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A Model Predictive Control Strategy for Load Shifting in a Water Pumping Scheme with Maximum Demand Charges

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Abstract—**This paper defines and evaluates a model predictive control strategy with binary integer programming optimization for load shifting in a water pumping scheme. Both time-of-use and maximum demand charges are considered in the control model. The control model yields near optimal switching times which reduce time-of-use and maximum demand based electricity costs. Both the time-of-use and maximum demand charges have a significant effect on the optimal switching times.**

Index Terms—**Demand side management, Load shifting, Maximum demand, Model predictive control, Optimal control, Time-of-use, Water pumping.**

I. Introduction

LOAD shifting is an aspect of demand side management (DSM) where electricity demand is shifted out of peak demand periods to off-peak demand periods. To encourage load shifting utilities have structured electricity tariffs with time-of-use (TOU) and/or maximum demand (MD) charges [1][2]. MD charges are also used by utilities to represent infrastructure costs due to high peak demands.

TOU charges are based on higher kWh rates during high demand periods, whilst MD charges are based on fixed fees per maximum kVA or kW for a month in high demand periods [2]. MD is measured as the highest averaged demand in kVA or kW during any integrating period –the integrating period is generally 30 minutes, and it coincides with the TOU periods [2].

Various techniques have been used to solve load shifting problems in different applications. For example, fuzzy logic is used in [3] for load shifting of a domestic hot water cylinder, A neural network is used in [4] for load shifting in a petrochemical plant, and in [5]-[14] load shifting problems are modelled as optimal control problems.

The focus of this paper is on the optimal control techniques [5]-[14]. In [5]-[7] load shifting problems are modelled as open-loop optimal control problems and then formulated into discrete time linear or discrete time non-linear programming problems [15]-[17]. The same technique was used in [8] for load shifting in a colliery, but feedback was included in the model by re-optimizing on the arrival of a coal train. In [9] the open-loop control model for a domestic hot water cylinder is formulated as a continuous time switched optimal control model –based on a technique in [18].

TOU and MD charges were used in the objective functions of the open-loop control models in [10]-[14], and the MD charges had a significant affect on the optimal switching times.

However, due to the added complexity to optimize the combined TOU and MD charges, more advanced optimization algorithms were considered in [10]-[14], as oppose to binary integer programming (BIP) or linear programming (LP) optimization. For example, mixed integer LP in [10], repeated LP optimization in [11], a Markov model in [12], dynamic programming in [13], and a generalized reduced gradient algorithm in [14].

The open-loop control models are valuable starting points to quantify the potential for load shifting. However, an open-loop model cannot compensate for changes from disturbances or model deficiencies, because no feedback and subsequent re-optimization is implemented. Therefore, a closed-loop model is required if active control is considered.

Closed-loop models are used in [8] and [10]. The model in [8] is re-optimized on the arrival of a new train, and the model in [10] is re-optimized daily. Ideally the models should be optimized more frequently to handle disturbances properly.

Therefore, little evidence could be found that proves the applicability of closed-loop optimal control for load shifting in different applications where MD charges are included in the objective functions of the control models.

The aim of this paper is to define and simulate a closed-loop optimal control model for a specific plant that is charged on both MD and TOU. Promising preliminary results are reported that might be updated in a follow-up publication.

A model predictive control (MPC) strategy with BIP optimization has been selected to model and simulate the closed-loop model [19][20]. An MPC strategy has been selected, because it is an optimal control strategy, and the periodic re-optimization characteristic of an MPC model provides stability during external disturbances and it compensates for inaccurate or simplified system models.

This paper is organized as follows. In section II a general control model is defined. In section III the general control model is applied to the selected case study, the applied model is simulated, and the results are discussed. Concluding remarks are covered in section IV.

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II. Control Model

The control model will first be formulated as a linear discrete time open-loop optimal control model, and thereafter it will be converted to a closed-loop (MPC) model.

A. Objective function

The aim is to optimize the switching of pumps to reduce the cost of both TOU and MD charges over a control horizon (*H*), for example 24 hours. The objective to minimize is defined as

$$J = \sum_{n=1}^{N} \left(\sum_{t=1}^{T} u_{tn} \cdot p_n \cdot c_t + \sum_{s=1}^{S} p_n \cdot z_{sn} \cdot C \right), \tag{1}$$

where:

N	Total number of pumps.
n	The n-th pump, where $n = 1,\ldots,N$.
S	The number of switching intervals in an MD integrating period.
s	The s-th switching interval in any MD integrating period.
T	Total number of discrete switching intervals in the control horizon i.e. $T = 2 \cdot H \cdot S$, assuming the MD periods are 30 minutes each and H is measured in hours.
t	The t-th discrete switching interval in the control horizon, where $t = 1,\ldots,T$.
u_{tn}	The binary switching status of the n-th pump at the t-th switching interval, where $u_{tn} = 0$ when the pump is off and $u_{tn} = 1$ when the pump is on.
p_n	Power consumption of the n-th motor.
c_t	The TOU energy cost in the t-th switching interval.
z_{sn}	The s-th discrete MD variable for the n-th pump in an MD integrating period. Discrete variables are used to support BIP.
C	The MD charge.

The variables u_{tn} and z_{sn} needs to be solved by the optimization algorithm over the control horizon (*H).*

The technique to represent the MD costs with a variable in the objective function is based on [10], where a mixed integer variable is used, and the problem is solved with mixed integer LP. However, discrete z_{sn} variables are used in this paper so that the problem can be solved with the standard BIP function in Matlab [21] –Matlab does not have a standard mixed integer LP function. Given the motor power grades, the resulting MD value from mixed integer LP will be realized by further scheduling the motors' operation within the MD integrating interval. An additional advantage of a binary LP is that the resulting MD optimization avoids this later step.

B. Constraints

The constraints are modeled as linear inequality constraints denoted as,

$$A \cdot x \le b \,, \tag{2}$$

where A is a matrix with the coefficients of the linear inequality constraints, b is a vector, and x represents the variables u_{tn} and z_{sn} that need to be solved.

One of the constraints is an MD constraint that is required to force z_{sn} to represent the highest demand across all MD integrating periods, formulated as,

$$\sum_{n=1}^{N} \left(\sum_{t=1+k\cdot S}^{k\cdot S+S} u_{tn} \cdot p_n - \sum_{s=1}^{S} p_n \cdot z_{sn} \right) \le 0$$

$$\text{for } k = 0,\ldots,\left(\frac{T}{S} - 1\right). \tag{3}$$

where k represents the number of MD periods in the control horizon. The purpose of (3) is to constrain each individual MD period to the maximum value of the MD that is represented by z_{sn} in the objective function in (1).

C. Closed-loop (MPC) model

In an MPC model, the closed-loop control is obtained by solving an open-loop optimal control problem over a finite control horizon at each sampling interval, while only the open-loop optimal control in the first sampling interval is executed at the next control step [19]. The model uses the current state of the plant as the initial state for the next optimization step. In this paper the MPC sampling intervals are chosen to coincide with the switching intervals of the pumps, and the choice of the control horizon is covered in section III.

III. Case Study

A. Plant overview

A South African water purification plant has been selected for the case study. The plant can be divided into the purification plant itself and the pumping scheme of the purified water (see Fig. 1).

This section focuses on the water pumping scheme, because it consumes the bulk of the electricity and has potential for load shifting.

Water flows from the dam through the purification plant into a reservoir (R1) at 40 mega litre (ML)/day. R1 is also supplied with water from a fountain at 5 ML/day. R1 has a capacity of 1.4 ML, which is 70% of the total capacity of the reservoir. R1 cannot operate at a level higher than 70%, because the inflow pipe from the purification plant is at the 70% level.

The water from R1 is pumped to two reservoirs: R2 and R3, with a capacity of 120 ML and 60 ML respectively.

The water to R2 is pumped by motors K1, K2 and K3; each rated at 300 kW with the ability to pump 22 ML/day per motor. The water to R3 is pumped by motors G1, G2 and G3; each rated at 275 kW with the ability to pump 10 ML/day per motor. R3 is also supplied by boreholes at a rate of 10 ML/day.

The remainder of this section focuses on the water pumping scheme at the purification plant, which includes reservoirs R1 and motor K1, K2, K3, G1, G2 and G3.

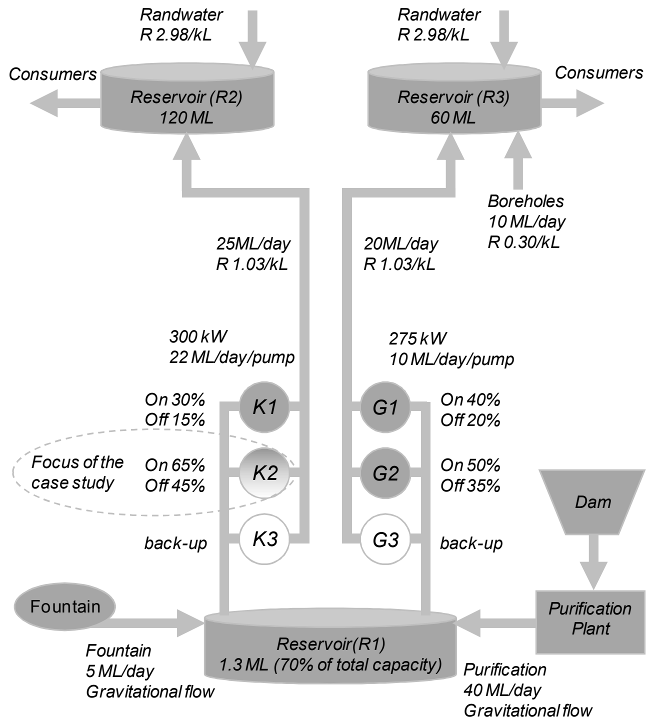

Fig. 1: Pumping scheme of the water purification plant.

The constraints of R1 and the relevant pumps are listed below:

1) At least one of the pumps to both R2 and R3 must always run, otherwise the water in the pipes flows back into R1.
2) As much water as possible must be pumped to R3, because the reservoir is small and therefore, the risk of running out of water is high.
3) Running three pumps to either R2 or R3 is not effective, because the mechanical losses are too high.
4) A motor should not be started more than three times per hour, although the motors have soft starters installed.
5) The level of R1 must always be below 70% of the total capacity.
6) There is no limit on the amount of water that can be pumped to R2 and R3.

In the current configuration, G1, G2 and K1 are set to always run, whilst K2 is used to control the level. This has been implemented with different switching levels for the relevant pumps (see Fig. 1). For example, K2 switches on and off in a narrow band at 65% and 45% respectively.

Note that Randwater supplies most of the water to R2 and R3 at a cost of R 2.98/M^3. The boreholes and the purification plant are used as alternative supplies with significant lower costs i.e. R 0.30/M^3 and R 1.03/M^3 respectively. This means that the maximum amount of water from R1 is pumped to R2 and R3, irrespective of the electricity costs –including peak electricity periods.

Therefore, the most viable load shifting option for the pumping scheme at the purification plant, within the listed constrains, and without large infrastructure expenditure, are the switching times of K2. The problem with the current switching times of K2 is that it operates approximately 6 times per day, for approximately 30 minutes at a time. This means that the maximum demand per month equals the maximum capacity of the motor. With the current control model K2 could also operate in peak demand periods, because the motor simply starts when the reservoir level is too high.

Therefore, it would be more desirable if K2 can run more frequently, for shorter periods, and preferable not in peak times.

The purification plant is supplied with electricity from the Tshwane municipality on the standard 11 kV tariff. This tariff includes a flat energy charge and an MD charge that is applicable at all times [2]. However, for this paper the 11 kV TOU tariff from the Tshwane municipality is considered. The 11 kV TOU tariff includes a TOU and an MD charge [2]. The variable fees of this tariff are summarized in Table I. Note that the 11 kV standard and TOU tariffs results in approximately the same annual costs if no load shifting is done.

TABLE I
SUMMARY OF THE TSHWANE 11 kV TOU TARIFF

Off-peak (0h00 to 6h00 and 22h00 to 24h00)	
High demand (winter)	11.87 cents
Low demand (summer)	10.49 cents
Standard (6h00 to 7h00 and 10h00 to 18h00)	
High demand (winter)	14.11 cents
Low demand (summer)	13.83 cents
Peak (7h00 to 10h00 and 18h00 to 22h00)	
High demand (winter)	82.05 cents
Low demand (summer)	26.28 cents
Maximum demand charge (applicable in peak and standard times)	R 66.50

B. *Assumptions for the control model*

1) Only K2 is considered in the optimal control model. Pumps K1, K3, G1, G2 and G3 is assumed to be controlled with the existing level based control model.
2) The switching levels for all the motors are modified to provide K2 with a wider operating band to support load shifting i.e. the upper limit for K2 is kept at 65% (1.3 ML) and the lower limit for K2 is reduced from 45% to 10% (0.2 ML).
3) The high demand season (winter) tariffs are used for the simulations.
4) The off-peak, standard and peak times for all days is considered the same as a week day. This is to simplify the objective function over a 30 day period. Although this is not a true reflection of the reality, the relative cost savings is similar.

C. *Objective function*

Since there is only one pump (K2) to consider, the objective function is simplified as

$$J = \sum_{t=1}^{T} u_t \cdot p \cdot c_t + \sum_{s=1}^{S} p \cdot z_s \cdot C , \tag{4}$$

where C = R 66.5 for the maximum kW over any 30 minute integrating period, p = 300 kW, S = 2, and $T = 2 \cdot H \cdot S$.

The cost of energy in Rands for the high demand season is

$$c_t = \begin{cases} \frac{0.1187}{2S}, t \in 2S \cdot ([0,6] \cup [22,24]) \\ \frac{0.1411}{2S} t \in 2S \cdot ([6,7] \cup [10,18]) \\ \frac{0.8205}{2S} t \in 2S \cdot ([7,10] \cup [18,22]) \end{cases}. \quad (5)$$

D. Constraints

The level of reservoir R1 at the *t*-th switching interval is defined as

$$L_t = L_0 + \sum_{t=1}^{t} FLOWIN_t - FLOWOUT_t \cdot u_t \ , \quad (6)$$

where:

L_0 — The initial level of reservoir R1. The upper level limit is used (i.e. L_0=1.3 ML).

$FLOWIN_t$ — The relative inflow to R1 over the *i*-th switching interval. This is a constant value from the fountain and the purification plant, minus the outflows from the level controlled motors i.e. FLOWIN = purification plant + fountain - G1 - G2 - K1 = 3 ML/day.

$FLOWOUT_t$ — The outflow of K2 for the *i*-th switching interval. This is a constant value of 22 ML/day.

The upper level constraint is defined as

$$L_t \le 1.3 \text{ ML for } t = 1,...,T \ . \quad (7)$$

The lower level constraint is defined as

$$L_t \ge 0.2 \text{ ML for } t = 1,...,T \ . \quad (8)$$

The MD constraint (applicable in peak and standard times) is defined as

$$\sum_{t=1+k\cdot S}^{k\cdot S+S} u_t \cdot p - \sum_{s=1}^{S} p \cdot z_s \le 0$$
$$\text{for } k = 0,...,\left(\frac{T}{S} - 1\right). \quad (9)$$

E. The closed-loop (MPC) model

The open-loop model is converted to a closed-loop model by solving repeatedly at each switching interval the open-loop model. Only the first control step u_1 is implemented after each iteration. At the next switching interval the state of the plant is re-sampled and the calculated result for u_t from the previous step is used as a suggestion to the optimization algorithm.

F. Choice of the switching intervals

The switching intervals is chosen to coincide with the TOU and MD periods i.e. at least one switching interval per 30 minute integrating period (*S*=1). To reduce the MD charge *S* needs to be bigger than 1 to divide the MD integrating period into smaller intervals. However, the size of *S* is a trade-off between computational time and cost saving.

For example, with S=6 the switching intervals are only 5 minutes long. This enables fine optimization, especially for the MD charges, because a motor can be started every 30 minutes for only 5 minutes. However, with S=6 the number of variables over a 24 hour period is very high, which has a significantly affect on computation time and it could make the solution impractical.

For S=2 the switching intervals are 15 minutes long, which results in higher MD charges compared to a larger S, but the total number of variables are not too many over a 24 hour period, which makes the computation time much faster.

For the simulation in this paper a value of S=2 has been selected.

G. Choice of the control horizon

Like the switching interval, the control horizon (*H)* is a trade-off between computational time and cost saving. For the open-loop simulation a control horizon of 24 has been selected, and for the closed-loop simulation a control horizon of 4 hours has been selected. The closed-loop model is simulated over a 30 day period though. A shorter prediction horizon was selected for the closed-loop model to limit the computation to 1 minute for online optimization. The results in section I show that this is a reasonable compromise.

H. Solving the optimizing problem with Matlab

The optimization problem has been solved with the Matlab LP and BIP functions, called *bintprog* and *linprog* respectively [21]. Although, the required solution is a binary switching sequence, which is solved with *bintprog*, *linprog* has been included to calculate a benchmark for comparison to evaluate the effectiveness of the binary solution. The LP model is not considered as a control solution for the specific case study, because the pumps are binary controlled. The LP model solves very quickly and gives optimal efficiency, because the variables are not constrained to binary values.

The *bintprog* and *linprog* functions are defined as

$$\frac{\min}{x} f^T \cdot x \ \text{ such that } \ A \cdot x \le b \text{ and } \ Aeq = beq \ , \quad (10)$$

where *f*, *b*, and *beq* are vectors; *A* and *Aeq* are matrices; and the solution *x* is a binary integer vector for *bintprog* and a decimal value for *linprog*.

I. Results

Table II compares the results of the current control model with three alternative optimal control models: an open-loop model with LP optimization, an open-loop model with BIP optimization and a closed-loop (MPC) model with BIP optimization. Fig. 2 shows a graphical representation of the results in Table II.

As mentioned, the open-loop models, with LP and BIP optimization is simply used as benchmarks to determine the efficiency of the closed-loop BIP model.

The open-loop model with BIP optimization was simulated for 24 hours only, because the number of variables over 30 days is too many, but the results for the energy charges was scaled to 30 days for comparison.

Table II shows that all three optimal control models save approximately 55% on energy costs for K2. This proves that the closed-loop model is very efficient, and the shorter prediction horizon (*H*=4) is an acceptable compromise.

The open-loop model with LP optimization saves 91% on MD costs for K2 whilst the open- and closed-loop models with BIP optimization saves 50% on MD costs for K2. The difference is related to the resolution of the switching intervals *S* i.e. more shorter intervals will result in lower MD costs. In this case it is half of the maximum (S=2). For example, with S=4 the saving can be 75%.

Table II also shows that the MD saving is higher than the energy saving i.e. 80% on the open-loop LP model, 68% on the open-loop BIP model and 69% on the closed-loop BIP model. This confirms that MD charges needs to be included in the objective function of the optimal control model where applicable.

TABLE II
MONTLY COST COMPARISON FOR K2 OVER 30 DAYS (S = 2)

	Energy costs	MD costs	Total costs	MD save portion
Current model	R 8,351	R19,950	R 28,301	
Open-loop with LP	R 3,694 -56%	R 1,781 -91%	R 5,475 -81%	80%
Open-loop with BIP	R 3,724 -55%	R 9,975 -50%	R 13,699 -52%	68%
Closed-loop with BIP	R 3,852 -54%	R 9,975 -50%	R 13,872 -51%	69%

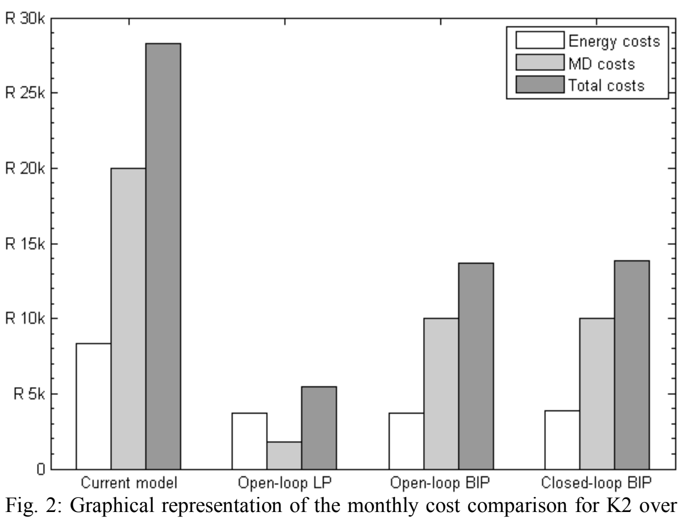

Fig. 2: Graphical representation of the monthly cost comparison for K2 over 30 days (*S*=2).

Fig. 3 and Fig. 4 show a snapshot of the simulated result for the current model on the 1st and 30th day. From the figures it is clear that the current model results in an undesirable high MD, which also falls within peak TOU periods. The resulting MD is registered in the applicable 30 minute MD integrating period with the highest average usage. As mentioned, MD charges are applicable in standard and peak TOU times i.e. between 6h00 and 22h00. Note that the first MD period represents the maximum if more than one period results in the same MD. In Fig. 3 K2 runs for the full 30 minutes in an applicable MD period, which result in the highest possible MD for K2. Note that the current model has been simulated with the increased switching band for K2 –the same increased band is used on the optimal control models.

Fig. 5 shows the simulated result of the open-loop model with LP optimization. The load is moved out of the peak periods, and the load in the standard periods is reduced. The MD is also reduced by spreading the load evenly over the applicable 30 minute MD periods.

Fig. 6 shows the open-loop model with BIP optimization. The load is moved out of the peak periods, and the load in the standard periods is also reduced. The MD is reduced by 50%, because K2 runs more frequently, but for shorter times i.e. only 15 minutes per 30 minute MD period.

Fig. 7 shows the closed-loop model with BIP optimization for the 1st day. The load is moved out of the peak periods, and the load in the standard periods is also reduced. The MD is reduced by 50%, because K2 runs more frequently, but for shorter times i.e. only 15 minutes per 30 minute MD period.

Although the final energy saving over 30 days between the open and closed-loop models with BIP optimization is similar, the switching sequences are different. This is firstly caused by the shorter prediction horizon (*H)* for the closed-loop model, and secondly by the fact that the closed-loop model looks beyond the last interval as it approaches the end, therefore the final reservoir limits are different on a one day snapshot.

The closed-loop model converges to a value slightly higher than the open-loop model result. However, it is expected that the closed-loop model will converge to the same value as the open-loop model if the control horizon (*H*) for both models is 24 hours.

Fig. 8 and Fig. 9 show the open- and closed-loop control models (BIP optimization) with a positive random inflow disturbance i.e.

$$FLOWIN_t = FLOWIN_t + 0.2 \cdot FLOWIN_t \cdot r(m)\,, \tag{11}$$

where *r(m)* is a random number between 0 and 1. Fig 8 shows that the level of R1 exceeds the maximum constraint in the open-loop model, whilst Fig. 9 shows that the closed-loop control model compensates for the disturbances and keeps the level of R1 close to the maximum level constraint.

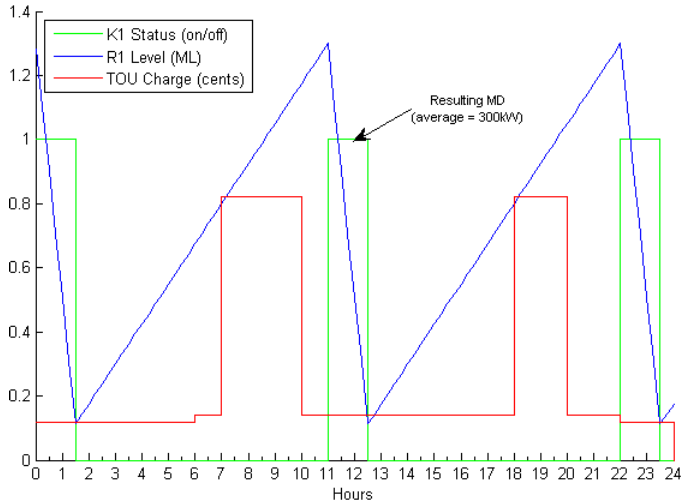

Fig. 3: Current control model with no optimization for K2 on the 1st day.

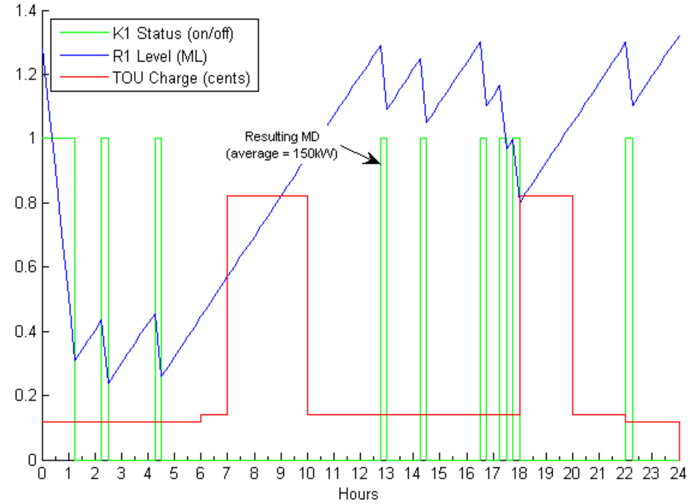

Fig. 6: Open-loop control model with BIP optimization for K2.

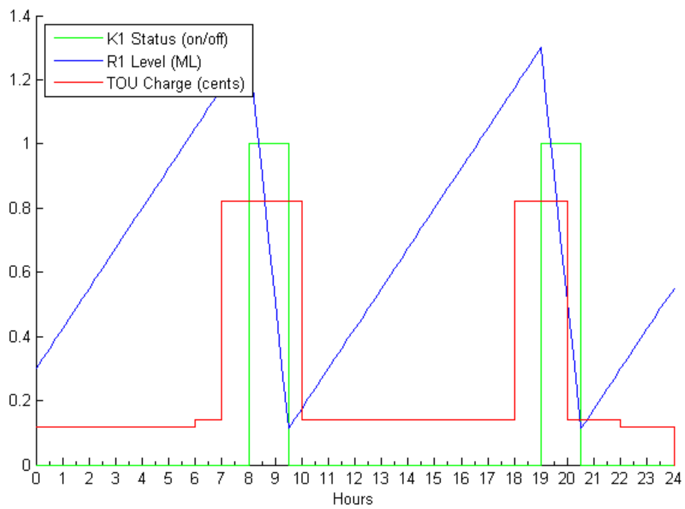

Fig. 4: Current control model with no optimization for K2 on the 30th day.

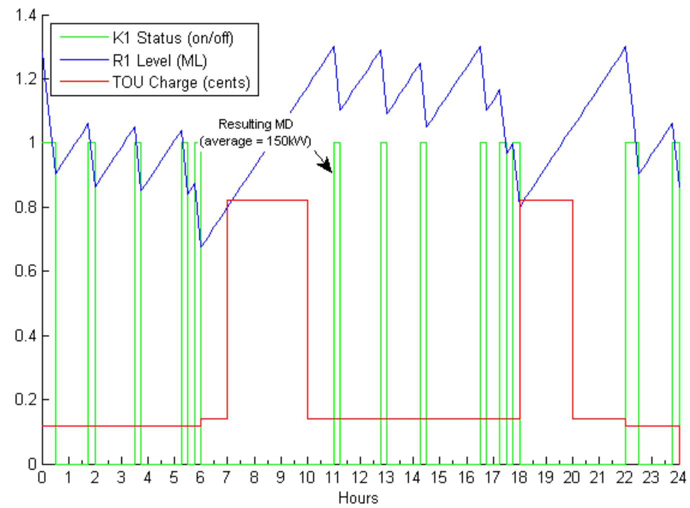

Fig. 7: Closed-loop (MPC) control model with BIP optimization for K2.

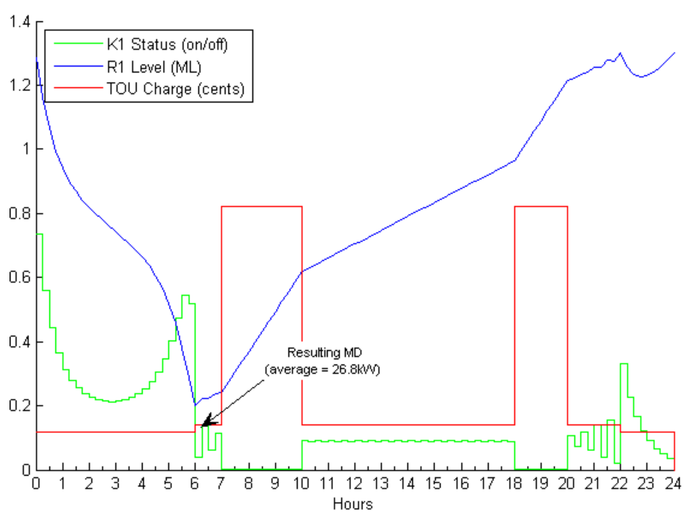

Fig. 5: Open-loop control model with LP optimization for K2.

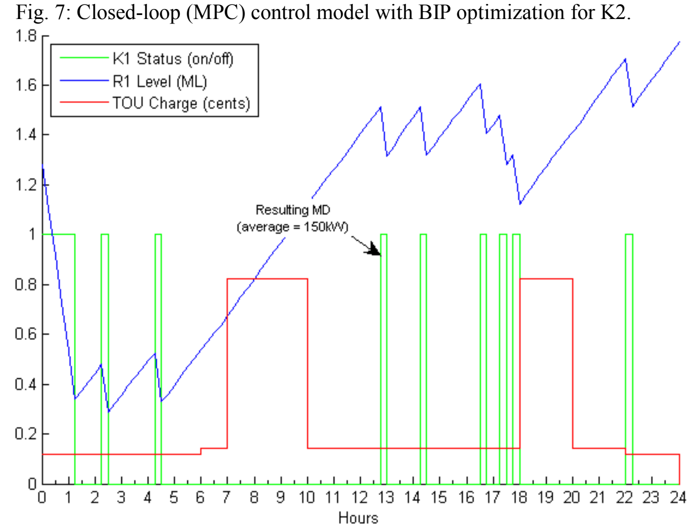

Fig. 8: Open-loop control model with BIP optimization for K2 with inflow disturbances.

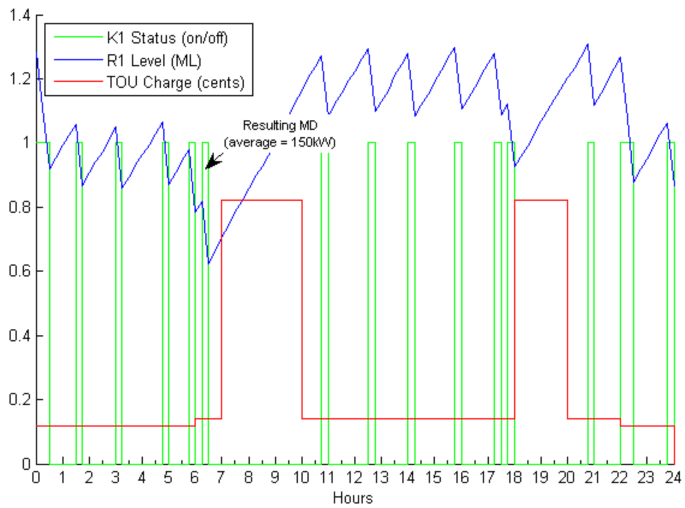

Fig. 9: Closed-loop (MPC) control model with BIP optimization for K2 with inflow disturbances.

IV. Conclusion

This paper evaluated the efficiency of a closed-loop (MPC) control strategy for load shifting in a specific plant with TOU and MD charges. The following four control models were compared to measure the efficiency of the closed-loop model: the current model with no optimization, an open-loop model with LP optimization, an open-loop model with BIP optimization, and the closed-loop model with BIP optimization.

The open-loop LP model was included as an optimal benchmark to compare the efficiency of the BIP models. The LP model gives the optimal solution within all possible real solutions which is not practical to implement, while the BIP models give binary integer solutions which can be practically implemented. Although a solution of the LP model is not practically implementable, it provides a lower bound for all the possible costs and therefore a benchmark for the efficiency of the BIP models.

The results showed that the closed-loop model saves 54% on energy costs and 50% on MD costs. The energy saving of the closed-loop model compared well with the open-loop LP and BIP models i.e. 56% and 55% respectively. The MD saving of the closed-loop model matched the saving of the open-loop BIP model, but it is not as good as the MD saving of the open-loop LP model i.e. 91%. It is expected that the MD saving for the open- and closed-loop BIP models will increase when more switching intervals per MD period is used. This case study affirms the applicability of a closed-loop (MPC) control strategy for load shifting in industrial plants that are charged on TOU and/or MD.

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