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A Unified Algorithm for Observability and Redundancy Analysis

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***Abstract*--This paper proposes a new algorithm that enables observability analysis (restoration and identification of observable islands) and identification of Critical Measurements and Critical Sets for observable or unobservable systems. The proposed algorithm is based on path graph concepts and triangular factorization of the Jacobian matrix, and has the following suitable characteristics for real-time operation: it is fast, simple, easy to implement and does not require solutions of any algebraic equation. Small numerical examples, using a 7 bus test system, that show the application of the proposed algorithm are presented. The IEEE 14, 30 and 57 bus systems with different measurement scenarios were used to evaluate the performance of the proposed algorithm.**

***Index Terms*—Critical Measurements, Critical Sets, Observability Analysis, Path Graph Concepts, Redundancy Analysis, State Estimation.**

I. Introduction

THE observability analysis consists in determining whether enough real-time measurements are available to make state estimation possible in the entire power system. Observability is related not only to the number of measurements but also to their types and locations. If the system is observable as a whole, it is important to verify the existence of critical measurements (CMs) and critical sets (CSs). On the other hand, if the system is unobservable, it is necessary to determine which parts of the system contain states that can still be estimated (observable islands), as well as where pseudo-measurements (based on data prediction) can be added so that the system becomes observable as a whole (observability restoration) [1]. However, it is necessary to highlight that sometimes it is not possible to restore the system observability through the addition of pseudo-measurements. In such cases, it is necessary to identify observable islands.

Several methods of observability analysis and restoration, as well as for identification of CMs and CSs have been developed [1]-[3]. Reference [4] provides a very interesting unified approach that allows observability analysis and restoration, CMs identification, and the determination of observable islands. However, that approach does not allow the identification of CSs.

Based on the analysis of the H_Δ matrix, which is obtained via triangular factorization of the Jacobian matrix, a method that allows, in a straightforward manner, observability analysis and restoration, as well as identification of CMs and CSs was proposed in [1]; however, this method does not allow the identification of observable islands.

In reference [5] a method of observability analysis and identification of observable islands based on path graph concepts [6] and triangular factorization of the Jacobian matrix is presented. The main characteristic of this method is to allow observable islands identification in a very simple and fast way, without demanding solutions of any algebraic equation.

This paper proposes an algorithm that enables observability analysis (restoration and identification of observable islands) and identification of CMs and CSs for observable or unobservable systems. The proposed algorithm is based on the methods proposed in references [1] and [5].

The proposed algorithm has the following suitable characteristics for real-time operation: it is fast, simple, easy to implement and does not require solutions of any algebraic equation.

The paper is organized as follows. Section II provides some relevant concepts related to the state estimation process. Section III presents the proposed algorithm and appropriate background. Section IV shows, through a 7 bus test system, how the proposed algorithm works. Section V provides some results of the tests in the IEEE-14 and 30 bus systems. Section VI presents some relevant conclusions.

II. Relevant State Estimation Concepts

Usually in observability analysis it is assumed that power measurements occur in active and reactive pairs and a sufficient number of voltage magnitude measurements are available. As a consequence, the linearized and decoupled state estimation model, or $P\theta$ model, is adopted to perform observability analysis [2],[3].

The linearized state estimation model relates the *(mx1)* active power measurement vector z to the *(nx1)* voltage phase angles vector θ as

$$z = H_{P\theta}\theta + w, \tag{1}$$

where: $H_{P\theta}$ is the *(m x n)* Jacobian matrix; and w is the *(m x 1)* measurement noise vector.

Hereafter, for the sake of simplicity, the $P\theta$ model (active power measurements – voltage phase angles) will be used in this paper, and the corresponding Jacobian matrix will be called just H matrix.

According to [7], a power system with n buses is said to be $P\theta$ - algebraically observable if the following equation holds:

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$$rank(H) = n - 1, \quad (2)$$

in which (n - 1) is the number of voltage phase angles to be estimated (one of the buses is chosen as the phase angle reference and its voltage phase angle is set equal to zero).

From the above considerations, observability analysis can be carried out by the triangular factorization (*LU* decomposition) of the *H* matrix [1].

III. Proposed Algorithm

In a general way, the proposed algorithm comprises two phases: *Phase 1*: Observability Analysis (Restoration and Identification of Observable Islands); and *Phase 2*: Redundancy Analysis (Identification of CMs and CSs).

Fig. 1 illustrates the flowchart of the proposed algorithm:

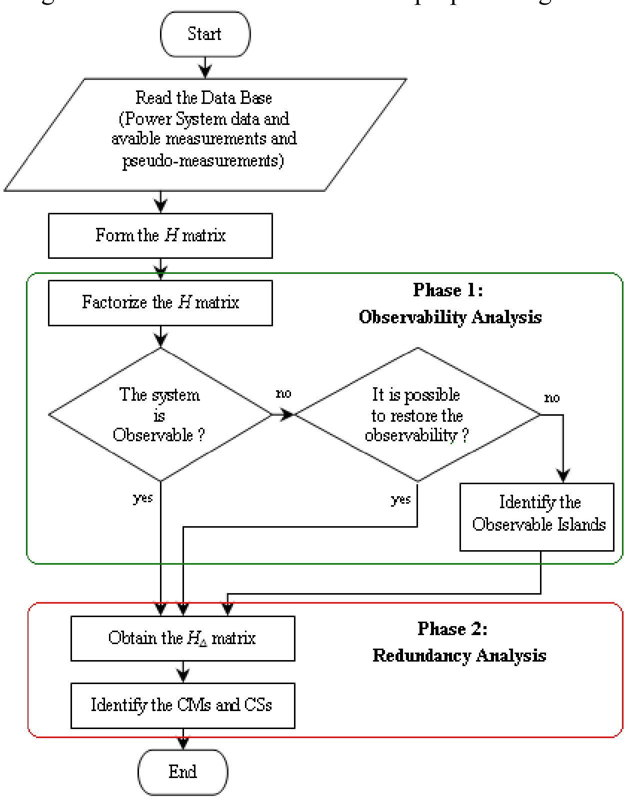


Fig. 1. Flowchart of the proposed unified algorithm for observability and redundancy analysis

A. Phase 1: Observability Analysis

The first step is to investigate if the power system is observable as a whole. For this analysis, the triangular factorization of the corresponding *H* matrix is necessary, and must be made through the combination of the columns of *H*, as presented in [1]. As the columns of *H* correspond to the system states, the columns of the factored matrix correspond to equivalent states, which are linear combinations of the system states.

If the system is $P\theta$ observable, the factorization results in only one zero pivot, in the diagonal (n,n), been n the number of buses of the system. During the course of the factorization, row permutations may be necessary to avoid zero pivots.

However, if the system was not observable as a whole, during the factorization of the *H* matrix one would find a zero pivot in the diagonal (i,i), where i < n, and there would be no other row in *H* with non-zero element in the column of the zero pivot. This indicates the absence of any measurement giving information of the equivalent state corresponding to that column.

When the system is not observable as a whole, the proposed algorithm enables: (i) Observability restoration, if there were the required pseudo-measurements; or (ii) Observable islands identification.

In order to restore the system observability, the proposed algorithm search for a pseudo-measurement that gives the information of the equivalent state corresponding to the column of the zero pivot, using the triangular factors obtained during the factorization of *H*, exactly as presented in [1].

This search is made in the following form: (i) Create one new row in the *H* matrix, which is being factorizated, where the first available pseudo-measurement will be stored; (ii) Apply to this new row the triangular factors; (iii) In case a non-zero element appears in that row in the column of the zero pivot, then that pseudo-measurement gives the required information.

If there are not the required pseudo-measurements to restore the system observability, the proposed algorithm enables the identification of observable islands.

To identify observable islands, the proposed algorithm uses path graph concepts, as presented in references [2] and [5]. In order to do this, it is necessary to substitute the zero pivot for non-zero value of 1.0, and the factorization process will be continued up to diagonal *(n,n)*. Observe that this is equivalent to insert a critical pseudo-measurement of angle [3],[5], becoming the system "artificially observable". Fig. 2 shows the matrices *L* and *U* from the *LU* decomposition of the *H* matrix associated with an unobservable system having m available measurements, after the inclusion of one pseudo-measurement of angle (θ_1^m).

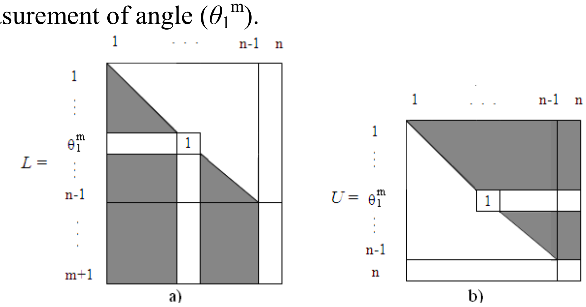


Fig. 2. The *LU* decomposition associated with an unobservable system: a) Lower-trapezoidal matrix L; b) Upper-triangular factors matrix U

Analyzing the path graphs associated with the resulting *U* factors matrix, the proposed algorithm allows observable islands identification through the following steps [5]:

Step 1: If in the available measurement set there is no injection measurement relating nodes (or equivalent states) of different path graphs, then the system as a block is unobservable and each subnetwork associated with each isolate path graph constitutes one observable island of the network;

Step 2: If in the available measurement set there are injection measurements relating nodes of different path graphs, then the system as a whole is not observable and it is not possible to assure that each subnetwork associates with each isolated path graph constitutes observable island. In order to find observable islands, identify those measurements and discard them (these are irrelevant measurements regarding state estimation to the observable islands). Re-factorize the new matrix.

Repeat the process of discarding and re-factorization until there is no injection measurement relating nodes of different path graphs. The resulting path graphs will indicate isolate observable islands [5].

B. Phase 2: Redundancy Analysis

This phase enables the identification of CMs or CSs associated with system observable as a whole or with each one of the observable islands.

This phase is carried out by the analysis of the H_Δ matrix, as presented in reference [1]. As a consequence, the first step of this phase is to obtain the H_Δ matrix, from the H matrix partially factored in phase 1. In order to do that, the L matrix from the LU decomposition of that H matrix is factorized again (LDU).

Considering the $P\theta$ - model, for an observable system (or for an artificially observable system) with n buses, the corresponding H matrix has rank equal to n-1, and the resulting H_Δ matrix presents the following structure [1]:

$$\boldsymbol{H}_{\Delta} = \left[\begin{array}{c|c} \boldsymbol{I}_{(n-1)} & \begin{matrix}0\\ .\end{matrix} \\ \hline \boldsymbol{R} & \begin{matrix}.\\ 0\end{matrix} \end{array}\right] \quad (3)$$

where: I is a identity matrix of dimension (n-1)x(n-1); R is a sub-matrix of dimension $[m_T-(n-1)]\mathrm{x}(n-1)]$; and m_T is the number of available measurements including the selected pseudo-measurements of angle.

Remark 1: The last column of H_Δ is composed only of zeroes and corresponds to the bus taken as reference of voltage phase angle.

The measurements corresponding to the rows of sub-matrix I are called Basic measurements, in the sense they assure the system observability. The measurements corresponding to the rows of matrix R are called Supplementary measurements.

Analyzing the H_Δ matrix structure, it is possible to identify the critical p-sets of measurements [1].

Definition 1: A critical p-set of measurements for $p \geq 1$, of an observable power system, is the set of p measurements which, when removed from the measurement set, makes the system unobservable; however, the removal of any set of k measurements of this set, with $k < p$, does not make the system unobservable.

Observe that a CM is a critical p-set with $p = 1$.

The p measurements corresponding to the p nonzero elements of a column of H_Δ form a critical p-set containing only one Basic measurement. Consequently, through the nonzero elements that appear in the H_Δ matrix, it is possible to identify all the critical p-sets formed only by one Basic measurement. To identify the critical p-sets formed by more than one Basic measurement an iterative process interchanging one non-critical Basic measurement with one redundant Supplementary measurement becomes necessary. It is important to highlight that this process requires only re-factorization in a very sparse matrix.

Considering Definition 1, in [1] it was demonstrated that a CS[1] of an observable-measurement set is composed of measurements which belong to critical 2-sets of measurements. As a consequence, the identification of CMs and CSs can be made by the analysis of the H_Δ matrix through the following steps:

Step 1: Analyzing the nonzero elements in the structure of the H_Δ matrix, find the CMs (or critical 1-set) and the critical 2-sets formed by only one Basic measurement.

Step 2: Among the critical 2-sets identified in step 1, select those without any Supplementary measurement in common. The two measurements of each one of these critical 2-sets form, in an isolated way, a CS composed of just two measurements.

Step 3: Among the critical 2-sets identified in step 1, select groups of these critical 2-sets having one Supplementary measurement in common. The measurements of each group of these critical 2-sets constitute, in an isolated way, a CS with more than two measurements.

Step 4: For each non-critical Basic measurement that does not belong to the already identified critical sets, remove the row corresponding to that Basic measurement from the original H_Δ. Obtain the new H_Δ matrix. The measurements now classified as critical constitute a CS together with the removed Basic measurement.

IV. EXAMPLES

Two examples to characterize the proposed algorithm are presents in the following (see the flowchart of the proposed algorithm illustrated in Fig. 1).

Example 1: Consider the system of Fig. 3, with the available pseudo-measurements: PF:1-6 (flow from bus 1 to bus 6) and PF:2-4 (flow from bus 2 to bus 4).

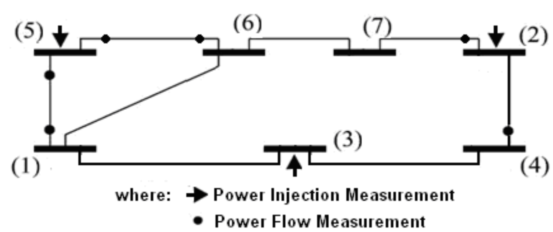


Fig. 3. 7-bus system associated with a metering system

<u>Phase 1: Observability Analysis:</u> From the measurement set shown in Fig. 3, the following H matrix is obtained (considering all branches reactances equal to unit):

[1] A critical set, also called as minimally dependent set, is well known as being a set of redundant measurements such that the removal of any one of them makes the remaining measurements critical.

$H =$

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F:1-5	1				-1		
F:5-1	-1				1		
F:4-2		-1		1			
F:2-7		1					-1
F:5-6					1	-1	
F:6-5					-1	1	
I:2		2		-1			-1
I:3	-1		2	-1			
I:5	-1				2	-1	

Through the *LU* decomposition of that *H* matrix (with the necessaries row permutations) was verified that the system is not observable as a whole (one zero pivot appears in row 6).

a) Observability restoration

To search by the required pseudo-measurements, the available pseudo-measurements are analyzed one by one. However, by space limitation, in this paper they will be analyzed in the same time. Thus, rows 10 and 11 are created to store pseudo-measurements PF:1-6 and PF:2-4 respectively. Applying the factors to those new rows one obtains the following H_F matrix:

$H_F =$

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F:1-5	1				1		
F:4-2		-1		1			
I:3	-1		2	0.5	0.5		
F:2-7		1		1			1
F:5-6					1	1	
F:6-5					-1	0	
I:2		2		1			
F:5-1	-1						
I:5	-1				1		
PF:1-6	1				1	0	
PF:2-4		1				0	

Remark 2: The white area corresponds to the upper-triangular factors matrix *U*, and the shaded area corresponds to lower-trapezoidal submatrix *L*.

As $H_F(10,6)=0$ and $H_F(11,6)=0$, none of these pseudo-measurements gives the information required to restore the system observability. As a consequence, it is not possible to restore the system observability.

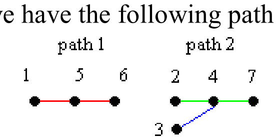
b) Identification of observable islands:

As one zero pivot appears in row 6, the angle pseudo-measurement θ_1^m is introduced in the same row (this pseudo-measurement gives information of the equivalent state θ_6). Continuing the factorization process one obtains the following H_F matrix:

$H_F =$

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F:1-5	1				1		
F:4-2		-1		1			
I:3	-1		2	0.5	0.5		
F:2-7		1		1			1
F:5-6					1	1	
$\boldsymbol{\theta_1^m}$						**1**	
F:6-5					-1		
I:2		2		1			
F:5-1	-1						
I:5	-1				1		

With the upper-triangular factors (white area) obtained in the previous step, we have the following path graphs:



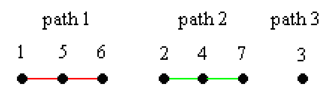
The injection measurement at bus 3 relates equivalent states (variables) of path graphs 1 and 2. Thus, this measurement is discarded and the H_F matrix is updated. Repeating the factorization process, one obtains:

$H_F =$

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F:1-5	1				1		
F:4-2		-1		1			
$\boldsymbol{\theta_1^m}$			1				
F:2-7		1		1			1
F:5-6					1	1	
$\boldsymbol{\theta_2^m}$						**1**	
F:5-1	-1						
F:6-5					-1		
I:2		2		1			
I:5	-1				1		

Note that the angle pseudo-measurements θ_1^m and θ_2^m were introduced in row 3 and row 6, respectively.

Now, there are three path graphs associated with the upper-triangular factors matrix *U*:



There is no injection measurement relating nodes (or equivalent states) of different path graphs. Therefore, there are three observable islands: Island 1:{1, 5, 6}; Island 2:{2, 4, 7}; and Island 3:{3}.

Phase 2 – Redundancy Analysis: Continuing the factorization process of the H_F matrix partially factored in phase 1 one obtains the following H_Δ matrix:

$H_\Delta =$

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F:1-5	1						
F:4-2		1					
$\boldsymbol{\theta_1^m}$			1				
F:2-7				1			
F:5-6					1		
$\boldsymbol{\theta_2^m}$						1	
F:5-1	-1						
F:6-5					-1		
I:2		-1		1			
I:5	-1				1		

Verifying the non-zero elements in the columns of the H_Δ matrix, one obtains: Column 1: critical trio: [F:1-5, F:5-1, I:5]; Column 2: critical pair: [F:4-2, I:2]; Column 3: critical measurement: [θ_1^m]; Column 4: critical pair: [F:2-7, I:2]; Column 5: critical trio: [F:5-6, F:6-5, I:5]; and Column 6: critical measurement: [θ_2^m].

Considering these critical p-sets and the algorithm presented in section III.B, only one CS is identified: [F:4-2; F:2-7; I:2].

Remark 3: Observe that this Critical Set belonging to the observable island 2, and the two pseudo-measurement of angle were correctly identified as critical measurements.

Example 2: Consider the system of Fig. 3 again, but now with the available pseudo-measurements: PI:1 (injection at bus 1) and PF:2-4 (flow from bus 2 to bus 4).

Phase 1: Observability Analysis: As in the previous example, through the *LU* decomposition of the corresponding *H* matrix (with the necessaries row permutations) it is verified that the system is not observable as a whole (one zero pivot appears in row 6).

a) Observability restoration

Row 10 is created to store the first available pseudo-measurement (PI:1). Applying the factors to row 10, a non-zero element appears, in that row, in the column of the zero pivot (columns 6). As a consequence, the pseudo-measurement PI:1 gives the necessary information to restore the system observability. After row permutations one obtains the following H_F matrix:

H_F =

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F:1-5	1				1		
F:4-2		-1		1			
I:3	-1		2	0.5	0.5		
F:2-7		1		1			1
F:5-6					1	1	
PI:1	3		-1	-0.5	1.5	0.5	1
F:6-5					-1		
I:2		2		1			
F:5-1	-1						
I:5	-1				1		

Phase 2 – Redundancy Analysis: Continuing the factorization process of the H_F matrix, one obtains the following H_Δ matrix:

H_Δ=

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F:1-5	1						
F:4-2		1					
I:3			1				
F:2-7				1			
F:5-6					1		
PI:1						1	
F:6-5					-1		
I:2		-1		1			
F:5-1	-1						
I:5	-1				1		

Verifying the non-zero elements in the columns of the H_Δ matrix, one obtains: Column 1: critical trio: [F:1-5, F:5-1, I:5]; Column 2: critical pair: [F:4-2, I:2]; Column 3: critical measurement: [I:3]; Column 4: critical pair: [F:2-7, I:2]; Column 5: critical trio: [F:5-6, F:6-5, I:5]; and Column 6: critical measurement: [PI:1].

Considering these critical p-sets and the algorithm presented in section III.B, only one CS is identified: [F:4-2; F:2-7; I:2].

V. Test Results

To verify the efficiency of the proposed algorithm, it has been applied to the IEEE 14, 30 and 57 bus systems. However, owing to space limitations, only some tests results in the IEEE 14 and 30 bus systems will be presented.

A. Test with the IEEE-14 bus system

Case A: Consider the measurement set associated with the IEEE 14 bus system illustrated in fig. 4, and that there is no available pseudo-measurement.

Phase 1: Observability Analysis: The system is not observable as a whole.

a) Restoration: It is not possible (there is no available pseudo-measurement).

b) Identification of observable islands: Island 1: {1, 2, 3, 4, 5, 7, 8, 9}; Island 2: {6, 12, 13}; Island 3: {10}; Island 4: {11}; and Island 5: {14}.

Irrelevant measurements: I:6, I:9, and I:14.

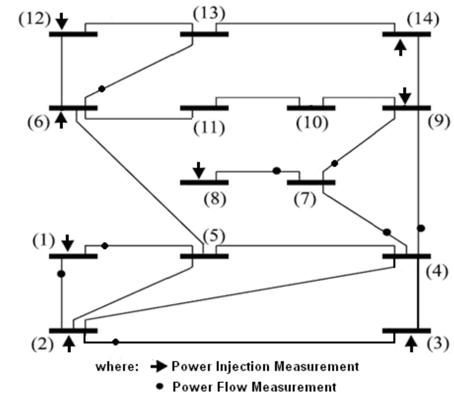


Fig. 4. The 14-bus system associated with a metering system: Cases A and B

Phase 2: Redundancy Analysis

Critical Measurements: F:6-13, I:12 and the angle pseudo-measurements in the rows 9, 10, 11 and 13;

Critical Sets: CS 1-[F:2-3, I:2, I:3]; CS 2 – [F:4-9, F:7-9].

Case B: Consider the measurement set associated with the IEEE 14 bus system illustrated in fig. 4 again, but now with the available pseudo-measurements: PI:5 (injection at bus 5) and PF:2-4 (flow from bus 2 to bus 4).

Phase 1: Observability Analysis: The system is not observable as a whole.

a) Restoration: Using the pseudo-measurement PI:5, the system observability is restored.

Phase 2: Redundancy Analysis

Critical Measurements: PI:5, F:6-13, I:6, I:9, I:12, I:14;

Critical Sets: CS 1-[F:2-3, I:2, I:3]; CS 2 – [F:4-9, F:7-9].

Case C: Consider the measurement set associated with the IEEE 14 bus system illustrated in fig. 5, and that there is no available pseudo-measurement.

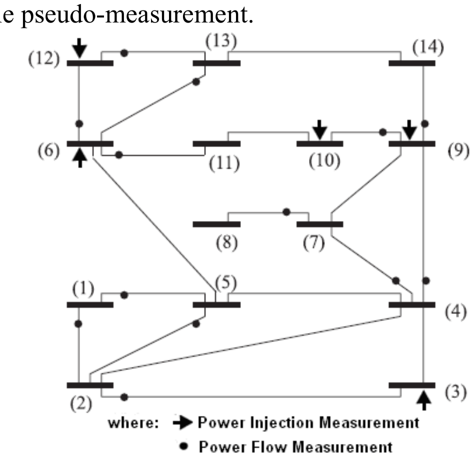


Fig. 5. The 14-bus system associated with a metering system: Case C

Phase 1: Observability Analysis: The system is observable as a whole.

Phase 2: Redundancy Analysis

Critical Measurements: F:7-8;

Critical Sets: CS 1-[F:2-3, I:10, I:3, I:6, F:6-11]; CS 2 – [F:4-7, I:9, F:9-14]; CS 3 – [F:1-2, F:1-5].

B. *Test with the IEEE-30 bus system*

Case A: Consider the measurement set associated with the IEEE 30 bus system illustrated in fig. 6, and that there is no available pseudo-measurement.

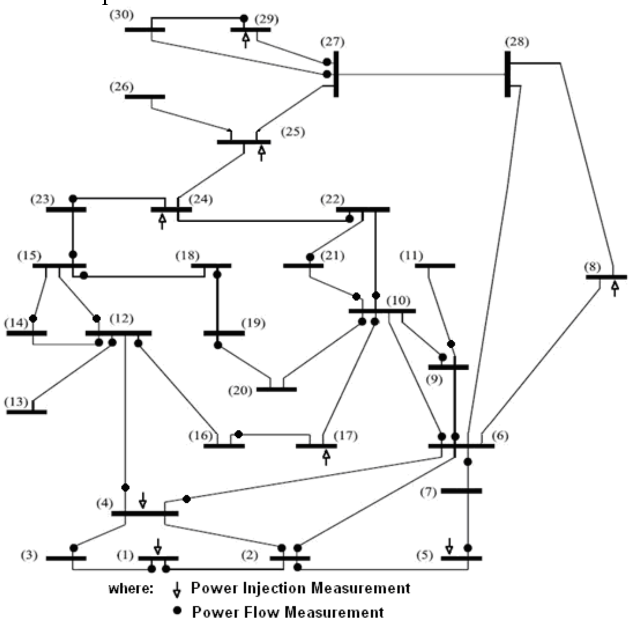


Fig. 6. The 30-bus system associated with a metering system

<u>Phase 1: Observability Analysis:</u> The system is not observable as a whole.

a) Restoration: It is not possible (there is no available pseudo-measurement).

b) Identification of observable islands: Island 1:{1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}; Island 2: {8}; Island 3: {26}; Island 4: {27, 29, 30}; and Island 5: {28}.

Irrelevant measurements: I:8 and I:25.

<u>Phase 2: Redundancy Analysis</u>

Critical Measurements: F:9-11, F:12-13, I:24 and the angle pseudo-measurements in the rows 8, 25, 26 and 28.

Critical Sets: CS 1-[F:6-9, F:9-10]; CS 2 – [F:10-20, F:19-20, F:15-18, F:18-19]; CS 3 – [F:12-14, F:14-15]; CS 4 – [F:10-21, F:21-22]; CS 5 – [F:15-23, F:23-24, F:22-24].

Case B: Consider the measurement set associated with the IEEE 30 bus system illustrated in fig. 6 again, but now with the available pseudo-measurements: PI:6 (injection at bus 6), PF:4-2 (flow from bus 4 to bus 2), and PI:26 (injection at bus 26).

<u>Phase 1: Observability Analysis:</u> The system is not observable as a whole.

a) Restoration: Using the pseudo-measurements PI:6 and PI:26, the system observability is restored.

<u>Phase 2: Redundancy Analysis</u>

Critical Measurements: PI:6, PI:26, I:8, F:9-11, F:12-13, I:24 and I:25;

Critical Sets: CS 1-[F:6-9, F:9-10]; CS 2 – [F:10-20, F:19-20, F:15-18, F:18-19]; CS 3 – [F:12-14, F:14-15]; CS 4 – [F:10-21, F:21-22]; CS 5 – [F:15-23, F:23-24, F:22-24].

VI. Conclusions

This paper proposed a fast and simple algorithm for observability and redundancy analysis, which enables observability restoration, identification of observable islands, and identification of Critical Measurements and Critical Sets for observable or unobservable systems. The proposed algorithm is based on the analysis of H_Δ matrix and on path graph concepts.

As the proposed algorithm is based on the analysis of linear dependence (or independence) among the system equations (H_Δ matrix rows), so it can be applied to any kind of equations. As a consequence, it can be extended to analysis metering system composed of conventional measurements (power and voltage magnitude measurements) and synchronized phasor measurements (provided by Phasor Measurement Units [8]).

The IEEE 14 and 30 bus systems and different measurement scenarios were used to test the performance of the proposed algorithm. The results have shown that in all scenarios the new algorithm worked successfully and presented the expected results.

VII. References

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VIII. Biographies

Raphael Augusto de Souza Benedito received the B.S.E.E. degree in 2005 from UNESP, Brazil, and the M.S.E.E. in 2007 from EESC/USP - University of Sao Paulo, Brazil. He is currently working toward the Ph.D. degree at the EESC/USP - University of Sao Paulo. His research interest includes state estimation in electrical power systems.

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Newton G. Bretas received in 1981 the Ph.D. degree from University of Missouri, Columbia, USA. He became Full Professor of the University of Sao Paulo, Brazil, in 1989. His work has been primarily concerned with power system analysis, transient stability using direct methods as well as power system state estimation. Dr. Bretas has also interest in energy restoration using ANN and GA for distribution systems.