

# A New Transformer Hysteresis Model in MATLAB™ Simulink

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**Abstract-** This paper, which is an extended version of Referenced paper[1], presents a new transformer model displaying hysteresis using MATLAB™ Simulink environment. The proposed model displays a complete scheme for the simulation of three phase transformers. The new model is mainly based on transmission line model (TLM) of power system using lumped parameters. The developed model is particularly suitable for the fault analysis and protective relaying studies under harmonic conditions where the transformer is driven into the nonlinear regime.<sup>1234</sup>

**Index Terms:** Jiles-Atherton (J-A), transformer, hysteresis, modeling, Simulink

## I. INTRODUCTION

Power transformer is one of the most important elements of power systems, since the continuity of transformer operation has a vital importance in maintaining the reliability of power supply. Therefore various modeling schemes have been developed for decades to provide a proper protective method for the transformers. This paper presents a three phase transformer model displaying hysteresis based on J-A magnetization with a modified Langevin function.

The development and the validation of protective algorithms for the transformers require the preliminary determination of the transformer model. The model used is required to simulate the power system normal and faulted conditions and to estimate the behavior of the protection algorithm under abnormal conditions. In particular, it must allow for the simulation of internal and external fault conditions. Most of the present electromagnetic transient programs are able to accurately simulate some of the phenomena occurring in the transformer like magnetizing inrush current, excitation current and transformer saturation [2]. However, the implementations of the present monitoring methods [3,4] tend to cost too much to be applied to distribution transformers. Besides, such an analysis usually requires commercial software like FEMLAB, MAGNET, and ANSOFT's Maxwell etc. However, most of present simulation programs are not able to properly simulate hysteresis characteristic of the core material [5].

In this paper, a complete model of three phase two winding transformer is presented using MATLAB™ Simulink environment. The proposed model is based on TLM method for simulating transformer which can consider nonlinear hysteresis. The proposed modeling technique is suitable to model the transformer related phenomenon such as magnetizing inrush, internal faults, and loading conditions. To validate the proposed technique, a small three phase two winding laboratory type transformer (1KVA, 380/220 V, star-star connected, 50 Hz) is modeled using TLM method including J-A technique.

## II. THREE PHASE TWO WINDING TRANSFORMER MODEL

The TLM method was first developed in early 1970s for modeling two dimensional field problems. Since then it has been extended to cover three dimensional problems and circuit simulations. For circuit simulation, the TLM method can be used to develop a discrete circuit model directly from the system without setting up any integro-differential equations. The TLM algorithm is discrete in nature and ideally suited for implementation on computer based systems [6].

The hysteresis specs of the transformer are based on J-A technique in TLM simulations. The J-A describes the relationship between magnetic moment  $M$  and magnetic field intensity  $H$  using the current physical theories of magnetic domains in ferromagnetic materials [7]. The J-A model requires the following input parameters: magnetization saturation, thermal energy parameter, domain flexing constant, domain anisotropy constant, inter-domain coupling parameter. These are not the parameters that transformer manufacturers or manufacturers of transformer magnetic material can provide for the users. In fact they can also not even be determined directly through measurements. The various core hysteresis parameters required in this model are theoretical and can be calculated from experimental measurements of coercivity, remanence, saturation flux density, initial anhysteretic susceptibility, initial normal susceptibility, and the maximum differential susceptibility [8]. More details about the J-A technique can also be found in [9, 10].

Conversions between B-H loop and an M-H loop is straightforward and is calculated using Eq. (1).

$$B = \mu_0(H + M) \quad (1)$$

The magnetization relationship between  $B$  and  $H$  is replaced by an hysteric magnetization curve between  $H_c$  and  $M$  as follows:

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$$M_{an} = M_s \cdot f(H_e) \quad (2)$$

where,  $H_e = H + \alpha M$ , and  $\alpha$  is a parameter which represents interdomain coupling,  $M_s$  is the saturation magnetization. A three phase two winding transformer is modeled by the proposed technique. Fig.1 shows the transformer core type under modeling consideration.

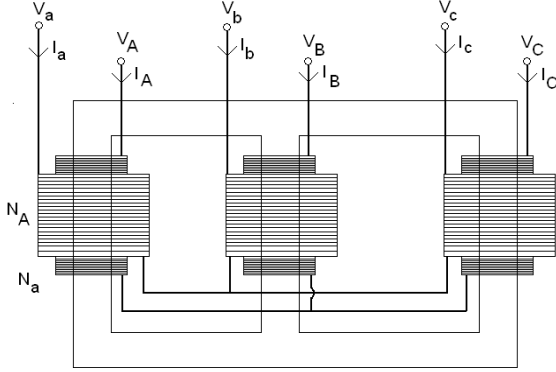


Figure 1. A three phase two winding star / star connected transformer

In Fig. 1,  $N_a$  and  $N_b$  represent number of turns of primary and secondary windings, respectively. Through the paper small letter subscripts represent the primary side parameters, while capital letter subscripts represent secondary side ones. Linear transformer model is shown in Fig. 2.

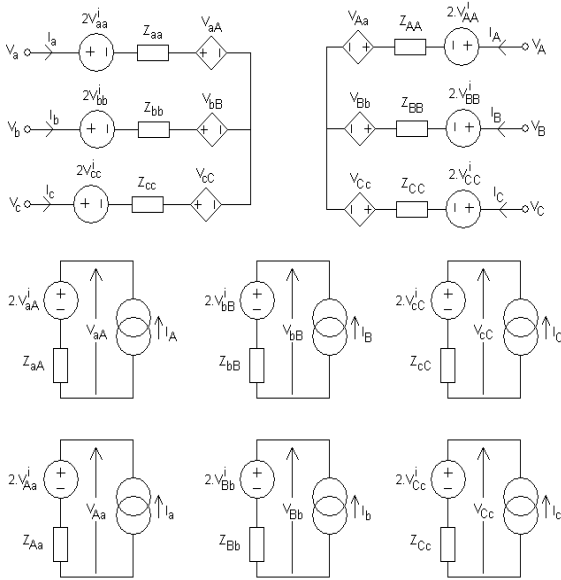


Figure 2. Linear modeling of the three phase two winding transformer

The main modeling equations such as  $V_{a-b-c}$ ,  $V_{A-B-C}$  (primary and secondary voltages), and  $M_{a-b-c}$  (magnetization intensities of the phases) are defined in Eq. (3), respectively.

$$\begin{aligned} V_a &= L_{aa} dI_a / dt + L_{aA} dI_A / dt + N_a L_m dI_{ma} / dt \\ V_b &= L_{bb} dI_b / dt + L_{bB} dI_B / dt + N_b L_m dI_{mb} / dt \\ V_c &= L_{cc} dI_c / dt + L_{cC} dI_C / dt + N_c L_m dI_{mc} / dt \\ V_A &= L_{AA} dI_A / dt + L_{aA} dI_a / dt + N_A L_m dI_{ma} / dt \\ V_B &= L_{BB} dI_B / dt + L_{bB} dI_b / dt + N_B L_m dI_{mb} / dt \\ V_C &= L_{CC} dI_C / dt + L_{cC} dI_c / dt + N_C L_m dI_{mc} / dt \end{aligned} \quad (3)$$

Where,  $L_{aa} = (\mu_0 N_a^2 A) / l$ ,  $L_{AA} = (\mu_0 N_A^2 A) / l$ ,  $L_m = (\mu_0 A) / l$ ,  $M_{a-b-c} = I_{m(a-b-c)} / l$ .  $A$  is the cross-sectional area in  $m^2$  and  $l$  is the length of the magnetic path in m.

These are initially calculated as simulation starts. The characteristic impedances of the primary and secondary sides are then calculated as follows:

$$\begin{aligned} Z_{aa} &= 2L_{aa} / dt = Z_{bb} = Z_{cc} \\ Z_{AA} &= 2L_{AA} / dt = Z_{BB} = Z_{CC} \end{aligned} \quad (4)$$

(since  $N_a = N_b = N_c$ )

Similarly, the characteristic impedances of the magnetizing branches are calculated using following equations.

$$\begin{aligned} L_{aA} &= (\mu_0 N_a N_A A) / l \\ Z_{aA} &= 2L_{aA} / dt \end{aligned} \quad (5)$$

where,  $Z_{aA} = Z_{bB} = Z_{cC}$ .

Leakage inductances and related characteristic impedances are calculated using Eq. (6).

$$\begin{aligned} Z_{LaS} &= 2L_{aS} / dt \\ Z_{LAS} &= 2L_{AS} / dt \end{aligned} \quad (6)$$

where,  $Z_{LaS} = Z_{LbS} = Z_{LcS}$  and  $Z_{LAS} = Z_{LBS} = Z_{LCS}$ .

$Z_{LaS}$  and  $Z_{LAS}$  are the leakage characteristic impedances of the primary and secondary sides respectively. For a lossless linear transformer, the mutual TLM voltages are defined in Eq. (7) and Eq. (8) as follows.

$$\begin{aligned} V_{aA} &= Z_{aA} I_A + 2V_{aA}^i \\ V_{bB} &= Z_{bB} I_B + 2V_{bB}^i \\ V_{cC} &= Z_{cC} I_C + 2V_{cC}^i \\ V_{AA} &= Z_{AA} I_a + 2V_{AA}^i \\ V_{BB} &= Z_{BB} I_b + 2V_{BB}^i \\ V_{CC} &= Z_{CC} I_c + 2V_{CC}^i \end{aligned} \quad (8)$$

where  $Z_{aA} = 2L_{aA} / dt$ ,  $Z_{bB} = 2L_{bB} / dt$ ,  $Z_{cC} = 2L_{cC} / dt$ , and  $dt$  is the sampling interval.

After these calculations steps, the primary and secondary currents  $I_{a-b-c}$  and  $I_{A-B-C}$  are calculated by using Eq. (9) and Eq. (10).

$$\begin{aligned} Z_{aa} I_a + Z_{aA} I_A &= V_a - 2(V_{aa}^i + V_{aA}^i) \\ Z_{bb} I_b + Z_{bB} I_B &= V_b - 2(V_{bb}^i + V_{bB}^i) \\ Z_{cc} I_c + Z_{cC} I_C &= V_c - 2(V_{cc}^i + V_{cC}^i) \\ Z_{aA} I_a + Z_{AA} I_A &= -2(V_{AA}^i + V_{aA}^i) \\ Z_{bB} I_b + Z_{BB} I_B &= -2(V_{BB}^i + V_{bB}^i) \\ Z_{cC} I_c + Z_{CC} I_C &= -2(V_{CC}^i + V_{cC}^i) \end{aligned} \quad (9)$$

where  $Z_{aa} = 2L_{aa} / dt$  and the other characteristic impedances are calculated in this way.

In Eq. (9),  $V_{a-b-c}$  is the source voltage of the  $a$ - $b$ - $c$  phases. The calculations are repeated till the end of simulation time. Fig. 3 shows a three phase two winding transformer model displaying hysteresis.

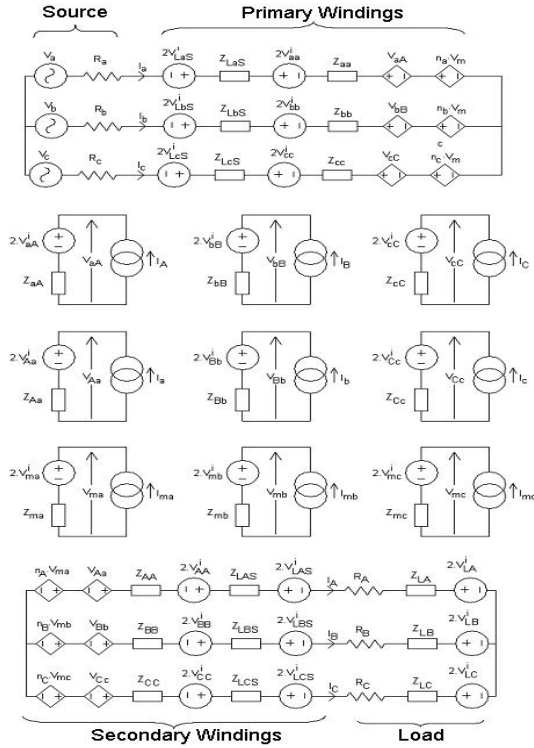


Figure 3. A three phase – two winding transformer model displaying hysteresis.

The required calculations for hysteresis model of a three phase transformer are a bit complicated then the linear model. To have a complete model, source and load impedances are added to the simulation. To simplify the whole procedure, the equations for only one phase are given below:

Step 1.  $V_a^k$  is the  $k$ . sampled source voltage.

Step 2. Phase – neutral voltage is calculated as  $V_{sa}^k = V_a^k + R_a I_a^k$ .  $R_a$  is the source resistance.

Step 3. J-A calculations are started from this step. Total ampere turns is calculated as  $I_{ha}^k = N_a I_a^k + N_A I_A^k$ .

Step 4. Calculation of exciting current as

$$I_{ea}^k = I_{ha}^k + \alpha I_{ma}^k$$

Step 5. Updating the total ampere turns of the phases as

$$dI_{ha}^{k+1} = I_{ha}^k - (N_a I_a^{k-1} + N_A I_A^{k-1})$$

Step 6. Directional flag is defined according the  $dI_{ha}^k$  as

$$\text{If } dI_{ha}^k < 0 \text{ then } \Delta_a^k = -1 \text{ else } \Delta_a^k = 1$$

Step 7. In order to refrain numerical instability, the following condition is defined in TLM calculations for anhsyteretic current.

$$\text{If } |I_{ea}^k| > 0.0001 I_a \text{ then}$$

$$I_{ana}^k = I_{sc} \left( \coth \frac{I_{ea}^k}{I_{ac}} - \frac{I_{ac}}{I_{ea}^k} \right) \text{ and}$$

$$\left( \frac{dI_{ana}^k}{dI_{ha}^k} \right)^k = \frac{I_{sc}}{I_{ac}} \left( -\frac{1}{\sin(I_{ea}^k / I_{ac})^2} + \left( \frac{I_{ac}}{I_{ea}^k} \right)^2 \right)$$

Else

$$I_{ana}^k = \frac{I_{sc} I_{ea}^k}{3 I_{ac}} \text{ and } \left( \frac{dI_{ana}^k}{dI_{ha}^k} \right)^k = \frac{I_{sc}}{3 I_{ac}}$$

Step 8. Irreversible current is defined as

$$dI_{irra}^k = dI_{ha}^k \frac{I_{ana}^k - I_{irra}^k}{\Delta_a I_{cc} - \alpha (I_{ana}^k - I_{irra}^k)}$$

Step 9. A migration flag is defined for each phase as

If  $\Delta_a^k > 0$  and  $I_{ana}^k > I_{irra}^k$  or  $\Delta_a^k < 0$  and  $I_{ana}^k < I_{irra}^k$  then

$$mig_a^k = 1$$

Else

$$mig_a^k = 0$$

Step 10. Updating the irreversible current as

$$I_{irra}^{k+1} = mig_a^k dI_{irra}^k + I_{irra}^k$$

Step 11.

If  $mig_a^k = 1$  and  $dI_{ha}^k = 0$  then

$$\left( \frac{dI_{ma}^k}{dI_{ha}^k} \right)^k = \beta_c \left( \frac{dI_{ana}^k}{dI_{ha}^k} \right)^k + mig_a^k (1 - \beta_c) \left( \frac{dI_{irra}^k}{dI_{ha}^k} \right)^k$$

Else

$$\left( \frac{dI_{ma}^k}{dI_{ha}^k} \right)^k = \beta_c \left( \frac{dI_{ana}^k}{dI_{ha}^k} \right)^k$$

Step 12. Magnetization current is defined as

$$I_{ma}^{k+1} = (1 - \beta_c) I_{irra}^{k+1} + \beta_c I_{ana}^k$$

Step 13. The following procedure is required for calculation of phase current.

$$U_a^k = V_{sa}^k - (S_{aa}^k - S_{aA}^k + N_a S_{ma}^k)$$

$$U_A^k = -(S_{Aa}^k + S_{AA}^k + N_a S_{ma}^k)$$

Voltage components are written as column vector in the following equation.

$$f = \begin{bmatrix} I_a^k (R_a + Z_{aa} + Z_{LaS}) + I_A^k Z_{aA} + N_a I_{ma}^{k+1} Z_{ma} - U_a^k \\ \dots \\ I_A^k (R_A + Z_{AA} + Z_{LaS} + Z_{LA}) + I_a^k Z_{aA} + N_a I_{ma}^{k+1} Z_{ma} - U_A^k \\ \dots \end{bmatrix}$$

Jacobian matrices is defined as

$$J = \begin{bmatrix} R_a + Z_{LaS}(1 + (\frac{dI_{ma}}{dI_{ha}})^k) & 0 & 0 & Z_{aA}(\frac{dI_{ma}}{dI_{ha}})^k & 0 & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots \\ Z_{aA}(\frac{dI_{ma}}{dI_{ha}})^k & 0 & 0 & R_A + Z_{LaS} + Z_{LA} + Z_{AA}(\frac{dI_{ma}}{dI_{ha}})^k & 0 & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 & 0 & \dots \end{bmatrix}$$

Newton Raphson iteration method is applied to calculate the next sample of phase currents.

$$\begin{bmatrix} I_a^{k+1} \\ \dots \\ I_A^{k+1} \end{bmatrix} = \begin{bmatrix} I_a^k \\ \dots \\ I_A^k \end{bmatrix} - J^{-1} f$$

Step 14. Updating the magnetization current is defined as

$$f_{ma} = I_a^{k+1}(R_a + Z_{aa}) + I_A^{k+1}Z_{aA} + N_a I_{ma}^{k+1}Z_{ma} - U_a^k$$

$$I_{ma}^{k+1} = I_{ma}^k - \frac{f_{ma}}{N_a Z_{ma}}$$

Step 15. Calculation of the simulation outcomes

$$\Psi_a = dt \frac{V_a^k - V_a^{k-1}}{2}$$

$$B_a^k = \frac{\mu_0}{l} (N_a I_a^{k+1} + N_A I_A^{k+1} + I_{ma}^{k+1})$$

$$H_a^k = \frac{(N_a I_a^{k+1} + N_A I_A^{k+1})}{l}$$

These equations are executed in Matlab™ Simulink environment. The simulation parameters and rated value of the modeled transformer can easily be changed. Fig. 4 shows the Simulink application of the proposed method. The left side of the Fig. 4 represents the primary side where it is connected to the source and the right side represents the secondary side where it is connected directly to the load ( $Z_A, Z_B, Z_C$ ). The user can easily change the required parameters by double clicking the each block. The main TLM transformer block is suitable to implement different type of three phase connections on both sides. The ‘powergui’ option in the proposed model is used for waveform analysis.

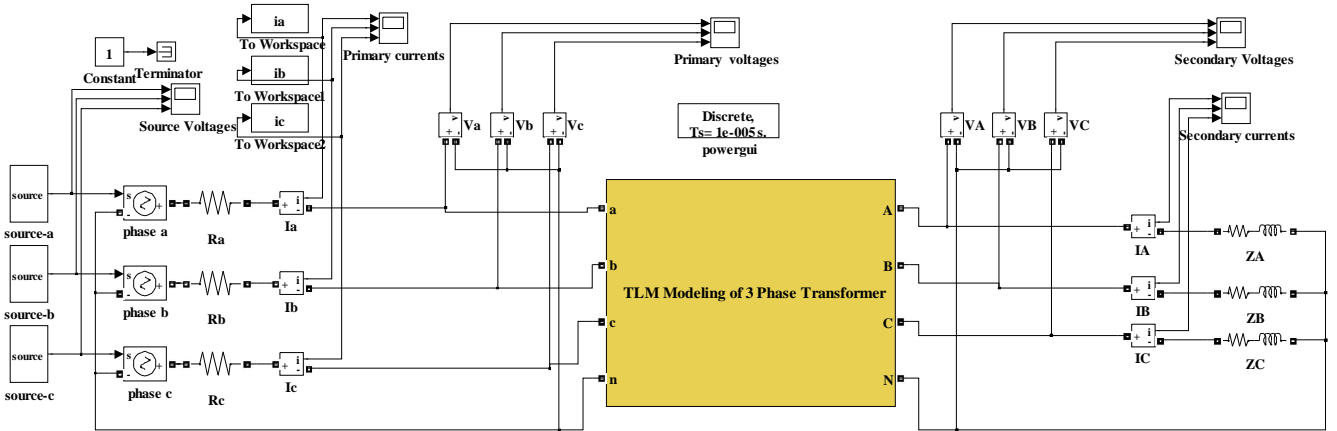


Figure 4. Matlab™ Simulink application of the proposed modeling technique

Fig. 5 shows the inside of the TLM model. Basically, it consists of voltage measurement unit, J-A implementation unit, and Jacobean matrix calculation module (subsystem4) for each phase. Fig. 6 shows the inside of the J-A block for phase a. This unit includes the calculation procedures Step 3 to Step 12 summarized above. The units named J-A1 and J-A2 are used for phase b and phase c, respectively.



parameters: source frequency, magnetic path length (m), core area ( $m^2$ ), number of turns for primary, number of turns for secondary, saturation magnetization ( $m^{-1}$ ), anhysteretic form factor ( $m^{-1}$ ), interdomain coupling coefficient, coercive field magnitude ( $m^{-1}$ ), magnetization weighting factor, permeability of free-space, leakage inductance of primary winding, leakage inductance of secondary winding, source resistors, load impedances and load factor.

Fig. 7 shows primary phase currents obtained using the TLM with J-A model.

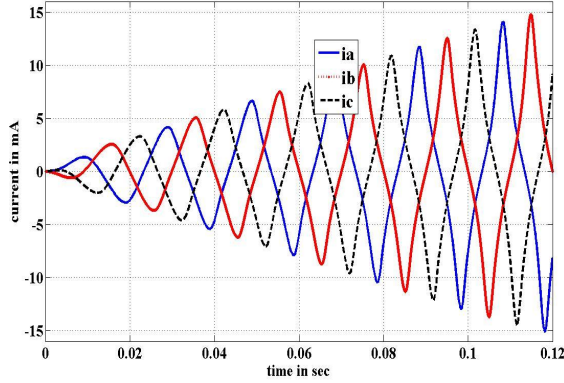


Figure 7. Simulated phase currents ( $i_a$ ,  $i_b$ ,  $i_c$ )

In this particular example, the three phase source voltage is defined as  $V_{peak} \sin(\omega t + \phi)(1 - e^{-20t})$ , where  $\phi = 0$ ,  $\phi = 2\pi/3$ , and  $\phi = -2\pi/3$ .

The following Fig.8 shows the B-H curve of phase a.

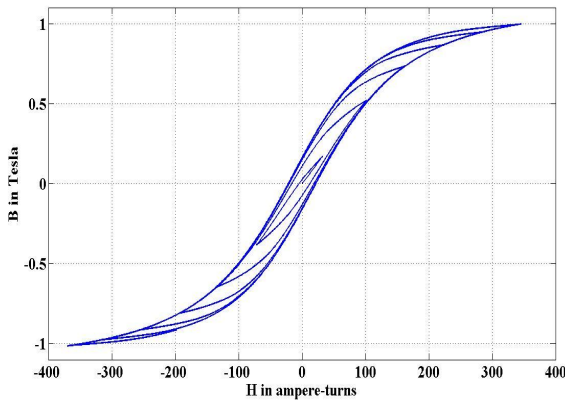


Figure 8. Simulated B-H curve of the transformer

To simulate the over-excitation condition, an exponential dc supply ( $0.1622 pu * \exp^{-0.1t}$ ) is added to the source of phase a. Fig. 8 shows the B-H curve of the phase a with a dc offset.

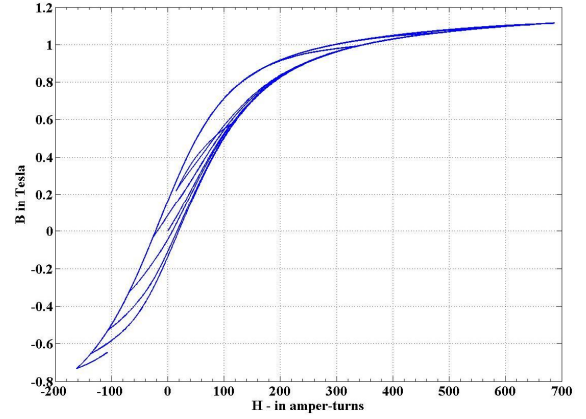


Figure 8. Simulated B-H curve of the transformer for a supply with a dc pulse

## V. CONCLUSION

This paper introduces a time domain model of a three phase two winding transformer with nonlinear and hysteretic behavior. It is a complete three phase transformer model in Matlab™ Simulink environment. The user can easily change the parameters by double clicking the “TLM Model of 3 Phase Transformer Model” block. The hysteretic model using modified Langevin function is based on J-A model of ferromagnetic hysteresis. The simulation results produce an acceptable transformer transient response and shows that the proposed overall technique is ideal for simulating three phase transformers.

The proposed modeling technique displaying hysteresis is suitable for most type of three phase two windings transformers. Apart from that, the suggested modeling technique is able to simulate the transformers during single or multiple internal faults.

## APPENDIX A

Data for 380/220V, 1KVA test transformer

$l_m = 24.10^{-3} m$ , magnetic path length,

$a = 45.4.10^{-6} m^2$ , core area,

$N_a = 550$ , primary turns

$N_A = 265$ , secondary turns,

$M_s = 275.10^5 m^{-1}$ , saturation magnetization,

$H_a = 14100 m^{-1}$ , anhysteretic form factor,

$\alpha = 5,10^{-6}$ , interdomain coupling coefficient,

$H_c = 17800 m^{-1}$ , coercive field magnitude,

$\beta_c = 0.55$ , magnetization weighting factor,

$R_a, R_b, R_c = 10\Omega$ , source resistances,

$\mu_0 = 4\pi.10^{-7} Hm^{-1}$ , permeability of the free space.

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### Bibliography

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