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CMUT with mechanically coupled plate actuators - Linearized electrostatic modeling

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***Abstract*—This paper derives an analytical model of the electrostatic transduction of a capacitive micromachined ultrasonic transducer (CMUT) with mechanically coupled plate actuators. The mass of the coupling geometry significantly affects the dynamic deflection shape of the plate actuators. As the electrostatic transduction of a CMUT depends on the distance between both electrodes of these capacitive systems, the static and dynamic deflection shapes are taken into account for the small signal electrostatic behavior. The analysis of the electrostatic lumped elements, varying with the bias voltage, reveals a dependence on the deflection shape. A comparison with finite element simulations shows good agreement with the analytical model.**

***Index Terms*—CMUT, modeling, superposition, electrostatics**

I. Introduction

Capacitive micromachined ultrasonic transducers (CMUTs) are multiphysical systems, exchanging energy between different physical domains. The most important domains enabling the principle operation of a CMUT are mechanics, electrostatics and acoustics. The mechanical structure is the connection between the other two physical fields. The electrostatic field directly converts electrical energy into mechanical energy or vice versa.

A CMUT is realized as a variable capacitor consisting of a fixed and flexible electrode. An applied voltage above both electrodes causes an electrostatic field generating an attractive force in transmit mode. The resulting deformation of the flexible electrode can generate acoustic waves. When the CMUT works in receive mode, an incoming pressure wave deforms the flexible electrode, which is measurable as capacitance change. Obviously, the transmit and receive behavior, e. g. the sensitivity, depends on this electro-mechanical transduction mechanism [1].

In this paper we derive a model for the electrostatic small signal operation of a CMUT with mechanically coupled plate actuators (Fig. 1). The structure consists of two levels: several plate actuators and a roof structure. The electro-mechanical conversion takes place at the plate actuators, which are excited by the force of the electrostatic field in transmit mode. The motion of the flexible actuators is transferred to the roof structure via the pillars at the actuator centers. The roof plate acts as sound source, separating the electrostatic and acoustic geometry in comparison to a conventional CMUT.

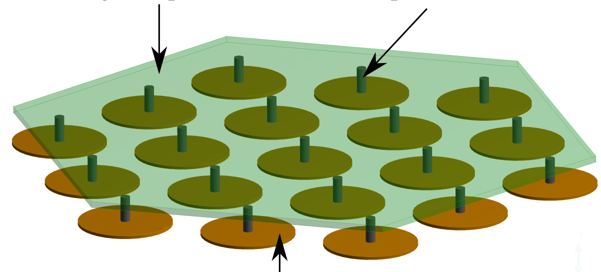

Fig. 1. Schematic representation of a CMUT with mechanically coupled plate actuators, transferring their motion to the sound transmitting roof structure in transmit mode.

II. 1D analytical model

An electro-mechanical system can be described as spring mass damper system, consisting of the mass m, the spring constant k and the damping constant b. It is excited by an electrostatic force

$$F_{\mathrm{el}} = \frac{U^2}{2}\frac{\partial C}{\partial w}, \tag{1}$$

i. e. the force depends on the electrical voltage U, the capacitance C and the deflection w of the flexible electrode, affecting the gap of the capacitor. The linearized dynamic force equilibrium

$$m\ddot{w}_{\mathrm{AC}} + b\dot{w}_{\mathrm{AC}} + kw_{\mathrm{AC}} = Nu_{\mathrm{AC}} + k_{\mathrm{el}}w_{\mathrm{AC}} \tag{2}$$

is determined by the small signal amplitude w_{AC} around the static operation point of the system [1]. The transformation factor

$$N = \left.\frac{\partial F_{\mathrm{el}}}{\partial U}\right|_{U_{\mathrm{DC}},w_{\mathrm{DC}}} = U_{\mathrm{DC}}\left.\frac{\partial C}{\partial w}\right|_{U_{\mathrm{DC}},w_{\mathrm{DC}}} \tag{3}$$

and the electrostatic stiffness

$$k_{\mathrm{el}} = \left.\frac{\partial F_{\mathrm{el}}}{\partial w}\right|_{U_{\mathrm{DC}},w_{\mathrm{DC}}} = \frac{U_{\mathrm{DC}}^2}{2}\left.\frac{\partial^2 C}{\partial w^2}\right|_{U_{\mathrm{DC}},w_{\mathrm{DC}}} \tag{4}$$

are the lumped electrostatic parameters in case of a small signal operation. These parameters depend on the static deflection

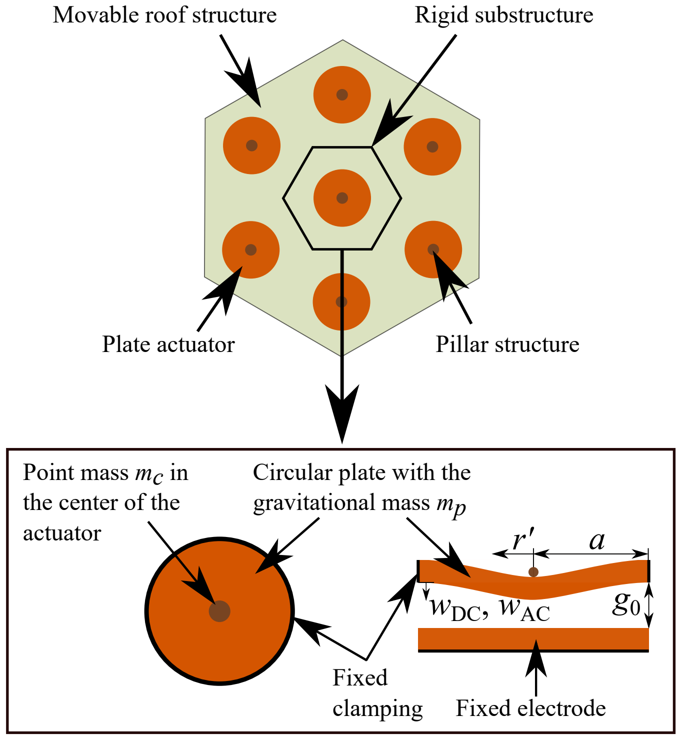

Fig. 2. Schematic representation of the analytical CMUT model. Approximation of the mechanically coupled structure (top) as a single electrostatic plate actuator with a point mass m_C, representing the inertial impact of a hexagonal roof substructure (bottom).

w_{DC}, the DC voltage U_{DC} and the capacitance $C(w)$ of the system as function of the deflection. We present a model, calculating the electrostatic stiffness and the transformation factor for a CMUT, according to Fig. 1.

A. Deflection shape of a circular plate actuator

The deflection shape of the fundamental actuator mode can be calculated analytically if the mechanically coupled CMUT is approximated as single plate actuator with a point mass m_C (Fig. 2). As explained in [2], the following assumptions enable the calculation of the dynamic deflection shape:

i) Uniform motion of the roof plate;
ii) The point mass represents the inertial impact of the hexagonal roof substructure, that is transferred to the plate center via the pillar;
iii) The same inertial force on each actuator;
iv) Thin circular actuator plate with small deflections;
v) Local stiffening of the actuator due to the pillar is neglected;
vi) Fixed clamping of the actuator.

With these assumptions the axisymmetric shape of deflection can be expressed by means of an approximation function

$$\begin{aligned} w_{dyn}(r,c) &= w_{AC}\, w_{dyn,0} \\ &= w_{AC}\left(\frac{2}{c-2}r^{\,c} - \frac{c}{c-2}r^{\,2} + 1\right), \end{aligned} \tag{5}$$

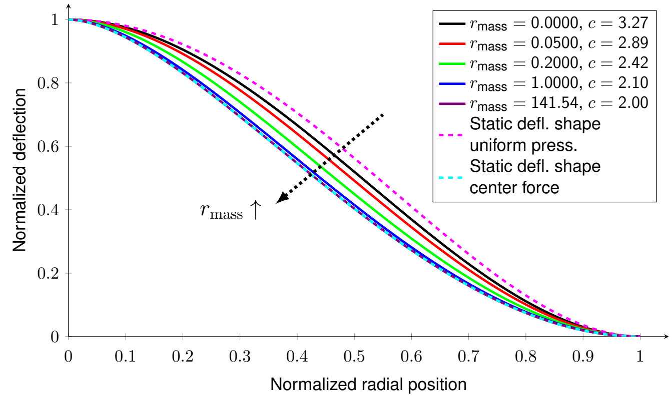

Fig. 3. Actuator deflection shape in dependence of the mass ratio r_{mass}.

separated into a constant deflection shape $w_{dyn,0}$ and a time varying amplitude w_{AC} [3]. The optimization parameter c depends on the mass ratio

$$r_{mass} = \frac{m_C}{m_P} \tag{6}$$

of the point mass m_C and the geometrical mass m_P of the circular plate. The shape of deflection is defined as function of the normalized radial coordinate $r = r'/a$ with the radius a of the circular plate (Fig. 2). With increasing mass ratio the shape of deformation changes significantly (Fig. 3). For large mass ratios ($r_{mass} > 1$) the dynamical deformation converges to the static deflection shape of a circular plate loaded with a center force (Fig. 3) [2].

B. Electrostatic superposition model

The electrostatic lumped parameters are derived by calculating the capacitance C in dependence of a single deflection shape, e. g. the static or dynamic deflection shape of a circular plate [4]. Afterwards the transformation factor and electrostatic stiffness can be determined with the first and second derivation of the capacitance according (3) and (4).

As the dynamic deflection shape of a CMUT with mechanically coupled plate actuators depends on the roof mass, static and dynamic deflection shapes can vary. In the static case, the inertial impact of the roof mass can be neglected. According to assumptions iv) – vi) the static shape of deflection defining the operation point of a circular plate is [4]:

$$w_{stat}(r) = w_{DC}\, w_{stat,0} = w_{DC}(1-r^2)^2. \tag{7}$$

The static shape function consists of a constant shape $w_{stat,0}$ and a static amplitude w_{DC}, which depends on external loads, e. g. the electrostatic pressure [4].

For the model description of a CMUT with mechanically coupled plate actuators (Fig. 2), we assume that the actuator plate vibrates according to (5) around the static deflection shape. The deflection superimposed yields the total capacitance

$$C = 2\pi a^2 \epsilon_0 \int_0^1 \frac{r}{g_0 - w_{stat}(r) - w_{dyn}(r,c)}dr \tag{8}$$

in dependence of the electrostatic gap g_0 and the absolute permittivity ϵ_0 for the axisymmetric vibration [4].

As the plate oscillates with the small signal amplitude w_{AC} the first derivation

$$\frac{\partial C(w_{\mathrm{DC}}, w_{\mathrm{AC}}, c)}{\partial w_{\mathrm{AC}}} = 2\pi a^2 \epsilon_0 \int_0^1 \frac{r w_{\mathrm{dyn},0}^2(r,c)}{[g_0 - w_{\mathrm{stat}}(r)]^2} dr \quad (9)$$

and second derivation

$$\frac{\partial^2 C(w_{\mathrm{DC}}, w_{\mathrm{AC}}, c)}{\partial w_{\mathrm{AC}}^2} = 2\pi a^2 \epsilon_0 \int_0^1 \frac{r w_{\mathrm{dyn},0}^2(r,c)}{[g_0 - w_{\mathrm{stat}}(r)]^3} dr \quad (10)$$

of (8) according to w_{AC} has to be calculated for the transformation factor N and electrostatic stiffness k_{el}.

With (3) and (9), a normalized transformation factor

$$N^{\mathrm{norm}} = \frac{N g_0^2}{U_{\mathrm{DC}} 2\pi a^2 \epsilon_0} = \int_0^1 \frac{r w_{\mathrm{dyn},0}^2(r,c)}{\left[1 - \frac{w_{\mathrm{DC}}}{g_0} w_{\mathrm{stat},0}(r)\right]^2} dr \quad (11)$$

that depends only on the deflection shapes and the normalized DC deflection $w_{\mathrm{DC}}/w_{\mathrm{PI}}$ can be defined.

In the same manner a normalized electrostatic stiffness is defined

$$k_{\mathrm{el}}^{\mathrm{norm}} = \frac{k_{\mathrm{el}} g_0^3}{U_{\mathrm{DC}}^2 \pi a^2 \epsilon_0} = \int_0^1 \frac{r w_{\mathrm{dyn},0}^2(r,c)}{\left[1 - \frac{w_{\mathrm{DC}}}{g_0} w_{\mathrm{stat},0}(r)\right]^3} dr. \quad (12)$$

The pull-in deflection w_{PI} is the maximum deflection before the flexible electrode moves unrestrained to the fixed electrode. In case of a thin circular plate, the pull-in occurs at 46 % of the electrostatic gap g_0. This deflection point can be determined by solving [4], [5]

$$\frac{\partial E_{\mathrm{m}}}{\partial w_{\mathrm{DC}}} \frac{\partial^2 C}{\partial w_{\mathrm{DC}}^2} - \frac{\partial^2 E_{\mathrm{m}}}{\partial w_{\mathrm{DC}}^2} \frac{\partial C}{\partial w_{\mathrm{DC}}} = 0 \quad (13)$$

with the mechanical energy

$$E_{\mathrm{m}} = \frac{1}{2} k_{\mathrm{stat}} w_{\mathrm{DC}}^2. \quad (14)$$

The mechanical stiffness in case of a static plate deformation is

$$k_{\mathrm{stat}} = 20.87 \frac{\pi D}{a^2}, \quad (15)$$

with the flexural rigidity D [2].

The normalized electrostatic stiffness increases in dependence of the normalized DC deflection for the conventional approach with a single deflection shape as well as for the superposition approach (Fig. 4). This can be observed for both a small ($r_{\mathrm{mass}} < 1$) and large mass ratio ($r_{\mathrm{mass}} \geq 1$).

The relative deviation

$$\frac{k_{\mathrm{superpos}}^{\mathrm{norm}} - k_{\mathrm{stat}}^{\mathrm{norm}}}{k_{\mathrm{stat}}^{\mathrm{norm}}} 100\,\% \quad (16)$$

between the electrostatic stiffnesses $k_{\mathrm{superpos}}^{\mathrm{norm}}$ and $k_{\mathrm{stat}}^{\mathrm{norm}}$ based on the superposition approach and solely on the static deflection shape provides a detailed insight in the difference between both modeling approaches.

In case of a circular plate without a center mass ($r_{\mathrm{mass}} = 0$), the stiffness based on the superposition approach is less than 10 % smaller, but for a plate with a large center mass ($r_{\mathrm{mass}} = 10.13 > 1$) the stiffnesses differ by up to −35 % (Fig. 5). Even at the pull-in point, the electrostatic stiffness with superimposed deflections is about 29 % smaller. After the pull-in the presented model is not valid, because the vibration behavior [6]. In both cases the deviations reduces as the deflection increases.

Similarly an evaluation of the transformation factor can be done, revealing also a reduced transformation factor for a plate with a large center mass.

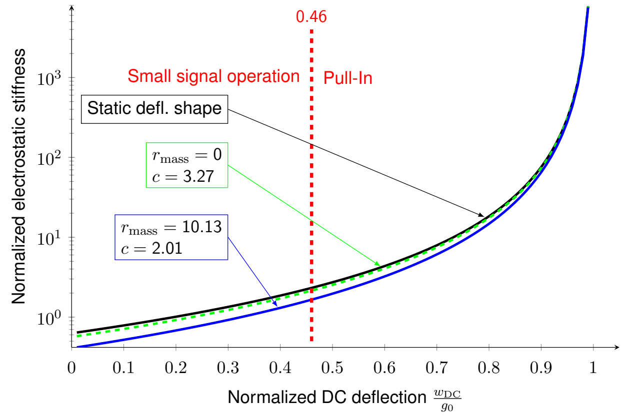

Fig. 4. Normalized electrostatic stiffness in dependence of the static deflection.

III. Comparison with FEM results and measured deflections

The analytical results are compared with numerical prestressed eigenfrequency calculations. A substructure approach with one sixth of a single plate actuator and a roof structure was used for the finite element model (FEM) (Fig. 6). An modal analysis was performed after prestressing the structure with a static analysis in ANSYS (17.2, USA). The electrostatic excitation was realized with Trans126 elements.

The obtained numerically calculated results are compared with the analytical calculations based on a single deflection

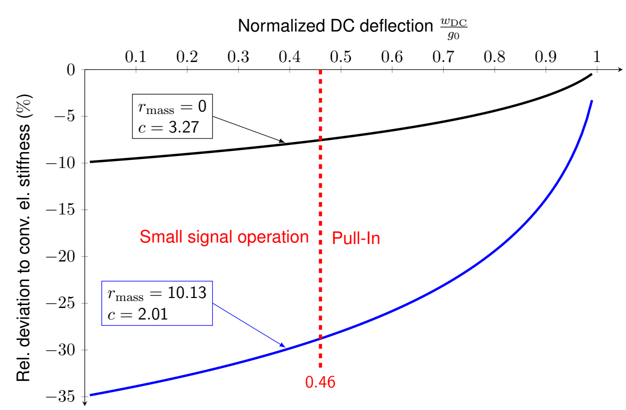

Fig. 5. Relative deviation between the normalized electrostatic stiffnesses based solely on the static deflection shape and on the superposition approach.

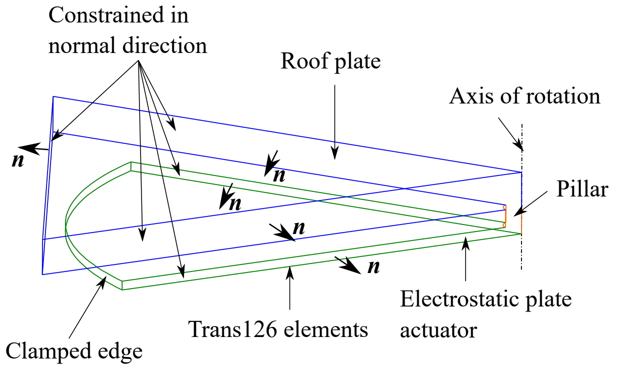

Fig. 6. Substructure FEM model of a CMUT with mechanically coupled plate actuators.

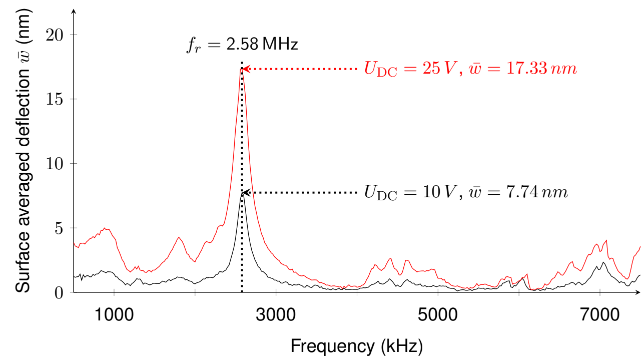

Fig. 8. Surface-averaged deflection of a mechanically coupled CMUT measured with a DHM.

shape and the superposition approach. The mechanical lumped parameters are calculated according to [2]. The eigenfrequencies reduce with increasing DC voltage in all investigated cases as result of the electrostatic stiffness (Fig. 7). The reduction of the eigenfrequency calculated with the single deflection shape approach is larger than for the numerical and superposition based results. FEM and superposition eigenfrequencies differs with less than $1.5\,\%$, while the deviation is up to $18\,\%$ for the large DC voltages and the conventional approach.

The surface-averaged deflection of a mechanically coupled CMUT was measured with a digital holographic microscope (DHM) (LynceeTec, R-Series, Switzerland) [2]. The experimental results show a DC-voltage-dependent amplitude at $2.58\,\text{MHz}$, increasing from $7.74\,\text{nm}$ to $17.33\,\text{nm}$ if the DC voltage is increased by a factor of 2.5. A frequency reduction with increasing DC voltage is not observed. Possible reasons are deviations from the model assumptions, e. g. a non-uniform roof motion, or small applied DC voltages in comparison to the pull-in voltage. Low DC voltages with small DC deflection correspond to small electrostatic stiffnesses.

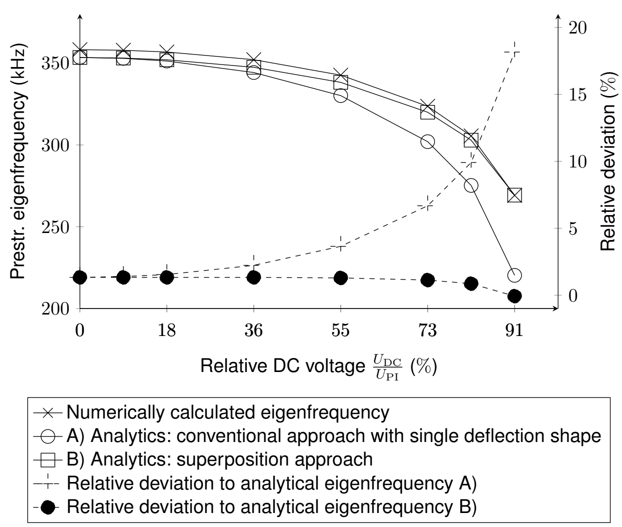

Fig. 7. Comparison between numerically and analytically calculated eigenfrequency in dependence of the DC voltage.

IV. Conclusion

An approach modeling the small signal electrostatics of a CMUT with mechanically coupled plate actuators was presented. The model takes the difference between static and dynamic deflection shape of a circular plate actuator into account. Both the electrostatic transformation factor and the electrostatic stiffness reduce when the superposition approach is utilized. For a CMUT with a large roof mass the electrostatic stiffness is from about $29\,\%$ to $35\,\%$ smaller than a conventional single deflection shape modeling approach would predict. Exemplary FEM simulations confirm a smaller eigenfrequency reduction with less than $1.5\,\%$ deviation. Experimental DHM results verify electrostatic behavior with a DC voltage dependent amplitude.

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