

Distance Protection Algorithm for Power Transmission Lines based on Monte-Carlo method

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Abstract— This paper is focused on the development of the new principles and operational algorithms of the transmission lines distance protection. The proposed distance protection algorithms utilize statistical information about the undefined and random parameters and quantities such as equivalent impedances of the systems at the unmonitored end of the power transmission line and current and voltage phasor measurements errors. Knowledge about the distribution of these parameters and values results in more secure and dependable procedure for the decision on the line tripping or restraining in case of the appearance of the fault conditions in the network.

The proposed algorithm is based on the modeling of the faulted line and the Monte-Carlo method. The algorithm computes expected value of the distance to the fault, fault resistance in the fault point and the standard deviation of the distance to the fault. This leads to the inherent adaptive features of the approach.

Index Terms-- Distance protection, Statistics, Transmission Lines, Adaptive protection.

I. INTRODUCTION

Distance protection is one of the most widely used methods to protect transmission lines [1]. The fundamental principle of distance relaying is based on the local measurements of voltages and currents where the relay responds to the calculated impedance value between the relay terminal and the fault location in the transmission network.

Due to impact of the several unmonitored parameters that are uncertain in the physical nature, such as errors of the currents and voltage measurements, fault resistance, loading and topology of the system at fault inception instance, the estimate of the impedance Z contains some error ΔZ that can lead to the incorrect protection operation. The operation zones of relays help to prevent financial losses and/or technical jeopardizing that might occur due to misoperation of the relays [1].

These zones are normally selected based on a worst case scenario [5] of the combination of factors that impact performance of the relay, seeing that the probabilistic problem definition would introduce complexity. Thus, selecting the operation zones, two main requirements of the protection should be compromised: dependability and security, and simultaneously both should be satisfied.

The stochastic nature of the power system parameters or the unforeseen changes in the topology of power system, the distance protection can operate undesirably. As a recent example, undesired operation of protection contributed to the development of many blackouts [4].

The increase of the efficiency of the protection operation is possible utilizing the adaptive approach [5]. For example, the adaptive protection depending on the parameters of the monitored process can change the boundaries of the protection zones. The implementation of the adaptive features of the relays became possible as a result of breakthrough of the microprocessor technique. Further fast development of this technique insures the potential implementation of the more and more sophisticated procedures for the decision making on tripping or restraining.

Particularly, it becomes realistic to exchange the deterministic approach by the more advantageous probabilistic one that requires real-time implementation of the Monte-Carlo method.

This paper discusses the theoretical background of the latter approach and shows its efficiency. As an example, the first zone protection operation is investigated for the most frequent fault type: the single phase to ground, assuming that for the controlled line the phasors of the current and voltage are available as the result of analog-digital conversion and the signal processing.

In previous publications [6], [7] the authors have proposed the application of a statistical approach to fault location. This paper discusses its extension showing principally new possibilities in the protections area.

II. THE THEORETICAL BACKGROUND

Let us consider a symmetrical three-phase transmission line connecting two power systems with known equivalent impedances (Fig.1). To determine the fault point let us apply the method based on power computation.

For known (measured) currents and voltages of the monitored end of the transmission line the complex values for positive, negative and zero sequences of power S_i at the

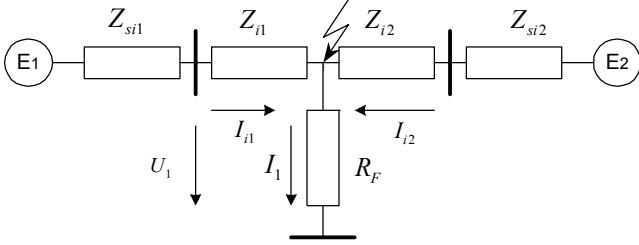


Fig. 1. One-line diagram of the transmission system equivalent for the SLG fault conditions.

faulted point can be expressed:

$$S_i = U_i I_i^* = U_{il} I_{il}^* - I_{il} \cdot Z_{il} I_{il}^* - R_F |I_i|^2 \quad (1)$$

where S_i is power of sequence i , U_{il} , I_{il} is measured voltage and current of sequence i respectively, I^* is conjugate value of current I and R_F is transient resistance of the fault.

Taking into consideration that in the fault point, in general case behind the fault impedance, there is an equality of sequences powers S_i [1]:

$$\sum_{i=0}^2 S_i = U_1 I_1^* + U_2 I_2^* + U_0 I_0^* \quad (2)$$

On (1) and (2) and assuming that the fault impedance is purely resistive and therefore consumed power is also active, it can be obtained:

$$\sum_{i=0}^2 Q_i = \text{Im} \left\{ \sum_{i=0}^2 \frac{U_{il} I_{il}^*}{K_i^*} - \sum_{i=0}^2 \frac{|I_{il}|^2 Z_{il}}{K_i^*} - \dots \right. \\ \left. \dots - \frac{U_{1l} \cdot I_L^*}{K_1^*} + \frac{I_{1l} I_L^* Z_{1l}}{K_1^*} \right\} = 0 \quad (3)$$

where I_L is pre-fault current, while the current distribution coefficients K_i are determined from the equation:

$$K_i = \frac{Z_{i2} + Z_{si2}}{Z_{si1} + Z_{i1} + Z_{i2} + Z_{si2}} \quad (4)$$

K_i^* is the conjugate value of K_i , Z_{si1} and Z_{si2} are the Thevenin equivalents of power system. The impedances Z_{i1} and Z_{i2} depend on distance to the fault point:

$$Z_{i1} = L_F Z_{isp}, \\ Z_{i2} = (L - L_F) Z_{isp} \quad (5)$$

where L is the length of the protected line, L_F is distance to the fault point and Z_{isp} is the i -sequence impedance per line length unit.

Taking into account that in case of single phase to ground fault there is an equality of currents in the fault point:

$$I_1 = I_2 = I_0, \quad (6)$$

the equation (3) can be rewritten in more convenient for the implementation form:

$$\sum_{i=0}^2 Q_i = \text{Im} \left\{ \frac{I_0^*}{K_0^*} \cdot \sum_{i=0}^2 (U_{il} - I_{il} Z_{il}) \right\} = 0 \quad (7)$$

Hence, (7) is a quadratic equation for unknown distance L_F that can be solved analytically for the assumed impedance Z_{si2} .

If the distance L_F is known, the resistance may be determined expressing voltage U_F via current I_F of the faulted phase as follows:

$$U_F = \left(I_F + \frac{Z_{0sp} - Z_{1sp}}{Z_{1sp}} \cdot I_{01} \right) Z_{1sp} L_F + \frac{3R_F I_{01}}{K_0} \quad (8)$$

If the remote end of the transmission line is unmonitored, the equivalent impedance Z_{si2} can be considered to be a random variable.

If a communication channel exists between the ends of the line that is to be protected, and synchronous measurements are conducted, the procedure for determining the coefficient K_i will be simplified:

$$K_i = \frac{I_{01}}{I_{01} + I_{02}} \quad (9)$$

In this case, for determining the unknown distance L_F and resistance R_F , it is sufficient to solve the linear equation (3).

III. THE STOCHASTIC APPROACH

Summarizing the stated above equations, one can declare that the distance to the fault L_F and R_F are linked to the measured phasors of the currents I and voltages U and unknown equivalent impedances Z_{i1} of the remote transmission line end system by relation of the following form:

$$L_F = \Phi(I, U, Z_{si2}) \quad (10)$$

where Φ is for some procedure of the distance L_F and R_F calculation. The procedure employs the measurement results of the controlled currents and voltages and information of the impedance Z_{si2} values.

On the other hand, taking into account that the measured current and voltage data contain random errors - correspondingly ΔI and ΔU , and, in general, values of the uncontrolled impedance Z_{si2} can also be treated as random, the equation (10) could be considered as basic one to determine the distribution law of the estimate of the distances to the fault L_{Fest} and R_{Fest} or its numerical characteristics.

To determine distribution density $g(L_{Fest})$ of the L_{Fest} on the base of (10), it is necessary to know the distribution relative density function $g(I, U, Z_{si2} / I_{est}, U_{est})$ - the density of the current, voltage and impedance distribution under obtained

measurement results I_{est}, U_{est} . Theoretically, Bayes' theorem could be involved [2]. Determination of the desired distribution function $g(I, U, Z_{si2} / I_{est}, U_{est})$ is possible on the base of faulted line processes simulation and Monte-Carlo method utilization. For this purpose, significant number of trials should be performed, and consequently, notable processing time will be needed.

However, more effective procedure could be obtained, using the linearization method, taking into account the physical nature of the measured values and relatively small values of measurement errors, and supposing that measurements errors are additive with the zero value of mathematical expectation, it can be stated that [2]:

$$\begin{aligned} E[\Phi(I, U, Z_{si2})] &\equiv E[\Phi(I_{est}, U_{est}, Z_{si2})] \\ \sigma[\Phi(I, U, Z_{si2})] &\equiv \sigma[\Phi(I_{est} + \Delta I, U_{est} + \Delta U, Z_{si2})] \end{aligned} \quad (11)$$

where $E(\dots)$ is the mathematical expectation and $\sigma(\dots)$ is the standard deviation.

Statement (11) is strictly true for linear functions. In considered non-linear case, it is deemed permissible for practical applications. On the other hand it becomes possible to employ more efficient procedure for Monte-Carlo method application.

IV. SIMULATIONS

The algorithm for estimation of the mathematical expectation $E[L_F]$, $E[R_F]$ and standard deviation $\sigma[L_F]$, $\sigma[R_F]$ values based on the Monte-Carlo simulations are shown in Fig. 2.

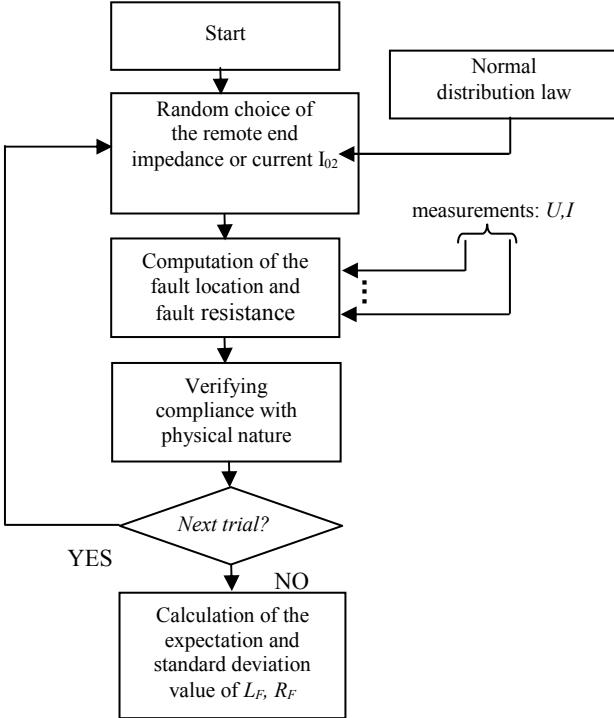


Fig. 2. Algorithm for the distance to fault and fault resistance estimation.

First, the remote end equivalent impedance, for example Z_{S02} or current I_{02} , is randomly selected. Using the measurements, the fault distance is computed, then the compliance with the physics of the problem is verified.

Let us show that mathematical expectation of the distance to the fault and fault resistance as well as standard deviation of these parameters are applicable for the conclusion regards the required protection action.

For comparison, next section contains results obtained by the conventional algorithm based on the following equation:

$$Z_F = \frac{U_f}{I_f + I_0} \frac{\frac{Z_{0sp} - Z_{1sp}}{Z_{1sp}}}{\text{real}(Z_F)} \quad (12)$$

This algorithm is affected by the pre-fault current, fault conditions and the impact of the remote end infeed, particularly strong in case of high resistance faults, and therefore, makes necessary certain restrictions in selection of the operational zone for the conventional protection.

Commonly, the extent of the characteristic along the X axis is chosen depending on the parameters of the line. The impedance loci projection on the R axis depends strongly on the computation conditions, such as fault impedance and pre-fault power flow.

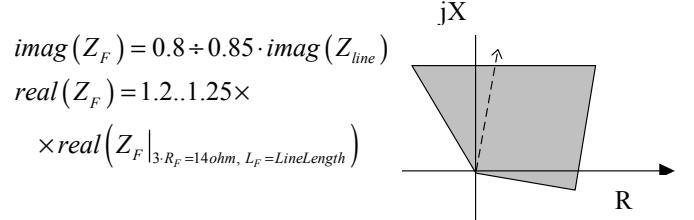


Fig. 3. Illustration of the protection quadrilateral operation zone selection; Z_{line} is the positive sequence impedance of the protected line.

The coefficient 0.8 in Fig. 3 is a commonly used [10] safety margin that is derived from practical experience, the value of the fault resistance and the coefficient 1.2 are another examples of empirical assumptions that, generally, tend to limit instantaneous protection operation to relatively low resistance faults, thus avoiding sympathetic trips. However, as shown below, protection operation could be improved with the more complex but simultaneously more adjusted to the particular conditions approach.

V. RESULTS

A. Test system

For the full power system model the combination of the parameters is limited and, in addition, simulations require more computational effort. In reality, the conditions of the protection operation vary significantly depending on the particular line within the power system, and also for each particular line the conditions are changing within certain limits as the power system operation changes. Therefore, we show more results of the algorithm performance on the reduced system equivalents and larger variation of operation conditions, simulated by Monte-Carlo method.

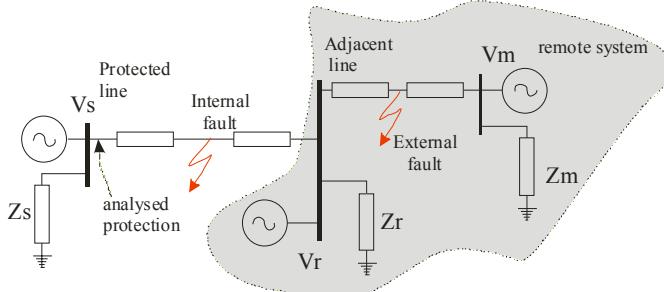


Fig. 4. The studied 3-bus test system diagram

Let us define a fault that occurs on the protected line as internal, and a fault on the adjacent line as external. Fig. 5 - 11 represent the modeling results of the described above algorithms. The results are discussed in more details in the subsequent subsections. The simulations were run for the following test system in Fig. 4:

- the nominal voltage of the modeled system is 330 kV;
- Operational voltage of the buses varies from 0.95 to 1.05 pu in magnitude;
- the pre-fault power flow through the line varies from 0 to 1000 MW in both directions and determined by the bus voltage angle;
- fault resistance varies from 0 to 50 ohm;
- the line protected line length is 150 km;
- the adjacent line length is 100 km
- line apparent parameters are:
 $Z_{l,2} = 0.042 + j0.324$ [ohm/km],
 $Z_0 = 0.187 + j0.7838$ [ohm/km];
- The system equivalent impedances at the monitored line end, except for the particularly mentioned cases, vary in magnitude and angle:

$$|Z_{s1,2}| \in [16, 80] \text{ ohm}, \quad \angle Z_{s1,2} \in [76, 85] \text{ deg},$$

$$|Z_{s0}| \in [20, 90] \text{ ohm}, \quad \angle Z_{s0} \in [76, 84] \text{ deg};$$

- Current random errors varies 5% in magnitude and 5 deg in angle
- The impedances of the system r and system m varies in the same range as the system s ; Thus, transformed for the two bus model as in Fig. 1, these impedances result in equivalent impedance at the remote line end:

$$Z_{ei} = \frac{Z_{ri}(Z_{il,2} + Z_{mi})}{Z_{ri} + Z_{il,2} + Z_{mi}},$$

$$|Z_{e1,2}| \in [12.04, 46.9] \text{ ohm}, \quad \angle Z_{e1,2} \in [76.24, 84.91] \text{ deg},$$

$$|Z_{e0}| \in [16.67, 59.03] \text{ ohm}, \quad \angle Z_{e0} \in [76.02, 83.63] \text{ deg};$$

The distance protection algorithms operation is frequently significantly impacted by the relation of the sending (local) and receiving (remote) end impedance relations. Generally, distance protection algorithms produce better results for lines with the strong (low impedance) local end, since the current infeed impact from the remote end is lower. In the following studies, we will distinguish these two cases, taking the following variation limits:

- sending end is strong:
 $|Z_{s1,2}| \in [16, 20] \text{ ohm}, \quad |Z_{e1,2}| \in [30.1, 36.5] \text{ ohm},$
 $|Z_{s0}| \in [18, 25] \text{ ohm}, \quad |Z_{e0}| \in [50.7, 38.5] \text{ ohm},$
- sending end is weak
 $|Z_{s1,2}| \in [80, 130] \text{ ohm}, \quad |Z_{e1,2}| \in [18.4, 27.6] \text{ ohm},$
 $|Z_{s0}| \in [90, 150] \text{ ohm}, \quad |Z_{e0}| \in [23.6, 36.2] \text{ ohm},$

The uniform distribution law was assumed for all the parameters variations.

Let us determine the maximal possible operational zone of the relay. The multiple computations of the standard deviation and the mathematical expectation of the fault resistance and distance to the fault at the various short-circuit conditions on the protected line determine the desired area in the Z-plane. On the other hand, by modeling the external faults and load conditions, we can determine the ensemble of points that correspond to the non-operation requirements.

The knowledge of these ensembles shall be the basis for the selection of the reach zone of the relay.

B. Operation under internal faults

Fig. 5. shows result of the computations for the faults that are simulated on the protected line. For the comparison results of the conventional (squares) algorithm (12) are presented. It can be observed that the proposed algorithm (circles) is less influenced by the parameter variations and provides accurate results on the large line extent. This advantage, the ability to provide less distorted picture, for particularly high impedance faults on the large power line part can be used to extend the reach zone of the protection.

In addition to mathematical expectation of the impedance to the fault, for each particular fault, the new algorithm will estimate standard deviation (small circles) of the impedance and thus, determine if the fault “belongs” to the operational zone.

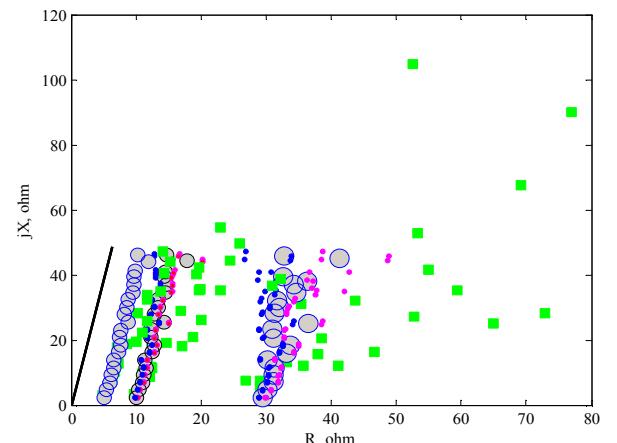


Fig. 5. Comparison of conventional and proposed algorithms operation for the internal faults with 5, 10 and 30 ohms resistance and fault position 0-100% of the line length. Line impedance (black line) and for each of the faults is shown impedance seen by conventional algorithm (green squares), mathematical expectation (gray circles) and \pm standard deviation (small circles) of the impedance determined by proposed algorithm.

C. Operation under external faults

The conditions for the protection misoperation¹ can be derived for the power system equivalent in Fig. 4. At the certain parameters combination, namely systems equivalent impedances, fault resistance, distance to the fault, power injection in intermediate and remote busses, line impedances, the currents and voltages measured by the protection will coincide for the external and internal faults. That is a general limitation of the distance protection approach and the use of the local measurements only.

The number of parameter combinations is large and which of those would result in misoperation cannot be determined analytically. However, the probability of the occurrence of the unfavorable conditions can be determined numerically for the particular power line.

Fig. 6 - Fig. 9 show the computed impedances by the conventional and the expectation determined by proposed algorithm in case of the external faults. Moreover, the probability densities are shown for the results of both methods.

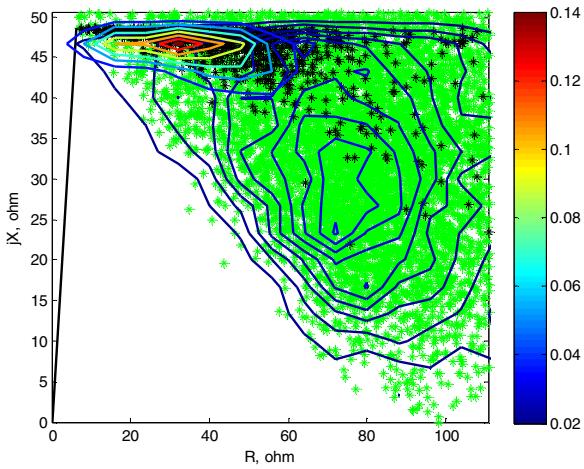


Fig. 6. Fault impedance seen by the conventional (green) and proposed (black) algorithms in cases of the external faults. Proposed algorithm sees the fault as internal in 3.2 % of cases (total number of trials 30 000);

The conventional algorithm will always determine the impedance, only the points that are close to the possible operational zone are shown and taken into account for the probability density estimation. The proposed algorithm would determine the impedance in limited number of cases that would satisfy its equations. In these relatively low probability cases, the determined impedance would correspond to remote high impedance faults. It can be observed that the conventional algorithm has higher standard deviation. In addition, its probability distribution shape implies stricter restrictions, i.e. it limits possible instantaneous protection operation to close, low resistance faults. Simultaneously, proposed method allows much more flexibility.

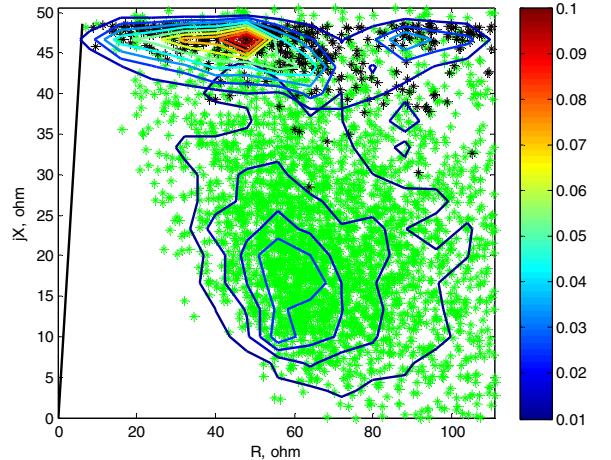


Fig. 7. Fault impedance seen by the conventional (green) and proposed (black) algorithms in cases of the external faults, when the sending end system is weak and the maximal power transfer increased up to 1.5 GW; Probability of misoperation¹ of the proposed algorithm 4.2 % (total number of trials 10 000);

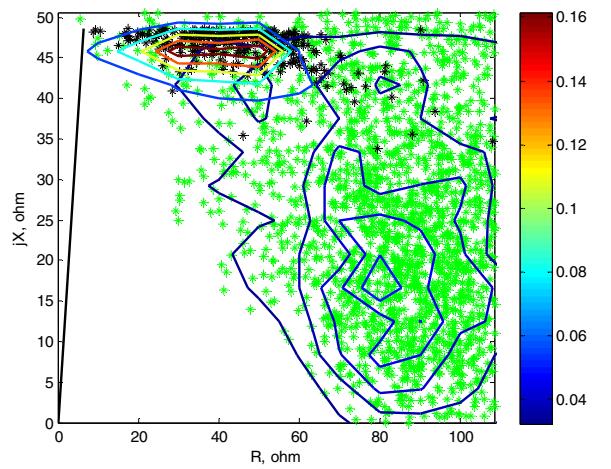


Fig. 8. Fault impedance seen by the conventional (green) and proposed (black) algorithms in cases of the external faults, when the sending end system is weak and the maximal power transfer is 1 GW. Probability of misoperation of the proposed algorithm 2.8% (total number of trials 10 000);

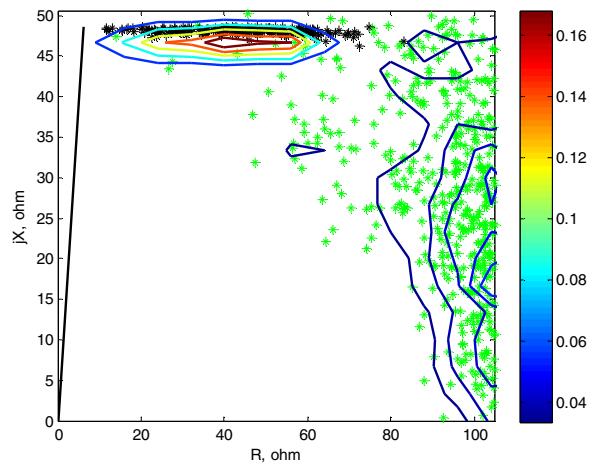


Fig. 9. Fault impedance seen by the conventional (green) and proposed (black) algorithms in cases of the external faults, when the sending end system is strong and the maximal power transfer is 1 GW. Probability of misoperation of the proposed algorithm 2.07% (total number of trials 10 000);

¹ In this context by misoperation we understand the case when the proposed algorithm identifies an external fault as an internal one. The algorithm real performance will be determined by the selected operational zone, which should avoid such impedances.

System equivalent impedances ratio does not significantly influence the mathematical expectation of the impedance determined by the proposed algorithm. It may be noticed however, that it could be influenced by the extent of the variations of the equivalent impedances.

Commonly, when more than two lines are connected to the substation that means there are several equivalents for the same protection depending on which neighboring line is faulted. However, this does not significantly affect the results represented in Fig. 6 - Fig. 9, as the simulated variations of the remote systems impedances were already relatively large.

D. Performance under no-fault conditions

The widely recognized restriction of the distance protection is the possible misoperation under heavy transfers and lower voltage conditions, when the impedance seen by the relay may move into the operation zone. Thus, the operational zone should be reduced to avoid these occurrences. This drawback can be improved by introducing the zero sequence current check that would block protection operation under symmetrical (no-ground-fault) conditions. However, under the unsymmetry of the measured currents and voltage or measurement errors the operation may be allowed and the misoperation may still occur.

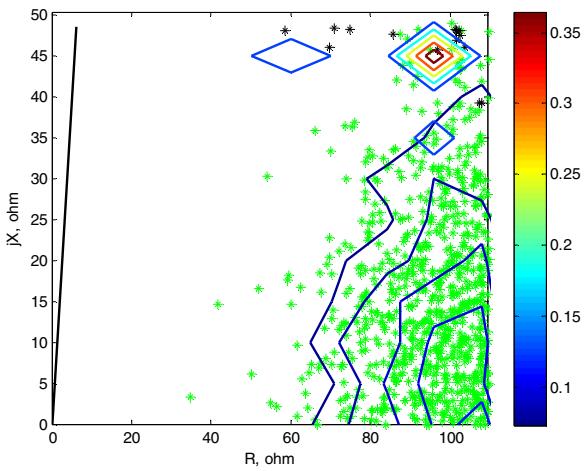


Fig. 10. Impedance determined by the proposed and the conventional algorithm in no fault conditions. The new algorithm has misoperated 0.033% (for 10^5 trials) and shows high impedance remote faults.

Fig. 10 shows the algorithms performance under unsymmetrical load conditions (or erroneous measurements). The currents and voltages for each phase are simulated as normally distributed with the standard deviation of 1 degree and 5% of magnitude for the random power transfers from 0 to 1000 MW. The figure does not show higher values of the determined by the conventional algorithm impedances, focusing on the most interesting cases (or closest to the implicit reach zone). For the proposed algorithm, however, all

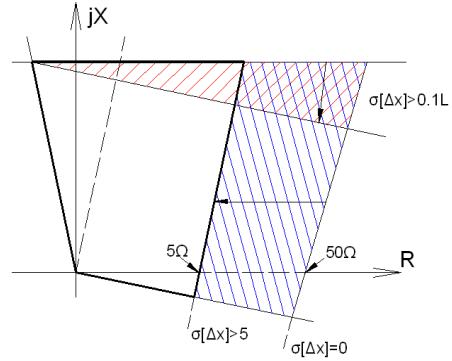


Fig. 11. Adaptive zone of the distance protection

the cases of misoperation are shown. Clearly, the values of the impedance seen by the proposed algorithm correspond to more distant and high resistance faults, which shall allow the increase the reach zone of relay.

To exclude the potential misoperation of the relay at external short circuits, the operational zone of the protection shall be defined depending on the value of the standard deviation of the resistance $\sigma[R]$ and reactance $\sigma[X]$ as in Fig. 11. Therefore, in unfavorable but relatively rare cases (high values of $\sigma[R]$ and $\sigma[X]$) the operational zone is limited. If these values are relatively small, the distance protection trips instantaneously even at large values of the fault resistance. The operation zone, thus in the majority of cases, extends up to 30-50 ohm fault resistances as shown in Fig. 11 and only at unlikely [11] adverse combination of the short circuit parameters it narrows down till 5 ohms faults.

VI. CONCLUSIONS

Distance protection operation is influenced by a number of random parameters that may lead to incorrect or failed operation and unnecessary delay in fault clearing.

The application of a probabilistic approach that takes into account parameter variations to distance protection provides new opportunities for decision of the relay action. Simulations confirm reliability and high efficiency of the proposed method that allows an increase of the relay operational zone for high resistance and remote faults.

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