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Study of angular gain in lobe-like modes for annular ring bulk acoustic wave (BAW) gyroscopes

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***Abstract*—Wineglass modes are the modes of choice in bulk acoustic wave (BAW) micro-ring gyroscopes owing to their high angular gain and high quality factor in vacuum. The quality factors of most mechanical modes, including the wineglass modes, are significantly lower in liquid medium ambience. While acoustic damping in fluids adversely impacts quality factor in modes with longitudinal motion, certain in-plane flexural modes in such resonators retain higher quality factors in liquid media. In this paper, we study the angular gain of a family of such in-plane flexural modes, termed as lobe-like modes. We present a simulation framework to validate the high quality factors of these modes in liquid medium ambience. We study the angular gain for these modes, and present a design insight to optimize the geometry of the micro-rings to maximize the angular gain.**

***Index Terms*—BAW gyroscope, offset drift, liquid medium operation, lobe-like mode, wineglass mode**

I. Introduction

In addition to finding widespread acceptance in industrial, automotive and consumer applications, MEMS inertial sensors continue to push the state-of-the-art in vibratory gyroscopes [1]. Two important specifications for tactical and navigation grade MEMS gyroscopes are the bias instability and rate ramp, which are significantly influenced by resonator self-heating, and other environmental sources of offset drift. Conventionally MEMS gyroscopes require vacuum packaging to ensure high quality factors and thereby low noise, which leaves no method to address the resonator self-heating. A possible solution is to immerse the resonator in a liquid environment. However, viscous energy losses tend to greatly reduce the quality factor of resonance in these environments [2], [3]. Recent work by Ali et al. [4] suggests that certain in-plane flexural shear modes, termed as button-like (BL) modes, in micro-disk resonators fabricated in a piezo-on-silicon platform retain a higher quality factor in liquid media as compared to the conventionally used wineglass (WG) modes. The authors primarily focused on timing applications, and did not study the angular gain for such modes. In this work, we have studied the angular gain of the BL modes, and another family of in-plane flexural modes that we call lobe-like (LL) modes. Figure 1 illustrates the various mode shapes studied in this paper. We present a finite element method (FEM) simulation framework to validate the high quality factors in such modes as compared to WG modes in a fluid environment, and extract the angular gain from eigenmode simulations. The layout of this paper is as follows: the next section describes the design intuition underlying the angular gain study conducted in this work. Section III supports the design intuition through a mathematical construction for analyzing the hypothesis, describing the methodology to calculate angular gain through an eigenmode FEM simulation. Section IV describes the solid geometry chosen in this work and the different simulation studies conducted. Section V describes the results of the quality factor simulations in fluid media for the WG, BL and LL modes, and provides additional insights for modifying the geometry to optimize the angular gain for the LL mode family.

Fig. 1. This work simulates and compares the liquid medium quality factors and angular gains of the wineglass (WG), button-like (BL) and lobe-like (LL) modes. In the case of the LL modes, the angular gain is also studied for annular ring geometries, to identify the optimal geometry for maximum angular gain.

II. Design Intuition

This work is inspired by previous work by Ali et al. [4], where high quality factors for in-plane flexural BL modes in micro-disk resonators immersed in liquid media were investigated. This study prompted us to explore other flexural shear modes that possess high angular gain in addition to retaining higher quality factor than WG modes in liquid media. For a vibrating body rotating about a stationary axis, the angular gain (A_g) is the the ratio of the angle rotated by the vibration pattern to the angle rotated by the body about the axis in a certain time. The angular gain is also referred to as Bryan's Factor, named after G. H. Bryan who discovered this phenomenon in 1890 [5]. While this interpretation is useful to understand the dynamics of the disk, another intuitive interpretation of the angular gain is a measure of the coriolis mass, or the degree of 'perpendicularity' of the displacement in the corresponding mode shapes of the orthogonal modes chosen for gyroscopy [6]. Tuning fork gyroscopes, with the orthogonal gyroscopy modes aligned along perpendicular axes, have the maximum possible angular gain $= 1$. The angular gain is also inversely proportional to the effective mass of the resonance mode [6]. Therefore, to improve the angular gain

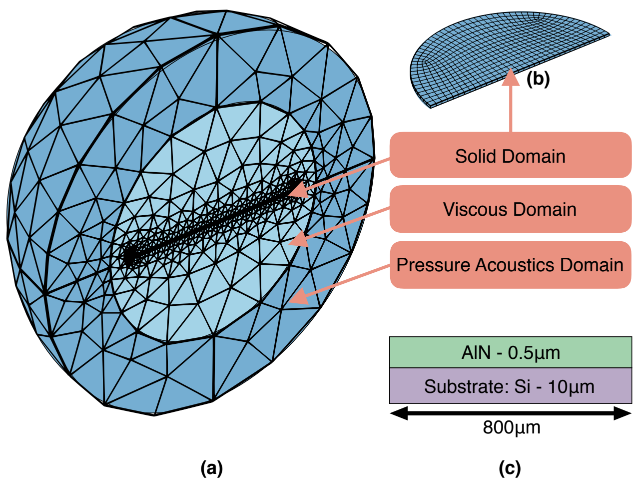

Fig. 2. (a) Isometric view of the simulated geometry. (b) Single layer distributed swept mesh of solid domain (disk). The mesh element size is chosen to be comparable to the viscous penetration depth of the respective medium. (c) Lateral cross-section of the solid piezo-on-Si disk.

of a mode, we must find ways to either increase coriolis mass or to reduce its effective mass, or both.

This intuition led us to investigate LL modes that are flexural shear modes (see Figure 1). The low displacement region around the center allows us to introduce a central hole to convert the geometry into an annular ring without altering the high displacement areas of the mode shape. The hole radius is an additional design parameter. Introducing the hole allows us to tune the coriolis and effective masses, thereby modifying the angular gain. This study focuses on identifying a methodology to appropriately tune and maximize the angular gain of the LL modes.

III. Angular Gain

In this section, the mathematical framework to calculate angular gain is presented [6], [7]. For a given eigenmode, the displacement (u) along the i^{th} direction is denoted as $u_i(x, y, z)$, and can be expressed as a superposition of generalized displacements due to the two orthogonal modes $(q_j, j = 1, 2)$. The proportionality constants in the superposition sum are called shape functions (alternatively, the modeshapes), denoted as $\phi_{ij}(x, y, z) \in [0, 1]$:

$$u_i(x, y, z) = \sum_{j=1,2} \phi_{ij}(x, y, z) q_j \tag{1}$$

The shape function $\phi_{ij}(x, y, z)$ denotes the displacement (normalized to maximum displacement at corresponding antinode) due to the j^{th} mode along the i^{th} direction at a coordinate (x, y, z).

The velocity of a body rotating in inertial frame (with angular rate $\overline{\Omega}$) can be written as:

$$\overline{v}_{inertial} = \dot{\overline{u}}(x, y, z) + \overline{u}(x, y, z) \times \overline{\Omega} \tag{2}$$

The displacement is specified as a vector $\overline{u}(x, y, z)$, comprising of components u_i along each direction in the coordinate system (this paper deals with planar modes, so only X and Y axes are considered). The velocity $\overline{v}_{inertial}$ is used to compute the kinetic energy, that in turn has three terms: two due to contributions from the individual modes themselves, and the third due to the coriolis coupling between the two modes. Expressing each kinetic energy term in the form of $\frac{1}{2}\mathrm{Mv}^2$, we thus get three masses: two effective masses $(\mathrm{M_{eff}})$ for the individual modes and a coriolis mass (γ):

$$\mathrm{M_{eff}} = \int \rho \left(\phi_{xi}^2 + \phi_{yi}^2 + \phi_{zi}^2\right) dV \tag{3}$$

$$\gamma = \int \rho \left(\phi_{x1}\phi_{y2} - \phi_{y1}\phi_{x2}\right) dV \tag{4}$$

The intuitive description for $\mathrm{M_{eff}}$ and γ is presented in the previous section. The two modes have the same effective mass, as we have assumed degeneracy. The angular gain is then derived as [6], [7]:

$$A_g = \frac{\gamma}{n\mathrm{M_{eff}}} \tag{5}$$

Here n is the mode order, which in case of a disk resonator, determines the number of angular nodes and the roots of the radial and circumferential displacements [8].

IV. Geometry & Simulation Setup

For the finite element method (FEM) simulation in COMSOL Multiphysics software, we use the same Piezoelectric-On-Insulator geometry described in [4] sans the anchors and support structure. The geometry is a microdisk with a $400\,\mu\mathrm{m}$ radius, comprising of a $0.5\,\mu\mathrm{m}$ thick layer of aluminium nitride (AlN) atop a $10\,\mu\mathrm{m}$ thick layer of silicon (Si). The cross-section of the disk is shown in Figure 2(c). For the FEM simulation, we need to discretize the continuous geometry using a mesh. Therefore, the integrals in equations 3 and 4 are re-written as summations over all mesh elements. The displacement contributions due to all eigenmodes at each node in the mesh are obtained by performing an eigenfrequency analysis on the respective geometry. The eigenmodes of this structure are computed in two environments:

Fluid medium (Air/Water): Our objective is to validate that the LL and BL modes retain higher quality factor in water as compared to the WG modes. A fluid (air/water) domain is setup around the solid disk to compute the quality factor of the WG, LL and BL modes, as shown in Figure 2.

Loss-less: We wish to achieve as high an A_g as possible to improve the sensitivity of the LL mode gyroscope. Equation 5 suggests that A_g increases when the coriolis mass increases. Trimming the non-moving regions of the resonator by introducing an annular hole in the center of the disk, should result in increase in the coriolis mass, with minimal changes in the effective mass, thereby increasing the angular gain. This hypothesis is investigated through eigenmode simulations that do not consider any losses.

A. *Simulation of quality factor in liquid medium*

Figure 2(a) illustrates the simulation framework set up in COMSOL Multiphysics. A spherical fluid domain is set up around the bi-layered disk. Thermoviscous effects are modelled in this domain using the Thermoviscous Acoustics node in COMSOL. The bulk vibrations of the disk induce pressure acoustic waves in the fluid. Hence an encapsulating pressure acoustics domain is created using the Pressure Acoustics node. Discretization of the solid domain is performed using a quadrilateral swept mesh with a single element distribution through each layer as shown in Fig. 2(b). Since we are only interested in in-plane flexural modes, higher mesh resolution along the thickness is unnecessary. The mesh element size is chosen to be comparable to the viscous penetration depth in the respective fluid ($0.22\,\mathrm{mm}$ and $0.057\,\mathrm{mm}$ at $100\,\mathrm{Hz}$, $1\,\mathrm{atm}$ pressure and 20°C for air and water respectively). At other frequencies (f), the penetration depth is expressed as:

$$d_{visc}(f) = d_{visc}^{100}\sqrt{\frac{100}{f}} \tag{6}$$

where d_{visc}^{100} is the viscous penetration depth or boundary-layer thickness at $100\,\mathrm{Hz}$ [9]. Within the solid disk geometry, thermoelastic damping physics is included in the material through a multiphysics coupling between the Heat Transfer and Solid Mechanics modules in COMSOL.

A spherical fluid domain of radius $0.5\,\mathrm{mm}$ is setup around the bi-layered disk to account for the thermoviscous interaction between the solid and the fluid domain. This radius is sufficient considering the penetration depths of air and water at frequencies of our interest. A pressure acoustics domain of thickness $0.3\,\mathrm{mm}$ is setup around the thermoviscous domain. This domain is assigned a spherical radiation boundary condition so that any outgoing acoustic waves pass through with minimal reflections. These two domains are meshed using a free tetrahedral mesh as shown in Figure 2(a). To speed up the computation, 2-fold symmetry of the targeted elliptical modes are exploited and only half the geometry is simulated. Eigenfrequency analysis is performed to calculate the quality factor for the WG, BL and LL modes, based on the complex eigenfrequencies obtained as result of simulation of acoustic losses in the fluid.

B. *Simulation of angular gain*

The angular gain simulation requires obtaining the spatial displacement profiles for the set of orthogonal eigenmodes, and can be obtained from the same eigenfrequency simulation described above. Since the damping effects need not be considered for calculation of angular gain, the surrounding fluid environment is not incorporated. For modes with mode order $n = 2, 3, 4, 5$, a hole with radius R_{in} is introduced as shown in Figure 3. The annular radius R_{in} is varied from $0\,\mu\mathrm{m}$ to $120\,\mu\mathrm{m}$ in steps of $10\,\mu\mathrm{m}$, while maintaining R_{out} at $400\,\mu\mathrm{m}$. The eigenmodes are computed using the Solid Mechanics module and displacement data for all mesh nodes for the orthogonal eigenmodes are exported to a comma separated value (csv) file, which is then processed in MATLAB to obtain the angular gain using equations 3, 4 and 5.

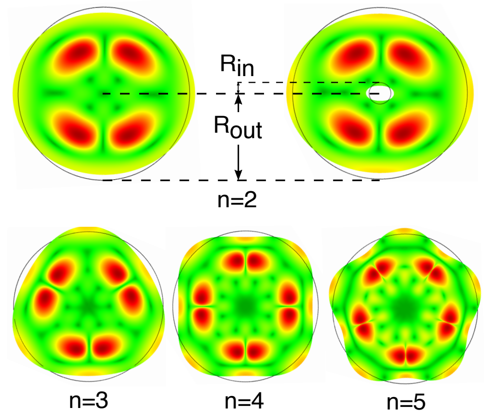

Fig. 3. Surface displacement profiles of multiple orders (n) of LL modes (green lowest, red highest). In this work, we studied the impact of introducing a hole of radius R_{in} on the angular gain of these modes.

V. Results

The simulation results for thermoelastic and viscous damping limited quality factors for the $n = 2$ mode of all three families are provided in Table I. As expected, the viscous damping limited quality factor in water for the BL and LL modes (shear-modes) are higher than that of the WG mode (bulk longitudinal-mode). The quality factor for the BL mode obtained through simulation is in the same order of magnitude as experimental results reported by Ali et al. [4]. The BL and LL modes are observed to have quality factors in same order of magnitude in the simulation results. The high quality factor in fluid media paves the way for a discussion about the angular gain of these modes for potential use in gyroscopy.

TABLE I
Simulation results in air and water medium

Parameter	Medium	WG mode	BL mode	LL mode
Frequency (MHz)	Air	5.05	16.33	18.78
	Water	4.54	14.77	16.96
Quality factor*	Air	4.3×10^4	1.25×10^5	1.94×10^5
	Water	88.17	191.79	233.36

*The quality factor simulation only accounts for losses due to thermoelastic and viscous damping.

Figure 4(a) shows the variation in angular gain with R_{in} for $n = 2, 3, 4, 5$ ordered modes of LL mode family. The curves for $n = 2$ and $n = 3$ modes show a peak at a certain radius value and then decrease for increasing R_{in}. For higher order modes, this peaking behavior is not observed. The peak A_g for $n = 3$ mode is observed at higher R_{in} as compared to $n = 2$ mode ($70\,\mu\mathrm{m}$ v/s $40\,\mu\mathrm{m}$).

Figure 4(b) and (c) show the plot of coriolis and effective masses versus R_{in} for the $n = 2$ and $n = 3$ modes respectively. As described in section IV, the effective mass for the $n = 2$ mode does not show large variation for $R_{in} > 0$,

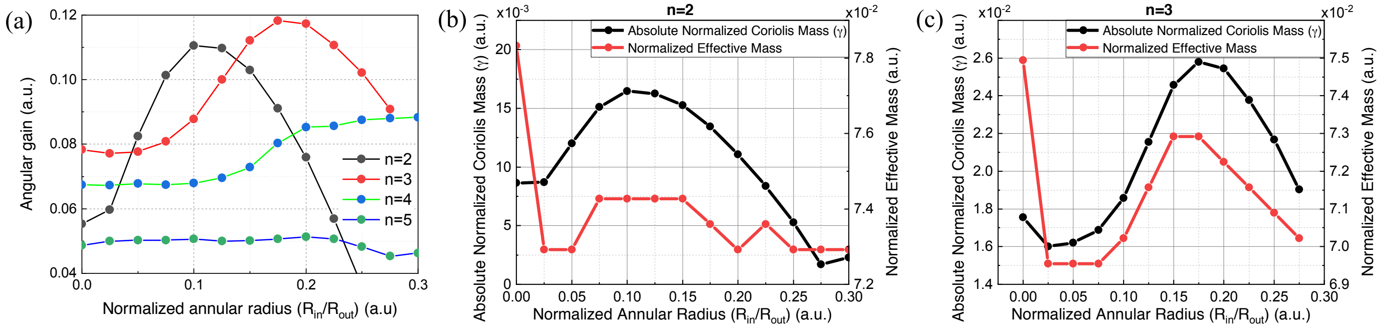

Fig. 4. (a) Variation of angular gain of various orders of LL modes, with increasing inner radius (R_{in}) of the annulus. (b) Variation in coriolis and effective masses of the $n = 2$ LL mode with increasing inner radius (R_{in}) of the annulus. (c) Variation in coriolis and effective masses of the $n = 3$ LL mode with increasing inner radius (R_{in}) of the annulus.

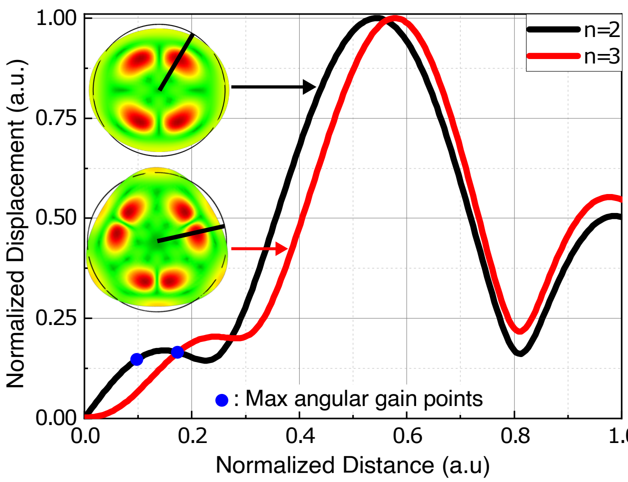

Fig. 5. Displacement profile along a radial segment passing through antinode (see insets) for $n = 2$ and $n = 3$ LL modes, plotted versus the radial distance normalized to R_{out}. Introducing an annular hole leads to increasing coriolis mass, as well as distortion of the mode-shape, thereby resulting in reducing angular gain beyond a critical value of R_{in}. The critical value of R_{in} is higher for the $n = 3$ mode since the antinode is located at a larger radial distance from the center of the disk, as compared to the $n = 2$ mode.

while the correlation between the the variation of coriolis mass and angular gain with R_{in} is clearly noticed. While the effective mass for $n = 3$ mode does show a peaking trend for increasing R_{in}, the coriolis mass peak occurs at a larger value of R_{in} than the effective mass peak and thus the peaking trend is also observed in angular gain (panel (a)). Higher modes do not show this trend because increasing R_{in} alters the mode shape and negates the benefit of introducing an annular hole.

The relative difference in optimal R_{in} values for maximum angular gain for $n = 2$ and $n = 3$ LL modes can be explained using Figure 5. The radial displacement for a disk geometry $(R_{in} = 0)$ is plotted along a segment passing through one of the high displacement lobes for these modes, as shown in the insets. The antinode for the $n = 3$ mode is located at a large radial spacing as compared to the $n = 2$ mode, and thus the $n = 3$ mode benefits from introducing a larger annular hole for maximum angular gain as compared to the $n = 2$ mode.

VI. Conclusion

The study presented in this paper explores the suitability of the LL modes for designing BAW gyroscopes with the resonator immersed in a liquid medium for potentially managing offset drifts due to resonator self heating. The LL modes exhibit in-plane shear deformations, and are validated with FEM simulations to exhibit higher quality factors in liquid medium as compared to the conventionally employed WG modes. We also present a systematic study of the optimization of angular gain in such LL modes by introducing an annular hole. Our insights suggest that micro-ring geometries are better suited for LL mode gyroscopes as compared to micro-disks, and we present a methodology for optimization of the annular ring radius to maximize angular gain.

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