

# A Novel Mathematical Model for Transit-time Ultrasonic Flow Measurement

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**Abstract** — The calculation of the averaged flow velocity along an ultrasonic path is the core step in ultrasonic transit-time flow measurement. The conventional model for calculating the path-averaged velocity does not consider the influence of the flow velocity on the propagation direction of the ultrasonic wave and can introduce error when the sound speed is not much greater than the flow velocity. To solve this problem, a new mathematical model covering the influence of the flow velocity is proposed. It has been found that the same mathematical expressions of the path-averaged flow velocity, as a function of the absolute time-of-flight (ToF) of ultrasonic waves travelling upstream and downstream, can be derived based on either of the models. However, the expressions as a function of the time difference (the relative ToF) between the ultrasonic waves travelling upstream and downstream derived by the two models are completely different. Flow tests are conducted in a calibrated flow rig utilising air as flowing medium. Experimental results demonstrate that the path-averaged flow velocities, calculated using either the relative or the absolute ToFs based on the new model, are much more consistent and stable, whereas those calculated based on the conventional model have shown evident and increasing discrepancy when the flow velocity exceeds 15 m/s. When the flow velocity is around 39.45 m/s, the discrepancy is as high as 0.38 m/s. As the relative ToF can be more accurately, reliably and conveniently measured in real applications, the proposed mathematical model has a great potential for the increase of the accuracy of the ultrasonic transit-time flowmeters, especially for the applications such as the measurement of fluids with high flow velocities.

**Keywords**—ultrasonic transit-time flow measurement, ultrasonic flowmeter, time-of-flight, mathematical model, ultrasonic transducer.

## I. INTRODUCTION

Ultrasonic transit-time flowmeters have many advantages, including no moving parts, high rangeability, bidirectional flow capability and fully piggyback, and have been successfully utilised for decades in various fields, such as custody transfer in the oil and gas industry [1, 2]. In ultrasonic transit-time flow measurement, two contra-propagating ultrasonic waves are

transmitted and received through a same path by a pair of ultrasonic transducers which are placed respectively upstream and downstream of the flow. By measuring either the absolute times-of-flight (ToF) of the ultrasonic signals travelling upstream and downstream through the path or the time difference (relative ToF) between the upstream and downstream signals, the averaged flow velocity along the path can be predicted. By using multiple pairs of transducers and various configurations of ultrasonic paths, together with appropriate calibration factors and correction factors or numerical integration algorithms, the area-averaged flow velocity and the flow rate can be obtained based on the measured path-averaged flow velocities.

One core step directly determining the accuracy and the uncertainty of the ultrasonic transit-time flow measurement is the measurement of the path-averaged flow velocities. A conventional model for the calculation of the path-averaged velocities has been broadly adopted by most ultrasonic transit-time flowmeters [1, 2]. In this model, the sound speed is assumed to be much greater than the flow velocity, and thus the propagation direction of the ultrasonic waves are assumed to be the same as the direction of the sound speed. In theory, this assumption only holds true when ultrasonic wave propagates in a zero-flow medium, and it can introduce errors due to the sound beam drift effect in the applications of high-velocity (such as over 20 m/s) flow measurement.

To improve the accuracy of the model, a novel mathematical model with a consideration of the influence of the flow velocity is proposed in this paper. In both of the conventional and the new models, the path-averaged flow velocity can be expressed either as a function of the absolute ToFs or as a function of the relative ToF. Consequently, four mathematical expressions of the path-averaged flow velocity as a function of either the absolute or the relative ToFs are derived and compared, and their performances are experimentally investigated based on our developed flowmeter [3, 4].

## II. METHODOLOGY

The conventional model broadly utilised in ultrasonic transit-time flowmeters for calculating the path-averaged flow velocity is shown in Fig. 1 [1, 2], where the centre of the front face of the ultrasonic transducers are represented by a red dot for simplicity. Note that in this model, the sound speed is in the same direction of the propagation of the ultrasonic wave.

Assuming the length of the path is  $L$ , the angle between the ultrasonic path and the inner wall of the pipe is  $\varphi$ , the sound speed is  $c$  and the averaged flow velocity along the ultrasonic path is  $v_L$ , this conventional model can be mathematically expressed by (1).

$$\begin{cases} L = (c + v_L \times \cos\varphi) \times t_d \\ L = (c - v_L \times \cos\varphi) \times t_u \end{cases} \quad (1)$$

where  $t_d$  and  $t_u$  is the absolute ToF of ultrasonic waves traveling downstream and upstream respectively.

The average flow velocity along the ultrasonic path can be deduced by removing the sound speed  $c$  in (1), as shown by (2).

$$v_L = L^2(t_u - t_d)/(2Xt_u t_d) \quad (2)$$

where  $X$  is the axial distance between the two transducers.

Equation (2) indicates that the accuracy of the measurement of flow velocity  $v_L$  is largely influenced by the accuracy of the measurement of the absolute ToFs  $t_u$  and  $t_d$  of ultrasonic waves and the relative ToF between  $t_u$  and  $t_d$ .

In practice, it is difficult to accurately measure the absolute ToF of ultrasonic waves. During the measurement of the ToF of the ultrasonic waves, time delays are introduced by the transducers and the electronic systems in the measurement of the absolute ToF; the time delays of the transducers are influenced by the dynamic performance of the transducers and can vary under different operating conditions. Moreover, mechanical and electrical noises exist in the measurement, and makes it difficult to accurately identify the real time of arrival of the ultrasonic waves.

In contrast, the relative ToF between  $t_u$  and  $t_d$  can be more accurately and easily measured by comparing the difference between the ultrasonic signals traveling upstream and downstream utilising algorithms such as cross correlation. Therefore, the averaged flow velocity along the ultrasonic path is  $v_L$  can also be derived based on the conventional model from (1), as a function of the relative ToF, which is shown by (3) and (4).

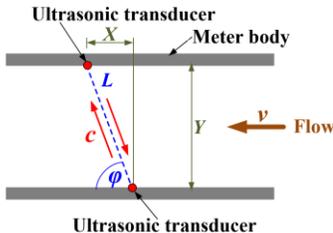


Fig. 1. Conventional model of the calculation of the averaged flow velocity along the ultrasonic path, showing that the sound speed is in the same direction of the propagation of the ultrasonic wave.

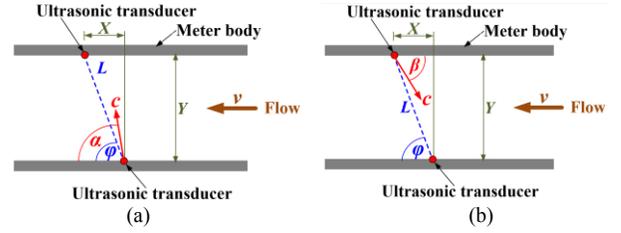


Fig. 2. A new model, considering the influence of both sound speed and flow velocity on the direction of ultrasonic path, for ultrasonic transit-time flow measurement, showing ultrasonic wave travels (a) downstream, and, (b) upstream.

In reality, the propagation direction of ultrasonic waves in flowing media is determined by both the sound speed and the flow velocity. Assuming the propagation direction is the same as the direction of the sound speed, errors can arise in the conventional model, which is particularly true when the sound speed is not much greater than the flow velocity.

Consequently, a new model for ultrasonic transit-time flow measurement is proposed which considers the influence of both sound speed and flowing fluids on the direction of the ultrasonic path. The new model is shown in Fig. 2, where the direction of the sound speed is different from the propagation direction of the ultrasonic waves due to the influence of the flow.

In the new model, when ultrasonic wave travels downstream, we obtain (5).

$$\begin{cases} X = (c \times \cos\alpha + v_L) \times t_d \\ Y = (c \times \sin\alpha) \times t_d \end{cases} \quad (5)$$

And when ultrasonic wave travels upstream, we have (6).

$$\begin{cases} X = (c \times \cos\beta - v_L) \times t_u \\ Y = (c \times \sin\beta) \times t_u \end{cases} \quad (6)$$

Solving (5) and (6), the averaged flow velocity along the ultrasonic path as a function of the absolute ToF is shown by (7).

$$v_L = L^2(t_u - t_d)/(2Xt_u t_d) \quad (7)$$

Likewise, the averaged flow velocity along the ultrasonic path can also be expressed as a function of the relative ToF, which is shown by (8).

$$v_L = [\sqrt{X^2 + c^2(t_u - t_d)^2} - X]/[(t_u - t_d)] \quad (8)$$

When comparing (2) and (7), one interesting outcome is that the expressions of the averaged flow velocity as a function of the absolute ToF, respectively derived based on the conventional and the new model, are exactly the same. This suggests that if the absolute ToF is known, there should not be any difference in the calculated path-averaged flow velocity respectively derived based on the conventional and the new models. However, the expressions as a function of the relative ToF which are shown in (4) and (8) are completely different, which indicates that if the relative ToF is adopted to predict the average flow velocity along the ultrasonic path, errors can be

introduced if the influence of the flow velocity on the direction of ultrasonic path is not considered.

### III. EXPERIMENTS

To evaluate the performance of the new model, both of the absolute and the relative ToFs are obtained based on our previously developed flowmeter. The path-averaged flow velocities are then respectively calculated using either the absolute ToF or the relative ToF based on the two models. Finally, the calculated velocities will be compared.

The developed 6-inch flowmeter incorporates a flexural ultrasonic phased array transducer and a single transducer. The single transducer faces the array transducer at a 30° angle, as shown in Fig. 3 [3, 4]. Sixteen independent ultrasonic paths are thus formed between each array element and the single transducer, and the paths are numbered from 17 to 32 according to the specific numbering of their corresponding array element.

The absolute ToF in the flowmeter at zero-flow state is firstly calculated based on the path length and the theoretical sound speed. The medium in the flowmeter is dry air, whose temperature is continuously monitored during tests. The temperature at the zero-flow state is 20°C, and the sound speed can be calculated based on (9) [5].

$$c \approx 331.45 \sqrt{1 + \frac{T}{273}} \quad (9)$$

where  $T$  is the absolute temperature.

Therefore, the theoretical absolute ToF for any given length of ultrasonic paths at the zero-flow state can be calculated by (10).

$$t = \frac{L_i}{c} \quad (10)$$

where  $L_i$  represents the length of the ultrasonic paths between each array element and the single transducer.

Flow tests of the flowmeter have been conducted with a commercial flow rig at Honeywell Process Solutions, Mainz, Germany. The flow rig is an open loop using air as the flowing medium. Ultrasonic signals travelling upstream and downstream between the single transducer and the sixteen array elements are recorded at different flow rates ranging from 0 to 2500 m<sup>3</sup>/h with a step of 100 m<sup>3</sup>/h. The time difference in the ToFs between non-zero and zero flow conditions is calculated using cross correlation algorithm. It has been found that the temperature of the air flowing through the flowmeter does not obviously change during the tests. Consequently, the difference in the ToF is primarily due to the change of the flow rates of air, and in this experimental set-up, the absolute ToFs at different flow velocities can thus be deduced based on the measured time difference and the theoretical absolute ToF at the zero-flow state.

The ultrasonic paths 22 and 23 respectively defined by array elements 22 and 23 are taken as examples in this study. The absolute ToFs of the ultrasonic waves travelling upstream and downstream along the two ultrasonic paths at different flow velocities are shown in Fig. 4, where the relative ToFs between the ultrasonic signals travelling upstream and downstream can also be derived based on Fig. 4.

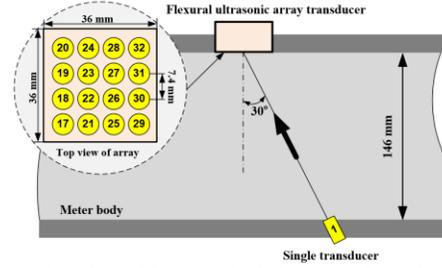


Fig. 3. Cross-section view of the meter body incorporating a flexural ultrasonic phased array transducer and a single transducer.

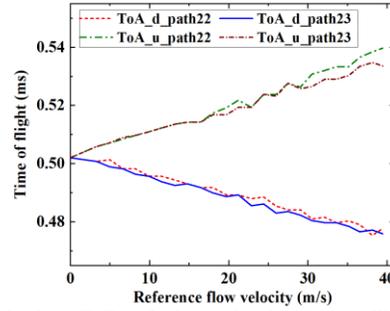


Fig. 4. The absolute ToFs of ultrasonic signals travelling upstream and downstream along paths 22 and 23 at different flow velocities. The relative ToFs between ultrasonic signals travelling upstream and downstream can also be calculated based on the absolute ToFs.

Averaged flow velocities along the two paths are respectively calculated using either the absolute or the relative ToFs, based on the conventional and the new models. As paths 22 and 23 are symmetrical about the diametral plane of the meter body, the arithmetic mean of the path-averaged velocities of the two paths are further calculated to suppress the influence of the circumferential velocity of the flow, and are shown in Fig. 5, where Vel\_Abs\_ToF represents the path-averaged flow velocity calculated utilising the absolute ToFs based on either of the models, Vel\_Rel\_ToF\_New\_Model denotes the path-averaged flow velocities calculated utilising the relative ToFs based on the new model, and Vel\_Rel\_ToF\_Conv\_Model denotes the path-averaged flow velocities calculated utilising the relative ToFs based on the conventional model. All of the calculated flow velocities shown in Fig. 5 strongly correlate with the reference velocity.

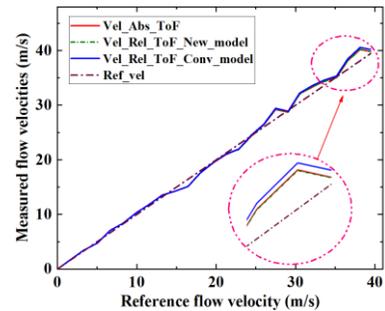


Fig. 5 The path-averaged flow velocities respectively calculated utilising either the absolute or the relative ToFs based on the conventional and the new models, showing that all calculated flow velocities strongly correlate with the reference velocity, but evident differences among the calculated path-averaged flow velocities can be found at high flow velocities.

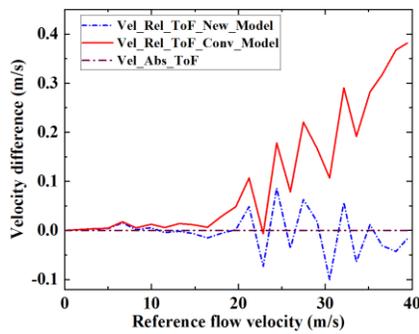


Fig. 6 The deviation of path-averaged flow velocity calculated utilising the relative ToF from the velocity calculated utilising the absolute ToF. This figure shows that the discrepancy between the calculated velocities using the relative ToF and using the absolute ToF, based on the conventional model, progressively increase with the rise in flow velocity.

There are always some differences between the path-averaged flow velocity and the area-averaged flow velocity (the reference flow velocity in Fig. 5) largely due to the uneven distribution of the flow velocity profile in a pipe. However, if the reference flow velocity is regarded as the accurate flow velocity of the flow, calibration factors can be drawn to compensate for the differences. Moreover, evident differences among the calculated path-averaged flow velocities can be found at high flow velocities, as shown in Fig. 5.

The flow velocities calculated utilising the absolute ToFs based on either of the models are the same, and the differences between the path-averaged velocities calculated using the relative ToF and using the absolute ToF are further compared, as shown in Fig. 6. This figure demonstrates that when we use the conventional model to calculate the path-averaged velocity, the discrepancy between the calculated velocities using the relative and the absolute ToFs progressively increases, which is particularly true in this experimental set-up when the flow velocity is greater than 15 m/s. When the flow velocity is around 39.45 m/s, the discrepancy is approximately 0.38 m/s, which is 0.96% of the flow velocity, and the root mean square deviation from the path-averaged velocities calculated based on the absolute ToF is 0.17 m/s. In comparison, when the calculation is conducted through the new model, the discrepancy is generally smaller than 0.1 m/s, fluctuating around zero most likely due to the influence of the chaotic turbulence of the flow at different flow velocities, and the root mean square deviation from the velocities calculated based on the absolute ToF is only 0.04 m/s.

In theory, the path-averaged flow velocities calculated either via the absolute or relative ToFs should be the same. As the conventional model fails to consider the influence of the flow velocity and assumes the direction of the sound speed is the same as the ultrasonic propagation direction, a larger discrepancy between the velocities calculated based on the absolute and the relative ToFs emerges. Comparing with the conventional model, it is more reliable and accurate to calculate

the path-averaged flow velocity using the relative ToF information based on the proposed new model. In practice, the relative ToF between the ultrasonic waves travelling upstream and downstream can be more reliably and accurately measured, and the influence of the time delays due to the dynamic characteristic of the transducers on the relative ToFs can be greatly suppressed. Consequently, the new model is more feasible and accurate for the calculation of the averaged flow velocity along an ultrasonic path in ultrasonic transit-time flow measurement.

#### IV. CONCLUSIONS

A new mathematical model considering the influence of the flow velocity on the propagation direction of ultrasound for ultrasonic transit-time flowmeters is proposed. Comparing with the conventional model, the same mathematical expression of the path-averaged flow velocity as a function of the absolute ToFs of ultrasonic waves travelling upstream and downstream can be derived based on the new model. However, the expression as a function of the time difference (the relative ToF) between the ultrasonic waves travelling upstream and downstream derived by the two models are completely different. Flow tests have demonstrated that the flow velocities respectively calculated using the relative and the absolute ToFs based on the conventional model exhibit an increasing discrepancy when the flow velocity exceeds 15 m/s, demonstrating that using the relative ToF to predict the flow velocity based on the conventional model can introduce error for high flow-velocity flow measurement. In comparison, the new model incorporating the influence of the flow velocity produces more consistent predictions of the path-averaged flow velocity using the relative ToF information, showing a great potential for increasing the accuracy of ultrasonic transit-time flowmeters for high-velocity flow measurement.

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