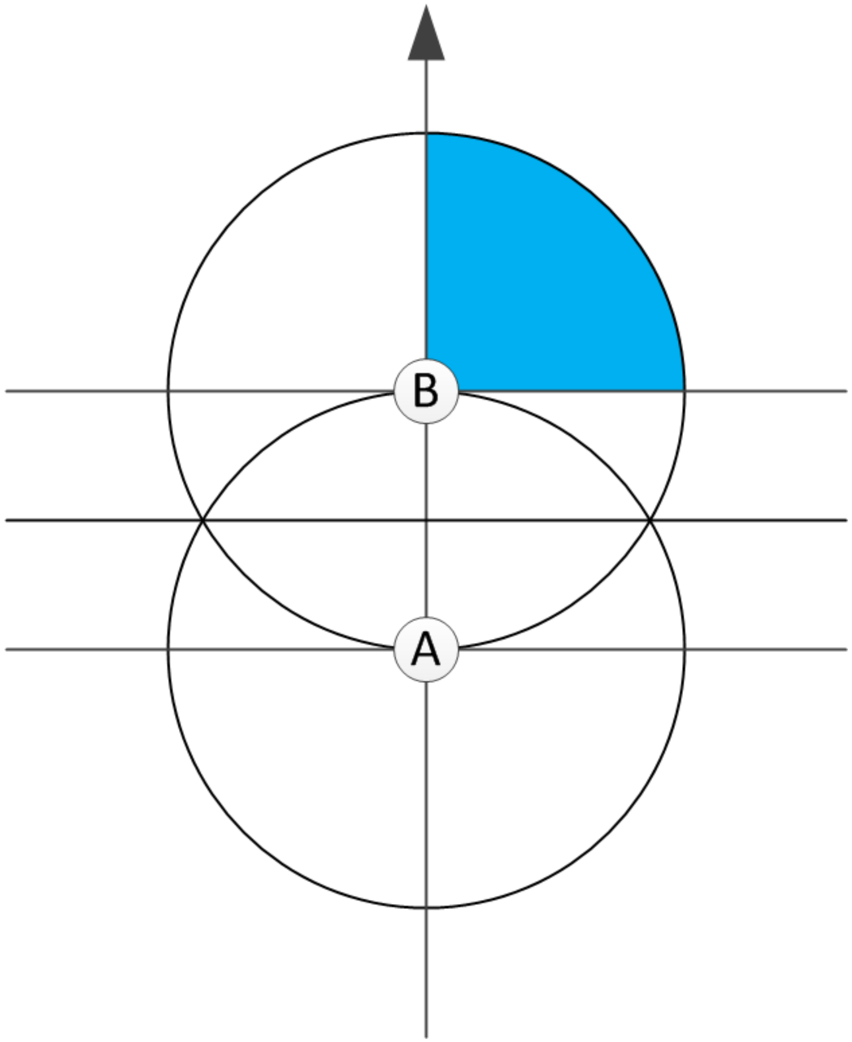
AB:C **->** **CA:B**

Qualitative Inference via Composition

- The Problem: What can we infer about landmark combinations we have not directly observed?
  - Constrain states of landmark triples never jointly observed
  - Update old observations with new constraints
- Solution: The composition operator
  - Given a state for **AB:C** and **BC:D**, we can determine a set of potential states for **AB:D**
  - Build a truth table for every possible combination of states
  - During operation, compositions are just table lookups

Geometrical Interpretation of Compositions





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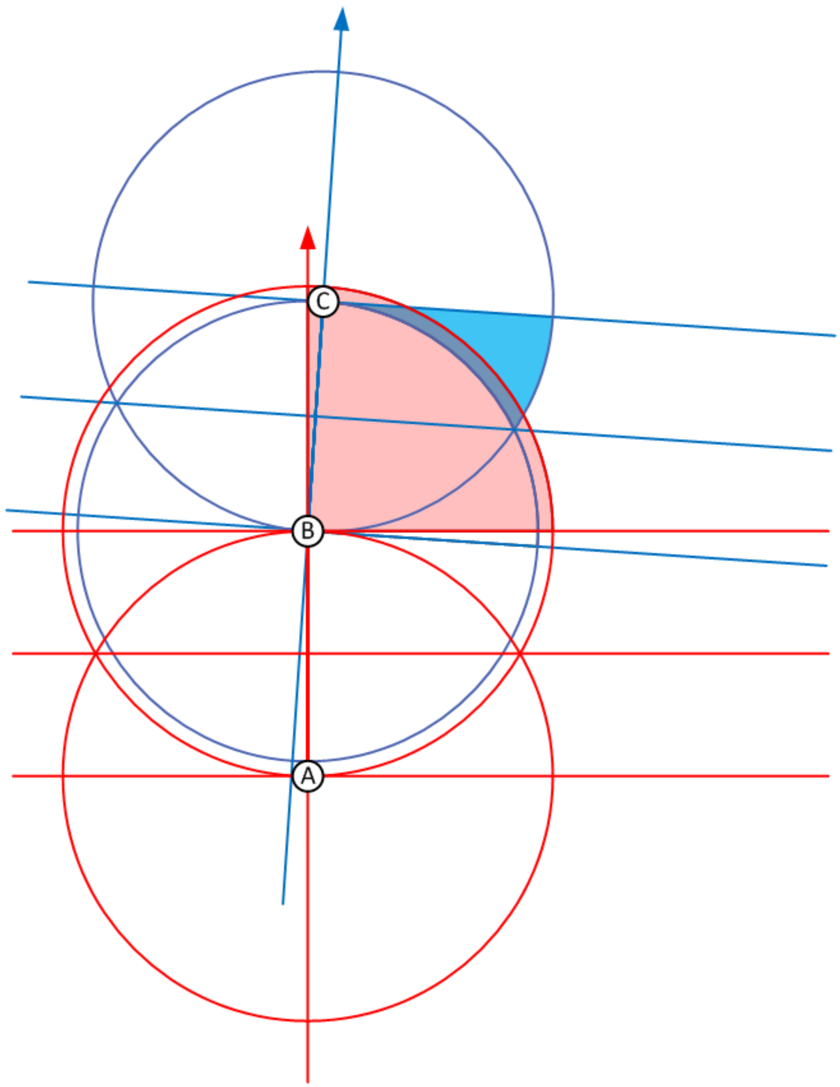

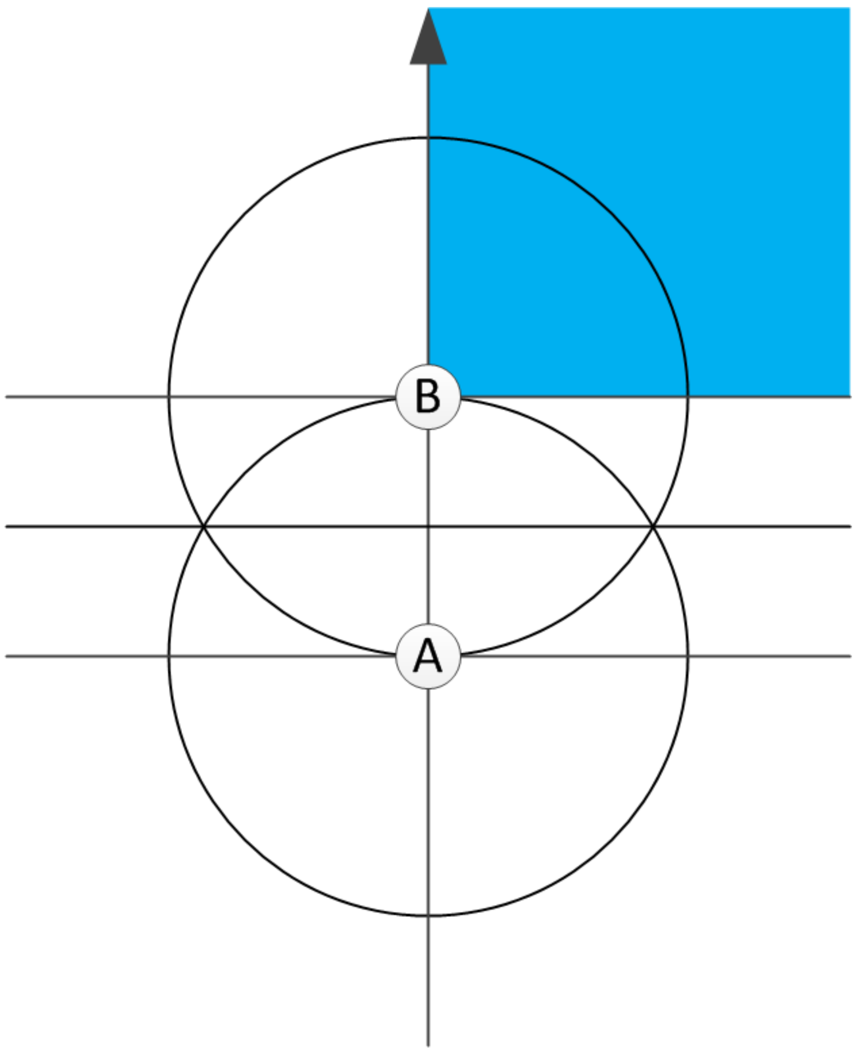

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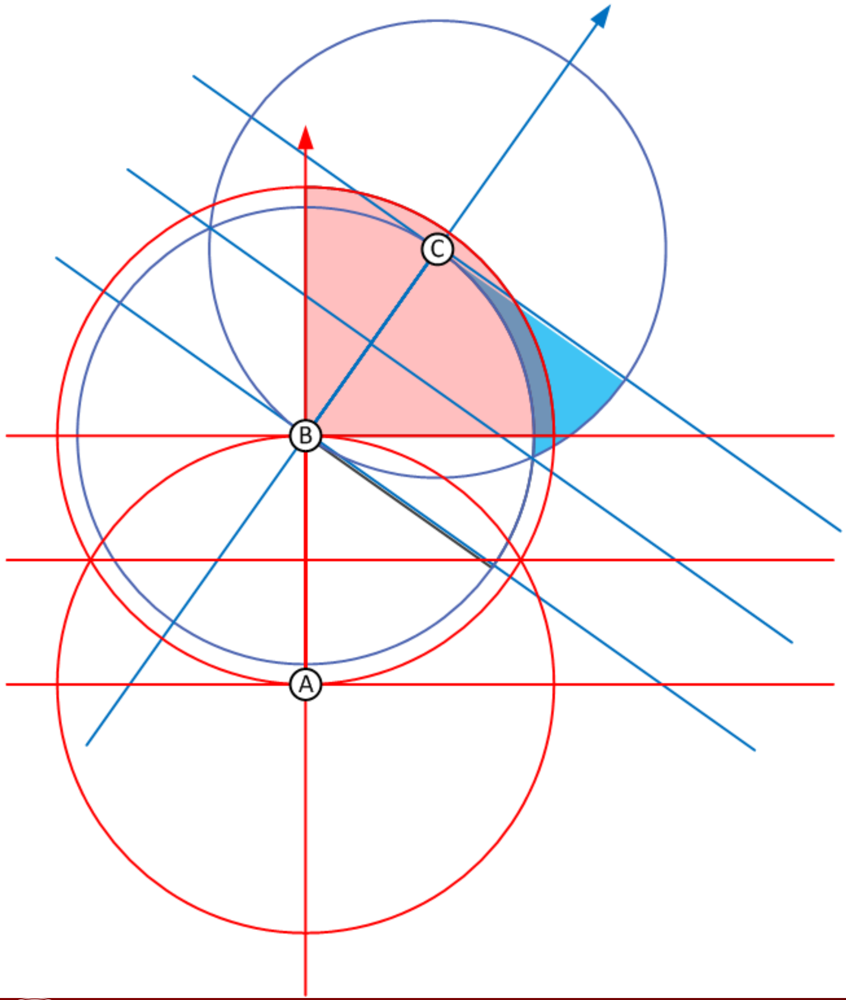

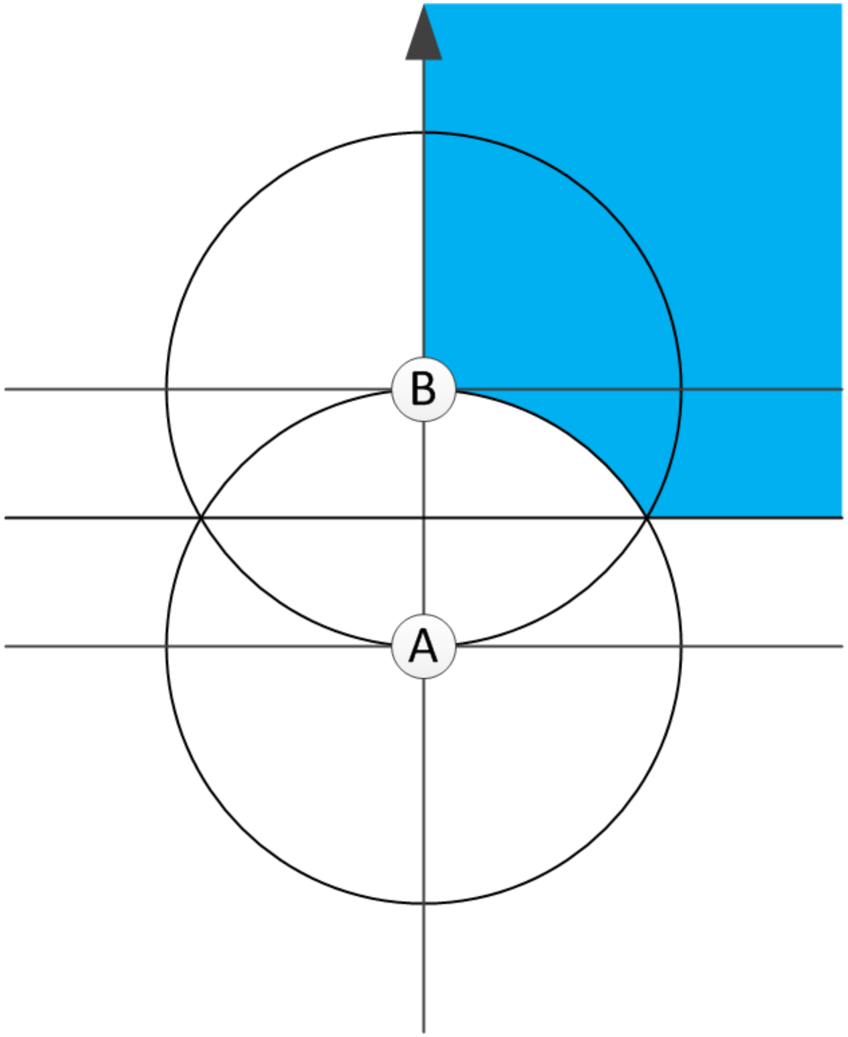

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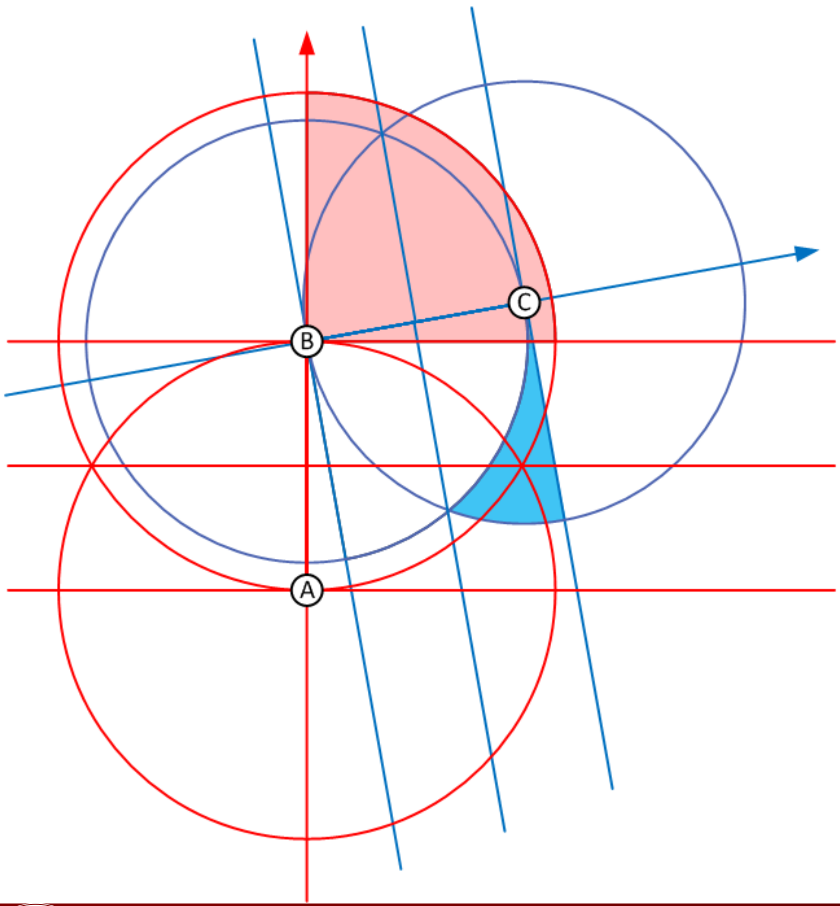

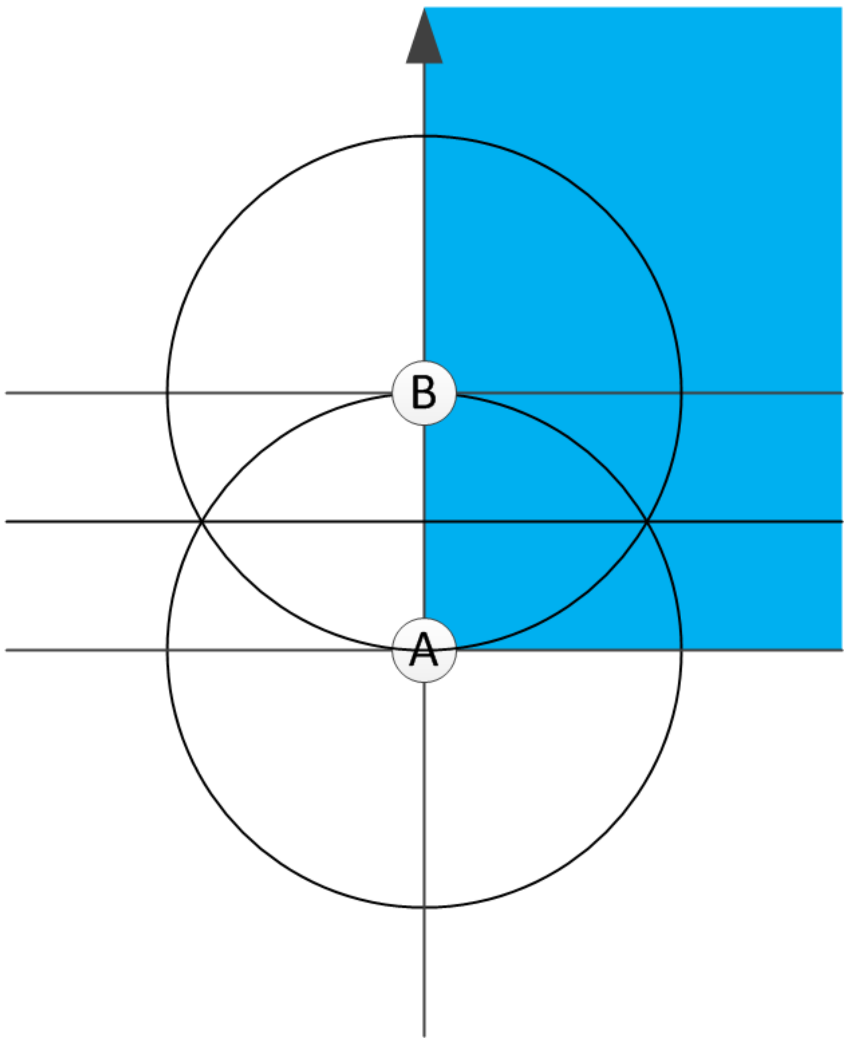

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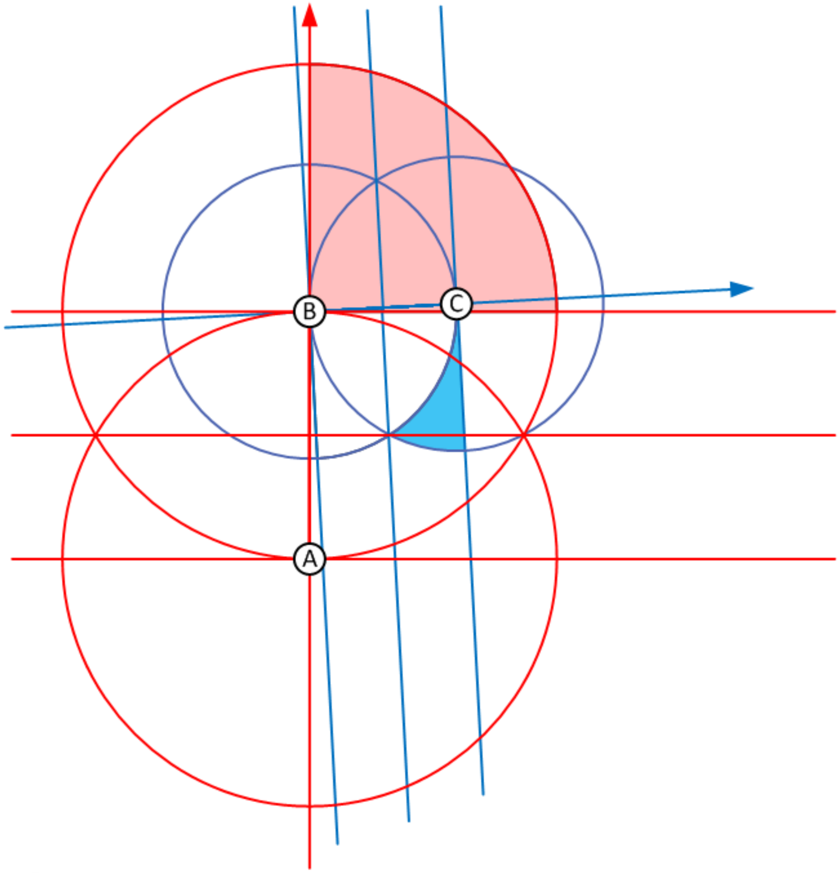
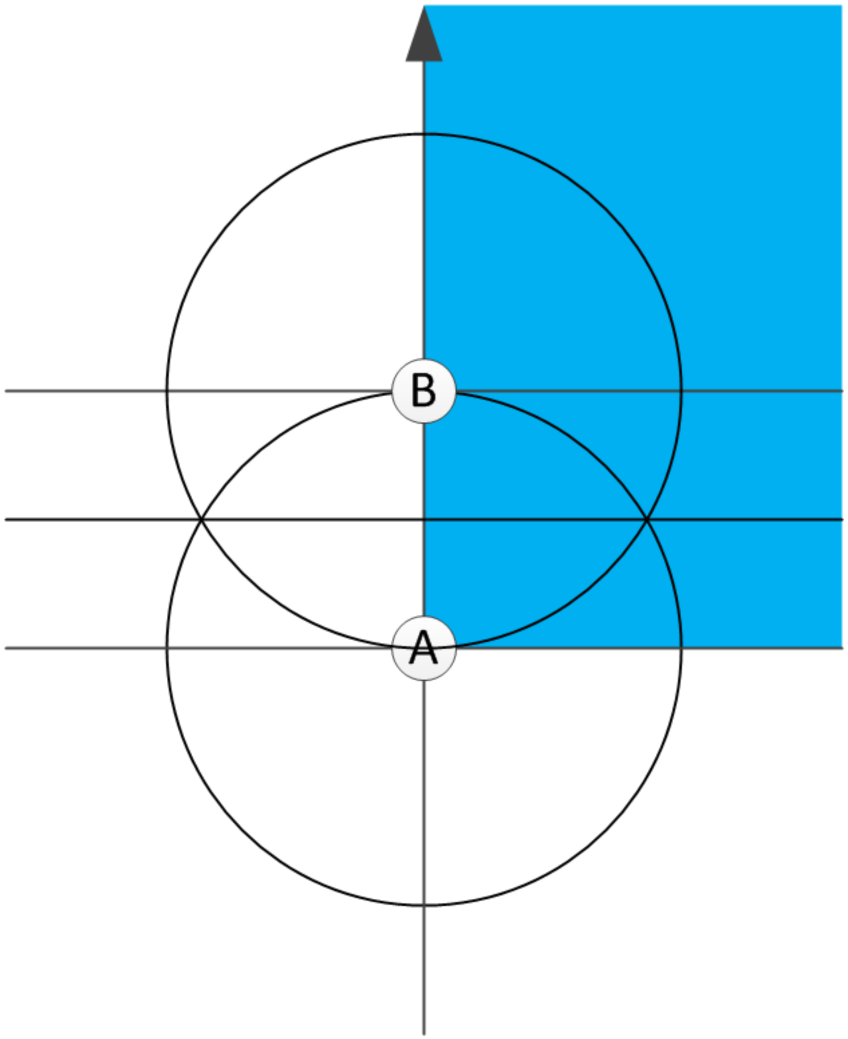




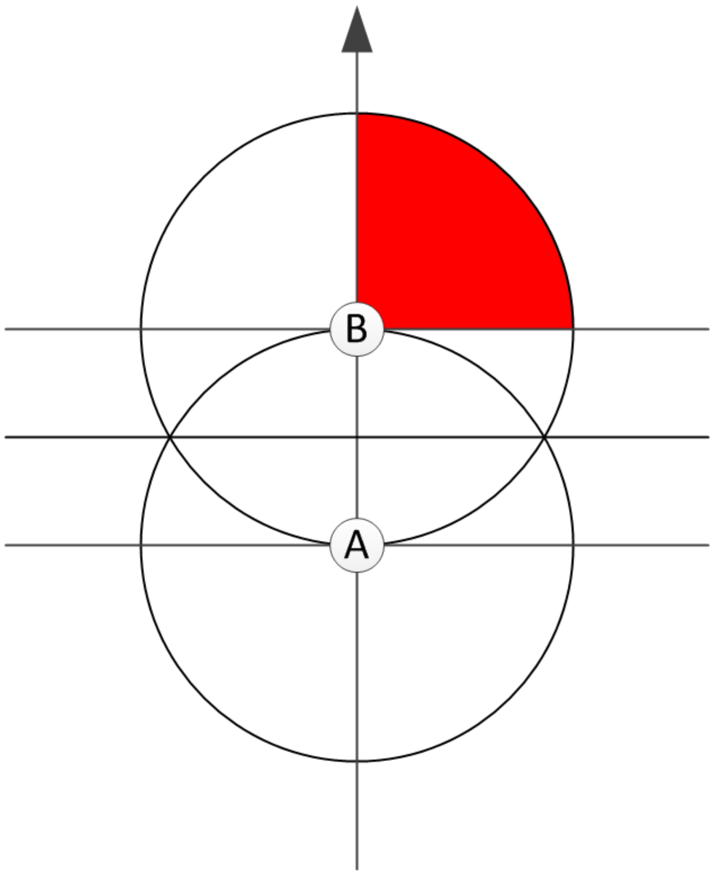
Algebraic Interpretation of Compositions

- A=(0,0)
- B=(1,0),
- C=(α, β)
- D=(γ, δ)
- **AB:C**=4 is then equivalent to the constraints

$$0 < \alpha$$

$$0 > 1 - \beta$$

$$0 < 2\beta - (\alpha^2 + \beta^2)$$



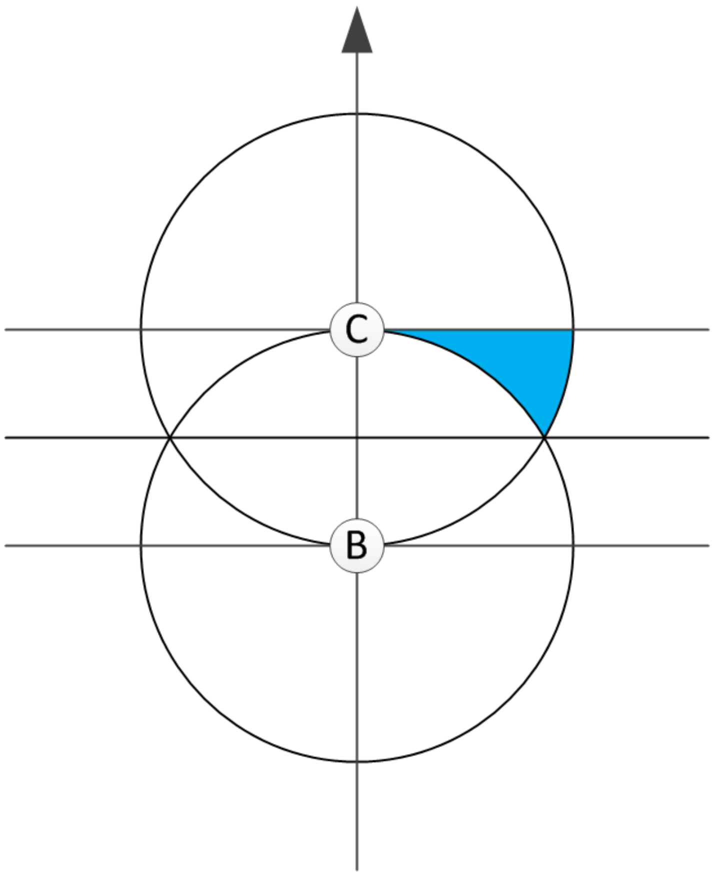
Algebraic Interpretation of Compositions

- A=(0,0)
- B=(1,0)
- C=(α, β)
- D=(γ, δ)
- **BC:D=9** is then equivalent to the constraints

$$0 < (\alpha^2 + \beta^2 + \delta) - (\beta\delta + \alpha\gamma + \beta)$$

$$0 > (\alpha^2 + \beta^2 + 2\delta) - (\gamma^2 + \delta^2 + 2\beta)$$

$$0 < (2\alpha\gamma + 2\beta\delta + 1) - (\gamma^2 + \delta^2 + 2\beta)$$



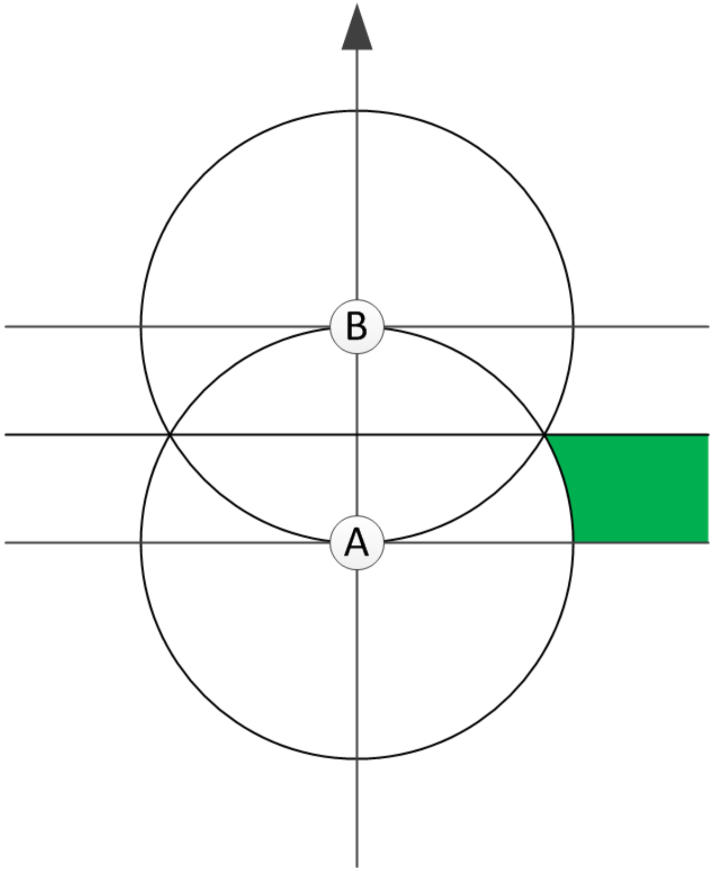
Algebraic Interpretation of Compositions

- A=(0,0)
- B=(1,0)
- C=(α, β)
- D=(γ, δ)
- **AB:D**=16 is then equivalent to the constraints

$$0 < \delta$$

$$0 < 1 - 2\delta$$

$$0 > 1 - (\gamma^2 + \delta^2)$$



Algebraic Interpretation of Compositions

- So the table entry for {AB:C=4, BC:D=9, AB:D=16} is true if there is some point (α, β, γ, δ) satisfying the system of nonlinear inequalities
- This is equivalent to non-convex global optimization
- Solve by branch-and-bound over a sufficiently large search space

$$0 < \begin{cases} \alpha \\ \beta - 1 \\ \beta - (\alpha^2 + \beta^2) \\ (\alpha^2 + \beta^2 + \delta) - (\beta\delta + \alpha\gamma + \beta) \\ (\gamma^2 + \delta^2 + 2\beta) - (\alpha^2 + \beta^2 + 2\delta) \\ (2\alpha\gamma + 2\beta\delta + 1) - (\gamma^2 + \delta^2 + 2\beta) \\ \delta \\ 1 - 2\delta \\ (\gamma^2 + \delta^2) - 1 \end{cases}$$

Feasibility Search via Branch-and-Bound

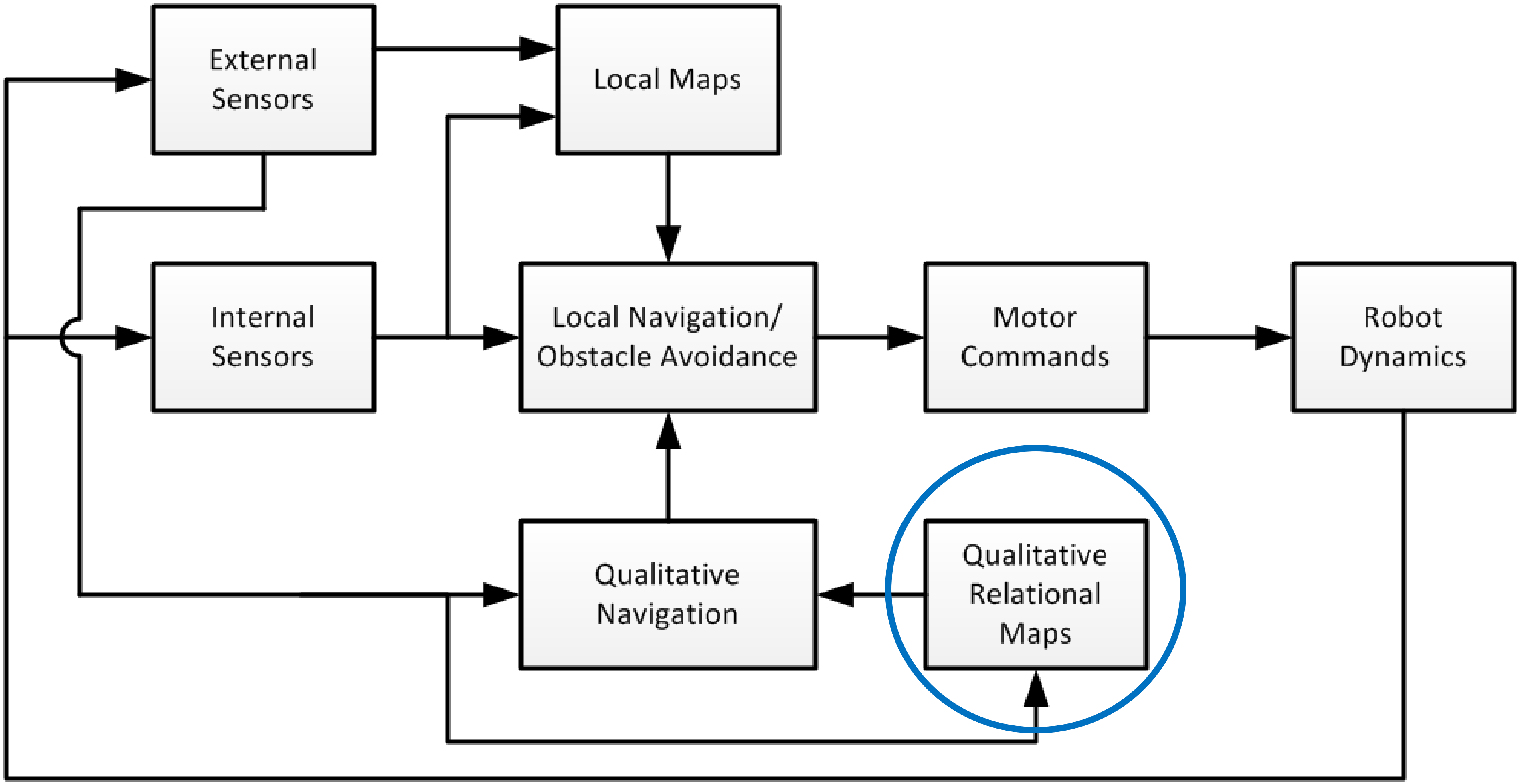
```
add rectangle $r_0 = [l_b, u_b]$ to search queue $S$;
while $S \neq 0$ do
    pop rectangle $r$ from $S$;
    if $DEPTH(r) > maxDepth$ then
        return FALSE;
    else
        choose random $x^* \in r$;
        evaluate constraints $q(x)_j = x^T A_j x + c_j^T x + d_j$;
        if $q(x^*)_j < 0, \forall j \in \{1, M\}$ then
            return TRUE;
        else
            for $j \leftarrow 1$ to $M$ do
                find $\underline{q}_j$ which lowerbounds $q(x)_j$ on $r$;
            if $\underline{q}_j < 0, \forall j \in \{1, M\}$ then
                split $r$ into $r_l$ and $r_u$;
                add $r_l$ and $r_u$ to $S$;
            else
                continue;
return FALSE;
```

EDC Compositions

- 8000 element table too large for hand-computation
- Solve feasibility given C=(α, β), D=(γ, δ)
- A table element is true iff a feasible solution exists

Expression	Interpretation of Expression < 0
$-\alpha$	C is to the right of $\overline{AB}$
$-\beta$	C is in front of A wrt $\overline{AB}$
$1-\beta$	C is in front of B wrt $\overline{AB}$
$1-2\beta$	$\lvert AC\rvert > \lvert BC\rvert$
$1-(\alpha^2+\beta^2)$	$\lvert AC\rvert > \lvert AB\rvert$
$2\beta-(\alpha^2+\beta^2)$	$\lvert BC\rvert > \lvert AB\rvert$
$(\alpha\delta+\gamma)-(\alpha+\beta\gamma)$	D is to the right of $\overline{BC}$
$(\beta+\delta)-(\beta\delta+\alpha\gamma+1)$	D is in front of B wrt $\overline{BC}$
$(\alpha^2+\beta^2+\delta)-(\beta\delta+\alpha\gamma+\beta)$	D is in front of C wrt $\overline{BC}$
$(\alpha^2+\beta^2+2\delta)-(2\beta\delta+2\alpha\gamma+1)$	$\lvert BD\rvert > \lvert CD\rvert$
$(\alpha^2+\beta^2+2\delta)-(\gamma^2+\delta^2+2\beta)$	$\lvert BD\rvert > \lvert BC\rvert$
$(2\alpha\gamma+2\beta\delta+1)-(\gamma^2+\delta^2+2\beta)$	$\lvert CD\rvert > \lvert BC\rvert$
$-\gamma$	D is to the right of $\overline{AB}$
$-\delta$	D is in front of A wrt $\overline{AB}$
$1-\delta$	D is in front of B wrt $\overline{AB}$
$1-2\delta$	$\lvert AD\rvert > \lvert BD\rvert$
$1-(\gamma^2+\delta^2)$	$\lvert AD\rvert > \lvert AB\rvert$
$2\delta-(\gamma^2+\delta^2)$	$\lvert BD\rvert > \lvert AB\rvert$

Qualitative Relational Mapping



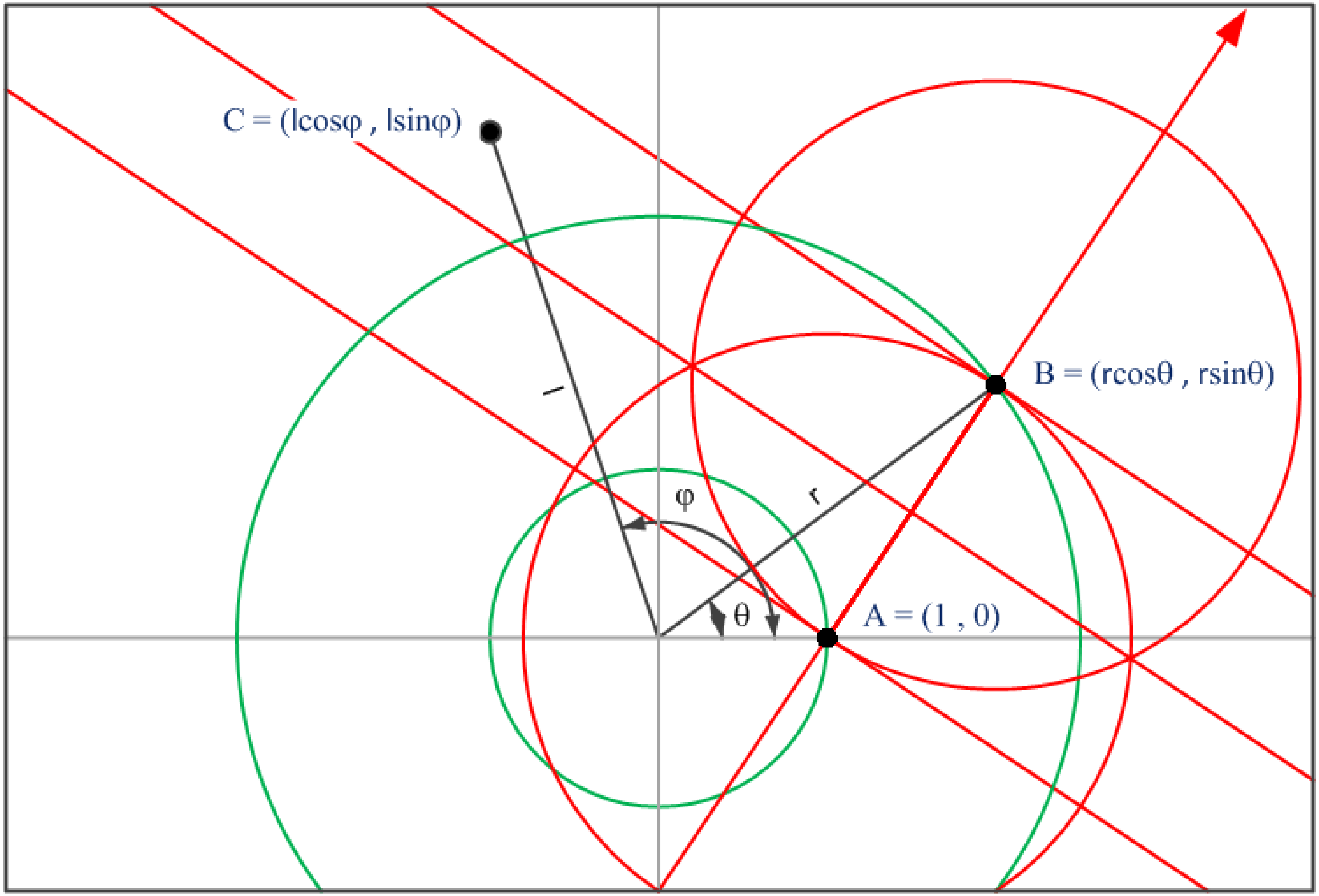
Qualitative Relational Mapping

- Qualitative states represent constraints on landmark relative positioning
  - Graph edges link sets of three landmarks
  - Each edge defines relations **AB:C**, **BC:A**, **CA:B**
  - Every state corresponds to a set of 2 or 3 nonlinear inequalities
- Generate measurements from unknown robot positions that can observe at least 3 landmarks
- Update appropriate graph edge
- Use compositions to generate "new" measurements for the edges of all connected nodes (**AB:C** $\cap$ **BC:D=AB:D**)

Extracting State Estimates from Images

- Assumptions:
  - Landmarks can be uniquely identified
  - Cameras provide exact angles to landmarks
  - Low-level image processing gives an ordering of landmark distances from camera position
- For any three points seen, the angles and range order restrict the possible qualitative states
  - Write qualitative states as sets of nonlinear inequalities
  - Use branch-and-bound algorithm to determine satisfiability of each potential qualitative state
- Edge updates are intersections of sets of qualitative states

EDC Measurements

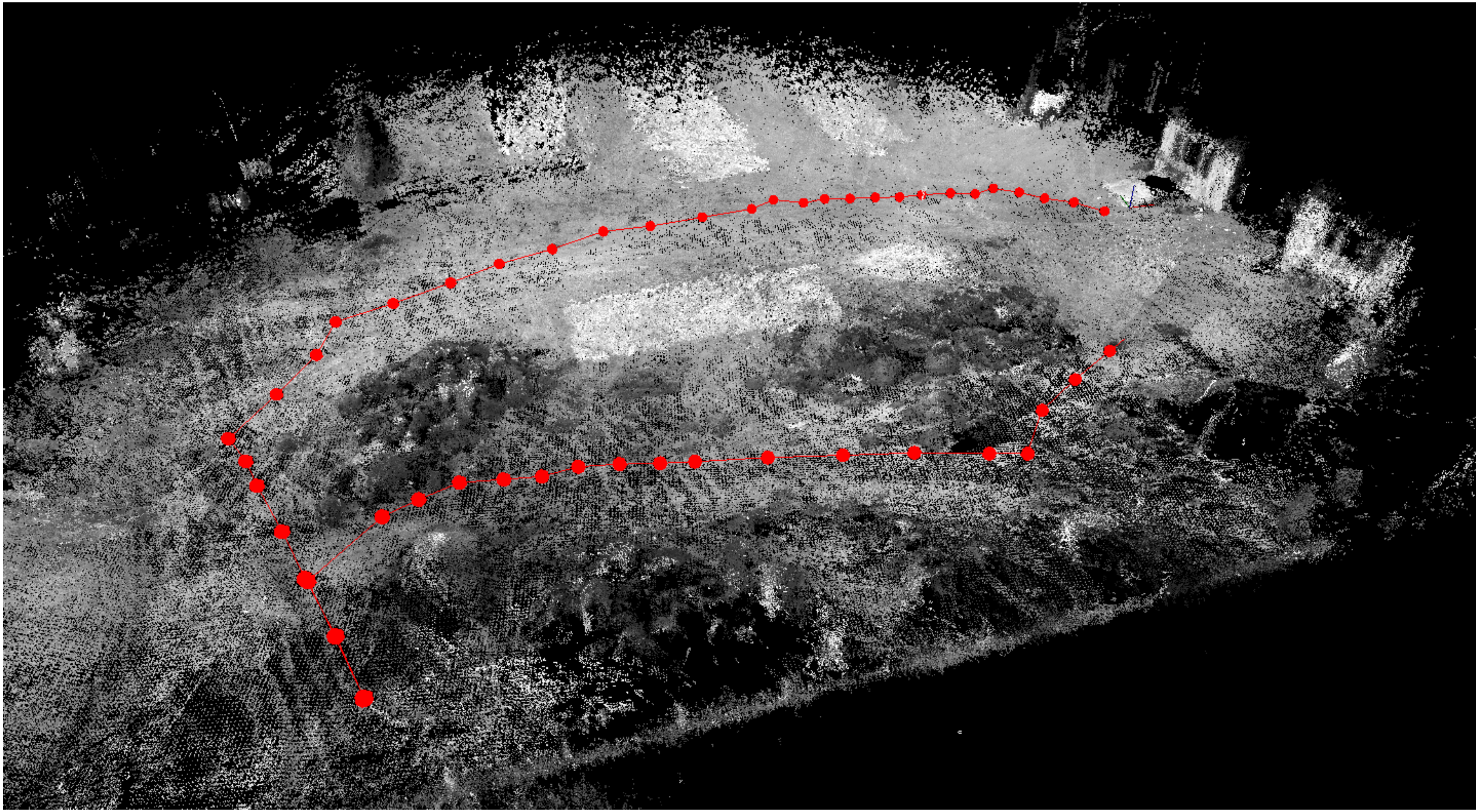


EDC Measurement Constraints

- Write EDC states as sets of nonlinear inequalities in (r, l) given known angles
- EDC state is consistent with measurement if there is a feasible solution
- Solve feasibility by branch-and-bound

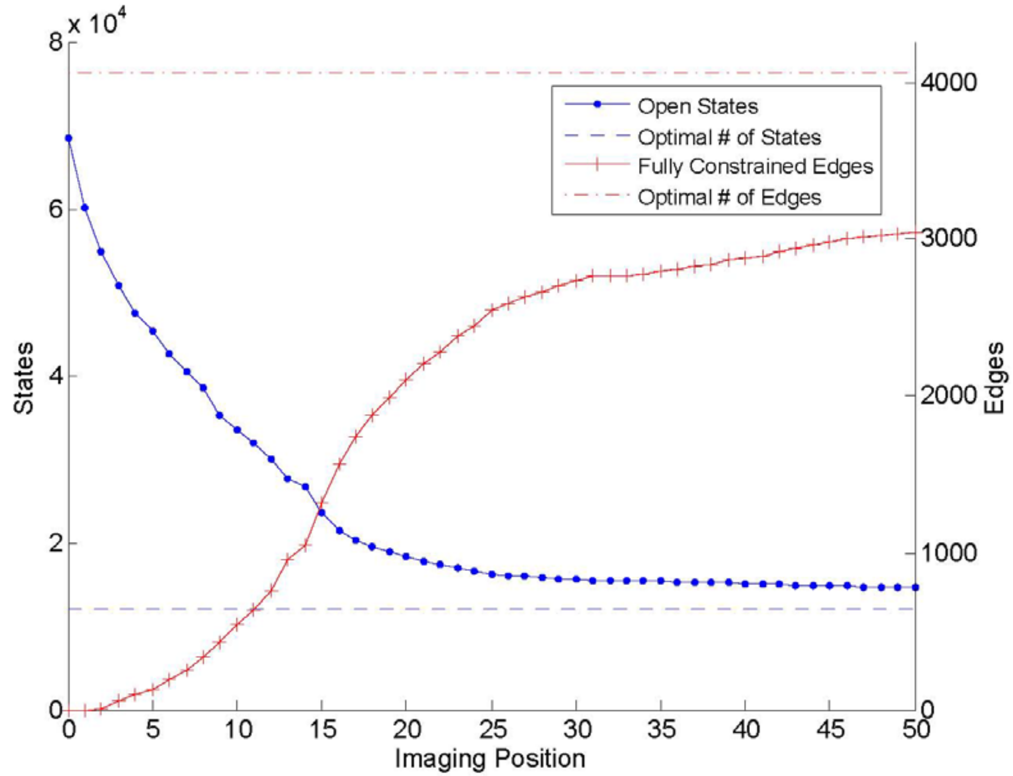
Expression	Interpretation of Expression < 0
$(\sin(\phi)\cos(\theta) - \cos(\phi)\sin(\theta))lr - \sin(\phi)l + \sin(\theta)r$	C is to the right of $\overline{AB}$
$-(\sin(\phi)\sin(\theta) + \cos(\phi)\cos(\theta))lr + \cos(\phi)l + \cos(\theta)r - 1$	C is in front of A wrt $\overline{AB}$
$r^2 - (\sin(\phi)\sin(\theta) + \cos(\phi)\cos(\theta))lr + \cos(\phi)l - \cos(\theta)r$	C is in front of B wrt $\overline{AB}$
$r^2 - 2(\sin(\phi)\sin(\theta) + \cos(\phi)\cos(\theta))lr + 2\cos(\phi)l - 1$	$\lvert BC\rvert < \lvert AC\rvert$
$l^2 - r^2 - 2\cos(\phi)l + 2\cos(\theta)r$	$\lvert AC\rvert < \lvert AB\rvert$
$l^2 - 2(\sin(\phi)\sin(\theta) + \cos(\phi)\cos(\theta))lr + 2\cos(\theta)r - 1$	$\lvert BC\rvert < \lvert AB\rvert$
$1 - l$	A is closer than C
$1 - r$	A is closer than B
$r - l$	B is closer than C

Test Case: JPL Mars Yard

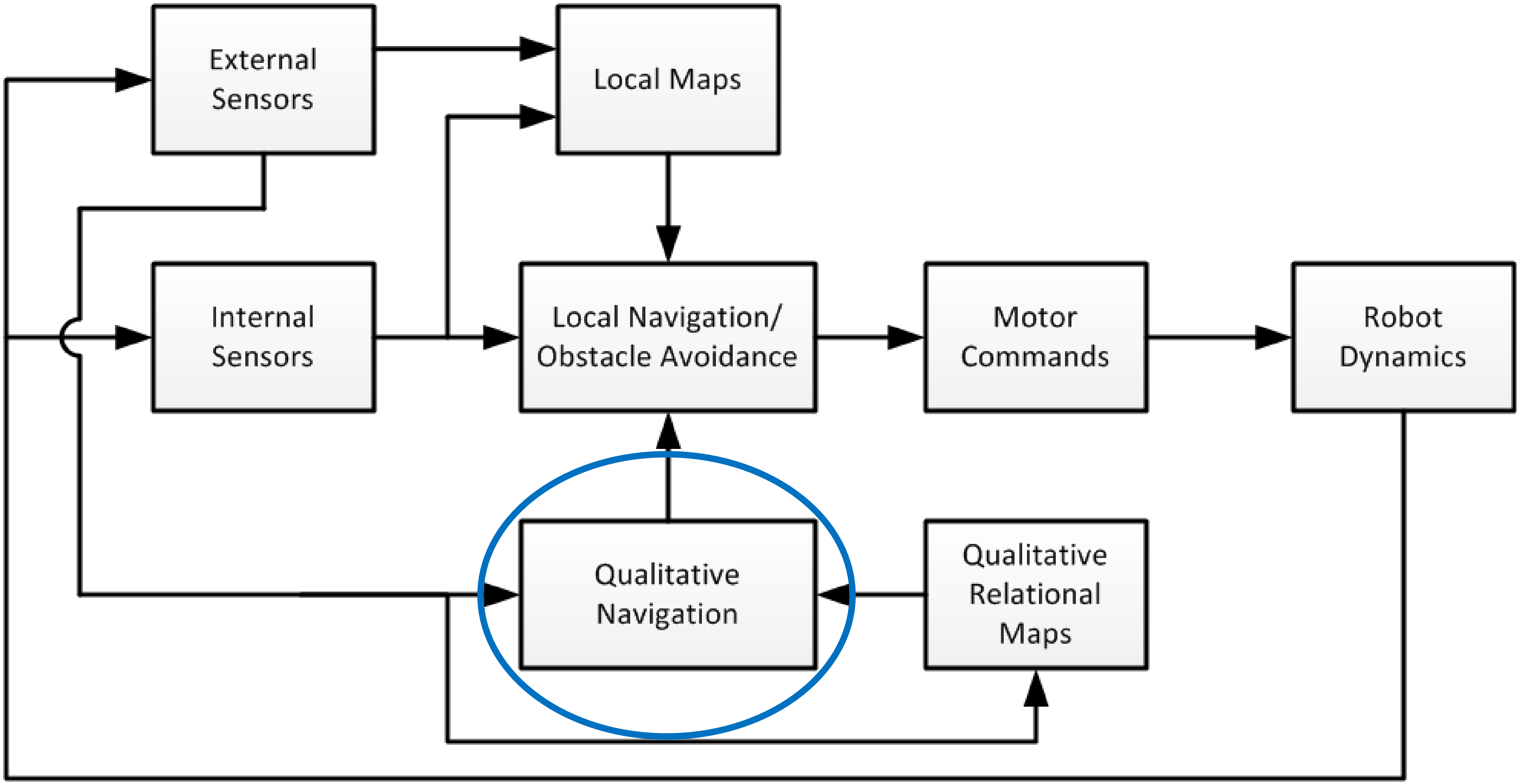


Mars Yard Mapping Results

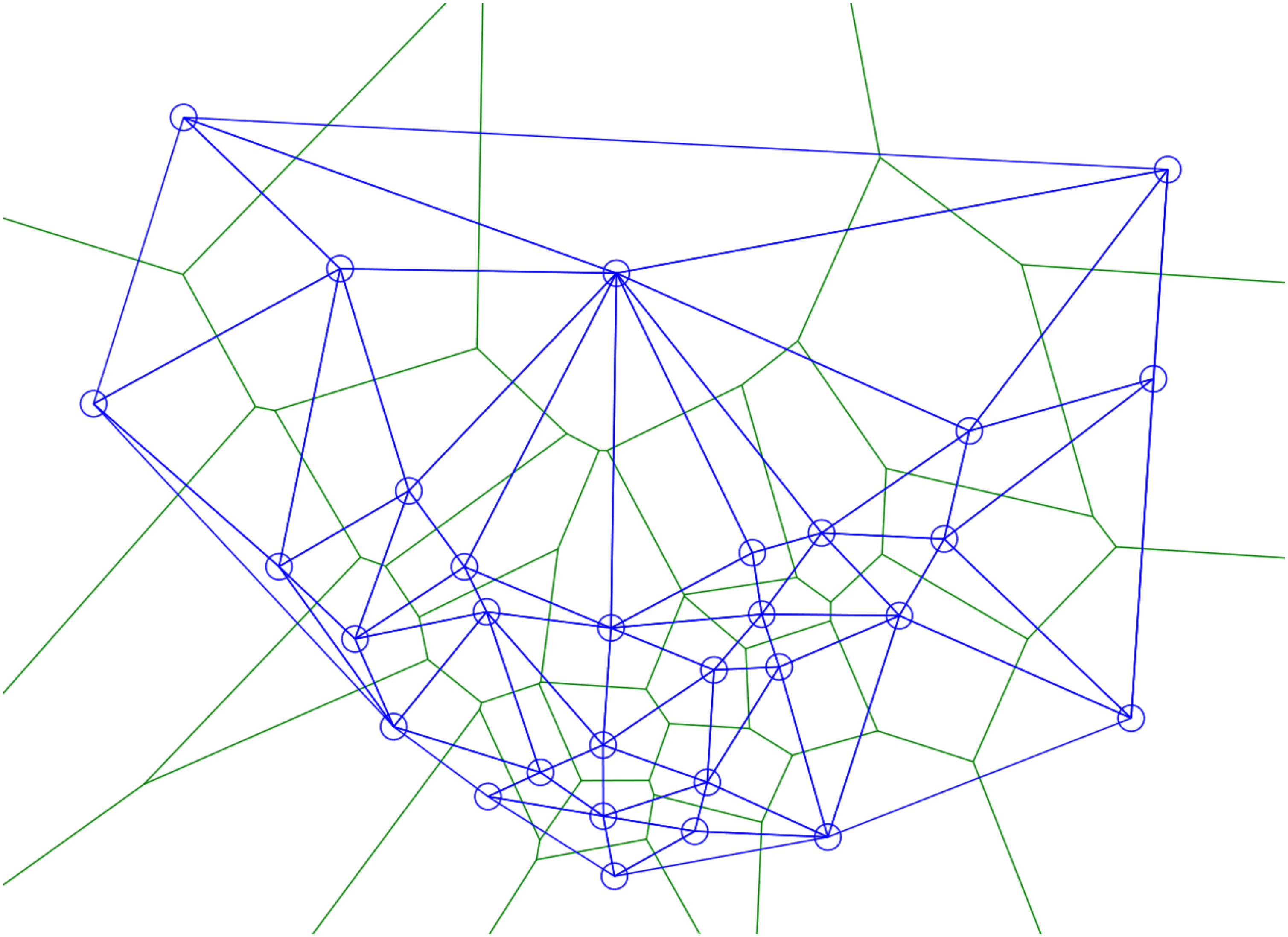
- 30 Landmarks (Tagged Manually)
- 4060 Edges
- Max of 243,600 states before first measurement (Not shown)



Qualitative Relational Navigation

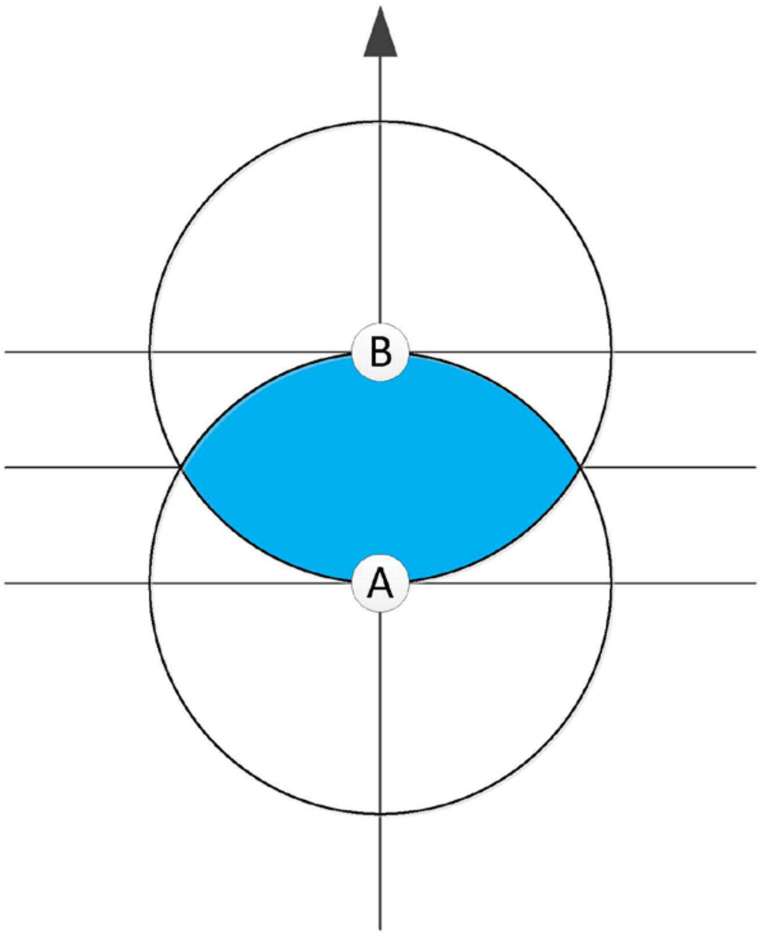


The Voronoi Diagram / Delaunay Graph

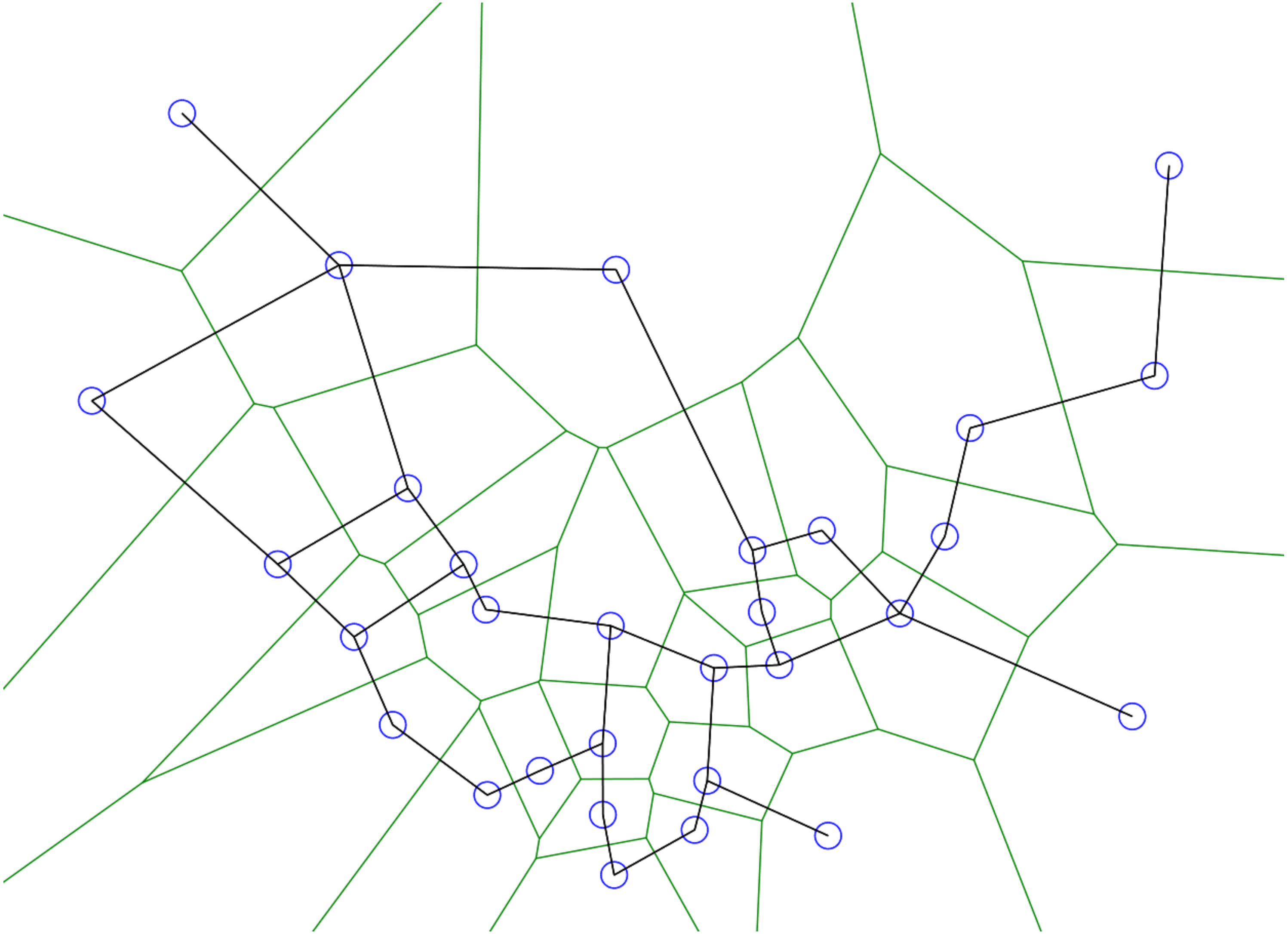


Finding the Relative Neighborhood

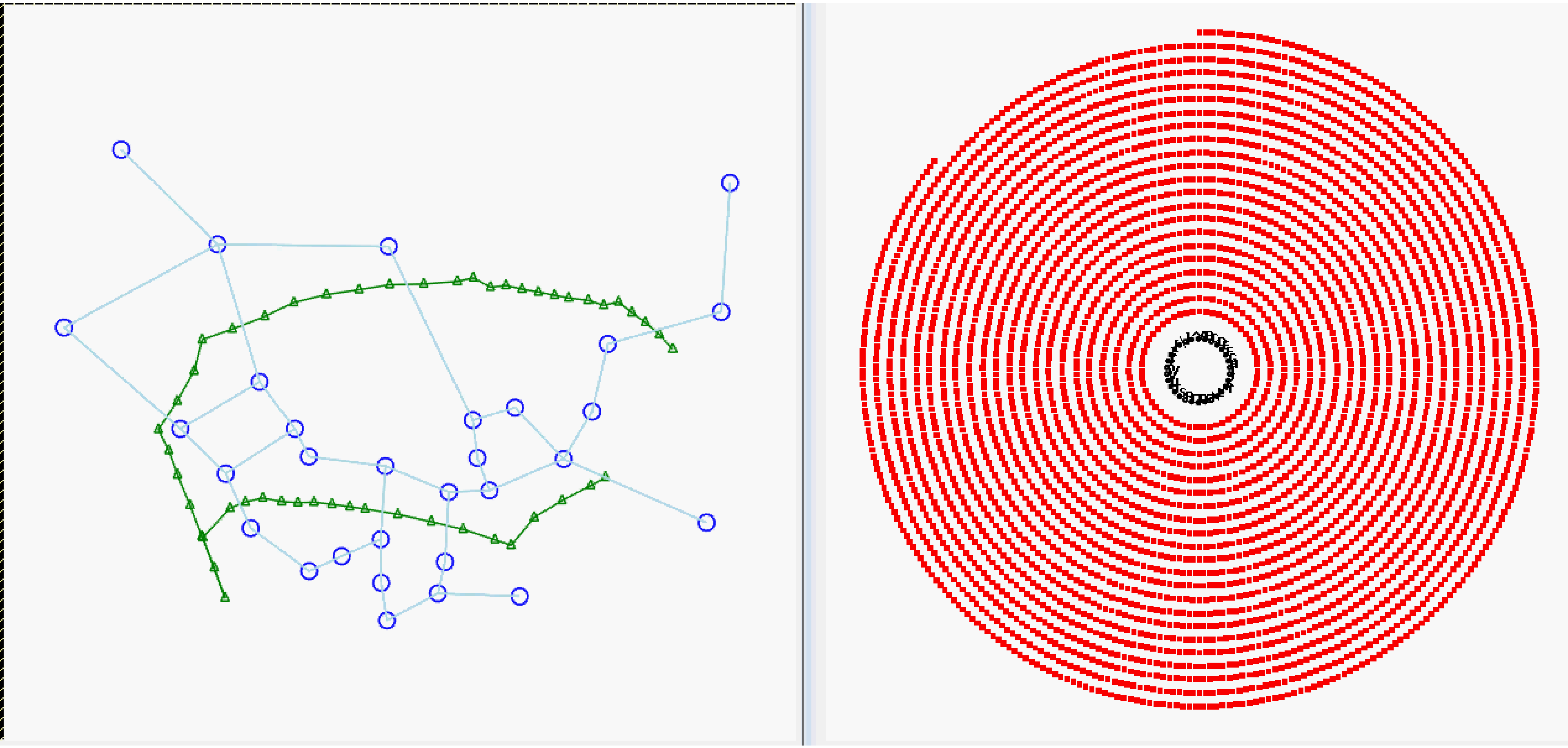
- The EDC graph does not contain enough information to find the Delaunay Triangulation
- But, we can find the Relative Neighborhood Graph (RNG)
  - Connected subgraph of the Delaunay graph
  - Points are linked if no third point lies in the lune of circles of radius AB centered at A and B
- We can also find the convex hull
  - Also a subgraph of the Delaunay



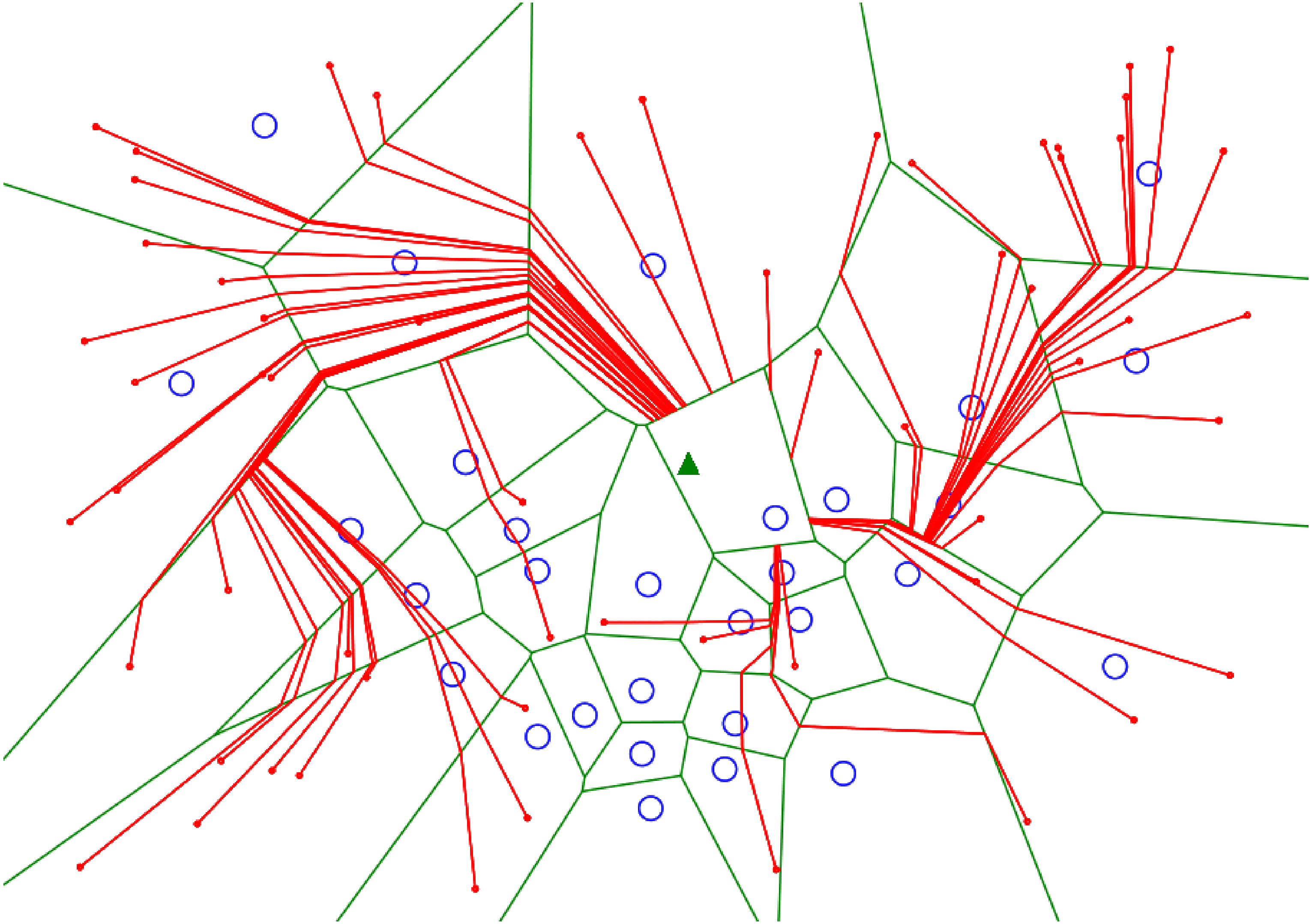
The Relative Neighborhood Graph



Building a Relational Map



Navigating with the RNG



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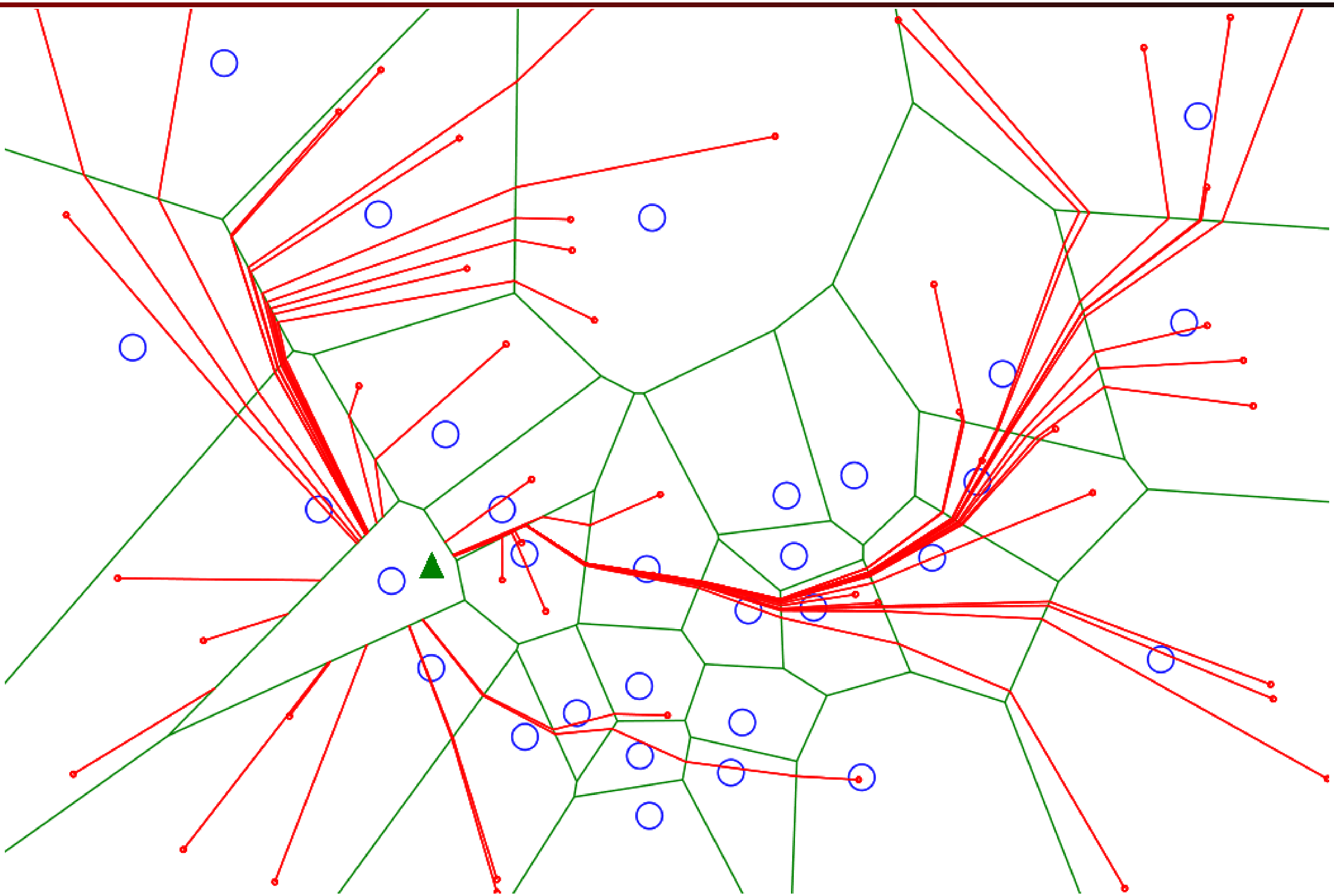

Current Limitations and Future Work

- Deductive reasoning leads to map inconsistency after a data-association mistakes
  - Track multi-hypotheses for delayed information fusion
  - Move to a probabilistic framework with discrete distributions
- Graph scales as n^3 with the number of landmarks
  - Hierarchical maps: cluster landmarks into local groups
  - Reason over extended meta objects (rock clusters, craters, etc)
- Dependence on observing most landmarks in each image
  - Improve simulation system to handle mixtures of local and distant features
  - Implement automatic rock detection to check visibility of mars yard landmarks
  - Run algorithm on data gathered by MER

Conclusions

- Qualitative Relational Mapping
  - Builds a network of geometrical constraints on possible landmark positions
  - Measurements rely only on knowing angles to landmarks and relative range ordering
  - Mapping requires no information about imaging locations
  - For any set of landmarks there is a guaranteed finite image sequence generating a fully constrained graph
  - Maps can be used for simple long-distance navigation using relative neighborhood graphs

Acknowledgements

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