Effects of Data Density of Echo Fourier Domain on Quality of High Frame Rate Imaging

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Abstract **– Based on the X wave theory, a high-frame rate (HFR) imaging method was developed and extended. In this paper, the effects of spatial data density of echo Fourier domain on the quality of the HFR imaging method is studied experimentally. In the experiment, echo data were acquired with a home-made HFR imaging system from an ATS539 tissue-mimicking phantom. Results show that with a two-fold densification of radio-frequency echo data of a fully sampled phased array transducer, one achieves the best compromise between the quality of reconstructed images and the amount of computations required due to the densification of echo data.**

Keywords - ***High Frame Rate Imaging, Fourier Transform, Data Densifying, Interpolation***

I. INTRODUCTION

The theory of X waves was developed in early 1990s [1]-[4]. Based on this theory, a high frame rate (HFR) imaging method was developed in 1997 [5]-[9]. The method can be implemented with either limited-diffraction beam or steered plane wave transmission [10]-[13]. Recently, the HFR imaging method has been extended and further developed [14]-[20]. Motion artifacts and phase aberration effects on the HFR imaging have been studied [21]-[23]. Applications of the HFR imaging method for blood flow velocity vector imaging [24] and high-speed wave-field computation [25] have been investigated.

Unlike the conventional delay-and-sum (D&S) beamforming method [26], the HFR imaging method can produce high-quality images (dynamic focusing in both transmit and receive) at a high frame rate while dramatically reducing the amount of computations due to the use of Fast Fourier transform (FFT).

Although the amount of computation of the HFR imaging method could be a few thousands times less than that required by the conventional D&S method to reconstruct a high-frame rate and high-quality three-dimensional (3D) image, the amount of computation is still larger than the current electronics could handle in a small space and a low-power environment of a transducer probe. In this paper, the trade-off between the image quality and the densification of the radio-frequency (RF) echo data is studied to avoid an unnecessary increase of computation.

In the study, experimental data from an ATS539 multi-purpose tissue-mimicking phantom (ATS Laboratory, Inc) were used. RF echo data were acquired at 40 MHz and 12-bit resolution with a home-made high-frame rate imaging system [14], [27]-[28]. A 2.5-MHz, 19.2 mm aperture, 128-element, and broadband phased-array transducer was used. One-and-a-half cycle sine waves were transmitted and both limited-diffraction beams [10]-[11] and steered plane waves [12]-[13] were used in transmissions. Two-dimensional (2D) images were reconstructed over a +/-45-degree field of view and 120 mm depth with 11 transmissions, at a frame rate of 486 frames/second. Results show that a two-fold increase of echo data is adequate for the reconstruction of HFR images.

II. THEORETICAL PRELIMINARY

To reconstruct a high-frame rate image, a relationship between the echo Fourier domain and the object Fourier domain is used [14] and [15]:

$$\begin{aligned}\tilde{R}_{k_{x_R}+k_{x_T},k_{y_R}+k_{y_T}}(\omega) &= \frac{A(k)T(k)H(k)}{c^2}\int_V f(\vec{r}_0)e^{i(\vec{K}^R+\vec{K}^T)\cdot\vec{r}_0}d\vec{r}_0 \\ &= \frac{A(k)T(k)H(k)}{c^2}F(\vec{K}^R+\vec{K}^T) \\ &= \frac{A(k)T(k)H(k)}{c^2}F'\left[(\vec{K}^R+\vec{K}^T)\Theta\right] \\ &= \frac{A(k)T(k)H(k)}{c^2}F'(k_{x_{R_2}},k_{y_{R_2}},k_{z_{R_2}}+k),\end{aligned} \qquad (1)$$

where $\tilde{R}_{k_{x_R}+k_{x_T},k_{y_R}+k_{y_T}}(\omega)$ is the Fourier transform of the time-varying RF echo signal when the transducer receive aperture is weighted at the spatial frequencies k_{x_R} and k_{y_R}, while the transmission wave vector is specified by k_{x_T} and k_{y_T}; $\vec{K}^T=(k_{x_T},k_{y_T},k_{z_T})$ and $\vec{K}^R=(k_{x_R},k_{y_R},k_{z_R})$ are the transmission and reception wave vectors, respectively, where $k_{z_T}=\sqrt{k^2-k_{x_T}^2-k_{y_T}^2}$ and $k_{z_R}=\sqrt{k^2-k_{x_R}^2-k_{y_R}^2}$; $\vec{r}_0=\left[x_0,y_0,z_0\right]^t$ and $\vec{r}_2=\left[x_2,y_2,z_2\right]^t$ are the original and rotated coordinates, respectively (the rotation angles are ζ_T and θ_T in Fig. 1); the superscript, t, represents a transpose of a vector or matrix; $H(\omega/c)=\{1,\ \omega\ge 0;\ 0,\ \omega<0\}$ is the Heaviside step function [29]; $A(k)$ and $T(k)$ are the transmit and receive transfer functions, respectively [30], where $k=\omega/c$ is the wave number, $\omega=2\pi f$ is the angular frequency, f is the temporal frequency, and c is the speed of sound; the relationship between the original coordinates, $\vec{r}_0$, and the rotated ones, $\vec{r}_2$, is given by $\vec{r}_0 = \Theta\vec{r}_2$ or $\vec{r}_2 = \Theta^{-1}\vec{r}_0$; the relationships between the rotated and original wave vectors are given by $\vec{K}_2^T = (k_{x_{T_2}}, k_{y_{T_2}}, k_{z_{T_2}}) = \vec{K}^T\Theta = (0,0,k)$ (assuming the transmit wave vector is aligned with a rotated axis $k_{z_{T_2}}$, see Fig. 1) and $\vec{K}_2^R = (k_{x_{R_2}}, k_{y_{R_2}}, k_{z_{R_2}}) = \vec{K}^R\Theta$, where $k_{z_{T_2}} = \sqrt{k^2 - k_{x_{T_2}}^2 - k_{y_{T_2}}^2}$ and $k_{z_{R_2}} = \sqrt{k^2 - k_{x_{R_2}}^2 - k_{y_{R_2}}^2}$; $f(\cdot)$ and $F(\cdot)$ are a 3D object function and its Fourier transform, respectively; $F'(\cdot)$ is the Fourier transform of $f(\cdot)$ at the rotated coordinates; V is the volume of the object; Θ and Θ^t are the coordinate rotation matrix and its inverse, respectively:

$$\Theta = \begin{bmatrix} \cos\zeta_T \cos\theta_T & -\sin\theta_T & \sin\zeta_T \cos\theta_T \\ \cos\zeta_T \sin\theta_T & \cos\theta_T & \sin\zeta_T \sin\theta_T \\ -\sin\zeta_T & 0 & \cos\zeta_T \end{bmatrix} \quad (2)$$

From Eq. (1), one can reconstruct images from RF echo data after both temporal and spatial samplings. In the following, 2D images are reconstructed with $\zeta_T = \theta_T = 0$ and $k_{y_T} = k_{y_R} = 0$ in Eq. (1) and various folds of densifications of data along transducer aperture in echo Fourier domain.

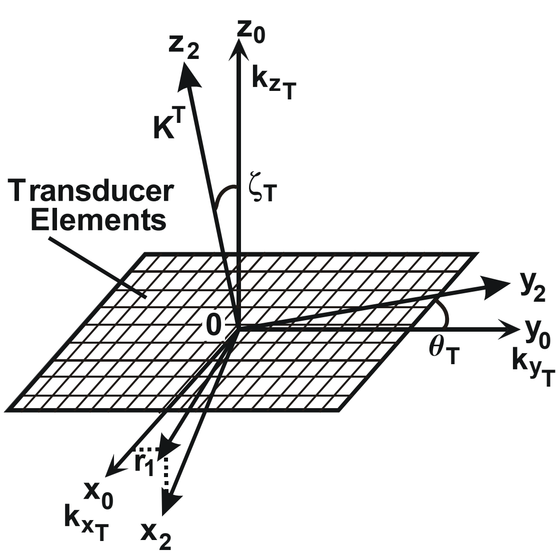

Figure 1. Coordinates before ($\vec{r}_0$) and after rotation ($\vec{r}_2$). (Fig. 1 of [15])

III. Experiments and Results

The following are images reconstructed with different folds (16, 4, 2, and 1) of densifications of RF echo data that were acquired with a home-made high-frame rate imaging system [14], [27]-[28] as described in the Introduction section. The images were reconstructed with limited-diffraction beam (LDB) (Figs. 2-5) (please click on the icon for an animation showing subtle differences among images) and steered plane wave (SPW) (Figs. 6-9) (please click on the icon for an animation showing subtle differences among images) transmissions, respectively.

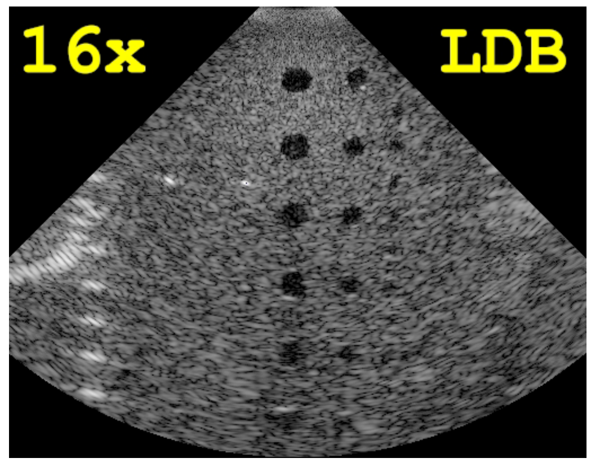

Figure 2. ATS539 phantom Image reconstructed with Eq. (1) and limited-diffraction beam in transmission (see Reference [14] for details of image reconstructions). Data in the echo Fourier domain are densified 16 folds.

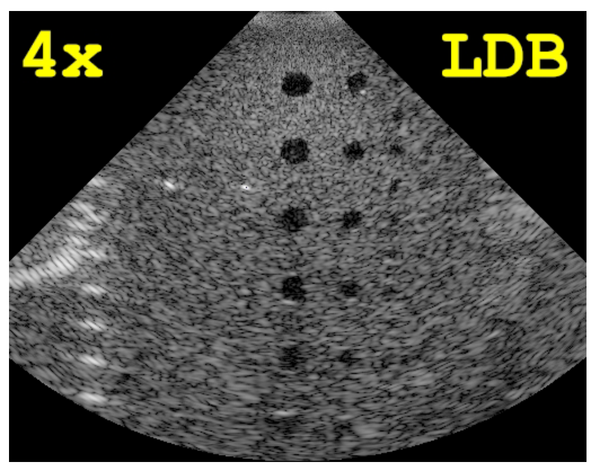

Figure 3. ATS539 phantom Image reconstructed with Eq. (1) and limited-diffraction beam in transmission (see Reference [14] for details of image reconstructions). Data in the echo Fourier domain are densified 4 folds.

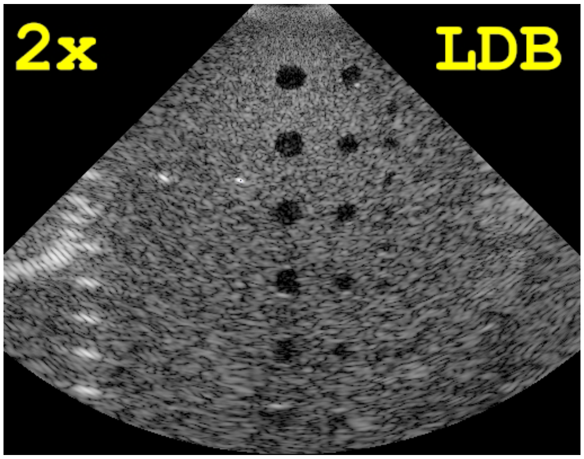

Figure 4. ATS539 phantom Image reconstructed with Eq. (1) and limited-diffraction beam in transmission (see Reference [14] for details of image reconstructions). Data in the echo Fourier domain are densified 2 folds.

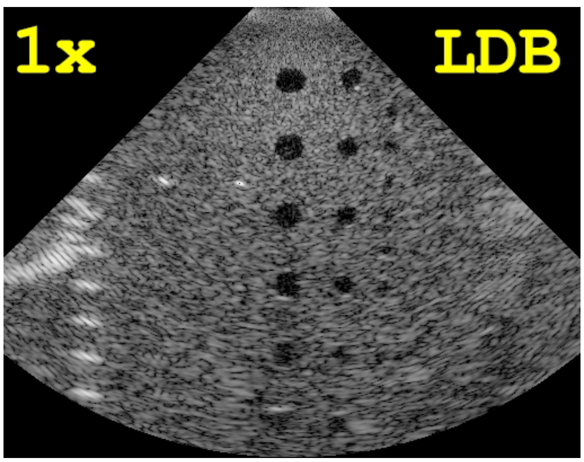

Figure 5. ATS539 phantom Image reconstructed with Eq. (1) and limited-diffraction beam in transmission (see Reference [14] for details of image reconstructions). Data in the echo Fourier domain are not densified.

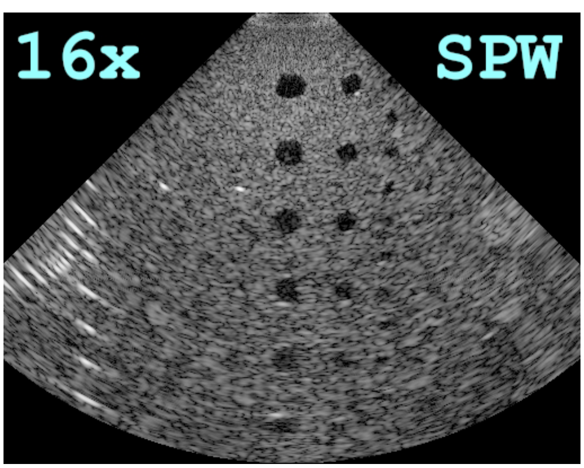

Figure 6. ATS539 phantom Image reconstructed with Eq. (1) and steered plane wave in transmission (see Reference [14] for details of image reconstructions). Data in the echo Fourier domain are densified 16 folds.

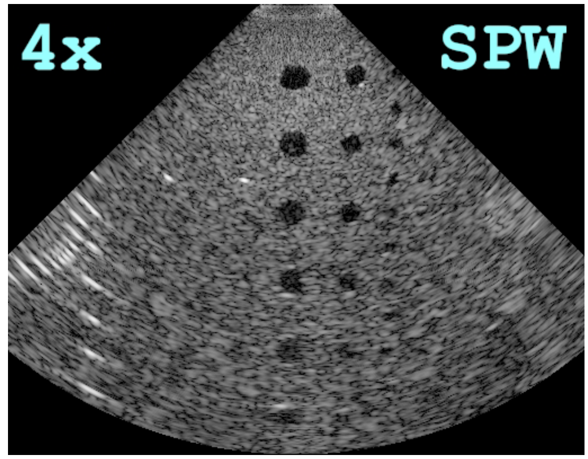

Figure 7. ATS539 phantom Image reconstructed with Eq. (1) and steered plane wave in transmission (see Reference [14] for details of image reconstructions). Data in the echo Fourier domain are densified 4 folds.

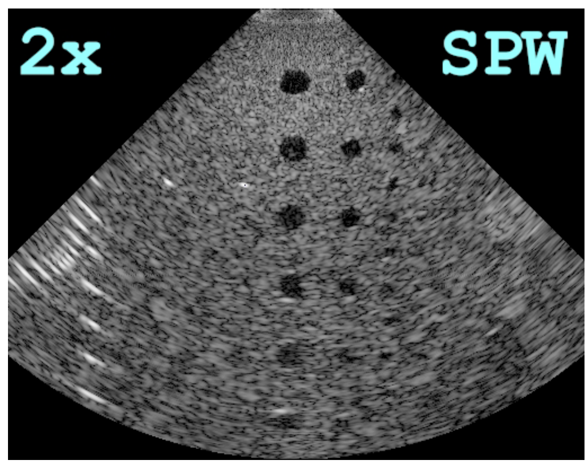

Figure 8. ATS539 phantom Image reconstructed with Eq. (1) and steered plane wave in transmission (see Reference [14] for details of image reconstructions). Data in the echo Fourier domain are densified 2 folds.

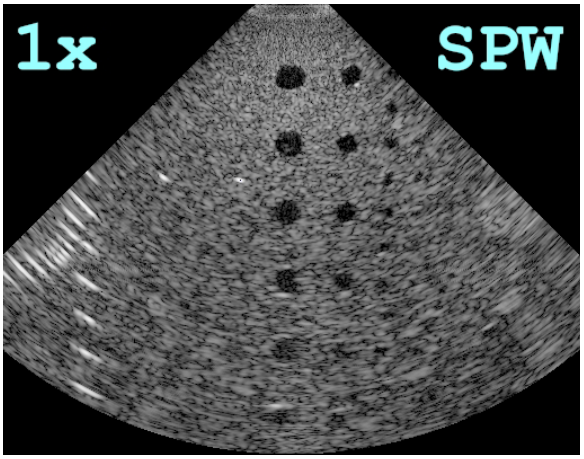

Figure 9. ATS539 phantom Image reconstructed with Eq. (1) and steered plane wave in transmission (see Reference [14] for details of image reconstructions). Data in the echo Fourier domain are not densified.

IV. CONCLUSION

It is found from this study that a two-fold densification of RF echo data in the echo Fourier domain gives the best compromise between quality of reconstructed images and the data increase due to the densification in HFR imaging. This finding will be especially significant for future HFR 3D imaging where any densification of echo data will be squared.

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