Program Digest, 2019 IEEE International Ultrasonics Symposium (IUS)
Glasgow, Scotland, October 6-9, 2019

Negative refraction in conventional and additively manufactured phononic crystals

Lorenzo Astolfi
School of Engineering
University of Warwick
Coventry, UK
lorenzo.astolfi@warwick.ac.uk

Richard L. Watson
School of Engineering
University of Warwick
Coventry, UK
r.watson.2@warwick.ac.uk

David A. Hutchins
School of Engineering
University of Warwick
Coventry, UK
d.a.hutchins@warwick.ac.uk

Peter J. Thomas
School of Engineering
University of Warwick
Coventry, UK
p.j.thomas@warwick.ac.uk

Meisam Askari
Department of Mechanical, Material and Manufacturing Engineering
University of Nottingham
Nottingham, UK
Meisam.Askari@nottingham.ac.uk

Adam T. Clare
Department of Mechanical, Material and Manufacturing Engineering
University of Nottingham
Nottingham, UK
Adam.Clare@nottingham.ac.uk

Luzhen Nie
School of Electronic and Electrical Engineering
University of Leeds
Leeds, UK
elln@leeds.ac.uk

Steven Freear
School of Electronic and Electrical Engineering
University of Leeds
Leeds, UK
s.freear@leeds.ac.uk

Stefano Laureti
Department of Informatics, Modelling, Electronics and System Engineering
University of Calabria
Rende, Italy
stefano.laureti@unipg.it

Marco Ricci
Department of Informatics, Modelling, Electronics and System Engineering
University of Calabria
Rende, Italy
m.ricci@dimes.unical.it

***Abstract*—Phononic crystals are acoustic metamaterials designed to manipulate sound when its wavelength is in the order of magnitude of the crystal lattice constant. Metallic phononic crystals for use in water were assembled using commercially available stainless steel rods with an average superficial roughness R_a=0.5±0.5 µm, while Laser Powder Bed Fabrication, an additive manufacturing technique, was employed to produce similar Inconel 718 structures with R_a=20±6 µm. Experiments in the 150 – 500 kHz frequency range indicated that acoustic band gaps and negative refraction were present in both cases, with similar features. This indicates that Laser Powder Bed Fabrication is a promising method for realising such phononic crystals.**

Keywords: **Metamaterial, Phononic Crystal, Laser Powder Bed Fabrication, Selective Laser Melting, Band Gaps, Negative Refraction.**

I. Introduction

When an acoustic wave crosses the interface between two media where one has a positive and the other a negative refractive index, it is refracted in the opposite direction to that normally expected. This effect is called negative refraction and can be observed in Phononic Crystals (PCs), designed to manipulate phonons to obtain band gaps, negative refraction, sound focusing and/or sub-wavelength imaging [1][2][3]. PCs typically contain periodic scattering sites such as cylinders or spheres. For example, Ke *et al.* used steel rods assembled in a triangular lattice to achieve negative refraction in water [4]. Constructive or destructive interference effects can occur due to Bragg scattering, weak elastic coupling or hybridization [5]. This, in turn, can produce band gaps, frequency ranges within which propagation is forbidden due to the dispersive effect of periodic lattices [6]. Band gap properties are determined by both the lattice geometry (for rods this would be their diameter and spacing) and the acoustic impedance mismatch between the scattering cylinders material and the surrounding medium.
It is important to maximise the acoustic impedance mismatch between the scatterer and the medium, in this case water; hence, metallic substrates were chosen. While polymers would have been more convenient, and indeed have been used widely for acoustic PCs in air [7][8], the acoustic impedance difference in water would not be sufficient to achieve efficient, wide band gaps (see Table 1).

Since the lattice is repeated, visualising the possible acoustic modes in a single cell is enough to describe the whole structure: this is done in reciprocal space, which is obtained by Fourier transforming the real space [9]. In the PCs reported here, scattering units in the form of cylinders were arranged in hexagonal Bravais lattices (the direct lattices) whose first Brillouin zone, in reciprocal space, is a hexagon and identifies the two main symmetry directions ΓM and ΓK [6].

TABLE I. ACOUSTIC IMPEDANCE OF COMMON MATERIALS

Material	Density ρ (kg m^{-3})	Longitudinal Sound Velocity c_l (m s^{-1})	Acoustic Impedance Z (Kg m^{-2} s^{-1} * 10^5)
Air	1.225	340.5	0.0042
Water	1000	1490	14.9
PLA polymer	1240	2220	27.5
Steel	7780	5825	453.2
Inconel 718	8190	5700	466.8

The Brillouin zone is the set of points closer to the origin (Γ) than to any other reciprocal lattice point and in this case corresponds to the blue hexagon of Fig. 1.

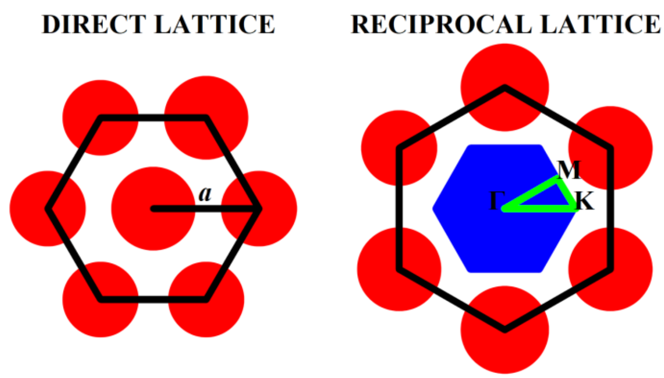

Fig. 1. (left) Direct and (right) reciprocal lattice. The lattice constant "a" is 4.8 mm long. Red circles represent an upper view of the rods. The blue region denotes the Brillouin zone. The unit of measure for reciprocal lattices is mm^{-1}.

If a = 4.8 mm is the crystal lattice constant, *i.e.* the spacing between adjacent rods, then we can write [9]:

$$\Gamma M = \frac{2\pi}{a\sqrt{3}} = \sim 0.76 \text{ mm}^{-1} \quad (1)$$

$$\Gamma K = \frac{\Gamma M}{\sin(60°)} = \sim 0.88 \text{ mm}^{-1} \quad (2)$$

$$MK = \frac{\Gamma K}{2} = \sim 0.44 \text{ mm}^{-1} \quad (3)$$

The aim of this paper is to determine whether AM is a suitable method for PCs fabrication. Samples manufacturing will be illustrated in Sec. II, band gaps and negative refraction experiments will be discussed in Sec. III and conclusions will be drawn in Sec. IV.

II. SAMPLE MANUFACTURE

Two types of crystals were investigated. Rectangular PCs were used in a transmission experiment to visualize their band gaps in the ΓK direction, whereas prismatic PCs were used to clearly display negative refraction. In order to assemble two conventional structures, we inserted commercially available stainless steel rods into laser-cut acrylic sheets. Also, Laser Powder Bed Fabrication (LPBF) was used to build replicas of the two structures using a Realizer SLM50 (DMG MORI limited). LPBF, also referred to as Selective Laser Melting, is an AM technique that uses a high powered laser to melt metal powder in successive layers of 40 μm thickness.

Figures 2(a)-(c) show the conventionally manufactured PCs, together with a close-up of the rod structure, whereas the AM equivalents are shown in Figures 2(d)-(f).

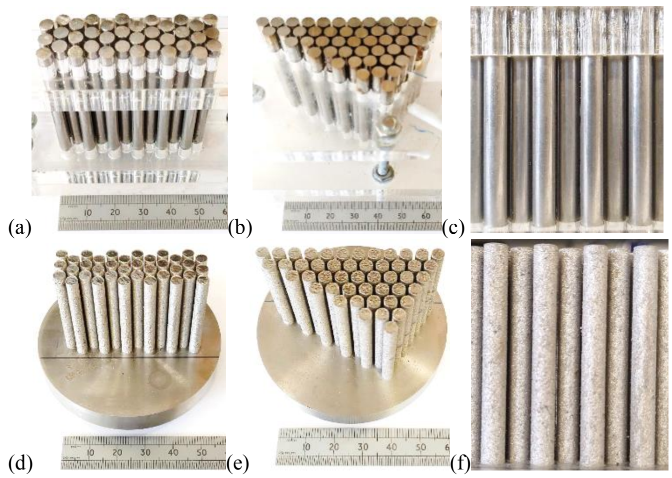

Fig. 2. 2D phononic crystals. (a,b) Conventionally assembled rectangular and triangle-shaped structure and (c) detail of rods showing surface roughness. (d,e) Selective laser melted replicas and (f) respective rods detail.

It can be seen that the AM cylinders are rougher. The average roughness parameter R_a was thus measured using a Bruker Contour GT to be 0.5±0.5 μm for the commercially available rods and 20±6 μm for the additive manufactured equivalent, as presented in Fig. 3.

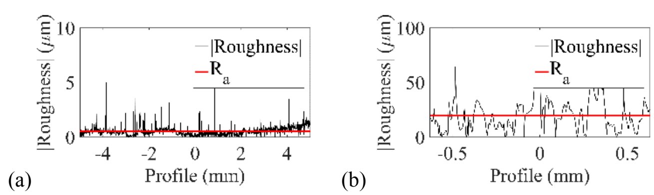

Fig. 3. Roughness profile of (a) conventional and (b) additively manufactured rods used to calculate R_a values.

The PCs were designed to be used with a transducer centred at 300 kHz, where the wavelength (λ) in water is ~5 mm. This means that the surface roughness is two orders of magnitude less than λ in the AM case. Another factor to take into consideration is that the diameter of the AM rods (3.91±0.05 mm) was different to those obtained commercially (3.99±0.02 mm).

III. EXPERIMENTAL RESULTS

Both the rectangular and the prismatic structures were designed to have a hexagonal Bravais lattice, using a lattice constant a = 4.8 mm for a superficial filling fraction of 63%.

The two rectangular crystals of Figs. 2(a) and 2(d) were made of 6 layers, orientated in the ΓK direction (cf. Fig. 1), a configuration stated to be efficient for imaging purposes using a 2D crystal [5]. This is important, as negative refraction makes sound waves focus within two separate zones - one inside and one outside the crystal, depending on the frequency and/or signal properties. The structures were immersed in a water tank to be used in a transmission experiment to visualize the band gaps. A pulse was emitted and received by identical 25.4 mm diameter piezocomposite transducers centred at 300 kHz at a fixed distance of 150 mm, depicted in Fig. 4.

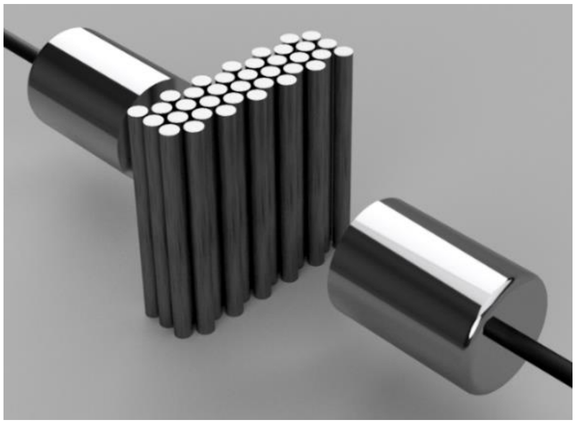

Fig. 4. The through-transmission experiment for rectangular PCs.

The experiment was repeated three times: one for each type of rectangular crystal and one without any sample to use water as a baseline reference. A 6 mm thick foam layer around the specimens was used to prevent edge effects; the samples were placed in the far-field of the ultrasonic source (50 mm) to approximate plane wave incidence. The signals acquired in the time domain were Fourier transformed to confirm the presence of a strong band gap from 230 to 310 kHz, as shown in Fig. 5.

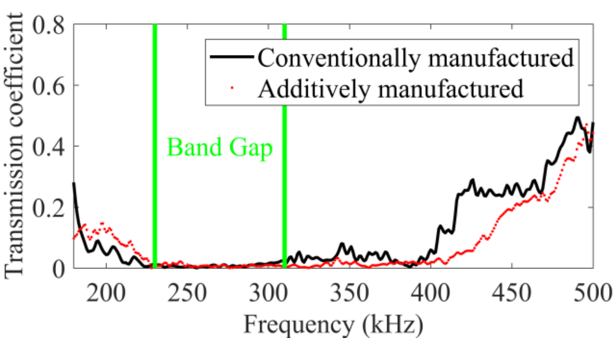

Fig. 5. Transmission coefficient for the additive and conventionally manufactured rectangular crystals normalized to the water only signal.

The effect of the increased roughness parameter R_a has shown that the AM structure exhibited a more attenuating and wider band gap. Note that band gaps can be "stop band", if they only block transmission in one direction or "absolute" if they are omnidirectional i.e. valid for ΓM, MK and ΓK directions. Wave transport inside the band gaps occurs by tunnelling [10]. Multiple band gaps are possible for a given PC structure.

Negative refraction is a consequence of the existence of band gaps and it can happen in two cases: strong Bragg scattering near the Brillouin zone end in the first band (without necessarily having a negative refractive index) or in the second band, as in this case, where wave vectors k and group velocity v_g have opposite direction [2]. The prismatic PCs had internal angles of 30°, 60° and 90°. The shortest and the longest sides were normal to ΓM, while the third one was perpendicular to ΓK. A chirp signal from 150 to 500 kHz was generated by a 1 inch diameter piezocomposite transducer centred at 300 kHz. The input was incident normally on the shortest side of the triangle-shaped crystal (see Fig. 6). The input wave propagates into the PC without changing its direction until it gets refracted. Inside the crystal, the wave vector's direction is opposite to the group velocity, which represents the energy direction. Spatial variations in the refracted ultrasonic field were detected using a 0.5 mm diameter Precision Acoustics hydrophone (λ/10 at 300 kHz) attached to a 3D motorized stage. It was scanned in a 2D horizontal plane aligned to the transducer beam axis, as illustrated in Fig. 6.

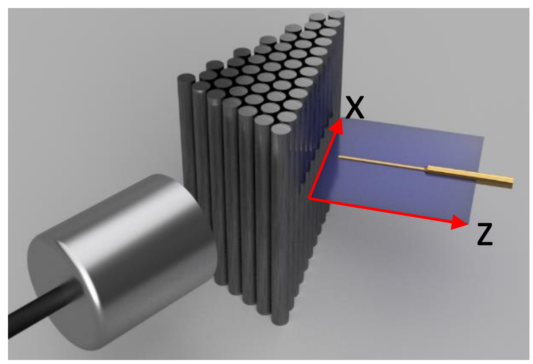

Fig. 6. A pulse was sent by a 1-inch diameter transducer and detected by a Precision Acoustics hydrophone following the blue path highlighted.

Band gaps depend on the direction: ΓM should have its first one at around 150 kHz, hence negative refraction is expected at a slightly higher frequency. The pressure field amplitudes of the outgoing pulses were Fourier transformed and digitally filtered at 190 kHz. Figure 7 shows that this revealed two separated beams at similar angles for both cases.

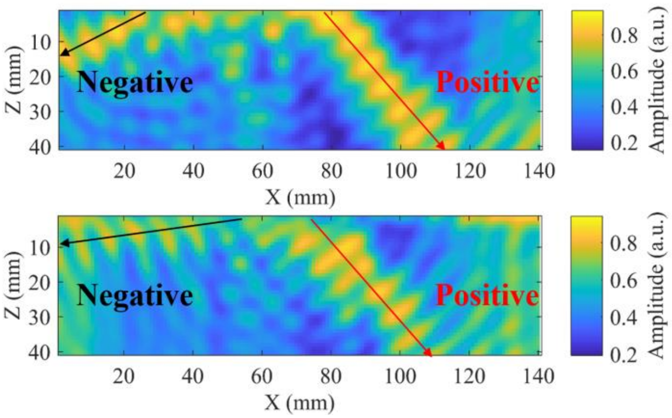

Fig. 7. Images revealing positive and negative refraction using the (top) conventional and the (bottom) additively manufactured crystals.

To sum up, negative refraction occurs in a similar form for both the conventional and the additively manufactured crystal.

IV. Conclusions

PCs have many exciting properties that make them ideal for sound manipulation: most of them are due to band gaps that are a consequence of their periodicity. Negative refraction is one of these phenomenon that was observed in both the conventionally manufactured prismatic crystal and the AM replica, regardless of their superficial roughness or difference in material properties or rod dimension. This work thus validates the use of the AM Laser Powder Bed Fabrication process to build phononic crystals for use at ultrasonic frequencies in water.

Acknowledgment

This work was funded via grants from the UK Engineering and Physical Sciences Research Council (EPSRC), grant numbers EP/N034163/1, EP/N034201/1 and EP/N034813/1.

References

[1] A. Sukhovich, L. Jing, and J. H. Page, "Negative refraction and focusing of ultrasound in two-dimensional phononic crystals," *Phys. Rev. B - Condens. Matter Mater. Phys.*, vol. 77, no. 1, 2008.

[2] J. H. Page, "Focusing of ultrasonic waves by negative refraction in phononic crystals," *AIP Adv.*, vol. 6, no. 12, 2016.

[3] S. Yang, J. H. Page, Z. Liu, M. L. Cowan, C. T. Chan, and P. Sheng, "Focusing of sound in a 3D phononic crystal," *Phys. Rev. Lett.*, 2004.

[4] M. Ke *et al.*, "Negative-refraction imaging with two-dimensional phononic crystals," *Phys. Rev. B - Condens. Matter Mater. Phys.*, 2005.

[5] C. Croënne, E. J. S. Lee, H. Hu, and J. H. Page, "Band gaps in phononic crystals: Generation mechanisms and interaction effects," *AIP Adv.*, 2011.

[6] A. Khelif and A. Adibi, *Phononic crystals: Fundamentals and applications*. 2015.

[7] F. Lucklum and M. J. Vellekoop, "Realization of Complex 3-D Phononic Crystals with Wide Complete Acoustic Band Gaps," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 63, no. 5, pp. 796–797, 2016.

[8] M. H. Lu, L. Feng, and Y. F. Chen, "Phononic crystals and acoustic metamaterials," *Mater. Today*, vol. 12, no. 12, pp. 34–42, 2009.

[9] D. Thompson, "The reciprocal lattice as the Fourier transform of the direct lattice," *Am. J. Phys.*, vol. 64, no. 3, pp. 333–334, 1996.

[10] J. H. Page, S. Yang, Z. Liu, M. L. Cowan, T. C. Che, and P. Sheng, "Tunneling and dispersion in 3D phononic crystals," *Zeitschrift fur Krist.*, 2005.