# Lamb waves Direction of Arrival estimation based on wavelet decomposition

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Abstract—Guided waves in Structural Health Monitoring permit to detect different types of damage inside a structure, such as cracks, as well as external impacts. A precise localisation of the damage or the impact point is necessary in order to improve the reliability and accuracy of monitoring approaches. A localisation algorithm based on the estimation of the waves Direction of Arrival (DoA) by means of a DFT-Based Continuous Wavelet Transform decomposition is presented in this work. The approach exploits a multiresolution DFT-Based CWT decomposition applied to the signals acquired by cluster of three piezoelectric (PZT) transducers in 60 degrees configuration. The multiple frequency-based filtering of the signals in the wavelet domain counteracts the complexity caused by the dispersive characteristics of the material. Subsequently the cross-correlation method and the angle estimation are applied for each computed scale. Finally, the Direction of Arrival is computed by an averaging procedure across scales. The approach is validated by means of an experimental setup. The results show the accuracy of the proposed algorithm, enabling to estimate the DoA with an average error of 1.15 degrees and a maximum error of 1.74 degrees.

Index Terms—SHM, GWs, localization algorithm, wavelet decomposition

# I. INTRODUCTION

In the last decades, Guided Waves (GWs) are rising as a widely adopted Structural Health Monitoring (SHM) inspection technique, especially in the specific case of guided stress waves within thin structures, i.e. Lamb-like Waves. Studies have shown that it is possible to perform damage detection, damage identification, e.g. cracks, delaminations, corrosion damages, impacts and others [1], and localisation through GWs methodologies [2]. In particular, reliable and precise localisation algorithms are fundamental in order to enhance the monitoring infrastructure, maximise time and thus reduce its cost. In *passive-only* sensor networks, in which only passive transducers are used, different strategies for damage localisation can be exploited, such as the direct approach, in which *ad hoc* transducers capable of revealing the wave Direction of Arrival (DoA) are developed, inverse optimisaNicola Testoni ARCES-Advanced Research Center on Electronic Systems 40123 Bologna, Italy nicola.testoni@unibo.it

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tion methods based on updating neural networks and genetic algorithms, or hyperbolic positioning methods [3]. In the latter case, the uncertainty about the Time of Arrival estimation of the stress wave inhibits a precise and reliable localisation if the dispersive characteristics of the material are not known in advance.

In this work, a localisation system which exploits a multiresolution Continuous Wavelet Transform (CWT) based algorithm, in conjunction with an ad hoc cluster of three circular closely-located PZT transducers is proposed. The cluster is able to acquire three different signals, one for each embedded PZT transducer. To overcome the limitations due to the waves dispersion, the algorithm performs a narrow band filtering on the acquired signals in the time-frequency domain. Thus, the Difference Time of Arrival (DToA) between all the combinations of the signals is evaluated by means of the crosscorrelation of the CWT coefficients. The specific disposition of the three transducers in the cluster permits the evaluation of the Direction of Arrival (DoA) of the GWs source by simple geometric calculations, reducing the computational cost of the algorithm and without any information about the material characteristics. The iteration of this procedure multiple times at different scales automatically permits to exploit an averaging procedure which allows a more reliable and precise DoA estimation.

# II. PZT CLUSTER DESCRIPTION

A novel PZT transducer has been exploited for the GWs acquisitions. The transducer is a cluster of three circular closelylocated PZT elements with 10 mm of diameter each, forming an angle of 60°, as shown in Fig. 1a. The peculiar disposition of the PZT elements leads to the following considerations. Defining as  $S_1$ ,  $S_2$  and  $S_3$  the three circular PZT active areas, the distances between their centroids, d, is constant. Assuming that an impact occurs in a generic point P on the plate, it is possible to define the distance between P and the centroid of the first PZT element hit by the guided wave as

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Fig. 1: PZT Cluster (top) and its output waveforms after an average procedure among 128 different acquisitions (bottom)

 $R_0$ . If the Fraunhofer approximation for far fields is valid, the incident wavefront may be assumed as planar. Consequently, the DoA is determined by means of the triangulation procedure described in [4] by Kundu et al., applying specific adjustments for the PZT configuration here presented. In particular the angle associated with the DoA can be expressed as:

$$\theta \simeq \operatorname{atan}\left(\sqrt{3} \ \frac{1 - D_1/D_2}{1 + D_1/D_2}\right)$$
 (1)

where  $D_1$  and  $D_2$  are the distances the waves travel between the first hit sensor  $S_1$  and  $S_2$ ,  $S_3$  respectively. Because of the strict relationship between the distance travelled by the stress wave, its velocity and the time taken to cover such distance, the previous equation can be rearranged in terms of the DToAs  $\Delta t_{1,2}$  and  $\Delta t_{1,3}$  as follows:

$$\theta \simeq \operatorname{atan}\left(\frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}}\frac{\Delta t_{1,2}}{\Delta t_{1,3}}\right)$$
 (2)



(b) Direction of Arrival estimation result in terms of degrees

Fig. 2: The experimental setup exploited to test the DoA estimation algorithm (top) and the final result (bottom)

#### **III. DOA ESTIMATION ALGORITHM**

In order to estimate the DoA of the incident wave, the evaluation of the DToAs among the active areas of the cluster is needed, as shown in the previous equation. Unfortunately, conventional techniques based on *cross-correlation* in time domain lack in reliability and precision. In fact, stress waves are affected during their propagation by distortion and broadening due to the different velocities of the different wave harmonics, aka dispersion. Thus, the evaluation of the DToA by just cross-correlating the PZT raw signals results to be not sufficiently accurate.

To limit the dispersion effect, a multiresolution isofrequential analysis employing DFT-Based CWT has been adopted in this work. The algorithm performs a narrow band filtering by exploiting CWT decomposition as a filtering technique. Furthermore, the DoA is extracted by applying firstly a recursive multiresolution analysis, and secondly an average procedure on the estimated angles for each computed scale.

Let  $s_i(t)$  be the signal associated to the *i*-th active area  $S_i$  of

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the PZT cluster, the CWT, denoted by  $W_i$ , can be defined as:

$$W_{i}(\psi; a, b) = \int_{-\infty}^{+\infty} s_{i}(t)\psi_{a,b}^{*}(t) dt$$
 (3)

where  $\Psi_{a,b}(t) = |a|^{-\frac{1}{2}} \Psi \frac{t-b}{a}$  is called *mother wavelet*, *a* is the scale factor and *b* is the translation parameter. By simple mathematical considerations, it is possible to rearrange the previous equation as an inverse Fourier transform of the product between the Fourier transform of the signal and the Fourier transform of the mother wavelet at scale *a* and location *b* [5], as follows:

$$W_i(\Psi; a, b) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_i(\omega) \Psi_{a,b}^*(\omega) \ d\omega \tag{4}$$

where  $\Psi_{a,b}^*(\omega) = \sqrt{a}\psi^*(a\omega)e^{i\omega b}$ . Hereby, it is evident that the CWT can also be described as a band pass filter of the signal in the frequency domain. By Cross-correlating the CWT coefficients for each scale *a*, the DToA between two PZT sensors is computed, as shown in the following equations.

$$C_{i,j}(a,t) = \int_{-\infty}^{+\infty} W_i^*(\Psi, a, b) W_j(\Psi, a, t+b) \ db$$
 (5)

$$\Delta t_{i,j}(a) = \max_{t} (C_{i,j}(a,t)) \tag{6}$$

Then, the DoA is estimated by applying Eq. 2 for each a value. Finally, the dependence on the scale factor is overcome by means of an averaging procedure which provides to the user the final estimation of the DoA in angular terms [6], [7].

### IV. EXPERIMENTAL INVESTIGATION AND RESULTS

The proposed algorithm has been tested in a laboratory experimental setup. The PZT cluster was fixed on an aluminium 1050A square plate 1000 mm wide and 3 mm thick, as depicted in Fig. 2a. Signals from the cluster were acquired by a Tektronix 3014 digital oscilloscope at a sampling frequency of 10 MHz. A PZT sensor was used in order to actuate GWs within the plate. Connected to the piezoelectric emitter, an Agilent 33220A function generator and a Tegam 2350 amplifier were exploited to obtain repeatable, precise and accurate control on the actuated signals. In particular, a burst of one sine wave of  $1 V_{pp}$  at 4 KHz was emitted by the function generator every  $50 \,\mathrm{ms}$  to simulate impacts on the plate. Then, the sinusoidal pulses were amplified by means of the Tegam 2350 with a gain factor of 50. The PZT actuator was placed along a circumference of radius 20 cm centred at the cluster position. The circle was divided into 24 angular intervals, each one  $15^{\circ}$ wide. The GWs generation was conducted by employing 128 sinusoidal bursts on each angle of the quantized circumference. Thus, an average among all the 128 acquisitions was performed automatically by the oscilloscope to enhance the signal to noise ratio. An acquisition example after the average procedure is depicted in Fig.1b. Then, the signals so obtained were processed by the localisation algorithm and the DoA was estimated for each angle. The result of the localisation is shown in Fig.2b. By comparing the actual DoA with the estimated angle, the high level of accuracy achieved by the algorithm is evident. In fact, the maximum error observed was  $1.74^{\circ}$ , with an average error of  $1.15^{\circ}$ . It's worthy to notice that, excluding the preliminary noise suppression, no pre-processing of the signal was needed. Moreover, the DFTbased CWT implementation approach permits to reduce the computational cost of the entire algorithm. In fact, just 0.2 seconds were needed to estimate all the 24 angles of arrival.

#### V. CONCLUSIONS

In this work, a new localisation system composed by an ad *hoc* piezoelectric cluster and an algorithm able to estimate the Direction of arrival of the incident GWs by limiting the effect of the dispersion is presented. The peculiar 60° configuration of the PZT cluster allows to compute the DoA by simple geometric considerations in terms of Difference Time of Arrival. In order to achieve high precision and reliability, an algorithm based on narrow band filtering by means of Continuous Wavelet Transform is used. In fact, for the estimation of the Difference Time of Arrival of the incident waves between different sensors, the cross-correlation in the wavelet domain for multiple scale factors is exploited. Then, for each scale factor the DoA is easily computed because of the 60° configuration of the cluster. Finally, the DoA in terms of angular degrees is computed by an averaging procedure. Experimental tests has been performed to test the localisation system, achieving high accuracy levels, with errors below 2° for the entire acquired dataset. Moreover, the low computational cost of the algorithm allows to exploit the proposed method in real time applications.

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#### REFERENCES

- H. M. Elwalwal, S. Mahzan and A. N Abdalla, "Crack Inspection Using Guided Waves(GWs)/Structural Health Monitoring(SHM): Review," J. Appl. Sci., vol. 17, pp. 415–428, July 2017.
- [2] S. Zhongqing, Y. Lin, L. Ye, "Guided Lamb waves for identification of damage in composite structures: A review," J. SOUND. VIB., vol.295, pp.753–780, 2006.
- [3] L. De Marchi, A. Marzani, N. Speciale and E. Viola, "A passive monitoring technique based on dispersion compensation to locate impacts in plate-like structures," Smart Mater. Struct., vol. 20, pp. 035021, 2011.
- [4] T. Kundu, S. Das and K. Jata, "Point of impact prediction in isotropic and anisotropic plates from the acoustic emission data," J. Acoust. Soc. Am., vol. 122, pp.2057–66, 2007.
- [5] D. Komorowski and S. Pietraszek, "The use of continuous wavelet transform based on the fast fourier transform in the analysis of multichannel electrogastrography recordings," J Med Syst, vol.40, pp.10, 2016.
- [6] A. Garofalo, N. Testoni, A. Marzani and L. De Marchi, "Multiresolution wavelet analysis to estimate lamb waves direction of arrival in passive monitoring techniques," IEEE Workshop on Environmental, Energy, and Structural Monitoring Systems (EESMS), pp. 1–6, IEEE Press, Milan(IT) (2017)
- [7] A. Garofalo, N. Testoni, A. Marzani and L. De Marchi, "Waveletbased Lamb waves direction of arrival estimation in passive monitoring techniques," IEEE International Ultrasonic Symposium (IUS), pp.1–4, 2016.