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FEM-based Resonant Ultrasound Spectroscopy Method for Measurement of the Elastic Properties of Irregular Solid Specimens

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***Abstract*—Resonant ultrasound spectroscopy (RUS) is a well-established method of identifying the elastic coefficients of solid materials. The Rayleigh-Ritz method (RRZ) is usually used to calculate the natural frequencies of a specimen, where the specimen needs to be processed into a certain regular body. This requirement is hard to meet for some materials with size limitations or special physical/chemical properties. Our objective was to address this limitation with a new approach adapted to specimens of arbitrary geometry by combining RUS with micro computed tomography (μ-CT) and finite element modeling (FEM). And also the accuracy of the proposed approach was assessed using titanium irregular specimens. The elastic coefficient results showed good agreements (below 2%) with values of a rectangular parallelepiped specimen measured by the RRZ-*based RUS*. This study overcomes the limitation of RUS to specimen geometry and allows identification of the elastic properties of irregular specimens with good accuracy.**

Keywords—resonant ultrasound spectroscopy, finite element method, irregularly shaped solid materials, elastic coefficients

I. Introduction

The elasticity measurement of solid materials is of great significance in industrial applications, as well as in physics and material science. Traditional measurement methods [1] mainly include quasi-static mechanical testing methods (macroscopic stretching and compression, etc.), sound velocity method and resonant ultrasound spectroscopy (RUS). In these methods, RUS is considered by physicists to be the most accurate method for measuring elastic coefficients of the solid materials with high Q (quality factor) values, such as metal and crystalline materials [2].

RUS method is a combination of the experimental measurement and numerical calculation. For a specimen with determined material properties (including elastic coefficients, symmetry and orientation), geometry and mass density, a mathematical model can be established to calculate the specimen's natural frequencies and the corresponding modes. Therefore, provided other parameters, the elastic coefficients of the specimen material can be inversely obtained from the experimental resonant frequencies measured in the ultrasound resonance experiments.

In RUS, the mathematical model for calculating the resonant frequencies of the specimen is generally solved by the Rayleigh-Ritz method (RRZ). This method is based on the integration of the specimen volume, which requires the specimen to be certain regular geometries with sufficient dimension accuracy, such as rectangular, cylinder, etc [2]. This requirement is hard to meet for some materials with size limitation or special physical/chemical properties [3], [4]. Instead, the finite element method (FEM) provides a feasible solution for the theoretical resonant frequency calculation of irregular specimens [4].

However, due to the fact that the FEM model has large degrees of freedom, researchers mentioned that the classic Levenberg-Marquart (LM) algorithm [2] is no longer suitable for solving the inverse problem when FEM is adopted to calculate the theoretical resonant frequencies [5], [6]. To address this problem, Plesek and Maletta proposed alternative methods: the fixed point iteration method [5] and the genetic algorithm [7]. Subsequently, the FEM-based RUS technique are applied to estimate the elastic coefficients of arbitrary contoured sheets [8] and an imaginary irregular body [6], but the above studies remain in the simulation stage.

To deal with the limitation that the RRZ-based RUS cannot be applied to irregular specimens, this paper proposes a new

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approach combining RUS with micro computed tomography (μ-CT) and FEM. The feasibility and accuracy of this approach are verified by the titanium material.

II. METHOD

A. RUS combined with FEM and μ-CT

The mathematical model for calculating the theoretical resonant frequencies of a specimen is usually described using the following generalized eigenvalue problem [2]:

$$\boldsymbol{\Gamma a} = (2\pi f)^2 \boldsymbol{Ea} \tag{1}$$

Where f is the natural resonant frequency, and $\boldsymbol{a}$ is the corresponding resonant mode. $\boldsymbol{\Gamma}$ and $\boldsymbol{E}$ are the specimen's stiffness matrix and mass matrix, respectively. In (1), $\boldsymbol{\Gamma}$ is the only term containing the elastic coefficients to be tested.

For specimens with complex geometry, the FEM method is adopted to solve the above mathematical model. First, the specimen is scanned using a μ-CT (Skyscan 1272, Bruker Micro-CT NV, Belgium). The scanned images are imported into the Mimics software (Materialise NV, USA) to initially reconstruct the specimen's three-dimensional (3-D) geometry, which is then optimized in Geomagic studio (3D Systems Corporation, USA). With the specimen's geometry acquired, the natural frequencies of the specimen can be calculated using the linear perturbation-frequency module in a commercial finite element software: Abaqus (Dassault Systèmes Simulia Corp., USA).

Given initial guesses of the elastic coefficients, the resonant frequencies and modes of the specimen can be obtained through the FEM model built in Abaqus. Also, a custom-made RUS experimental setup is used to measure the resonant spectra of the specimen, from which the experimental resonant frequencies are extracted [4]. Thereby, an inverse problem can be constructed by fitting the theoretical frequencies and the experimental ones to optimize the elastic coefficients in the FEM model. The method flowchart is shown in Fig. 1.

B. Inverse problem solving using LM

The cost function of the inverse problem is as follows:

$$F(C_{ijkl}) = \sum_{i=1}^{n}\left[(f_i^{cal}(C_{ijkl}) - f_i^{exp}) \;/ f_i^{exp}\right]^2 \tag{2}$$

The key step of using the LM algorithm to solve this inverse problem is to calculate the derivatives of the resonant frequencies to the elastic coefficients [2]:

$$\frac{\partial f_i^{cal}}{\partial C_\alpha} = \boldsymbol{a}_i^{\mathrm{T}} \frac{\partial \boldsymbol{\Gamma}}{\partial C_\alpha} \boldsymbol{a}_i \,/\, 8\pi^2 f_i^{cal} \boldsymbol{a}_i^{\mathrm{T}} \boldsymbol{E} \boldsymbol{a}_i \tag{3}$$

Where f_i^{cal} , $\boldsymbol{E}$, $\boldsymbol{a}_i$ are all known and only the $\frac{\partial \boldsymbol{\Gamma}}{\partial C_\alpha}$ term needs to be calculated. Due to the fact that a linear relationship exists between the stiffness matrix and the independent elastic coefficients [2]:

$$\boldsymbol{\Gamma} = \frac{\partial \boldsymbol{\Gamma}}{\partial C_\alpha} C_\alpha \tag{4}$$

$\frac{\partial \boldsymbol{\Gamma}}{\partial C_\alpha}$ depends only on the geometry, mass density and material symmetry of the specimen. It can be calculated in advance without giving the exact value of C_α, which greatly reduces the calculation cost. And $\frac{\partial \boldsymbol{\Gamma}}{\partial C_\alpha}$ is the specimen's stiffness matrix when C_α equals 1 and other elastic coefficients equal 0.

III. METHOD VALIDATION

The method validation was carried out using the titanium material. First, a rectangular parallelepiped specimen was prepared, and its elastic coefficients were measured by the RRZ-based RUS, which served as the reference values. At the same time, the elastic coefficients of five irregular titanium specimens were measured by the FEM-based RUS and compared with the reference values.

For irregular specimens, the precision of the CT image was 4 microns for an isotropic voxel. In Abaqus, the specimen is discretized into multiple quadratic tetrahedral elements (C3D10). The material symmetry of titanium in the LM optimization was assumed to be orthotropic and the initial values of the elastic coefficients were $C_{11} = C_{22} = C_{33} = 162.8$ GPa, $C_{44} = C_{55} = C_{66} = 42.9$ GPa, $C_{12} = C_{13} = C_{23} = 77.0$ GPa [9].

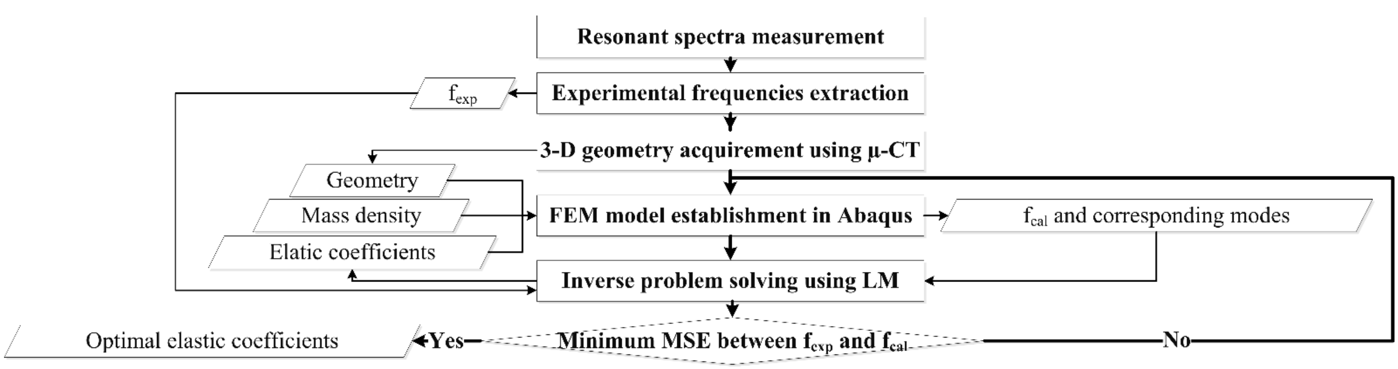

Fig. 1. The flowchart of RUS combined with FEM and μ-CT.

TABLE I. ELASTIC COEFFICIENTS OF THE TITANIUM SPECIMENS

Method	Specimen	Elastic coefficient (GPa) (|Difference|%)								
		C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}	C_{12}	C_{13}	C_{23}
RRZ-based RUS	1	162.20	166.95	169.71	47.22	42.49	40.32	83.40	79.58	74.57
FEM-based RUS	2	160.17	166.71	171.29	47.12	42.64	39.79	83.1	79.05	75.9
		-1.25	-0.15	0.93	-0.21	0.35	-1.31	-0.36	-0.66	1.79
	3	161.14	167.27	170.12	47.49	42.98	39.68	84.02	79.15	75.46
		-0.66	0.19	0.24	0.58	1.15	-1.58	0.74	-0.54	1.20
	4	161.67	168.21	170.36	47.58	42.78	40.28	84.27	78.61	74.66
		-0.33	0.75	0.38	0.77	0.68	-0.09	1.04	-1.22	0.13
	5	161.59	164.88	170.83	47.58	42.72	40.07	82.53	79.11	73.3
		-0.38	-1.24	0.66	0.77	0.54	-0.61	-1.05	-0.59	-1.70
	6	161.48	166.48	169.22	47.24	42.33	39.92	83.61	78.54	74.18
		-0.45	-0.28	-0.29	0.05	-0.38	-0.99	0.25	-1.30	-0.52

The elastic coefficient results of the specimens are shown in Tab. 1. The results show that the relative differences between the elastic coefficients measured by the proposed method and the reference values are within 2%.

IV. CONCLUSIONS

In this paper, a new approach combining RUS with FEM and μ-CT is proposed to estimate the elastic coefficients of irregularly shaped solid materials. The feasibility and accuracy of this method are verified using the titanium material. This study provides a feasible solution for the elasticity measurement of irregular solid materials.

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