

F-K Migration for Photoacoustic Tomography Imaging Simulation

1st Jian An
*Academy for Advanced
Interdisciplinary Studies
Peking University
Beijing, China
anjian@pku.edu.cn*

2nd Hanjing Kong
*Academy for Advanced
Interdisciplinary Studies
Peking University
Beijing, China
1501111537@pku.edu.cn*

3rd Shuo Huang
*Academy for Advanced
Interdisciplinary Studies
Peking University
Beijing, China
huangshuo@pku.edu.cn*

4th Feng Feng
*College of Engineering
Peking University
Beijing, China
fengfeng95@pku.edu.cn*

5th Jingyi Yin
*Academy for Advanced
Interdisciplinary Studies
Peking University
Beijing, China
jingyi_1997@pku.edu.cn*

6th Shijie Zhang
*Academy for Advanced
Interdisciplinary Studies
Peking University
Beijing, China
zhang_shijie@foxmail.com*

7th Jue Zhang
*Academy for Advanced
Interdisciplinary Studies &
College of Engineering
Peking University
Beijing, China
zhangjue@pku.edu.cn*

Abstract—A fundamental assumption in traditional photoacoustic computed tomography (PACT) reconstruction methods is that the medium is acoustically homogeneous. However, this assumption is often inconsistent with reality. Seismic migration imaging based on the exploding reflector model (ERM) has many similarities with photoacoustic imaging principles, and there exist valuable references in reconstruction strategies. In this paper, Stolt's F-K Migration technique is introduced to reconstruct photoacoustic images. Furthermore, in order to solve the non-uniform speed of sound (SOS) problem in the practical application of PACT, we utilized the Margrave's strategy in seismic imaging which considers the change of arbitrary vertical SOS in the object to be measured for reference, and we proposed a PACT reconstruction approach based on prior SOS. Moreover, k-Wave toolbox was used for performance comparison via simulation, and the reconstruction results of point sources and vascular network model show that F-K method based on prior SOS has better reconstruction performance than traditional FFT method.

Keywords—PACT, seismic migration imaging, speed of sound

I. INTRODUCTION

Photoacoustic computed tomography (PACT), which combines high optical image contrast and sensitivity with excellent ultrasound penetration is a rapidly emerging hybrid imaging modality [1-3]. Both structural and functional information can be provided and thus PACT has been employed for human and small animal imaging applications [4, 5]. Comparing with other imaging modalities, PACT has concrete advantages. For instance, PACT offers faster image acquisition at lower costs than MRI, and it does not involve the use of ionizing radiation compared with PET and CT. Besides, PACT has the ability to obtain high-resolution images at large penetration depths compared with other optical imaging modalities [6].

In PACT, the wideband ultrasonic transducers are used to measure the acoustic wave and later reconstructing the photoacoustic image through various algorithms. Because the initial amplitudes of the induced acoustic wavefields are proportional to the spatially variant absorbed optical energy density distribution within the object, the reconstructed image from the collected acoustic amplitude signal reflects the absorption distribution of specific light inside the object, which is the source of the contrast in photoacoustic imaging [7].

A fundamental assumption in traditional PACT image reconstruction methods is that the medium is acoustically homogeneous, so the speed-of-sound (SOS) is determined as a constant during image reconstruction. However, this assumption is often violated for biomedical applications since the complex properties of the imaged tissue [8-10]. For example, the presence of bone and/or gas pockets in PACT small animal imaging can strongly perturb the photoacoustic wavefield. Also, in transcranial PACT brain imaging of primates, the photoacoustic wavefields can be strongly attenuated and aberrated by the skull [8]. In biomedical applications of PACT, the reconstructed images can contain significant artifacts and distortions if the inhomogeneous acoustic properties of the object are not considered for in the reconstruction algorithm [11].

In seismic imaging, the acoustic data from deep underground is collected using large acoustic transducer arrays placed on the surface of the ground to reconstruct the image of underground structures. Inspired by the similarity between seismic imaging and medical ultrasound imaging, Damien et al. introduced the classical Stolt's F-K Migration technology, which is based on the migration imaging principle of seismic imaging, for image reconstruction of an ultrasonic plane wave [12]. Their results showed that the quality of F-K offset image was similar to that of the most advanced dynamic focusing mode, and the Stolt's F-

K Migration technology could provide a competitive reconstruction speed.

Unlike seismic imaging, ultrasonic planar wave imaging requires active emission of multi-angle acoustic waves. Therefore, some pre-processing is needed to correct the delay effect of the acoustic wave when using F-K migration imaging technology [12]. On the contrary, PACT is more similar to seismic migration imaging technology in its working principle. As such, in this study, we attempted to introduce Stolt's F-K Migration technology to reconstruct photoacoustic images. Furthermore, in order to solve the non-uniform SOS problem in the practical application of PACT, we considered the change of arbitrary vertical SOS in the object to be measured for reference, and introduced the Margrave's strategy [13] as a method of PACT reconstruction based on prior SOS. The reconstruction image of the traditional Fourier method and F-K method was compared by simulation based on K-wave toolbox.

II. METHODS

A. Stolt's F-K Migration technology for reconstruction in photoacoustic signal

The photoacoustic signal $\psi(x, z, t)$ is a scalar wavefield satisfying the exploding reflector model (ERM) [14] and the two-dimensional linear wave equation [15]. We want to determine the ERM wavefield at the time of the explosion, i.e., $\psi(x, z, t = 0)$, knowing the wavefield on the surface $\psi(x, z = 0, t)$. Let $\phi(k_x, z, \omega)$ denote the Fourier transform of $\psi(x, z, t)$ over (x, t) , so that

$$\psi(x, z, t) = \iint \phi(k_x, z, \omega) \exp(ik_x x - i\omega t) dk_x d\omega \quad (1)$$

where k_x stands for the spatial wavenumber related to x and ω is the temporal frequency.

According to the two-dimensional scalar wave equation and the boundary condition at zero depth, and assuming that v is a constant [16], we can get:

$$\phi(k_x, z, \omega) = \phi(k_x, 0, \omega) \exp(-ik_z z) \quad (2)$$

$$k_z^2 = \frac{\omega^2}{v^2} - k_x^2 \quad (3)$$

Combing (1) and (2), the wavefield at depth z then can be expressed as an extrapolation from the wavefield at $z=0$,

$$\psi(x, z, t) = \iint \phi(k_x, 0, \omega) \exp(ik_x x - ik_z z - i\omega t) dk_x d\omega \quad (4)$$

The migration result is assumed to be the wavefield before the waves begin to propagate, i.e., the wavefield at time 0. Set $t = 0$ in (4), then

$$\psi(x, z, 0) = \iint \phi(k_x, 0, \omega) \exp(ik_x x - ik_z z) dk_x d\omega \quad (5)$$

To fully benefit from the Fourier transforms, Stolt proposed to change the variable k_z by (3).

$$\omega(k_z) = v \text{sign}(k_z) \sqrt{k_x^2 + k_z^2} \quad (6)$$

Using the change of (6), the Stolt's migration solution is finally

$$\psi(x, z, 0) = \iint \frac{vk_z}{\sqrt{k_x^2 + k_z^2}} \phi(k_x, 0, \omega(k_z)) e^{ik_x x - ik_z z} dk_x dk_z \quad (7)$$

As shown by (7), the migrated solution (initial photoacoustic signal intensity) is basically the inverse Fourier transform of

$$\frac{vk_z}{\sqrt{k_x^2 + k_z^2}} \phi(k_x, 0, \omega(k_z)) \quad (8)$$

B. Extension of F-K method to vertical SOS variation

To obtain the wavefield at any depth z , (2) cannot be directly used because the materials between surface and depth z may have different velocities. Gazdag assumed that in each infinitesimally thin layer with thickness dz , the wave propagation speed does not change [17]. Using this assumption, (2) can be applied recursively to obtain

$$\begin{aligned} \phi(k_x, z, \omega) &= \phi(k_x, 0, \omega) \exp\left(-i \int_0^z \sqrt{\frac{\omega^2}{v^2(z')} - k_x^2} dz'\right) \\ &= \phi(k_x, 0, \omega) m(k_x, z, \omega) \end{aligned} \quad (9)$$

$$k_z^2 = \frac{\omega^2}{v^2(z')} - k_x^2 \quad (10)$$

Combing (1) and (9), and set $t = 0$, then

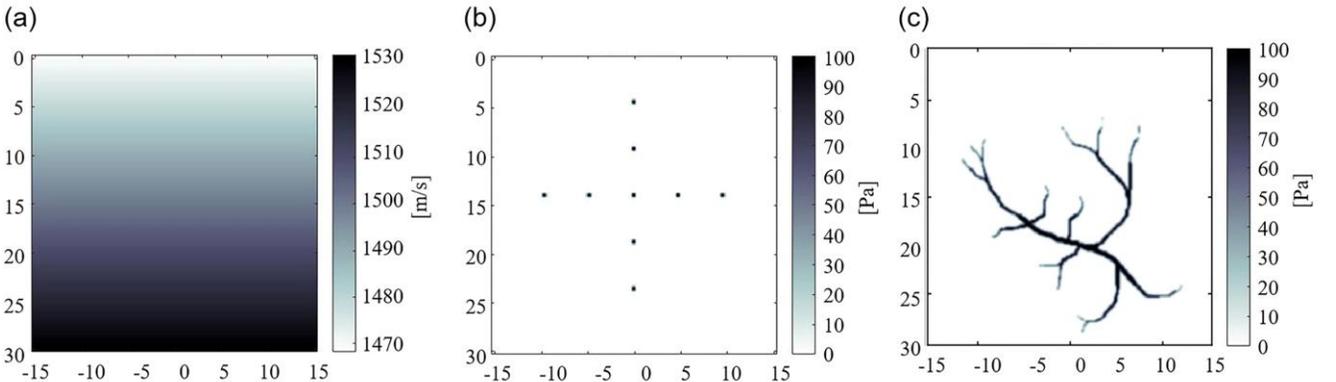


Fig. 1. The simulated SOS sound distribution and initial pressure diagram.

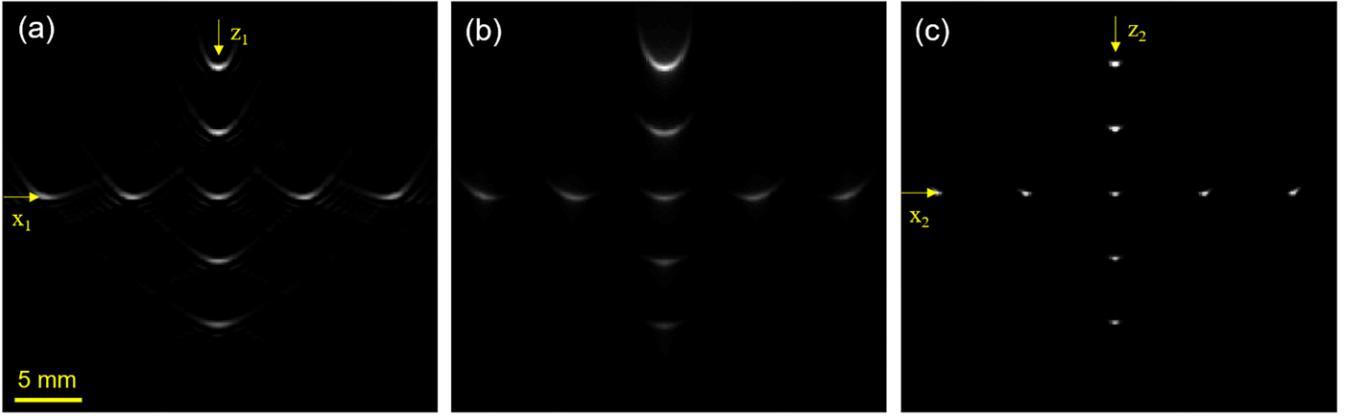


Fig. 2. The point sources reconstruction through FFT in k-Wave and two F-K migration imaging methods.

$$\psi(x, z, 0) = \iint \phi(k_x, 0, \omega) m(k_x, z, \omega) \exp(ik_x x) dk_x d\omega \quad (11)$$

Then ignore the Fourier transform over k_x

$$\phi(k_x, z, 0) = \int \phi(k_x, 0, \omega) m(k_x, z, \omega) d\omega \quad (12)$$

Fourier transform of (12) from z to k_z gives the following

$$\phi(k_x, k_z, 0) = \int \phi(k_x, 0, \omega) M(k_x, k_z, \omega) d\omega \quad (13a)$$

Where

$$\begin{aligned} M(k_x, k_z, \omega) &= \int m(k_x, z, \omega) \exp(-ik_z z) dz \\ &= \int \exp\left(-i \int_0^z \sqrt{\frac{\omega^2}{v^2(z')} - k_x^2} dz'\right) \exp(-ik_z z) dz \end{aligned} \quad (13b)$$

And the migrated solution (initial photoacoustic signal intensity) is basically the inverse Fourier transform of $\phi(k_x, k_z, 0)$.

C. Simulation in k-Wave toolbox

The $v(z)$ F-K Migration technology was achieved by use of the MATLAB k-Wave toolbox [18]. The SOS is shown in Fig. 1 (a). The point source and the blood vessel are created in Fig. 1 (b) and (c) to compare reconstruction performance between F-K migration and the Fourier reconstruction algorithm in k-Wave toolbox. Set the first row as an ultrasonic transducer to simulate a line-array probe. The center frequency of the transducer is 3MHz and the bandwidth is 70%.

III. RESULTS/DISCUSSION

As shown in Fig. 2, (a) and (b) are the results of point sources reconstruction through FFT in k-Wave and Stolt's F-K. Since the heterogeneity of SOS is not considered, (a) and (b) are reconstructed according to the average speed of 1500m/s, so there are relatively serious artifacts. In addition, the preset SOS decreases gradually from the shallow layer to the deep layer, and the shallow layer SOS is higher than the reconstructed value, while the deep layer SOS is lower than the reconstructed value. Therefore, the SOS of mismatch is more serious during the reconstruction of the shallow layer, and the artifact is also stretched more seriously. When $v(z)$ F-K is used for reconstruction in Fig. 2 (c), the preset speed of sound is taken as the prior condition, so the reconstruction performance is the best.

Fig. 3 is obtained by extracting the lateral and axial signal intensity of figure (a) and (c) according to the yellow arrow. The two reconstruction methods have the same axial resolution, and the intensity of $v(z)$ F-K is slightly lower than FFT reconstruction, while in terms of lateral resolution, $v(z)$ F-K is obviously better than FFT reconstruction. The point spread function of FFT reconstruction is not only enlarged many times horizontally but also has many side-lobe artifacts.

Fig.4(a) is the vascular network based on FFT reconstruction in the k-Wave toolbox, and obvious aliasing artifacts can be observed at the yellow arrow. Fig. 4 (b) is Stolt's F-K reconstruction, which exhibits few artifacts, and the overall image is blurred and out of focus due to speed mismatch. Fig. 4(c) is the vascular network reconstruction of $v(z)$ F-K, which has the best resolution and artifact suppression. The main deficiency is the poor signal-to-noise ratio (SNR) in the reconstruction of vessels perpendicular to the transducer direction, which is a common problem at present.

In this study, we introduce two migration techniques in seismic imaging and apply them to the reconstruction of PACT. Simulation results suggest that the $v(z)$ F-K migration technique has better performance than the traditional FFT reconstruction method.

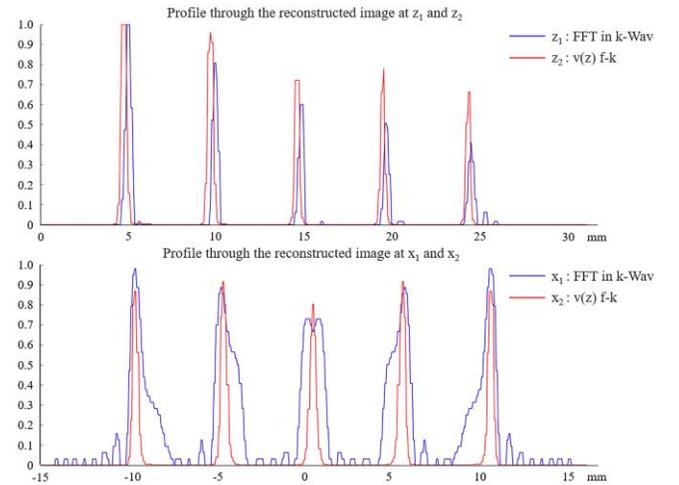


Fig. 3. The lateral and axial signal intensity of point sources reconstruction

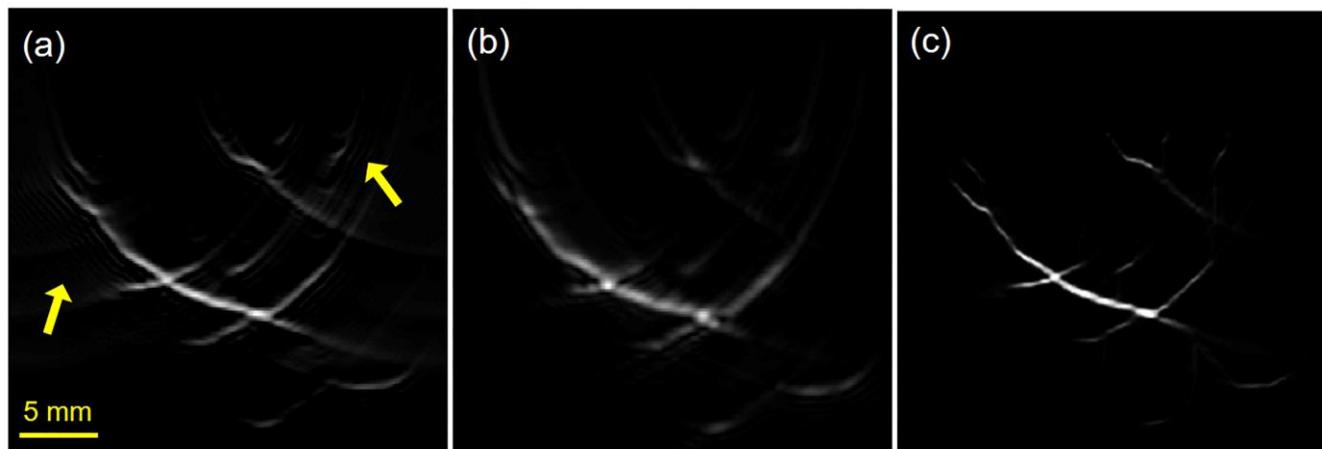


Fig. 4. The vascular network reconstruction through FFT in k-Wave and two F-K migration imaging methods.

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