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Dynamic Equivalent Model of Distribution Network Cell Using Prony Analysis and Nonlinear Least Square Optimization

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***Abstract*—The paper presents initial results in development of dynamic equivalent model of Distributed Network Cell (DNC) comprising various types of loads and distributed energy resources. The method used for development of the model is based on Prony analysis and Nonlinear least square optimization. The dynamic equivalent model is developed in MATLAB based on simulated measurement data of DNC. The model is given in the form of an equivalent second order transfer function that can be used in dynamic stability studies. The model estimation procedure is evaluated on a case study and the initial simulation results show very good performance of the proposed dynamic equivalent model.**

***Index Terms*—Dynamic equivalent, Prony analysis, Nonlinear least square optimization.**

I. INTRODUCTION

THE increased penetration of distributed generation (DG) observed over last several years started to alter perception about distribution networks as passive terminations of transmission networks. The small-scale generation technologies, using mainly renewable energy sources, have been improved in the recent years, allowing their wider integration into distribution networks and at the same time resulting in requirement for different types of studies to be performed in distribution network.

The future distribution systems are now widely envisaged as active networks that might be subdivided in autonomous distribution network cells [1]-[4], with local management of power flows between the local generators, loads and adjacent cells.

Detailed study of the DNC characteristics and the effects they would have on the power system operation are therefore, of very high importance if these concepts are to be implemented in the future. So far, the performance of micro grid/DNC has been investigated, with special emphasis on island operation [5]-[6], transient behaviour [7]-[8], as well as on control and protection schemes. An adequate equivalent dynamic model, representing the DNC, has not yet been proposed even though it is necessary for more thorough assessment of the performance of power systems with DNC.

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The equivalent dynamic models of DNC are highly needed so that power system operators can estimate DNC impact on power system dynamic behaviour. The model should be able to account appropriately for the DNC dynamics seen by the external system through DNC interconnection. The main goal of dynamic equivalent therefore is to eliminate a part of distribution network and to replace it by simple equivalent model which has the same dynamic characteristics. The detailed modelling of the whole DNC is not practical due to the size of the system and computational time constrains associated with dynamic simulations of large power networks.

Most the work done on the dynamic equivalents has been focused on the distribution network that contains wind farms [9]-[12]. There are a few papers on the dynamic equivalent of DNC [13]-[16]. A dynamic equivalent of DNC using Hankel norm approximation is reported in [13]-[14]. The model was developed based on calculating the specific operating point data using the load flow calculation. A linearized model is produced by combining the state space model of generator and the model of the network into one linear model. The model reduction is then performed using Hankel norm approximation based on the specified error boundary. However, the dynamic equivalents produced are valid only for a given operating condition. Therefore, the procedures for obtaining them need to be repeated for different operating conditions.

Another dynamic equivalent of DNC is developed using system identification approach [15]-[16]. This approach treated the DNC as a black box due to the lack of detailed information on the network structure and parameters. The black box approach means that the dynamic equivalent of the distribution network is obtained based on some observed input and output data. The voltage and frequency are used as the input, and real and reactive power as the output. The parameter identification is then performed by importing the input and output data into the MATLAB System Identification toolbox. Developed model is in the form of state space and Auto-regressive model with exogenous input (ARX). This method offers the simplicity in the implementation as there is no necessity for detailed information about the network. However, the equivalent model produced is highly dependent on the type and location of the disturbance. This paper deals with the development of simple equivalent DNC model using detailed models of different types of renewable energy sources (RES) different levels and type of DG and different type and size of DNC load. The paper is continuation of the research reported in [8], [17]-[18] and it developed simple equivalent dynamic model of DNC in the transfer function form to analyse qualitatively the effects of cell structure and composition on its voltage and power responses.

II. THE ESTIMATION PROCEDURE

The method used for model development in this study is based on combination of the Prony analysis and the Nonlinear least square optimization.

Prony analysis has been shown to be a viable technique to model a linear sum of complex exponentials of signals that are uniformly sampled. Let $f(t)$ be a signal consisting of N evenly spaced samples. Prony's method fits a function

$$\hat{f}(t)=\sum_{i=1}^{N} A_i e^{\sigma_i t}\cos\left(2\pi f_i t+\phi_i\right) \quad (1)$$

to the observed function $f(t)$. After some manipulation utilizing Euler's formula, the following is obtained:

$$\hat{f}(t)=\sum_{i=1}^{N} A_i e^{\sigma_i t}\cos\left(2\pi f_i t+\phi_i\right)=\sum_{i=1}^{N}\frac{1}{2}A_i e^{\phi_i j}e^{\lambda_i t} \quad (2)$$

where:

$\lambda_i=\left(-\sigma_i \pm j\omega_i\right)t$ are the eigenvalue of the system, σ_i is the damping, φ_i is the phase angle, f_i are the frequency, A_i is the amplitude of the series, and $j=\sqrt{-1}$ [19].

The Prony analysis is used for initial estimates. The initial estimates are further optimized by an iterative nonlinear least square optimization procedure. Nonlinear least square is a general technique used to fit a curve through data. It fits data to any equation that defines ***Y*** as a function of ***X*** and one or more parameters. It finds the values of those parameters that generate the curve that come closest to the data (minimizes the sum of the squares of the vertical distances between data points and curve) [20]. This technique requires a model of the analysed signal. In this research, the signal model is defined by (1). The nonlinear least square optimization is generally used where the goal is to minimize the difference between the physical observation and the prediction from mathematical model. More precisely, the goal is to determine the best values of the unknown parameters amplitude (A), damping factor (α), frequency (ω) and phase (ϕ) in order to minimize the squared errors between the measured values of the signal and the computed ones.

All of the estimation methods used in this paper are implemented using MATLAB software. Initially, the PowerFactory DIgSILENT software is used to simulate dynamic response of the network and the appropriate active power responses are recorded. These responses are then imported into MATLAB software to determine the transfer function of the network.

The purpose of this proposed method is to represent the model of the response $y(t)$ as a sum of an initial step, K and a damped sinusoid response. Therefore, the DNC responses are represented as follows:

$$y(t)=\left[K+Ae^{\alpha t}\sin\left(\omega t+\phi\right)\right]u(t) \quad (3)$$

where $y(t)$ is active power response, K is the initial step, A is amplitude, α is damping factor, ω is frequency (in radian) and ϕ is phase angle (in radians).

The equation (3) is represented in MATLAB/Simulink as shown in Fig. 1.

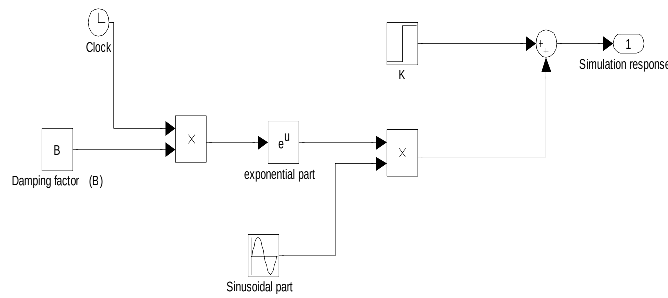

Fig.1. MATLAB/Simulink representation of equation (3)

A Prony analysis algorithm is written in MATLAB software and the active power responses are imported into MATLAB in order to obtain the parameters of (3). Once the parameters are derived, they are used as the initial values for the nonlinear least square optimization algorithm. The estimation of parameters is performed using Simulink Parameter Estimation. The Simulink model shown in Fig. 2 is then used to obtain model responses with estimated parameters and to compare those with actual DNC responses. Once the parameters of the model are tuned the model is converted into transfer function form using the Laplace transformation. By applying trigonometric identities to (3), it becomes:

$$\begin{aligned}\frac{y(t)}{u(t)}&=K+Ae^{\alpha t}\sin\omega t\cos\phi+Ae^{\alpha t}\sin\phi\cos\omega t\\&=K+\left[A\cos\phi\right]e^{\alpha t}\sin\omega t+\left[A\sin\phi\right]e^{\alpha t}\cos\omega t\\&=K+Be^{\alpha t}\sin\omega t+Ce^{\alpha t}\cos\omega t\end{aligned} \quad (4)$$

where $\begin{aligned}B&=A\cos\phi\\C&=A\sin\phi\end{aligned}$

and then after transformation into s-domain by using Laplace transform, the transfer function form of the model is obtained.

$$\begin{aligned}H(s)=\frac{Y(s)}{U(s)}&=K+B\left(\frac{\omega}{\left(s-\alpha\right)^2+\omega^2}\right)+C\left(\frac{s-\alpha}{\left(s-\alpha\right)^2+\omega^2}\right)\\&=K+\frac{B\omega+C\left(s-\alpha\right)}{\left(s-\alpha\right)^2+\omega^2}\end{aligned} \quad (5)$$

The accuracy of the estimation procedure was checked by calculating the Root Mean Square Error (RMSE) values. The RMSE is a frequently used measure of the differences between values predicted by a model or an estimator, and the values actually observed from the subject being modeled or estimated. These individual differences are also called residuals and the RMSE serves to aggregate them into a single measure of predictive power. In this research, RMSE is used to measure the differences between the estimated responses and the actual responses obtained from measurement.

Let the estimated response be $\theta_1 = \begin{bmatrix} x_{1,1} \\ x_{1,2} \\ \vdots \\ x_{1,n} \end{bmatrix}$ and the actual response $\theta_2 = \begin{bmatrix} x_{2,1} \\ x_{2,2} \\ \vdots \\ x_{2,n} \end{bmatrix}$. The RMSE is then calculated as:

$$\begin{aligned} RMSE(\theta_1,\theta_2) &= \sqrt{MSE(\theta_1,\theta_2)} = \sqrt{E\left((\theta_1-\theta_2)^2\right)} \\ &= \sqrt{\frac{\sum_{i=1}^{n}\left(x_{1,i}-x_{2,i}\right)^2}{n}} \end{aligned} \tag{6}$$

III. DNC STRUCTURE

The DNC study system shown in Fig. 2, is broadly based on the UK 11 kV distribution network. The DNC is connected to 33 kV external grid, represented by equivalent synchronous generator source. The grid supplies three 11 kV feeder system through 33/11.5 kV, 12/24 MVA transformer with 21 % impedance, Dy11 connection and voltage regulation at the low voltage side. The tap range is ±10 % of the nominal voltage, with 1.25% step change. The 11 kV feeders are connected to the point of common coupling (Bus 2) via fixed tap 11/0.433 kV transformers, with rating varying between 0.5-2.5 MVA and impedances between 4-6% depending on the load size. The converter connected (CCG) and fixed speed induction generators (FSIG) are connected on feeder 1. Two synchronous generators (SG) connected to Bus 2, are driven by gas turbine units modelled as IEEE GAST type. Further details on DNC modelling, in DigSILENT PowerFactory software, can be found in [8].

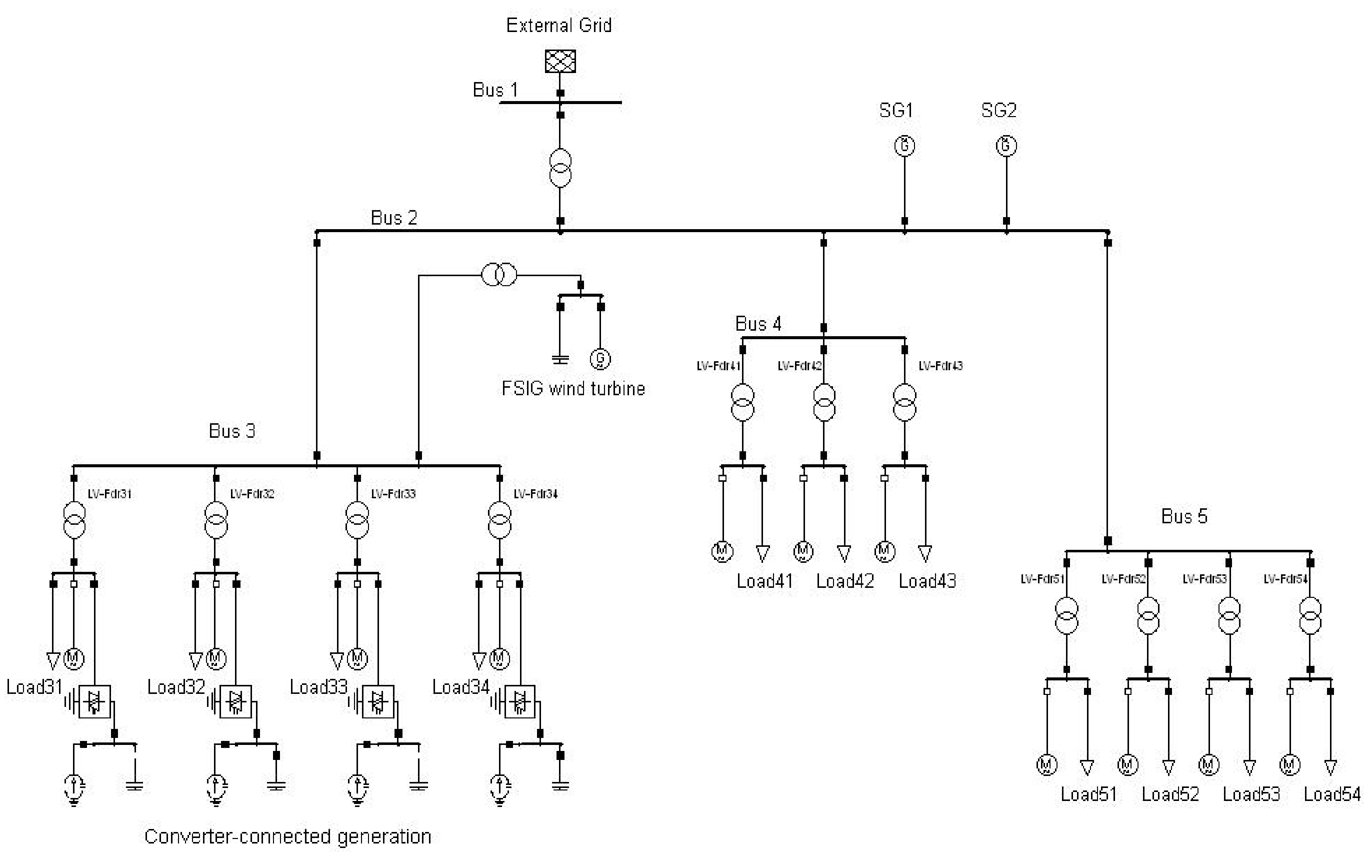

Fig.2. Single line diagram of the test network

IV. CASE STUDIES AND RESULTS OF SIMULATIONS

In case studies considered in this paper the total installed local generation, i.e., generation connected at Bus 1 (see Fig 2) is equal to total load in DNC. So, there is no exchange of real power with the rest of distribution network through Bus 1. The generation mix consisted of 85% of synchronous generators and 15% of renewable generation. The renewable generation included fixed speed wind turbines (modeled as conventional induction generators) and converter connected, photovoltaic, generation. The load mix consisted of 50% of static load (modeled as constant power load) and 50% of dynamic load (modeled as a mix of conventional small and large induction motors).

The responses of active power to a various small disturbances are measured at the point of connection (Bus 1). Only two types of disturbances are considered at this stage, namely small increase in DNC load and torque reduction of synchronous generator.

The disturbances are first simulated and the active power responses at the point of connection recorded. All the responses are simulated for 10 seconds after the initial disturbance. The sampling rate was 0.01s. Fig. 3 to Fig. 7 show the comparison between the actual DNC responses (obtained using DIgSILENT) and responses obtained with the equivalent model using estimated parameters. Table I shows the estimated model parameters and corresponding RMSE in all case studies.

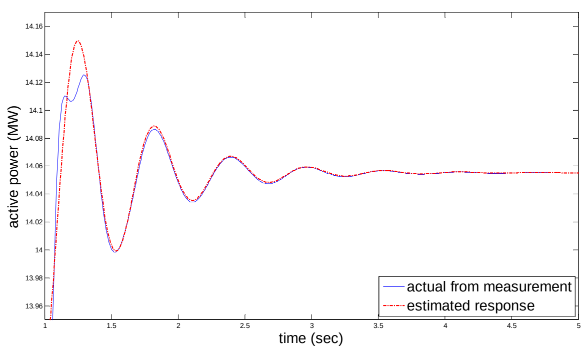

Fig. 3. Response following 5% load increase

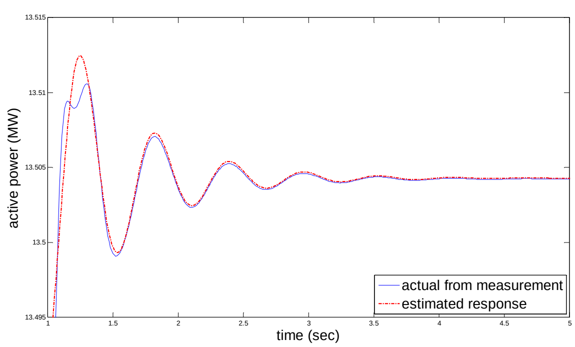

Fig. 4. Response following 5% increase in Load31

Fig. 3 to Fig. 7 show that the equivalent model responses match very closely the actual DNC responses even for large disturbances. The estimation method works particularly well if the response is purely sinusoidal, e.g., Fig. 6. In case of non-sinusoidal system response, e.g., Fig. 3 to Fig. 5, the model responses are slightly different from the simulated ones during the first swing and then very quickly (about 0.5 s after the disturbance, when higher order frequency mode gets damped) resume the same form as original system response.

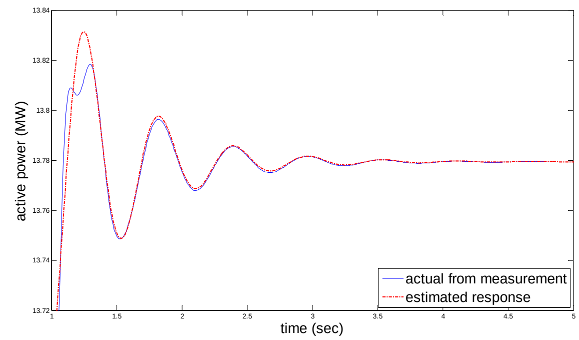

Fig. 5. Response following 30% increase in Load54

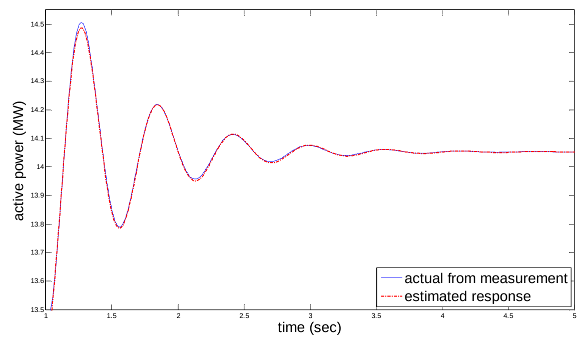

Fig. 6. Response following 10% reduction in torque of SG1

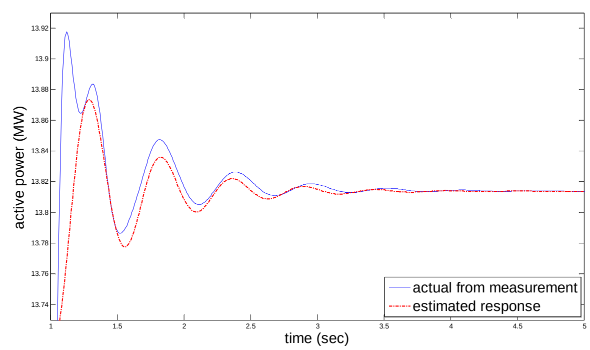

Fig. 7. Response following 25% reduction in torque of SG2

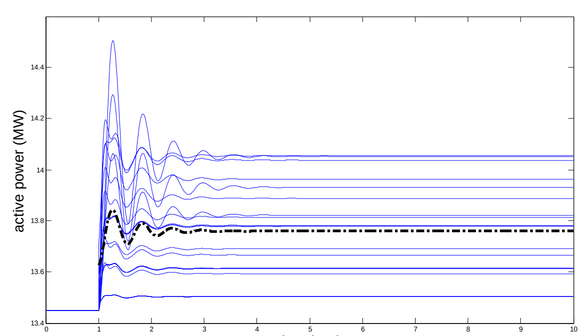

Fig. 8. Model response with averaged parameters (thick dashed line) and responses with individual sets of parameters

Fig. 8 shows equivalent model response (thick dashed line) with average values of parameters obtained from all individual responses along with all the individual responses. It can be seen that even with the average values of parameters the model captures qualitatively the response of the DNC and that by adjusting parameters a whole range of responses can be easily obtained.

TABLE I
ESTIMATION PARAMETERS OBTAINED BY THE PROPOSED ESTIMATION PROCEDURE

Disturbance	K	A	α	ω	ϕ	RMSE
Load43 increase 5%	13.5037	0.0787	-1.7891	11.003	6.5066	0.0011
Load54 increase 5%	13.5038	0.0888	-1.8538	11.0115	6.4958	0.0011
Load31 increase 5%	13.5043	0.0733	-1.745	10.999	6.5083	0.0019
Load43 increase 15%	13.6136	0.2518	-1.821	10.9983	6.5076	0.0033
Load54 increase 15%	13.6137	0.2647	-1.8474	10.9961	6.5178	0.0034
Load31 increase 15%	13.6154	0.2585	-1.8284	10.9989	6.5003	0.0035
All load increase 2%	13.6911	0.3781	-1.8308	10.9923	6.5159	0.005
Load43 increase 30%	13.7792	0.4972	-1.8111	10.9886	6.5172	0.0067
Load54 increase 30%	13.7794	0.5248	-1.8395	10.9850	6.5299	0.0068
Load31 increase 30%	13.7827	0.5128	-1.8212	10.9822	6.5221	0.0071
All load increase 5%	14.0554	0.9281	-1.8176	10.9692	6.542	0.0141
SG1 torque reduction 0.1pu	14.0524	3.7412	-1.6866	10.9301	6.3847	0.0061
SG2 torque reduction 0.1pu	13.5933	0.3862	-2.0844	11.9165	4.8038	0.0076
SG1 and SG2 torque reduction 0.05pu	13.8223	2.156	-1.7419	10.9045	6.4602	0.0032
SG2 torque reduction 0.4pu	14.0377	0.5086	-1.4576	11.3136	5.8819	0.0315
SG2 torque reduction 0.25pu	13.8138	0.6607	-1.8531	11.7337	5.1067	0.0207
SG2 torque reduction 0.15pu	13.6663	0.299	-1.7143	11.723	5.1237	0.0112
SG2 torque reduction 0.35pu	13.9628	0.9593	-1.8612	11.6828	5.189	0.0284
SG2 torque reduction 0.3pu	13.8882	0.8079	-1.857	11.7042	5.1563	0.0245
SG1 torque reduction 0.08pu	13.9315	3.116	-1.7127	10.9424	6.3733	0.0082
Average parameter	**13.7605**	**0.8246**	**-1.7987**	**11.1887**	**6.1072**	

V. CONCLUSIONS

Initial stages of development of the dynamic equivalent model of distribution network cell using Prony analysis and Nonlinear least square optimization are described in this paper.

The model is intended primarily for the use in small disturbance stability studies of large distribution and transmission networks.

The paper is primarily concerned with the estimation method to establish the deterministic model of reasonably complex DNC in a simple transfer function form. The results obtained so far are promising and they gave a significant confidence in applicability of the method for determining the dynamic equivalent of the DNC.

The equivalent model developed is measurement-based rather than component-based as the exact composition and structure of DNC is generally not known.

Since the model is of a simple second order transfer function form it could fully capture only one oscillatory mode of the system and such it is particularly useful for modeling DNC where one oscillatory mode clearly dominates its dynamic response.

The work is currently underway to develop low order equivalent model that would represent more accurately network responses following large disturbances (power system faults) where there is more than one dominant oscillatory mode.

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