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A Model Predictive Control Approach to Dynamic Economic Dispatch Problem

Xiaohua Xia*, Jiangfeng Zhang* and Ahmed Elaiw*

***Abstract*—There are two ways in literature to formulate the optimal power dispatch problem of generators with ramp rate constraints. One way is the optimal control dynamic dispatch (OCDD) approach based on control theory, the other way is the dynamic economic dispatch (DED) based on optimization theory, and the two formulations are usually believed to be the same. In this paper we show the difference between the two formulations and also propose a Model Predictive Control (MPC) approach to the dynamic dispatch problem by the OCDD framework. This MPC approach provides solutions converging to the optimal solution of an extended version of the DED problem and the MPC algorithm is also robust under certain disturbances and uncertainties.**

I. Introduction

In order to solve the dynamic dispatch problem of the committed generating units' outputs to meet the load demand at minimum operating cost while satisfying ramp rate constraints and various other constraints, a control system framework was introduced in the 1970's ([2], [11]). The obtained optimal control dynamic dispatch (OCDD) formulation is to model the power system generation by means of state equations where the state variables are the electrical outputs of the generators and the control inputs are the ramp rates of the generators.

Since 1980's, the dynamic dispatch problem has been formulated as a minimization problem of the total cost over the dispatch interval, and has been known as the dynamic economic dispatch (DED) problem (see, *e.g.*, [18], [9], [17], [1], [8], [7], [3]).

The two formulations have great similarities. For example, both of them are subject to similar sets of constraints, and the solutions are to be implemented repeatedly and periodically due to the cyclic consumption behavior and seasonal changes of the demand. Exactly due to this periodic implementation, both formulations have the same technical deficiencies as we will illustrate later in the paper. Furthermore, we will show in this paper that the two formulations are actually different.

After the formulation of OCDD and DED, the thrust of research has been focused on various optimization techniques and procedures incorporating with extended and complex objective functions or constraints. However both formulations and their solving algorithms suffer from the deficiency of not allowing to compensate for inaccuracies originating from modeling uncertainties, external disturbances, and unexpected reactions of some of the power system components. In the terminology of control theory, both formulations are in fact open-loop systems and there is no way to feedback the inaccuracy information to the systems so that the solutions can be compensated. In other words, the two formulations are not closed-loop systems. In this paper we will introduce a closed-loop approach to compensate those inaccuracies. To this end, the Model Predictive Control (MPC) technique is adopted.

MPC method has emerged and been successfully applied particularly in the process control industry since 1970's. Theoretical properties such as stability and robustness of the MPC have been studied by many authors since the early work of Kleinman [10]. MPC is a feedback control technique that uses an explicit model of the plant to predict the future response of the plant over a finite horizon. The feedback controller is constructed by solving a finite horizon optimal control problem at each sampling instant using the current state of the plant as the initial state for the optimization and applying only "the first part" of the optimal control [12]. Up to present, MPC has become one of the most widely used multivariable control algorithms in various industries including chemical engineering, food processing, automotive, aerospace applications [14], and recently in power systems [13]. This is due to its facility of handling constraints, being able to use simple models, and its closed-loop stability and inherent robustness. To the best knowledge of the authors, the application of MPC in the dynamic dispatch problem has never been studied before.

The main purpose of this paper is to propose an MPC approach to the dynamic dispatch problem. We start with a critical review and comparison of OCDD and DED, and show that they are essentially different, which is a fact overlooked in the literature, and yet both suffer from real practical implementability problems. An extended version of the DED formulation is then given. The MPC approach is introduced through algorithmic procedures. The MPC approach, based upon the OCDD framework, with a moving horizon, is by its nature a closed-loop design. Furthermore, the MPC solutions asymptotically approach the optimal solution of the extended version of the DED problem. The robustness of the MPC algorithm can also be shown. The MPC approach therefore provides a bridge between the OCDD and DED formulations, apart from the additional advantages of reduced dimensionality, easy implementation and the requirement of simple modelling. The MPC approach is then shown through two applications of the dynamic dispatch problem with ten and six generators respectively under load demand balance, ramp rate constraints and generation capacity constraints.

*: Centre of New Energy Systems, Department of Electrical Electronic and Computer Engineering, University of Pretoria, Pretoria, 0002, South Africa, E-mails: xxia@postino.up.ac.za(corresponding author), jfzhang@up.ac.za, a_m_elaiw@yahoo.com

The layout of the paper is as follows: In Section II, we introduce the OCDD and DED formulations. In Section III, we propose the MPC algorithms to the dynamic dispatch problem. Section IV presents the simulation results. The last section is the conclusions.

II. Problem formulation

In this section we introduce the OCDD and DED formulations. We shall consider simple forms of OCDD and DED problems involving three types of constraints, equality, dynamic and inequality constraints. The general form or even more extended forms can be considered in a similar fashion. To this end, some notations are introduced first.

For a sampling period T, the dynamic dispatch problem is considered over time intervals, or dispatch intervals, $[iT,(i+N)T))$ where the optimization is considered, for $i\geq 0$. N is a fixed positive integer, and NT is the dispatch period. For simplicity, we make the convention throughout the paper that $[i,j)$ denotes the time interval $[iT,jT)$. Assume that n is the number of committed units, P_i^k is the generation of unit i during the k-th time interval $[k-1,k)$; $C_i(P_i^k)$ and $R_i(P_i^k)$ are the generation and ramping costs respectively for unit i to produce P_i^k; D^k is the demand at time k (*i.e.*, the k-th time interval); the control variable u_i^k is the ramp rate of the unit i at time k; UR_i and DR_i are the maximum ramp up/down rates for unit i; $P_i^{\min}$ and $P_i^{\max}$ are the minimum and maximum capacity of unit i respectively; the notation $(P_i^k: 1\leq i\leq n, l\leq k\leq l+j)$ denotes the vector $(P_1^l,P_2^l,\cdots, P_n^l, P_1^{l+1}, P_2^{l+1},\cdots,P_n^{l+1},\cdots, P_1^{l+j},P_2^{l+j},\cdots, P_n^{l+j})$, and $C(P_i^k: 1\leq i\leq n, l\leq k\leq l+j)$ denotes the cost (objective) function C with variables $\{P_i^k: 1\leq i\leq n, l\leq k\leq l+j\}$. Define $D=(D^1,D^2,\cdots,D^N)^T$, $P^k=(P_1^k,P_2^k,\cdots,P_n^k)^T$, $u^k=(u_1^k,\cdots,u_n^k)^T$, $k\geq 0$. The function $[C_i(P_i^k)+R_i(P_i^k)]$ is assumed to be a quadratic function $a_i(P_i^k)^2+b_iP_i^k+c_i$ with known positive constants a_i, b_i and c_i.

The demand D^k is assumed to be periodic with period N. This periodic assumption is made to reflect the cyclic consumption behavior and seasonal changes over the dispatch interval. The following convention is also made:

$$\sum_{i=j}^{k} x_i = \begin{cases} 0, & \text{if } j>k;\\ x_j+x_{j+1}+\cdots+x_k, & \text{if } j\leq k.\end{cases}$$

A. OCDD formulation

We consider the dynamics of the power system as a discrete-time control system (see [15] and [16]):

$$P_i^{k+1}=P_i^k+Tu_i^k,\qquad k\geq 0,\quad i=1,2,\cdots,n. \tag{1}$$

The equations in (1) actually define a coordinate transformation between the variables $\{P_i^k: 1\leq i\leq n, 1\leq k\leq N\}$ and the variables $\{u_i^j: 1\leq i\leq n, 0\leq j\leq N-1\}$. It is obvious that the inverse coordinate transformation is given by

$$P_i^k=P_i^0+\sum_{j=0}^{k-1}Tu_i^j, k=1,2,\cdots,N. \tag{2}$$

The OCDD problem is formulated as follows: given a set of generators, load demand D^k, and initial generation P_i^0, find a set of control actions u_i^k to minimize the total generation cost, and to meet the load demand of a power system over the dispatch period:

Problem OCDD$_{P^0}(u,\ [0,N))$ Given n,N,DR_i,UR_i, $P_i^{\min}$, $P_i^{\max}$, $1\leq i\leq n$, P^0, and D, solve the following minimization problem:

$$\begin{aligned}
\min\quad & C(u_i^j: 1\leq i\leq n, 0\leq j\leq N-1)\\
& = \textstyle\sum_{k=1}^N\sum_{i=1}^n[C_i(P_i^0+\sum_{j=0}^{k-1}Tu_i^j)+\\
& \quad R_i(P_i^0+\textstyle\sum_{j=0}^{k-1}Tu_i^j)]\\
\text{subject to}\quad & \textstyle\sum_{i=1}^n(P_i^0+\sum_{j=0}^{k-1}Tu_i^j)=D^k,\\
& -DR_i\leq u_i^j\leq UR_i,\\
& P_i^{\min}\leq P_i^0+\textstyle\sum_{j=0}^{k-1}Tu_i^j\leq P_i^{\max},\\
& (1\leq i\leq n, 0\leq j\leq N-1, 1\leq k\leq N).
\end{aligned} \tag{3}$$

When the OCDD problem is solvable, it gives an open-loop optimal controller denoted by $\bar{u}=(\bar{u}_1^0,\ \bar{u}_2^0,\cdots, \bar{u}_n^0,\bar{u}_1^1,\bar{u}_2^1,\cdots,\bar{u}_n^1,\cdots,\bar{u}_1^{N-1},\bar{u}_2^{N-1},\cdots,\bar{u}_n^{N-1})$ and the corresponding optimal generation is given by $\bar{P}=(\bar{P}_1^1,\bar{P}_2^1,\cdots,\bar{P}_n^1,\bar{P}_1^2,\bar{P}_2^2,\cdots,\bar{P}_n^2,\cdots,\bar{P}_1^N,\bar{P}_2^N,\cdots,\bar{P}_n^N)$, where $\bar{P}_i^k=P_i^0+\sum_{j=0}^{k-1}T\bar{u}_i^j, k=1,2,\cdots,N$.

Note that the assumption of a given initial generation is more of a technical one than a practical one. For a given initial generation P^0, the above model optimizes $\bar{u}^0,\bar{u}^1,\cdots,\bar{u}^{N-1}$ and obtains the optimal generation $\bar{P}^1,\bar{P}^2,\cdots,\bar{P}^N$. The OCDD solution gives the optimal generation dispatch for the dispatch interval $[0,N)$. Due to the periodicity of the demand over the dispatch intervals, it is expected that a cyclicity in the optimal dispatch power will result. Therefore, this optimal solution will be hopefully executed by a simple repetition for other dispatch intervals, say, $[N,2N)$, *i.e.*, $\bar{P}^{N+1}=(\bar{P}_1^{N+1},\bar{P}_2^{N+1},\cdots,\bar{P}_n^{N+1})=(\bar{P}_1^1,\bar{P}_2^1,\cdots,\bar{P}_n^1)=\bar{P}^1$, and so on. However one must be very careful when move from the interval $[0,N)$ to the other interval $[N,2N)$. In fact, due to same physical reasons, the difference between $\bar{P}^{N+1}$ (or $\bar{P}^1$ in case of periodic implementation) and $\bar{P}^N$ must satisfy the ramp rate constraint. Note that there is no ramp rate constraint between $\bar{P}^1$ and $\bar{P}^N$ in the last inequalities of (3) in the OCDD formulation. Therefore when executing $\bar{P}$ over the next dispatch interval $[N,2N)$, $\bar{P}^{N+1}$ cannot be actually implemented if the ramp rate constraint between $\bar{P}^1$ and $\bar{P}^N$ are violated. This issue has been overlooked in the original OCDD formulation. Our MPC approach developed in the next section will include such constraints in the consideration.

B. DED formulation

Normally, the DED problem can be formulated as follows:

Problem DED$(P,\ [0,N))$ Given $n, N, DR_i, UR_i,\ P_i^{\min}$, $P_i^{\max}$, $1 \le i \le n$, and D, solve the following minimization problem:

$$\begin{aligned} \min \quad & C(P_i^k : 1 \le i \le n, 1 \le k \le N) \\ & = \textstyle\sum_{k=1}^N \sum_{i=1}^n [C_i(P_i^k) + R_i(P_i^k)] \\ \text{subject to} \quad & \textstyle\sum_{i=1}^n P_i^k = D^k, \\ & -DR_i \cdot T \le P_i^{j+1} - P_i^j \le UR_i \cdot T, \\ & P_i^{\min} \le P_i^k \le P_i^{\max}, \\ & (1 \le i \le n, 1 \le j \le N-1, 1 \le k \le N) \end{aligned} \tag{4}$$

The notation DED$(P, [0, N))$ means that the variables and the dispatch interval under consideration are $\{P_i^k : 1 \le i \le n, 1 \le k \le N\}$ and $[0, N)$, respectively.

Again, after obtaining the optimal solution of the DED problem, this solution is executed repeatedly for every following dispatch interval. Note that the above optimization problem does not consider the initial generation $P^0 = (P_1^0, \cdots, P_n^0)$, that is, the initial generation before the time interval $[0, 1)$ and up to the time instant $t = 0$. There is no ramp limits on the increase from the initial generation P_i^0 to the generation P_i^1 at time instant $t = 0$ in (4). If the optimal solution, denoted as $(\bar{P}_i^k : 1 \le i \le n, 1 \le k \le N)$, of (4) does not satisfy the ramp limits for the increase from P_i^0 to P_i^1, then such an optimal solution is not able to be practically implemented, or in other words, the solution is not practically feasible. Similar to the increase/decrease from P^0 to P^1, and also similar to the problem encountered in the OCDD model, the ramp rate constraint may be violated when the generators are moved from the time $t = jN$ to $t = jN+1$, $j \ge 1$. This problem will be resolved in our MPC approach. Before that the following Extended DED (EDED) problem is introduced to include this ramp limit between the time $t = N$ and $t = 1$. We will show that our MPC solutions converge to the optimal solution of the EDED problem.

Problem: Extended DED$(P, [0, N))$ Given $n, N,\ DR_i$, UR_i, $P_i^{\min}$, $P_i^{\max}$, $1 \le i \le n$, and D, solve the following minimization problem:

$$\begin{aligned} \min \quad & C(P_i^k : 1 \le i \le n, 1 \le k \le N) \\ & = \textstyle\sum_{k=1}^N \sum_{i=1}^n [C_i(P_i^k) + R_i(P_i^k)] \\ \text{subject to} \quad & (P_i^k : 1 \le i \le n, 1 \le k \le N) \in \Omega_{EDED}, \end{aligned} \tag{5}$$

where the feasible domain Ω_{EDED} is defined to be the set of $(P_i^k : 1 \le i \le n, 1 \le k \le N)$ satisfying

$$\begin{aligned} & \textstyle\sum_{i=1}^n P_i^k = D^k, \\ & -DR_i \cdot T \le P_i^{j+1} - P_i^j \le UR_i \cdot T, \\ & -DR_i \cdot T \le P_i^1 - P_i^N \le UR_i \cdot T, \\ & P_i^{\min} \le P_i^k \le P_i^{\max}, \end{aligned}$$

for all $1 \le k \le N, 1 \le i \le n, 1 \le j \le N-1$.

The only difference between the EDED problem (5) and the classical DED problem (4) is that the constraints

$$-DR_i \cdot T \le P_i^1 - P_i^N \le UR_i \cdot T, (1 \le i \le n) \tag{6}$$

do not appear in the classical problem (4). These constraints will mean that the difference between P^{N+1} and P^N, due to the difference between P^1 and P^N, is bounded by some given constants, therefore the anticipated periodic and repeated implementation is practically feasible. The optimal solution of the EDED problem can be executed on each whole dispatch interval. Note that the EDED problem has the same cost (objective) function with the classical DED problem but more constraints, therefore the cost determined by the optimal solution of the EDED is expectedly greater than or equal to that of the classical DED.

The differences between OCDD and DED are listed below:

1) The OCDD formulation produces an optimal solution for a given initial value P^0, and the optimal solution also depends on P^0; while the DED problem does not consider the initial generation P^0, and is totally independent of P^0.

2) The OCDD formulation has the ramp limit for u^0, that is, the differences between P_i^1 and P_i^0 must satisfy the ramp constraints; however the DED formulation considers the ramp rate constraints only for $P_i^2 - P_i^1$, $P_i^3 - P_i^2, \cdots, P_i^N - P_i^{N-1}$, and has ignored the ramp limit for $P_i^1 - P_i^0$, where $i = 1, \cdots, n$.

III. MPC Approach to DED

This section proposes an MPC approach based on the OCDD framework in order to obtain a closed-loop system. For this purpose, we first do a few steps of mathematical transformations. We also introduce dummy variables to avoid handling more mathematical notations.

By the transformation defined in (1), the EDED problem (5) can be transformed into the following equivalent form.

Problem EDED$(P^1, u, [0, N))$ Given $n, N, DR_i,\ UR_i$, $P_i^{\min}$, $P_i^{\max}$, $1 \le i \le n$, and D, solve the following minimization problem:

$$\begin{aligned} \min \quad & C(P_i^1, u_i^j : 1 \le i \le n, 1 \le j \le N-1) \\ & = \textstyle\sum_{k=1}^N \sum_{i=1}^n [C_i(P_i^1 + \sum_{j=1}^{k-1} T u_i^j) + \\ & R_i(P_i^1 + \textstyle\sum_{j=1}^{k-1} T u_i^j)] \\ \text{subject to} \quad & (P_i^1, u_i^j : 1 \le i \le n, 1 \le j \le N-1) \in \\ & \Omega_D(P^1, u), \end{aligned} \tag{7}$$

where the notation EDED($P^1, u, [0, N)$) denotes the variables and the dispatch interval under consideration are $\{P_i^1, u_i^j : i = 1, \cdots, n, j = 1, \cdots, N-1\}$ and $[0, N)$ respectively, and $\Omega_D(P^1, u)$ is the set of $(P_i^1, u_i^j : i = 1, \cdots, n, j = 1, \cdots, N-1)$ satisfying the following constraints:

$$\sum_{i=1}^{n}(P_i^1 + \sum_{j=1}^{k-1} Tu_i^j) = D^k, (1 \le k \le N),$$

$$-DR_i \le Tu_i^j \le UR_i, (1 \le i \le n, 1 \le j \le N-1),$$

$$-DR_i \cdot T \le -\sum_{j=1}^{N-1} Tu_i^j \le UR_i \cdot T, (1 \le i \le n),$$

$$P_i^{\min} \le P_i^1 + \sum_{j=1}^{k-1} Tu_i^j \le P_i^{\max}, (1 \le i \le n, 1 \le k \le N).$$

Obviously $\Omega_D(P^1, u)$ is equivalent to Ω_{EDED} under the coordinate change defined by (1). The optimal solution of this EDED problem is implemented repeatedly at instants which equal to multiples of N. To introduce the MPC approach, let us consider the EDED problem starting at an arbitrary instant $t = m$ and over a dispatch interval $[m, m+N)$. Then the EDED problem can be rewritten in the following general form.

Problem EDED($P^{m+1}, u, [m, m+N)$) Given n, N, DR_i, UR_i, $P_i^{\min}$, $P_i^{\max}$, $1 \le i \le n$, D, let $P_i^1 := P_i^{m+1}, u_i^j := u_i^{m+j}, D^k := D^{m+k}, 1 \le i \le n, 1 \le j \le N-1$, $1 \le k \le N$, and solve the minimization problem (7).

In an MPC approach, a finite-horizon optimal control problem is repeatedly solved and the input is applied to the system based on the obtained optimal open-loop control. In our case, the horizon is chosen to be N. Instead of solving the EDED($P^{m+1}, u, [m, m+N)$) over the time period $[m, m+N)$, the MPC algorithm solves the following problem:

Problem MPCEDED$_{P^{m+1}}(u, [m, m+N))$ Given n, N, DR_i, UR_i, $P_i^{\min}$, $P_i^{\max}$, $1 \le i \le n$, D, P^{m+1}, let

$$\begin{aligned} & P_i^1 := P_i^{m+1}, u_i^j := u_i^{m+j}, D^k := D^{m+k}, \\ & 1 \le i \le n, 1 \le j \le N-1, 1 \le k \le N, \end{aligned} \tag{8}$$

and solve the following minimization problem

$$\begin{aligned} \min \quad & C(u_i^j : 1 \le i \le n, j = 1, 2, \cdots, N-1) \\ & = \sum_{k=1}^{N}\sum_{i=1}^{n}[C_i(P_i^1 + \sum_{j=1}^{k-1} Tu_i^j) \\ & + R_i(P_i^1 + \sum_{j=1}^{k-1} Tu_i^j)] \\ \text{subject to} \quad & (P_i^1, u_i^j : 1 \le i \le n, j = 1, 2, \cdots, N-1) \in \\ & \Omega_D(P^1, u), \end{aligned} \tag{9}$$

where the notation MPCEDED$_{P^{m+1}}(u, [m, m+N))$ denotes the optimization problem is solved over the interval $[m, m+N)$ with variables u_i^j and for known inputs P_i^{m+1}, $1 \le i \le n, j = m+1, \cdots, m+N-1$.

For a given P^{m+1}, the corresponding $\Omega_D(P^1, u)$, which is obtained after the change of variables in (8), may be an empty set, and the above MPCEDED problem may be unsolvable. This is the feasibility problem considered in [8], [7]. We shall not indulge in the feasibility discussion, and we make the following feasibility hypothesis to exclude the possibility of an empty $\Omega_D(P^1, u)$. The hypothesis is easily fulfilled if the supplier has enough capacity to meet the demand and the demand does not change too much over adjacent sampling periods, and actually most of the practical dispatch problems satisfy this hypothesis.

Feasibility Hypothesis 1 ([8], [7]): After the change of variables in (8) over any dispatch interval $[m, m+N)$ with $m \ge 0$, the set $\Omega_D(P^1, u)$ is not empty.

This hypothesis ensures the solvability of the problem MPCEDED$_{P^{m+1}}(u, [m, m+N))$. Denote the optimal solution of (9) by $(\tilde{\mathbf{u}}_i^j : 1 \le i \le n, 1 \le j \le N-1)$, then we use the inverse of (8) to change the dummy variables back by letting

$$\bar{u}_i^{j+m}|_m := \tilde{\mathbf{u}}_i^j, 1 \le i \le n, 1 \le j \le N-1.$$

Now the optimal solution of MPCEDED$_{\bar{P}^{m+1}}(u, [m, m+N))$ is denoted by

$$\bar{u}|_m = \left(\bar{u}_i^j|_m : i = 1, 2, \cdots, n; j = m+1, \cdots, m+N-1\right),$$

where the notation $\cdot|_m$ denotes the value obtained at the dispatch interval $[m, m+N)$. Denote $\bar{u}^j|_m = (\bar{u}_1^j|_m, \bar{u}_2^j|_m, \cdots, \bar{u}_n^j|_m)$. The optimal solution $\bar{u}|_m$ is applied only in the first sampling period $[m, m+1)$, that is, $\bar{u}_i^{m+1}|_m$ is applied to the state $\bar{P}_i^{m+1}$ to obtain $\bar{P}_i^{m+2} = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m$, and this $\bar{P}_i^{m+2}$ is actually executed over the time period $[m+1, m+2)$, where $i = 1, 2, \cdots, n$. Note that the optimal controller $\bar{u}|_{m+1}$ of the dispatch interval $[m+1, m+N+1)$ is indeed a function of the initial value $\bar{P}^{m+2}$, thus a closed-loop feedback is obtained. The above ideas can be strictly formulated into the following MPC algorithm.

MPC Algorithm 1 Initialization: Input the initial status $\bar{P}^1 \triangleq P^1 = (P_1^1, P_2^1, \cdots, P_n^1)$ and let $m = 0$.

(1) Compute the open-loop optimal solution $\bar{u}_i^j|_m$ to the problem MPCEDED$_{\bar{P}^{m+1}}(u, [m, m+N))$, where $i = 1, \cdots, n$, $j = m+1, \cdots, m+N-1$.

(2) The (closed-loop) *MPC controller* $\bar{u}_i^{m+1}|_m$ is applied to the plant in the sampling interval $[m, m+1)$ (the remaining $\left\{\bar{u}_i^j|_m : i = 1, 2, \cdots, n; j = m+2, m+3, \cdots, m+N-1\right\}$ are discarded) to obtain the closed-loop *MPC solution*

$$\bar{P}_i^{m+2} = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m \tag{10}$$

over the period $[m+1, m+2)$.

(3) Let $m := m+1$ and go to step (1).

Figure 1 shows how the MPC controller is updated at each step for the case of $n = 1, N = 5$. The first dotted column in each (a), (b) and (c) of Fig. 1 denotes the initial generation $\bar{P}^1 = P^1$, and the other columns denote the controllers. At the first iteration, the MPC algorithm does the optimization over four time periods (from 1 to 5), and obtains a 4-dimensional optimal controller, however only the first (shaded) component of this solution will be actually executed at time instant $t = 1$, and this shaded controller is the MPC controller $\bar{u}^1|_0$. The MPC controller $\bar{u}_0^1$ is added to $\bar{P}^1$ to obtain the MPC solution

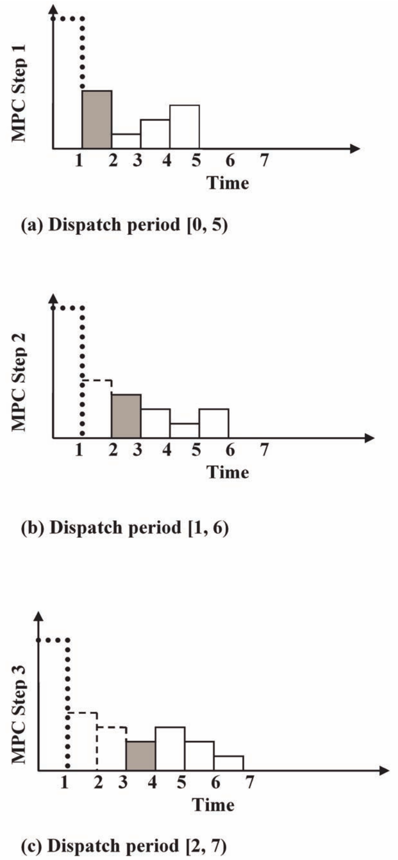

Fig. 1. Implementation of the MPC algorithm

$\bar{P}^2$ at $t=1$. After this, the optimization horizon moves one step forward, and optimization is then done on the next four time intervals (from 2 to 6) with the initial value $\bar{P}^2$. Now a new 4-dimensional optimal controller is obtained, and only the first (shaded) component, which is the MPC controller $\bar{u}^2|_1$, will be implemented at time instant $t=2$ to obtain the MPC solution at this time instant (see In Fig. 1 (b)). The dashed column in Fig.1(b) denotes the controller $\bar{u}^1|_0$, and the two dashed columns in Fig. 1 (c) denote the MPC controller $\bar{u}^1|_0$ and $\bar{u}^2|_1$ respectively. As the optimization horizon recedes, a sequence of MPC solutions (the dashed columns) will be designed up to the current implementation time instant.

Generally, the above MPC algorithm never stops, and it updates the controller at each time interval $[m, m+1)$ to include feedback information. This algorithm actually computes the optimal solution $(\bar{u}_1^1|_0, \cdots, \bar{u}_n^1|_0, \cdots, \bar{u}_1^N|_0, \cdots, \bar{u}_n^N|_0)$ for the EDED problem over the period $[0, N)$, however only $\{\bar{u}_i^1|_0 : i = 1, \cdots, n\}$, the MPC controller, is executed to obtain the MPC solution $\{\bar{P}_i^2 : i = 1, \cdots, n\}$. Now the algorithm continues to compute over the period $[1, N+1)$ to find the optimal solution $(\bar{u}_1^2|_1, \cdots, \bar{u}_n^2|_1, \cdots, \bar{u}_1^{N+1}|_1, \cdots, \bar{u}_n^{N+1}|_1)$ with measured generation powers $\bar{P}^2$ as the initialization, and only the first component, the MPC controller $\{\bar{u}_i^2|_1 : i = 1, \cdots, n\}$ is executed to obtain the MPC solution $\{\bar{P}_i^3 : i = 1, \cdots, n\}$. This process is repeated, and each time the control horizon moves only one step from $[m, m+N)$ to $[m+1, m+N+1)$. Let $u_{MPC}^* = (u_{MPC}^1, u_{MPC}^2, u_{MPC}^3, \cdots,)$ be the optimal MPC controller that MPC actually executes. Then $u_{MPC}^1 = \bar{u}^1|_0$, $u_{MPC}^2 = \bar{u}^2|_1$, $\cdots$, $u_{MPC}^{m+1} = \bar{u}^{m+1}|_m$, $\cdots$.

It is noticed that the MPC approach has the following advantages.

(1) Reduced Dimension:
The classical OCDD or DED problem must solve an optimization problem with nN number of variables for the dispatch power over the dispatch period, however there are only $n(N-1)$ number of variables in each iteration step of the MPC Algorithm.

(2) Convergence and robustness:
Theorem 1 and 2 show that one can start the MPC Algorithm with any P^1 satisfying $\sum_{i=1}^n P_i^1 = D^1$ and the optimal solution at each step will converge to the optimal solution of the EDED problem. Furthermore, if there is any disturbance in the initial value of any iteration, the MPC algorithm will detect the disturbed value and the obtained optimal MPC controller will make the compensation and correction automatically. When there is disturbance in the demand, it can be transformed into the case studied in Theorem 2 and thus the MPC algorithm is still robust ([19]).

(3) Easy Implementation:
The optimal solution of the MPC algorithm can be executed at any time $m \geq 0$, while other open-loop algorithms can only be executed from the time instant iN, $i \geq 0$.

Thus the MPC algorithm is more favorable for practical applications than other open-loop algorithms.

The proofs of the following theorems are omitted here due to the page limit, and they can be found in [19].

Theorem 1: Suppose Feasibility Hypothesis 1 holds, P^* is the globally optimal solution of the EDED problem (5), $\tilde{P}^*$ is the globally optimal solution of the DED problem (4), then

(i) MPC Algorithm 1 converges to P^* if, furthermore, the initial power output P^1 at time $t=1$ satisfies $\sum_{i=1}^n P_i^1 = D^1$;

(ii) the value of the objective function $C(P_i^k : 1 \leq i \leq n, 1 \leq k \leq N) = \sum_{k=1}^N \sum_{i=1}^n [C(P_i^k) + R(P_i^k)]$ at the point P^* is greater than or equal to that at the point $\tilde{P}^*$.

Now consider the robustness of the MPC algorithms. For simplicity, suppose that disturbance happens only in the execution of the controller. That is, the disturbance happens only in Step (2) of MPC Algorithm 1 so that when the control $\bar{u}_i^{m+1}|_m$ is applied to the plant in the sampling interval $[m, m+1)$, the system actually execute $\bar{P}_i^{m+2} = F(\bar{P}_i^{m+1}, \bar{u}_i^{m+1}|_m, w_i^{m+1})$ over the period $[m+1, m+2)$, where F is a function, w_i^{m+1} is a disturbance vector satisfying $||w_i^{m+1}|| < e$ and e is a positive constant. Although F is written in a general form to include general disturbances in nonlinear MPC ([4]), it is often written in the addition form ([6]) as $F(\bar{P}_i^{m+1}, \bar{u}_i^{m+1}|_m, w_i^{m+1}) = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m + Tw_i^{m+1}$, therefore

$$\bar{P}_i^{m+2} = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m + Tw_i^{m+1} \tag{11}$$

When we talk about the robust version of MPC Algorithm 1, it always means that (10) is replaced by (11) during its execution.

Theorem 2: Suppose Feasibility Hypothesis 1 holds, P^* is the globally optimal solution of the EDED problem (5), $\tilde{P}^*$ is the globally optimal solution of the DED problem (4), Ω_{EDED} is the feasible domain of problem (5), the norm of the gradient of the cost function of problem (5) has the upper bound L on Ω_{EDED}, ϵ is a small enough positive constant, c is a positive constant which is less than ϵ, (11) is executed in Step (2) of MPC Algorithm 1 in stead of (10), the constant disturbance w_i^k satisfies $||w_i^k|| < e$, and e is small enough so that $e < \min\{c/L, (\epsilon - c)/L\}$, then there exists an integer N_0 such that for any $k > N_0$, the optimal MPC solution $\bar{P}^{k+1}$ of the k-th loop in MPC Algorithm 1 belongs to the domain $\overline{\Omega} := \{P : \ ||P - P^*|| < c\}$.

IV. Simulation results

In this section we present two examples. The first example shows the difference between the OCDD and DED formulations. The second example shows the convergence and robustness properties of the proposed MPC algorithm. All computations are carried out by MATLAB program.

Example 1 Consider the ten-unit power system in [1]. The dispatch period is chosen as one day with one hour sampling period. The sum of the generation and ramping costs are given by a quadratic function $C_i(P_i) + R_i(P_i) = a_i + b_i P_i + c_i P_i^2$. The initial P_i^0 is chosen such that $\sum_{i=1}^{10} P_i^0 = D^0$. Here, P_i^0 and D^0 are the initial generation and load demand, respectively, during the interval $[-1, 0)$. Figure 2 shows the optimal outputs of units 1 and 5 of the ten-unit system for both OCDD and DED problems. It can be observed that the solutions from the OCDD problem for the given initial P_i^0 and that of the DED problem are different in the beginning of the dispatch period, but they coincide from the 7-th time instant. Now we show the technical deficiencies that DED and OCDD suffer from. Figure 3 shows the optimal solution of unit 2 for the DED problem. The results show that, the difference between $\bar{P}_2^1$ and P_2^0 does not satisfy the ramp rate constraint, since $\bar{P}_2^1 - P_2^0 = -90 < -DR_2$. Therefore, this optimal solution cannot be implemented at time instant $t = 1$ for the second generation unit. One may start at an initial condition P_i^0 that the ramp rate limits between $\bar{P}_i^1$ and P_i^0 can be satisfied. However, the optimal solution can be only implemented over the interval $[0, 24]$, it cannot be implemented for the following 24 hours by a simple repeating, since $\bar{P}_2^{24j+1} - \bar{P}_2^{24j} = -85 < -DR_2, j \geq 1$.

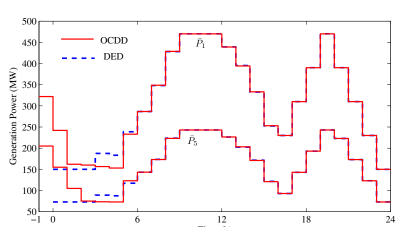

Fig. 2. Optimal trajectories of units 1 and 5 for OCDD and DED

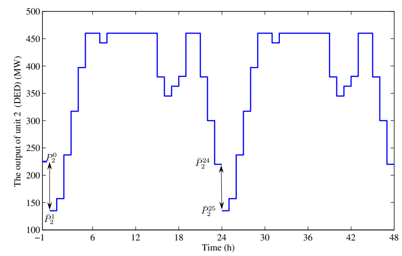

Fig. 3. Optimal trajectory of unit-2 for DED

Example 2 Consider the system of six units in [5]. The load demand is assumed to be periodic over a dispatch period of one day and the sampling period is chosen to be one hour. Figure 4 shows that the MPC closed-loop solutions asymptotically approach the optimal solutions of the EDED problem.

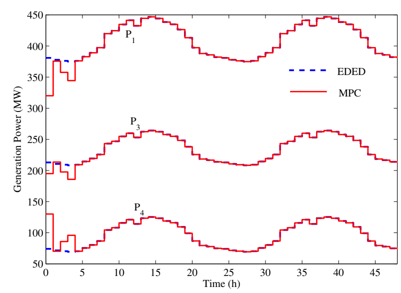

Fig. 4. Convergence of the closed-loop MPC solutions to those of EDED

To show the robustness of the MPC algorithm, (11) is executed, and the disturbance w_i^m is generated by $w_i^m = -\varepsilon_i + 2\varepsilon_i r(m)$, where the parameters $r(m)$'s are uniformly distributed random numbers on $[0, 1]$ and ε_i's are chosen as $\varepsilon_1 = 5$, $\varepsilon_2 = 5$, $\varepsilon_3 = 4$, $\varepsilon_4 = 4$, $\varepsilon_5 = 5$ and $\varepsilon_6 = 3$. To show the effectiveness of the MPC, the following cases are shown in Figure 5:

(i) the optimal solutions of the EDED problem ($\bar{P}_i^k : 1 \leq i \leq n, 1 \leq k \leq 72$),

(ii) the (closed-loop) MPC solutions with disturbances,

(iii) the solutions of the disturbed system $P_i^{k+1} = P_i^k + T\bar{u}_i^k + Tw_i^k$ with the open-loop controller ($\bar{u}_i^k : 1 \leq i \leq n, 1 \leq k \leq 72$) obtained by OCDD problem. Note that this open-loop controller is implemented by a 24-hour repetition procedure.

In cases (ii) and (iii) the initial P_i^1 is chosen as the optimal solution of the EDED problem at $t = 1$, *i.e.*, $P_i^1 = \bar{P}_i^1$. From Figure 5 we can see that, the MPC can keep the disturbed system around the optimal solution of EDED while the open-loop optimal controller ($\bar{u}_i^k : 1 \leq i \leq n, 1 \leq k \leq 72$) can lead to undesired results (the generation capacity constraints of unit 1 are violated) when applied to the system in the presence of disturbances.

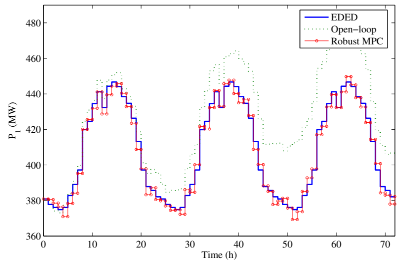

Fig. 5. Generation output of unit-1 under EDED, open-loop controller and MPC

V. CONCLUSIONS

The main purpose of this paper is to propose an MPC approach to the dynamic dispatch problem. The differences between the OCDD and DED approaches are discussed and also illustrated on a ten-unit system. The convergence and robustness of the MPC algorithms are demonstrated through the application of MPC to the dynamic dispatch problem with six units. The results show promising applications of MPC method in the dynamic dispatch problem.

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