Program Digest, 2019 IEEE International Ultrasonics Symposium (IUS)
Glasgow, Scotland, October 6-9, 2019

Numerical investigation of mode conversion of SH guided waves in plates reflected from discontinuities with different symmetries

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***Abstract*—Shear horizontal guided waves are commonly used in non-destructive evaluation of plates and pipes. When interacting with a thickness discontinuity, the reflected and transmitted wavefields can be composed of several modes due to mode conversion. It is known that in a plate with a symmetric discontinuity, with respect to the plate's mid-plane, mode conversion is restricted to modes that share the same symmetry as the incident mode. In this paper, we analyse mode conversion due to reflection from non-symmetric and symmetric discontinuities and investigate different types of discontinuity's symmetry through numerical analysis. We show that in a discontinuity with symmetric geometry but with one of its sides constrained fixed and the other one free, mode conversion is virtually restricted to modes with the opposite symmetry of the incident mode, acting as a symmetry inverter discontinuity. The latter can be efficiently simulated by filling one of the halves of the discontinuity with a material with at least tenfold the plate's acoustic impedance. Additionally, the scattering behaviour generally depends on whether the discontinuity is placed on the plate's surface or in the middle of the plate.**

***Index Terms*—SH guided waves, mode conversion, discontinuity, symmetry.**

I. Introduction

Interaction of shear horizontal (SH) modes with discontinuities in plates has been studied previously [1], [2] aimed at the interpretation of scattered waves from wall thinning, such as corrosion-like defect, which is relevant for nondestructive evaluation of plates and pipes [3], [4].

In the high frequency-thickness regime, several SH modes are able to propagate. Therefore, in this regime, whenever an SH guided wave mode impinges upon a feature in the plate, such as a wall thinning section, the scattered field may be composed of other modes, i.e. the incident mode may suffer mode conversion [5]–[10]. The mode-conversion behaviour is complex [10], for instance, in a gradual thickness reduction section, the intensity of the converted modes heavily depends on the shape of the thinning section, i.e., on its depth and edge angle [5], [10].

Some recent work investigated the interaction with symmetric discontinuity. Pau et al. [7] analysed the reflection and transmission coefficients for SH waves in plates with a thickness reduction section with a right-angle edge and also with different geometries [8], such as thin ellipses. Two symmetric cases were studied, namely on both surfaces of the plate, or voids in the middle of the plate's cross-section. Only the case in which the incident mode was the symmetric SH0 mode was analysed. It was verified that in the case of a symmetric discontinuity, mode-conversion to an antisymmetric mode was not allowed. Thus, when interacting with a symmetric discontinuity, the SH0 mode was converted only to the SH2, due to the considered frequency-thickness, whereas, when interacting with a non-symmetric discontinuity, it was converted to either the SH1 or SH2 modes. Interestingly, for both low and high frequency-thickness cases, it was shown that the symmetric superficial notch and a void in the middle of the plate, present the same coefficient's values and that the elliptical and the rectangular notch presented similar results.

In this paper, we analyse the interaction of the fundamental and higher-order SH modes with discontinuities in plates, drawing attention to the mode-conversion due to the reflection from different types of discontinuities, and how the discontinuity symmetry affects mode-conversion with regards to the symmetry characteristics of the SH modes across the plate's cross-section.

Authors would like to thank the Brazilian National Council for Scientific and Technological Development, CNPq, for financial support.

II. SH GUIDED WAVES IN PLATES

Shear horizontal (SH) guided waves [11] have only one non-null displacement component, in the z direction, perpendicular to the propagation direction, x, and to the coordinate of the plate thickness, y. The coordinate system and plate orientation is shown in Fig.1. The displacement field is given by:

$$u_z(x,y,t) = A_n U_n(y) e^{j(\omega t - \kappa_n x)}, \tag{1}$$

where ω is the angular frequency, n is the mode order, κ_n, A_n and $U_n(y)$ are the wavenumber, amplitude and displacement profile of mode n, respectively. SH modes are classified as symmetric and antisymmetric according to their displacement profile, which can be described by:

$$U_n(y) = \begin{cases} \cos(n\pi y/h) & \text{, for } n = 0,2,4,6,... \\ \sin(n\pi y/h) & \text{, for } n = 1,3,5,7,... \end{cases} \tag{2}$$

where h is the plate thickness. In the low frequency-thickness regime, below the cut-off frequency-thickness of the SH1 mode, only the SH0 mode can propagate, whereas in the high frequency-thickness regime, up to the n-th order mode can propagate if the operating frequency-thickness is above the cut-off value for mode n.

Considering that when an incident mode n impinges upon a discontinuity, the reflected field is composed of N modes, due to mode-conversion, then the reflected field can be expressed as:

$$u_r(x,y,t) = \sum_{m=0}^{N} R_{nm} A_n U_m(y) e^{j(\omega t - \kappa_m x)}, \tag{3}$$

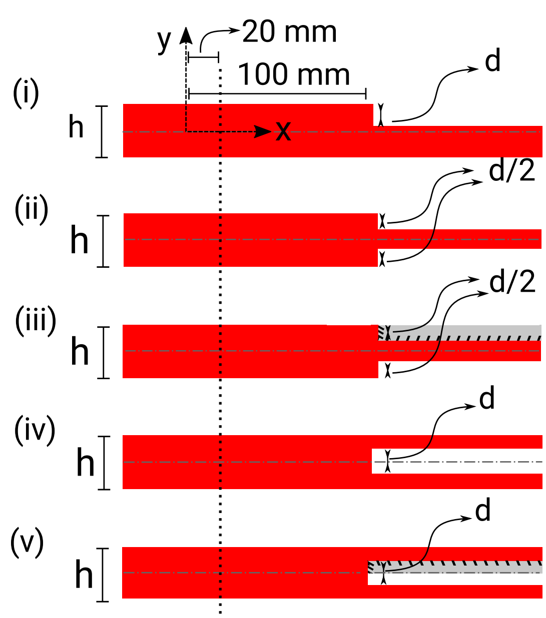

Fig. 1. Discontinuities types studied in this paper: (i) non-symmetric; (ii) symmetric; (iii) symmetric with free boundary at one surface and rigid boundary at the opposite surface; (iv) symmetric slot in the middle of the plate's cross-section; and (v) a slot in the middle of the plate with free boundary at one half of the slot and rigid boundary at the opposite half. Generation is imposed in the origin, the discontinuity starts at $x = 100\,mm$, the vertical dotted line represent the reception point, at $x = 20\,mm$.

where R_{nm} is the reflection coefficient from incident mode n to reflected mode m.

III. FINITE ELEMENT SIMULATION OF SH WAVES REFLECTED FROM DISCONTINUITIES

Numerical analysis was performed using a commercial time-domain Finite Element Method (FEM) solver, PZFlex©, which allows simulation of SH waves in a two-dimensional model. An $8\,mm$ thick aluminium plate was modelled with transverse wave speed $c_T = 3111\,m/s$ and density $\rho = 2698\,kg/m^3$ and a subsequent shear wave acoustic impedance of $Z = 8.4\,MRayl$. The left- and right-most ends were terminated with absorbing boundary conditions to minimise reflections.

Five types of discontinuity were modelled, namely (i) non-symmetric; (ii) symmetric; (iii) symmetric with free boundary at one surface and rigid boundary at the opposite surface; (iv) symmetric slot in the middle of the plate's cross-section; and (v) a slot in the middle of the plate with a free boundary at one half of the slot and rigid boundary at the opposite half. Fig.1 shows the discontinuities types (i) to (v). Different modelling was adopted for treating cases (iii) and (v): either by imposing a fixed boundary condition in the vertical and horizontal walls, marked with a hatched line in Fig.1(iii) and (v), or by filling the void space with a material with high-impedance compared with the plate's acoustic impedance, marked with a grey rectangle in Fig.1(iii) and (v). In all cases generation was imposed in the origin, and discontinuities were located at $x = 100\,mm$, as shown in Fig.1.

Generation was performed by imposing a force in the z-direction, modulated in time by an 8-cycle tone burst at the frequency of interest, applied to all nodes along the cross-section of the mesh, i.e. y-axis, following the displacement profile given by (2), according to the intended mode to be generated. Since modes' profiles form an orthogonal basis, this procedure ensures that only the mode that matches the applied profile is generated.

Reception was done by acquiring the response field along all nodes in the y-axis at a fixed longitudinal position, $80\,mm$ before the discontinuity. This position is represented by the vertical dotted line in Fig.1. Also due to the mode's orthogonality, each SH mode n that composes the response field in (3) was extracted [12], [13] by integrating the product of the response field with $U_n(y)$, along the plate's thickness. After the reflected modes were separated, the amplitude of the reflection coefficients were calculated by the ratio of the peak-to-peak amplitudes of the reflected modes' signal to the incident one, following:

$$R_{nm} = A_m^- / A_n^+, \tag{4}$$

where A_m^- is the peak-to-peak amplitude of the reflected mode m and A_n^+ the peak-to-peak of the reflected mode n.

IV. RESULTS

In order to analyse the interactions in the high frequency-thickness regime, the operating frequency was set to

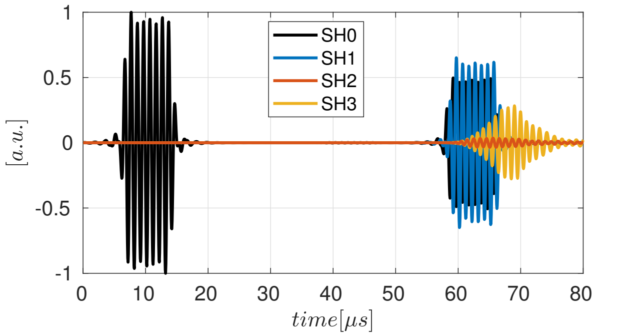

Fig. 2. Interaction of the SH0 mode with a half-thickness non-symmetric discontinuity. Incident mode, at $\sim 10\mu s$, and reflected modes at $\sim 60\mu s$.

$1000kHz$, so modes SH0 to SH3 could propagate with relatively low dispersion. These modes were individually generated and, in each case, separated in the received signal. Fig.2 shows the received mode's waveform when the SH0 mode was generated in a plate with non-symmetric discontinuity with $d/h = 50\%$. At around $10\,\mu s$, one can see the signal corresponding to the excitation of only the SH0 mode, whereas at around $60\,\mu s$ it is evident the presence of reflected modes SH0 to SH3.

All modes (SH0 to SH3) were generated due to mode conversion, being the SH2 the one with lower amplitude. From these modes' signal the reflection coefficients $R_{0m}, 0 \leq m \leq 3$, were calculated. Their values are shown in the first row of Fig. 3(a). The same was done for the remaining modes and discontinuities shapes.

Fig.3 summarises the reflection coefficients for discontinuities types (i), (ii) and (iv). In a symmetric discontinuity there is conversion only to modes that share the same symmetry as the incident one, i.e., from an even (resp. odd) order mode, only even (resp. odd) order modes are created. This is clearly seen from the checkerboard pattern in Fig.3(b) and (c).

Observing the first and third lines of Fig.3(b) and (c) one can see that the reflection coefficient from the SH0 and SH2 are equal for both symmetric discontinuities, type (ii) and (iv). This behaviour for the SH0 mode was observed previously by Pau et al. [8]. This happens because $U_n(y)$ is symmetric in these two cases. Thus interaction with the discontinuity wall is the same either if it is superficial or internal, because $U_n(y)$ along the discontinuity has the same absolute value in both cases. On the other hand, this is not true for the antisymmetric modes, where $U_n(y)$ is zero in the middle of the plate, i.e, $y = 0$.

Fig.4(a) and (b) show the reflection coefficients for discontinuities types (iii) and (v). Due to the opposite boundary condition on both surfaces of the plate (fixed and free), discontinuity type (iii) behaves virtually as a symmetry inverter. That is, when the incident mode is symmetric (resp. antisymmetric), the reflected modes are antisymmetric (resp. symmetric), as it can be seen from the reverse checkerboard pattern of Fig.4(a). It should be noticed that the values of converted modes with same symmetry as the incident mode are very low, but are not zero. Observing Fig.4(b), one can see that interaction with discontinuity (v) presents a trend for inverting symmetry, but far less effective than discontinuity (iii).

In order to assess the capability of a high-impedance material to simulate a fixed boundary in the plate's discontinuity, the root mean square error of the reflection coefficient obtained from simulating discontinuities (iii) and (v) with a fixed boundary constraints and with high-impedance material filling the void of discontinuity was calculated. This was done for all the incident and converted modes as a function of the filling material impedance. As it can be seen in Fig.5, the higher the filling material's acoustic impedance, Z_d, the lower the error; at about an acoustic impedance twenty-fold the plate's impedance, the difference is below 2%. In each cell of Fig.4, the lower triangles stand for the results with fixed boundary condition, whereas the upper triangles when using a filling material with normalised impedance $Z_d/Z = 10$. As it can be seen, this level of impedance mismatch between plate's and discontinuity is enough to efficiently provide the symmetry inversion pattern. This is a relevant result for enabling experimentally evaluation of a symmetry inverter discontinuity.

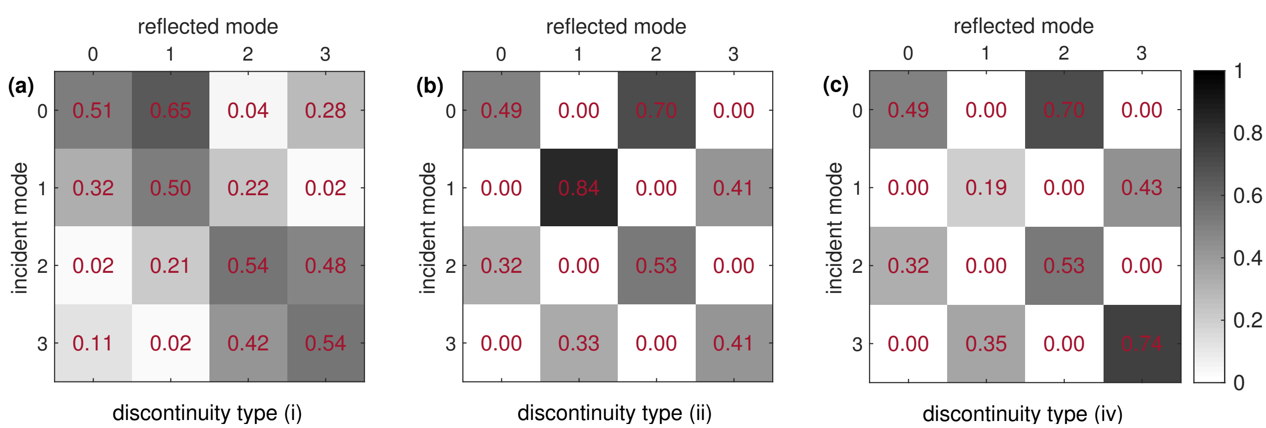

Fig. 3. Reflection coefficients due to the interaction of SH guided wave modes with discontinuities types (i) (ii) and (iv). The colormap represents the intensity of the reflection coefficient. The numerical values of the reflection coefficients are shown in each cell.

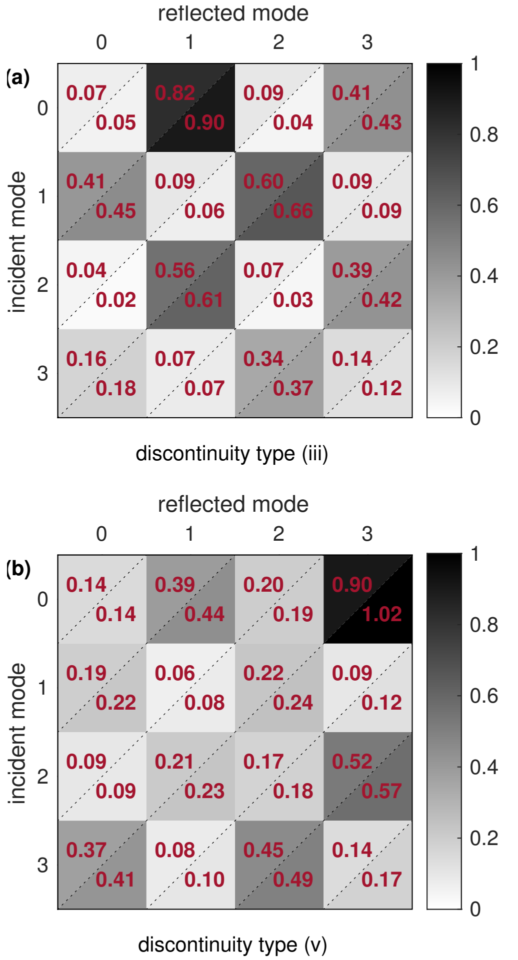

Fig. 4. Reflection coefficients due to the interaction of SH guided wave modes with discontinuities types (iii) and (v). The colormap represents the intensity of the reflection coefficient. Upper triangles represent the case in which the fixed half of the discontinuity is simulated with a filling material whose impadance is $Z_d = 10 \times Z$, whereas lower triangles when using fixed boundary conditions.

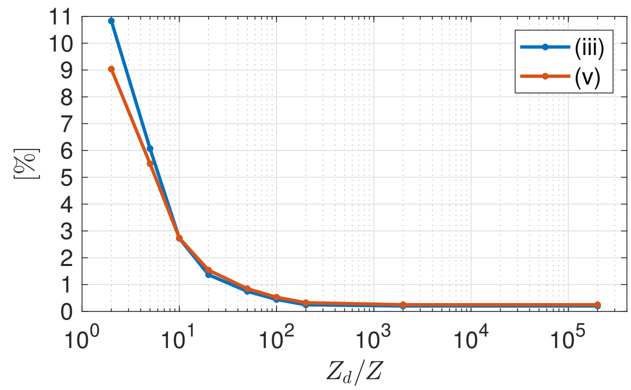

Fig. 5. Root means square error between fixed boundary and discontinuity impedance for discontinuities (iii) and (v) as a function of the discontinuity impedance to plate impedance ratio.

V. Conclusions

Numerical analysis was performed to model the interaction of up to the third order SH guided wave mode with discontinuities in plates. A single SH mode was generated, and the reflected modes were separated by post-processing the received wave field. A symmetric discontinuity restricts mode-conversion, preserving the incident mode symmetry, either if the discontinuity is placed on the plate's surface or in the middle of the plate, because identical boundary conditions are present in both halves of the plate. For symmetric modes, the reflection coefficient is the same for internal and superficial cases, whereas for antisymmetric modes, the scattered field is different due to the mode's displacement profile along the plate's thickness. The investigation of a symmetric discontinuity with opposed free and fixed boundary conditions revealed that a superficial discontinuity virtually restricted mode-conversion to modes with opposite symmetry as the incident one, being more effective as a symmetry inverter than an internal discontinuity with opposite fixed and free boundaries. The fixed boundary condition could be effectively simulated by filling one of the voids used to model a discontinuity with a material with at least ten times the plate's impedance.

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