Program Digest, 2019 IEEE International Ultrasonics Symposium (IUS)
Glasgow, Scotland, October 6-9, 2019

Non-Classical Second-Order Nonlinear Elastic Wave Interactions

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Abstract—We report a novel ultrasonic measurement technique based on non-classical nonlinear evanescent field interactions. We demonstrate significant enhancement in sensitivity of contactless measurements at interfaces, with the potential to detect material degradation, such as fatigue and ageing, which is currently not possible using linear ultrasonics.

Index Terms—evanescent fields, nonlinear ultrasonics, momentum conservation

I. Introduction

Immersion and air-coupled contactless linear ultrasonics are widely used for non-destructive testing. Different wave incidence/sensing configurations have been used in the industry with the aim to keep measurement setup simple (normal incidence [1]) or to excite a specific wave mode (e.g. Rayleigh wave [2], single guided wave mode [3], transverse wave [4]). For example, it is often preferable to excite a single wave mode, so the generation of transverse waves occurs above the first critical angle θ_{L} whilst localized longitudinal evanescent fields are generated just below the interface, see Fig. 1. In linear ultrasonic measurements, these evanescent fields can be ignored in most cases without affecting the results, due to the wave fields separation in time and space.

Nonlinear ultrasonic measurements, based on the noncolinear wave mixing of bulk elastic waves, have been studied previously and used in a number of material characterization applications [5]–[7]. These measurements are based on the nonlinear interaction between two initial propagating waves $\mathbf{k_1}$ and $\mathbf{k_2}$ and fulfil of the classical resonance conditions defined by the energy and momentum conservation laws: $\omega_1 \pm \omega_2 = \omega_3$ and $\mathbf{k_1} \pm \mathbf{k_2} = \mathbf{k_3}$, where ω_3 and $\mathbf{k_3}$ are the frequency and wave-vector of the nonlinear wave, respectively. When the two initial waves are well separated at the interface, i.e. the initial beams do not overlap, and the nonlinear interaction occurs in a homogeneous semi-space, the classical resonance conditions enable to control the measurements. However, when the two initial beams overlap at the interface (as is the case when a test specimen is thin or there is a need to evaluate surface or subsurface properties), the classical resonance conditions are not sufficient to describe possible nonlinear interactions caused by the localized evanescent field [8]. Using modelling based on an angular spectral representation of plane waves and experimental validation, we studied second order nonlinear elastic wave interactions to enhance contactless nonlinear ultrasonic measurements, by using non-collinear wave mixing. Our methods enabled us to characterize non-classical interactions involving the

This work was supported in part by the Engineering and Physical Sciences Research Council Fellowship under Grant EP/K027611/1 and in part by the European Research Council advanced investigator award under Grant 340117.

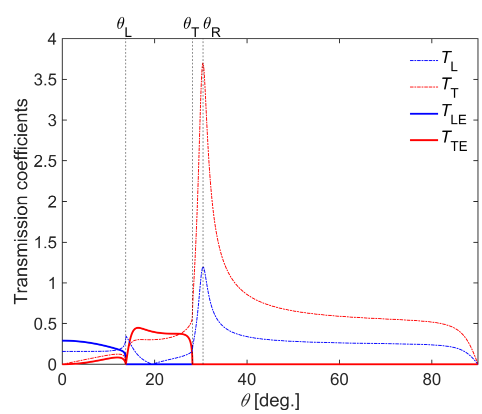

Fig. 1. Amplitude ($T_{\mathrm{L,T}}$) and energy ($T_{\mathrm{LE,TE}}$) of the displacement transmission coefficients for longitudinal (blue) and transverse (red) polarisation plane waves between water and aluminum semi-spaces. The dotted lines indicate critical angles for the three waves (longitudinal θ_{L}, transverse θ_{T} and Rayleigh θ_{R}).

evanescent fields and we show that the localized evanescent fields generate nonlinear waves at interfaces which cannot be ignored in experiments.

II. Analytical method

Non-classical nonlinear wave interactions were analyzed using an analytic model described by a volume integral involving nonlinear plane wave amplitude A_3 in the far-field zone [8]:

$$A_3 = \frac{a_1 a_2 \mathbf{u_3} \cdot \mathbf{f^{(12)}}(\mathbf{u_1}, \mathbf{k_1}, \mathbf{u_2}, \mathbf{k_2})}{\mathrm{i} c_3 \rho \omega_3} \times \int_V e^{\mathrm{i}(-\mathbf{k_1}-\mathbf{k_2}+\mathbf{k_3})\cdot \mathbf{r}} dx^3 - \frac{a_1^* a_2^* \mathbf{u_3} \cdot \mathbf{f^{(12)}}(\mathbf{u_1^*}, \mathbf{k_1^*}, \mathbf{u_2^*}, \mathbf{k_2^*})}{\mathrm{i} c_3 \rho \omega_3} \times \int_V e^{\mathrm{i}(\mathbf{k_1^*}+\mathbf{k_2^*}+\mathbf{k_3})\cdot \mathbf{r}} dx^3, \tag{1}$$

where V is the interaction volume, a_1 and a_2 are the spatially dependent amplitudes of the two interacting waves with polarisations $\mathbf{u_1}$ and $\mathbf{u_2}$. $\mathbf{f}^{(12)}(\mathbf{u_1}, \mathbf{k_1}, \mathbf{u_2}, \mathbf{k_2})$ determines the interaction strength between the two initial propagating or evanescent infinite plane waves [8]. The vectors $\mathbf{u_3}$ and $\mathbf{k_3}$ are the polarisation and wave-vector of the nonlinear wave at the sum frequency, respectively. This integral does not diverge and is finite when considering the interaction between evanescent and propagating waves. However, it is important to note that the evanescent waves break the translational symmetry and therefore classical momentum conservation rules ($\mathbf{k_1} + \mathbf{k_2} = \mathbf{k_3}$) do not apply.

Fig. 2 shows modeling of the amplitude intensity of the generated nonlinear wave when the incidence angle θ_1 is varied in the range $0° - 80°$, and the second angle is $\theta_2 = 20.8°$. Both beams propagate through the water and overlap on the aluminum surface. The frequency ratio between the incident waves is 1.5. The results show three distinct peaks in the sum-frequency longitudinal wave intensity. The first two peaks correspond to classical resonance conditions for the transverse and longitudinal wave interaction (TL:L), and the transverse and transverse wave interaction (TT:L). The unexpected third peak is observed above the second critical angle θ_T and this peak shows the non-classical wave interaction between propagating transverse wave and evanescent longitudinal and transverse waves (TE:L). Moreover, we observe that at this point not only is a longitudinal wave generated, but the transverse wave has a similar intensity as this longitudinal wave.

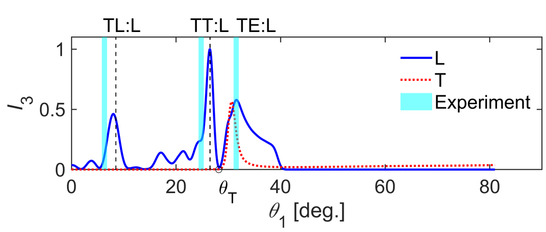

Fig. 2. Normalized intensity I_3 of the nonlinear longitudinal and transverse waves. The normalization is performed scaling intensities by absolute maximum. The dotted lines indicate resonance conditions for the transverse and longitudinal wave interaction with the generated longitudinal wave (TL:L), and transverse and transverse wave interaction with the generated longitudinal wave (TT:L). TE:L denotes the transverse and evanescent wave interaction with the resultant longitudinal wave. The blue columns depict the experimental measurements.

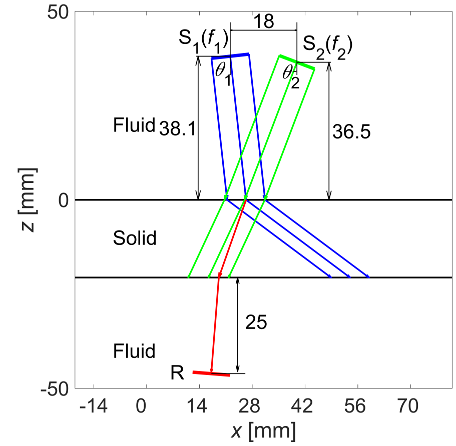

Fig. 3. Experimental configuration for the LT:L interaction.

III. Experimental results

Immersion ultrasonic experiments in transmission mode were carried out to verify the modelling results depicted in Fig. 2. 60 cycles sinus tone bursts with center frequencies of 4 MHz and 6 MHz were used to generate two initial ultrasonic waves for nonlinear wave mixing.

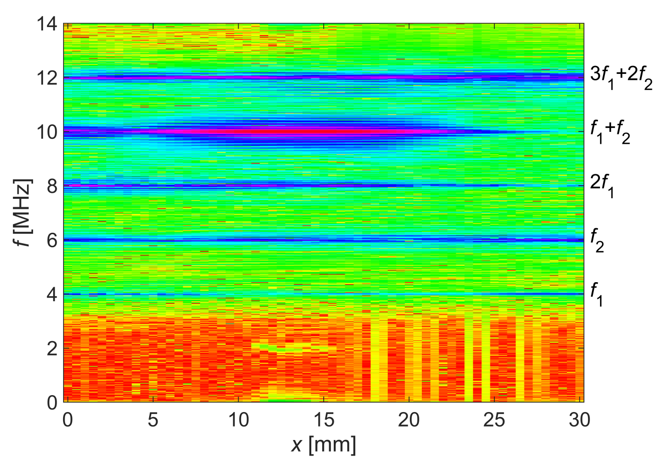

Fig. 4. Measured sum-frequency wave generation at the water and aluminum interface for the LT:L interaction.

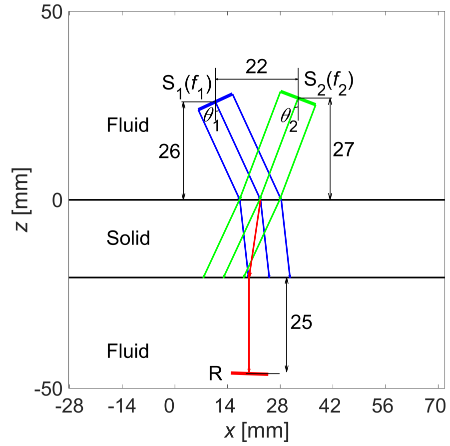

Fig. 5. Experimental configuration for the TT:L interaction.

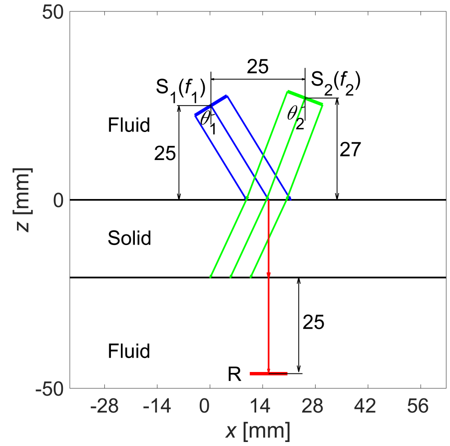

Fig. 7. Experimental configuration for the ET:L interaction.

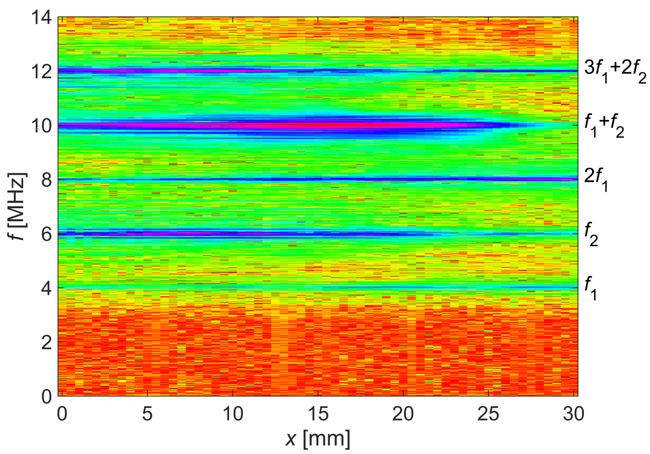

Fig. 6. Measured sum-frequency wave generation at the water and aluminum interface for the TT:L interaction.

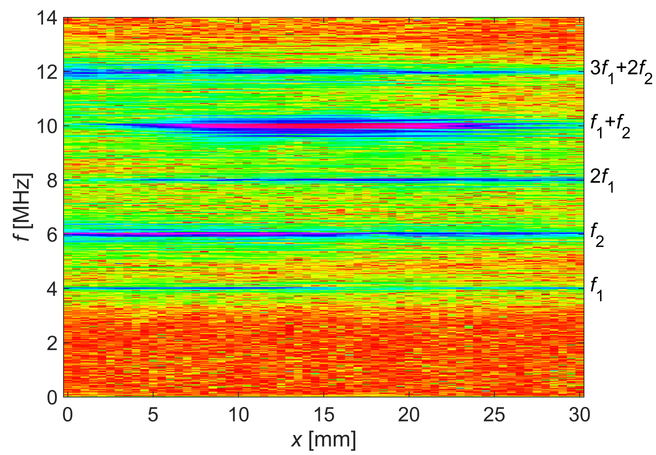

Fig. 8. Measured sum-frequency wave generation at the water and aluminum interface for the TT:L interaction.

Figs. 3-8 show experimental configuration and measurements for the three cases, where E denotes the evanescent wave (in this case longitudinal and transverse):

- LT:L interaction when $\theta_1 = 6.3°$ and $\theta_2 = 20.8°$, $f_1 = 6$ MHz and $f_2 = 4$ MHz, see Fig. 3 and 4;
- TT:L interaction when $\theta_1 = 24.8°$ and $\theta_2 = 20.8°$, $f_1 = 4$ MHz and $f_2 = 6$ MHz, see Fig. 5 and 6;
- ET:L interaction when $\theta_1 = 31.5°$ and $\theta_2 = 20.8°$, $f_1 = 4$ MHz and $f_2 = 6$ MHz.

IV. Conclusions

This analytical and experimental study shows that a wide range of non-classical nonlinear wave interactions occur in the presence of standing evanescent fields. This gives rise to unexpected wave mixing combinations, propagating when the resonance conditions are not fulfilled (e.g. propagating and evanescent, evanescent and evanescent). These interactions have the potential to enable new, highly sensitive measurements, to detect either defects or discontinuities in sub-surfaces and coatings, including e.g., non-destructive characterisation of aging in composites.

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